An Equilibrium Search Model with Co-worker Discrimination

by

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Abstract: This paper analyzes the effect of co-worker discrimination on wage and unemployment differentials between males and females using a search model. In the presence of asymmetric co-worker discrimination, no female-dominated firm emerges in the labor market. An increase in female participation drives up the wage offer to female workers and raises female employment Moreover, an increase in the degree of discrimination by males results in gains to them in terms of higher wages and lower unemployment but results in losses to females in terms of lower wages and higher unemployment. The benefit to males provides an explanation for the persistence of discrimination**.**

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1. Introduction

Discrimination still appears to persist in the labor market and to give rise to wage and unemployment differentials between minority and non-minority workers[1] [2]. This paper analyzes the impacts of co-worker discrimination on these wage and unemployment differentials using a search theoretical approach.

Wage and unemployment differentials between non-minority and minority workers have been attributed to co-worker discrimination as well as to consumer and employer discrimination. Starting with the seminal work of Becker (1972), it has been argued that non-minority workers receive a premium to compensate them for working with minority workers in integrated firms even though a compensating wage premium need not be paid to non-minority workers in segregated firms. There is some empirical support for this view. Using data from the National Longitudinal Survey, Ragan and Tremblay (1988) found that whites were compensated for working alongside blacks in racially mixed firms. Similarly, using data from the CPS Displayed Workers Survey, Nord and Ting (1994) found that the average duration of an unemployment spell was 4.58 weeks longer for blacks than for whites. Their empirical results suggest that 3.81 of these 4.58 weeks are attributable to discrimination, and that 3.11 of the 3.81 weeks result from co-worker discrimination. Whites are

[1]For example, using data from the 1984 Survey of Income and Program Participation, Baldwin and Johnson (1996) found that 62 percent of the offer wage differential and 67 percent of the observed wage differential were not attributed to the difference in productivity characteristics, and, in addition, that the effect on these wage differentials reduced the relative employment rate of blacks to whites from 89 to 82 percent.

[2]Using March 1990 Current Population Survey Data, Stratton (1993) argued that only 20 to 40 percent of the difference in unemployment rates between white and black males was attributable to differences in population characteristics.

encouraged to accept wage offers more quickly by a compensating wage premium, which shortens their unemployment spells.

There are two other papers that analyze labor market discrimination using equilibrium search models. Borjas and Bronars (1989) examined consumer discrimination, and Black (1995) examined employer discrimination. In the Borjas and Bronars model, consumers search for a low price but dislike buying from minority sellers. Consumer discrimination cuts the return to self-employment for high productivity minority sellers and, in addition, leads to a lower variance of income for minority sellers than for non-minority sellers.

In Black's model of employer discrimination, there is a pool of potential employers, who are either prejudiced or unprejudiced. Prejudiced firms hire both types of worker, but pay lower wages to minority workers. In equilibrium, the proportions of prejudiced and unprejudiced firms that are active in the market are determined endogenously. He assumes that potential employers are heterogeneous with respect to productivity (a fixed cost). He shows that an increase in the measure of minority workers leads to an increase in the fraction of unprejudiced firms. More minority participation in the labor market raises the probability of matching with a minority applicant. This enables inactive unprejudiced firms to enter the market whereas the marginal prejudiced firms are forced to exit. Black also shows that unprejudiced firms earn higher profits than their counterpart prejudiced firms at the same level of productivity. That is, the prejudiced firm has to sacrifice productivity rents to carry out discrimination.

This paper examines asymmetric co-worker discrimination in an equilibrium search model. We assume that male workers, who sequentially search for a job, dislike working with females; hence, they are less likely to accept an offer from a firm that also hires females[3]. Although there can in principle be three types of firm, male-dominated, female-dominated, and mixed, the assumption of asymmetric co-worker discrimination, in the sense that males discriminate against females but not vice versa, eliminates female-dominated firms in equilibrium. The reason is that the expected value of having a female worker is, no matter what wages are offered, lower for a female-dominated firm than for a mixed firm.

One of the main findings is that increased female participation in the labor market results in an increase in the proportion of mixed firms. The reason is that an increase in the likelihood of meeting female applicants encourages firms to hire more women. This result distinguishes the search model from the neoclassical approach in which more female labor participation shifts their labor supply rightward and causes their equilibrium wage to decline.

An additional feature of this model is that more severe discrimination by males leads to an increase in the proportion of male-dominated firms, higher wage offers to males, and lower wage offers to females. As male workers' distaste for working with female workers increases, men gain rents, but women must give up rents. The group that is discriminated against incurs a utility loss.

This model is most directly comparable to Black's. The same comparative statics effects on the minority value of unemployment are obtained in both models: increased minority participation raises the minority value of unemployment, and more severe discrimination by males (in Black's model, an increase in the proportion of potential prejudiced firms) reduces the minority value of

[3]Co-worker discrimination as well as employer discrimination is more prevalent in the Japanese labor market than those of other industrialized countries. In this paper, minority is referred to as female, and non-minority is referred to as male to capture the essence of discrimination in the Japanese case, particularly in mind.

unemployment. On the other hand, the two models give different results regarding the effects on the non-minority value of unemployment. In Black's model, the value of unemployment for non-minority workers remains unchanged even if more minority workers participate in the labor market and even if the proportion of potential prejudiced firms rises, whereas in this model, increased female participation reduces the male value of unemployment, and more severe discrimination by males raises the male value of unemployment.

The results presented in this paper are consistent with the long-run persistence of discrimination, that is, the discriminators benefit from discrimination. Specifically, non-minority workers gain with high wage and lower unemployment while minority workers lose with low wages and high unemployment. This is consistent with some earlier theoretical models, e.g., Arrow (1972) and Goldberg (1988) and with recent empirical results. Cotton (1988), Neumark (1988) and Oaxaca and Ransom (1994) found that not only did minority workers receive lower wages than they would in the absence of discrimination, but also non-minority workers received higher wages than they would in the absence of discrimination.

In contrast, in Black's model, a prejudiced firm has to sacrifice rents to discriminate. If firms are profit maximizers, employer discrimination should be eliminated in the long run, and wage and unemployment differentials would also be eliminated between two groups. That is, Black's model has no explanation for the long run persistence of discrimination. As noted above, in our model, if males discriminate against females, not only must female workers incur a utility loss, but also male workers gain utility. Male workers are then discouraged from supporting anti-discrimination policies. The long run persistence of discrimination seems more consistent with our model of co-worker discrimination than with Black's model of employer discrimination.

This paper is organized as follows. In the next section, worker decision rules are developed using a search model for each type of worker that is similar to that of Albrecht and Vroman (1992). Firms' decision rules are derived in Section 3. The profit maximization problem of each type of firm is solved, and the value of a vacancy for each firm type is constructed using zero profit conditions. In Section 4, we show that there can be no female-dominated firms in equilibrium. Comparative statics analysis is then carried out to examine the effects of increases in female labor participation and in the degree of discrimination by males on the overall equilibrium of the system. Finally, the main findings of the model are given in the last section.

2. Workers

The decision rule of a worker is developed using a simple discrete-time search model in which the worker lives forever, and the discount factor is δ. Then the probabilities that applicants of each type will accept firms' wage offers are derived, and the unemployment rates and the proportion of female workers in a representative mixed firm are computed using steady state conditions.

2.1 Basic Search Model

Workers are of two types in this model, male and female, with measures fixed to N_m and N_f, respectively. Both types of worker are identical with respect to productivity. In any period, an individual worker is employed or unemployed (searching).

The market consists of at most three types of firm, male-dominated, female-dominated, and mixed. A male-dominated firm hires only male workers, a female-dominated only female workers, but a mixed firm accepts both types of workers. The decision problem of an individual worker is as follows. A worker who is unemployed, whether male or female, randomly draws a wage offer along with a match-specific, non-pecuniary benefit from one of the three types of firm. The unemployed worker knows the proportions of each type of firm, which is common knowledge, but not the type of firm that will be drawn. An individual who has a job offer has two options: to accept this offer and work until exogenously separated from the job, or to reject the offer and search for a job, staying unemployed.

We assume that male workers prefer working with other males to females , but female workers do not care about the gender of their co-workers. We also assume that males are homogeneous in the degree of co-worker discrimination. This implies that the model does not allow for adverse selection and assures that the equilibrium wage offer distribution will be degenerate in each sector.[4] There exists no co-worker discrimination in male or female dominated firms. Male-dominated firms are assumed to offer a wage of zero to a female applicant, hence she always rejects offers from them. Likewise, unemployed males are assumed to reject offers from female-dominated firms since these would be unattractive.

A worker's optimal behavior is derived by examining the following value functions: (1) the value to each type of worker of working in the corresponding dominated firm, (2) the value to each type of worker of working in a mixed firm, (3) the value to each type of being unemployed.

An individual's instantaneous utility is the sum of the wage, a non-pecuniary benefit, and for males in mixed firms, an exogenous disutility incurred by working with female workers. Let w_m,

[4]Albrecht and Vroman (1992 b) proved that there does not exist a single wage equilibrium in a search model with adverse selection. Had male workers been assumed to be heterogeneous in their degree of discrimination, then the present model would have to be far more complicated.

w_{mx}, w_f and w_{fx} denote wage offers to males from male-dominated and mixed firms, to females from female-dominated and mixed firms, respectively. The non-pecuniary benefit, ξ is idiosyncratic, hence it is considered a match-specific random variable. It is drawn from a continuously differentiable distribution function of $G(\xi)$, whose inverse hazard function, $[1-G(\xi)]/g(\xi)$, is strictly decreasing in ξ.[5] Finally , the instantaneous disutility which an individual male suffers in a mixed firm is α, an exogenous value. Regardless of the number of female workers in a mixed firm, a male worker incurs a constant value of disutility. The instantaneous utility of a male worker is thus $w_m + \xi$ if employed in a male-dominated firm, $w_{mx} + \xi - \alpha$ in a mixed firm, and 0 if unemployed[6]. Likewise, the instantaneous utility of a female worker is: $w_f + \xi$, $w_{fx} + \xi$ and 0 if employed in a female-dominated, a mixed firm and unemployed, respectively.

Suppose that all workers are separated from their present jobs with the identical exogenous rate of μ. Let γ_m, γ_f, and $(1- \gamma_m - \gamma_f)$ denote the proportions of male-dominated, female-dominated, and mixed firms, respectively. Let V_m, V_{mx}, and U_m denote the value functions for a male of working in a male-dominated and in a mixed firm, and of unemployment, respectively. By standard dynamic programming, these are,

$$V_m(w_m, \xi) = \frac{w_m + \xi}{1 - \delta(1-\mu)} + \frac{\delta\mu}{1-\delta(1-\mu)} U_m \tag{1}$$

$$V_{mx}(w_{mx}, \xi) = \frac{w_{mx} + \xi - \alpha}{1 - \delta(1-\mu)} + \frac{\delta\mu}{1-\delta(1-\mu)} U_m \tag{2}$$

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[5]The upper bound of the distribution of the non-pecuniary benefit must be finite. Otherwise, an applicant may accept a job offer from a firm dominated by the opposite gender despite a zero wage offer when s/he receives the extremely high value of a non-pecuniary benefit.

[6]Unemployment insurance is not considered in this model for simplicity.

$$U_m = \delta[\gamma_m Emax(V_m, U_m) + (1-\gamma_m-\gamma_f)Emax(V_{mx}, U_m) + \gamma_f U_m] \qquad (3)$$

Correspondingly, the value functions for a female worker of working in a female-dominated (V_f) and in a mixed firm (V_{fx}), and of unemployment (U_f), respectively are computed,

$$V_f(w_f, \xi) = \frac{w_f + \xi}{1-\delta(1-\mu)} + \frac{\delta\mu}{1-\delta(1-\mu)}U_f \qquad (4)$$

$$V_{fx}(w_{fx}, \xi) = \frac{w_{fx} + \xi}{1-\delta(1-\mu)} + \frac{\delta\mu}{1-\delta(1-\mu)}U_f \qquad (5)$$

$$U_f = \delta[\gamma_f Emax(V_f, U_F) + (1-\gamma_m-\gamma_f)Emax(V_{fx}, U_f) + \gamma_m U_f] \qquad (6)$$

The expectations in the value functions, U_m and U_f ((3) and (6)) are taken with respect to the distribution $G(\xi)$ of the non-pecuniary benefit across all matches.

There are two critical values of ξ for each type of worker. These critical values play a key role in analyzing an applicant's decision to accept a job offer or not. The critical value of $\xi_m^*(w_m)$ at which a male worker is indifferent between working in a male-dominated firm and staying unemployed is given by equating $V_m(w_m, \xi_m^*(w_m))$ and U_m:[7]

[7]Note that the critical value ξ_m^* depends indirectly on w_{mx} through the male value of unemployment U_m. Since the male value of unemployment is taken as given by an individual firm, this critical value ξ_m^* is expressed as a function of only w_m.

$$\xi_m^*(w_m) = -w_m + (1-\delta)U_m$$

A random job offer is accepted by a male applicant if $\xi \geq \xi_m{}^*(w_m)$ and rejected if $\xi < \xi_m{}^*(w_m)$.

Equating the values for a male worker of employment in a mixed firm versus of unemployment gives the critical value:[8]

$$\xi_{mx}^*(w_{mx}) = -(w_{mx} - \alpha) + (1-\delta)U_m$$

An individual male applicant accepts an offer from a mixed firm if $\xi \geq \xi_{mx}{}^*(w_{mx})$ and rejects if $\xi < \xi_{mx}{}^*(w_{mx})$.

With the same procedure, two critical values for female workers are developed. The critical values $\xi_f{}^*(w_f)$ for indifference between employment in a female-dominated firm versus unemployment, and $\xi_{fx}{}^*(w_{fx})$ for indifference between employment in a mixed firm versus unemployment are:[9]

$$\xi_f^*(w_f) = -w_f + (1-\delta)U_f$$

$$\xi_{fx}^*(w_{fx}) = -w_{fx} + (1-\delta)U_f$$

A job offer from a female-dominated firm is accepted by a female applicant if $\xi \geq \xi_f{}^*(w_f)$, and one

[8]As in Footnote 7, the critical value $\xi_{mx}{}^*$ depends indirectly on w_m through the value of unemployment.

[9]Similar to above two footnotes, the critical values $\xi_f{}^*$ and $\xi_{fx}{}^*$ depend indirectly on w_{fx} and w_f through the female value of unemployment U_f.

from a mixed firm is accepted if $\xi \geq \xi_{fx}*(w_{fx})$, otherwise the unemployed female chooses to stay unemployed in the present period.

2.2 Acceptance Probabilities

In this section, we derive the probabilities that each type of applicant will accept a job offer from the different firm types, using the critical values given above.[10]

Let $q_m(w_m)$ denote the probability that an unemployed male accepts a job offer from a male-dominated firm. This probability is expressed as one minus the distribution function of ξ, evaluated at $\xi_m*(w_m)$, that is,

$$q_m(w_m) = 1 - G[\xi_m^*(w_m)] \tag{7}$$

Let $q_{mx}(w_{mx})$ be the probability that an unemployed male accepts a wage offer w_{mx} from a mixed firm. It is given by the same expression, evaluated at $\xi_{mx}*(w_{mx})$,

$$q_{mx}(w_{mx}) = 1 - G[\xi_{mx}^*(w_{mx})] \tag{8}$$

Let $q_f(w_f)$ and $q_{fx}(w_{fx})$ denote the probabilities that a female applicant accepts the offers w_f from a female-dominated firm and w_{fx} from a mixed firm, respectively. These are derived using the same procedure, that is,

$$q_f(w_f) = 1 - G[\xi_f^*(w_f)] \tag{9}$$

[10] Since the random variable ξ is independently and identically distributed across all matches, a firm cannot control the value of a non-pecuniary benefit received by an applicant. The realization of ξ is a worker's private information.

$$q_{fx}(w_{fx}) = 1 - G[\xi^*_{fx}(w_{fx})] \tag{10}$$

2.3 Steady State Conditions

In this section, We consider steady state conditions to compute the unemployment rates of each type of worker and the proportion of female workers in a representative mixed firm. Steady state conditions require that (i) the flow of each type of worker into unemployment from the correspondingly dominated firm must be equal the reverse flow, and that (ii) the flow of each type of worker from the mixed firm must be equal the reverse flow.

Let a_i denote the probability that a type i worker (i = m (male), f (female)) is employed in a corresponding dominated firm, let b_i denote the probability that a type i worker is employed in the mixed firm, and let u_i denote the probability that a type i worker is unemployed. Note that u_i is the unemployment rate of type i workers.

The flow out of unemployment consists of new hires by firms. The flow of job offers from male-dominated firms to unemployed males is $\gamma_m(1-a_m-b_m)$. In order to compute the flow rate of newly hired males, this arrival rate of job offers to unemployed males has to be multiplied by the acceptance probability of that applicant type, $q_m(w_m)$. Hence the flow rate of unemployed males into male-dominated firms is given by $\gamma_m(1-a_m-b_m)\, q_m(w_m)$. On the other hand, the flow rate of males working in male-dominated firms into unemployment is simply the exogenous separation rate times the probability that an individual male applicant is employed in a male-dominated firm, μa_m.

Likewise, the flow rate of newly hired males into mixed firms from unemployment is expressed as $(1-\gamma_m-\gamma_f)(1-a_m-b_m)q_{mx}(w_{mx})$, whereas the reverse flow rate of incumbent male workers is μb_m. The steady state conditions in which the flow rates of male workers into and out of unemployment are equated are:

$$\mu a_m = \gamma_m(1-a_m-b_m)q_m(w_m)$$

$$\mu b_m = (1-\gamma_m-\gamma_f)(1-a_m-b_m)q_{mx}(w_{mx})$$

$$a_m + b_m + u_m = 1$$

This steady state system can be solved for a_m, b_m, and u_m. The probabilities that an individual male is employed in the male-dominated firm, in the mixed firm, and is unemployed are derived as follows,

$$a_m = \frac{\gamma_m q_m(w_m)}{\mu + (1-\gamma_m-\gamma_f)q_{mx}(w_{mx}) + \gamma_m q_m(w_m)} \tag{11}$$

$$b_m = \frac{(1-\gamma_m-\gamma_f)q_{mx}(w_{mx})}{\mu + (1-\gamma_m-\gamma_f)q_{mx}(w_{mx}) + \gamma_m q_m(w_m)} \tag{12}$$

Likewise, the corresponding probabilities for females can be derived using the steady state conditions in which the flow rates of females into and out of unemployment are equated:

$$a_f = \frac{\gamma_f q_f(w_f)}{\mu + (1-\gamma_m-\gamma_f) q_{fx}(w_{fx}) + \gamma_f q(w_f)} \tag{14}$$

$$b_f = \frac{(1-\gamma_m-\gamma_f) q_{fx}(w_{fx})}{\mu + (1-\gamma_m-\gamma_f) q_{fx}(w_{fx}) + \gamma_f q_f(w_f)} \tag{15}$$

$$u_m = \frac{\mu}{\mu + (1-\gamma_m-\gamma_f) q_{mx}(w_{mx}) + \gamma_m q_m(w_m)} \tag{13}$$

$$u_f = \frac{\mu}{\mu + (1-\gamma_m-\gamma_f) q_{fx}(w_{fx}) + \gamma_f q_f(w_f)} \tag{16}$$

Using the probabilities that each individual type is employed in a representative mixed firm, b_m and b_f, the proportion of female workers in the mixed firm can be computed. Let R be the proportion of females in the mixed firm. R is invariant over all mixed firms in equilibrium because all jobs among mixed firms are homogenous ex post, and each type of worker is identical with respect to the degree of discrimination (no adverse selection). The measure of type i workers working in all mixed firms is $b_i N_i$ (i = m, f). Then the proportion of female workers in the representative mixed firm is:

$$R = \frac{b_f N_f}{b_m N_m + b_f N_f} \tag{17}$$

The effect of a rise in wage offers of mixed firms on this proportion can be easily obtained. R is decreasing in w_{mx}, but increasing in w_{fx}. The rise in w_{mx} induces more male applicants to accept job offers from mixed firms, and as a result, R decreases. In contrast, more female applicants accept job offers from them when facing a rise in w_{fx}, resulting in a higher R.

3. Firms

The decision problem of each type of firm is considered below, and it is verified that these problems are well defined. We first set up the job profit maximization problem of each type of firm, taking into account the matching technology. Then zero profit conditions are developed using the values of a vacancy of each firm type.

3.1 Profit Maximization Problem

A firm of any type consists of a large number of jobs, that is, a firm is viewed as a collection of individual jobs. Each job is either occupied or vacant and incurs a fixed cost of c regardless of occupancy. For simplicity, labor is the only input. Since workers, either male or female, are identical with respect to productivity, a single job produces a constant output of λ in all types of firm when it is occupied. Because jobs in a firm are assumed to be independent of each other, the aggregate output of a firm is the sum of the outputs produced by the firm's jobs. Hence the profit maximization problem of a firm is carried out job-by-job.

Let v denote the measure of vacancies across all firm types, which is endogenous and positive. In equilibrium, some jobs are not occupied. Let ψ_i be the probability that a vacancy in any type of firm draws an unemployed type i (i =m, f). Since the measure of each type of unemployed is u_iN_i, the probability that a vacancy draws an unemployed type i is the measure of unemployed type i divided by that of vacancies across sectors, $\psi_i = u_i N_i / v$. Note that these probabilities must be common across all types of firm.

In equilibrium, the measure of vacancies must exceed that of overall unemployment because otherwise inactive firms would be encouraged to enter the market. Hence, the equilibrium condition ensures that the probability of drawing an unemployed type i workers lies between zero and one.[11] [12]

Let Π_j be the maximum value of a vacancy in a type j firm (j = m (male-dominated), f (female-dominated), x (mixed)). A job's entry/exit cost is assumed zero for convenience. New jobs are created if the value of a vacancy is positive, while they are eliminated if it is negative.

The decision rule of a firm is as follows. A male-dominated firm draws an unemployed male with probability ψ_m , and offers a wage of w_m. The offer is accepted with probability $q_m(w_m)$. If he accepts this offer, the value to the male-dominated firm of having this worker in a job at wage w_m, $S_m(w_m)$ is expressed as:

$$S_m(w_m) = \frac{\lambda - w_m - c}{1 - \delta(1 - \mu)} + \frac{\delta \mu}{1 - \delta(1 - \mu)} \Pi_m$$

This male applicant rejects the offer with probability $(1-q_m(w_m))$. If this occurs the firm keeps the value of a vacancy, Π_m. Moreover, the firm fails to draw a male applicant with probability $(1-\psi_m)$, and it then retains the value of a vacancy, Π_m. Thus the expected profit per job in a representative

[11]The equilibrium condition $u_m N_m + u_f N_f \leq v$ implies $(u_m N_m / v) + (u_f N_f / v) \leq 1$, or $\psi_m + \psi_f \leq 1$. This is a sufficient condition for $\psi_i \in [0, 1]$. i = m, f.

[12]A vacancy in any type does not necessary draw an unemployed in one period. In contrast, it implies that an applicant, whether male or female, always draws a job offer in one period.

male-dominated firm at wage w_m, $\pi_m(w_m)$ is,

$$\pi_m(w_m) = \psi_m\{q_m(w_m)S_m(w_m) + [1-q_m(w_m)]\Pi_m\} + (1-\psi_m)\Pi_m$$

A positive value of Π_m induces inactive, and incumbent female-dominated and mixed firms to become male-dominated. It leads to job creation in male-dominated firms. On the other hand, a negative value of Π_m forces incumbent male-dominated firms to exit the market or to become other firm types. Hence the free entry/exit condition assures that the value of a vacancy Π_m is zero in equilibrium. Then, in equilibrium, $\pi_m(w_m)$ becomes,

$$\pi^*_m(w_m) = \psi_m q_m(w_m)\left[\frac{\lambda - w_m - c}{1-\delta(1-\mu)}\right] \tag{18}$$

The firm maximizes $\pi_m^*(w_m)$ with respect to w_m.[13] The first-order condition is,

$$q_m{'}(w^*_m)[\lambda - w^*_m - c] = q_m(w^*_m) \tag{19}$$

where w_m^* is the equilibrium wage offered to males by the male-dominated firm. Since the

[13]The first-order condition is derived with respect to the wage offer on an individual job, and not by the common wage. All jobs play the normal one-shot game within each firm type. The

$$q_m{'}(w^i_m; w^{-i*}_m)[\lambda - w^i_m - c] = q_m(w^i_m; w^{-i*}_m)$$

first-order condition of the i th job given the optimal strategy of other jobs is, where w_m^i is the wage offered by i th job, and w_m^{-i*} is the optimal strategies by all other jobs in the male-dominated firms. Since all jobs are identical within each firm type, they offer the common wage , w_m^*, which is the Nash equilibrium.

probability that a male-dominated firm draws an unemployed male, ψ_m depends on the common wage offered by all male-dominated firms, and not on the wage that an individual firm offers, each firm does not take into account its effect on ψ_m in solving the profit maximization problem.

Similarly, a female-dominated firm draws an unemployed female with probability ψ_f, and offers a wage of w_f. It is accepted by a female applicant with probability $q_f(w_f)$. The value to the female-dominated firm of having a female worker, $S_f(w_f)$ is,

$$S_f(w_f) = \frac{\lambda - w_f - c}{1 - \delta(1-\mu)} + \frac{\delta\mu}{1-\delta(1-\mu)}\Pi_f$$

This offer is rejected by a female applicant with probability $(1-q_f(w_f))$, and the firm fails to draw a female applicant with probability $(1-\psi_f)$. If the job remains vacant, the female-dominated firm retains the value of vacancy Π_f. The expected unit profit function of a representative female-dominated firm offering wage w_f, $\pi_f(w_f)$ is,

$$\pi_f(w_f) = \psi_f\{q_f(w_f)S_f(w_f) + [1-q_f(w_f)]\Pi_f\} + (1-\psi_f)\Pi_f$$

Since the value of a vacancy, Π_f is zero in equilibrium due to the free entry/exit condition, the expected profit per job in equilibrium, π_f^* is,

$$\pi_f^*(w_f) = \psi_f q_f(w_f)\left[\frac{\lambda - w_f - c}{1-\delta(1-\mu)}\right] \qquad (20)$$

The profit maximization problem per job in a representative female-dominated firm is solved with respect to w_f:[14]

$$q_f'(w_f^*)[\lambda - w_f^* - c] = q_f(w_f^*) \tag{21}$$

where w_f is the equilibrium wage offered to females by the female-dominated firm. Note that since ψ_f is taken as given by an individual firm, this probability does not affect its decision rule.

A mixed firm draws an unemployed individual with probability $(\psi_m+\psi_f)$. It offers w_{mx} to a male and w_{fx} to a female, and the offer is accepted by the corresponding applicant with probability q_{mx} or q_{fx}, respectively. The values to the mixed firm of having each type of worker at wages w_{mx} and w_{fx}, respectively are,

$$S_{mx}(w_{mx}) = \frac{\lambda - w_{mx} - c}{1-\delta(1-\mu)} + \frac{\delta\mu}{1-\delta(1-\mu)}\Pi_x$$

$$S_{fx}(w_{fx}) = \frac{\lambda - w_{fx} - c}{1-\delta(1-\mu)} + \frac{\delta\mu}{1-\delta(1-\mu)}\Pi_x$$

Each type of unemployed rejects this offer with probability $(1-q_{mx}(w_{mx}))$ or $(1-q_{fx}(w_{fx}))$. Moreover, this job draws no applicant with probability $(1-\psi_m- \psi_f)$. If the job is unfilled, the value of a vacancy, Π_x is incurred by the mixed firm. Therefore, the expected profit per job of a representative mixed firm at wages w_{mx} and w_{fx} , $\pi(w_{mx}, w_f)$ is expressed as,

[14]See Footnote 13.

$$\pi_x(w_{mx},w_{fx}) = \psi_m\{q_{mx}(w_{mx})S_{mx}(w_{mx}) + [1-q_{mx}(w_{mx})]\Pi_x\}$$
$$+ \psi_f\{q_{fx}(w_{fx})S_{fx}(w_{fx}) + [1-q_{fx}(w_{fx})]\Pi_x\} + (1-\psi_m-\psi_f)\Pi_x$$

The free entry/exit condition implies $\Pi_x = 0$. Maximization is done with respect the wages offered to both types of applicants:[15]

$$\pi_x^*(w_{mx},w_{fx}) = \psi_m q_{mx}(w_{mx})\left[\frac{\lambda-w_{mx}-c}{1-\delta(1-\mu)}\right] + \psi_f q_{fx}(w_{fx})\left[\frac{\lambda-w_{fx}-c}{1-\delta(1-\mu)}\right] \tag{22}$$

$$q_{mx}'(w_{mx}^*)[\lambda-w_{mx}^*-c] = q_{mx}(w_{mx}^*) \tag{23}$$

$$q_{fx}'(w_{fx}^*)[\lambda-w_{fx}^*-c] = q_{fx}(w_{fx}^*) \tag{24}$$

where w_{mx}* and w_{fx}* are the equilibrium wages offered to males and females by the mixed firm, respectively. The probabilities of drawing an unemployed, ψ_m and ψ_f, are considered constant by an individual firm, so they are not involved in deriving the first-order conditions.

The second-order conditions for maximization in each firm type are satisfied by the assumption that the inverse hazard function of the distribution function of the non-pecuniary benefit is strictly decreasing in ξ.

3.2 Value of a Vacancy

[15]See Footnote 13.

In this section, we obtain the zero profit conditions using the values of a vacancy. These conditions assure that there is no incentive for inactive or incumbent firms to change their decisions about whether or not to enter the market and about which sector to belong to.

A vacant job in a male-dominated firm is considered first. When a job offer is rejected by a male applicant, this vacant job incurs a fixed cost of c. In the next period, it yields the discounted value of the expected unit profit. Then the value of a vacancy is $\Pi_m = -c+\delta\pi_m(w_m^*)$. The assumption of no entry/exit cost ensures that the value of a vacancy is zero in equilibrium. In male-dominated firms, this implies,

$$c = \delta\pi_m^*(w_m^*) = \delta\psi_m q_m(w_m^*)\left[\frac{\lambda - w_m^* - c}{1-\delta(1-\mu)}\right] \tag{25}$$

Similarly, the values of a vacancy in female-dominated and in mixed firms consist of a fixed cost of c plus the discounted value of the expected unit profit for each firm type, that is, $\Pi_f = -c+\delta\pi_f(w_f^*)$ and $\Pi_x = -c+\delta\pi_x(w_{mx}^*, w_{fx}^*)$, respectively. By the free entry/exit condition, the zero profit conditions of these firm types are,

$$c = \delta\pi_f^*(w_f^*) = \delta\psi_f q_f(w_f^*)\left[\frac{\lambda - w_f^* - c}{1-\delta(1-\mu)}\right] \tag{26}$$

$$c = \delta\pi_x^*(w_{mx}^*, w_{fx}^*) = \delta\left\{\psi_m q_{mx}(w_{mx}^*)\left[\frac{\lambda - w_{mx}^* - c}{1-\delta(1-\mu)}\right] + \psi_f q_{fx}(w_{fx}^*)\left[\frac{\lambda - w_{fx}^* - c}{1-\delta(1-\mu)}\right]\right\} \tag{27}$$

.

4. Characterization of Equilibrium

The most interesting features of equilibrium in this model are the non-existence of female-dominated firms in the market, and the effects of changes in female participation and in the degree of discrimination on wages, unemployment rates, the proportions of firm types, and the proportion of females in mixed firms. First, we show that a female-dominated firm can not exist in the market with asymmetric co-worker discrimination. Then comparative statics analysis is performed in order to observe the change in equilibrium caused by incremental changes in two of the exogenous variables. These comparative statics results highlight the difference between this equilibrium search model and the neoclassical discrimination models and Black's model.

Below, for convenience, ξ is assumed to be distributed as a uniform function over [0, 1]. [16] This satisfies the condition that the inverse hazard function is strictly decreasing in ξ.

4.1 Non-Existence of Female-Dominated Firms

With asymmetric co-worker discrimination between male and female workers, the labor market consists of male-dominated and mixed firms: no female-dominated firms are established.

Proposition 1: *Suppose that discrimination is asymmetric between gender groups, that is, male workers discriminate against females, but not vice versa. There will not exist any female-dominated firms in the market.*

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[16] Since the upper bound of the uniform distribution is finite, the assumption that an applicant does not accept an offer from the opposite sex dominated firm holds.

Proof: The first-order conditions for a female-dominated firm, (21), and for the wage offered to females by a mixed firm, (24), are rewritten as follows.

$$g[-w_f^*+(1-\delta)U_f](\lambda-w_f^*-c) = 1-G[-w_f^*+(1-\delta)U_f]$$

$$g[-w_{fx}^*+(1-\delta)U_f](\lambda-w_{fx}^*-c) = 1-G[-w_{fx}^*+(1-\delta)U_f]$$

Because each of these equations is solved by a unique wage, w_f^* and w_{fx}^* are equal. This means that the acceptance probabilities $q_f(w_f^*)$ and $q_{fx}(w_{fx}^*)$ are also equal. Let π_{mx}^* and π_{fx}^*denote the expected values of having each type of worker in a representative mixed firm. Let $\pi_x^*=\pi_{mx}^*+\pi_{fx}^*$. With $w_f^*=w_{fx}^*$, $q_f(w_f^*)=q_{fx}(w_{fx}^*)$, from equations (20) and (22), $\pi_f^*=\pi_{fx}^*$. Then $\pi_x^*= \pi_{mx}^*+\pi_f^*$,i.e, $\pi_x^*-\pi_f^*=\pi_{mx}^*>0$. Hence $\pi_x^*>\pi_f^*$, and no firm chooses to be female-dominated.

Q.E.D.

Intuitively, since the expected values of having a female worker in a female-dominated and in a mixed firm are equal, and mixed firms also have male workers, the expected profit per job is higher in a mixed firm than in a female-dominated firm in equilibrium. Thus, no firm has an incentive to be female-dominated, which makes $\gamma_f=0$, that is, females can only work in mixed firms. From here on, we redefine $\gamma_m=\gamma$ (The proportion of mixed firms is then $1-\gamma$).

4.2 Comparative Statics

4.2.1 The Impact of an Increase in Female Participation

In many countries, and particularly in industrialized countries, an increasing number of females has participated in the labor market. It is thus interesting to observe how an increase in female participation affects the composition of firms and the values of unemployment for both type of workers.

As a result of an increase in the number of female participants N_f, the proportion of male-dominated firms (γ) and the male value of unemployment decrease while the female value of unemployment increases.

Proposition. 2: *As more female applicants enter the labor market,*

(a) the proportion of male-dominated firms decreases,

(b) the male value of unemployment decreases, and as a result, the acceptance probabilities for males $q_m(w_m*)$ and $q_{mx}(w_{mx}*)$ increase, and the offered wages w_m* and $w_{mx}*$ fall,

(c) the female value of unemployment increases, the acceptance probability for females $q_{fx}(w_{fx}*)$ decreases, and the offered wage $w_{fx}*$ rises.

Proof: see Appendix.

An increased number of female applicants in the market enables firms to draw a female applicant more frequently. The probability of drawing a male applicant is lower. This gives firms an incentive to be mixed in order to hire more females. Therefore, the proportion of male-dominated firms decreases as a result of an increased female participants.

The increased difficulty for firms of drawing male applicants lowers the expected value of having a male worker in both types of firm, and thus the firms offer lower wages to male workers. From the point of view of unemployed males, less frequent access to male-dominated firms lowers their expected value of search for males and their value of unemployment. This encourages male applicants to accept job offers more readily. Therefore, the acceptance probabilities $q_m(w_m*)$ and $q_{mx}(w_{mx}*)$ rise.

On the other hand, an increased number of female applicants in the market raises the expected value of having a female worker in a job, and a higher wage $w_{fx}*$ is offered to them. This drives up the expected value of search for females and their value of unemployment. As she has more opportunities to meet firms, an unemployed female is more selective in her acceptance decision, that is, the acceptance probability $q_{fx}(w_{fx}*)$ decreases.

The essential result in this search model is that the wage offered to females w_{fx} rises while the wages offered to males w_m* and $w_{mx}*$ fall as a result of an increase in female participants of N_f; this is the opposite of what a simple Walrasian equilibrium model with labor demand and supply would predict in the female labor market, but the same in the male labor market.

As a result of the increase in female participation, the female unemployment rate decreases, but the effect on the male unemployment rate is ambiguous. The sign of the change depends on the initial equilibrium.

Proposition. 3: *As more female applicants enter the market,*

(a) the effect on the male unemployment rate is ambiguous, but if the proportion of male-dominated firms is originally high in equilibrium, then the male unemployment rate rises.

(b) the female unemployment rate falls.

Proof: see Appendix

The change in the male unemployment rate depends on two opposite effects: the negative effect through a decrease in the male value of unemployment and the positive effect through a decrease in the proportion of male-dominated firms. The decreased male value of unemployment leads to higher acceptance probabilities from Proposition 2. Unemployed males are more likely to accept offers from both types of firms, which contributes to a decrease in male unemployment. On the other hand, the lower proportion of male-dominated firms, holding the acceptance probabilities $q_m(w_m^*)$ and $q_{mx}(w_{mx}^*)$ constant, results in a decrease in the *expected* acceptance probability for unemployed males, $(1-\gamma)q_{mx}(w_{mx}^*)+\gamma q_m(w_m^*)$, which means that male applicants are less likely to accept offers from firms on average. Hence more males are unemployed. Which effect is dominant is uncertain. If the proportion of male-dominated firms is high in the original equilibrium, the effect of the value of unemployment is dominated by that of the proportion of male-dominated firms. Overall, the male unemployment rate rises due to the increase in N_f.

Likewise, there are two opposite effects on the female unemployment rate. An increase in the female value of unemployment lowers the acceptance probability for females. They are less likely to accept offers from mixed firms, which raises the female unemployment rate. On the other hand, female unemployment decreases because of the decrease in the proportion of male-dominated firms. The increased fraction of mixed firms raises the *expected* acceptance probability for females, $(1-\gamma)q_{fx}(w_{fx}^*)$, holding the acceptance probability $q_{fx}(w_{fx}^*)$ constant, which means that female applicants are more readily induced to accept offers from mixed firms on average. Thus female unemployment is reduced. Using the uniform distribution for the non-pecuniary benefit, the effect of the proportion of male-dominated firms dominates the one of the female value of unemployment. The overall effect of the increase in female participation on female unemployment is negative, that is, female unemployment is lower.[17]

The interesting feature of this model is not only that there is unemployment in equilibrium (there is no unemployment in the neoclassical model), but that female unemployment is lower while male unemployment may be higher when more females participate in the labor market.

These results from Proposition 2 and 3 contrast with those from Black's model of employer discrimination regarding the value of unemployment for males (non-minority workers). In Black's model, since prejudiced and unprejudiced firms offer the same wages to non-minority workers, the increased proportion of unprejudiced firms due to an increase in minority labor participation does not affect the non-minority value of unemployment. On the other hand, in this model of co-worker discrimination, since male-dominated and mixed firms offer different wages to males in order to compensate men working with females, the increased proportion of mixed firms that results from increasing female labor participation reduces the male value of unemployment.

Finally, the effect of an increase in N_f on the proportion of female workers in a representative mixed firm, R, is examined. Comparative statics analysis indicates that the result is ambiguous. However, it is still interesting to observe the indirect effects of the proportion of male-dominated firms and the values of unemployment and the direct effect of increased female participation on the change in the proportion of female workers in a mixed firm.

Proposition. 4: *When more females participate in the market, if* $\partial u_m / \partial N_f \geq 0$*, the indirect effects through the proportion of male-dominated firms and the values of unemployment for both types of*

[17] The change of female unemployment may be ambiguous, if other distributions of the non-pecuniary benefit are adopted.

workers are negative, but the direct effect through the increase in female participation N_f is positive on the proportion of female workers in a representative mixed firm. The overall effect is ambiguous.

Proof: see Appendix.

As the proportion of mixed firms rises, it is relatively difficult for mixed firms to draw female applicants unless there is a large female participation rate. Moreover, the increased female value of unemployment makes females more selective in their acceptance decisions, and on the other hand, the lower male value of unemployment encourages males to accept offers from mixed firms more frequently. As a result, the proportion of female workers in a mixed firm decreases indirectly. If enough female applicants enter the market to compensate for the indirect negative effects, the overall effect is positive and the proportion of female workers in a mixed firm increases.

4.2.2 Impact of an Increase in the Degree of Discrimination

In this sub-section, comparative statics analysis demonstrates the effect of an increase in the degree of discrimination by males (α) on equilibrium values.

Proposition. 5: *When the degree of discrimination by male workers increases,*

(a) the proportion of male-dominated firms increases,

(b) the male value of unemployment increases, and as a result, the acceptance probabilities of $q_m(w_m^*)$ and $q_{mx}(w_{mx}^*)$ decrease, and the offered wages of w_m^* and w_{mx}^* rise.

(c) the female value of unemployment decreases, so the acceptance probability of $q_{fx}(w_{fx}^*)$ increases

and the offered wage of w_{fx} falls.*

Proof: see Appendix.

The increase in the degree of discrimination by males widens the difference between the acceptance probabilities for males being offered from male-dominated and mixed firms. This indicates that as the degree of discrimination increases, unemployed males are less likely to accept offers from mixed firms. They are more likely to stay unemployed and to wait for offers from male-dominated firms than to accept from mixed firms. Additional distaste among males for mixed firms induces more firms to become male-dominant and thus causes an increase in the proportion of male-dominated firms.

As a result of the increase in the proportion of male-dominated firms, unemployed males have more opportunities to match with those firms. This implies that the male value of unemployment increases. On the other hand, the increased degree of discrimination by males reduces their acceptance probability of job offers from mixed firms, holding the proportion of male-dominated firms fixed, which means that they are more likely to be unemployed. This implies that the male value of unemployment decreases. Using the uniform distribution for the non-pecuniary benefit, the former effect dominates the latter one. That is, the male value of unemployment increases with the degree of discrimination by males. Since an increase in the male value of unemployment makes males more selective in their acceptance decisions, the acceptance probabilities $q_m(w_m^*)$ and $q_{mx}(w_{mx}^*)$ fall. Firms will thus offer higher wages to males to attract them.

As more firms become male-dominant, female applicants become relatively unattractive, which implies that the expected value of having a female worker in a job is lower. Unemployed females have fewer chances to meet mixed firms because of the lower proportion of such firms. As a result, the female value of unemployment decreases, which drives up the acceptance probability for females $q_{fx}(w_{fx}*)$. Mixed firms are induced to cut the wage offer to females.

In the following, the impact on unemployment for each gender type is analyzed.

Proposition. 6: *When the degree of discrimination by male workers increases,*

(a) the change of the male unemployment rate is ambiguous,

(b) the female unemployment rate rises.

Proof: see Appendix.

The effect through the increase in the male value of unemployment raises male unemployment while the one through the increase in the proportion of male-dominated firms lowers it. Since the acceptance probabilities for males are decreasing in the male value of unemployment, they are less likely to accept offers when facing each type of firm. It contributes to higher male unemployment. In contrast, the *expected* acceptance probability is increasing in the proportion of male-dominated firms, holding the acceptance probabilities $q_m(w_m*)$ and $q_{mx}(w_{mx}*)$ constant. This indicates that wage offers are more likely to be accepted by unemployed males on average, which contributes to the decrease in male unemployment.

The change in the female unemployment rate also consists of two opposite effects through the female value of unemployment and the proportion of male-dominated firms. The decrease in the female value of unemployment due to the increase in the degree of discrimination by males drives up the acceptance probability for females. Since females are more likely to accept offers when facing mixed firms, female unemployment is lower. In contrast, the increase in the proportion of male-dominated firms (the decrease in the proportion of mixed firms) lowers the *expected* acceptance probability for females, holding the acceptance probability of $q_{fx}(w_{fx}*)$ fixed, which means that they are less likely to accept offers from firms on average. It contributes to higher female unemployment. Using the uniform distribution for ξ, the effect of the increased proportion of male-dominated firms dominates the one of the decreased female value of unemployment. As a result, the overall effect is positive: the female unemployment rate rises as a result of the increased degree of discrimination by males.

The results from Proposition 5 and 6 also contrast with Black's regarding the value of unemployment for males (non-minority workers). In Black's model, since prejudiced and unprejudiced firms treat males identically, the increased proportion of prejudiced firms does not affect the non-minority value of unemployment. In this model, since male-dominated and mixed firms treat males differently, that is, offer different wages, the increased proportion of male-dominated firms raises the male value of unemployment.

More discrimination by males against females gives males a utility gain (high wage offers and possible low unemployment) and gives females a utility loss (low wage offers and high unemployment). This result is consistent with discrimination as a long term phenomenon. In contrast, Black's model and Beckerian models of employer discrimination imply that employers must give up rents in order to discriminate. In the long run, firms that fails to maximize profit should be eliminated from the market; thus, models of employer discrimination cannot explain the persistence of discrimination.

Finally, we will examine how the proportion of female workers in a mixed firm is affected by the increase in the degree of discrimination by males.

Proposition. 7: *If $\partial u_m/\partial \alpha \leq 0$, that is, if the male unemployment rate depends negatively on the degree of discrimination by males, then the proportion of female workers in a mixed firm is increasing in α.*

Proof: see Appendix.

As males have more disutility from working with females, they are more likely to reject offers from mixed firms. On the other hand, the acceptance probability for females rises due to the lower female value of unemployment. It then becomes difficult for mixed firms to obtain male workers relative to female ones. This increases the proportion of female workers in a mixed firm.

5. Concluding Remarks

This paper examines an equilibrium search model of labor market discrimination in asymmetric co-worker discrimination and investigates the impacts of co-worker discrimination on wage and unemployment differentials between males and females. Because the expected unit job profit of the mixed firm exceeds that of the female-dominated firm over all possible offered wages, it is shown that no firm will be female dominated. Hence in equilibrium, the labor market consists only of male-dominated and mixed firms in equilibrium.

The model presented by this paper shows that as more females enter the labor market, female workers receive higher wages, female unemployment can be lower, and the proportion of mixed firms increases. The increased probability of matching with a female applicant due to the increase in female participation raises the expected unit profit of having a female worker in a job in the mixed firm. Increased female participation induces some male-dominated firms to become mixed ones because the likelihood of drawing a female applicant is raised. An increase in the female value of unemployment raises their reservation wages, which induces mixed firms to raise wage offers to them. Employing the uniform distribution for the non-pecuniary benefit, the effect of the increase in the proportion of mixed firms that causes lower female unemployment outweighs the counterpart effect through the increase in the female value of unemployment that raises female unemployment. Hence, this model predicts lower female unemployment. On the other hand, increased female participation reduces the male value of unemployment. This is in contrast to Black's model of employer discrimination in which the non-minority value of unemployment does not depend on the measure of the minority participation in the labor market.

Another result is that with an increased degree of co-worker discrimination by males not only does the female value of unemployment decrease, but also the male value of unemployment increases at least if we assume a uniform distribution for the non-pecuniary benefit. That is, the discriminators gain collectively from their action. This is again in contrast to Black's result in which the non-minority value of unemployment remains the same regardless of the degree of employer discrimination by prejudiced firms (the proportion of potential prejudiced firms).

Much of literature on discrimination emphasizes employer discrimination and ignores co-worker discrimination. This paper sheds light on the impacts of asymmetric co-worker discrimination on males and females using a search theoretic approach and shows that co-worker discrimination is a more convincing mechanism than employer discrimination for explaining the persistence of discrimination in labor market.

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Appendix

In this appendix, the comparative statics results are given.

Proposition. 2

Since the uniform distribution of $\xi \in [0, 1]$ is used for a non-pecuniary benefit distribution, the wages derived from the first-order conditions, Equations (19), (23) and (24) are given by,

$$w_m^* = \frac{\lambda - c - 1 + (1-\delta)U_m}{2}$$

$$w_{mx}^* = \frac{\lambda - c - 1 + \alpha + (1-\delta)U_m}{2}$$

$$w_{fx}^* = \frac{\lambda - c - 1 + (1-\delta)U_f}{2}$$

Note that $w_m^* < w_{mx}^*$, that is, mixed firms have to offer higher wages to males to compensate for their disutility from working with females.

The acceptance probabilities are as follows,

$$q_m = \frac{\lambda - c + 1 - (1-\delta)U_m}{2}$$

$$q_{mx} = \frac{\lambda - c + 1 - \alpha - (1-\delta)U_m}{2}$$

$$q_{fx} = \frac{\lambda - c + 1 - (1-\delta)U_f}{2}$$

Probability q_i is evaluated at the equilibrium wage w_i* (i = m, mx, fx). The acceptance probabilities for males, q_m and q_{mx} depend on the male value of unemployment while the acceptance probability for females, q_{fx} depends on the female value of unemployment. Note that $q_m > q_{mx}$, that is, unemployed males are more likely to accept offers from male-dominated than from mixed firms because they dislike working with females.

The system of equilibrium equations is obtained from Equations (3), (6), (17), (25), and (27).

$$\frac{u_m N_m q_{mx}}{u_f N_f q_{fx}} = \frac{1-R}{R} \tag{A-1}$$

$$u_m N_m (q_m^2 - q_{mx}^2) = u_f N_f q_{fx}^2 \tag{A-2}$$

$$2[1-\delta(1-\mu)](1-\delta)U_f = \delta(1-\gamma)\, q_{fx}^2 \tag{A-3}$$

$$2[1-\delta(1-\mu)](1-\delta)U_m = \delta\gamma\, q_m^2 + \delta(1-\gamma)\, q_{mx}^2 \tag{A-4}$$

(A-1) is the equilibrium proportion of female workers in a mixed firm from Equation (17); (A-2) comes from the zero profit conditions, eliminating vacancies (v) from Equations (25) and (27); (A-3) and (A-4) are the individual equilibrium search conditions for males and females from Equations (3) and (6), respectively. From Equations (13) and (16), the male unemployment rate u_m depends on the male value of unemployment U_m and the proportion of male-dominated firms γ. On the other hand, the female unemployment rate u_f depends on the female value of unemployment U_f and the proportion of male-dominated firms γ. Thus, this system consists of four equations with four unknown variables U_m, U_f, γ, and R.

Equations (A-2), (A-3), and (A-4) do not depend on the proportion of females in a representative mixed firm, R. Hence the comparative statics analysis demonstrates the effects of increases in female participation and in the degree of co-worker discrimination on the male and female values of unemployment and the proportion of male-dominated firms, U_m, U_f, and γ. Then from Equation (A-1), the effect on the proportion of females in a representative mixed firm R is analyzed using the comparative statics results for U_m, U_f, and γ.

Taking the total derivative of this system that consists of Equation (A-2) - (A-4) with respect to U_m, U_f, γ, and N_f:

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & 0 \\ a_{31} & 0 & a_{33} \end{bmatrix} \begin{bmatrix} \frac{d\gamma}{dN_f} \\ \frac{dU_f}{dN_f} \\ \frac{dU_m}{dN_f} \end{bmatrix} = \begin{bmatrix} b_1 \\ 0 \\ 0 \end{bmatrix} \qquad \text{(A-5)}$$

$$a_{11} = -\frac{N_m (q_m + q_{mx})(q_m - q_{mx})^2}{[\mu + (1-\gamma) q_{mx} + \gamma q_m]^2} - \frac{N_f q_{fx}^3}{[\mu + (1-\gamma) q_{fx}]^2} < 0$$

$$a_{12} = \frac{N_f q_{fx}(1-\delta)\mu + \frac{1}{2} N_f q_{fx}^2 (1-\delta)(1-\gamma)}{[\mu + (1-\gamma) q_{fx}]^2} > 0$$

$$a_{13} = \frac{N_m (1-\delta)(q_{mx} - q_m)[\mu + (\frac{1}{2} - \gamma) q_{mx} + (\gamma - \frac{1}{2}) q_m]}{[\mu + (1-\gamma) q_{mx} + \gamma q_m]^2}$$

$$a_{21} = -\delta q_{fx}^2 < 0$$

$$a_{22} = -\delta (1-\delta)(1-\gamma) q_{fx} - 2[1 - \delta(1-\mu)](1-\delta) < 0$$

$$a_{31} = \delta (q_m - q_{mx})(q_m + q_{mx}) > 0$$

$$a_{33} = -\delta(1-\delta)\gamma q_m - \delta(1-\delta)(1-\gamma) q_{mx} - 2[1-\delta(1-\mu)](1-\delta) < 0$$

$$b_1 = \frac{q_{fx}^2}{\mu + (1-\gamma) q_{fx}} > 0$$

Note that the sign of a_{13} is ambiguous; $a_{13} > 0$ if $\alpha > 2\mu/(1-2\gamma)$ and $a_{13} \leq 0$ if $\alpha \leq 2\mu/(1-2\gamma)$. Hence, the sign of the Jacobian determinant, ∇ is ambiguous as well. We assume that equilibrium derived from Equations (A-2) - (A-4) is locally stable. This assumption ensures that the Jacobian determinant is negative, i.e, $\nabla<0$.

By Cramer's rule,

$$\frac{d\gamma}{dN_f} = \frac{a_{33} b_1 a_{22}}{\nabla} < 0 \qquad \frac{dU_f}{dN_f} = \frac{-b_1 a_{21} a_{33}}{\nabla} > 0 \qquad \frac{dU_m}{dN_f} = \frac{-b_1 a_{22} a_{31}}{\nabla} < 0$$

Proposition 3 (a):

the male unemployment rate is given from Equation (13),

$$u_m = \frac{\mu}{\mu + (1-\gamma) q_{mx} + \gamma q_m}$$

Taking the partial derivative with respect to N_f,

$$\frac{\partial u_m}{\partial N_f} = \frac{\mu}{2\left[\mu + q_{mx} + \frac{1}{2}\gamma\alpha\right]^2} \left[(1-\delta)\frac{dU_m}{dN_f} - \alpha\frac{d\gamma}{dN_f}\right] \tag{A-6}$$

The effect through the proportion of male-dominated firms is positive on the male unemployment rate from the second term in brackets in the numerator, but the effect through the male value of unemployment is negative from the first term in these brackets.

Substituting the comparative statics results from Proposition 2 into Equation (A-6), the brackets in the numerator in Equation (A-6) are rewritten as follows,

$$-\frac{b_1}{\nabla}[(1-\delta)a_{31}+\alpha a_{33}]$$

$$= -\frac{\delta(1-\delta)b_1}{\nabla}\left\{(q_m-q_{mx})(q_m+q_{mx})-\alpha[\gamma q_m+(1-\gamma)q_{mx}]-\frac{2\alpha[1-\delta(1-\mu)]}{\delta}\right\}$$

Since $q_m>q_{mx}$, the higher γ is, the more likely the brackets are to be positive. Therefore, if the equilibrium proportion of male-dominated firms is initially high, Equation (A-6) tends to be positive, that is, an increased female participation results in a rise in the male unemployment rate.

(b) From Equation (16), the female unemployment rate is,

$$\frac{\partial u_f}{\partial N_f} = \frac{\mu}{[\mu+(1-\gamma)q_{fx}]}\left[q_{fx}+\frac{(1-\gamma)(1-\delta)}{2}\frac{dU_f}{d\gamma}\right]\frac{d\gamma}{dN_f} \qquad \text{(A-7)}$$

From Equation (A-3), the derivative of the female value of unemployment with respect to the proportion of male-dominated firms is,

$$\frac{dU_f}{d\gamma} = -\frac{\delta q_{fx}^2}{2[1-\delta(1-\mu)](1-\delta)+\delta(1-\delta)(1-\gamma)q_{fx}} < 0 \qquad \text{(A-8)}$$

Substituting Equation (A-8) into brackets in the numerator of Equation (A-7), it is positive even though $dU_f / d\gamma$ is negative. Hence since $d\gamma/dN_f < 0$ from Proposition 2, $\partial u_f/\partial N_f < 0$, i.e, the female unemployment rate is falling in female participation.

Proposition 4 Suppose h(R) = (1 - R)/R. From Equation (A-8), the derivative of h(R) with respect to female participation is as follows,

$$\frac{\partial h(R)}{\partial N_f} = \frac{1}{u_f N_f q_{fx}} \left[\frac{\partial u_m}{\partial N_f} N_m q_{mx} - \frac{1-\delta}{2} u_m N_m \frac{dU_m}{dN_f} \right]$$

$$- \frac{u_m N_m q_{mx}}{(u_f N_f q_{fx})^2} \left[\frac{\partial u_f}{\partial N_f} N_f q_{fx} - \frac{1-\delta}{2} u_f N_f \frac{dU_f}{dN_f} \right] - \frac{u_m N_m q_{mx}}{u_f N_f q_{fx}^2} \qquad \text{(A-9)}$$

The first four terms represent the indirect effects through the male and female values of unemployment and the proportion of male-dominated firms, U_m, U_f, and γ. If the male unemployment rate depends positively on female participation, i.e., $\partial u_m/\partial N_f > 0$, the function h(R) is increasing in female participation, i.e, the proportion of female workers in a representative mixed firm is decreasing in female participation. The final term represents the direct effect via an increased female participation. The function h(R) is decreasing in female participation, i.e, the proportion of female workers in a representative mixed firm is increasing in female participation.

Proposition 5 As in the proof of Proposition 2, taking the total derivative of the system that consists of Equations (A-1) - (A-4) with respect to the degree of co-worker discrimination α is given

$$
\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & 0 \\ a_{31} & 0 & a_{33} \end{bmatrix}
\begin{bmatrix} \dfrac{d\gamma}{d\alpha} \\ \dfrac{dU_f}{d\alpha} \\ \dfrac{dU_m}{d\alpha} \end{bmatrix}
=
\begin{bmatrix} b_2 \\ 0 \\ b_3 \end{bmatrix}
\qquad \text{(A-10)}
$$

where

$$
b_2 = \frac{-q_{mx} N_m [\mu + (1-\gamma) q_{mx} + \gamma q_m] - \frac{1}{2}(q_m^2 - q_{mx}^2) N_m (1-\gamma)}{[\mu + (1-\gamma) q_{mx} + \gamma q_m]^2} < 0
$$

$$
b_3 = \delta (1-\gamma) q_{mx} > 0
$$

By Cramer's rule,

$$
\frac{d\gamma}{d\alpha} = \frac{a_{22}[b_2 a_{33} - a_{13} b_3]}{\nabla} > 0 \qquad \frac{dU_f}{d\alpha} = \frac{-a_{21}[b_2 a_{33} - b_3 a_{13}]}{\nabla} > 0
$$

If we assume that $a_{13} < 0$, that is, the degree of discrimination by male is relatively small, then the proportion of male-dominated firms and the female value of unemployment increase with the degree of discrimination by males. On the other hand, the sign of $dU_m/d\alpha$ is undetermined.

From Equation (A-5), $dU_m/d\alpha$ can be decomposed into two effects as follows,

$$
\frac{dU_m}{d\alpha} = \theta \left[(q_m + q_{mx})(q_m - q_{mx}) \frac{d\gamma}{d\alpha} - (1-\gamma) q_{mx} \right]
$$

where $\theta = \delta/\{2[1-\delta(1-\mu)](1-\delta)\}$. The first term represents the job arrival effect and is positive. As males' distaste for working alongside females increases, the proportion of male-dominated firms rises because unemployed males are more likely to reject job offers from mixed firms. This leads to an increase in the job arrival rate from male-dominated firms to unemployed males. The second term represents the acceptance effect and is negative. An increase in the degree of discrimination reduces the acceptance probability for males of job offers from mixed firms. Then unemployed males are less likely to obtain jobs and more likely to be unemployed, which reduces the male value of unemployment. The overall effect is thus ambiguous.

Using the uniform distribution for the non-pecuniary benefit, the male value of unemployment depends positively on the degree of discrimination by males, that is, the arrival job effect dominates the acceptance effect.

$$\frac{dU_m}{d\alpha} = \frac{a_{11}a_{22}b_3 - a_{12}a_{21}b_3 - b_2a_{22}a_{31}}{\nabla} > 0$$

The details are available from the author upon request.

Proposition 6

(a) the male unemployment rate

Taking the derivative of the male unemployment rate (Equation (13)) with respect to the degree of co-worker discrimination α,

$$\frac{\partial u_m}{\partial \alpha} = \frac{\mu}{2[\mu + (1-\gamma)q_{mx} + \gamma q_m]}\left[(1-\gamma) + (1-\delta)\frac{dU_m}{d\alpha} - \alpha\frac{d\gamma}{d\alpha}\right] \qquad \text{(A-11)}$$

From the third term in the brackets in the numerator, the effect through the proportion of male-dominated firms is negative on the male unemployment rate, but from the second term in the brackets, the effect via the value of unemployment is positive. The overall effect is thus ambiguous.

(b) the female unemployment rate

Taking the derivative of the female unemployment rate (Equation (16)) with respect to the degree of discrimination, α:

$$\frac{\partial u_f}{\partial \alpha} = \frac{\mu}{[\mu + (1-\gamma)q_{fx}]^2}\left[q_{fx} + \frac{(1-\delta)(1-\gamma)}{2}\frac{dU_f}{d\gamma}\right]\frac{d\gamma}{d\alpha} \qquad \text{(A-12)}$$

From Proposition 2, the sign of brackets in the numerator is positive, Therefore, since $d\gamma/d\alpha > 0$ from Proposition 5, $\partial u_f/\partial\alpha > 0$, i.e, the female unemployment rate is rising in the degree of discrimination.

Proposition. 7

As in the proof of Proposition 4, suppose that h(R) = (1 - R)/R. Then the derivative of h(R) with respect to the degree of discriminationα is derived as follows,

$$\frac{\partial h(R)}{\partial \alpha} = \frac{1}{u_f N_f q_{fx}}\left[\frac{\partial u_m}{\partial \alpha}N_m q_{mx} - \frac{1}{2}u_m N_m - \frac{1-\delta}{2}u_m N_m \frac{dU_m}{d\alpha}\right]$$

$$- \frac{u_m N_m q_{mx}}{(u_f N_f q_{fx})^2}\left[\frac{\partial u_f}{\partial \alpha}N_f q_{fx} - \frac{1-\delta}{2}u_f N_f \frac{dU_f}{d\alpha}\right] \qquad \text{(A-13)}$$

This indicates that if the male unemployment rate depends negatively on the degree of discrimination $\partial u_m/\partial\alpha < 0$, as male workers have more distaste to work with female workers, the function h(R) decreases, i.e, the proportion of female workers in a representative mixed firm increases.