Divide and Conquer*

Zhijun Chen
School of Economics, Zhejiang University, China P.R, and
GREMAQ, Toulouse University of Social Science
Email: chenzj1219@hotmail.com, chenzj1219@vip. sina.com,
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Abstract

Tournaments are well known to be vulnerable to collusion as shown by the impossibility theorem in Ishiguro (2004), which asserts that efficient effort levels are impossible to be implemented through a collusion-proof contract. However, we argue that this impossibility is a product of simple mechanisms that prevail in collusion-proof mechanism design. In this paper, we explore more sophisticated mechanisms with discrimination and asymmetric information to prevent collusion, outlining the principle of "divide and conquer". As a result, we establish a possibility theorem of implementing efficient effort levels, and thus break down the impossibility theorem in Ishiguro (2004).

Keyword: Collusion, Discrimination, Moral Hazard, Tournament Model
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Zhijun Chen

1. Introduction

Rank-order tournaments prevail in organizations where the performance of agents is hard to verify, such as R&D sectors and bureaucratic institutions. They are proven to be effective in resolving the moral hazard of agents due to non-observable actions, as well as the credibility of the principal due to non-verifiable performance[1].

However, it is also well known that tournaments are vulnerable to collusion[2]. By shirking cooperatively rather than working individually, agents can maintain their rank orders, and thus save on their effort costs. Indeed, the phenomenon of collusion in tournaments is quite prevalent in bureaucratic environments, poring a permanent threat to efficiency and bringing an austere challenge, both in practice and theory, to politicians and scholars.

Recent research on mechanism design under collusion has made much progress, while little attention has been paid to tournaments. Ishiguro (2004) is the first attempt to resolve this problem. He introduces discrimination policy in a collusion-proof contract and shows that only asymmetric effort levels, in which the favored agent works and disfavored shirks, can be implemented. Furthermore, the paper presents an impossibility theorem, which asserts that efficient effort levels are impossible to be implemented in a collusion-proof contract.

We argue that this impossibility is the result of too simple mechanisms used in collusion-proof mechanism design. In this paper, we explore more sophisticated mechanisms and establish a possibility theorem of preventing collusion in tournaments. The sophisticated mechanisms are based on the principle of "divide and conquer" which asserts that to prevent collusion, the principal should create conflicts between the agents and thus undermine the coalition. The implication of this principle comes from the old wisdom of "divide and conquer" in the history of politics, which plays a key role in preventing collusion of cliques in bureaucracy and is thus preferred by the monarch.

In this paper, we introduce two kinds of instruments for preventing collusion: discrimination and manipulation of information, both are quite commonly used in organizations.

Discrimination, in which people having the same qualifications are treated differently, is an effective measure to bring on conflicts. Being discriminated in organizations, agents have different status quo utility levels when participating in the coalition and thus will claim different stakes of collusion. This causes conflicts among agents. By re-allocating stakes of collusion inside the coalition, these conflicts can be reconciled at the cost of shrinking the stakes of collusion if side transfers entail

[1] See Green and Stokey (1983), Lazear and Rosen (1981), Nalebuff and Stiglitz (1983) about the basic ideas of tournament model.
[2] For instance, Mookherjee (1984) has pointed out this vulnerability of collusion in tournament.

transaction costs. This leads to a loss of efficiency in the coalition and thus helps the principal in preventing collusion.

As the first step, we introduce an open discrimination mechanism into the tournament, where agents' identities of being favored or not are common knowledge. Under the perfect collusion assumption, we show that the open discrimination mechanism is not sufficient to prevent collusion and thus efficient effort levels cannot be implemented. This result coincides with Ishiguro (2004)'s impossibility theorem.

The intuition of this impossibility appears quite simple. The perfect collusion assumption implies that a side contract would be enforceable and side transfers would be costless, and thus reallocation of the stakes would be costless as well. As a result, an open discrimination mechanism creates no loss of efficiency under perfect collusion and thus cannot prevent collusion.

To break down this impossibility result, a more powerful mechanism must be explored, which involves in another important instrument of preventing collusion: manipulation of information.

Asymmetric information in organizations causes distortions of efficiency. As a established result, it can be utilized to undermine the coalition. By controlling the revelation of information, the principal can introduce asymmetric information between agents, and thus bring on conflicts in the coalition. The trade-off between the rent extraction and the distortion of efficiency leads to the inefficient outcome of the coalition, and this can improve the principal's welfare.

Based on this principle, we introduce a hidden discrimination mechanism into the tournament, in which agents' identities are hidden information known only by the principal. By revealing this piece of information only to one agent, which we called the informed agent, the principal brings asymmetric information into the coalition. The informed agent, who now owns private information on identities, wants to claim the information rents for truth-telling. This causes a trade-off between rent-extraction and distortion of efficiency, which shrinks the stakes of collusion and thus leads to inefficiency of the coalition. As a result, it is possible to implement efficient effort levels, as shown in proposition 4, which breaks down the impossibility theorem by Ishiguro (2004).

The possibility theorem, which hinges on the principle of "divide and conquer", induces us to review the literature in collusion-proof mechanism design, which is pioneered by Tirole (1986), and developed by Laffont & Martimort (1997, 2000) and Faure-Grimaud, Laffont & Martimort (2003). Many of the key contributions to this literature are based on the Principle of Collusion-Proofness, which asserts that, for any initial contract which is not collusion-proof, the principal can replicate the same outcome by offering some collusion-proof contract. Under the collusion-proof mechanism, no agent can benefit from joining the coalition, and thus no side contract arises in equilibrium. As a result, without loss of generality, the principal need only concentrate on designing a collusion-proof mechanism. In general, the principle holds under the circumstances where the principal can benefit from preventing collusion, which are the cases being looked at in this paper.

In spite of this powerful principle, little attention has been paid to further developments of new instruments and mechanisms, especially in the arena of moral hazard. A brief investigation of the literature shows that the existing mechanisms for collusion-proofness are quite simple. Under these mechanisms, the coalition is viewed as an entity or black box, and its efficient allocation is attained through an enforceable side contract. By the Principle of Collusion-Proofness, the principal can design a grand contract to replicate this efficient outcome for the coalition, leaving the coalition sufficient stakes of collusion for the sake of collusion-proofness. This incurs huge costs of preventing collusion if the stakes of collusion are very large, and leads to inefficient outcomes, as illustrated by the impossibility result in the tournament.

Therefore, more sophisticated mechanisms, such as hidden discrimination mechanism based on the principle of divide and conquer, have appeal to be developed, and this motivates the further research in future.

The rest of this paper is organized as follows. In section 2, a typical story of a bureaucracy is illustrated to highlight the basic idea of "divide and conquer". In section 3, the basic framework of the paper is presented. We analyze the case of the simple mechanism without discrimination as a bench-mark in section 4, and then introduce the open discrimination mechanism and prove the impossibility result in section 5. As our key contribution, in section 6, we introduce the hidden discrimination mechanism, and prove the possibility theorem. Finally, concluding remarks are drawn in section 7.

2. Hidden discrimination Mechanism: A Story

In this section, we illustrate the theory through a story drawn from bureaucratic environments. There are two offices in a bureaucratic department, and each has one dean (principal) and two employees (agents). The story takes place in office A. Dean A (Mr. Chen) assigns two identical and independent tasks to the agents respectively. Good performance will be achieved with higher probability if agents work hard, and it helps the dean to be promoted to the level of minister of the department. However, the dean cannot observe the agents' effort levels. In order to resolve moral hazard, he creates a position of deputy dean for better-performing agent. As a result, one and only one agent will be promoted at the end of the game on the basis of agents' relative performance.

However, agents have incentives to collude under this tournament mechanism. By shirking cooperatively rather than working hard, they maintain their relative performance, and thus benefit by saving as their effort costs. To sustain the collusion, the agents must seek a benevolent third party to design and enforce a side contract. To this end, they find that Dean B (Mr. Zhang) is the best candidate of the "benevolent" third party. As a competitor of Dean A for the position of minister, Dean B has the incentive to help the agents form a coalition, and thus benefits from undermining office A. As a result, Dean B is delighted to design a side contract, which is aimed at maximizing the total welfare of the agents. The side contract has the form of mutual insurance: both agents are assigned to shirk and whoever wins the high position must pay side payments to his losing peer.

While expecting the ongoing collusion between agents, the principal (Mr. Chen) will design a sophisticated mechanism to prevent collusion. From his vast experience in bureaucracies, Mr. Chen is familiar with the notion of “divide and conquer”, and employs a hidden discrimination mechanism.

In this department, it is common knowledge that agent Mr. Li is Mr. Chen’s brother in law (see structure of department in Fig 1 below). At the beginning, Mr. Chen declares the rules of game as follows: by turning a prepared roulette, he will select a favored agent who is granted a higher chance of promotion; the realization of the random choice will be kept as a secret. However, Mr. Li can acquire this piece of information from his sister, Mr. Chen’s wife, and thus become an informed agent. Indeed, Mr. Chen is motivated to reveal this information only to Mr. Li, since he can introduce asymmetric information between agents in this way and thus bring conflicts into the coalition. The structure of the department is illustrated in Figure 1.

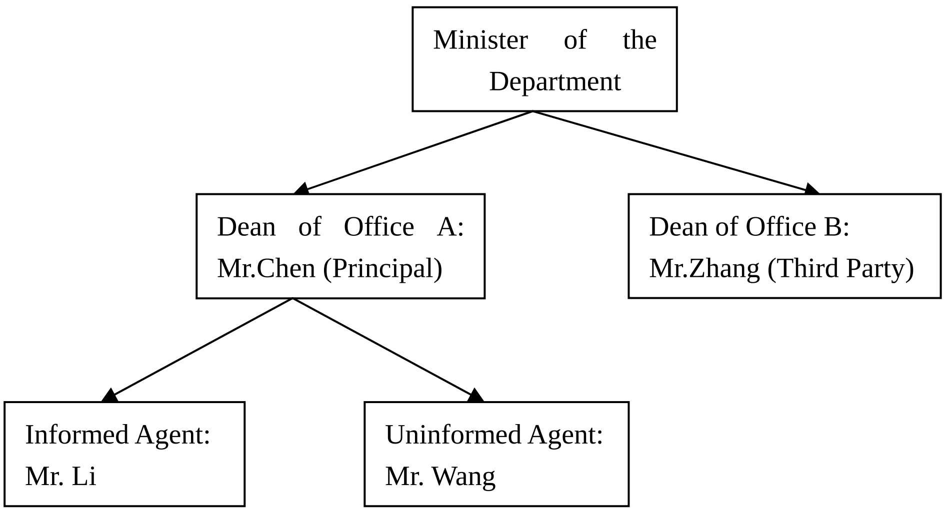

Figure 1 Structure of the Department

Due to discrimination, the lucky agent who happens to be favored by chance has a higher status quo utility level than his peer, the unlucky one, and thus claims more stakes of collusion. As a result, the informed agent, Mr. Li, if he is unlucky, has incentives to mimic the lucky type. To prevent his misreporting, information rents must be given to Mr. Li, which brings a trade-off between the rents extraction and the distortion of efficiency in the coalition. This trade-off shrinks the stakes of collusion and incurs a loss of efficiency, and thus makes it possible to implement efficient effort levels. In this way, Mr. Chen beats his rival, Mr. Zhang.

3. Basic Model

We now present the basic model. A principal employs two agents to complete two identical and independent tasks. Agent i, $i=1,2$, has two effort levels, low (shirk) and high (work), as denoted by $e^i \in \{0,1\}$, which cannot be observed by others. The output of agent i is observable but non-verifiable, with two values, good or bad, as denoted by $q_i \in \{\overline{q}, \underline{q}\}$, with $\overline{q} > \underline{q}$. The probability of good performance is $\pi(e^i)=\pi_1$ (resp. $\pi(e^i)=\pi_0$) when the agent works (resp. shirks), with $1>\pi_1>\pi_0>0$. Let $\Delta\pi = \pi_1 - \pi_0$.

The principal will devise a tournament to resolve the agents' moral hazard, due to the non-verifiability of performance. He sets two positions, high and low, with prize T and 0 respectively, and then assigns the positions to agents based on their relative performance.

The principal, who benefits only from agents' good performance, is risk-neutral with his utility function expressed by:

$$V(e)=[\pi(e^i)+\pi(e^j)]R-T,$$

where R is the principal's revenue from agents' good performance.

Agents are also risk-neutral and protected by limited liability. The utility function of agent i is expressed as follows:

$$U_i(e)=p_i(e)T-\psi(e^i),$$

where $\psi(e^i)$ is the disutility of effort e^i, with $\psi(1)=\psi$, $\psi(0)=0$ and $\psi>0$, and where $p_i(e)$ represents the winning probability of agent i, given the effort pair $e=(e^i,e^j)$. For simplicity, we employ the following notation for different effort pairs:

$$\underline{e}=(0,0),\quad \breve{e}=(1,0),\quad \tilde{e}=(0,1),\quad \overline{e}=(1,1).$$

That is, $\breve{e}$ represents the effort pair that agent i works and his peer shirks, and *vice versa* for $\tilde{e}$.

4. Benchmark: Simple Mechanism

4.a Contract without Collusion

In the absence of collusion, if the prize of the high position is sufficiently high, then the efficient effort pair $\overline{e}=(1,1)$ can be implemented through the following simple symmetric mechanism: if agent i has better performance than agent j, that is, $q_i > q_j$, then agent i obtains the high position, and his peer, agent j gets the low position; if both agents achieve the same performance, that is, $q_i = q_j$, then each receives the high position with probability $\frac{1}{2}$. Given this fair promotion rule, agent i's probability of winning is

$$p_i(e^i, e^j) = \Pr\{q_i > q_j \mid e\} + \frac{1}{2}\Pr\{q_i = q_j = \overline{q} \mid e\} + \frac{1}{2}\Pr\{q_i = q_j = \underline{q} \mid e\}$$
$$= \pi(e^i)(1-\pi(e^j)) + \frac{1}{2}\pi(e^i)\pi(e^j) + \frac{1}{2}(1-\pi(e^i))(1-\pi(e^j)) \quad .$$

For the efficient effort pair $\overline{e} = (1,1)$ to be implemented at the equilibrium, each agent must benefit from working rather than shirking. Under this mechanism, if agent i works rather than shirks, he can obtain utility $\frac{1}{2}\Delta\pi T - \psi$, which equals his expected prize of winning given his peer works, $\frac{1}{2}\Delta\pi T$, minus his effort cost ψ. This leads to the following proposition:

[Proposition 1]: In the absence of collusion, the efficient effort pair $\overline{e}$ can be implemented by the following optimal contract:

$$T = \frac{2\psi}{\Delta\pi}.$$

[Proof]: The proof is obvious and is thus omitted.

Denote T as the incentive power of the contract, which is the spread of the prizes of the two positions. Proposition 1 asserts that, to implement the high effort levels, the principal must provide high enough incentive power to meet agents' incentive compatibility conditions.

4.b Collusion under Simple Mechanism

Unfortunately, this contract is vulnerable to collusion. If agents collude on the effort pair $\underline{e}$, they can save their effort costs 2ψ, while keeping their relative performance or rank order. The existence of stakes of collusion, which equal to 2ψ, thus provides incentives for collusion.

To fix ideas, we will consider the case of perfect collusion. Assume a benevolent third party, who aims to maximize total welfare of agents, and can design and enforce a side contract. The side contract has the following form: whoever wins the prize must pay a side payment s to his losing peer. Under this side contract, the ex post prizes of the high and low position become $T-s$ and s respectively, and agent i's expected utility can be then expressed as $\hat{U}_i(e)=p_i(e)[T-s]+[1-p_i(e)]s-\psi(e^i)$, where $\hat{U}_i(e)$ is the utility level of agent i in the coalition with effort pair e.

Given any incentive feasible grand contract $T\geq\frac{2\psi}{\Delta\pi}$, the side payment $s=\frac{1}{2}T$ ensures the utility level of the agent $\hat{U}_i(e)=\frac{1}{2}T-\psi(e^i)$, which leads to the agents' optimal effort pair $(e^i,e^j)=\underline{e}$. Both agents are fully insured under collusion, and thus have no incentives to work. This immediately results in the following proposition:

[Proposition 2]: Under the simple mechanism, only low effort levels can be implemented under collusion.

The implication is quite clear. Under the simple mechanism, agents are symmetric in the grand contract and thus have the same status quo utility when participating in the coalition. The coalition can therefore be sustained through a simple anonymous side contract, which offers full insurance for agents and thus provides no incentives for working.

5. Sophisticated Mechanism with Open Discrimination

The impossibility result in proposition 2 appeals to us to introduce new principles and explore more sophisticated mechanism for preventing collusion. While rethinking the methodology, it appears obvious that the most effective way of preventing collusion is to undermine the coalition directly through some exogenous force such as law. For instance, the principal may employ a spy to look for evidence of collusion, and then punish the collusive agents if the spy delivers hard evidence. However, this is mission-impossible in general, due to the non-verifiability of the evidence and some other difficulties in law.

For this reason, the mechanism designer must seek out endogenous conflicts, and utilize it to break down the coalition. This is possible if he can design more sophisticated mechanisms and create conflicts in the coalition, which brings on the principle of "divide and conquer".

Discrimination–the act of treating people with the same qualifications differently in either an explicit or implicit way–is one way to bring conflicts. Being treated differently, agents have different status quo utility levels while participating in the coalition. The favored agent, who has a higher status, will claim more stakes of collusion due to his higher participation costs. This creates a conflict between the agents. To sustain the coalition, the stakes of collusion must be redistributed through a side transfer, and this will shrink the virtual stakes of collusion if the side transfer entails transaction costs.

Discrimination in organizations can be classified into two kinds, open and hidden discrimination. In open discrimination, the identities of agents, that is, who is favored and who is not, are common knowledge, while in hidden discrimination, they are hidden information. We concentrate on the open discrimination mechanism in this section, and will proceed to hidden discrimination mechanism in the next section.

5.a Open Discrimination Mechanism

Discrimination in promotion opportunity is quite a common act in bureaucracies. The favored agent has priority to get to the high position even if his performance is not better than his peer's. The open discrimination rule in tournaments can be illustrated as follows:

If agent i has better performance than his peer, say $q_i > q_j$, then agent i gets the high position, and his peer, agent j, gets the low position, and *vice versa.* If both agents perform poorly, that is, if $q_i = q_j = \underline{q}$, then each receives the high position with probability $\frac{1}{2}$. However, when both have a good performance, that is, when $q_i = q_j = \overline{q}$, then the tie is broken through a biased lottery, which gifts probability $v > \frac{1}{2}$ of winning to the favored agent, probability $1 - v < \frac{1}{2}$ of winning to the disfavored agent. We assume $v < 1$ to guarantee that the lottery is fully stochastic.

At the beginning of the game, the principal tosses an unbiased coin to select the favored agent and declares the outcome to both agents. The identities of agents are thus common knowledge in this mechanism.

Denote the lucky agent, who is selected to be favored at random, as agent l, and the unlucky agent, who is selected to be disfavored, as agent u. Given the effort pair $e = (e^i, e^j)$, the probability of wining for the lucky and unlucky agent, denoted as p_l and p_u respectively, can be expressed as follows:

$$
\begin{aligned}
p_l(e^i, e^j) &= \Pr\{q_i > q_j \mid e\} + v\Pr\{q_i = q_j = \overline{q} \mid e\} + \frac{1}{2}\Pr\{q_i = q_j = \underline{q} \mid e\} \\
&= \pi(e^i)(1 - \pi(e^j)) + v\pi(e^i)\pi(e^j) + \frac{1}{2}(1 - \pi(e^i))(1 - \pi(e^j)) \quad ,
\end{aligned}
$$

$$p_u(e^i,e^j)=\Pr\{q_i>q_j\mid e\}+(1-v)\Pr\{q_i=q_j=\bar{q}\mid e\}+\frac{1}{2}\Pr\{q_i=q_j=\underline{q}\mid e\}$$
$$=\pi(e^i)(1-\pi(e^j))+(1-v)\pi(e^i)\pi(e^j)+\frac{1}{2}(1-\pi(e^i))(1-\pi(e^j)) \quad .$$

Under the open discrimination mechanism, the side contract designed by the third party can be non-anonymous, based on different identities. The timing of game is as follows:

[Timing of Open discrimination Mechanism]:

1. The principal designs the grand contract, which includes: the lottery for the lucky and unlucky agents, and the prize for the high position;
2. Both agents decide whether or not to accept the grand contract. The grand contract will be approved if no agent vetoes, and the game continues;
3. The principal tosses an unbiased coin to select a lucky agent, and declares the outcome in public;
4. A benevolent third party designs the non-anonymous side contract based on agents' identities. The contract contains: the coalition effort levels and the side transfers from high position to low position, for both lucky and unlucky agent respectively;
5. Both agents decide whether or not to accept the side contract. The side contract will be ratified if no agent vetoes, and will be enforced by the third party;
6. Both agents choose effort levels simultaneously, which are privately observed;
7. Each agent's output realizes. The principal announces the winner according to their relative performance and identities. Both grand transfer and side payment are enforced.

5.b Impossibility of Collusion-Proofness

The principal wants to implement efficient effort pair $\bar{e}$ under the open discrimination mechanism. The set of incentive feasible grand contracts he designs is denoted as G^{ED}. Given the grand contract, the third party can design a non-anonymous side contract, with different side payment s_l and s_u for the lucky and unlucky agent respectively. Denote $[S^{ED}(e)]$ as the coalition which assigns effort pair $e=(e^i,e^j)$ to agents, and $S^{ED}(e)$ as the set of incentive feasible side contracts for the coalition $[S^{ED}(e)]$. The optimization program of side contracting for the coalition $[S^{ED}(\underline{e})]$ is as follows:

$[S^{ED}(\underline{e})]:\ \max_{\{s_l,s_u\}}\hat{U}_l(\underline{e})+\hat{U}_u(\underline{e})$

Subject to:

$[CIR_l(\underline{e})]$: $\hat{U}_l(\underline{e}) \geq U_l(\bar{e})$

$[CIR_u(\underline{e})]$: $\hat{U}_u(\underline{e}) \geq U_u(\bar{e})$

$[CIC_l(\underline{e})]$: $\hat{U}_l(\underline{e}) \geq \hat{U}_l(\breve{e})$

$[CIC_u(\underline{e})]$: $\hat{U}_u(\underline{e}) \geq \hat{U}_u(\breve{e})$

$[CLL]$: $T \geq s_l, s_u \geq 0$

where, subscript l and u represent the lucky and unlucky type respectively. In the program, $[CIR_l(\underline{e})]$ and $[CIR_u(\underline{e})]$ are the coalition participation constraints for the lucky and unlucky agent respectively; $[CIC_l(\underline{e})]$ and $[CIC_u(\underline{e})]$ are the coalition incentive compatibility constraints for the lucky and unlucky agent respectively.

Expecting the on-going collusion game, the principal must design a collusion-proof grand contract in order to implement the efficient effort pair $\bar{e}$. However, this is impossible under the open discrimination mechanism, as claimed by the following proposition:

[Proposition 3](Impossibility of Collusion-Proofness): Under the open discrimination mechanism, it is impossible to implement efficient effort pair $\bar{e}$ in a collusion-proof contract.

[Proof]: Rewrite the program of $[S^{ED}(\underline{e})]$ as follows:

$[CIR_l(\underline{e})]$: $p_l(\underline{e})(T - s_l) + p_u(\underline{e})s_u \geq p_l(\bar{e})T - \psi$

$[CIR_u(\underline{e})]$: $p_u(\underline{e})(T - s_u) + p_l(\underline{e})s_l \geq p_u(\bar{e})T - \psi$

$[CIC_l(\underline{e})]$: $[p_l(\breve{e}) - p_l(\underline{e})][T - s_l - s_u] \leq \psi$

$[CIC_u(\underline{e})]$: $[p_u(\breve{e}) - p_u(\underline{e})][T - s_u - s_l] \leq \psi$

$[CLL]$: $0 \leq s_l, s_u \leq T$

It is easy to check that given any incentive feasible prize T, the following side contract: $s_u = p_l(\bar{e})T$, $s_l = p_u(\bar{e})T$ satisfies all the constraints above. The feasibility of coalition $[S^{ED}(\underline{e})]$ thus implies the impossibility result. Q.E.D

Further intuitions of the proposition can be drawn. Under the open discrimination mechanism, the identities of the agents, that is, who is favored or not, are common knowledge and thus contractible in the grand contract as well as in the side contract. Therefore, the third party can design a non-anonymous side contract based on agents' identities to reconcile the conflicts in the coalition due to discrimination. As incentive power T increases, the coalition participating constraints $[CIR_u(\underline{e})]$ and coalition incentive constraints $[CIC_l(\underline{e})]$ become more stringent, which implies that the unlucky type has more incentive to quit the coalition, and the lucky type has more incentive to misbehave. However, the trade-offs can be resolved by designing the side payments s_l and s_u based on agents' identities: $s_u = p_l(\overline{e})T$, $s_l = p_u(\overline{e})T$. Under this side contract, agents' utility levels are $\hat{U}_l(\underline{e}) = p_l(\overline{e})T$ for the lucky type and $\hat{U}_u(\underline{e}) = p_u(\overline{e})T$ for the unlucky type respectively, which ensure that the coalition participation constraints are upheld. Furthermore, the virtual incentive power $T - s_l - s_u$ is reduced to zero, and this ensures that the coalition incentive compatibility constraints are met. Under perfect collusion, reallocating stakes through a side transfer is costless, and entails no loss of efficiency in the coalition.

Ishiguro (2004) obtains the same result in tournaments. The discrimination mechanism he employs is quite different from ours: agents are discriminated in reward schemes rather than in promotion chances, according to their exogenous and explicit characteristics, such as sex. Under the open discrimination mechanism he introduces, Ishiguro (2004) proves that the efficient effort pair $\overline{e}$ is impossible to be implemented in a collusion-proof contract. Furthermore, he shows that, only the less efficient effort pair $\breve{e}$, which assigns the favored agent to work and disfavored agent to shirk, can be implemented if the principal reduces the incentive power of the disfavored agent to zero. In this discrimination mechanism, the disfavored agent has no incentive to work at all under the grand contract, and is thus not induced to participate in the coalition.

6. Sophisticated Mechanism with Hidden Discrimination

6.a Hidden Discrimination Mechanism

The impossibility conclusion in proposition 3 shows that the open discrimination mechanism is not powerful enough to prevent collusion. As we pointed out, under the open discrimination mechanism, the third party can design a non-anonymous side contract, based on agent's identities, to reallocate the stakes of collusion, and thus reconcile the conflicts in the coalition.

Therefore, revealing the information of agents' identities publicly is not an optimal strategy for the principal. This suggests that the principal should hide the piece of information and reveal it only to one agent, by which he can introduce asymmetric information between agents. By the Revelation Principle, the informed agent who now owns the information of agents' identities, wants to claim information rents for truth-telling to the uninformed third party. The trade-off between rent extraction and distortion of efficiency creates a loss of efficiency in the coalition, which makes it possible to implement efficient effort levels. Therefore, controlling information structure sheds additional light on the principle of "divide and conquer".

The time sequence of this hidden discrimination mechanism is as follows:

[Timing of Hidden discrimination Mechanism]:

1. Principal designs the grand contract, which specifies who the informed agent is, the lottery for the lucky and unlucky agents, and the prize for the high position;
2. Both agents decide whether or not to accept the grand contract. The grand contract is approved if no agent vetoes, and the game continues;
3. Principal wheels a roulette to select a lucky agent, which assigns probability $0<\alpha<1$ of being lucky to the informed agent and probability $1-\alpha$ of being lucky to the uninformed agent. The outcome will be revealed to only the informed agent, say, agent i;
4. A benevolent and uninformed third party designs the side contract, which specifies effort levels and non-anonymous side transfers from high position to low position;
5. Both agents decide whether or not to accept the side contract. The side contract is ratified if no agent vetoes, in which case it is enforced by the third party;
6. Both agents choose effort levels simultaneously, which is privately observed;
7. Agents' outputs realize; the principal announces the winner based on their relative performance and status. Both grand and side contract are enforced.

[Remark]: In the final stage, the principal will only announce the winner, and no information about agents' identities will be revealed. This ensures that no side contract based on the ex post information of identities can be drawn. Moreover, the assumption $v<1$ ensures that the third party cannot infer agents' identities from the final outcome.

6.b Program of Side Contracting

When designing the side contract under hidden discrimination, the third party now faces two kinds of agents: the informed agent with adverse selection and moral hazard, and the uninformed agent with moral hazard only. By the Revelation Principle, without loss of generality, the third party can devise a direct side contract, which assigns a menu of effort levels $E=\{E_l,E_u\}=\{(e_l^i,e_u^j),(e_u^i,e_l^j)\}$ and side payments $S=\{S_l,S_u\}=\{(s_l^i,s_u^j),(s_u^i,s_l^j)\}$ for the agents, where the superscripts i and j represent the informed and uninformed agent respectively, and the subscripts l and u represent the lucky and unlucky type. For instance, e_l^i represents the effort level assigned to the informed lucky agent, and s_l^i is the side payment from the informed lucky agent to his peer if he wins.

Suppose the third party wants to form the coalition $[S^{ID}(e)]$ and implement the effort pair $e=(e^i,e^j)$. For the sake of truth-telling by the informed agent, the third party must ensure that the following truth-telling constraints are satisfied:

For the lucky type:

$[TT_l^i(e)]$: $\hat{U}_l^i(e_l^i,e_u^j)\geq \hat{U}_{l,u}^i(e_u^i,e_l^j)$

$[TM_l^i(e)]$: $\hat{U}_l^i(e_l^i,e_u^j)\geq \hat{U}_{l,u}^i(\hat{e}_u^i,e_l^j)$

For the unlucky type:

$[TT_u^i(e)]$: $\hat{U}_u^i(e_u^i,e_j^j)\geq \hat{U}_{u,l}^i(e_l^i,e_u^j)$

$[TM_u^i(e)]$: $\hat{U}_u^i(e_u^i,e_l^j)\geq \hat{U}_{u,l}^i(\hat{e}_l^i,e_u^j)$

where, the superscript i represents the informed agent, and the subscript l and u represent the lucky and unlucky type respectively. Denoting $\hat{U}_{l,u}^i(e_u^i,e_l^j)$ as the utility level of the informed lucky agent when he misreports his type as unlucky, constraint $[TT_l^i(e)]$ thus guarantees his truth-telling of the lucky type; denoting $\hat{U}_{l,u}^i(\hat{e}_u^i,e_l^j)$ as the utility level of the informed lucky agent when he misreports as well as misbehaves, constraint $[TM_l^i(e)]$ thus guarantees his truth-telling and right-behaving. Similar notation is employed for the unlucky type.

Furthermore, to form the coalition $[S^{ID}(e)]$, the third party must ensure that the side contract satisfies the following participation, incentive compatibility and limited liability constraints of the coalition:

For the informed agent i:

$[CIR_l^i(e)]$: $\hat{U}_l^i(e_l^i,e_u^j)\geq U_l(\overline{e})$

$[CIR_u^i(e)]$: $\hat{U}_u^i(e_u^i,e_l^j)\geq U_u(\overline{e})$

$[CIC_l^i(e)]$: $\hat{U}_l^i(e_l^i,e_u^j)\geq \hat{U}_l^i(\hat{e}_l^i,e_u^j)$

$[CIC_u^i(e)]$: $\hat{U}_u^i(e_u^i,e_l^j)\geq \hat{U}_u^i(\hat{e}_u^i,e_l^j)$

$[CLL(e)]$: $0\leq s_l^i, s_u^i, s_l^j, s_u^j \leq T$, For the uninformed agent j:

$[CIR^j(e)]$: $(1-\alpha)\hat{U}_l^j(e_l^j, e_u^i) + \alpha\hat{U}_u^j(e_u^j, e_l^i) \geq (1-\alpha)U_l(\overline{e}) + \alpha U_u(\overline{e})$

$[CIC_l^j(e)]$: $\hat{U}_l^j(e_l^j, e_u^i) \geq \hat{U}_l^j(\hat{e}_l^j, e_u^i)$

$[CIC_u^j(e)]$: $\hat{U}_u^j(e_u^j, e_l^i) \geq \hat{U}_u^j(\hat{e}_u^j, e_l^i)$

Denoting $S^{ID}(e)$ as the set of side contracts which satisfy all these constraints, we can present the optimization program of side contracting as follows:

Program $[S^{ID}(e)]$:

$$\max_{\{(E,S)\}} \hat{U}^i(e) + \hat{U}^j(e) = \alpha\hat{U}_l^i(e_l^i, e_u^j) + (1-\alpha)\hat{U}_u^i(e_u^i, e_l^j) + (1-\alpha)\hat{U}_l^j(e_l^j, e_u^i) + \alpha\hat{U}_u^j(e_u^j, e_l^i)$$
$$= T - [\alpha\psi(e_l^i) + (1-\alpha)\psi(e_u^i) + (1-\alpha)\psi(e_l^j) + \alpha\psi(e_u^j)]$$

Subject to $(E,S) \in S^{ID}(e)$.

The agents' utility levels in the coalition can be expressed as follows:

$\hat{U}_l^i(e_l^i, e_u^j) = p_l(e_l^i, e_u^j)(T - s_l^i) + [1 - p_l(e_l^i, e_u^j)]s_u^j - \psi(e_l^i)$

$\hat{U}_u^i(e_u^i, e_l^j) = p_u(e_u^i, e_l^j)(T - s_u^i) + [1 - p_u(e_u^i, e_l^j)]s_l^j - \psi(e_u^i)$

$\hat{U}_l^j(e_l^j, e_u^i) = p_l(e_l^j, e_u^i)(T - s_l^j) + [1 - p_l(e_l^j, e_u^i)]s_u^i - \psi(e_l^j)$

$\hat{U}_u^j(e_u^j, e_l^i) = p_u(e_u^j, e_l^i)(T - s_u^j) + [1 - p_u(e_u^j, e_l^i)]s_l^i - \psi(e_u^j)$

Denoting $T_l^i = T - s_l^i - s_u^j$ and $T_u^i = T - s_u^i - s_l^j$ as the virtual incentive power of the informed lucky agent and informed unlucky agent respectively, and $\Psi(e) = \alpha\psi(e_l^i) + (1-\alpha)\psi(e_u^i) + (1-\alpha)\psi(e_l^j) + \alpha\psi(e_u^j)$ as the expected effort costs, we can rewrite the program as follows:

Program $[S^{ID}(e)]$:

$$\max_{\{E,S\}} \hat{U}^i(e) + \hat{U}^j(e) = T - \Psi(e)$$

Subject to:

$[TT_l^i(e)]$: $\hat{U}_l^i(e_l^i, e_u^j) \geq \hat{U}_u^i(e_u^i, e_l^j) + [p_l(e_u^i, e_l^j) - p_u(e_u^i, e_l^j)]T_u^i$

$[TT_u^i(e)]$: $\hat{U}_u^i(e_u^i, e_l^j) \geq \hat{U}_l^i(e_l^i, e_u^j) - [p_l(e_l^i, e_u^j) - p_u(e_l^i, e_u^j)]T_l^i$

$[TM_l^i(e)]$: $\hat{U}_l^i(e_l^i, e_u^j) \geq \hat{U}_u^i(e_u^i, e_l^j) + [p_l(\hat{e}_u^i, e_l^j) - p_u(e_u^i, e_l^j)]T_u^i + \psi(e_u^i) - \psi(\hat{e}_u^i)$

$[TM_u^i(e)]$: $\hat{U}_u^i(e_u^i, e_l^j)) \geq \hat{U}_l^i(e_l^i, e_u^j) - [p_l(e_l^i, e_u^j) - p_u(\hat{e}_l^i, e_u^j)]T_l^i + \psi(e_l^i) - \psi(\hat{e}_l^i)$

$[CIR_l^i(e)]$: $\hat{U}_l^i(e_l^i, e_u^j) \geq U_l(\overline{e})$

$[CIR_u^i(e)]$: $\hat{U}_u^i(e_u^i, e_l^j) \geq U_u(\overline{e})$

$[CIR^j(e)]$: $\alpha\hat{U}_l^i(e_l^i, e_u^j) + (1-\alpha)\hat{U}_u^i(e_u^i, e_l^j) \leq T - [(1-\alpha)U_l(\overline{e}) + \alpha U_u(\overline{e})] - \Psi(e)$

$[CIC_l^i(e)]$: $[p_l(\hat{e}_l^i, e_u^j) - p_l(e_l^i, e_u^j)]T_l^i \leq \psi(\hat{e}_l^i) - \psi(e_l^i)$

$[CIC_u^i(e)]$: $[p_u(\hat{e}_u^i, e_l^j) - p_u(e_u^i, e_l^j)]T_u^i \leq \psi(\hat{e}_u^i) - \psi(e_u^i)$

$[CIC_l^j(e)]$: $[p_l(\hat{e}_l^j, e_u^i) - p_l(e_l^j, e_u^i)]T_u^i \leq \psi(\hat{e}_l^j) - \psi(e_l^j)$

$[CIC_u^j(e)]$: $[p_u(\hat{e}_u^j, e_l^i) - p_u(e_u^j, e_l^i)]T_l^i \leq \psi(\hat{e}_u^j) - \psi(e_u^j)$

$[CLL(e)]$: $-T \leq T_l^i, T_u^i \leq T$.

6.c Possibility of implementing $\overline{e}$

We now proceed to the grand contract. To implement the efficient effort pair $\overline{e}$, the principal must ensure that all the coalitions $[S^{ID}(e)]$ ($e \neq \overline{e}$) are not feasible under the grand contract, and this can be expressed as the following collusion-proof condition:

$[CP^{ID}]$: $\bigcup_{e \neq \overline{e}} S^{ID}(e) = \phi$.

The following proposition claims that, under the hidden discrimination mechanism, the efficient effort pair can be implemented with probability α :

[Proposition 4](Possibility Theorem): Under the hidden discrimination mechanism, it is possible to implement the efficient effort pair $\overline{e}$.

[Proof]: In the program $[S^{ID}(e)]$ ($e \neq \overline{e}$), consider the truth-telling constraint of the unlucky type as follows:

$[TT_u^i(e)]$: $\hat{U}_u^i(e_u^i, e_l^j) \geq \hat{U}_l^i(e_l^i, e_u^j) - [p_l(e_l^i, e_u^j) - p_u(e_l^i, e_u^j)]T_l^i$.

Denoting $\Delta\hat{U}(e) = \hat{U}_l^i(e_l^i, e_u^j) - \hat{U}_u^i(e_u^i, e_l^j)$ as the spread of utilities in the coalition, and $\Delta p(e) = p_l(e) - p_u(e)$ as the probability premium, $[TT_u^i(e)]$ then implies the lower bound of the virtual incentive power of the lucky type:

$$T_l^i \ge \frac{\Delta\hat{U}(e)}{\Delta p(e_l^i, e_u^j)}.$$

Moreover, by combing two coalition participation constraints $[CIR_l^i(e)]$ and $[CIR^j(e)]$ yields the lower bound of the utility spread:

$$\Delta\hat{U}(e) \ge [p_l(\overline{e}) - p_u(\overline{e})]T - \frac{2\psi - \Psi(e)}{1-\alpha}.$$

These two inequalities imply the lower bound of T_l^i as the function of prize T:

$$T_l^i \ge \frac{\Delta p(\overline{e})}{\Delta p(e_l^i, e_u^j)}T - \frac{2\psi - \Psi(e)}{\Delta p(e_l^i, e_u^j)(1-\alpha)}.$$

On the other hand, the coalition limited liability constraint $[CLL(e)]$ implies the upper bound of the virtual incentive power of the lucky type:

$$T_l^i \le T$$

From the discrimination rule of the mechanism yields $\Delta p(e_l^i, e_u^j) = (2v-1)\pi(e_l^i)\pi(e_u^j)$, $\Delta p(\overline{e}) = (2v-1)\pi_1^2$ and $\Delta p(\overline{e}) > \Delta p(e_l^i, e_u^j)$, $\forall e \ne \overline{e}$, which implies that the upper bound contradicts the lower bound if and only if:

$$T \ge T^*(e_l^i, e_u^j) = \frac{2\psi - \Psi(e)}{[\Delta p(\overline{e}) - \Delta p(e_l^i, e_u^j)](1-\alpha)}$$

$$= \frac{2\psi - \Psi(e)}{(2v-1)[\pi_1^2 - \pi(e_l^i)\pi(e_u^j)](1-\alpha)}$$

Denoting $T^*(v,\alpha) = \max_{e \ne \overline{e}} T^*(e_l^i, e_u^j)$ as the maximal threshold of the incentive power, simple calculations yield:

$$T^*(v,\alpha) = \frac{(1+\alpha)\psi}{(1-\alpha)[\Delta p(\overline{e}) - \Delta p(\breve{e})]} = \frac{(1+\alpha)\psi}{(2v-1)\pi_1\Delta\pi(1-\alpha)}.$$

We thus claim that, as $T \ge T^*(v,\alpha)$, all the coalitions $[S^{ID}(e)]$ ($E_l \ne \overline{e}$) are not feasible.

Now the remaining optimal coalition is $E_l = \overline{e}, E_u = \underline{e}$, which assigns both agents to work when the informed agent reports he is lucky, and to shirk otherwise. Therefore, the efficient effort pair $\overline{e}$ can be implemented with probability α under this coalition, which is the assigned probability of the informed agent being lucky. Q.E.D

Two kinds of trade-offs exist in the coalition. The first one exists in the coalition participation constraints due to discrimination, which implies that the lucky agent will claim more stakes of collusion. The spread of utilities $\Delta\hat{U}(e)$ measures this trade-off, and its lower bound is determined by the coalition participation constraints:

$$\Delta\hat{U}(e) \geq \Delta p(\overline{e})T - \frac{2\psi - \Psi(e)}{1-\alpha} = \Delta U(\overline{e}) - \frac{2\psi - \Psi(e)}{1-\alpha},$$

where $\Delta U(\overline{e}) = U_l(\overline{e}) - U_u(\overline{e}) = \Delta p(\overline{e})T$ is the discrimination rent.

The second trade-off results from the asymmetric information between agents and exists in the truth-telling constraints. Rewrite the truth-telling constraint of unlucky type as follows:

$$[TT_u^i(e)]:\ \hat{U}_u^i(e_u^i, e_l^j) \geq \hat{U}_l^i(e_l^i, e_u^j) - \Delta p(e_l^i, e_u^j)T_l^i,$$

where, the utility level of the unlucky type when he misreports, $\hat{U}_u^i(e_l^i, e_u^j)$, contains two terms: the first term, $\hat{U}_l^i(e_l^i, e_u^j)$, is the utility level of the lucky type who he wants to mimic; and the second term is the expected cost for his misreporting, which equals to the probability premium times the virtual incentive power of the lucky type. To ensure the truth-telling of the unlucky type, the cost of misreporting must be raised. This gives rise to the lower bound for the virtual incentive power, which is proportional to the spread of utility levels $\Delta\hat{U}(e)$:

$$T_l^i \geq \frac{\Delta\hat{U}(e)}{\Delta p(e_l^i, e_u^j)}.$$

On the other hand, the increased virtual incentive power T_l^i will meet its upper bound implied by the coalition limited liability constraint. Combing two bounds yields

$$\Delta p(e_l^i, e_u^j)T \geq \Delta p(\overline{e})T - \frac{2\psi - \Psi(e)}{(1-\alpha)}.$$

This relation fails if the prize T becomes large enough, since the marginal rate $\frac{\Delta p(\overline{e})}{\Delta p(e_l^i, e_u^j)}$ is greater than one for $E_l = (e_l^i, e_u^j) \neq \overline{e}$, and this leads to a break-down of the coalition. To resolve this conflict, the third party has to assign coalition $E_l = \overline{e}$, and thus distorts the efficiency of the coalition. Defining the stakes of collusion as:

$$S(e) = [\hat{U}^i(e) + \hat{U}^j(e)] - [U_l(\overline{e}) + U_u(\overline{e})] = 2\psi - \Psi(e),$$

it then shrinks to $2(1-\alpha)\psi$ under the hidden discrimination mechanism. As a result, the efficient effort levels can be implemented with probability α, and the optimal coalition at the equilibrium is $E_l = \overline{e}, E_u = \underline{e}$.

The possibility theorem sheds light on the principle of “divide and conquer”. Discrimination in the grand contract gives rise to different status quo utility levels for agents, which implies different participation costs in the coalition. As the prize increases, the spread of the utility levels between lucky and unlucky type is widened due to discrimination, and this provides the unlucky type more incentive to mimic the lucky one. As the prize becomes large enough, the effects of countervailing incentives come into play, and the truth-telling constraint of the unlucky type becomes more stringent. This causes a trade-off between the rent extraction of the lucky type and the distortion of efficiency and shrinks the stakes of collusion as a result.

The optimal value of collusion-proof probability α can be determined endogenously. The principal wants to assign to the informed agent a higher value of α in *ex ante* in order to obtain the higher probability of collusion-proofness. However, increasing α also incurs higher implementation costs $T^*(v,\alpha)$. At the equilibrium, the optimal value of α is determined by solving the principal’s optimization program, which is expressed as follows:

$$V(v) = \max_{\{\alpha\}} 2[\alpha\pi_1 + (1-\alpha)\pi_0]R - T^*(v,\alpha)$$

Employing the first order condition yields the optimal value of collusion-proof probability:

$$\alpha^*(\lambda, v) = 1 - \frac{1}{\sqrt{(2v-1)\lambda\pi_1\Delta\pi}},$$

where $\lambda = \frac{\Delta\pi R}{\psi}$ is the benefit-cost ratio of agent’s effort, which measures the average benefit-cost of implementing the high effort level.

The relations $\frac{\partial\alpha^*(\lambda,v)}{\partial\lambda} > 0$ and $\frac{\partial\alpha^*(\lambda,v)}{\partial v} > 0$ imply that the optimal collusion-proof probability increases with the benefit-cost ratio, as well as the discrimination factor v. Therefore, if $\lambda \gg 1$, that is, if high effort levels create much more benefits than costs, then the efficient effort levels can be implemented virtually under hidden the discrimination mechanism.

6. d Extension to the Mutually Observable Case

In the basic framework of section 3, we assume that the agents’ effort levels are only privately observable, which excludes the possibility of mutual monitoring. The assumption of privately observable effort brings forth further constraints of moral hazard in the coalition, which may incur the loss of efficiency. However, under the hidden discrimination mechanism, this assumption can be relaxed, as the coalition incentive compatibility constraints are irrelevant in the program of side contracting. This implies that the validity of the hidden discrimination mechanism can be extended to the case of mutually observable effort.

In the situation where we have mutual observable efforts and verifiable performance, Itoh (1993) has shown that the Collusion-Proof Revelation Principle does not hold. By permitting side contracting between agents and rewarding the coalition with collective bonus, the principal can benefit from mutual monitoring and risk sharing between agents. However, this result falls apart in tournaments with non-verifiable performance, as the principal cannot reward agents based on their joint performance.

Assume that agents can mutually observe their effort levels, and consider the hidden discrimination mechanism in section 6. The program of side contracting can be reduced by eliminating four coalition incentive compatibility constraints $[CIC_l^i(e)]$, $[CIC_u^i(e)]$ and $[CIC_l^j(e)]$, $[CIC_u^j(e)]$ as well as two other truth-telling constraints $[TM_l^i(e)]$ and $[TM_u^i(e)]$. All these constraints are irrelevant in the proof of the possibility theorem, which implies that the theorem still holds in this case:

[Proposition 5]: Assume agents can mutually observe their efforts. Under the hidden discrimination mechanism, it is possible to implement the efficient effort pair $\overline{e}$.

The result that moral hazard in the coalition is irrelevant to collusion-proofness should not be surprising. While moral hazard imposes further constraints on the coalition and brings on conflicts, the trade-off can be resolved by reducing the virtual incentive power, which entails no cost under perfect collusion. In hidden discrimination mechanism, only asymmetric information plays a key role in preventing collusion.

7. Concluding Remarks

Tackling collusion is a permanent challenge faced by politicians and scholars, and therefore deserves more attention in both practice and research. The research on collusion-proof mechanism design has made much progress after being pioneered by Tirole (1986), but still leaves many questions unanswered, as argued in Tirole (1992). A brief investigation of the literature shows that the existing mechanisms for collusion-proofness are too simple, especially under the framework of moral hazard. Therefore, more sophisticated mechanisms have appeal to be explored to tackle the issue, which constitutes the main purpose of this paper.

The main contribution of this paper is to introduce the principle of “divide and conquer" as an effective way to deter coalition formation. Discrimination and manipulation of information are introduced into a simple tournament model and are proven to be effective to prevent collusion. As a result, efficient effort levels are possible to be implemented under the hidden discrimination mechanism, which breaks down the impossibility theorem by Ishiguro (2004).

Although the principle of "divide and conquer" is an old idea to politicians and sociologists, it is relatively new to economists. Therefore, further developments of this principle and its applications in different settings are expected in future.

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