Discrimination and job-uncertainty

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Abstract

In this paper I look at the possibility of encorporating group behaviour into a model of the labour market by showing that discrimination can be the result of competition between coalitions of workers and bosses for a scarce amount of jobs. Coalitions can form either on the basis of the productivity of the individual members or on the basis of a recognisable characteristic. If the probability of correctly assessing the productivity of individual workers decreases, coalition-formation on the basis of recognisable characteristics becomes relatively more rewarding than coalition forming on the basis of productivity. I thus identify the conditions under which each individual in the endogeneously defined group actively discriminates persons with different recognisable characteristics, independent of productivity.

1 Introduction

Within the economic literature on discrimination there has been a broad acceptance of the idea that real discrimination is not the result of competitive forces. The argument is that any employer considering any other trait than the productivity of a worker will be not be profit-maximizing and thus will not remain under perfect competition. In this paper I will try to argue that discrimination can occur between completely identical groups if we allow for uncertainty for both employers and employees as to whether they will retain their current jobs in future periods. This uncertainty reflects an assumed perceived scarcity in the amount of jobs or rents.

Although the idea that discrimination is the result of the struggle over scarce resources in the presence of uncertainty comes from sociology and anthropology, this paper tries to add insight into the dynamics of discrimination by constructing a simple infinite-period game whose defining feature is that it allows for insecurity over the division of jobs. This is done by distinguishing the total population between bosses, workers and unemployed. Each period, the bosses are selected from the entire population according to a probability-distribution which itself is dependent on the outcome in the previous period. Each boss must then select one worker from the rest of the population, whereafter boss and worker share the value of their relationship, which is a function of their individual productivities. Because selecting a worker increases the chance of that individual to be a boss next period, this simple mechanism encourages coalition forming: if a group of individuals only selects workers from its own group but allows its members to be workers for bosses from other groups, this group increases the number of jobs it will probably possess next period. As long as the cost of selecting workers on the basis of group identity as opposed to worker-productivity is not too big, a discriminating group will thus prosper at the expense of others. Each potential group has to solve the problem of how to recognize its members. Because of this identification problem, the amount of information that individuals have about each other determines the eventual outcome of the game. The possibility of mistaking low-productivity workers for high-productivity workers is what allows coalitions to form on the basis of recognizable characteristics in stead of on reasons of productivity: coalitions who form on the basis of productivity are not able to obtain all the jobs in a market because they will mistake some low-productivity workers for high-productivity workers, thereby allowing some of the jobs to flow to others. Coalitions on the basis of recognizable characteristics will however obtain and keep all the scarce positions and thus discrimination between groups of workers can occur on no other basis than on recognizable characteristics. The possibility that any recognizable trait can be the basis of a group seems to confirm well to reality, where racial and ethnic identity are often found not to be rooted in a common genetic or historical basis (Lewontin et al. (1984), Harris (1993)) but are still used as a focal point for current group identity and group behaviour.

The organisation of the paper is as follows. Section 2 critiques some of the theoretical models that are currently used to explain observed differences in wages and job-opportunities for different recognizable groups. Section 3 presents the model and highlights some special cases. Section 4 concludes.

2 Theories of discrimination

The definition of discrimination I will use in the paper mirrors the literature: real (job)-discrimination occurs whenever a job-applicant is not hired for a position because of any characteristic not related to that applicant's productivity in the current position or any other that may follow as a result of hiring the worker[1]. As standard competitive theory predicts that productivity related variables are the only relevant variables a profit maximizing decision maker should consider, the question arises what the reason could be for discrimination if it is nevertheless observed within competitive markets[2]. How can one observe discrimination when there should be none? In order to explain the "anomaly" of discrimination within competitive markets, a number of explanations have been suggested. The dominant theories of the moment explain observed wage-differentials as being the outcome of differences in expected productivity. One type of these "statistical discrimination" models assumes there is something wrong with the discriminated group, be it a higher probability of women for leaving the labour market

[1]This definition does not necessarily imply that employers have a "taste for discrimination" (Becker (1991)), as it does not rule out profit-maximizing behaviour.

[2]Although I will make no attempt at rigourously defending my assumption that real discrimination indeed takes place in competitive markets, references to the recent empirical literature on this topic can be found in Botwininck (1993) who argues that no neo-classical theory as yet has explained observed wage-differentials between different groups of workers.

(Polachek (1995)), be it a greater difficulty of observing the quality of the workers, or be it a comparative advantage in a different field of activity (Renes and Ridder (1995), Lazear and Rosen (1990) and Becker (1991)). A second type of these statistical discrimination models explains discrimination as a self-fulfilling prophesy (Arrow (1973), Lang (1986), Coate and Loury (1993), Kremer (1993)), whereby low expectations of the average productivity of a group lead individuals in that group to undertake actions which make the expectation come true, such as making lower investments in human capital (Kremer (1993)).

Apart from these general explanations of discrimination, there are many theories which can explain discrimination as the result of severe market failures, such as monopsonists taking advantage of different labour-supply elasticities of different groups by offering one group a lower wage rate than the other group, higher transaction costs occurring for one group, or the existence of segmented labour markets. However, even in these models, the common opinion prevails that discrimination is the optimal market response to differences in group-characteristics related to productivity, tastes, labour supply and expectations which are formed outside the labour market.

There are two problems with this common opinion I want to highlight. Firstly, current theories of discrimination insufficiently incorporate group- behaviour: the group which is not ”discriminated” is not expected to benefit from the lower wages of the group which is discriminated. This seems to me to contradict reality, where groups of individuals fight bitter wars over scarce resources, such as land, rents and high-paying jobs. The second problem concerns the irrelevance of labour market uncertainty in current models of discrimination. I argue instead that insecurity on the part of workers and employers as to whether they will obtain a part of the scarce resources encourages groups of individuals to form coalitions against other groups of individuals to ensure future labour market success.

A small step in the direction of incorporating group behaviour and uncertainty, whilst assuming no differences at all between different groups, into a theory of discrimination is then the aim of this paper.

3 A model of discrimination

3.1 A simple model

In this section I will construct a model of a competitive market with job-insecurity. The way that individuals act within that market given their own productivity will be determined by the information available to them. To get the flavor of the general model, I start with an exceedingly unrealistic small model, which incorporates the notion of group competition over scarce resources and insecurity over the division of jobs.

Take a population of four individuals, 2 of type A and 2 of type B. These types denote some recognizable characteristic, whereas individuals are otherwise completely anonymous for each other: in each period, player one faces two indistinguishable individuals of type B and one individual of type A, whom he will recognize merely by virtue of being ’the other one of type A”. The four players have a fixed identical productivity, which is described by a one-dimensional quality variable and observable to all players, termed q_i:

$$q_1 = q_2 = q_3 = q_4 = 1$$

The four individuals play an infinitely-repeated game whereby there are only two jobs available each period; in the first period one of the four workers is randomly selected to be the boss for the first period. In every subsequent period, the boss is selected randomly from the worker and the boss in the previous period. Being a boss or a worker in one period thus gives one a 50% chance of becoming boss in the next period. This boss then has to hire one of the remaining three individuals as his worker. This makes the probability of being a boss in period t:

$$P_{i,t} = \frac{wage_{i,t-1}}{2}$$

The pay-offs in each period for the boss, the worker and the unemployed, are as follows:

$$\begin{aligned} Wage_{unemployed} &= 0 \\ Wage_{boss} &= Wage_{worker} = (q_{boss} + q_{worker})/2 \end{aligned}$$

No time-preference is assumed in this model and throughout. Persons are thus interested in the undiscounted total flow of pay-offs, which can be interpreted as an assumption of constant real discounted wages. I now define a group of N individuals to be a coalition of size N if each individual in that group will only try to hire individuals from within the group should they become a boss. The model above gives only one perfect coalition proof Nash-equilibrium: the two players of type A will form a coalition, as will the two players of type B. This can easily be seen if we reflect that should one of the individuals in these two coalitions of two be appointed boss in period t, that this coalition will keep the two jobs available for all subsequent periods, as the two other individuals will never get a chance to become boss. Thus the other two workers will have a long-run expected wage of zero unless they keep the two jobs between themselves should they become boss, thus forming a coalition themselves. The ability of keeping a set of jobs within a coalition is what creates the incentive for individuals to form coalitions, whereas the impossibility of recognizing individuals makes it impossible for a stable coalition to form on any other basis than type in this model.[3]

The general model will try to generalize the circumstances under which this coalition formation on the basis of type will happen, but will be driven by the same principle.

3.2 The general model

Consider a labour market with 2N persons, N of type A and N of type B. Apart from their type, individuals have a productivity known only to themselves. The distribution

[3]The superiority of forming a coalition on the basis of type as opposed to picking a worker at random or of the opposite type, holds even when the probabilty of being a boss next period is partially random and partially related to previous wages: as long as $0 \leq \varepsilon < 1$, $P_{it} = \frac{\varepsilon}{4} + \frac{(1-\varepsilon)Wage_{i,t-1}}{2}$, produces the same coalitions.

of productivity over the two types is identical, whereby half of each group has a low-productivity (=1) and the other half a high productivity (=$\delta > 1$):

$$q_1 = q_2 = .. = q_{N/2} = 1$$
$$q_{N/2+1} = q_{N/2+2} = .. = q_N = \delta$$
$$q_{N+1} = q_{N+2} = .. = q_{3N/2} = 1$$
$$q_{3N/2+1} = q_{3N/2+2} = .. = q_{2N} = \delta$$

In stead of assuming that productivities are perfectly observed, I assume that the actual productivity of an individual is correctly observed with a probability γ, which lies between 0.5 and 1. Thus a high productivity worker has a probability γ of being assessed as a high productivity worker in one period, and a probability $(1-\delta)$ of being assessed as a low-productivity worker in that period[4]. Otherwise, each player is anonymous.

As before, we obtain a situation of scarcity by restricting the number of bosses each period to $\frac{N}{2}$ which implies that N persons will have a job each period and N persons will be unemployed. The pay-offs are still:

$$Wage_{unemployed} = 0$$
$$Wage_{boss} = Wage_{worker} = (q_{boss} + q_{worker})/2$$

In the first period, every worker has a 25% chance of becoming a boss, but in each subsequent period, the chances of becoming a boss for individual i are completely dependent on his wage relative to the average wage:

$$P_{i=boss,t} = \frac{Wage_{i,t-1}}{4WAGE_{t-1}},$$
whereby $WAGE_{t-1} = \frac{1}{2N}\sum_{i=1}^{2N} Wage_{i,t-1}$

Given these probabilities and pay-offs, we can represent the model as an infinite repetition of the following period game:

Stage 1. $\frac{2}{N}$ jobs are devided over the whole population according to the probability schedule described above.
Stage 2. each person is assessed to either high-productivity or low-productivity, based on their actual productivity and γ
Stage 3. each boss offers a job to one person. The person accepts or not.
Stage 4. Stage 3 is repeated until all bosses have found a worker who accepts a job-offer
Stage 5. worker and boss receive pay-off and the partnership is dissolved

Coalitions between workers can now form on the basis of both type and quality, as quality can be (imperfectly) observed and type is perfectly observed. The following

[4] it doesn't make a difference for the results if we interpret this to mean that one worker is assessed the same by everybody else in a period, or whether each individual is assessed differently by different persons. The first option is however how we interpret this in the text.

theorem identifies the nature of these coalitions and the circumstances under which coalitions on the basis of type will occur.

Theorem 1 *In the game described above, there are at least two Nash-equilibria for the set* $\delta \in \{1.05, 1.1, .., 99.95\} \times \gamma \in \{0.51, 0.52, .., 0.99\}$. *The first Nash-equilibrium, the quality equilibrium, is where each of the 2N players, should they become boss in any period, will try to hire a worker of high-productivity, independent of his type. The second Nash-equilibrium, the type-coalition, which is a Nash-equilibrium under any value of* γ *and* δ, *occurs when each player of type A (B) will only hire type A (B) workers and will try to hire high-productivity type A (B) workers. The expected pay-off to the high-productivity workers in a quality equilibrium equals* $X_1(\gamma,\delta)$, *whereas the expected pay-off to high-productivity workers of type A (B), should all bosses be of type A (B), equals* $Y_1(\gamma,\delta)$. *If* $Y_1 > X_1$ *then a type-coalition will not be coalition-proof. If* $X_1 > Y_1$ *then a quality-coalition will not be coalition proof.*

The proof is long and is given in the appendix.

The intuition behind the proof is this: simply due to the law of large numbers, we get that if all individuals of one type form a coalition and the other group not, the coalition on the basis of type will secure all the jobs available. For a coalition of quality to emerge, it must thus hold that for some of the persons in the type-coalition, a quality coalition must give them a higher expected pay-off than a succesful type-coalition would. The workers who will most benefit from a quality coalition are going to be the high-productivity individuals who in a type-coalition would often have to work together with a low-quality worker. In a quality-coalition however, it will be impossible for the high-quality workers to secure all jobs due to the impossibility of perfectly recognizing the productivity of its members. In a quality-coalition-equilibrium, there will thus be a number of jobs (depending on δ and γ) which will be held by low-productivity workers whilst some high-productivity workers will be unemployed. Thus for every γ, δ has to be higher than some number for quality-coalitions to give the higher-productivity workers higher expected pay-offs then a type coalition would if the individuals of the other type do not form a coalition on the basis of type. The reason why we do not have to concern ourselves with the low-quality workers is because they will, under any coalition, try to hire high-productivity workers should they become boss. As the lower-productivity workers have a chance of one of getting a job in a type-coalition, where they will also have higher chances of working with a high-productivity person, the lower-productivity persons will prefer the type-coalition if the higher-productivity persons prefer it. Due to the fact that everyone in a type-coalition will have expected wages of zero if only one person in that type deviates, the resulting equilibrium is also optimal for each individual to adhere to.

By calculating the expected pay-offs for the high and low productivity workers of either a type-coalition or a quality-coalitions for many combinations of δ and γ, we can construct the following graph, whereby a quality coalition will occur in the area above the line and a type coalition will occur below the line.

We may note two things: these simulations show that if $\gamma < 0.8$, a quality coalition cannot be a coalition proof Nash-equilibrium, independent of δ. Secondly, the necessary quality-differential between low-productivity workers and high productivity

Figure 1:

workers (=δ), rises quite sharply if the quality coalition is to be preferred above the type-coalition: if $\gamma = 0.99$, δ already has to be 1.345 for a quality coalition to occur. With $\gamma = 0.95$, δ has to be 2.303, and with $\gamma = 0.90$, δ already has to surpass 3.502. Thus even small probabilities of mistaking high-productivity workers for low-productivity workers and vice versa, lead to coalition forming on the basis of type, resulting in overall losses to the economy and discrimination.

There are several ways one can interpret the model. First of all one can label the model as an extreme case of job-insecurity in the labour market during the life-time of an individual, as is done in this paper. Secondly one can view the given model as a representation of the labour dynamics within an organisation, in which individuals compete for the best jobs within that organisation and where individuals can help each other to obtain the best jobs within the organisation. Unemployed in this case would then mean having a low-paying job within the organisation. Thirdly one could interpret the different persons not as individuals but as units (e.g. families). This would describe the case where the progeny of a boss will not automatically become a boss themselves but may be helped by the actions of its altruistic parent in a previous period. Lastly, one could see the model as being no more than an unrealistic game in which one gets discrimination on the basis of recognizable characteristics.

The analysis thus covers more relationships than strictly that of group dynamics in an extremely uncertain labour market but also some aspects of internal labour market dynamics and intergenerational family decision making.

4 Conclusions

In this paper I have argued that discrimination can be the result of the struggle of groups to obtain a scarce amount of resources, called jobs or rents. In the model of this paper, the defining feature is that each period there are a group of persons (called bosses) who are in a position to do others favours by hiring them as workers, thereby making it more likely for these persons to be bosses next period. This leads to a situation whereby it is optimal for a group to internalize this favour, i.e., only to give favours to persons within one's coalition and thereby to capture the entire market for jobs if other groups do not do the same. As this behaviour is dependent upon the ability to recognize persons from one's own group, recognizable traits unrelated to productivity can become important. In the model, a decrease in the ability to correctly identify someone's productivity reduces the relative pay-off of forming a coalition on the basis of productivity. Even very small probabilities (5%) of mistaking a low-productivity worker for a high productivity worker, mean that the productivity differential between high-productivity workers and low-productivity workers has to be 130% if coalitions are to be based on productivity instead of a recognizable characteristic.

In the light of the necessity of coalition members to recognize each other, we could interpret networking, old-boys-clubs, free-masons, rotary clubs and other distinction-creating groups as an attempt by individuals to provide such a recognition for its members, whereby the individuals in such groups increase their expected pay-offs.

Off course the idea that discrimination is caused by a struggle over scarce resources in the presence of uncertainty is not new and has been put forward in other fields of science before and can command some empirical support (see e.g. Harris (1993), Cohen (1975), North and Thomas (1975)). This paper highlights the importance of uncertainty for both the employers (called ”bosses”) and employees as to whether they will have a job next period. This is somewhat in line with well-studied historical examples where uncertainty in the labour markets has been connected to discrimination and racism, such as the 1930's in Germany and South Africa (see Lipton (1989) and Sparks (1990)).

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Appendix 1: theorem one.

The proof proceeds in four steps. Firstly, I show that a situation where all the persons of type A form a coalition and all the persons of type B form a coalition is a Nash-equilibrium. I compare this with the second Nash-equilibrium, which is the situation when everyone who becomes a boss tries to hire a high-productivity worker, independent of type. I then show that the expected pay-offs of following a quality-strategy or a type-strategy for the four groups (high-productivity type A, low productivity type A, high-productivity type B, low-productivity type B) have the following form:

Table 1	Type B, type-strat.	Type B, qual. strategy
Type A, type-strategy	$\begin{pmatrix} Y_1/2, & Y_2/2 \\ Y_1/2, & Y_2/2 \end{pmatrix}$	$\begin{pmatrix} Y_1, & Y_2 \\ 0 & 0 \end{pmatrix}$
Type A, qual. strategy	$\begin{pmatrix} 0 & 0 \\ Y_1 & Y_2 \end{pmatrix}$	$\begin{pmatrix} X_1, & X_2 \\ X_1, & X_2 \end{pmatrix}$

I then identify the conditions under which a type-coalition is not a coalition-proof Nash-equilibrium (arising out of a prisoner’s dilemma between groups) and when the quality-coalition is not a coalition-proof Nash-equilibrium (the quality coalition is not coalition proof if $Y_1 > X_1$ and $Y_2 > X_2$).

If all type A workers form a coalition and all type B workers form a coalition, then simply due to the law of large numbers, it will occur once that all the bosses will be either all of type A or all of type B, whereafter the type that obtained all the bosses will keep all the jobs till eternity. If all players are thus only hiring workers from within their own type should they become boss, each individual i of type X will thus have to maximize the following:

$$\lim_{T\to\infty}\left\{E[\frac{1}{T}\sum_{t=1}^{T} wage_i]\right\} = P(X = winning)E[wage_i|X = winning]$$

Which means that each individual will first maximize the probability that his type will obtain all the jobs and then maximize his wages if his type has secured all the jobs. Consider now what happens with the probability of obtaining all the jobs if one individual of type A deviates by not hiring a worker of his own type. We can immediately reason from this equation that only when it is in the interest of the type B coalition, i.e. if it increases the chance that type B will obtain all the jobs, will someone from type B accept a job from a type A boss. As soon as it is in the interest of type B however to have some of its individuals working for type A, it must be in the interest of all type A individuals not to do so. Thus we may say that deviating behaviour can never occur if one person wants to deviate. Thus the situation where all bosses hire workers from their own type is a Nash-equilibrium. As to the division of jobs within a coalition, something curious will happen in the time when both types still have jobs. As it is in each person’s interest to give the type to which he belongs as big a total of wages as possible (which increases the probability of obtaining all jobs), a low-productivity worker will reveal to each boss his true productivity so that a boss will always find a high-productivity worker if they are not already employed. Thus, until one type ”wins”, low-productivity workers will voluntarily be unemployed.

To examine the expected pay-offs of an individual if his type wins, we now have to look at what the strategies of individuals will be after his type has won all the jobs. Once a type obtains all the jobs, then it becomes in each persons interest to hire a high-productivity worker and to be hired by a high-productivity worker (because the chance of a job is then 1, deviating behaviour cannot change the chance of a job next period and will only have an effect on one’s own wages this period). As higher productivity persons receive higher average wages than lower productivity persons, the number of high-productivity bosses will be higher than low-productivity bosses. From the remaining persons, those looking for a job, there will be a lower number of individuals assessed as high-productivity than there are actually bosses who are taken to be high-quality. Therefore the individuals who are taken to be high-quality in a period will only accept job-offers from bosses who are assessed as high quality bosses. Although the chances of being a boss in one period or of managing to hire a high-productivity worker are dependent on the labour market outcome last period, we can calculate the long-term expected pay-off by assuming that expected pay-offs remain constant (we will cheque whether this model does yield a stable outcome later) . We therefore term D the fraction of bosses who are assessed as high-quality and who manage to hire a worker who is assessed to be high-quality.

We name the chance of a high-quality boss to obtain a high-quality worker P. The chance of a high-quality boss to obtain a low-quality worker is 1-P. The chance of a high-quality individual to be hired by a high-quality boss is termed K whereas the chance of a high-quality worker to be hired by a low-quality boss is 1-K. Similarly, these probabilities for a low quality boss\worker are termed F and G. The probability of being a boss when one is of high-quality equals $\frac{Y_1}{Y_1+Y_2}$. In the steady state, this set of probabilities gives expected pay-offs to high-quality workers of Y_1 and to low-productivity workers a pay-off of Y_2, which are the values arising from:

$$
\begin{aligned}
Y_1 &= \frac{Y_1}{Y_2+Y_1}(P\delta+(1-P)(1+\delta)/2)+\frac{Y_2}{Y_2+Y_1}(K\delta+(1-K)(1+\delta)/2) \\
Y_2 &= \frac{Y_2}{Y_2+Y_1}(F(\delta+1)/2+(1-F))+\frac{Y_1}{Y_2+Y_1}(G(\delta+1)/2+(1-G)) \\
D &= \frac{Y_2\gamma+Y_1(1-\gamma)}{Y_1\gamma+Y_2(1-\gamma)} \\
P &= \gamma\frac{DY_2\gamma}{Y_2\gamma+Y_1(1-\gamma)}+((1-\gamma)+\gamma(1-D))\frac{Y_2(1-\gamma)}{Y_2(1-\gamma)+Y_1\gamma} \\
K &= \gamma\frac{Y_1\gamma}{Y_1\gamma+Y_2(1-\gamma)}+(1-\gamma)(\frac{Y_1(1-\gamma)+Y_1\gamma(1-D)}{Y_2(1-\gamma)+Y_1\gamma}) \\
F &= (1-\gamma)\frac{DY_2\gamma}{Y_2\gamma+Y_1(1-\gamma)}+(\gamma+(1-\gamma)(1-D))\frac{Y_2(1-\gamma)}{Y_2(1-\gamma)+Y_1\gamma}
\end{aligned}
$$

$$G = (1-\gamma)\frac{Y_1\gamma}{Y_1\gamma+Y_2(1-\gamma)}+\gamma(\frac{Y_1(1-\gamma)+Y_1\gamma(1-D)}{Y_2(1-\gamma)+Y_1\gamma})$$

For the moment we will assume this system, which is actually iterative, to yield stable expected pay-offs.

In the second Nash-equilibrium each boss will try to hire a high-productivity individual of any type. First we calculate the steady state outcome if the strategy of each boss is to try to hire a high-productivity individual and then show that it actually is a Nash-equilibrium.

Because more individuals who are assessed as being high-productivity can be hired in a period then there are bosses, one only has a chance to become employed if one is either a boss or assessed as high-productivity in a period. To calculate X_1,X_2, we have to calculate the steady state of the pay-offs should there be a quality-equilibrium, when each boss will try to hire a high-productivity worker. This leads to the following set of equations:

$$\begin{aligned}
X_1 &= \alpha(\delta+U)/2+(1-\alpha)\gamma j(\delta+E)/2 \\
X_2 &= \beta(1+U)/2+(1-\beta)(1-\gamma)j(1+E)/2 \\
\beta &= \frac{X_2}{2X_2+2X_1} \\
\alpha &= \frac{X_1}{2X_2+2X_1}=\frac{1}{2}-\beta \\
U &= \frac{\gamma(1-\alpha)}{\gamma(1-\alpha)+(1-\gamma)(1-\beta)}\delta+\frac{(1-\gamma)(1-\beta)}{\gamma(1-\alpha)+(1-\gamma)(1-\beta)} \\
j &= 2(\gamma(1-\alpha)+(1-\gamma)(1-\beta)) \\
E &= \frac{\alpha}{\alpha+\beta}\delta+\frac{\beta}{\alpha+\beta}=2\alpha\delta+2\beta
\end{aligned}$$

whereby U equals the expected quality of the worker when one is a boss, E the quality of the boss of a high-quality individual and j the chance of being hired when one is taken to be a high quality worker.

This is a Nash-equilibrium because no single worker can affect the steady-state if the steady state is stable, and thus each worker will try to maximize their expected pay-offs given the steady state, which automatically means they will try to hire high-quality individuals should they become boss, as that gives them the highest possible pay-off this period and the highest chance of working next period. The stability of the steady state can be shown by looking at what a deviation from the steady state does. To see what happens, consider what happens if the steady-state β is decreased by ε, and the steady-state α is thus increased by ε. We want to show that if we calculate the changes for one period, that α_t and β_t in the next period, say α^* and β^* deviate less than ε, thereby reducing the distance to the steady-state and thus returning to it. Then there must hold:

$$\alpha^*=\frac{X_1-\varepsilon\frac{dX_1}{d\beta}}{2(X_2+X_1)-2\varepsilon(\frac{dX_1}{d\beta}+\frac{dX_1}{d\beta})}<\alpha+\varepsilon=\frac{X_1+2\varepsilon(X_2+X_1)}{2(X_2+X_1)} \tag{1}$$

Which is the case if:

$$0 < -(\frac{dX_2}{d\beta})X_1 + (X_2 + X_1)2(X_2 + X_1) + o(\varepsilon) \quad (2)$$

which reduces to $\frac{dX_2}{d\beta} < 2(X_1 + 2X_2 + \frac{X_2^2}{X_1})$

Now, if we explicitly calculate this condition, we get a six-degree polynomial of β for which we can only cheque numerically if it indeed satisfies this condition of stability for particular values of δ and γ . Doing this for the set $\delta \in \{1.05, 1.1, .., 99.95\} \times \gamma \in \{0.51, 0.52, .., 0.99\}$, we can indeed confirm that for these values, (2) is satisfied. Indeed, a similar excercise for the type-coalition confirms that that also has stable expected pay-offs. α is thus a stable parameter under small deviations. Now, as $\alpha = \frac{1}{2} - \beta$, stability will then also hold for β. As all other parameters only vary because of variations in α, and β, the pay-offs will also be stable. Thus, as small errors 'dampen out', the whole quality-equilibrium is stable and individuals will thus take the steady-state as given as their individual actions will not change the steady-state. Thus individuals will maximize their expected wages, which will mean trying to hire a high-productivity worker should they become boss and accepting all job-offers. As this is essentially a flow-model with two distinct groups (high-productivity and low-productivity), who can either become boss or not, it does not seem likely that this does not hold for all the intermediate values of δ and γ.

We may note in passing that the reason why low-productivity workers will not form a sub-coalition by trying to hire only low-productivity workers is because they face a free-riders problem in that each individual's probability of obtaining jobs in future is not affected by his hiring decision now, making it optimal to deviate from what otherwise might be beneficial for the low-productivity individuals as a whole: because workers of the same productivity cannot perfectly recognize each other, it is not possible for either the high-productivity workers or the low-productivity workers to obtain all jobs. A deviation from low-productivity workers from the quality coalition might thus be an improvement for their pay-offs but is not self-enforcing.

However, these two models define Y_1,Y_2,X_1, and X_2, as high-degree polynomials of γ and δ which are not easily solvable or manipulable. Given the structure of this game, we can see that the quality-coalition cannot be a perfect coalition proof Nash-equilibrium if $Y_1 > X_1$, and $Y_2 > X_2$, as that would mean that a type-coalition gives higher expected pay-offs to all the individuals of one type should the workers from the other type not form a type-coalition. It is always the case that $Y_2 > X_2$ because in a type-coalition the lower-productivity workers both have a higher probability of a job and will have a higher probability of working with a high-productivity individual. If $Y_1 > X_1$, then the high-quality individuals can increase their expected pay-offs by forming a high-quality coalition, leaving the low-quality bosses no better alternative but also to try to hire high-productivity workers of any type. By calculating the pay-offs for many combinations, we can however get an idea of when the conditions for one of the two Nash-equilibria not to be perfect coalition-proof, prevail, which is done in figure 1.