Intergenerational Earnings Mobility: Mechanism and Measurement

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April 1997

I would like to thank Professors John E. Craig, V. Joseph Hotz, Robert T. Michael, Casey Mulligan, Yoram Weiss, and Robert J. Willis for valuable comments. Financial support from the University of Chicago, the Hewlett Foundation, and the Spencer Foundation is acknowledged. All errors are my own.

I. Introduction and Major Findings

In recent years interest in intergenerational economic mobility has increased greatly (Buron, 1994; Cao, 1997; Mulligan, 1993; Navarro-Zermeno, 1993; Solon, 1992; and Zimmerman, 1992a, 1992b). New sources of panel data have become available, allowing for mobility measurement based on individuals' longer term -- rather than single year -- experience. The debates about the economic and statistical models of mobility (Becker, 1989; Goldberger, 1989; and Mulligan, 1995), as well as on the question "how equal the United States and other developed countries are?" (Becker and Tomes, 1986; Becker, 1989; Solon, 1992; and Zimmerman, 1992b) have apparently attracted many economists into the field. Some sociologists who traditionally focused on occupational mobility have also realized that income or earnings mobility maybe offer a better description of the degree to which an individual's socioeconomic status is transmitted from his parents. For, among other things, people tend to rank one another more in terms of income than in terms of occupation (Jencks, 1988).

A. Linear Transmission Equation

Increased interest in intergenerational earnings or income mobility, however, has not settled all issues in mobility studies. First, while both roles of the family and the society in determining mobility are acknowledged, very few studies have addressed the two roles empirically simultaneously. With its origin back at least to the last century (Galton, 1886), the statistical model neglects or neutralizes family effects. In doing so, it usually specifies a linear mechanism for status transmission. For example, in a typical model of intergenerational earnings mobility between fathers and sons, the transmission equation is specified as

$$y_s = \theta y_f + \epsilon, \qquad (1.1)$$

where y_s and y_f , measured away from their respective means, are the son's and father's earnings statuses; ϵ is a random disturbance with zero mean and uncorrelated with y_f ; and θ is the coefficient measuring intergenerational immobility: the effect of the father's status on the son's. Here, the transmission process is treated as a "black-box". The measurement of mobility is simply a measurement of the association between the two earnings variables, without asking why there should be any associations between them.

The economic model, which was pioneered by Gary Becker (1981) and refined in Becker and Tomes (1986), recognizes that both the family and the society play important roles in transmitting status from one generation to the next. However, it does so only theoretically. Like the statistical model, almost all empirical work based on the economic model has also followed a linear status transmission equation, assuming a constant effect of the parents' statuses on the children's across families (Behrman and Taubman, 1986; Peters, 1992). There are two drawbacks with the linear specification. First, it is hard to believe that the intergenerational relationship upholding the very notion of mobility is generally linear across families. After all, rich families may not prefer high mobility as much as poor families, for high mobility tends to move poor families up while forcing rich families down. Unless the effects of the society and the family work in exactly opposite directions, a linear transmission equation would be a mis-specification of the intergenerational relationship.

A linear transmission equation also reduces the empirical power of the economic model. One strength of the economic model over the statistical model is that the former dissects the "black-box" neglected in the latter, and potentially allows for examining the relative importance of the family to the society in determining mobility. But a linear transmission equation would make it difficult, if not impossible, to decompose observed mobility into the components due to the family and the society, as the equation makes the two models empirically indistinguishable.

B. The Errors-in-Variables (EIV) Problem

Another unsettled issue in previous mobility studies is related to measurement of individual status. Ideally, an individual's status should be measured in such a way that it indicates the individual's permanent or lifetime position on the social ladder. There exists no such an ideal measure, however, because no data set contains information on an individual's experience over his entire life cycle. In fact, except for educational attainment, all of the status measures used in previous mobility studies, such as occupation and earnings in some particular ages, are short-run variables, subject to the time of observation. But the use of short-run variables in place of permanent status would result in a classic errors-in-variables (EIV) problem, and would generally lead to biased estimates of intergenerational mobility (Becker and Tomes, 1986; Solon, 1992; and Zimmerman, 1992b).

Two major strategies have been suggested to deal with the EIV problem. The first, taking advantage of the availability of panel data, approximates individuals' permanent statuses by averaging repeated observations on their single-year status measures. Since averaging single-year measures reduces transitory noise relative to permanent status, this strategy is supposed to reduce the errors-in-variables bias. The second strategy seeks to use "instrumental variables" to get a consistent estimate of mobility, a very popular method to address general EIV problems.

Both strategies, however, have a common shortcoming: It regards an individual's permanent status as a constant stream of earnings or income over his working span. But the working span itself is a choice variable. It results from the individual's utility maximizing decisions on education and/or retirement, and is not identical across individuals. The use of the constant stream as permanent status thus ignores the impact on the status of differential working spans (Cao, 1997).

C. An Outline of the Present Paper

This paper investigates intergenerational earnings mobility of men, with two specific objectives: (i) overcoming or alleviating the errors-in-variables problem; and (ii) exploring non-linear intergenerational relationships that may be helpful in examining the relative effect of the family to society. To achieve the objectives, I use an individual's lifetime earnings as a proxy of his permanent or long-term status, and set up a model permitting imputation of an individual's lifetime earnings on the one hand, and justifying a non-linear relationship between fathers and sons in lifetime earnings on the other.

The baseline model envisions an individual's life cycle as consisting of two stages: young and adult. When young, he makes decisions on human capital investment so as to maximize his lifetime earnings. He does this based on his genetic endowments and parental investment. At the adult stage, the individual becomes altruist father of his own son. Given his lifetime earnings, he maximizes his lifetime utility by splitting the lifetime earnings between own consumption and the investment in his son. At any time, the family, which is the basic unit of analysis, may thus be considered as a group of two individuals, one at the young stage and the other at the adult stage.

The intergenerational relationship in lifetime earnings is determined not only by the genetic connection between the father and the son, as well as the investment provided by the father, but also by some non-family variables, as neither the father's nor the son's decision is made in the vacuum. Therefore the observed relationship between fathers and sons in lifetime earnings must be the product of the family and the society.

In Section II, I follow the spirit of the Becker-Tomes (1986) analysis to show how the lifetime earnings are related intergenerationally. Unlike their linear approximation, I emphasize that the lifetime earnings transmission equation is generally non-linear. This result offers an implicit guide to our empirical tour. In Section III, I derive a profile for a representative individual's intertemporal earnings as well as an index for his lifetime earnings. Central to the exercise is a simplified Ben-Porath (1967) model of lifetime human capital investment, which was a benchmark in the early development of human capital theory (Blinder and Weiss, 1976; Haley, 1973, and 1976; Heckman, 1976; and Rosen, 1976). After discussing empirical strategies to test my theory in Section IV, I report simulation results in Section V, using the data from the Panel Study of Income Dynamics (PSID). Throughout the rest of the work, I shall use "mobility" and "the elasticity of the son's lifetime earnings with respect to the father's" interchangeably. Smaller elasticity, however, implies higher mobility.

The major empirical findings are as follows. The lifetime earnings transmission equation is not linear. Using log lifetime earnings as baseline variable, a cubic equation is the best in describing the intergenerational relationship. The elasticity of the son's lifetime earnings with respect to the father's, mostly between zero and unity, is not constant across families. Specifically the relationship between the elasticity and the father's log lifetime earnings is of an inverted U shape: The elasticity is relatively small for both rich and poor families, with the maximum in the middle. Consistent with my predictions about the lifetime earnings transmission equation, the evidence not only supports a familiar result of "regression to the mean", but establishes a "self-reinforcement" mechanism for the result. That is, "regression to the mean" is stronger, the further a father's status is from the mean.

Based on the cubic transmission equation, the average elasticity, or the elasticity for an average family, is around .38. This number is greater than those based on short-run proxies *and* a linear transmission equation, which are usually around .2 (Becker and Tomes, 1986). It is also greater than those based on a linear transmission equation and at the same time with imputed lifetime earnings as permanent status (.27). This suggests that both the EIV problem and a linear transmission equation in the conventional approach contribute to the underestimation of the "average" degree to which socioeconomic status is transmitted from one generation to the next.

The analysis about the relative effects of the family and the society on earnings mobility is not conclusive. But the exercises that compare mobility patterns in the real society with those in some hypothesized worlds do indicate the importance of several identifiable non-family variables in affecting mobility.

II. Lifetime Earnings Transmission

Consider a representative family in a world of full information. Unlike a real family, the representative family is a group of two persons: the father and the son. The son inherits genetic endowments from the father, who cares about not only his own consumption but also his son's lifetime earnings.

A representative individual has a life cycle of two stages: young and adult. When young, the individual maximizes his lifetime earnings, $W[\beta_0, K_0]$, based on two personal parameters: initial human capital stock (K_0) and learning efficiency (β_0). The initial human capital stock is the "amount" of human capital embedded in the individual when he begins to make decisions on his lifetime human capital investments, while the learning efficiency determines his learning speed. The two parameters are assumed to be determined sequentially by an exogenous rule of genetic transmission and his father's decision on the human capital investment in the individual: $\beta_0 = \beta_0(I;\ m)$, and $K_0 = K_0(I;\ m)$. Here m indexes genetic endowments; and I is the father's investment in the individual's human capital. An individual is smarter, the larger m. The effects of the father's investment on the parameters are contingent on the genetic endowments. Given the endowments and the investment, the parameters maximize the individual's lifetime earnings.[1]

At the adult stage, the individual becomes altruistic father of his own son. Specifically, he maximizes his lifetime utility

$$U = U(C,\ W_s[\beta_0(I;\ m),\ K_0(I;\ m)]) = U(C,\ W_s[I;\ m]) \tag{2.1}$$

by allocating his own lifetime earnings, W_f, between his own consumption, C, and the human capital investment in his son, I, based on the son's genetic endowments. $W_s[.]$ in equation (2.1) is the son's lifetime earnings. It is assumed to be increasing and concave in the investment I, and increasing in the genetic endowments m. The average return to the investment, defined as $R \equiv \frac{W_s}{I}$, is therefore always greater than the marginal return $\frac{\partial W_s}{\partial I}$, and decreasing in I.

The individual faces a "quasi-perfect" physical capital market: He faces a common and

[1] The assumption that the parameters are not completely determined by genetic endowments needs elaborations. Basically, it is consistent with a well-known theory in the modern developmental psychology that an individual's development results from the interaction between "nature" (genetic endowments) and "nurture" (environmental forces). But this observation may cause inconsistency within the model, because the assumption implies that the parameters should not be understood as constants at the very beginning of the individual's life cycle: It takes time for environmental forces to influence the individual's development. To avoid this inconsistency, one may mandate that an individual's life cycle and hence, his young stage, does not begin until he reaches a certain age. The two-stage life cycle modeled here therefore truncates some early part of experience from an individual's real life cycle.

constant interest rate at which he can freely borrow and lend within his own lifetime earning capacity. However, he may not use the market to finance the human capital investment in his son, as human capital is poor collateral and he cannot leave any debt to the son.[2] One implication is that poor fathers are not able to invest in their sons as much as rich fathers.

Unlike many similar models, financial bequests from the father to the son are not considered here, with an assumption that the bequest decisions either are independent of or have no significant impact on the father's human capital investment in the son. There are several reasons for this assumption. First, the primary purpose of the work is to investigate intergenerational earnings -- not income -- mobility. Second, financial bequests on average are small compared to lifetime earnings.[3] Third, both propositions to be presented would still be valid without this assumption, although for different reasons. The corollary from the propositions, while sensitive to the assumption, may lead to an empirical way to demonstrate how serious the assumption is to the model.

Finding solution to the above utility maximizing problem is trivial. The optimal choices of the investment and own consumption should be such that the marginal rate of substitution between own consumption and the son's lifetime earnings, *MRS*, is equal to the marginal return to the investment. That is,

$$MRS \equiv \frac{\partial U/\partial C}{\partial U/\partial W_s} = \frac{\partial W_s}{\partial I}. \tag{2.2}$$

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[2] Economists have long argued "that human capital is poor collateral to lenders. Children can 'default' on the market debt contracted for them by working less energetically or by entering occupations with lower earnings but higher psychic income. ... Moreover, most societies are reluctant to collect from children debts that were contracted by their parents, perhaps because the minority of parents who do not care much about the welfare of their children would raise their own consumption by leaving large debts to children." (Becker and Tomes, 1986, p. S10.)

[3] According to Mulligan (1995), who used the PSID to investigate the financial transfer from parents to children, only 24% of the overall sample people have received or expect to receive positive inheritance. Of those received or expecting to receive positive inheritance, only 12% have received or expect to receive inheritance of more than $25,000. This means that, overall, only 3% of the people have received or expect to receive inheritance of more than $25,000. Further, even $25,000 is very small compared to an individual's lifetime earnings, which, according to our calculation later on, is more than one million dollars in comparable value.

The interior solution is guaranteed due to the assumption neutralizing financial bequests. Generally both the investment and own consumption are functions of the father's lifetime earnings as well as the son's genetic endowments. Denote the optimal investment as $I^* = I(W_f\,;\, \boldsymbol{m})$, where $\frac{\partial I^*}{\partial W_f} > 0$. [4] From equation (2.2), one has

$$\frac{\partial W_s}{\partial W_f} = MRS\,\frac{\partial I^*}{\partial W_f}. \tag{2.3}$$

The elasticity of the son's lifetime earnings with respect to the father's, θ, can be calculated as

$$\theta \equiv \frac{\partial lnW_s}{\partial lnW_f} = MRS\,\frac{\partial I^*}{\partial W_f}\frac{W_f}{W_s} = MRS\,\left(\frac{\partial I^*}{\partial W_f}\frac{W_f}{I^*}\right)\left(\frac{I^*}{W_s}\right) = \frac{\theta_I\,MRS}{R}, \tag{2.4}$$

where R, as defined earlier, is the average return to the father's investment; and $\theta_I = \frac{\partial I^*}{\partial W_f}\frac{W_f}{I^*}$ is the elasticity of the father's investment with respect to his lifetime earnings. Since the father values both his own consumption and his son's lifetime earnings, $\theta_I \in (0, 1)$.

The elasticity of the son's lifetime earnings with respect to his father's is positively related to the father's willingness to invest in his son (θ_I) and the marginal rate of substitution (*MRS*), but negatively related to the average return to the father's investment ®). Holding constant *MRS* and *R*, a greater θ_I implies more investment by the father in the son; holding constant θ_I and *R*, a greater *MRS* implies that the son is smarter; and, finally, holding constant θ_I and *MRS*, a greater *R* implies that the son is less smart. Consequently, equation (2.4) *seems* to claim that earnings mobility is determined only by family-related factors.

But non-family factors also have a say on earnings mobility. Examples include the valuation scheme of human capital and the "quasi-perfect" physical capital market. The former, apparently dependent on labor demand and supply, is important in determining an individual's lifetime earnings. The later not only influences the real values of individuals' lifetime earnings, but may also limit poor

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[4] If financial bequests were allowed in the model, $(\partial I^*/\partial W_f)$ would be zero for those fathers with very large lifetime earnings. Given their sons' genetic endowments, the best strategy for them would be to fix $\mathbf{I}^*$ at the level where $(\partial W_s/\partial I)$ is equal to the interest rate in the physical capital market, and then transfer to their sons financial bequests.

fathers' abilities to "optimize" human capital investment in their sons.[5]

Since θ_I, *MRS* and *R* in equation (2.4) are all equilibrium results corresponding to the individual's maximized utility, neither of them is generally independent of his lifetime earnings. This leads to the following proposition.

Proposition 1: Intergenerational lifetime earnings transmission equation, $W_s = F(W_f)$, is generally not linear or log-linear.

The proposition has an important empirical implication. Other things being equal, fathers with different lifetime earnings may have different impact on their children's earnings. Generally the elasticity of the son's lifetime earnings with respect to the father's is not constant across families. The linearity convention in previous studies of intergenerational earnings mobility is therefore not necessarily legitimate.

Proposition 1 gives no clue about how the elasticity varies across families. In fact, the lifetime earnings transmission equation may take any forms. To see this, define $D \equiv \theta_I \, MRS$ as "modified elasticity of the father's investment in the son". Find the elasticity of θ with respect to W_f from equation (2.4): $\frac{\partial \ln\theta}{\partial lnW_f} = \theta_D - \theta_R$, where θ_D and θ_R are elasticities of **D** and **R** with respect to W_f. In general both θ_D and θ_R can be of any signs, and so can $\frac{\partial \ln\theta}{\partial lnW_f}$. Formally,

Proposition 2: Intergenerational earnings mobility may not only vary across families, but also show a non-monotonic relationship with the father's lifetime earnings.

The magnitude of θ often has special interpretations. Particularly, when its absolute value is less than unity, one says that "regression to the mean" -- as opposed to "regression away from the mean" -- happens, in the sense that sons of rich families would tend to be less rich, whereas sons of poor families would tend to be less poor. Since the elasticity of the father's investment, θ_I, is between

[5] If the physical capital market could be used to finance investment in his son, a poor father would invest more in the son's human capital than he does in the "quasi-perfect" capital market.

zero and unity, and the average return to the investment, *R*, is always greater than the marginal return, which in equilibrium is equal to ***MRS***, the following corollary is therefore obvious.

Corollary: Equation (2.4) implies "regression to the mean".

That the elasticity of the father's investment θ_I is greater than zero but less than unity plays a crucial role in this corollary. If financial bequests were added to the model, this θ_I would be equal to zero for some very rich families, and so would the elasticity θ. For those rich families, the sons' lifetime earnings would be completely independent of the fathers', and the corollary would not be valid. However, the two propositions would still hold water, as the intergenerational relationship in lifetime earnings would not be continuous over all families.

On the other hand, if the marginal return to the father's investment was independent of the investment, and, simultaneously, the utility function was homothetic, θ_I would be equal to one, while *MRS* would be equal to *R*. The earnings elasticity θ would be reduced to exactly unity. Neither of the above propositions or corollary would hold. Therefore, empirical evidence on "regression to the mean" or "regression away from the mean" may demonstrate how serious these alternative assumptions would be to the model.[6]

III. Intertemporal and Lifetime Earnings

The representative individual's lifetime earnings is a key to our enterprise. While fixed at the adult stage, the earnings results from his first stage calculation.

Imagine the following scenario. Given the personal parameters, β_0 and K_0, the individual tries

[6] One might argue that the above results are valid only for representative families, which, relative to real families, ignore uncertainty and the roles of the mothers, simplify the families' fertility behavior, and when a family has more than one child, ignore differential preferences that the parents might give to different children. This argument is not true (Cao, 1997) The basic results would only be reinforced in the context of real families.

to develop a strategy for the adult stage, allocating his time between learning and earning activities, and maximizing his lifetime earnings evaluated at the beginning of the stage or the base period, 0,

$$W = \int_0^T y_t e^{-rt} dt = \int_0^T a[K_t - s_t K_t] e^{-rt} dt. \tag{3.1}$$

Here W is the individual's discounted lifetime earnings; y_t is his intertemporal earnings at period t; K_t is his intertemporal human capital stock, and $s_t \in [0, 1]$ is the fraction of his time -- normalized to be unity -- devoted to the learning activity; a is the human capital price he faces, assumed to be constant over his planning horizon $[0, T]$; and, finally, r is the rate of interest prevailing in the "quasi-perfect" capital market depicted in the last section. Note that the intertemporal earnings is defined as the difference between the potential earnings, $\boldsymbol{aK_t}$, and the foregone earnings due to the learning activity, $a(s_t K_t)$. Also human capital price is the value of one unit of human capital. Although it is constant for the individual over his life cycle, the price may vary across individuals of different cohorts, capturing some effects of the non-family variables.

The individual has a simplified Ben-Porath (1967) type of human capital production function

$$Q_t = \beta_0 (s_t K_t)^{\alpha}, \tag{3.2}$$

where Q_t is the amount of human capital produced at t, and $\alpha \in (0, 1]$ is a parameter identical for all individuals, reflecting the human commonality. The equation of motion is familiar,

$$\frac{\partial K_t}{\partial t} = Q_t - \delta K_t, \tag{3.3}$$

where δ is a positive depreciation rate of human capital. Notice that, with equation (3.3), the model implies that the intertemporal human capital stock is continuous over the adult stage.

Following Ben-Porath (1967), three phases of human capital investment are suggested by the constraint on s_t. The first corresponds to $s_t = 1$. The available human capital stock, even fully allocated to produce human capital, is not large enough to yield optimal human capital stock. In this phase the individual specializes in the learning activity, and, if the human capital depreciation rate is not too high, the available human capital stock should keep on increasing. This tendency continues until some point when the available human capital stock becomes so large that $\boldsymbol{s_t}$ is no longer equal to 1. Consequently, he switches some of his time from learning to earning. The third and last phase is $s_t = 0$, when the stock of human capital is too large to produce more human capital. Obviously, the first and last phases correspond to corner solutions, while the second to an interior one.

The optimal path of the human capital investment is derived in Appendix A. With appropriate assumptions and restrictions, the path allows us to determine the individual's intertemporal and lifetime earnings, as well as the "transition period" that is probably empirically observable. Among the results most relevant to the present paper are:

1. **The Schooling Equation**. Define the transition period, t^*, as one when the individual switches from the first phase of human capital investment to the second. Since people usually specialize in human capital production in the regular schools, earning nothing before entering the labor market, the period may be measured as regular schooling years. Mathematically, it is the solution to the following equation -- called the "schooling equation" later,

$$1 - \left(1-\frac{\alpha\delta}{r+\delta}\right)e^{(1-\alpha)\delta t^*} - \frac{\alpha\delta}{r+\delta}e^{-(r+\delta)(T-t^*)+(1-\alpha)\delta t^*}$$

$$\equiv z(t^*) = \frac{\delta K_0^{1-\alpha}}{\beta_0}. \qquad (3.4)$$

2. **The Admissibility of** α: $\alpha = n/(n+1)$, with n a positive integer. Without this restriction, one cannot find closed form expressions for the individual's intertemporal and lifetime earnings, two building blocks of the work.

3. **Intertemporal Earnings**:

$$y_t = 0 \qquad \textit{if } t \leq t^*,$$

$$= a\beta_0^{n+1}\left(\frac{\alpha}{r+\delta}\right)^n \Omega_t \equiv g\ \Omega_t \qquad \textit{otherwise}, \qquad (3.5)$$

where $n = \frac{\alpha}{1-\alpha}$, (3.5a)

$$g = a\beta_0^{n+1}\left(\frac{\alpha}{r+\delta}\right)^n, \quad (3.5b)$$

and

$$\Omega_t = \sum_{j=0}^{n} \frac{(-1)^j C_n^j}{j(r+\delta)+\delta}\left[e^{j(r+\delta)(t-T)} - e^{j(r+\delta)(t^*-T)+\delta(t^*-t)}\right]$$

$$- \sum_{j=0}^{n+1} \frac{(-1)^j C_{n+1}^j \alpha}{r+\delta}\left[e^{j(r+\delta)(t-T)} - e^{j(r+\delta)(t^*-T)+\delta(t^*-t)}\right]. \quad (3.5c)$$

The earnings has familiar features. It is zero when $t \leq t^*$, because the individual specializes in human capital production in that range. After the transition period, it becomes positive, and increasing in t -- at least initially. After reaching its maximum, the earnings may decline or fluctuate over the remaining periods, provided that the intertemporal human capital stock so behaves (Cao, 1997).

4. **Unadjusted Lifetime Earnings**:

$$W = \int_{t^*}^{T} y_t e^{-rt} dt = g\int_{t^*}^{T} \Omega_t e^{-rt} dt$$

$$= g\left[\sum_{j=0}^{n} \frac{(-1)^j C_n^j}{j(r+\delta)+\delta}\Psi_j - \sum_{j=0}^{n+1} \frac{(-1)^j C_{n+1}^j \alpha}{r+\delta}\Psi_j \right] \equiv g\ \Gamma, \quad (3.6)$$

where

$$\Gamma = \sum_{j=0}^{n} \frac{(-1)^j C_n^j}{j(r+\delta)+\delta}\Psi_j - \sum_{j=0}^{n+1} \frac{(-1)^j C_{n+1}^j \alpha}{r+\delta}\Psi_j, \quad (3.6a)$$

$$\Psi_j = T - t^* + \frac{e^{\delta(t^*-T)} - 1}{\delta} \quad \text{if j = 0 and r = 0}, \quad (3.6b)$$

and

$$\Psi_j = e^{-rt^*}\left(\frac{e^{r(t^*-T)} - e^{j(r+\delta)(t^*-T)}}{j(r+\delta)-r} + \frac{e^{(j+1)(r+\delta)(t^*-T)} - e^{j(r+\delta)(t^*-T)}}{r+\delta}\right) \text{ otherwise.} \quad (3.6c)$$

Equation (3.6) gives "unadjusted" lifetime earnings because the base period, 0, to which the earnings is discounted is generally not comparable from one individual to another. To adjust for this, one may set a common reference time, τ, and re-discount the unadjusted earnings to τ. Let the period difference between the individual's base period and the reference time τ be A_τ. The adjusted lifetime earnings may then be calculated as

$$W^* = W e^{-rA_\tau}. \qquad (3.7)$$

Notice that, given the population parameters and the ending period T, the transition period t^* is completely determined by the individual parameters (equation (3.4)), and so is the unadjusted lifetime earnings (equation (3.6)). Also, both the intertemporal and lifetime earnings are proportional to an individual-specific constant g, which builds a convenient bridge between the two earnings. For example, if one finds a way to impute the constant based on the intertemporal earnings, one should easily find the lifetime earnings.

IV. The PSID Data and Empirical Implementation

A. The PSID Data

I used intergenerational data from the Panel Study of Income Dynamics (PSID) to examine the validity of the model. A longitudinal survey, the PSID commenced in 1968 with a national probability sample of about 5,000 families. It emphasizes the dynamic aspects of economic and demographic behavior, and contains a wide range of sociological and psychological measures. The covered time span for this work is from 1968 to 1989.

Since the model concentrates on intergenerational relationships of men, and requires information on individuals' earnings, not all the families retained in the PSID were used in this work. Instead, a sub-sample of families was selected, with each family consisting of two persons, indexed as "father" and "son". To be eligible, a "father" must have been recorded as "family head" -- as opposed to "family member" -- at least once before 1972, while a "son" must not only have "father" who himself is eligible, but have also become "family head" within the covered time span. I chose the sub-sample in this way because (i) the PSID collects information on "total annual labor income", the variable containing information on an individual's intertemporal earnings, only for a family head; and (ii) for a "father" who became "family head" after 1972, there was little information about his "son". In case an selected family had several "sons", the oldest was chosen to maximize information for the family. Further, the families that had never reported positive annual labor income were dropped. Using the above criteria, I obtained 663 families.[7]

The relationship between intertemporal earnings, y_t, and total annual labor income, indexed as Y_t, should be clearly understood. Total annual labor income or annual earnings is the amount of money that an individual earns within a year. Mathematically,

$$Y_t = \int_t^{t+1} y_t dt = g\int_t^{t+1} \Omega_t dt \equiv g\ \Pi_t, \tag{4.1}$$

where

$$\Pi_t \equiv \int_t^{t+1} \Omega_t dt, \tag{4.1a}$$

and Ω_t is defined in equation (3.5c). Notice that total annual labor income Y_t is also proportional to the individual-specific constant g.

I defined the base period in the model, when an individual begins to make decisions on his own human capital investment, as equivalent to age 6, for several reasons. First, it is consistent with the fact that people usually begin formal education at age 6. One might argue that a child of 6 years old cannot

[7] The best known panel survey carried out in the United States by the Institute for Social Research at the University of Michigan (Atkinson et al., 1992), the PSID has been used by many other people (notably, Hill and Duncan, 1987; Mulligan, 1993; and Solon, 1992) to exploit intergenerational relationships. Most of these people limit their samples to the Survey Research Center (SRC) component of the PSID, because the other component -- the Survey of Economic Opportunity (SEO) component -- is believed to over-represent the low income population. A recent development on individual weights, however, makes it possible to combine the two components together (Hill, 1992).

make his own decisions. But given the nature of the model, which bars uncertainty, a 6 years old boy would make decisions just in the same way that an individual of any other age would.

The choice of 6 years old as the base period is also consistent with the assumption about an individual's personal parameters. Recall that those parameters are not predetermined, but dependent on various environmental forces in general and his father's investment in the individual in particular. It takes time for environmental forces to influence the formation of the parameters. Six years seem to be an appropriate time span for the task.[8]

Aware of potential criticisms, which will be discussed in the next section, I assumed that all individuals retire at age 65. Their ending periods (T) were therefore uniformly equal to 59. In addition, as discussed earlier, I measured their transition periods (t^*) by their regular schooling years. Their working spans were thus equal to ($59-t^*$), which may differ from one individual to another. Among other important information from the PSID used in this work was a vector of age or cohort dummies, and a vector of exogenous variables reflecting an individual's *early* family background, including early residency, early economic condition, father's and mother's education, father's early occupation, and number of siblings. A statistical summary and the definitions of the above variables are in Table 1.

B. Imputing Lifetime Earnings and Cohort Human Capital Prices

The analysis in Section III provides the base for imputing individual-specific lifetime earnings and cohort-specific human capital prices. The former will be used as proxy of an individual's permanent status, while the latter helps examine the effect of non-family factors on earnings mobility.

To impute lifetime earnings, I resorted to repeated observations in the PSID on total annual labor income Y_t, as well as the relationship between lifetime earnings and the total annual labor income.

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[8] Both initial human capital stock and learning efficiency are sensitive to the timing of the base period. But learning efficiency, which determines an individual's learning speed, is more sensitive. A relevant question is: Is an individual's learning speed about constant around 6 years old? While there is no agreement on this question, one piece of evidence from the developmental psychology seems to support a positive answer: The development of an individual's neural system reaches its full potential around age 10. By age 6, it reaches about 90 percent of the potential.

Table 1. The Sample Characteristics

	Pooled		Father		Son	
Variable Definition	Mean	S.D.	Mean	S.D.	Mean	S.D.
Race						
white	.918	.275	.927	.260	.909	.287
nonwhite	.082	.275	.073	.260	.091	.287
Religion						
protest	.592	.492	.642	.480	.549	.498
catholic	.190	.392	.178	.383	.201	.401
jewish	.084	.278	.112	.315	.060	.237
others	.134	.340	.069	.254	.191	.394
Early Residency						
rural	.239	.426	.337	.473	.151	.358
town	.438	.496	.343	.475	.522	.500
city	.324	.468	.320	.467	.327	.469
Early Economic Condition						
good	.241	.428	.128	.334	.343	.475
average	.446	.497	.378	.485	.507	.500
poor	.312	.464	.495	.500	.150	.358
Father's Education (Schooling Years)						
fed1 0 - 5	.092	.289	.145	.352	.045	.207
fed2 6 - 8	.355	.479	.590	.492	.145	.352
fed3 9 - 11	.104	.305	.071	.257	.132	.339
fed4 12	.229	.420	.097	.296	.346	.476
fed5 12 -	.221	.415	.097	.296	.332	.471
Mother's Education (Schooling Years)						
med1 0 - 5	.120	.325	.201	.401	.049	.215
med2 6 - 8	.217	.412	.367	.482	.084	.277
med3 9 - 11	.117	.321	.108	.310	.125	.331
med4 12	.375	.484	.260	.439	.477	.500
med5 12 -	.171	.377	.064	.246	.266	.442
Father's Occupation When Respondent Was Young						
focc1 professional & technical	.110	.313	.057	.232	.157	.364
focc2 manager & official	.073	.260	.053	.225	.091	.287
focc3 self-employed business	.054	.225	.077	.267	.032	.177
focc4 clerical & sales worker	.069	.253	.035	.185	.099	.299
focc5 craftsman & foreman	.222	.415	.203	.402	.238	.426
focc6 operatives	.158	.365	.161	.368	.156	.363
focc7 laborer & service worker	.071	.257	.082	.275	.061	.239

Table 1 continued

Variable Definition	Pooled Mean	Pooled S.D.	Father Mean	Father S.D.	Son Mean	Son S.D.
focc8 farmer & farm manager	.159	.365	.264	.441	.065	.247
focc9 all others	.085	.280	.067	.251	.101	.302
Number of Siblings						
sib01 0 - 1	.232	.422	.204	.403	.257	.438
sib24 2 - 4	.515	.500	.437	.496	.584	.493
sib5 5 -	.253	.435	.359	.480	.159	.366
Regular Schooling Years yrsch	12.97	2.50	12.12	2.68	13.72	2.05
Age in 1988 age88	45.75	15.46	60.24	8.22	32.85	6.19
Sample Size	1,326		663		663	
Annual Labor Income (1984 Dollars)	24,760	22,193	29,003	26,242	20,984	16,963
Number of Obs. for Income	18,591		12,562		6,689	

Note: All variables but "yrsch", "age88", "annual labor income", and sample size are dummies.

Let individual j of cohort i's unadjusted lifetime earnings be W_{ij}; and his total annual labor income at age (t+6) be Y_{ijt}. *Assume that the population parameters are known.*[9] W_{ij} may then be imputed as

$$W_{ij} = \hat{g}_{ij} \, \Gamma_{ij} = [\sum_t \frac{Y_{ijt}}{\Pi_{ijt}}]\Gamma_{ij}, \tag{4.2}$$

where the definition of Γ_{ij} is in equations (3.6a)-(3.6c), and the definition of Π_{ijt} in equation (4.1a). Note that the individual-specific constant g was imputed as an average over all observations available on total annual labor income:

[9] This assumption is in sharp contrast with most of the Ben-Porath type of studies on human capital investment (e.g., Heckman, 1976; Rosen, 1976). Those studies concentrated on the earnings profile for a representative individual, with an objective to estimate the population parameters. Since my interest is not in the representative earnings profile but in individual differences, I will not pay any attention to the estimation of the parameters.

$$\hat{g}_{ij} = \sum_t \frac{Y_{ijt}}{\Pi_{ijt}} \ . \tag{4.2a}$$

Each individual might have different number of observations. Generally the imputation was more accurate for an individual who had more observations on the income, and if the observations were more spread over an individual's life cycle.

The imputation of the cohort-specific human capital prices was based on the schooling equation (3.4), along with the following assumptions: (i) an individual's initial human capital stock is determined by a vector of exogenous variables reflecting his early family background, *X*; and (ii) the cohort-specific human capital price can be estimated by a vector of cohort dummies, *d*. The detailed imputation strategy is given in appendix B.

C. Intergenerational Earnings Transmission Equations

I measured an individual's lifetime or permanent status by the logarithm of his adjusted lifetime earnings. Following Proposition 1 in Section II, I used a set of polynomial equations to describe the process of intergenerational earnings transmission. In particular, I was interested in the following specifications:

Linear: $\ln W_s^* = \theta_0 + \theta_1 \ln W_f^*;$ (4.3a)

Quadratic: $\ln W_s^* = \theta_0 + \theta_1 \ln W_f^* + \theta_2 [\ln W_f^*]^2;$ (4.3b)

Cubic: $\ln W_s^* = \theta_0 + \theta_1 \ln W_f^* + \theta_2 [\ln W_f^*]^2 + \theta_3 [\ln W_f^*]^3;$ and (4.3c)

Quadrinomial: $\ln W_s^* = \theta_0 + \theta_1 \ln W_f^* + \theta_2 [\ln W_f^*]^2 + \theta_3 [\ln W_f^*]^3 + \theta_4 [\ln W_f^*]^4;$ (4.3d)

where W^* was adjusted lifetime earnings; subscripts *s* and *f* indicated "son" and "father", and θ_0, θ_1, θ_2, θ_3, and θ_4 were coefficients to be estimated.[10]

Which of the above specifications is the best is purely an empirical question. Specification tests will be conducted in the next section to answer the question, based on the idea that the best

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[10] In theory, one can specify the transmission process as a polynomial equation of any order. But preliminary tests show that polynomial equations with orders higher than in (4.6d) would yield no new, if not worse, results.

specification should have the highest likelihood to match data.

The specification tests, together with the signs and the magnitudes of the earnings elasticities, have important implications for my model. If the linear specification is the best, my theory would gain nothing new or testable relative to the conventional approach to the process of intergenerational status transmission. If the quadratic specification is the best, Proposition 1 would be supported, while Proposition 2, which claims non-monotonicity of the relationship between θ and $\ln W_f^*$, would not. If the cubic or the quadrinomial specification is the best, both Propositions 1 and 2 would be supported. In any event, if the absolute values of the earnings elasticities are less than unity, the corollary of "regression to the mean" would be supported, and some key assumptions in the model, especially the assumption that financial bequests either are independent of or have no significant impact on the father's human capital investment in the son, would survive.

The baseline variable in the earnings transmission equations, log **adjusted** lifetime earnings, is dependent on the father's and the son's ages at reference time τ. The earnings elasticities calculated from the transmission equations are therefore subject to the age distributions of fathers and sons in the population, or "age structure" for short. In the cubic transmission equation, for example, age structure affects earnings elasticities in two ways. First, the four coefficients in the equation are subject to both fathers' and the sons' ages at reference time τ. Second, fathers' ages affect their log adjusted lifetime earnings, a variable directly determining earnings elasticities.

One important theme in the model is that intergenerational earnings mobility is determined by both family and non-family factors. For family factors, I have enumerated genetic connections between generations and parental altruism toward children. For non-family factors, I have listed the quasi-perfect physical capital market and cohort-specific human capital prices. Age structure just defined is another non-family factor. To be sure, the age difference between a father and his son results from the individual family's fertility decisions. But the age distribution of each generation, and the age differences between two generations are beyond any individual family's control.

V. The Results

I report empirical results for six sets of population parameters, $\pi \equiv (\alpha, r, \delta)$: α = ½, 2/3, or 3/4; r = .02 or .05; and δ = .05. The following notations are used: π_1 = (½, .02, .05); π_2 = (½, .05, .05); π_3 = (2/3, .02, .05); π_4 = (2/3, .05, .05); π_5 = (3/4, .02, .05); and π_6 = (3/4, .05, .05). The choice of the depreciation rate of human capital is based on estimates from Haley (1976) and Heckman (1976), who adopted models similar to Ben-Porath (1967). As required, the α's are in the admissible set. They guarantee closed form expressions for an individual's intertemporal and lifetime earnings. Relative to the estimates by Haley (1976), Heckman (1976) and Rosen (1976), the interest rates are low. However, since the most important role of the interest rate in my model is in converting earnings at different periods of the life cycle into comparable, a low interest rate seems to be reasonable.[11]

A. Lifetime Earnings: Unadjusted versus Adjusted

Table 2, panel A summarizes the unadjusted lifetime earnings for both the fathers and sons for the selected population parameters. The lifetime earnings were not very sensitive to α. But they seemed to be very sensitive to the interest rate. For the fathers, the mean earnings were about $450,000 when r = .02 while about $150,000 when r = .05. For the sons, the corresponding mean earnings were about $800,000 and $260,000. Alternatively, the mean earnings at r = .02 were three times as large as at r = .05. Considering the way that the lifetime earnings were imputed, however, the high sensitivity of the lifetime earnings to the interest rate was deceiving. To see this, I added to the panel constant annual earnings, $\bar{Y}$, that would yield the same amount of lifetime earnings as the means. For the fathers, the annual earnings would be about $19,000 if r = .02, and about $15,500 if r = .05. For the sons, the corresponding annual earnings would be about $33,000 and $26,000, respectively. Clearly, the sensitivity of the annual earnings to the interest rate was much lower than implied in the mean

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[11] One major objection to an interest rate of .02 or .05 might be that the interest rate should be equal to the rate of return to human capital investment, which is usually higher than .05. In my model, however, the interest rate is not required to be the same as the rate of return. As indicated in equation (2.2), with financial bequests neutralized, the rate of return may be of any value, and may vary from one individual to another.

Table 2. Summary of the Unadjusted and Adjusted Lifetime Earnings
(in thousand dollars)

		A. Unadjusted					B. Adjusted	
		Father		Son		Generational Premium	Father	Son
		W_f	$\bar{Y}_f$	W_s	$\bar{Y}_s$		W_f^*	W_s^*
π_1	$\alpha = ½$ r = .02 $\delta = .05$	458.5 (305.5)	19.2	804.7 (506.8)	33.7	.020	1,393.3 (913.6)	1,418.6 (852.6)
π_2	$\alpha = ½$ r = .05 $\delta = .05$	155.4 (101.5)	15.5	259,0 (161.0)	25.9	.018	2,639.0 (1,881.6)	1,100.0 (664.9)
π_3	$\alpha = 2/3$ r = .02 $\delta = .05$	450.3 (298.7)	18.9	803.0 (508.3)	33.6	.021	1,370.0 (896.3)	1,415.5 (852.9)
π_4	$\alpha = 2/3$ r = .05 $\delta = .05$	154.2 (100.6)	15.4	259.9 (161.8)	26.0	.019	2,619.5 (1,866.6)	1,103.4 (666.8)
π_5	$\alpha = 3/4$ r = .02 $\delta = .05$	443.5 (292.4)	18.6	782.6 (497.2)	32.8	.020	1,353.2 (885.8)	1,378.7 (832.4)
π_6	$\alpha = 3/4$ r = .05 $\delta = .05$	153.4 (99.7)	15.3	259.2 (161.7)	25.9	.019	2,612.0 (1,867.7)	1,100.0 (664.4)

Notes: ***W*** is the mean value of lifetime earnings. $\bar{Y} = rW/[(1+r)^{1-t^*} - (1+r)^{-T}]$. Generational premium is defined as annual growth rate of the sons' unadjusted lifetime earnings relative to the fathers': $(\ln W_s - \ln W_f)/28$, where 28 refers to the average age difference between the generations. The adjusted earnings are in the 1984 price. Standard errors are in parentheses.

lifetime earnings.

The differences between the fathers and the sons in the mean lifetime earnings seems to be huge. Over the life cycle, a son on the average earns about 70 percent more than a father does. However, the generational premium, defined as annual growth rate of the mean value of the sons' lifetime earnings relative to that of the fathers, is only around .02 (see the last column in the panel).[12]

[12] The 2% generational premium is consistent with evidence reported elsewhere. For example, according to Levy and Murnane (1992), who reviewed earnings levels and inequality in the United

In terms of real lifetime earnings, the sons are not better-off than the fathers. When the interest rate is .02, the 2% generational premium implies that the sons at best have the same real lifetime earnings as their fathers'; when the interest rate is .05, the sons even have lower real lifetime earnings. The higher the interest rate, the lower the sons' real lifetime earnings relative to their fathers'. In fact, one of the reasons why I did not choose an interest rate as high as, say, .10 is that a high interest rate would put the sons' real lifetime earnings much lower than their fathers', a result contradicting to common sense.

Adjusted lifetime earnings differ from the unadjusted in that the former are sensitive to the common reference time (see equation 3.7). Since the reference time may be set arbitrarily, the properties possessed by the adjusted lifetime earnings such as means and standard deviations are not generalizable. The earnings therefore may only be understood as indexes. Table 2, panel B summarizes the lifetime earnings when adjusted to year 1984. Both the sons' and the fathers' adjusted earnings were higher than their unadjusted, because their base periods were both earlier than 1984 (i.e., both the sons and the fathers were born before 1978). The sons' lifetime earnings were equal to or less than their fathers' because the fathers' earnings were discounted to the time when they were, on the average, 56 years old, while the sons' earnings to the time when they were only 28. It is important to note that, although a different reference time would lead to different adjusted lifetime earnings, reference time has a very marginal effect on our results about earnings mobility.

For both the fathers and the sons, the average adjusted lifetime earnings were more than one million dollars in the 1984 price. These figures were much larger than the average financial transfers from fathers to sons (Mulligan, 1995), suggesting that, as far as lifetime earnings is concerned, financial bequests may not be a very significant determinant of individual behavior.

States over the past fifty years, disposable income per full-time worker increased by 2.45% per year from 1947-73, but only by .67% per year from 1973-88.

B. Intergenerational Earnings Transmission

a. Non-linearities: cubic transmission equation

Four polynomial equations (4.6a - 4.6d) were estimated to examine the nature of intergenerational earnings transmission. For reasons that will be clear later, I used the cubic equation as baseline, testing the hypothesis that an alternative equation was better than the cubic in depicting the relationship between the fathers' and the sons' log lifetime earnings (Appendix C). The regression results are reported in Table 3a for the population parameters π_1, π_3, and π_5, and in Table 3b for the other selected population parameters.

For all specifications in Tables 3a and 3b, the dependent variables were $\log(W_s^*)$, and the independent variables were, $\log(W_f^*)$, $[\log(W_f^*)]^2$, etc. All regression were weighted. Regression coefficients were significant, except for the quadrinomial specification, where none of its five coefficients was significant.[13] The hypothesis that an alternative specification is better than the cubic was tested by an F-statistic listed in the last columns of the tables. For the linear specification, the null hypothesis was $\theta_1 = \theta_2 = 0$; for the quadratic specification, it was $\theta_3 = 0$; and for the quadrinomial specification, it was $\theta_4 = 0$. A rejection of a hypothesis would statistically support a claim that the cubic specification is the better than the relevant specification, while a non-rejection would support the otherwise. The F-statistics in Table 3a and Table 3b show that the cubic specification was better than both linear and quadratic, while at least not worse than the quadrinomial. In conjunction with the fact that none of the regression coefficients in the quartic was significant, this exercise suggested that the cubic specification was the best in describing the relationship between fathers' and sons' log lifetime earnings. And, as a byproduct, intergenerational lifetime earnings transmission equation was not linear or log-linear, supporting Proposition 1 in Section II.

[13] In fact, in "any" polynomial equation with order higher than cubic, regression coefficients would not be significant.

Table 3a. Intergenerational Earnings Transmission Equations
(Population Parameters: π_1, π_3, π_5)

Specification	Population Parameters	θ_0	θ_1	θ_2	θ_3	θ_4	Adjusted R^2	F Value
Linear	α=1/2	10.449 (.539)	.255 (.038)	--	--	--	.061	8.867[a]
	α=2/3	10.484 (.539)	.253 (.039)	--	--	--	.060	8.680[a]
	α=3/4	10.528 (.539)	.248 (.039)	--	--	--	.058	8.280[a]
Quadratic	α=1/2	32.381 (7.283)	-2.900 (1.046)	.113 (.038)	--	--	.073	8.515[a]
	α=2/3	32.285 (7.274)	-2.887 (1.046)	.113 (.038)	--	--	.071	8.232[a]
	α=3/4	31.656 (7.261)	-2.780 (1.045)	.110 (.038)	--	--	.068	7.965[a]
Cubic	α=1/2	233.33 (69.25)	-46.32 (14.92)	3.234 (1.07)	-.075 (.026)	--	.083	--
	α=2/3	228.31 (68.70)	-45.31 (14.82)	3.166 (1.07)	-.073 (.025)	--	.081	--
	α=3/4	222.88 (68.18)	-44.22 (14.72)	3.094 (1.06)	-.071 (.025)	--	.080	--
Quadri-nomial	α=1/2	-209.00 (652.53)	82.46 (189.50)	-10.78 (20.59)	.60 (.99)	-.01 (.02)	.082	.464
	α=2/3	-215.34 (644.66)	84.08 (187.52)	-10.94 (20.41)	.61 (.98)	-.01 (.02)	.080	.480
	α=3/4	-203.98 (636.63)	80.39 (185.37)	-10.50 (20.19)	.59 (.98)	-.01 (.02)	.077	.457

Notes: r = .02, and δ = .05. The dependent variables were $\log(W_s^*)$, while the independent variables were $\log(W_f^*)$, $[\log(W_f^*)]^2$, etc. All regressions were weighted. "F-Value" in the last column stands for the F statistic under a null hypothesis that an alternative specification is better than the cubic. The superscript "*a*" indicates that the null hypothesis is rejected at a significance level .01. The specification tests support that the cubic specification is better than any other polynomial specification listed here.

The cubic transmission equation implied that the elasticity of the son's lifetime earnings with respect to the father's was a quadratic function of the father's status or log lifetime earnings. In each of

Table 3b. Intergenerational Earnings Transmission Equations
(Population Parameters: π_2, π_4, π_6)

Specification	Population Parameters	θ_0	θ_1	θ_2	θ_3	θ_4	Adjusted R^2	F Value
Linear	α=1/2	9.608 (.533)	.284 (.036)	--	--	--	.083	5.845[a]
	α=2/3	9.631 (.532)	.282 (.036)	--	--	--	.082	5.879[a]
	α=3/4	9.659 (.530)	.281 (.036)	--	--	--	.082	5.770[a]
Quadratic	α=1/2	20.612 (7.614)	-1.229 (1.045)	.052 (.036)	--	--	.084	9.562[a]
	α=2/3	20.775 (7.605)	-1.250 (1.044)	.053 (.036)	--	--	.084	9.572[a]
	α=3/4	20.627 (7.556)	-1.228 (1.038)	.051 (.036)	--	--	.083	9.397[a]
Cubic	α=1/2	273.58 (82.15)	-53.34 (16.88)	3.622 (1.16)	-.081 (.026)	--	.096	--
	α=2/3	273.31 (81.97)	-53.30 (16.86)	3.621 (1.15)	-.081 (.026)	--	.096	--
	α=3/4	269.34 (81.49)	-52.51 (16.76)	3.569 (1.15)	-.080 (.026)	--	.095	--
Quadri-nomial	α=1/2	820.62 (851.05)	-204.29 (234.34)	19.201 (24.15)	-.794 (1.104)	.012 (.02)	.095	.415
	α=2/3	815.71 (846.71)	-203.09 (233.33)	19.093 (24.07)	-.790 (1.101)	.012 (.02)	.095	.412
	α=3/4	833.76 (839.41)	-208.46 (231.43)	19.684 (23.88)	-.819 (1.093)	.012 (.02)	.094	.456

Note: See notes for Table 3a.

the six cubic equations reported in Tables 3a and 3b, θ_2 was positive and θ_3 negative, implying that the elasticity curve was concave from below in the father's status. Further, the non-trivial magnitudes of θ_3's relative to θ_2's suggested that the elasticity curves were of an inverted *U* shape,[14]supporting

[14] Setting the first derivative of θ with respect to $\ln W_f^*$ equal to zero, one knows that the elasticity curve begins to fall with $\ln W_f^*$ when the father's status reaches a turning

Proposition 2 about non-monotonicity.

The non-linearities of the intergenerational transmission process may also be examined numerically. In Table 4, under the title "cubic equation", I report the earnings elasticities for six representative families: the families in which the fathers' statuses were at the 5th, 25th, 50th, 75th and 95th percentiles, and equal to the mean value of the sampled fathers' log lifetime earnings. The columns are arranged by the father's status. For instance, "25th" stands for the family in which the father's log lifetime earnings are at the 25th percentile, while "mean" stands for the family in which the father's log lifetime earnings are equal to the mean. The mean elasticities – or the elasticities corresponding to the "mean" family – were about .38. For most families -- more than 95 percent, the absolute values of the elasticities were less than unity, rendering strong support for the corollary of "regression to the mean". In addition, relative to the families with middle positions, both poor and rich families had lower elasticities or higher mobility, confirming the inverted U-shape of the elasticity curves.

Table 4. The Elasticities of the Son's adjusted Lifetime Earnings
with respect to the Father's: Cubic and Linear Status Transmission Equation

	Cubic Equation						Linear Equation
	Father's log earnings: Percentile						
	5th	25th	Mean	50th	75th	95th	
π_1	-.063	.285	.371	.380	.416	.394	.255
π_2	.002	.334	.402	.405	.410	.304	.284
π_3	-.064	.281	.367	.375	.411	.392	.253
π_4	-.002	.333	.400	.404	.409	.304	.282
π_5	-.006	.279	.360	.366	.401	.381	.248
π_6	.000	.331	.397	.400	.405	.299	.281

point $-\theta_2/3\theta_3$. The non-trivial θ_3 relative to θ_2 makes sure that the turning point is not an outlier, but in the middle of the father's status ladder.

b. The Elasticity Curves and Regression to the Mean

The inverted U-shape of elasticity curves is a very important finding[15] First, it supports my theoretical model, which indicates that intergenerational mobility is generally not the same across families. It also offers a new insight into the familiar result of “regression to the mean”. That is, “regression to the mean” is stronger, the further a father’s status is from the mean. Relative to the families with middle positions, rich families have lower elasticities, so their children’s statuses are less dependent on the fathers’ high statuses, pushing the children down toward to the middle. On the other hand, for poor families–those very poor with the absolute values of their elasticities greater than unity excluded, lower elasticities imply that their children’s statuses are less confined to their fathers’ low statuses, pushing the children up to the middle.

Evidence on "regression to the mean" or "regression away from the mean" has been suggested as an internal check for several alternative assumptions in the model. The strong evidence documented here on "regression to the mean" rejects the assumptions that the marginal return to the parental investment is independent of the investment and, at the same time, the individual utility function is homothetic. If those assumptions held water, the elasticities would be uniformly equal to unity. In addition, the fact that the elasticities for families as rich as at the 95th percentile are well above zero implies that the assumption that the bequest decisions either are independent of or have no significant impact on the parental investment may not be very strong. For otherwise one should expect the

[15] One might, however, suspect that the inverted U-shape of elasticity curves is economically spurious, nothing more than a statistical phenomenon that the probability of a poor (or rich) family becoming poorer (richer) is limited by its current position of being poor (rich). While detailed explanations about the shape are offered in later sections, I have two immediate reasons to dispel the suspicion. First, these are earnings elasticity curves–not earnings or log earnings curves. The potential effects of the father’s status–if any–on the son’s have already been controlled. Second, the curves have been found to be favored systematically over any other important alternative in describing the intergenerational relationship (see Tables 3a and 3b). Ever there were only statistical forces behind the curves, one has to explain why an inverted U-shape–rather than, say, a U-shape–is systematically favored.

elasticities to be very close to zero for those families.[16] The conclusion: my model about the mechanism of lifetime earnings transmission is empirically supported.

The inverted U shape of elasticity curves suggests that, in terms of lifetime economic status and one the average, sons from poor (rich) families have better chance to move up (down) than maintain their original statuses. This finding cannot be compared directly with other mobility studies, because those studies have never used a status measure covering individual lifetime earnings experience. Ignoring the differences in the method of status measurement, however, the finding is consistent with evidence reported by Brittan (1977), who found that for families whose relative status ranking at the top 10 percentile of the sample, the sons' expected income ranking was about 21, whereas for families whose percentile ranking was at 90, the sons' ranking was about 79 (p.58).[17] On the other hand, the elasticity curves reported here are in sharp contrast with some recent studies on non-linearities of intergenerational earnings mobility. For example, also using the PSID, Mulligan (1993) found no evidence on non-linearities. In their attempts to incorporate the family's neighborhood selection into the determination process of an individual's economic status, several other authors--including Cooper, Durlauf and Johnson (1994) and Durlauf (1996)--suggested that the intergenerational persistence of earnings was greatest at the extremes of the income distribution. In other words, they found elasticity curves to be of U-shape.[18]

One lesson from these conflicting results about intergenerational economic mobility is clear: the

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[16]The elasticities were decreasing for rich families, and they did reach and even fell below zero for some extremely rich families. However, those extremely rich families were so few and so far away from others that one may take it as outlier.

[17] I thank Casey Mulligan for bringing this to my attention.

[18] Strictly speaking, the studies by these authors are not the same as traditional mobility studies. In the study by Cooper, Durlauf, and Johnson (1993), for example, the focus was not on how an individual's status is determined, but on how measured mobility is sensitive to different neighborhoods. Non-linearities of intergenerational relationship were found in the study, as families residing in relatively affluent or poor communities exhibit more persistence in economic status than families residing in communities near the middle of the income distribution.

judgment about the extent to which socioeconomic status is transmitted from one generation to the next is greatly sensitive to the way individual status is measured. Except for the present study, most of the aforementioned studies approximated an individual's economic status by averaging a single-year variable such as earnings over a span of several years. But how long a time span should be and in what periods an individual's earnings should be used are often a question of data availability. Consequently, to have a better understanding of the intergenerational relationship, one first has to answer a more fundamental question: how should we measure an individual's status? Since lifetime earnings are a better measure of individual status than most of previously used short-run variables, the findings from this study may be of more significance.

c. Comparisons with the linear transmission equation

Attention to non-linearities of the intergenerational relationship is only a recent phenomenon. The dominant practice in the studies of intergenerational mobility has been confined to a linear transmission process. Although our specification tests have revealed that the linear transmission process is inferior to the cubic, it is worthwhile examining how mobility results based on a linear equation would change as one changes individual status measure from short-run variables to lifetime earnings.

Based on the linear equation (4.3a), the elasticity of the son's lifetime earnings with respect to the father's, listed in the last column of Table 4, would be around .27. This number is slightly higher than suggested in most studies surveyed in Becker and Tomes (1986), which used short-run variables as proxies for individual permanent status *and* did not correct the Errors-in-Variables (EIV) problem. It is not, nevertheless, as high as claimed by Solon (1992) or Zimmerman (1992b), who believed that, if an individual's permanent status itself, not any of its proxies, were used, the number might be above .4.

It is not surprising that the elasticities reported in the last column of Table 4 were higher than those collected in Becker and Tomes (1986). For, relative to short-run proxies, lifetime earnings clear out some transitory noises that might reduce elasticities. But why were those elasticities lower than claimed by Solon (1992) and Zimmerman (1992b)? One hypothesis is that, while clearing out some transitory noises essentially included in a short-run proxy, lifetime earnings as permanent status may bring in new sources of variation that were originally not in the proxy. In both Solon (1992) and Zimmerman (1992b), an individual's permanent status was defined as a constant stream of earnings over his working span. But the working span is not identical across individuals. Differential working spans may be a source contributing to lower elasticities.

A simple test with an assumption that all individuals have worked or will work for 51 years[19] is in favor of the hypothesis. As shown in Table 5, where I duplicated the calculations in Table 4 under the fixed working span assumption, all elasticities -- in absolute values -- were larger when the working

Table 5. The Elasticity of the Son's adjusted Lifetime Earnings with respect to the Father's: Cubic and Linear Status Transmission Equation, and Fixed Working Spans

	Cubic Equation						
	Father's log earnings: Percentile						Linear Equation
	5th	25th	Mean	50th	75th	95th	
π_1	-.078	.337	.434	.447	.485	.441	.303
π_2	-.022	.331	.408	.412	.428	.347	.299
π_3	-.088	.335	.439	.448	.490	.452	.303
π_4	-.037	.340	.421	.426	.442	.353	.300
π_5	-.089	.335	.440	.446	.493	.455	.303
π_6	-.053	.347	.433	.437	.453	.359	.305

Note: All elasticities in this table were calculated with the assumption that individuals have worked or will work for 51 years.

spans were fixed than when they were not. However, the elasticities based on the linear equation were still far from .4 claimed by Solon and Zimmerman. Further, the inverted U-shape of the elasticity curves

[19] This is the working span that an individual with 8 years of schooling retires at age 65.

remained intact with the alternative assumption, suggesting that the non-linear relationship between generations in lifetime earnings was not due to differential working spans.

The linear transmission equation predicts the "average" degree to which socioeconomic status is transmitted from one generation to the next. For the cubic equation, the "average" degree of transmission may be calculated at the mean (or median) value of the father's log lifetime earnings; and it was in neighborhood of .38 (Table 4), greater than both the elasticities in the Becker-Tomes survey (.2), and those following the linear transmission equation and at the same time using lifetime earnings as a proxy of individual status (.27). This suggests that both the EIV problem and the linear specification contribute to the underestimation of the "average" degree of earnings transmission.

C. The Effects of the Non-Family Factors on Mobility: Preliminary Results

It is known to us that intergenerational mobility is determined by both the family and the society. However, the relative effect of one factor to another has seldom been seriously discussed. One reason is that the conventional specification of the intergenerational transmission process has almost always been linear, which hinders attempts to separate the effect of one factor from another. The inverted U-shape of elasticity curves, along with hypotheses about the society, may help in this regard.

If the society is a continuum, and, as espoused in the “American Dream”, provides people with equal opportunities, different elasticities across families would be accounted for only by family- related variables. That is, it would be the family-related forces that make both poor and rich families experience higher mobility than those in the middle. Presumably, the most important force may be fathers’ investments in their sons, constrained by diminishing returns to the investments. Relative to “middle” families, poor families invest less in their sons, but their investments are more productive. The sons’ dependency on their fathers’ is alleviated by higher returns to the investments. By contrast, rich families invest more in their sons, but their investments are less productive. The sons’ dependency on their fathers is also alleviated, but by lower returns to the investments.

Of course, the society envisioned in the model presented in this paper does provide the same opportunities for poor families as for rich families. For example, while it has little adverse effect on rich fathers' investments in sons, the quasi-perfect physical capital market makes poor fathers invest less in their sons than they would if they could borrow to finance investments. In the latter scenario, poor fathers would–through "education loan"–make investment decisions in such a way as to equate the marginal rate of return to the investments with the interest rate in the market. Total returns captured by their sons would be larger; their sons' relative status would improve; and the earnings mobility for those families might be even higher. High mobility experienced by poor families is therefor a net result balancing a higher mobility potential due to the family forces, and the disadvantages that the quasi-perfect capital market imposes on those families.

The effects of the family and the society on earnings mobility may also be examined by comparing the real world with a hypothesized society where the effects of certain variables are neutralized. The difference in mobility patterns between the two worlds would then inform us of how the real society has affected mobility.

a. Human capital price

Human capital price is a variable determined by the demand for and supply of human capital in the labor market. A constant for an individual over his life cycle, the price may be regarded as an indicator of average *social* opportunities provided to the individual as well as his fellows in the same cohort. In response to macro changes in the society, the price varies across cohorts, and, supposedly, has affected the measured earnings mobility.

What would happen with mobility should the price not change over time? Table 6, Panel *A* answers the question. The elasticities in the panel were calculated for four representative families, and under the assumption that human capital price were equal to unity for all cohorts.[20] As in Tables 4 and

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[20] To calculate these elasticities, I first imputed the human capital price facing each cohort, based on the strategy illustrated in Appendix B. Using this price, I then imputed all other variables necessary for imputing lifetime earnings. Holding the imputations for other variables intact, and setting the human capital price equal to unity for all cohorts, I got lifetime earnings and earnings elasticities under the alternative assumption. Throughout the rest of the paper, I will not report results for the population parameters π_1 and π_2. The schooling regression (b.3) did not work well when $\alpha = ½$.

5, "25th" stands for the family in which the father's log lifetime earnings is at the 25th percentile, "50th" for the family at the 50th percentile, etc. The elasticity curves maintained an inverted U shape under the new assumption. Except for rich families (above the 85th percentile), the elasticities were greater -- but not very much -- than those in the real world, which are listed in panel *D*. Overall, the variation in human capital price has contributed to increasing earnings mobility, as expected.

Table 6. The Effects of Human Capital Price and Age Structure on Mobility

Control Variable(s)		α=2/3 r=.02 δ=.05	α=2/3 r=.05 δ=.05	α=3/4 r=.02 δ=.05	α=3/4 r=.05 δ=.05
(A)	25th	.314	.376	.305	.357
Human Capital Price	50th	.398	.473	.386	.439
	75th	.423	.480	.408	.438
	95th	.361	.329	.343	.282
(B)	25th	.278	.263	.280	.260
Age Structure	50th	.385	.356	.381	.354
	75th	.434	.402	.429	.401
	95th	.446	.414	.442	.415
(C)	25th	.278	.256	.274	.254
Human Capital Price and Age Structure	50th	.368	.348	.356	.343
	75th	.408	.394	.397	.384
	95th	.395	.393	.384	.380
(D)	25th	.281	.333	.279	.331
No Control (Selected from Table 4)	50th	.375	.404	.366	.400
	75th	.411	.409	.401	.405
	95th	.392	.304	.381	.299

Note: In panel (A), I assumed that human capital price did not change across cohorts. In panel (B), I eliminated age differences within generations, and assumed that the differences between generations were uniformly equal to 28, the average age difference between fathers and sons in the sample. In panel (C), I controlled both human capital price and age structure. In panel (D), neither human capital price nor age structure was controlled.

Poor and rich families respond to the variation in human capital price in different ways. While poor families have higher mobility in the real society than in the hypothesized world, rich families tend to have lower mobility. Since higher mobility is good for poor families, but not for rich families, the exercise shows that both poor and rich families have taken advantage of the variation in human capital price.

b. Age structure

Age structure in this work is referred to as (i) age distributions within generations; and (ii) age differences between generations. Although it results from individual families' fertility decisions, age structure is a social variable over which any single individual or family has no direct control.

To see how age structure affects earnings mobility, imagine a population in which all fathers' ages were the same, and became fathers at some age simultaneously.[21] The earnings elasticities for this imagined population were reported in Table 6, panel *B*. The differences between the real society and the hypothesized population in elasticity showed no uniform pattern. For example, for the population parameter set π_3, the elasticities were universally larger in the hypothesized world than in the real world; but for other population parameter sets, the elasticities were larger only for some families.

Unlike in panel *A* or *D*, the elasticity curves in the hypothesized world no longer maintained an inverted U shape. Instead, they were uniformly monotonically increasing with the family status: The richer a family, the larger elasticity, and the lower mobility.

The distinct mobility patterns in the real society and in the imagined world inform us of significant effects of age structure. This is surprising, for the age structure has been neglected in almost all empirical efforts to measure mobility. Usually, researchers take for granted whatever intergenerational samples available to them, without investigating how sensitive their findings on mobility are to the ways of sample construction. On the other hand, since age structure is related to families' fertility decisions, which may be an important argument in individual lifetime utility function, the significant effects of

[21] Apparently, in this hypothesized world, no age differences exist within generations, and the age differences between generations are identical for all families.

age structure should not be beyond expectation.

c. Human capital price and age structure

The effect of age structure on mobility seemed to be more significant than the effect of human capital price. But the two effects were in opposite directions. This result may be found in Table 6, panel *C*, where both human capital price and age structure were controlled. Except for some very rich families, the elasticities in panel *C* were lower than those not only in panel *A*, but also in panel *B*. Should human capital price and the age structure affect mobility in the same direction, the elasticities in panel *C* would be the largest among the three panels.

Except for rich families, the elasticities in panel *C* were even lower than those in panel *D* or in the real society. Since low elasticities are good for poor families but bad for rich families, the evidence implies that, in terms on earnings mobility, the real society is worse for all families than the world characterized in panel *C*. One possible explanation for this is the following: Human capital price and the age structure are correlated with each other; one cannot expect a world where both variables are controlled independently.

VI. Concluding Remarks

This paper supports a very general result: When an individual's status is measured by his lifetime earnings, intergenerational relationship in status is generally non-linear. The result is consistent with a two-stage life cycle model in which individuals maximize their lifetime earnings at the early stage, and maximize their lifetime utilities at the other. Intergenerational relationship in lifetime earnings is built on genetic connections between generations as well as parental altruism toward children. It is also qualified by social factors that are beyond the family's control.

Using data from the PSID, I have found that the elasticity of the son's lifetime earnings with respect to the father's is not constant across families. Earnings mobility is higher for both poor and rich families than for the families in the middle. In sharp contrast with some recent studies on non-linearities of intergenerational earnings mobility, this result suggests that mobility studies are very sensitive to the way in which an individual's status is measured.

One important feature of the work is that it essentially allows for examining the relative effect on mobility of non-family factors to family factors. While not conclusive, the attempts comparing mobility patterns in the real society with those in some hypothesized worlds do show potentials for further research on the relative effect.

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Appendix A. The Intertemporal and Lifetime Earnings

This appendix is to derive the intertemporal and lifetime earnings for the representative individual based on the model in Section III. The Hamiltonian associated with the problem is

$$H = a[1 - s_t]K_t e^{-rt} + q(t)[\beta_0(s_t K_t)^\alpha - \delta K_t], \tag{a-1}$$

where $q(t)$ is a non-negative costate variable indicating the shadow demand price of the human capital investment at t. For an interior solution (i.e., $0 < s_t < 1$), the following conditions must hold:

$$\frac{\partial H}{\partial s_t} = -aK_t e^{-rt} + q(t)\beta_0\alpha(s_t K_t)^\alpha / s_t = 0, \tag{a-2}$$

and

$$\frac{\partial q(t)}{\partial t} = -\frac{\partial H}{\partial K_t} = -a[1-s_t]e^{-rt} - q(t)[\beta_0\alpha(s_t K_t)^\alpha / K_t - \delta]. \tag{a-3}$$

Consequently,

$$q(t) = \frac{a}{r+\delta}e^{-rt} + Ce^{\delta t}, \tag{a-4}$$

where C is a constant of integration. Assuming $q(T) = 0$, that is, human capital is useless at the end of the planning horizon, equation (a-4) is reduced to

$$q(t) = \frac{a}{r+\delta}e^{-rt}[1 - e^{-(r+\delta)(T-t)}]. \tag{a-5}$$

Substituting (a-5) into (a-2), we get the dynamics of the human capital stock devoted to the human capital production,

$$s_t K_t = \left(\frac{\alpha\beta_0}{r+\delta}[1 - e^{-(r+\delta)(T-t)}]\right)^{\frac{1}{1-\alpha}}. \tag{a-6}$$

This dynamics implicitly determines the individual's profile of human capital stock.

Substituting the dynamics into the human capital production function (3.2) in the text, we know that a closed form expression for the intertemporal human capital stock K_t is available only when α takes on value $n/(n+1)$, with n a positive integer. We call the α's satisfying the restriction as admissible.

The above results are generally not valid for corner solutions. In the first phase, when $s_t = 1$, we have the following differential equation,

$$\frac{\partial K_t}{\partial t} = \beta_0 K_t^{\alpha} - \delta K_t. \tag{a-7}$$

It has a solution

$$K_t = [\frac{\beta_0}{\delta} + (K_0^{1-\alpha} - \frac{\beta_0}{\delta})e^{-(1-\alpha)\delta t}]^{\frac{1}{1-\alpha}}. \tag{a-8}$$

The transition period from the first phase to the second, t^*, is determined by equalizing (a-8) with (a-6), which gives rise to the "schooling equation" (3.4) in the text.

The third phase has no real implication for our purpose, as it occurs when and only when the individual reaches the end of the planning horizon: Check equation (a-5), $q(t) = 0$ if and only if $t = T$.

The representative has positive earnings only in the second phase. For any admissible α, and with the corresponding closed form expression for K_t, (a-6) directly leads to an intertemporal earnings profile as defined in equation (3.5). In turn, the intertemporal earnings yields an index for the unadjusted lifetime earnings, defined in equation (3.6).

Appendix B. Imputing the Cohort-Specific Human Capital Prices

Assume

$$K_{0ij} = b_1\, e^{X_{ij}\gamma + \epsilon_{ij}}, \text{ and} \tag{b.1}$$

$$a_i = b_2\, e^{d_i\lambda + \mu_i}. \tag{b.2}$$

Here, K_{0ij} is the initial human capital stock of individual j of cohort i; X_{ij} is his early family background variables; a_i is the human capital price shared by cohort i; d_i is the cohort dummy vector; b_1, b_2, γ, and λ are scaler or vector coefficients; and ϵ_{ij} and μ_i are zero mean disturbances independent of each other, and uncorrelated with (d_i, X_{ij}).

Substitute equations (b.1) and (b.2) into the schooling equation (4.3). After some manipulations, one should be able to get the following "schooling" regression,

$$Z_{ij} = c + d_i\lambda + X_{ij}\gamma - \ln(g_{ij}) + \upsilon_{ij}, \tag{b.3}$$

where $Z_{ij} = \frac{1}{1-\alpha}\ln[z(t^*)_{ij}]$, $c = \ln[b_1 b_2\ \delta^{\frac{1}{1-\alpha}}(\frac{\alpha}{r+\delta})^{\frac{\alpha}{1-\alpha}}]$, $\upsilon_{ij} = \epsilon_{ij} + \mu_i$, and $z(t^*)$ is defined in equation (3.14). Substituting $\hat{g}_{ij}$ imputed in equation (4.2a) into the regression, and normalizing b_1 to be unity, one should get

$$\hat{a}_i = \hat{b}_2\ e^{d_i\hat{\lambda}}, \tag{b.4}$$

where $$\hat{b}_2 = \delta^{\frac{1}{\alpha-1}}(\frac{r+\delta}{\alpha})^{\frac{\alpha}{1-\alpha}}e^{\hat{c}}. \tag{b.5}$$

Appendix C. Intergenerational Earnings Transmission: Specification Tests

Consider the following pair of linear regressions

$$lnW_s^* = \theta_0 + \theta_1 lnW_f^* + ... + \theta_k[lnW_f^*]^k + \theta_{k+1}[lnW_f^*]^{k+1} + ... + \theta_l[lnW_f^*]^l + \epsilon, \quad \text{©.1)}$$

$$lnW_s^* = \theta_0 + \theta_1 lnW_f^* + ... + \theta_k[lnW_f^*]^k + \epsilon, \tag{C.2}$$

where lnW_s^* and lnW_f^* are the logarithms of the son's and the father's adjusted lifetime earnings; ϵ is zero mean normal disturbance uncorrelated with the independent variables in each regression; l and k are integers indicating, in addition to constant terms, the numbers of independent variables in two regressions; and, of course, $l > k$. A test of the hypothesis that regression (C.2) is not worse than (C.1) may be conducted by testing the null hypothesis that

$$\theta_{k+1} = \theta_{k+2} = ... = \theta_l = 0. \tag{C.3}$$

Denote the sums of residual squares in two regressions as SSR_L and SSR_S, and the sample size as N. It can be shown that the following statistic

$$F \equiv \frac{N-l-1}{l-k}\frac{(SSR_S - SSR_L)}{SSR_L} \tag{C.4}$$

follows a F distribution with degrees of freedom (l-k) and (N-l-1).

If the linear specification is tested against the cubic, $l = 3$ and $k = 1$, as the null hypothesis becomes $\theta_2 = \theta_3 = 0$. If the quadratic specification is tested against the cubic, however, $l = 3$ but

k = 2, and the null hypothesis is $\theta_3 = 0$.

A comparison between the cubic and the quadrinomial specifications requires the quadrinomial being the baseline, when l = 4, k = 3, and the null hypothesis is $\theta_4 = 0$.