Dismissals and Match-Specific Rents

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Abstract

Labor contracts that result in dismissals are quite common in the real world. The question that arises is why employers do not just offer reduced wages instead of asking workers with low realized productivity to leave. This paper argues that such behavior can be explained by workers' understandable unwillingness to agree to contracts that an employer will not have an incentive to honor in the future. Specifically, we construct a matching model in which the employer and the worker are both uncertain about the value the other places on the match. Because the worker's match-specific productivity is the employer's private information, a commitment to pay a wage equal to the worker's value of marginal product is not enforceable. In the absence of a wage guarantee, the employer will offer retained workers wages below their value of marginal product, which causes quits to be inefficiently high. The employer can reduce quits by contractually promising a guaranteed wage to retained workers. Although this will lead to some involuntary dismissals, the loss from dismissals will be less than the gain from lower quits if the wage guarantee is not too high.

I. Introduction

Some time ago Johnson (1978) pointed out that workers often can only ascertain the nonpecuniary attributes of an employer's job by actually trying the job out for a while. Along similar lines, Jovanovic's (1979) seminal paper stresses that a worker's match-specific productivity often can be determined only after observing the worker's on-the-job performance over a period of time. In Jovanovic's model, an employer promises a new worker that he will pay him a future wage equal to his realized value of marginal product. This contract has the desirable feature that the employer will never dismiss the worker and the worker will quit only if his expected marginal product elsewhere exceeds that in his current match.

In Jovanovic's model, employers can make binding commitments conditional on the realization of the match-specific productivity component because this realization is assumed to be common information. Frequently, however, this information will be private to the employer (by the same token, the worker's assessment of the nonpecuniary attributes of the employer's job is also likely to be private information). In such a case, a contract where the employer promises to pay the worker a future wage equal to his realized value of marginal product will be unenforceable.

Economists have long recognized that contracts that specify payments contingent on private information are not generally feasible. Two alternative solutions to this problem have emerged in the literature. First, it is sometimes assumed that agents must choose incentive compatible or "self-enforcing" contracts that they will have not an incentive to break. Thus, for example, Thomas and Worrall (1988) and Bester (1989) analyze self-enforcing contracts that can accommodate workers' desire for consumption smoothing and MacLeod and Malcomson (1989) analyze whether employers can offer self-enforcing contracts that solve the moral hazard problem created by the unobservability of effort. In a similar vein, Black and Loewenstein (1991) determine the self-enforcing contract that results when workers find it costly to switch employers and Kuhn (1993) and Barron, Black, and Loewenstein (1993) shows how self-enforcing considerations can lead to women being rationed from high training jobs.

Alternatively, researchers have sometimes assumed that employers must specify all future wages at the beginning of the employment contract. Thus, Becker (1962, 1964), Parsons (1972), Pencavel (1972), Salop (1973), and Salop and Salop (1976), have examined how employers can reduce quits by choosing an appropriate wage profile. And Hashimoto (1981) has generalized this analysis by simultaneously considering both quits and dismissals.[1] Specifically, Hashimoto considers a two-period model and assumes that contracts specify future wages in advance. An employer dismisses any worker whose realized productivity falls below the specified second period wage, and a worker who can obtain a higher wage elsewhere quits. The optimal second period wage balances the loss from lower quits against the gain from increased dismissals, and the equilibrium first period wage sets expected profits to zero.

The assumption that the second period wage is specified *ex ante* implies that employers have minimal standards that workers must meet in order to be retained. This is something that seems to correspond fairly well with what we often see in the real world. In fact, some employers offer explicit "up-or-out" contracts that specify probationary periods at the end of which employees whose performance is unsatisfactory are let go. As all academics know, assistant professors who do not obtain tenure within a specified time period are terminated. And a similar fate often befalls junior members of law firms who do not become partners. Moreover, dismissals are not limited to those employers who offer formal up-or-out contracts. Using data from the Employment Opportunity Pilot Program, Barron and Loewenstein (1985) find that the monthly rate of fires is 0.91 percent, which implies an annual fire rate of about 11 percent. These dismissals were not simply to reduce employment in slack periods: over 90 percent of the employers dismissing workers reported hiring new workers within the same quarter.

The question that remains unanswered is why employers do not just offer reduced wages instead of asking workers with low realized productivity to leave.[2] This question cannot be answered from either the self-

[1]Mortensen (1978) shows that inefficient separations can be eliminated if the employee and employer post turnover bonds that provide compensation for losses imposed by the other's decision to separate. We do occasionally observe turnover bonds. For example, corporations such as Electronic Data Systems, General Dynamics, McDonald Douglas, and Northrop require employees to repay relocation costs if they quit within a specified period of time, usually one year, Lockheed requires employees to reimburse educational expenses if they quit within one year, and American Airlines requires pilots to reimburse on a prorated basis their $10,000 training expense. (See "Firms Forcing Employees to Repay Some Costs if They Quit Too Soon," *The Wall Street Journal*, Tuesday, July 16, 1985.) Still, there are at least two factors that limit the general use of turnover bonds. First, as Carmichael (1983) notes, they provide the worker and employer with an incentive to induce the other party to initiate turnover. In addition, turnover bonds are hard to implement when the exact value of the match to one party is not known by the other party. This second consideration plays a crucial role in the analysis to follow.

[2] Of course, in some cases, an employer may simply not bother to offer a reduced wage because he knows that mutually advantageous employment is not possible. For example, O'Flaherty and Siow (1992) show that if (a)

enforcing or the fixed wage model alone. A contract that sets the wage offered to retained workers at a level that is sometimes above their realized value of marginal product cannot be self-enforcing because the employer can increase his expected second period profit by offering to employ low productivity workers at a reduced wage instead of simply dismissing them. And by definition the fixed wage model does not give the employer the opportunity to offer a reduced wage when the worker's realized value of marginal product turns out to be low.

Because a contract that is "self-enforcing" does not need to be enforced by a third party and a contract that sets a future wage in advance should be relatively easy to enforce through either the judicial system or a reputation mechanism, this paper generalizes the previous literature by giving employers the choice of offering either contract or, if they so desire, a combination of the two. For simplicity, we consider a two-period model. In the first period, a worker discovers his valuation of the nonpecuniary attributes of an employer's job, and the employer determines the worker's match-specific productivity.[3] Because a worker's match-specific productivity is the employer's private information, a commitment to pay a second period wage equal to the value of marginal product is not enforceable. Instead the employer must offer either a self-enforcing *or* guaranteed wage. The optimal contract may generate dismissals for the following reason. In the absence of a wage guarantee, an employer will choose future wages so as to maximize his expected rents on retained workers. Because it will not pay the employer to offer retained workers a wage equal to their value of marginal product, quits will be inefficiently high. Quits can be reduced if the employer promises some minimum wage guarantee. Although this will lead to some involuntary dismissals, the loss from dismissals can be less than the gain from lower quits if the wage guarantee is not too high.

The remainder of the paper is organized as follows. In Section II, we characterize the incentive compatible, or self-enforcing, wage contract when the employer does not make a wage guarantee. In Section III,

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employers can ascertain a worker's match-specific productivity only by observing his on-the-job performance and (b) low ability workers are completely unproductive in senior positions, then within some specified time period a worker will either be promoted or dismissed. In a similar vein, Barron and Loewenstein (1985) show that employer-specific information coupled with comparative advantage can lead to dismissals. In both of these models, all dismissals are efficient *ex post*. This is not the case in the model we present.

[3] Formally, our model is similar to that of Hall and Lazear (1984). The uncertainty in our model, however, is entirely match-specific, stemming from variations among workers in both their productivity at a given employer and in the value they place on the nonpecuniary attributes of the employer's job. In their search model, Albrecht and Jovanovic (1986) also assume that workers differ in the satisfaction they receive from a given employer's job.

we expand the set of allowable contracts to include commitments by the employer to offer a guaranteed wage to any worker retained in the second period. Concluding remarks appear in Section IV.

II. A Simple Two-Period Matching Model

For simplicity, suppose that workers' lives last two periods and consider a match between a young worker and an employer. Letting m_1 denote the worker's productivity in period 1,

$$m_1 = V_1 + \varepsilon , \tag{1}$$

where V_1 is the worker's general productivity in period one and ε is a match-specific productivity component. Because neither the employer nor the worker knows ε at the time of hire, we take ε to be a random variable with density function $f(\varepsilon)$, distribution function $F(\varepsilon)$, and expectation $E(\varepsilon) = 0$. Let ε_0 and ε_1 denote the lower and upper support of $f(\varepsilon)$, respectively. During the first period of employment, the employer learns the exact value of ε. This realization remains the employer's private information. In contrast, the worker's general productivity is common knowledge.

If the worker remains with the employer in period two, the worker's productivity, m_2, is

$$m_2 = V_2 + \varepsilon , \tag{2}$$

where V_2 is the worker's general productivity in period two.

The utility, U_i the worker receives in the employer's job in period i depends on both the wage the employer offers in period i and on how the worker values the nonpecuniary characteristics of the employer's position. That is, letting w_i denote the wage the employer pays in period i and letting α denote the worker's valuation of the job's nonpecuniary characteristics,

$$U_i = w_i + \alpha. \tag{3}$$

Because α is unknown at the time of hire and not revealed until the end of the first period of employment, we take α to be a random variable with density function $\varphi(\alpha)$, distribution function $\Phi(\alpha)$, and expectation $E(\alpha) = 0$. Let a_0 and a_1 denote the lower and upper supports of $\varphi(\cdot)$ and let $M(\alpha) \equiv [1-\Phi(\alpha)]/\varphi(\alpha)$ denote the inverse hazard function.

At the end of the first period of employment, the employer makes a second-period wage offer and the worker decides whether to accept this offer or to quit.[4] The worker quits if he can obtain a higher expected utility elsewhere. Given a competitive labor market, the worker's alternative wage in period two is V_2, his expected marginal product elsewhere.[5] Because $E(\alpha) = 0$, the worker's expected utility in a job elsewhere is simply the alternative wage, V_2. The worker therefore quits his initial job if $w_2 + \alpha < V_2$.

An employer cannot condition his second period wage offer on a worker's valuation of the job's nonpecuniary characteristics because the realized value of α is the private information of the worker, but the employer can condition his second-period offer on the realized value of ε. Because the worker quits if the realization of α turns out to be less than the critical value $V_2 - w_2$, the worker's quit probability is $\Phi(V_2 - w_2)$. Given that the worker's realized match-specific productivity component is ε, the employer's expected profit in the second period is thus given by

$$\pi_2(\varepsilon) = [1-\Phi(V_2 - w_2)][V_2 + \varepsilon - w_2]. \tag{4}$$

The wage offer that maximizes $\pi_2(\varepsilon)$ satisfies the first-order condition:

$$[\varphi(V_2 - w_2)][V_2 + \varepsilon - w_2] - [1-\Phi(V_2 - w_2)] = 0, \tag{5}$$

[4]Provided workers will play along, a line of argument similar to that in Samuelson (1984) and Riley and Zeckhauser (1983) tells us that the employer can maximize his expected profit on retained workers by making them take-it-or-leave-it wage offers. Of course, without additional structure on the problem, this equilibrium would not be subgame perfect: because some separations impose losses on the employer, he could not credibly refuse to entertain counteroffers from workers. One can make the take-it-or-leave-it strategy credible by assuming that employers can develop a reputation for not haggling.

[5] Because ε is a purely match-specific variable, a worker's quit decision does not provide useful information to a new employer.

which can be rewritten as

$$(5') \quad V_2 + \varepsilon - w_2 - M(V_2 - w_2) = 0$$

Equation (5') implicitly defines the self-enforcing second period wage as a function of ε, say $w_2(\varepsilon)$.

According to equation (5'), the employer extracts part of the rents that accrue to a favorable match, as the self-enforcing second period wage is unambiguously lower than the worker's second period marginal product. Substituting equation (5') into equation (4), we find that the employer's expected second-period profit prior to the realization of ε is given by

$$(6) \quad \Pi_2 = \int_{\varepsilon_0}^{\varepsilon_1} \pi_2(\varepsilon) f(\varepsilon) d\varepsilon = \int_{\varepsilon_0}^{\varepsilon_1} M(V_2 - w_2(\varepsilon))[1 - \Phi(V_2 - w_2(\varepsilon))] f(\varepsilon) d\varepsilon > 0.$$

Letting b denote the discount factor, the employer's expected profit from hiring an entry level worker is

$$(7) \quad \pi = V_1 - w_1 + b\Pi_2 .$$

To attract workers in period one, the employer must offer a self-enforcing second period wage $w_2(\varepsilon)$ and first period wage w_1 that provide workers with the same expected utility $\overline{U}$ available elsewhere in the labor market. Because a worker receives an expected utility of V_2 if he switches employers in period two and the utility $w_2 + \varepsilon$ if he remains with his initial employer in period two, his expected second period utility is given by

$$(8) \quad E(U_2) = \int_{\varepsilon_o}^{\varepsilon_1} \{ \int_{a_0}^{V_2 - w_2} V_2 \varphi(\alpha) d\alpha + \int_{V_2 - w_2(\varepsilon)}^{a_1} (w_2(\varepsilon) + \alpha)\varphi(\alpha) d\alpha \} f(\varepsilon) d\varepsilon \; .$$

As a worker's expected utility in period one is simply w_1, the employer must choose w_1 so that

(9) $$w_1 + bU_2 = \overline{U} .$$

If the labor market is competitive, $\overline{U}$ and the entry-level wage will be bid up until expected profits are driven to zero, or

(10) $$w_1 = V_1 + b\Pi_2 .$$

Equations (5'), (6), and (10) completely characterize the equilibrium self-enforcing two period wage contract. Several characteristics of this contract deserve mention. First, as equation (10) indicates, entry level workers are paid more than their expected marginal product in period one. This simply reflects the fact that employers expect to earn positive profits on retained workers in period two, as these workers will be paid a wage below their marginal product. The positive expected profit in period two is paid out in the form of a higher starting wage in period one.[6]

The inability of employers to credibly commit to contracts that pay workers their expected marginal product in period two does not merely alter the timing of workers' wage receipts. It also has important effects on resource allocation. Efficiency dictates that a worker leave an initial employer when the marginal product in the employer's job in period two plus the worker's valuation of the nonpecuniary characteristics of the employer's position is below the worker's expected marginal product elsewhere ($V_2 + \varepsilon + \alpha < V_2$), or equivalently, when

(11) $$\varepsilon + \alpha < 0 .$$

A worker, however, leaves his initial employer if $w_2 + \alpha < V_2$. Using equation (5'), this occurs when

[6]For a more detailed discussion of this point, see Black and Loewenstein (1991) who analyze self-enforcing employment contracts when workers face differing mobility costs. See Parsons (1990) for alternative explanations of front loading and a review of other theoretical and empirical studies.

(12) $$\varepsilon + \alpha < M(V_2 - w_2).$$

Comparing equation (12) with equation (11), we see that there is too much turnover.[7]

Differentiating equation (5) and invoking the second order condition for the maximization of the employer's expected second period profit yields

(13) $$\partial w_2/\partial\varepsilon = -\phi/(\partial^2\pi_2(\varepsilon)/\partial w^2) > 0.$$

Thus, in order to retain well-matched workers, the employer offers a share, but generally not all, of the return to a favorable match-specific productivity shock.[8] This sharing rule is fundamentally different from that occurring in Jovanovic's (1979) well-known job matching model. In Jovanovic's model, employers can make binding commitments conditional on the realization of the match-specific productivity component because this realization is assumed to be common information. Labor market equilibrium in turn requires that employers promise to pay workers their realized marginal product, which ensures that all turnover is efficient. Such contracts, however, are not enforceable when transaction costs prevent third parties from discovering the value of ε. In the absence of a wage guarantee, employers expropriate a share of the rents, an action that leads to inefficient turnover.

III. Are Contracts with Dismissals Optimal?

Economists traditionally have taken one of two approaches when considering contracts between an employer and worker who have two-sided asymmetric information. As in the preceding section, it is sometimes

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[7]From Myerson and Satterthwaite (1983), we know that there does not exist an *ex post* efficient mechanism that ensures that employment will occur whenever $\varepsilon + \alpha > 0$, but there may exist an allocation mechanism that maximizes the expected gain from trade. When both valuations are distributed uniformly, this mechanism would take the following form: the employer announces a wage offer and the worker announces a reservation wage. If the employer's wage offer exceeds the worker's announced reservation wage, employment takes place at the average of the two; otherwise, the worker and employer separate. Of course, implementing this mechanism would most likely prove to be extremely difficult: after seeing each other's announcement, both agents would like to submit new offers.

[8] Many common distributions, including the normal, uniform, and exponential, have a nonincreasing inverse hazard function. Differentiation of (5') reveals that if M is nonincreasing, then $\partial w_2/\partial\varepsilon < 1$. Even in the unusual case where $\partial w_2/\partial\varepsilon > 1$, the employer still extracts part of the rents that accrue to a favorable match, as (5') tells us that the self-enforcing second period wage is unambiguously lower than the worker's second period marginal product.

assumed that all wage offers must be self-enforcing. Such contracts have the advantage that they do not require enforcement by third parties. Alternatively, authors sometimes assume (e.g., Hashimoto, 1981) that agents agree that if future exchange is to take place it will do so at a predetermined wage that is specified before the realization of private information. Enforcing such contracts is not difficult since third parties should be able to easily verify whether employers offer the contractually agreed upon wage. In this section, we consider a more general contract. This contract contains the other two contracting forms as special cases, so that we are able to determine the circumstances when each is optimal.

Our more general contract specifies a wage floor, which we term a wage "guarantee." If exchange takes place, the wage must be no lower than the wage guarantee. On the realization of his private information, however, an employer may elect to offer a self-enforcing wage above the wage guarantee. If the wage guarantee is set sufficiently low as to never be binding, then our more general contract reduces to a pure self-enforcing contract. And if the wage is set sufficiently high so as to always be binding, then the general contract reduces to a fixed wage contract. Since outsiders can readily observe whether or not an employer complies with a simple promise to pay a guaranteed wage to retained workers, a "guaranteed wage contract," like the simpler fixed wage contract, should be enforceable through the judicial system and/or a reputation mechanism.[9]

Because an employer offering a wage guarantee of w_g will always dismiss workers whose second period value of marginal product falls below w_g, a "guaranteed wage contract" will generally result in dismissals. The higher is the wage guarantee, the greater the chance that a worker will be dismissed. On the other hand, a higher wage guarantee will reduce quits. The optimal wage guarantee balances the loss from higher dismissals against the gain from reduced quits.

We wish to find the profit-maximizing wage guarantee. As noted above, an employer offering the wage guarantee w_g will dismiss a worker whose realized marginal product in period two, $V_2 + \varepsilon$, falls short of the wage guarantee. The employer will offer to retain the worker at a wage $w_r(\varepsilon)$ if $\varepsilon \geq w_g - V_2$. If the worker's realized value of ε is sufficiently high that $w_2(\varepsilon) \geq w_g$, then the wage guarantee does not constitute a binding

[9]If workers outside a firm can readily observe the firm's failure to honor the wage guarantee, then such a failure may have significant effects on the firm's reputation, which can dramatically increase future labor costs. For additional discussion of this point, see Carmichael (1989). Outsiders do not have to observe retained workers' realized productivity in order to determine whether or not a firm honors its guaranteed wage contract.

constraint and the worker receives the wage offer $w_r(\varepsilon) = w_2(\varepsilon)$. The wage guarantee, however, raises the employer's wage offer for in between values of ε. More precisely, if $w_g - V_2 < \varepsilon < w_2^{-1}(w_g)$, the wage guarantee raises the employer's wage offer from $w_2(\varepsilon)$ to w_g. Because a worker accepts a wage offer of $w_r(\varepsilon) = \max[w_g, w_2(\varepsilon)]$ if $\alpha \geq V_2 - w_r(\varepsilon)$, the employer's expected second-period profit prior to the realization of ε is given by

$$(14) \qquad \Pi_2 = \int_{w_g - V_2}^{\varepsilon^*} \int_{V_2 - w_g}^{a_1} (V_2 + \varepsilon - w_g)\varphi(\alpha)f(\varepsilon)d\alpha d\varepsilon + \int_{\varepsilon^*}^{\varepsilon_1} \int_{V_2 - w_2(\varepsilon)}^{a_1} (V_2 + \varepsilon - w_2(\varepsilon))\varphi(\alpha)f(\varepsilon)d\alpha d\varepsilon,$$

where $\varepsilon^* = \min(w_2^{-1}(w_g), \varepsilon_1)$.

A worker whose match-specific productivity ε is sufficiently high that he is not dismissed and whose valuation of job amenities α is sufficiently high that he does not quit receives a second period utility of $w_r(\varepsilon) + \alpha$. If the worker switches jobs in period two -- either because he is dismissed or because he quits -- then he receives the expected utility V_2. Given that he is dismissed if $\varepsilon < w_g - V_2$ and he quits if $\alpha < V_2 - w_r(\varepsilon)$, his expected second period utility is given by

$$(8') \qquad E(U_2) = pV_2 + \int_{w_g - V_2}^{\varepsilon^*} \int_{V_2 - w_g}^{a_1} (w_g + \alpha)\varphi(\alpha)f(\varepsilon)d\alpha d\varepsilon + \int_{\varepsilon^*}^{\varepsilon_1} \int_{V_2 - w_2(\varepsilon)}^{a_1} (w_2(\varepsilon) + \alpha)\varphi(\alpha)f(\varepsilon)d\alpha d\varepsilon,$$

where

$$p = \int_{\varepsilon_0}^{w_g - V_2} f(\varepsilon)d\varepsilon + \int_{w_g - V_2}^{\varepsilon^*} \int_{a_0}^{V_2 - w_g} \varphi(\alpha)f(\varepsilon)d\alpha d\varepsilon + \int_{\varepsilon^*}^{\varepsilon_1} \int_{a_0}^{V_2 - w_2(\varepsilon)} \varphi(\alpha)f(\varepsilon)d\alpha d\varepsilon$$

is the probability of a separation. As before, in order to attract workers in period one, an employer must offer a lifetime wage-employment contract that provides workers with the same expected utility $\overline{U}$ available elsewhere in the labor market. That is, the first-period wage w_1 and guaranteed wage w_g must be chosen so as to satisfy equation (9).

Substituting equations (14), (9), and (8') into equation (7), one can, after a little algebra, rewrite the expected profit π on an entry level worker as

(15)

$$(15)\quad \pi = V_1 + bpV_2 - \overline{U} + b\int_{w_g - V_2}^{\varepsilon^*}\int_{V_2 - w_g}^{a_1}(V_2 + \varepsilon + \alpha)\varphi(\alpha)f(\varepsilon)d\alpha d\varepsilon + b\int_{\varepsilon^*}^{\varepsilon_1}\int_{V_2 - w_2(\varepsilon)}^{a_1}(V_2 + \varepsilon + \alpha)\varphi(\alpha)f(\varepsilon)d\alpha d\varepsilon$$

$$= V_1 + b(V_2 + \int_{\varepsilon_0}^{\varepsilon_1}\int_{-\varepsilon}^{a_1}(\varepsilon + \alpha)\varphi(\alpha)f(\varepsilon)d\alpha d\varepsilon - L(w_g)) - \overline{U}\ ,$$

where $L(w_g)$ denotes the loss from inefficient turnover and is given by

(16)

$$(16)\quad L(w_g) = \int_{\varepsilon_0}^{w_g - V_2}\int_{-\varepsilon}^{a_1}(\varepsilon + \alpha)\varphi(\alpha)f(\varepsilon)d\alpha d\varepsilon + \int_{w_g - V_2}^{\varepsilon^*}\int_{-\varepsilon}^{V_2 - w_g}(\varepsilon + \alpha)\varphi(\alpha)f(\varepsilon)d\alpha d\varepsilon + \int_{\varepsilon^*}^{\varepsilon_1}\int_{-\varepsilon}^{V_2 - w_2(\varepsilon)}(\varepsilon + \alpha)\varphi(\alpha)f(\varepsilon)d\alpha d\varepsilon\ .$$

From equation (15), we see that in order to maximize expected profits an employer must choose the wage guarantee so as to minimize the loss from inefficient turnover. In interpreting this loss, the first term in equation (16) reflects the welfare loss created by the fact that the employer will not extend a second-period offer to a worker when $V_2 + \varepsilon < w_g$. The resulting turnover is inefficient when the worker's valuation of the job's amenities is sufficiently high to outweigh the low match-specific productivity -- that is, when $\alpha + \varepsilon > 0$. The second term in equation (16) reflects the loss due to inefficient quits when the worker is paid the guaranteed wage, while the third term reflects the loss due to inefficient quits when the wage guarantee is not binding on the employer.

Differentiating equation (16) yields:

(17) $$L'(w_g) = \int_{V_2 - w_g}^{a_1} (\alpha + w_g - V_2) f(w_g - V_2)\varphi(\alpha) d\alpha - \int_{w_g - V_2}^{\varepsilon^*} (\varepsilon + V_2 - w_g)\varphi(V_2 - w_g) f(\varepsilon) d\varepsilon \quad .$$

The first term in equation (17) represents the increase in the expected loss from inefficient terminations when the guaranteed wage increases, while the second term represents the decreased loss from inefficient quits. At the optimal wage guarantee, L' = 0 and these two effects exactly offset each other.[10] Let $\tilde{w}_g$ denote the optimal wage guarantee.

It is straightforward to show that the optimal wage guarantee is binding with probability greater than zero or, in other words, that it forces the employer to offer a higher wage to some workers than he would have otherwise. To demonstrate this, we first prove that the optimal wage guarantee is as least as large as $V_2 + \varepsilon_0$.

Lemma 1: The optimal wage guarantee, $\tilde{w}_g$, is at least as large as $V_2 + \varepsilon_0$ and is smaller than $V_2 + \varepsilon_1$.
Proof: Proofs of the lemmas are given in the Appendix.

The intuition behind Lemma 1 is straightforward. It is costless for the employer to offer a wage guarantee equal to a worker's minimum productivity because such a guarantee will never induce the employer to dismiss workers in the second period. Because a wage guarantee of $V_2 + \varepsilon_0$ will reduce inefficient quits, the optimal wage guarantee must be at as large as $V_2 + \varepsilon_0$. Similarly, the employer will never offer a wage guarantee as large as $V_2 + \varepsilon_1$ because it would then never pay for him to retain any workers in the second period.

In the absence of a wage guarantee, the employer offers a worker a second period (self-enforcing) wage that is less than the worker's value of marginal product (that is, $w_2(\varepsilon) < V_2 + \varepsilon$). Lemma 1 tells us that the optimal wage guarantee is between $V_2 + \varepsilon_0$ and $V_2 + \varepsilon_1$. It follows immediately that the wage guarantee is set

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[10]The second order condition for the loss minimization problem is

$$\frac{d^2 L}{d w_g^2} = \int_{V_2 - w_g}^{a_1} \{ f(w_g - V_2) + (w_g - V_2 + \alpha) f'(w_g - V_2)\} \varphi(\alpha) d\alpha + \int_{w_g - V_2}^{\varepsilon^*} \{\varphi(V_2 - w_w) + (V_2 + \varepsilon - w_g)\varphi'(V_2 - w_g)\} f(\varepsilon) d\varepsilon > 0$$

sufficiently high so as to raise the wage offer received by some workers. For example, in the absence of the wage guarantee, a worker whose marginal product is just equal to $\tilde{w}_g$ will be offered a second period wage below $\tilde{w}_g$. With the wage guarantee, he will be offered second period employment at the wage $\tilde{w}_g$. More precisely, we have:

Proposition 1: There exist values $\hat{\varepsilon}$ and ε^*, $\hat{\varepsilon} > \varepsilon^*$, between ε_0 and ε_1 such that any worker with realized value of ε that is between $\hat{\varepsilon}$ and ε^* will be offered second period employment at a wage that is greater than the self-enforcing wage.

Proof: Let $\hat{\varepsilon} = \tilde{w}_g - V_2$ and let $\varepsilon^* = \min(w_2^{-1}(\tilde{w}_g), \varepsilon_1)$. From Lemma 1, we know that $\varepsilon_0 \leq \hat{\varepsilon} < \varepsilon_1$. Since $w_2(\hat{\varepsilon}) < V_2 + \hat{\varepsilon} < \tilde{w}_g$, it must be the case that $\hat{\varepsilon} < \varepsilon^* \leq \varepsilon_1$. Consider a worker whose realized value of ε is in the interval $[\hat{\varepsilon}, \varepsilon^*]$. Since the worker's productivity of $V_2 + \varepsilon$ exceeds the guaranteed wage $\tilde{w}_g$, the employer is certainly willing to offer the worker employment in the second period at the guaranteed wage. Since $w_2(\varepsilon) < w_2(\varepsilon^*) = \tilde{w}_g$, the wage guarantee raises the worker's wage above the self-enforcing level.

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Proposition 1 tells us that it is *never* optimal to rely solely on a fully self-enforcing contract of the type outlined in the previous section. Since the optimal wage guarantee is at least as large as $V_2 + \varepsilon_0$, the wage guarantee will be binding in period two with probability strictly greater than zero in the sense that it will force the employer to offer employment to some workers at a wage higher than he would have otherwise offered. It does not necessarily follow, however, that there will be dismissals. A worker will be dismissed in the second period if and only if his realized productivity falls short of the guaranteed wage. Thus, if the optimal wage guarantee just equals $V_2 + \varepsilon_0$, then *ex post* it will be to the employer's advantage to offer all workers employment in the second period. There will be dismissals only if the guaranteed wage strictly exceeds $V_2 + \varepsilon_0$.

The underlying probability distributions f and ϕ determine whether or not the optimal contract will induce dismissals. We demonstrate below that a wide variety of plausible distributions generate dismissals. In fact, our results indicate that it may be more difficult to find distributions that do not lead to dismissals than distributions that induce dismissals.

We start the discussion by determining a simple condition that will lead to dismissals. In this regard, the following lemma is quite helpful.

Lemma 2: The optimal wage guarantee $\tilde{w}_g$ is greater than $V_2 - a_1$.

In interpreting Lemma 2, note that it is costless for the employer to offer a guaranteed wage of $V_2 - a_1$ since no worker would ever accept a wage lower that $V_2 - a_1$. Because a wage guarantee that is only a little higher than $V_2 - a_1$ will cause relatively few inefficient dismissals, we prove in the appendix that the optimal wage guarantee strictly exceeds $V_2 - a_1$.

It follows immediately from Lemma 2 that the optimal wage guarantee will generate dismissals if some workers have realized productivity of $V_2 - a_1$ or lower. More formally, we have

Proposition 2: If $-\varepsilon_0 \geq a_1$, then the optimal contract will generate dismissals. Some of these dismissals will be *ex post* inefficient.

Proof: If $-\varepsilon_0 \geq a_1$, then we have from Lemma 2 that $\tilde{w}_g > V_2 - a_1 \geq V_2 + \varepsilon_0$. Any worker whose realized value of ε is between $\tilde{w}_g - V_2$ and ε_0 will be dismissed. The dismissal will be inefficient if $\varepsilon + a > 0$. Since $\tilde{w}_g$ strictly exceeds $V_2 - a_1$, the set $S = \{(\varepsilon,a)|V_2 + \varepsilon < \tilde{w}_g, \varepsilon + a > 0\}$ has positive measure.

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Proposition 2 identifies a simple condition that is sufficient to ensure that there will be dismissals that are *ex post* inefficient: when $-\varepsilon_0 \geq a_1$, then $\tilde{w}_g > V_2 - a_1 \geq V_2 + \varepsilon_0$. Because a worker is relatively unlikely to have a match-specific productivity component ε that is slightly higher than $V_2 - a_1$, it pays to offer a guaranteed wage strictly above $V_2 - a_1$ even though this will result in some inefficient dismissals.

When $-\varepsilon_0 < a_1$, some workers with a low realized match-specific productivity will place a high enough value on the nonpecuniary features of the employer's job to justify their being retained on efficiency grounds, so that even a guaranteed wage that is only slightly above $V_2 + \varepsilon_0$ is not costless. It will be advantageous to raise the guaranteed wage above $V_2 + \varepsilon_0$ only if the ensuing loss from inefficient terminations is less than the ensuing gain from the reduction in inefficient quits. This in turn will depend upon the actual shapes of the density functions f and ϕ. More precisely, if we assume for the moment that $f(\varepsilon_0) \neq 0$, then we see from (17) that there will be dismissals if

$$(18) \qquad \int_{-\varepsilon_0}^{a_1} (\alpha + \varepsilon_0)(\varphi(\alpha) / \varphi(-\varepsilon_0))d\alpha < \int_{\varepsilon_0}^{\varepsilon^*} (\varepsilon - \varepsilon_0)(f(\varepsilon) / f(\varepsilon_0))d\varepsilon \ .$$

Inspection of (18) indicates that dismissals are more likely, the thinner is the lower tail of f. If f has a relatively thin lower tail, then a guaranteed wage above $V_2 + \varepsilon_0$ will induce a relatively small number of inefficient terminations and thus not be very costly. Of course, the lower tail of f is thinnest when $f(\varepsilon_0) = 0$. Given the argument above, we should thus expect dismissals to be most likely when $f(\varepsilon_0) = 0$. As the following proposition demonstrates, this is indeed the case. In fact, the proposition establishes a stronger result: no matter what the density of ϕ, the optimal contract will generate dismissals if $f(\varepsilon_0) = 0$.

Proposition 3: If $f(\varepsilon_0) = 0$, then the optimal contract will generate dismissals. Some of these dismissals will be inefficient.

Proof: In light of Proposition 2, we need only consider the case where $-\varepsilon_0 < a_1$. If $f(\varepsilon_0) = 0$ and $-\varepsilon_0 < a_1$, then it follows from (17) that

$$L'(V_2 + \varepsilon_0) = -\int_{\varepsilon_0}^{\varepsilon^*} (\varepsilon - \varepsilon_0)\varphi(-\varepsilon_0)f(\varepsilon)d\varepsilon \quad < 0.$$

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When $f(\varepsilon_0) = 0$, a worker is relatively unlikely to have a match-specific productivity that is slightly higher than $V_2 + \varepsilon_0$. Consequently, the optimal contract generates dismissals. Keeping the density of α unchanged, if we now change f so as to increase the thickness of the lower tail, we will find that the optimal guaranteed wage falls. If the lower tail becomes sufficiently thick, then the optimal wage guarantee will fall to $V_2 + \varepsilon_0$ and there will be no dismissals.

The shape of the ϕ distribution also affects the likelihood of dismissals. In general, the thicker the upper tail of ϕ, the greater is ε^* and thus the more likely are dismissals. The intuition behind this result is that the thicker is the upper tail of ϕ, the greater the rents that the employer would attempt to extract in the absence of a wage guarantee and thus the greater the loss from inefficient quits. Consequently, an increase in the thickness of ϕ's upper tail causes an an increase in the optimal wage guarantee, which makes dismissals more likely.[11]

The discussion above indicates that dismissals are more likely the thinner the tails of f relative to those of ϕ. This argument is illustrated by the following proposition, which covers the case where one density is uniform and the other is unimodal symmetric, the tails of the latter being thin relative to those of the former.

Proposition 4:

a) If f is uniform and φ is linear, symmetric, and unimodal, then the optimal contract does not generate dismissals.[12]

b) If φ is uniform, f is symmetric, unimodal, and $a_1 < 3\varepsilon_1$, then the optimal contract generates dismissals.

c) If φ is uniform, f is symmetric, unimodal, and $a_1 > 3\varepsilon_1$, then the optimal contract is a fixed wage guarantee. If the tails of f are sufficiently thin (fat), then the optimal fixed wage offer will (not) generate dismissals.

Proof: The proof of Proposition 4 is given in the Appendix.

In interpreting condition c) of Proposition 4, note that the match-specific productivity component ε varies over a smaller range than the preference component α when $a_1 > 3\varepsilon_1$. In this case, one can show

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[11] There is a partially offsetting effect: the thicker is ϕ's upper tail, the greater the likelihood that a dismissal caused by a guaranteed wage above $V_2 + \varepsilon_0$ will be inefficient. This effect shows up in (18) as an increase in the left hand side of the inequality.

[12] The assumption that ϕ is linear, although likely somewhat stronger than necessary, greatly simplifies the analysis because it ensures that there is a closed form expression for $M(\alpha)$.

(assuming that φ is uniform) that the self-enforcing wage is always below $V_2 + \varepsilon_0$. As a wage guarantee of $V_2 + \varepsilon_0$ will be binding with probability one, the optimal contract has the property that all workers are offered the same second period wage, no matter what their realized productivity. If the expected loss from inefficient dismissals is smaller than the gain from additional rent reduction, it will be optimal to raise the fixed wage above $V_2 + \varepsilon_0$.

Brown (1990) has observed that contracts offering a standard wage to all workers are common, especially for blue collar jobs. While Hashimoto *assumes* that employers commit to standardized wage contracts, our analysis is more general in that we allow employers to choose from a larger set of contracts, a set that includes a fixed wage contract as a special case. Our results demonstrate that unlike the case with purely self-enforcing contracts, there are times when a fixed-wage contract is optimal.[13]

IV. Conclusion

Economists have long recognized that wage contracts that specify payments contingent on private information are not generally feasible. This does not lead to any efficiency losses in a world with one-sided asymmetric information concerning the value of an employment match. If an employer could determine a worker's valuation of a job's nonpecuniary characteristics, the employer's self-enforcing wage offer would extract all the *ex post* gains to successful matches without inducing any inefficient quits by offering a wage equal to $V_2 - \alpha$ whenever $\varepsilon + \alpha \geq 0$ and a wage below $V_2 - \alpha$, when $\varepsilon + \alpha < 0$. Because workers anticipating this behavior would demand higher starting wages, the employer's appropriation of match-specific returns would affect the wage profile, but not lead to any efficiency losses. If a worker knew his marginal product and this information could be verified by a third-party, then an employer could credibly commit to paying the worker a future wage equal to his marginal product. In this case also, there would never be any dismissals and all quits would be *ex post* efficient.

When there is two-sided asymmetric information, there does not exist a contract that will deter all inefficient separations. Two alternative contracts have been discussed in the literature. Some authors assume that agents agree that if future exchange is to take place it will do so a predetermined wage that is specified

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[13] In contrast to our explanation of standardized wage contracts that emphasizes rent extraction considerations, Brown's explanation is based on the cost of monitoring an employee's performance.

before the realization of private information. Other researchers assume that that all wage offers must be self-enforcing so that no third party enforcement is necessary. In this paper, we consider a more general contract. This contract contains the other two contracting forms as special cases, so that we are able to determine the circumstances when each is optimal.

Our more general contract specifies a wage floor, which we term a wage “guarantee.” If exchange takes place, the wage must be no lower than the wage guarantee. On the realization of his private information, however, an employer may elect to offer a self-enforcing wage above the wage guarantee. If the wage guarantee is set sufficiently low as to never be binding, then our more general contract reduces to a pure self-enforcing contract. And if the wage is set sufficiently high so as to always be binding, then the general contract reduces to a fixed wage contract. Since outsiders can readily observe whether or not an employer complies with a simple promise to pay a guaranteed wage to retained workers, a "guaranteed wage contract", like the simpler fixed wage contract, should be enforceable through the judicial system and/or a reputation mechanism.

Our analysis demonstrates that there are conditions under which neither simple contract is optimal. In fact, we have shown that a purely self-enforcing contract is *never* optimal when there is two-sided asymmetric information, as it is *ex ante* efficient for employers to offer a wage guarantee to workers who are retained in the future. The wage guarantee has the desirable effect of limiting the amount of match-specific rents that the employer can attempt to extract. Furthermore, our analysis indicates that the wage guarantee will often be sufficiently high to lead to the dismissal of workers who would be willing to work for a lower wage. Unlike the purely self-enforcing contract, the fixed-wage contract is sometimes optimal. This result is consistent with Brown's (1990) observation that fixed-wage contracts are common for blue collar jobs.

Our finding that a wage floor is always optimal is similar in spirit to the result in the efficiency wage model (for example, see Shapiro and Stiglitz, 1984) that employers may find it advantageous to offer a wage above the market clearing level. However, there is an important difference between the two models. In the efficiency wage model, the unemployed worker's claim to be willing to put forth effort at a lower wage is not to be believed. In contrast, the dismissed worker in our model may be willing to work at a lower wage.

One other implication of our analysis is worth noting. The findings of a number of researchers suggest that employers incur a large share of the costs and returns to investments in on-the-job training (for example, see

Barron, Black, and Loewenstein (1989), Barron, Berger, and Black (1993), Lynch (1992), and Parsons (1989)). Loewenstein and Spletzer's results (1995) even indicate that employers share in the returns and costs to general training. Our model provides a possible explanation for this phenomenon. A binding wage guarantee in the future means that an increase in a worker's productivity caused by an increase in his stock of (specific or general) human capital will not fully be reflected in a higher wage.[14] Hence employers will share in the returns and costs to both specific and general training.

[14] For a more detailed discussion of this point, see Loewenstein and Spletzer (1995).

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Appendix

This Appendix contains the proofs of the lemmas and Proposition 4.

Proof of Lemma 1: Since f has the lower support ε_0, $f(w_g - V_2) = 0$ if $w_g < V_2 + \varepsilon_0$. It thus follows from equation (17), that if $\delta > 0$, then

$$\text{(A1)} \qquad L'(V_2 + \varepsilon_0 - \delta) = -\int_{\varepsilon_0-\delta}^{\varepsilon^*} (\varepsilon - \varepsilon_0 + \delta)\varphi(-\varepsilon_0 + \delta) f(\varepsilon) d\varepsilon$$

$$\leq 0 \, .$$

It follows immediately from (A1) that the optimal wage guarantee is at least as large as $V_2 + \varepsilon_0$. Similarly, if w_g is close to $= V_2 + \varepsilon_1$, then $\varepsilon^* = \varepsilon_1$. If δ is small and positive, it thus follows from (17) that

$$L'(V_2 + \varepsilon_1 - \delta) = \int_{-\varepsilon_1+\delta}^{a_1} (\alpha + \varepsilon_1 - \delta) f(\varepsilon_1 - \delta)\varphi(\alpha) d\alpha - \int_{\varepsilon_1-\delta}^{\varepsilon_1} (\varepsilon - \varepsilon_1 + \delta)\varphi(-\varepsilon_1 + \delta) f(\varepsilon) d\varepsilon$$

$$= f(\varepsilon_1 - \delta)\Big(\int_{-\varepsilon_1+\delta}^{a_1} (\alpha + \varepsilon_1 - \delta)\varphi(\alpha) d\alpha - \int_{\varepsilon_1-\delta}^{\varepsilon_1} (\varepsilon - \varepsilon_1 + \delta)[\varphi(-\varepsilon_1 + \delta) / f(\varepsilon_1 - \delta)] f(\varepsilon) d\varepsilon \, .$$

As

$$\lim_{\delta \to 0}\Big\{\int_{-\varepsilon_1+\delta}^{a_1} (\alpha + \varepsilon_1 - \delta)\varphi(\alpha) d\alpha - \int_{\varepsilon_1-\delta}^{\varepsilon_1} (\varepsilon - \varepsilon_1 + \delta)[\varphi(-\varepsilon_1 + \delta) / f(\varepsilon_1 - \delta)] f(\varepsilon) d\varepsilon\Big\} = \int_{-\varepsilon_1}^{a_1} (\alpha + \varepsilon_1)\varphi(\alpha) d\alpha$$

$$\geq 0 \, ,$$

$L'(V_2+\varepsilon_1-\delta) > 0$ if δ is sufficiently small. Thus, the optimal wage guarantee must be strictly less than $V_2+\varepsilon_1$. •

Proof of Lemma 2: If δ is small and positive, it follows from equation (17) that

$$\text{(A2)}\quad L'(V_2 - a_1 + \delta) = \int_{a_1-\delta}^{a_1} (\alpha - a_1 + \delta) f(-a_1 + \delta)\varphi(\alpha) d\alpha - \int_{-a_1+\delta}^{\varepsilon^*(\delta)} (\varepsilon + a_1 - \delta)\varphi(a_1 - \delta) f(\varepsilon) d\varepsilon$$

$$= \varphi(a_1 - \delta)\{ \int_{a_1-\delta}^{a_1} (\alpha - a_1 + \delta) f(-a_1 + \delta)[\varphi(\alpha) / \varphi(a_1 - \delta)] d\alpha - \int_{-a_1+\delta}^{\varepsilon^*(\delta)} (\varepsilon + a_1 - \delta) f(\varepsilon) d\varepsilon \},$$

where $\varepsilon^*(\delta) = \min(w_2^{-1}(V_2\text{-}a_1\text{+}\delta),\varepsilon_1)$. As

$$\lim_{\delta\to 0}\{ \int_{a_1-\delta}^{a_1} (\alpha - a_1 + \delta) f(-a_1 + \delta)[\varphi(\alpha) / \varphi(a_1 - \delta)] d\alpha - \int_{-a_1+\delta}^{\varepsilon^*(\delta)} (\varepsilon + a_1 - \delta) f(\varepsilon) d\varepsilon \} = -\int_{-a_1}^{\varepsilon^*(0)} (\varepsilon + a_1 - \delta) f(\varepsilon) d\varepsilon$$

$$< 0,$$

$L'(V_2 - a_1 + \delta) < 0$ if δ is sufficiently small.

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Proof of Proposition 4:

a) Let ε be distributed uniformly over the interval [-e,e]. Since f(ε) = 1/2e and $e \equiv \varepsilon_1 = -\varepsilon_0$, equation (17) gives us

$$\text{(A4)}\qquad L'(V_2\text{-}e) = \quad \phi(e)(1/2e)\left(\int_{e}^{a_1} (\alpha - e)(\phi(\alpha)/\phi(e)) d\alpha - \int_{-e}^{\varepsilon^*} (\varepsilon + e) d\varepsilon \right) \ .$$

Using equation (5'), we see that when $w_g = V_2 - e$,

$$\text{(A5)}\qquad \varepsilon^* = \min(w_2^{-1}(V_2\text{-}e),e) \geq w_2^{-1}(V_2\text{-}e) = -e + M(e) \ .$$

Substituting (A5) into (A4) one obtains:

(A6) $L'(V_2\text{-}e) \geq \phi(e)(1/2e)\left(\int_{e}^{a_1}(\alpha - e)(\phi(\alpha)/\phi(e))d\alpha - \int_{-e}^{-e+M(e)}(\varepsilon + e)d\varepsilon\right)$

or,

(A7) $L'(V_2\text{-}e) \geq \phi(e)(1/2e)\int_{e}^{a_1}(\alpha - e)(\phi(\alpha)/\phi(e))d\alpha - (M(e)^2/2)$.

If ϕ is a symmetric, unimodal density over the interval [-a,a], then ϕ must be given by

(A8) $\phi(\alpha) = (1/a) - x + ((2xa-1)/a^2)\alpha$ if $\alpha \geq 0$

$\phi(\alpha) = (1/a) - x - ((2xa-1)/a^2)\alpha$ if $\alpha < 0$,

where $0 \leq x < 1/2a$. To verify this, note that $\int_{-a}^{a}\phi(\alpha)d\alpha = 1$, $\phi(\alpha) > 0$, and $\phi(0) > \phi(\alpha)$ for all values of $\alpha \neq 0$ in the interval [-a,a]. Define $\phi(a) \equiv \phi(-a) \equiv x$, so x cannot be smaller than zero. Finally, ϕ would be uniform if x were equal to 1/2a and ϕ would be bimodal (v-shaped) if x were greater that 1/2a.

Substituting (A8) into the right-hand side of equation (A7), one finds after some tedious algebra that

(A9) $\phi(e)(1/2e)\left(\frac{-1}{6}\frac{-3a^2e + a^3 + xa^4 - e^3 + 2e^3xa + 3ae^2 - 3e^2xa^2}{-2xae + e - a + xa^2} - \frac{1}{8}\left[\frac{a^2 - 2ea + 2exa^2 - 2xae^2 + e^2}{-2xae + e - a + xa^2}\right]\right)^2$,

which, with some more algebra, can be shown to be greater than zero for all $x \varepsilon [0,1/2a]$.

b) Let α be distributed uniformly over the interval [-a,a]. Since $\phi(\alpha) = 1/2a$ and $a = a_1 = -a_0$, equation (17) gives us

(A10) $$L'(V_2+\varepsilon_0) = f(e)(1/2a)\left(\int_e^a (\alpha - e)d\alpha - \int_{-e}^{\varepsilon^*} (\varepsilon + e)(f(\varepsilon)/f(e))d\varepsilon\right).$$

As f is unimodal and symmetric, $f(\varepsilon) < f(e)$ if $-e < \varepsilon < e$. It thus follows immediately from (A10) that

(A11) $$L'(V_2+\varepsilon_0) < f(e)(1/2a)\left(\int_e^a (\alpha - e)d\alpha - \int_{-e}^{\varepsilon^*} (\varepsilon + e)d\varepsilon\right)$$

$$= f(e)(1/4a)(-2ea + a^2 -2ea - (\varepsilon^*)^2 \ .$$

It is easy to verify that $M(\alpha) = a - \alpha$ when $\phi(\alpha) = 1/2a$. Substituting this result into equation (5'), one finds that the self-enforcing wage is given by

(A12) $$w_2(\varepsilon) = V_2 + (\varepsilon - a)/2.$$

From equation (A12), we see that when $w_g = V_2 - e$, ε^* is given by

(A13) $$\varepsilon^* = \min(w_2^{-1}(w_g),e) = \min(2(w_g-V_2)+a,e) = \min(-2e+a,e) \ .$$

Thus, if $a < 3e$, then $\varepsilon^* = -2e+a$. Substituting $-2e+a$ for ε^* in (A11), we find that

$$L'(V_2 + \varepsilon_0) < 0 \ .$$

c) From the analysis in part b), we see that if ϕ is uniform density and $a > 3$, then $\varepsilon^* = e$. From Proposition 3, we know that the optimal wage guarantee exceeds $V_2 + \varepsilon_0$ if $f(\varepsilon_0)$ is sufficiently close to zero. If f is uniform, then

(A12) $$L'(V_2 + \varepsilon_0) = (1/2e)(1/2a)\left(\int_{e}^{a}(\alpha - e)d\alpha - \int_{-e}^{\varepsilon^*}(\varepsilon + e)d\varepsilon\right)$$

Using the fact that $\varepsilon^* = e$, we have that

$$L'(V_2 + \varepsilon_0) = (1/8ae)(a^2 - 2ea - 3e^2) = (1/8ae)(a+e)(a-3e) > 0.$$

•