SKILLS FORMATION AND WAGE INEQUALITY IN DEVELOPING COUNTRIES: A THEORETICAL ANALYSIS

Sarbajit Chaudhuri
Reader
Dept. of Economics
University of Calcutta
56A, B.T. Road
Kolkata 700 050
India.

Address for communication: Dr. Sarbajit Chaudhuri, 23 Dr. P.N. Guha Road, Belgharia, Kolkata 700083, India.
Tel: 91-33-2541-0455 (R), 91-33-2557-5082 (C.U.)
Fax: 91-33-2844-1490 (P)
E-mail: srbajitch@yahoo.com, srbajitchaudhuri@yahoo.com
http://papers.ssrn.com/author=294419

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ABSTRACT: The paper is purported to analyze the impact of skill formation on the skilled-unskilled wage inequality using a few variants of the HOS-type framework. It shows that the effect of skill formation on the wage inequality depends crucially upon the technologies of production of the economy and institutional nature of the markets for unskilled labour. In the extreme case when all unskilled labour markets are distorted any attempt of skill formation unequivocally accentuates the wage gap and may increase the level of unemployment of unskilled labour. These results point out that the empirical evidence as found in Beyer, Rojas and Vergara (1999) and the World Development Report (1995) that skill formation has contributed in reducing the skilled-unskilled wage gap in some developing countries lack solid theoretical bearing. The paper suggests that institutional reform programs, designed for the removal of labour market distortions, should be given high priority along with skill improvement measures to improve the skilled-unskilled wage inequality in the developing countries.

Keywords: Skilled labour, unskilled labour, wage inequality, skill formation, institutional reform programs.

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I. INTRODUCTION

Education is generally considered to have an equalizing effect on wages, which would indicate a positive relation between measures of wage inequality and of education inequality. According to the World Development Report (1995), combined with other human capital variable like experience and occupation, skills account for one-third to one-half of the variation in earnings observed across individuals within countries. Naturally, policies of spreading education among unskilled workers are likely to produce significant impact on the skilled-unskilled wage dispersion.

Educational attainment levels increased rapidly in most developing countries since the 1950s (Schultz 1988). As a consequence, the supply of workers with higher education is increasing, which is the desired market response to the increased wage premium on higher education. The increasing supply of more educated workers should eventually reduce the wage premium received by such workers and, thus, also tend to equalize the earnings distribution.

However, there has been a significant increase in earnings inequality in many of the developing countries over the last decade, accompanied by an absolute decline in the real incomes of the less educated, poorer members of society. This comes as something of a surprise in light of the great equalizing properties generally attributed to education.

For example, De la Torre (1997), Pánuco-Laguette and Székely (1996) and Lachler (2001) have noted that although remarkable advance in education attainment was taking place, the distribution of wage income in Mexico worsened considerably. Despite the observations of Beyer, Rojas and Vergara (1999) that the extent of wage inequality and the proportion of the labour force with college degrees in the post-liberalization period in Chile were negatively related and the World Development Report (1995) that in Columbia and Costa Rica increased educational opportunities exerted downward pressures on wage inequality, in reality in all these countries the wage inequality as a whole had deteriorated. The studies by Robbins (1994a, 1994b) covering Argentina, Columbia, Costa Rica, Chile and Uruguay, reveal that skill differentials in wages (by the level of education) have increased significantly in all these countries. Pedderson (1998) also points out increasing wage inequality in Chile over the 1980s. Khan (1998) and Tendulkar et al (1996) provide some indirect evidence in support of the rising inequality in South Asia in the post-liberalization period despite advancement in education attainment.
A few explanations have been provided in the empirical literature to explain why the wage inequality has deteriorated in these countries despite considerable improvement in educational facilities. Robbins (1995a,b), Feenstra and Hanson (1997), Harrison and Hanson (1999), Hanson and Harrison (1999), Currie and Harrison (1997) and Beyer, Rojas and Vergara (1999) have identified removal of tariff restrictions from the relatively unskilled labour intensive sectors, reduction in the relative prices of labour-intensive commodities; growth in foreign direct investment, which is positively correlated with the relative demand for skilled labour; skill-biased technological change etc. as the alternative factors responsible for the growing incidence of wage inequality in the Latin American countries.

In the circumstances, a pertinent question is whether one really needs to look for other factors to explain the increasing wage inequality despite growth in educational opportunities, or the relationship between skill formation and wage inequality is not so straightforward as it is thought to be. The present paper is purported to provide a satisfactory theoretical answer to the above question in a general equilibrium set-up using a few variants of the Hechscher-Ohlin-Samuelson (HOS) type framework. It shows that the effects of skill formation on the wage inequality depends critically upon the technologies of production of the economy and institutional nature of the markets for unskilled labour, which are generally plagued by various types of distortion. In the extreme case, when all the unskilled labour markets are distorted any attempt of skill formation unequivocally accentuates the wage inequality. The results of the paper are not only inclined to challenge the empirical findings of Beyer, Rojas and Vergara (1999) and World Development Report (1995) but also are indicative of the limitation of the HOS-type theories to explain the phenomenon of skill formation in a general equilibrium framework. The paper indicates that institutional reform programs for the removal of labour market distortions and skill improvement measures must go hand in hand to improve the skilled-unskilled wage inequality in a developing economy.

II. THE MODEL

Let us consider a small open economy with two broad sectors: rural and urban. The urban sector is further subdivided into two sub-sectors so that in all we have three sectors. All the sectors operate at close vicinity. The rural sector (sector 1) produces a primary export commodity using unskilled labour and capital of type 1. The first sub-sector within the urban sector (sector 2) produces a skill-intensive manufacturing commodity with the help of skilled labour and capital of type 2. The other urban sub-sector (sector 3) uses unskilled labour and capital of type 2 to produce a low-skill manufacturing product. So, capital of type 2 is mobile between sectors 2 and 3 and unskilled labour is mobile between sectors 1 and 3. But capital of type

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1 One might find the analysis of skill formation in a static model like this a limitation of the paper. In defense, it may be argued modestly that even a suitable static model can capture some of the essence of a dynamic model. The static nature of the model may be interpreted as a stationary state equilibrium where the system repeats itself perpetually. See, Marjit and Acharyya (2001), chapter 9 in this context.
1 and skilled labour are specific to sectors 1 and 2, respectively. Product prices are given internationally. We consider a trade situation where the economy exports commodities 1 and 2 and is a net importer of the other. A developing region which fits this pattern of comparative advantage is Latin America or India\(^2\) – a South Asian country. Production functions exhibit constant returns to scale with diminishing marginal productivity to each factor. Markets are perfectly competitive\(^3\) and resources are fully utilized. We shall also assume that the skilled wage is always greater than the unskilled wage as it is empirically supported. However, we do not provide any theory for such a difference.

The following symbols will be used in the equations.

\[
\begin{align*}
\alpha_{Li} &= \text{unskilled labour-output ratio in the } i\text{th sector, } i = 1, 3; \\
\alpha_{Ki} &= \text{capital-output ratio in the } i\text{th sector, } i = 1, 2, 3; \\
\alpha_{S2} &= \text{skilled labour-output ratio in sector 2}; \\
P_i &= \text{world price of the } i\text{th good, } i = 1, 2, 3; \\
W_S &= \text{skilled wage rate}; \\
W &= \text{unskilled wage rate}; \\
R_j &= \text{return to capital of the } j\text{th type}, j = 1, 2; \\
E &= \text{government expenditure on education}; \\
L^* &= \text{given labour endowment of the economy (skilled plus unskilled)}; \\
S &= \text{supply of skilled labour (an increasing function of } E); \\
L &= \text{supply of unskilled labour (a decreasing function of } E); \\
K_j &= \text{economy's endowment of the } j\text{th type of capital, } j = 1, 2; \\
\theta_{ji} &= \text{distributive share of the } j\text{th input in the } i\text{th industry, e.g. } \theta_{S2} = \frac{W_S\alpha_{S2}}{P_2}; \\
\lambda_{ji} &= \text{proportion of the } j\text{th input employed in the } i\text{th sector, } i = 1, 2, 3; \text{ and, } j = L, K_1, K_2, S; \text{ e.g. } \lambda_{L1} = \frac{(\alpha_{L1}X_1/L)}{\text{and } \lambda_{K1} = \frac{(\alpha_{K1}X_1/K)}{3}; \\
\sigma_i &= \text{the elasticity of substitution between capital and labour in the } i\text{th sector}; \\
^\Delta &= \text{proportional change, e.g. } \frac{\Delta X_i}{X_i}. \\
\end{align*}
\]

The general equilibrium is represented by the following set of equations.

\(^2\) The production structure, as depicted here, does not apply either to East or South Asia (whose countries are in general net importers of primary products, because they are land scarce), except India, or to Africa (which is a net exporter of primary products but not of skill-intensive − or indeed of any − manufactures), although variants on the model can be constructed to cover these other regional cases.

\(^3\) We shall, however, introduce imperfections in the markets for unskilled labour in the subsequent sections of the paper.
Equations (1), (2) and (3) state that unit cost of production of each commodity must equal its domestic price in equilibrium. In other words, these are the three competitive industry equilibrium conditions.

Equations (4) – (7) are the full employment conditions of different inputs of production.

There are seven equations to solve for seven unknowns – W, Ws, R1, R2, X1, X2 and X3. The system does not possess the decomposition property. So, the factor prices are affected by any changes in factor endowments. The following describes the general equilibrium property of the model.

From equations (5) and (6) we may respectively, write

\[ X_1 = \frac{K_1}{a_{K1}} \]  

(5.1)

\[ X_2 = \frac{S}{a_{S2}} \]  

(6.1)

Using (6.1) from (7) it is easy to check that

\[ X_3 = \left( \frac{1}{a_{K3}} \right) K_2 - \left( \frac{a_{K2}/a_{K3}a_{S2}}{a_{K1}} \right) S \]  

(7.1)

Using (4), (5.1) and (7.1) we obtain,

\[ (a_{L1}/a_{K1}) K_1 + \left( a_{L3}/a_{K3} \right) \{ K_2 - \left( a_{K2}/a_{S2} \right) S \} = L \]  

(8)

The four factor prices – W, Ws, R1 and R2 are determined by solving equations (1) – (3) and (8) simultaneously. Once the factor prices are known the factor coefficients ai are also known, as these are functions of the relative input prices only. Then X1, X2 and X3 are obtained from equations (5.1), (6.1) and (7.1), respectively.

Determination of the factor prices can be alternatively explained diagrammatically. Figure 1 shows the zero-profit curves for the three sectors. Figure 2 is a standard diagram used to illustrate the specific –factors model for sectors 1 and 3. Given R2 and the product prices, all other factor prices are determined. Moreover, given R2 and the endowment of skilled labour, the demand for capital (of type 2) in sector 2 is known. The remaining amount of capital of type 2 is then released to sector 3. This takes us to figure 2 where the unskilled wage rate, W, is found from the intersection point of the demand for labour curves for

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4 If in a production structure the unknown factor prices can be solved from the price-unit cost equality conditions only, independently of the factor endowments, the system is said to possess the decomposition property. Thus, any changes in factor endowments cannot produce any effect on the factor prices in such a system.
sectors 1 and 3. Actual solution of the equilibrium is a fixed-point problem, but assume we start at equilibrium where the return to capital of type 2 is the same in both sectors 2 and 3.

(Insert figures 1 and 2 here)

Effects of Skill Formation on Wage Inequality

By skilled-unskilled wage gap we simply mean the algebraic difference between the two wage rates. The wage gap rises if this difference rises and vice versa. Besides, the relative wage inequality increases (decreases) if the difference between the proportionate changes of the two wages rises (falls) algebraically.

We shall now examine the effects of skill formation\(^5\) on the wage inequality within a static framework where the size of the workforce (skilled plus unskilled), \(L^*\) is given. We assume that the endowment of skilled labour, \(S\), is a rising function of the amount of government expenditure on education and training of unskilled workers, \(E\). The workers do not spend anything of their own\(^6\) for that purpose. So we write \(S = S(E); S'(.) > 0\).

The labour endowment (skilled plus unskilled) equation for the economy is given by

\[
L + S(E) = L^* \tag{9}
\]

An increase in government’s spending on education, \(E\), implies improvement in educational and training facilities for unskilled workers that helps them to acquire the necessary skill to enable them to be labeled as skilled workers. As a consequence of skill formation, the endowment of skilled labour rises while that of unskilled labour falls.

To focus on the aspect of skill formation we assume that owing to government’s policy changes \(E\) takes a higher value while all the other parameters of the system remain unaltered. Owing to skill formation the endowment of skilled labour, \(S\), rises while that of unskilled labour, \(L\), falls. As \(L\) decreases, the distance \(OO'\) in figure 2 decreases. The MPL\(_1\) and MPL\(_3\) curve must intersect each other at a higher \(W\). As \(W\) rises, the return to capital of type 2, \(R_2\), in sector 3 falls. Capital flows from sector 3 to sector 2. This lowers \(R_2\) in sector 2 and raises the skilled wage rate, \(W_s\). It also shifts the MPL\(_3\) curve in figure 2 downward, exerting a partial offsetting downward pressure on \(W\). It also somewhat lowers \(W_s\) through an increase in \(R_2\). On the

\(^5\) See footnote 1 in this context.

\(^6\) Although this is an extreme assumption, it is not far from reality in the context of the developing countries. Governments all over the world devote substantial resources to their education sector. This is especially true in developing countries. In 1995, public spending on education accounted for 15.7% of total government expenditure in developing countries (see Bedi and Garg (2000)). Furthermore, the majority of students in developing countries are educated in publicly funded and publicly managed educational institutions. According to Jimenez and Lockhead (1995), almost 90% of all primary and 70% of all secondary enrollments in developing countries are in public schools.
The other hand, as S rises, sector 2 expands, drawing capital of type 2 from sector 3. MPL₂ in figure 2 shifts downward resulting in a fall in W, which in turn raises R₂ in sector 3. Thus, some amount of capital of type 2 moves back to sector 3 causing R₂ to rise in sector 2. Wₘ falls as a consequence. Thus, two opposite effects work on both W and Wₘ. Their relative strengths not only depend on the $θₖ_i$'s but also on the relative proportions of unskilled labour employed in sector 1 and 3 and the initial endowments of skilled unskilled labour of the economy\(^7\). This leads to the following proposition.

**PROPOSITION 1:** An expansion of educational facilities among the unskilled workers does not necessarily reduce the skilled-unskilled wage gap.

*Introduction of Imperfections in the Urban Market for Unskilled Labour*

The market for unskilled labour in the urban sector of a developing economy is plagued by distortion. We introduce wage rigidity in the market for unskilled labour in sector 3 (the low skilled urban sector) while the wage rate for unskilled labour in sector 1 (the rural sector) is flexible. The unskilled workers in the urban sector receive contractual wage, W₃*, while the remaining workers employed in the rural sector earn a wage, W₁ where W₃* > W₁. The allocation mechanism for unskilled labour is like this. The unskilled workers first rush for getting employment in sector 3 that offers the higher wage rate. Those who cannot get employment in that sector are automatically absorbed in the rural sector.

The general equilibrium is now characterized by the equations (2), (4) – (7) and the following two equations.

\[
\begin{align*}
W₁.aₗ₁ + R₁.aₖ₁ &= P₁ \quad \text{(1.1)} \\
W₃*.aₗ₃ + R₂.aₖ₃ &= P₃ \quad \text{(3.1)}
\end{align*}
\]

The system still remains indecomposable. However, R₂ is now directly solved from equation (3.1) as W₃* is given. Given R₂, Wₘ is obtained from (2). Using (4) – (7) one can again derive equation (8). W₁ and R₁ are determined by solving (1.1) and (8) simultaneously.

The general equilibrium of this modified system is represented by figures 3 and 4. In figure 3, the unionized wage for unskilled labour in sector 3 uniquely determines the return to capital of type 2, R₂, which in turn determines the skilled wage rate, Wₘ, given the relative product prices. The competitive unskilled wage rate is O′W₁ in figure 4.

(Insert figures 3 and 4 here)

\(^7\) See appendix I for the mathematical proof.
Unskilled workers in this system earn two different wages – either the contractual wage, \( W_3^* \) in sector 3 or a lower wage \( W_1 \) in sector 1. The average wage for unskilled labour is given by

\[
W_A = \frac{(W_1 L_1 + W_3^* L_3)}{L} = \frac{(W_1 a_{11} X_{1l} + W_3^* a_{l3} X_{3})}{L}
\]  \( (10) \)

In terms of figure 4, the average unskilled wage is given by the measure \((Otnq + O'qmW_3^*) / OO')\). In this case, the skilled—unskilled wage gap improves (worsens) if the gap between \( W_S \) and \( W_A \) falls (rises).

Due to skill formation, the endowment of skilled labour rises and that of unskilled labour falls. In figure 4 the distance \( OO' \) gets smaller. The competitive unskilled wage rate, \( W_1 \) rises. Sectors 2 and 3 and hence the skilled wage rate, \( W_S \), remain unaffected. On the other hand, as \( S \) rises, sector 2 expands and draws some capital from sector 3. MPL in figure 4 shifts downward leading to a fall in \( W_1 \). Thus, two opposite effects on \( W_1 \) are generated. The net effect crucially depends upon the relative proportions of capital of type 2 used in sectors 2 and 3 and also upon the proportion of unskilled labour employed in sector 3. Besides, as the higher (lower) unskilled wage-paying sector 3 (sector 1) now employs a larger proportion of the contracted unskilled labour force, the net effect on the average unskilled wage, \( W_A \), must depend upon the initial endowments of the two types of labour. It is easy to show\(^8\) mathematically, that \( W_A \) falls if \((S/L) \leq \left( \frac{\lambda L_3}{\lambda K_2/\lambda K_3} \right)\). Thus, the skilled-unskilled wage gap may rise owing to a spread of educational facilities although the skilled wage rate itself does not rise. However, unlike the perfect unskilled labour market case, the effect on the wage inequality does not depend upon the \( \theta_{Ki} \) s as \( R_2 \) and \( W_S \) cannot change in this case. This leads to the following proposition.

**PROPOSITION 2:** In the presence of distortions in the market for unskilled labour in the urban sector, the skilled-unskilled wage inequality increases owing to spread of educational facilities among the unskilled workers if \((S/L) \leq \left( \frac{\lambda L_3}{\lambda K_2/\lambda K_3} \right)\).

**Imperfections in both rural and urban markets for unskilled labour**

In this section of the paper, we shall consider imperfections both in the rural and urban markets for unskilled labour. There are sufficient reasons why the unskilled wage rate in the rural market may be rigid downwards. It can be explained either by the ‘wage efficiency hypothesis’ \(^9\) or the ‘collusive theory of

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\(^8\) This has been proved in appendix II.

\(^9\) According to the ‘wage efficiency hypothesis’, the nutritional efficiencies of the workers are positively related to their respective wage incomes (at least for some minimum levels). Thus, the employers in the rural labour market would prefer to pay efficiency wages to their respective workers than lower wages even if there is excess supply in the labour market. In this situation, competition would fail to lower the wage rates and clear the labour market. For surveys of the nutritional evidence from the perspective of economics, one can look at Dasgupta and Ray (1990) and Osmani (1990). On the other hand, Osmani (1991) has observed that in the casual labour market in the rural areas, workers generally refuse to
unemployment' (Osmani 1991) or simply by the minimum wage law of the government. If both the rural and urban sector unskilled wage rates are inflexible downwards, the possibility of unemployment of unskilled labour arises. The unskilled workers first compete for the higher-paid urban sector jobs and those who fail to get employment in the urban sector then turn to the rural sector. As the rural sector wage rate is now rigid, this sector may fail to absorb the remaining unskilled labour force if the demand for labour at the given wage rate falls short of its supply. Owing to this new feature, the production structure as depicted in section 2.2 undergoes only one change. Equation (1.1) is now replaced by the following equation.

\[ W_1^*a_{L1} + R_1a_{K1} = P_1 \]  

(1.2)

where \( W_1^* \) is the exogenously given unskilled wage rate in the rural sector. However, this modification makes the production structure a decomposable one. Factor prices are determined from the price system alone. \( R_1 \) and \( R_2 \) are obtained from equations (1.2) and (3.1), respectively as \( W_1^* \) and \( W_3^* \) are given. Given \( R_2 \), \( W_S \) is now solved from (2).

Determination of the three factor prices can be illustrated diagrammatically in terms of figures 5 and 6. \( W_1^* \) is now the institutionally given unskilled wage rate in the rural sector. Given \( W_3^* \), the return to capital of type 2, \( R_2 \), is directly read from the \( \pi_3 = 0 \) curve. Then, the skilled wage rate, \( W_S \), can be obtained from the \( \pi_2 = 0 \) curve. Finally, the return to capital of type 1 is found from the \( \pi_1 = 0 \) curve as \( W_1^* \) is given. As \( W_1^* \) is rigid, the unskilled labour market may not be cleared. The level of unemployment of unskilled labour is shown by \( gq \) in figure 6. The factor prices remain undisturbed despite changes in factor endowments.

\[ \text{(Insert figures 5 and 6 here)} \]

According to our definition, the average wage rate for unskilled labour is now given by

\[ W_A = \left[ \frac{W_1^*a_{L1}X_1 + W_3^*a_{L3}X_3}{a_{L1}X_1 + a_{L3}X_3} \right] \]  

(11)

Now if the endowment of skilled labour rises and that of unskilled labour falls owing to skill formation, sector 2 will expand, as skilled labour is specific to that sector. It draws more capital of type 2 from sector 3, forcing the latter to contract. Sector 1 remains undisturbed. Aggregate employment of unskilled labour falls as sector 3 contracts. Also the level of unemployment of unskilled labour may increase\(^\text{10}\) if the decrease in total employment of unskilled labour is greater than the reduction in the endowment of unskilled labour. As the higher (lower) wage-paying sector 3 (sector 1) contracts (remains unchanged), the former (latter) would now absorb a lower (higher) proportion of the contracted unskilled labour force, undercut other workers, even if they are unemployed, for fear that this would lower wages for everybody at present and in the future. In other words, they may prefer to remain unemployed in the hope that in the next period they would find employment at the prevailing high wage rates. Thus, the wage rates would not fall even when there is open unemployment in the labour market. Similar evidence is also found in Rudra (1981) and Dreze and Mukherjee (1989).

\(^{10}\) The condition under which the level of unemployment of unskilled labour rises owing to skill formation has been derived in appendix IV.
leading to a decrease\textsuperscript{11} in the average unskilled wage, $W_A$. Thus, the skilled-unskilled wage inequality unequivocally worsens even if the skilled wage rate does not increase directly. This establishes the following proposition.

**PROPOSITION 3:** In the presence of imperfections in both the rural and urban markets for unskilled labour the skilled-unskilled wage inequality unambiguously worsens due to skill formation. The level of unemployment of unskilled labour may increase as a consequence.

### III. CONCLUDING REMARKS

The paper analyzes how an increase in the proportion of skilled workers to unskilled workers affects the skilled-unskilled wage inequality in three types of small open economy models, where the numbers of the distorted labour markets for unskilled workers are 0, 1 and 2. A few variants of the H-O-S model have been considered for the purpose of the analysis, which are applicable to many Latin American countries and India. The conventional wisdom is that any attempt of skill formation must be inequality-reducing as it increases the endowment of skilled labour by lowering that of unskilled labour. The analysis of the present paper has shown that there is little substance in such conventional wisdom as the impact of skill formation on wage inequality crucially depends upon the technological specifications and nature of markets for unskilled labour of the economy. In the extreme case, when all the markets for unskilled labour are distorted skill formation not only worsens the wage inequality but may also raise the level of unemployment for unskilled labour. The results of the paper point out that with the increase in the number of distorted unskilled labour markets, the effects of skill formation on the skilled-unskilled wage inequality become increasingly undesirable. In other words, the paper shows that the parameter region in which the increase of the proportion of skilled workers improves the wage inequality is larger in the economy with less distorted labour markets. These results are significant, especially in the context where the empirical literature on skilled-unskilled wage inequality has pointed out different liberalized trade policies and skill-biased technological changes as the underlying factors responsible for deteriorating wage inequality in the Latin American countries in the liberalized trade and investment regime and has gone into prescribing policies like diffusion of educational facilities among unskilled workers to mitigate the problem. The analysis of the paper clearly indicates that institutional reform programs for the removal of labour market distortions\textsuperscript{12} and skill improvement measures must go hand in hand to improve the skilled-unskilled wage inequality. Any attempt of skill formation unless accompanied by necessary labour market reform programs might produce counterproductive result on the wage inequality.

\textsuperscript{11} It has been proved mathematically in appendix III.

\textsuperscript{12} For the removal of labour market distortions, the government of a developing economy, may undertake measures so as to make the labour markets more competitive by curbing the trade union power in determining the unionized wages, which can be achieved by partial or complete ban on resorting to strikes by the trade unions, reformation of employment security laws, etc.
References:


Appendix I:

Totally differentiating equations (1), (2), (3) and (8) and applying envelope conditions we get,

\[ \theta_{L_1} \dot{W} + \theta_{K_1} \dot{R}_1 = 0 \]  
(A.1)

\[ \theta_{S_2} \dot{W}_S + \theta_{K_2} \dot{R}_2 = 0 \]  
(A.2)

\[ \theta_{L_3} \dot{W} + \theta_{K_3} \dot{R}_3 = 0 \]  
(A.3)

and

\[ -A \dot{W} + B \dot{R}_1 + C \dot{R}_2 - D \dot{W}_S = [ \dot{L} + (\lambda_{L_3} \lambda_{K_2} / \lambda_{K_3}) (S'/E) \dot{E} ] \]  
(A.4)

where \( A = [\lambda_{L_1} \sigma_1 + \lambda_{L_3} \sigma_3]; B = (\lambda_{L_1} \sigma_1); C = [\lambda_{L_1} \sigma_3 + (\lambda_{L_3} \lambda_{K_2} / \lambda_{K_3}) \sigma_3]; D = (\lambda_{L_3} \lambda_{K_2} / \lambda_{K_3}), \lambda_{K_3} > 0. \)

Solving equations (A.1)–(A.4) by Cramer’s rule one obtains the following.

\[ \dot{W} = (\theta_{K_1} / \Delta) \theta_{S_2} \theta_{K_3} \]  
\[ \dot{W}_S = (\theta_{K_1} / \Delta) \theta_{S_2} \theta_{L_3} \]  
\[ \dot{L} = - (S'/E / L) \dot{E} \]  
(A.6)

Using (A.5) and (A.6) one can write

\[ (\dot{W}_S - \dot{W}) = (\theta_{K_1} / \Delta) \theta_{S_2} \theta_{K_3} \]  
\[ (\dot{L} + (\lambda_{L_3} \lambda_{K_2} / \lambda_{K_3}) (S'/E) \dot{E} ] \]  
(A.7)

As \( \Delta < 0, \) from (A.7) it follows that

\[ (\dot{W}_S - \dot{W}) > 0 \] when \( \dot{E} > 0 \) if (i) \( \theta_{S_2} > \theta_{K_3}; \) and, (ii) \( (\lambda_{L_3} \lambda_{K_2} / \lambda_{K_3}) < (S/L) \)

or if (i) \( \theta_{S_2} < \theta_{K_3}; \) and, (ii) \( (\lambda_{L_3} \lambda_{K_2} / \lambda_{K_3}) > (S/L) \)

(A.8)

Appendix II:

After totally differentiating equations (8) and (1.1) and noting that \( S = S(E) \) and \( dK_1 = dK_2 = 0, \) one can obtain the following expressions.

\[ -\lambda_{H_1} \dot{W}_1 + \lambda_{H_1} \dot{R}_1 = (\lambda_{H_1} \lambda_{K_2} / \lambda_{K_3}) (S'/E) \dot{E} + \dot{L} \]  
(A.9)

\[ \theta_{K_1} \dot{W}_1 + \theta_{K_1} \dot{R}_1 = 0 \]  
(A.10)

where \( \sigma_1 \) is the elasticity of substitution between capital of type 1 and unskilled labour in sector 1.
Solving (A.9) and (A.10) by Cramer’s rule we get

$$\hat{W}_1 = (\theta_{K1}/\Delta^*).G$$

and, $$\hat{R}_1 = - (\theta_{L1}/\Delta^*).G$$ \hspace{1cm} (A.11)

where $$\Delta^* = -\lambda_{L1}.\sigma_1 < 0$$; and, $$G = ([\lambda_{A1} \lambda_{K2}/\lambda_{K3}] \cdot (S'.E/S). \hat{E} + \hat{L})$$ \hspace{1cm} (A.12)

Equation (10) can be rewritten as

$$W_A = W_1.\lambda_{L1} + W_3*.\lambda_{L3} = W_3* - \lambda_{A1}.(W_3* - W_1)$$ \hspace{1cm} (A.13)

where $$\lambda_{Li}$$ is the proportion of unskilled workers employed in the $$i^{th}$$ sector, $$i = 1, 3$$; and since unskilled labour is fully employed in sectors 1 and 3, we have $$(\lambda_{A1} + \lambda_{A3} = 1)$$.

Total differentiation of (A.13) yields

$$dW_A = (a_{L1}.X_1/L^2).(W_3* - W_1).dL - (1/L).[(W_3* - W_1).dX_1 + X_1.dla_{L1} - a_{L1}.X_1.dW_1]$$

or, $$W_A.\hat{W}_A = \lambda_{A1}.[(W_3* - W_1).\hat{L} - (W_3* - W_1)(\hat{a}_{L1} + \hat{X}_1)] + W_1.\hat{W}_1$$

$$= \lambda_{A1}.[(W_3* - W_1).\hat{L} - (W_3* - W_1)(\hat{a}_{L1} - \hat{a}_{K1})] + W_1.\hat{W}_1$$

(from (5) we note that $$\hat{X}_1 = -\hat{a}_{K1}$$ as $$K_1$$ does not change.)

Substituting the value of $$\hat{L}$$ from (A.6) and noting that $$(\hat{a}_{K1} - \hat{a}_{L1}) = \sigma_1.(\hat{W}_1 - \hat{R}_1)$$, one may write

$$W_A.\hat{W}_A = \lambda_{A1}.[-(W_3* - W_1). (S'.E/L). \hat{E} + (W_3* - W_1).\sigma_1.(\hat{W}_1 - \hat{R}_1)] + W_1.\hat{W}_1 \hspace{1cm} (A.14)$$

From (A.6), (A.11) and (A.12) we may write

$$(\hat{W}_1 - \hat{R}_1) = (G / \Delta^*) \hspace{1cm} (A.15)$$

and, $$G = (S'.E/S).[(\lambda_{A1} \lambda_{K2}/\lambda_{K3}) - (S/L)]. \hat{E} \hspace{1cm} (A.16)$$

Using (A.11), (A.14) and (A.15) we get

$$W_A.\hat{W}_A = \lambda_{A1}.[-(W_3* - W_1). (S'.E/L). \hat{E} + (W_3* - W_1)(\sigma_1.G/\Delta^*) + W_1.(\theta_{K1}.G/\Delta^*)] \hspace{1cm} (A.17)$$

After substituting the values of $$\Delta^*$$ and $$G$$ from (A.12) and (A.16), (A.17) can be rewritten as the following.

$$W_A.\hat{W}_A = - \hat{E}.(S'.E).[(\hat{a}_{L1}/L).(W_3* - W_1) + (W_3* - W_1) + (W_1.\theta_{K1}/\sigma_1)].(1/S).[(\lambda_{A1} \lambda_{K2}/\lambda_{K3}) - (S/L)]$$

$$(\text{A.18})$$

So $$\hat{W}_A < 0$$ when $$\hat{E} > 0$$ if $$\{(\lambda_{A1} \lambda_{K2}/\lambda_{K3}) \geq (S/L)\}$$.

**APPENDIX III:**

Differentiating totally equations (5), (6) and (7) and noting the decomposition property of the production structure we can respectively write the following.
\( \hat{X}_1 = \hat{K}_1 = 0 \) \hspace{1cm} (A.19)

\( \hat{X}_2 = (S'.E /S). \hat{E} \) \hspace{1cm} (note that \( S = S(E) \).) \hspace{1cm} (A.20)

and, \( \lambda \lambda_k, \hat{X}_2 + \lambda \lambda_k, \hat{X}_3 = 0 \). Using (A.20) the latter expression can be rewritten as

\( \hat{X}_3 = - (\lambda \lambda_k / \lambda \lambda_k). (S'.E /S). \hat{E} \) \hspace{1cm} (A.21)

Now after totally differentiating (11) and using the decomposition property of the production structure, one gets

\[
\begin{align*}
\frac{dW_A}{dX} &= \left\{ \frac{1}{(aL_1.X_1 + aL_3.X_3)^2} \right\} \left[ (aL_1.X_1 + aL_3.X_3)(W_1* \cdot aL_1.dX_1 + W_3* \cdot aL_3.dX_3) - (W_1* \cdot aL_1.X_1 + W_3* \cdot aL_3.X_3)(aL_1.dX_1 + aL_3.dX_3) \right] \\
&= \left\{ \frac{1}{(aL_1.X_1 + aL_3.X_3)^2} \right\} \left[ (aL_1.aL_3.X_1.X_3)(W_3* - W_1*) \cdot (\hat{X}_3 - \hat{X}_1) \right]
\end{align*}
\]

After simplification this reduces to

\[
\begin{align*}
\frac{dW_A}{dX} &= \left\{ \frac{1}{(aL_1.X_1 + aL_3.X_3)^2} \right\} \left[ (aL_1.aL_3.X_1.X_3)(W_3* - W_1*) \cdot (\hat{X}_3 - \hat{X}_1) \right] \\
&= \left\{ \frac{1}{(aL_1.X_1 + aL_3.X_3)^2} \right\} \left[ (W_3* - W_1*) \cdot (\hat{X}_3 - \hat{X}_1) \right]
\end{align*}
\]

With the help of (A.19) and (A.21) it becomes

\[
\frac{dW_A}{dX} = - \left\{ \frac{1}{(aL_1.X_1 + aL_3.X_3)^2} \right\} \cdot (aL_1.aL_3.X_1.X_3)(W_3* - W_1*) \cdot (\hat{X}_3 - \hat{X}_1) \cdot (\lambda \lambda_k / \lambda \lambda_k). (S'.E /S). \hat{E}
\]

or, \( \hat{W}_A - \hat{W}_A = - \left\{ \frac{1}{(aL_1.X_1 + aL_3.X_3)^2} \right\} \cdot (aL_1.aL_3.X_1.X_3)(W_3* - W_1*) \cdot (\lambda \lambda_k / \lambda \lambda_k). (S'.E /S). \hat{E} \) \hspace{1cm} (A.22)

\(< 0 \) when \( \hat{E} > 0 \).

**APPENDIX IV:**

The level of unemployment of unskilled labour is given by

\[
L_4 = L - aL_1.X_1 - aL_3.X_3 = L^* - (aL_1.X_1 - aL_3.X_3) \quad (\text{note that } L = L^* - S(E)) \hspace{1cm} (A.23)
\]

Totally differentiating (A.23) one gets

\[
\frac{dL_4}{dX} = - S'.E. \hat{E} - aL_1.X_1. \hat{X}_3 \quad (\text{note that due to decomposition property of the production structure, } a_{ij} \text{s do not change and that } \hat{X}_1 = 0 \text{ (from (A.19)).})
\]

After using (A.21) the above expression becomes

\[
\frac{dL_4}{dX} = - S'.E. \hat{E} + aL_1.X_1. (\lambda \lambda_k / \lambda \lambda_k). (S'.E /S). \hat{E}
\]

\[
= S'.E. \hat{E} \cdot ([aL_3.X_3/S].(\lambda \lambda_k / \lambda \lambda_k) - 1] \hspace{1cm} (A.24)
\]

So, from (A.24) it follows that

\( \hat{L}_U > 0 \) \text{ when } \( \hat{E} > 0 \) \text{ if and only if } (aL_3.X_3/S).(\lambda \lambda_k / \lambda \lambda_k) > 1. \)
Figure 1: Determination of input prices in the indecomposable production structure with perfect unskilled labour market.

Figure 2: Determination of the unskilled wage rate in the perfect unskilled labour market case.
Figure 3: Determination of input prices in the indecomposable production structure with imperfect urban unskilled labour market.

Figure 4: Determination of the unskilled wage rate with imperfect urban unskilled labour market.
\[ \pi_1 = 0 \]
\[ \pi_2 = 0 \]
\[ \pi_3 = 0 \]

**Figure 5:** Determination of input prices in the indecomposable production structure with both urban and rural imperfect unskilled labour markets.

**Figure 6:** Determination of the level of unemployment of unskilled labour when both unskilled labour markets are imperfect.