Reflection on Microeconomic Adjustment Hazard Approach

Edlira Narazani

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Abstract

This paper studies the way the adjustment process takes place in labor demand when it is expressed as a Cox proportional hazard model. I use a simulated firm-level panel data based on a threshold model with periods of high and low frequency of employment fluctuations, which is consistent with the infrequent way the adjustment process takes place according to the new theories of adjustment. I model the probability that a firm adjusts its employment level during a time-period as a Cox proportional hazard function dependent on the deviation of its actual employment value variable from its target. I show that the aggregate employment change, based on a high proportion of firms experiencing large employment fluctuations, could be very well represented by the Cox proportional hazard and also could be very well approximated by the empirical mean of the product of the hazard function and the deviations. On the other hand, I show that the aggregate employment change based on a very low proportion of firms facing large employment adjustment can be well represented by a quadratic (nonlinear) adjustment hazard. Finally, I try to conclude that in order to construct a measure of deviation from the target level (which is the state variable of the model) the regression of the employment fluctuation on the wage fluctuation could be more helpful than the regression of the employment fluctuation on the hours fluctuation.

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[1] University of Turin.
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I. Introduction

The literature of labor markets recognizes the sluggish behavior of adjustment process, and the traditional justification is that the employment fluctuation is accompanied by adjustment costs.[2] This reason obliges firms not to choose the optimal employment level, but instead, to hire or fire as much as the previous optimal and real level of employment in addition to the present ones could stand for. This results in a nonlinear pattern of employment fluctuation in contrast with the earlier model of linear adjustment based on standard quadratic adjustment costs. Hamermesh and Pfann (1996) have prepared an essay about the adjustment cost in factor demand introducing all the forms the adjustment costs would theoretically take and drawing the conclusion that employment adjustment at firm level is slow and not characterized by symmetric quadratic costs. This could provide a good reason to give up using the quadratic adjustment cost for studying the behavior of employment fluctuation. But on the other side, this renouncement could make difficult the aggregation process across firms in opposition to the smooth quadratic adjustment cost could be able to do.

Caballero and Engel (1993) show through an adjustment hazard model that the average employment fluctuation is an increasing and nonlinear function of the so-called deviation of the desired level of employment from its previous real value. This nonlinear dependence stems from the cross-sectional distribution of the deviations in time, which in turn is settled by the aggregate and idiosyncratic shocks and the proportion of firms that adjust. They depict the aggregate adjustment process involving the heterogeneity of

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[2] Costs of adjusting labor demand are those related to the flows of workers, i.e., to changing the identity of the individuals filling a fixed number of jobs. They include among others: Search costs (advertising, screening and processing new employees), the cost of training (including disruptions to production as previously trained workers' time is devoted to on-the-job instruction of new workers); and the overhead cost of maintaining that part of the personnel function dealing with recruitment and worker outflows. All of these can be substantial even if $\Delta x=0$, as new workers must be hired and trained to replace those who depart (whose possibly involuntary departure also0 generates costs). Hamermesh and Pfann, 1996.

agents (firms) and idiosyncratic uncertainty and their approach seems attractive because it doesn’t need the assumption of the representative agent to allow for description of the aggregate employment dynamics.

In this paper I intend to: a) build a stochastic model for the adjustment process of employment in firm level, b) the factor demands (present at firm’s production function) evolution may be affected by productivity, demand and other shocks and this justifies the stochastic element of this model and c) will use simulated data in order to assess different estimation approaches.

Following recent empirical contributions[3] (Caballero and Engel 1993, Caballero et al 1995), my study consists on the following components. First, I simulate some data based on the pattern of no and full employment adjustment range at firm level. Second, following Caballero and Engel I analyze how firms respond to the deviations in the employment level. Third I analyze the relationship between the frequency of firms undergoing periods of lumpy adjustment and aggregate employment fluctuations in order to see if the bunching of employment micro changes is important for understanding macro outcomes. My simulated data allow me to confront new theories with data simulated according to a threshold behavior as the threshold effect should be particularly pronounced for an economy compound of small and medium size firms (as Italian labor market is) in a highly uncertain environment.

This paper is organized as follows: Section 2 discusses the analytical framework. Section 3 presents the aggregate form of the model under an empirical mean representation and also econometric tools to check the properties of infrequent adjustment process. In Section 4 I try to construct and measure the employment shortage by relating the changes in plants deviations between the actual and desired employment to the fluctuations in wages per worker that I recover from the aggregate and idiosyncratic shocks behavior.

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[3] See the literature on aggregate dynamics in the presence of fixed costs on microeconomic adjustment (S,s) models, Blinder (1981), Caplin (1985), Caplin and Spulber (1987), Caballero and Engel (1991, 1992,1993,1997), the literature related to the importance of lumpy changes in plant level employment Hamermesh (1989), Davis and Haltiwanger (1990,1992) , Bresnahan and Ramey (1991).

II. Analytical Framework

When shocks of different type (productivity, demand, wage shocks, etc) affect the labor market, firms might react to them. The first reaction is the creation of a desirable value of the major variables of the firms activity. The second reaction is the adoption of the current level to this desirable level. This process is called adjustment and seems difficult to be measured as long as it is difficult to measure precisely the desirable level. Probably because of the adjustment costs firms find costly to adjust employment.

Several studies have attempted to understand the structure of this adjustment. Most of the empirical studies before 1990s have assumed that the adjustment cost function is quadratic in the employment change rate, with marginal cost linearly increasing. Therefore firms adjust smoothly over a long time period by small units of adjustment in any period. During 1990s, another branch of literature was blooming under the assumption of fixed costs and piece-wise linear adjustment costs rather than quadratic ones. Under these costs, firms wait and adjust infrequently in order not to pay continuously the adjustment costs. This implies an inaction range where the firms renounce of adjusting their employment level until the deviation of the current employment level from the desired one reaches a threshold. Under no adjustment costs the employment fluctuates a lot over time reacting to the economic shocks. Under quadratic adjustment costs the employment will continuously adjust less than under no-adjustment costs. Under piece-wise linear costs, over some periods the employment will not adjust at all even if some shock is present. Uncertainty regarding the future (fear of an adverse future shock) makes the adjustment employment process reluctant . Under fixed adjustment costs only large shocks would convince firms to adjust and these costs become sunk at the moment of adjustment.

Another way to study the adjustment process is by expressing the probability of adjustment as a function of the deviation of the current value of employment from the optimal one. Caballero and Engel (1993, 1996) show that in an (S,s)[4] framework with random fixed costs, average employment fluctuation is an increasing and nonlinear function of the so-called deviations between desired and actual employment level implying that large deviations lead to larger changes in employment than the small ones. This relationship becomes linear[5] under quadratic costs as firms fill a constant part of the deviation between desired and actual employment level and linear with a size of inactions between for piece-wise[6] linear cost. Therefore on the axes of deviations, the constant hazard adjustment is equivalent to the quadratic adjustment costs and brings about smooth adjustment while the piecewise linear and quadratic hazard adjustment brings about infrequencies.

The economic intuition supporting the adjustment hazard theory is that when a random shock increases the deviation of the desired level from the current one some firms adjust the employment in a lumpy way. The others will not adjust until other shocks occur. Given the same size of shock (at absolute value), the fraction of firms disposed to adjust after a positive shock is smaller than the fraction of firms disposed to adjust after a negative shock. As a result if the hazard adjustment function is asymmetric (it is less likely to adjust positive deviations than negative ones at firm level) lumpy upward adjustment are less likely to happen than the downward ones.

Gap Definition

Consider a firm i that employs $e_{i,t}$ but would employ $e^*_{i,t}$ (in logs) workers if frictions were momentarily removed. The difference between these quantities (but in different time periods) is called the *gap* or the distance of the current value from the target level:

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[4] The (S,s) rule behaves as follows: an individual agent allows his state variable to fall freely until it reaches a certain critical level s; at this point abrupt action takes place and the state variable is reset to an upper value S from where the cycle starts again.

[5] See Rotemberg, Julio, 1989

[6] See Caballero, Engel, 1993

(1) $x_{i,t} \equiv e^{*}_{i,t} - e_{i,t-1}$

for any firm i at time t.

The gap $x_{i,t}$ is distributed across firms over time with a distribution function $F_t(x)$ which denote the cross-section distribution of gaps $x_{i,t}$ before adjustment is made. Thus the fraction of firms with gap between z and $z + dz$ is equal to $dF_t(x)$. To make things simple I work on discrete time and accordingly under a timing principle which can be defined as follows: each time period, t-1, ends with a gap $x_{i,t-1}$ with distribution[7] $F_{t-1}{}^{1}(x)$; then different shocks occur as the next period, t, starts resulting in a gap $x_{i,t}$ with distribution $F_t\ (x)$ and finally at the end of the period t and the beginning of the period t+1, firms adjust their employment level resulting in a gap with distribution $F_t^1(x)$.

Further I assume that firms put this gap equal to zero any time they adjust. This process happens with a given probability which is called adjustment hazard function and depends on the value of the gap. The adjustment hazard function could take different forms but I better consider it as an increasing function of its state variable. Therefore I first need to construct a measure of the gap (state variable) in order to compute the adjustment hazard function.

I try to construct a measure of gap starting from the overall assertion on a lumpy and infrequent adjustment of employment in firm level. The optimal level of employees the firm should employ is determined by the way the aggregate and idiosyncratic shocks affect the deviation of the actual level from what is called the optimal level and of the previous value of this deviation as well. Following this chain of key employment determinants and considering a large number of firms, I can study the macroeconomic of

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[7] The index at F(.) function holds for defining the distribution of shocks after the firms have adjusted their actual employment level to the desired one.

employment adjustment. But I start constructing first an employment picture that characterizes the aggregate behavior dynamics and by the end of the paper I try to measure the theoretical structure of deviations I need.

III

Aggregate Employment Dynamics

If I assume that employment shares are independent of gap distribution across firms, the average employment growth rate is equal to the growth rate of the aggregate employment. Therefore, the average aggregate employment change (equal to the expected gap, expectation taken over the gaps in all firms) in discrete time could be given as the sum of all employment deviations weighted by the their density after adjustment is made.

In the survival analysis, the hazard function is the rate at which a spell can be completed at a time *t* if it hasn't been changed up to that moment. In my case, the adjustment hazard function is the instantaneous rate at which a firm adjusts its employment level conditional on not having done this up to the moment that the gap was smaller than x. But differently from the traditional hazard function, this function depends indirectly on time through the variable x.

Then, the average aggregate employment change could be given as:

$$\Delta A_t = \int_x x\Lambda_t(x)dF_t(x) \tag{2}$$

where ΔA_t denotes the aggregate employment change, $F_t(x)$ is the cross sectional distribution of firms with deviation x at time t and before shocks are experienced, and $\Lambda_t(x)$ is the hazard function or the probability that each firm adjust its employment shortage x at a given period of time t. Thus the average employment change by firms with shortage x at time t is equal to $x\Lambda_t(x)$.

The hazard adjustment function could take several forms conditional on the shape of adjustment cost. The increasing hazard means that the probability that a firms adjust its employment level is increasing with the gap value and this feature is consistent with the non-convex theory of adjustment cost where firms follow an infrequent adjustment process. The decreasing hazard function behaves in the opposite way, in the meaning that smaller is the deviation of the employment level from the desirable one less likely it is that the firms adjust its employment level. This function stands for convex adjustment cost theory. The intermediate case is the constant hazard which is independent of gap value.

The name "hazard" motivates this paper which considers the hazard adjustment function as a one of the known hazard forms, Cox proportional. Starting from the above definition, I parameterize the hazard function as a Cox proportional hazard model written as:

$$(3 \qquad \Lambda_t(x) = 1 - \exp(-\beta x)$$

where β is a parameter which takes only positive values in order to allow the hazard function is increasing in x.

Caballero and Engel (1993) express the hazard function as a piece-wise linear function of the gap and also as a quadratic one. Their hazard representations $\Lambda_t(x) = \beta x$ or $\Lambda_t(x) = \beta x^2$ are only special cases of Cox proportional hazard functions. They are however confirmed for tiny values of the $-\beta x$. This restriction could be interpreted in the way that only for small values of weighted gap deviations, higher moments of the cross-sectional density of gaps affect the evolution of aggregate employment through mean-variance and variance-skewness interaction terms in a nonlinear style.

In contrast to Caballero and Engel I shall base the analysis on the exponential representation of hazard function and replacing it at the aggregate expression I'll get:

$$(4) \qquad \Delta A_t = \int_x x\left[1-\exp(-\beta x)\right]dF_t(x)$$

The variable x denotes the above-mentioned deviations across firms and over time. The expression (6) is equivalent to:

$$(5) \qquad \Delta A_t = E_t\left[x_{it}(1-\exp(\beta x_{it}))\right]$$

where E(.) is the expectation operator

Caballero and Engel transform the aggregate employment equation in a regression of the aggregate employment change variable on the moments of the gap distribution $F_t(x)$. In order to test which of the hazard expressions (linear piece-wise, quadratic or exponential) fits better the aggregate adjustment behavior and in the absence of some adequate data I make use of simulated data whose simulation method will be explained in Section III.

Below I consider two cases when gap distribution is known or unknown to the researcher.

A. Unknown Gap Distribution Case

If one does not have information on the distribution of the gap (if it is a normal or exponential) before adjustment is made but observes x_{it} for i = 1, 2, … N and t = 1, 2, …, T, a useful approximation of the expected value expression could be through the empirical mean representation:

$$(6) \quad \Delta A_t \cong \frac{1}{N}\sum_i^N x_{it}(1-\exp(-\beta x_{it}))$$

This approximation is good only if N is large. I make use of the simulated values[8] of employment and gap across firms and time periods and under a large number N of firms, I could estimate the parameter β by minimizing :

$$(7)\qquad \sum_t \left[\Delta A_t - \frac{1}{N}\sum_i^N x_{it}(1-\exp(-\beta x_{it})) \right]^2$$

Using the Gauss-Newton method I could obtain estimates by iterations. For a large number of observations also the asymptotic normal distributed behavior of errors is guaranteed[9]. The convergence is obtained after a small number of iterations. The following tables[10] contain some evidence on the statistics of the parameter and the goodness of fit.

Using simulated data I check how Cox proportional, linear and quadratic hazard functions perform differently. After estimating the mean nonlinear regression relating the gap with the aggregate employment fluctuations I regress the average employment change on the mean and variance of the gap before adjustment is made. I observe that in the case the adjustment is huge (there is a wide range of full adjustment generated), the nonlinear approximation through the exponential Cox proportional form fits much better the real values of average employment fluctuations than the linear regression of the aggregate employment fluctuation on the mean and variance of gap before adjustment is made. But when the adjustment is insignificant (the adjustment range is very small), the latter estimation affords a goodness of fit much better than the previous one. The Durbin-Watson test supports the lack of first order correlation in residuals in both cases when they are prevailing. I show below a table with simulated results of the estimated parameters of the three regressions, their goodness of fit, Durbin –Watson results on the serial correlation of their residuals.

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[8] At the end of this chapter I will describe largely how the data used in these estimations methods are generated.

[9] See W. Greene “Econometric Analysis”, Chapter 9 for a proof of the asymptotic normal behavior of error under a large number of observations.

[10] See Table 1 and 2 at the next pages.

Table 1

Aggregate Regressions Results (full adjustment case)

N=2000, T=50

	Exponential $\Lambda_t(x) = 1 - \exp(-\beta x)$ **(My function)**	**Mean linear** $\Lambda_t(x) = \beta x$ **(CE 1st function)**	**Mean variance linear** $\Lambda_t(x) = \beta x^2$ **(CE 2nd function)**
β **(s.e)**	24.8622 (1.81)		
β₁ **(s.e)**		1.602 (0.011)	0.3141 (0.013)
β₂ **(s.e)**			0.3616 (0.004)
R^2	0.9174	0.8591	0.9152
$R^{2\ \text{adjusted}}$	0.9157	0.8561	0.9116
DW	2.0770	2.2636	1.7084

Table 2

Aggregate Regressions Results (no adjustment case)

N=2000, T=50

	Exponential $\Lambda_t(x) = 1 - \exp(-\beta x)$ **(My function)**	**Mean linear** $\Lambda_t(x) = \beta x$ **(CE 1st function)**	**Mean variance linear** $\Lambda_t(x) = \beta x^2$ **(CE 2nd function)**
β **(s.e)**	0.4943 (0.082)		
β₁ **(s.e)**		1.0335 (0.0045)	-1.4694 (0.0119)
β₂ **(s.e)**			0.5021 (0.0023)
R^2	0.6125	0.325	0.8643
$R^{2\ \text{adjusted}}$	0.6044	0.540	0.8585
DW	3.6391	3.7389	2.0119

Below I show two graphs of the residuals behavior in both cases and for the three regressions.

Figure 1

No adjustment case

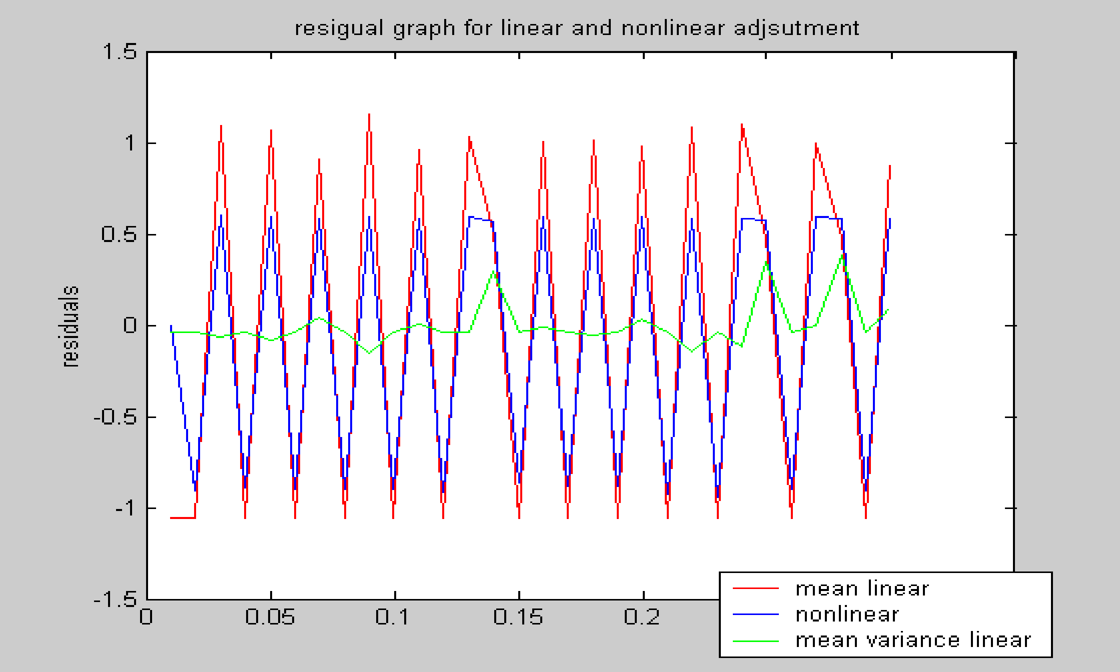
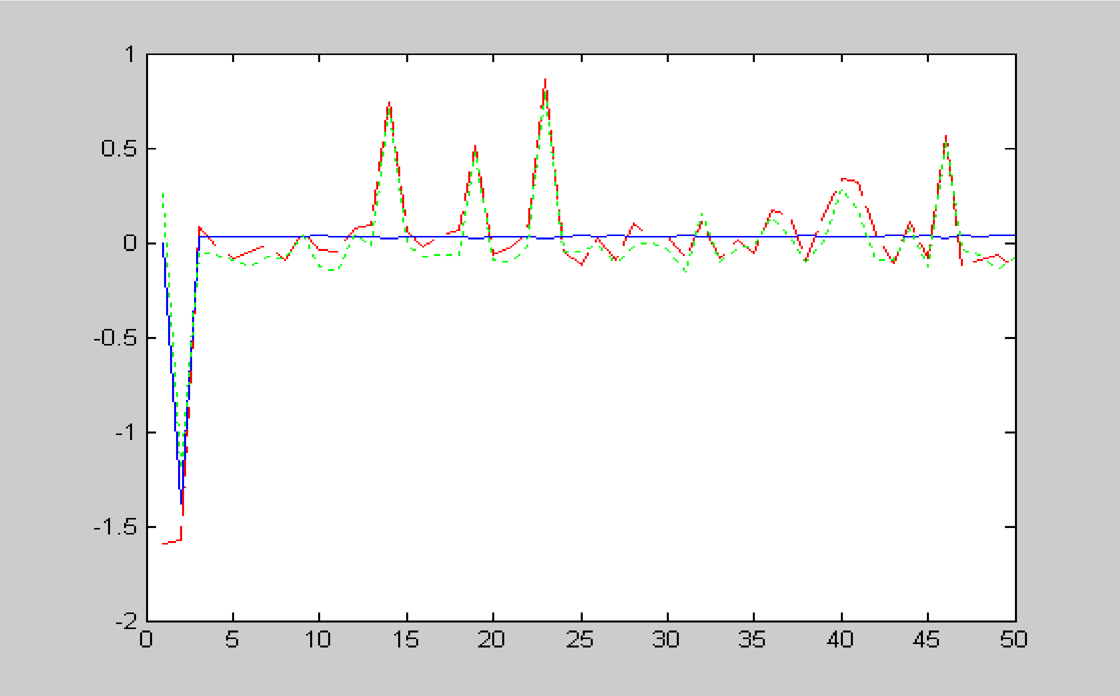


Figure 2

Adjustment case



B. A Theoretical Example Considering Gap as Normal Variable

If one does know that the gap follows from a well-known density (lets say a normal or gamma distribution) before adjustment is made, then the integral representing the aggregate employment change could be resolved directly.

Lemma 1[11]

Suppose that the variable x_{it} t is normally distributed with mean m and standard deviation s. Then

$$(8) \qquad E\left[x_{it}(1 - x_{it}\exp(-\beta x_{it})\right] = m - (m - \beta s^2)\exp(-\beta m + \frac{\beta^2 s^2}{2})$$

It follows that the aggregate employment integral will take the form as in equation (2), which means that I have a nonlinear regression function that looks like:

$$(9) \qquad \Delta A = m - (m - \beta s^2)E\left[\exp(-\beta x_{it})\right]$$

where $E\left[\exp(-\beta x_{it})\right]$ represents the mean probability of no adjustment
and $E\left[1 - \exp(-\beta x_{it})\right]$ represents the mean probability of adjustment

Considering the expression (9), if x_{it} represents the gap and ΔA represents the aggregate employment fluctuation then the relationship (9) means that an increase in the expected value of the probability of non adjusting will decrease the aggregate employment change. On the other hand, the expected value of non-adjustment probability increases if the fraction of firms that don’t adjust is augmented.

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[11] See Appendix 1 for a proof of Lemma 1

Now lets consider the situation when two different (in size) shocks occur. Due to the first shock, the gap variable is distributed normally with mean zero and a given variance s^2 and due to the second shock the gap is distributed normally with mean m and the same variance s^2 as in the first case. The difference between the first aggregate employment change and the second aggregate employment change ($\Delta A(0)$ and $\Delta A(1)$) represents the aggregate employment reaction to the shock size. This difference is given below as:

(10) $$\Delta A(1) - \Delta A(0) = E[\phi(1)](m - \beta s^2) + E[\phi(0)]\beta s^2$$

where $E[\phi(1)]$ and $E[\phi(0)]$ are the mean probability of adjustment in each case

Under the condition of a positive β and if ($m - \beta s^2$) is positive, the above difference is certainly positive. Otherwise it could be even zero which means no adjustment is made. The same story could be said for a negative shock. This implies that size doesn't matter for the aggregate employment change. What really matters is the distribution of shock determined by its moments.

C. Exponential Bound for the Aggregate Employment Change

In order to know something more on the properties of the aggregate employment change I study its exponential bound making use of the following Lemma.

Lemma 2[12]

Let x be an independent random variable. Suppose $0 \leq x \leq 1$ and let β be a real number. Then

(11) $$[\exp(\beta) - 1]E(x^2) \geq E[x(1 - \exp(-\beta x))] \geq [1 - \exp(-\beta)]E(x^2)$$

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[12] See Appendix 2 for a proof of Lemma 2

If I denote with x (as I have done so far) the value of the gap to be adjusted by the firm then the above inequation implies that the smallest value the expected value $E[x(1-\exp(-\beta x))]$ could take is zero, but this could happen only in case the mean of x (gap) is zero which is a value to be economically excluded.

In this way I could show that even for small values of x, the aggregate change in employment is non smaller than a certain value determined by the variance and the mean of the gap distribution suggesting that there is adjustment even for small values of x but it could be inconsiderable. So even for small deviations of the current employment value from the target there is a tiny adjustment process taking place.

But this result contradicts the overall theory and evidence of no adjustment for the small values of gap or the inaction range of adjustment. Therefore either I should abandon the exponential hazard model or I should deny the inaction range of adjustment process. It seems as this mathematical logics could not advocate the use of exponential Cox proportional hazard function and for the gap value smaller than one this complies with the empirical results as well.

The inaction range is symmetric around zero because the parameters of the adjustment hazard function were taken as equal either for positive gap or for negative one. If the asymmetric aspect will be considered by permitting the adjustment hazard coefficient to be different this could introduce skewness in the adjustment aggregate distribution by changing the inaction range.

D. Data Generating Process

In this part I will describe how the data used in the three estimation methods given above are generated. I start this process generating three different shock variables (productivity, wage and demand) as a random walk. Then, based on the first order conditions[13] of the production function, and some fixed parameters, I make use of shocks

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[13] See the next chapter for the right expression of the fist order conditions.

variables to construct the path of employment desired level. Assuming further that the adjustment process of employment is lumpy and infrequent at firm level, I derive the path of actual employment according to a lumpy order considering a given threshold such that if the deviation of the desired value of employment from the actual one is bigger than the given threshold I say the employment adjusts fully and set the actual value equal to the desired value. Otherwise I set it equal to the previous employment value.

After having derived the paths of desired and actual employment, I take their difference and call it the gap variable x_{it} as in the expression (1). To derive the variable ΔA_t (which denotes the aggregate employment change), I take the average of all generated gaps across firms either for positive or negative gap values and use them separately in the estimation procedure.

These generated data will be used to assess different estimation procedures. Therefore the final step is to regress in three different ways (using a Cox proportional, linear and quadratic hazard function) the constructed variable ΔA_t on the gap variable x_{it}. To distinguish between the three regression methods, I compare their respective goodness of fit and the residuals. It is obvious to opt for the regression that enjoys the best goodness of fit and the smallest residuals (Tables 1 and 2) and to conclude accordingly

IV.

Simulated Firm Model & Gap estimation

CE[14] model a sector with non-convex adjustment costs where the average employment change is an increasing function of the deviation of the desired employment value from the actual one. The desired level of employment is the number of employees the firm would keep in if adjustment costs were temporarily off and is equal to the frictionless level the firm would hold if it never faces adjustment cost plus a firm specific

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[14] Caballero and Engel (1993, 1997)

constant. They assume further that the deviation between the desired from the actual value of employment is likely to be stationary over time.

Accordingly, I model a sector with a large but fixed number of oligopolistic competitive firms where each firm faces isoelastic demand for its product under a Cobb-Douglas technology with constant returns in its factor demands, labor and hours. Both demand and technology are affected by shocks which follow a joint geometric random walk. Firms face a wage curve that increases in the average number of hours worked as:

$$w_{it} = w_t + g(h_{it}) \tag{12}$$

where w_t denotes the wage shock and g(.) is a nonlinear function of hours

Also firms always choose the same number of hours[15] worked per worker in the absence of the employment adjustment costs and adjust to the shocks only by varying employment. The firm production function[16] and demand at a given instant of time t are given by

$$y = (\alpha e + \beta h) + a \tag{13}$$

$$p = -\left(1 \big/ \eta \right) y + v \tag{14}$$

where y, e, h, p, a and v denote output, employment, hours per worker, price and productivity and demand shock, respectively while α and β denote the employment and hours share respectively

Under these assumptions, firms maximize the current revenues with respect to employment (facing no adjustment costs) given that h (hours) is at its frictionless optimal level. The first order condition of this maximization problem with respect to employment results in an expression of difference target employment levels in terms of productivity, wage and demand shocks as follows:

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[15] See Sargent (1978), Shapiro (1986) and Bils (1987) for a formalization of this idea

[16] For simplicity I exclude the capital form the production function.

(15) $$\Delta e^{*} = \frac{\gamma\Delta a + \Delta v - \Delta w}{1-\alpha\gamma} \qquad \text{where} \qquad 1-\alpha\gamma \neq 0$$

As I mentioned at the end of the previous chapter, the path of actual and desired employment values will be derived based on the expression (15). To obtain an expression for the hours I assume further there are no adjustment costs in average hours and the first order condition of firm maximization problem with respect to hours will yield:

(16) $$h = -\frac{e}{\theta} - \frac{a-w}{\beta\gamma-\mu} \qquad \text{where } \theta = \frac{\mu-\beta\gamma}{1-a\gamma} \text{ and } \beta\gamma-\mu \neq 0$$

and (μ-1) is the elasticity of marginal wage schedule with respect to average hours worked.

This implies that firms first fix the employment at its optimal level and then determines the level of hours to be performed as a function of employment. Given the path of changes in hours, the path of changes in wages could be derived as:

(17) $$\Delta w_{it} = \Delta w_{t} + \mu\Delta h_{it}$$

Both first order conditions yield an expression that links the change of employment with the change in hours and shock variables and also a relationship regarding the gap construction:

(18) $$x_{it} = \theta(h_{it} - \overline{h}_{i}) + \Delta e_{it}$$

(19) $$\Delta e_{it} = -\theta\Delta h_{it} + \Delta e^{*}{}_{it}$$

where Δe^*_{it} represents the sum of all the shocks primitively considered in this model. The expression (18) implies that the estimation of the parameter θ would help to measure further the gap x_{it}.

One recognize that the equation (19) resembles to a standard regression form if some replacements are made as :

$$(20) \qquad \Delta e_{it} = const_i - \theta \Delta h_{it} + \varepsilon_{it}$$

where ε_{it} corresponds to the overall shock after removing the (constant) individual effect and the coefficient $\theta = \frac{\mu - \beta\gamma}{1 - a\gamma}$ is increasing in the elasticity of marginal wage schedule with respect to average hours (μ-1).

The economic intuition behind the expression (19) suggests that when a firm faces two possibility of adjustment, it will prefer the less costly one. Hence, for a given value of shock, the higher the marginal cost of changing hours, the higher is the management of adjustment by changing employment than by changing hours. Then, if one possesses data on employment and hours change of a large set of firms and regresses the employment change variable on hours change variable, an estimated value of the parameter θ is obtained.

But this model suffers from the endogeneity problem because hours are used to hold part of the shock, at least in short run, when employment doesn't adjust fully as such is the case. This brings about a correlation between the change in hours and the error term. Therefore a simple OLS estimation of θ could yield biased estimates.

If I do not possess data on hours per worker per plant, I can substitute this variable by the wage variable and instead of the regression (19) I'll get:

$$(21) \qquad \Delta e_{it} = \frac{-\theta \Delta w_{it}}{\mu} + \frac{\theta \Delta w_t}{\mu} + \Delta e^*_{it}$$

This regression seems more attractive because it contains a smaller shock term compared to the shock term of the expression (19) as displayed below:

$$(22) \qquad \frac{\theta \Delta w_t}{\mu} + \Delta e^*_{it} \prec \Delta e^*_{it}$$

and by making the same replacement as above I'll get:

$$(23) \qquad \Delta e_{it} = \frac{-\theta \Delta w_{it}}{\mu} + error$$

where the error term is smaller than in the previous case and therefore even the OLS estimation method could produce better estimates of the coefficient θ / μ in spite of the endogeneity problem the regression still suffers from (but it is of lesser importance).

In the following part I'll see estimation techniques either dealing with the wage employment fluctuation regression or with the hour employment fluctuation regression.

IV Estimation

Differently from Caballero and Engel, I take into consideration the fact that another way to estimate the parameter "theta" in the regression (19) is through an instrumental variable, which should be highly correlated in time with the endogenous variable (change in hours) and not correlated at all with the "error term". Hence, I will consider two situations when the gap variable is constructed based on an one-threshold model and on a two-threshold model.

Assume there exists such a variable. Applying the law of iterated expectation, the lag value of this variable will not be correlated with the error term if this error term has a mean zero (the error represents the shock variable that by assumption follows a random walk). Under the same motivation, if the expected value of the change in hours is sufficiently different from zero, the lag value of IV variable would be correlated with it. According to the same statistical law, the higher the variability displayed by hours the bigger the correlation of the one-lag change in wage with the change in hours, the better IV estimation. Also the assumption made about the behavior of shocks as random walk guarantees the mean very close to zero of the error term.

Below I try to check the validity of this IV estimator and compare it with the OLS estimator as well. In the case of 2-option adjustment model (with only 1 threshold), the variable hours displays too little variability and therefore the expected value of change in hours is around zero. This would invalidate the use of one lag wage change as instrumental variable. For all values of shocks' variances, the OLS estimation provides bad estimates of theta. An increase (in digit) in the variance of shocks (demand, productivity and wage) will provide even worse theta estimates and higher full adjustment probability. This could be explained by the fact that both variables (hours and employment) display little variability in this model. When the variances are very small there is a high range of zero adjustment probability. Small variances of shocks lead to small values of employment (many of them less than the threshold) which is translated as non-adjustment. When the variance increases, the full adjustment range increases also (there are values of employment bigger than the threshold). Even the increase of one of the shocks' variances could be sufficient to increase the range of full adjustment.

Thus, I could say that for any variance of shocks, using one-lag change in wages as an instrumental variable gives a bad estimate of theta. IV estimation doesn't resolve the problem because of the low correlation of the instrumental variable with the endogenous one. The change of threshold value affects only the values of adjustment probabilities and the OLS and IV estimators still remain invalid.

When another order (with more than one threshold) is employed to describe the employment adjustment process, for any variance of shocks, OLS estimation offers better estimators than IV estimation. OLS estimator is valid because the variables display high variability in this model. IV estimator is valid because of the strong correlation between the IV variable and hours change variable. But independently of the validity of the estimators, IV estimation is not better than OLS estimation.

Thus, the results of the above models show that IV estimation via one lag wage change provides a poor estimation. The data are simulated taking into consideration 1000 observations and 1500 replications. The further increase of number of observations and replications does not affect the results.

To finish, I consider the expression (22) and regress the employment fluctuations on the wage fluctuations to estimate the coefficient θ / μ. Either in the full adjustment case or in the little adjustment case I obtain better estimates of the coefficient than through the other regressions and methods.

Below I display the OLS and IV theta estimators' statistics in each cases and some graphs of the deviation of the estimated coefficients from their real value for each adjustment case and for each estimation method.

Table 3

One option case

(Real θ = -5, real θ/μ =-2.6)

1500 replications

	Theta OLS	Theta IV	θ/μ
Mean	-3.1853	0.6962	-4.2395
Standard deviation	0.7006	5.3123	0.1456
Range	4.3467	32.7986	1.0560
Max	-1.0572	16.6913	-3.6995
Min	-5.4039	-16.1073	-4.7555
Kurtosis	2.8963	2.7449	2.9584
Percentile (25)	-3.6611	-3.0343	-4.3393
Percentile (75)	-2.7196	4.4307	-4.1410

Figure 3

Theta wage- θ / μ

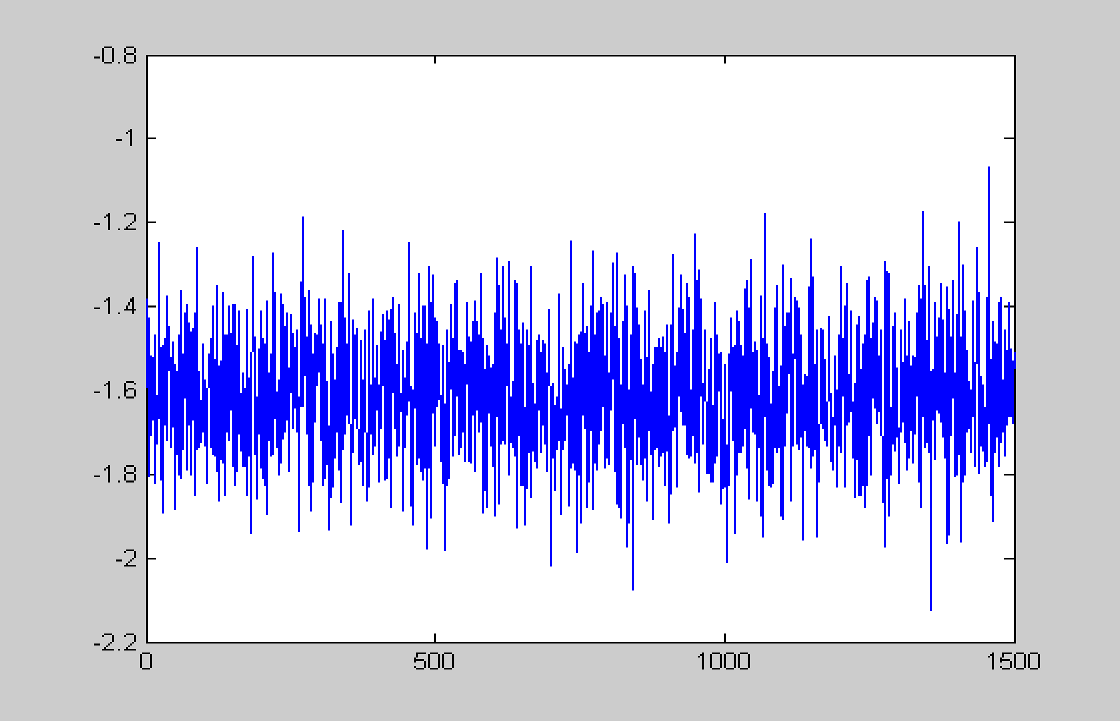


Figure 4

Theta OLS - θ

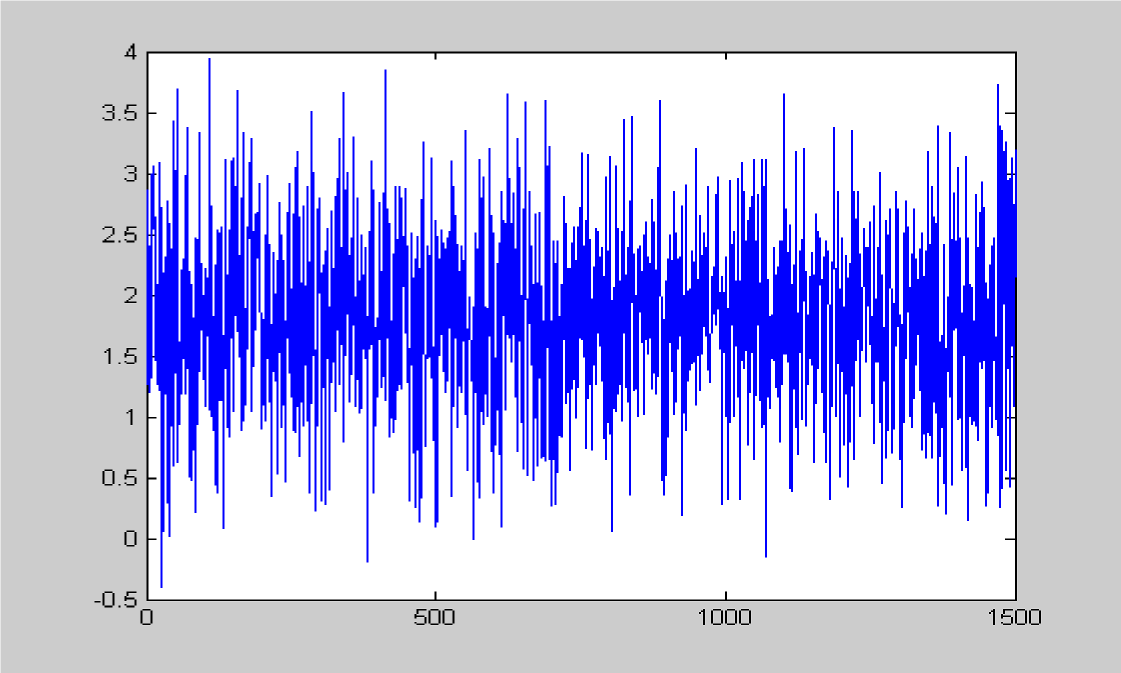


Figure 5

Theta IV- θ

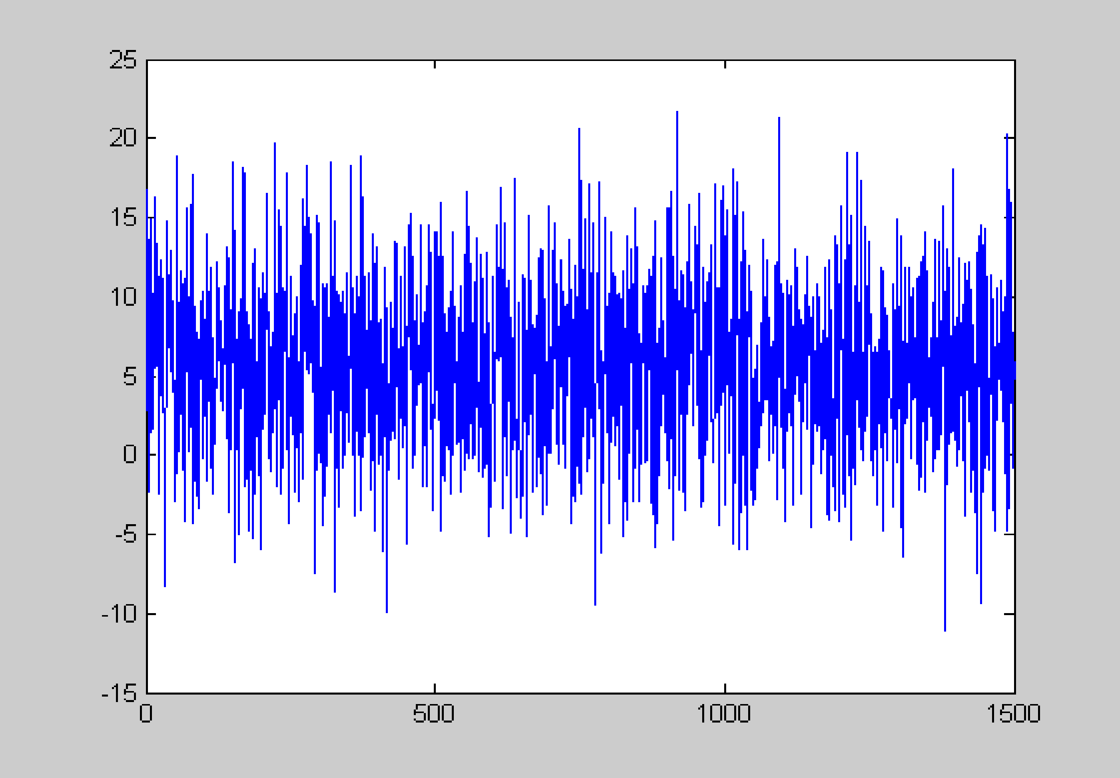


Table 4

Two option case

(real θ = -5, real θ/μ=-2.6)

1500 replications

	Theta OLS	Theta IV	θ/μ
Mean	--4.9567	-5.0477	-2.6154
Standard deviation	0.0137	0.6057	0.0060
Range	0.0834	4.2736	0.0385
Max	-4.9156	-2.6376	-2.5963
Min	-4.9990	-6.9112	-2.6348
Kurtosis	2.8141	-2.9778	2.9810
Percentile (25)	-4.9663	-5.4568	-2.6193
Percentile (75)	-4.9470	-4.6238	-2.6113

Figure 6

Theta wage- θ / μ

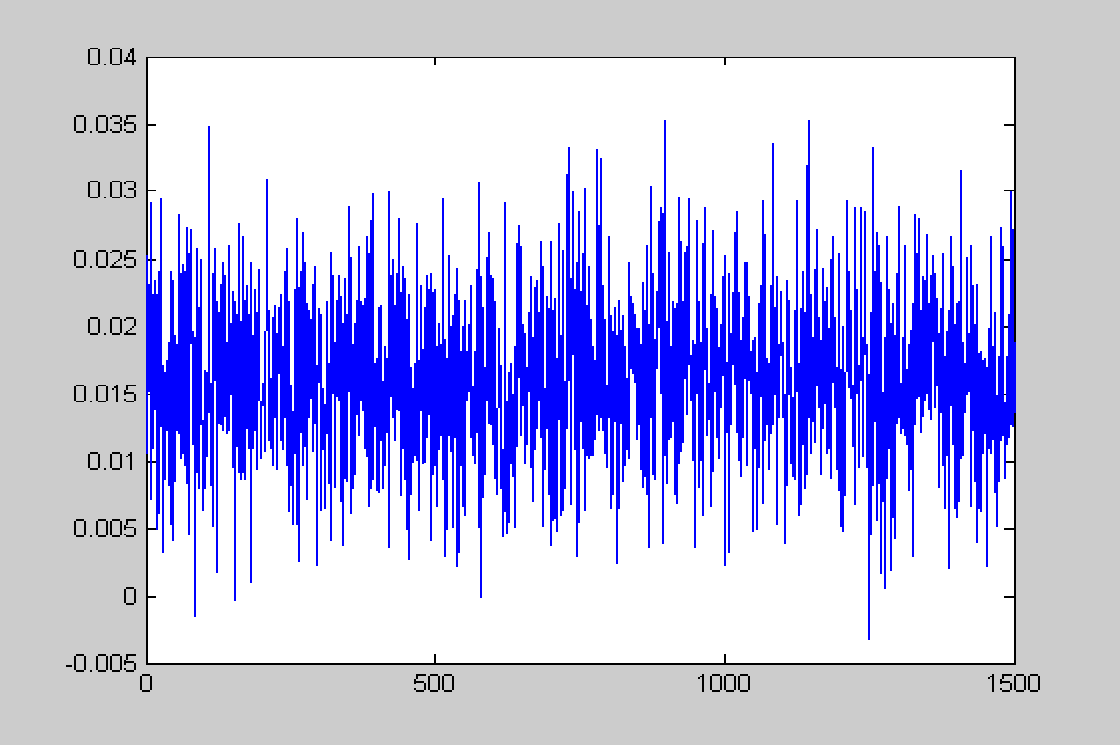


Figure 7

Theta OLS - θ

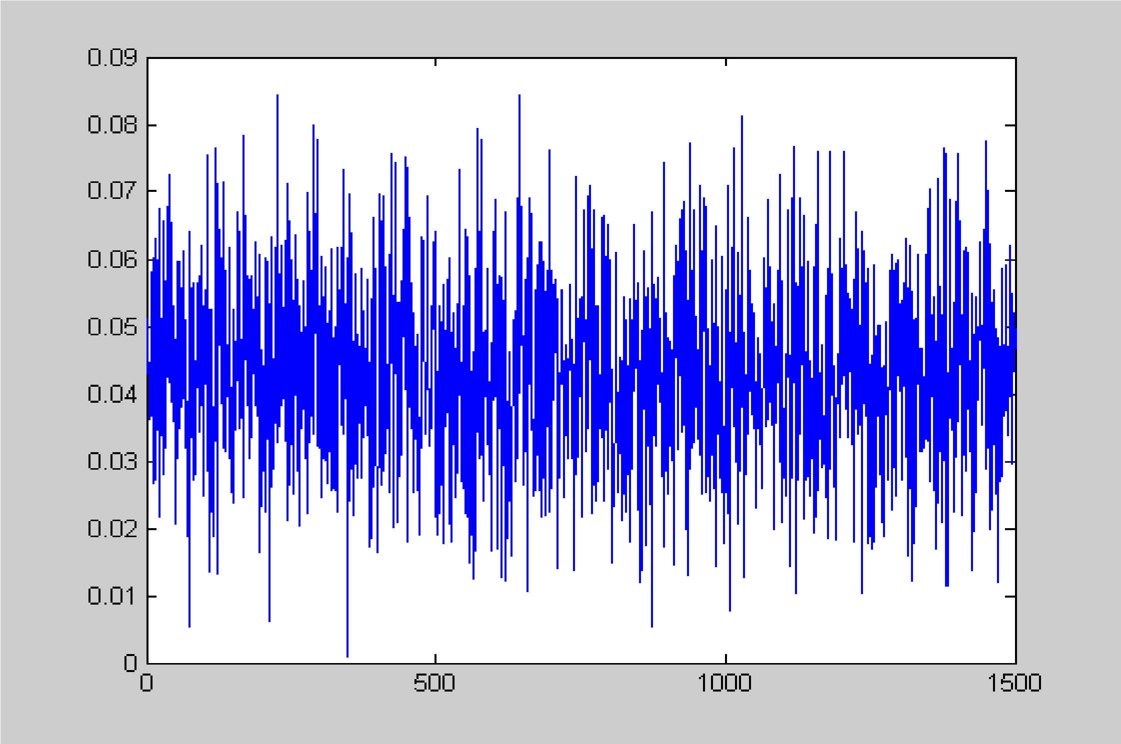
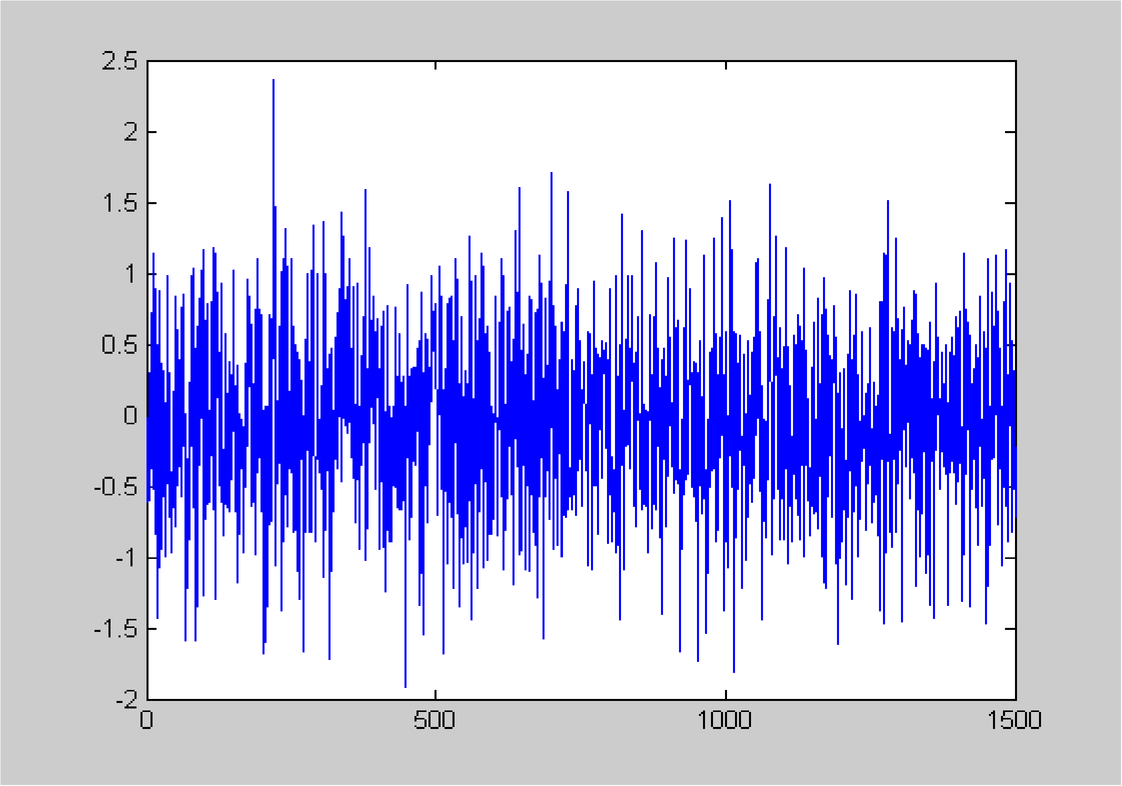


Figure 8

Theta IV- θ



Conclusion

In this paper I have analyzed the aggregate behavior of employment adjustment using simulated data according to a lumpy adjustment model at the firm level. Using the CE model I found that an Cox exponential hazard function dependant on the deviation of the desired level from the actual one would explain better the aggregate employment change than the CE quadratic hazard function in case that data simulation process complies with full adjustment process. But in case that data simulation process is in line with few adjustment process, CE quadratic hazard function performs much better. Also the simulated data has shown that the regression of the employment change on the wage change could give better estimates and measures of the deviations of the desired value of the employment from the actual one.

In sum, this paper suggests that would be better to measure the gap considering the relationship between the wage change and employment. Also it would suggest to use some exponential modeling of the hazard function in case that data exhibit full adjustment behavior and a quadratic modeling in the opposite case. Anyway this paper does not advocate any other form for the remaining range of employment adjustment.

References

Bils, M. (1987), “The Cyclical Behavior of Marginal cost and Price”, American Economic Review, 77, 838-855

Blinder, Alan S., “Retail Inventory Behaviour and Business Fluctuations,” Brookings Papers on Economic Activity, 2(1981), 443-505

Bresnahan, Timothy and Valerie A. Ramey (1994), “Output Fluctuations at the Plant Level Data” The Quarterly Journal of Economics, MIT Press, vol. 109(3), pages 593-624,

Caballero, R. and Engel, E. (1993), “Microeconomic Adjustment Hazards and Aggregate Dynamics”, Quarterly Journal of Economics,108, 359-83.

Caballero, R. and Engel, E. (1999), “Explaining Investment Dynamics in U.S. Manufacturing: A Generalized (S,s) Approach”, Econometrica, 67(4), 741-782 .

Caballero, R., Engel, E. and Haltiwanger, J. (1997), Aggregate Employment Dynamics: Building from Microeconomic Evidence”, American Economic Review, 87, 115-137.

Caplin Andrew, 1985, “Variability of Aggregate Demand with (S,s) inventory Policies”, Econometrica, 53, 1395-1409.

Caplin Andrew and Daniel Spulber, 1987, “Menu Costs and the Neutrality of Money”, Quarterly Journal of Economics, 102, 703-725.

Caplin Andrew and John Leahy, 1991, “State Dependent Pricing and the Dynamics of Money and Output”, Quarterly Journal of Economics, 106, 638-708.

Davis, Steven and John Haltiwanger, 1990, “Gross Job Creation and Destruction: Microeconomic Evidence and macroeconomic Implications,” NBER Macroeconomics Manual, V, 123-168

Davis, Steven and John Haltiwanger, 1990, “Gross Job Creation and Destruction and Employment Reallocation,” Quaterly Jurnal of Economics, 107, no.3, 819-863.

Davis, Steven and John Haltiwanger, 1999. "On the Driving Forces behind Cyclical Movements in Employment and Job Reallocation", American Economic Review, vol. 89(5), pages 1234-1258.

Hamermesh, D. (1989), “Labor Demand and the Structure of Adjustment Costs”, American Economic Review, 79, 674-689.

Hamermesh, D. (1993), “Labor Demand”, Princeton university Press, Princeton NJ.

Hamermesh, D. and G.A.Pfann (1996), “Adjustment Costs in Factor Demand”, Journal of Economic Literature, vol. XXXIV, 1264-1292.

Rotemberg, Julio, (1987), “The New Keynesian Microfoundations”, NBER Macroeconomics Annual

Sargent, T. (1978), “Estimation of Dynamic Labor Demand under Rational Expectations”, Journal of Political Economy, 86, 1009-44

Shapiro, M.D., (1986), “The Dynamic Demand for Capital and Labor”, Quarterly Journal of Economics, 101, 513-42

Appendix 1

Proof of Lemma 1

Let

(1) $$\varphi(\beta) = E\exp(-\beta x)$$

I can write $x = m + sU$, where U is standard normally distributed. For any real number b have that

(2)

$$E\exp(-bU) = \pi^{-1/2}\int \exp(-bu - \frac{u^2}{2})du = \pi^{-1/2}\int \exp\left[-\frac{(u+b)^2}{2} + \frac{b^2}{2}\right]du = \exp(\frac{b^2}{2})\int \pi^{-1/2}\exp\left[-\frac{(u+b)^2}{2}\right]du$$

The last integrand is a normal density so the integral is therefore equal to 1. Hence

(3) $$E\exp(-bU) = \exp(\frac{b^2}{2})$$

Now let b = sβ. Then I get from (3) that

(4)

$$\varphi(\beta) = E\exp(-\beta X) = E\exp(-\beta m - \beta sU) = \exp(-\beta m)E\exp(-\beta sU) = \exp(-\beta m + \frac{s^2\beta^2}{2})$$

Furthermore, I get from (3) and (4), by differentiating φ(β) that

(5) $$E[x\exp(-\beta X)] = \varphi'(\beta) = (m - s^2\beta)\varphi(\beta)$$

(6) $$E[x\exp(-\beta X)] = (m - \beta s^2\beta)\exp(-\beta m + \frac{\beta^2 s^2}{2})$$

(7) $$E\left[x(1-x\exp(-\beta X)\right]=E(x)-E(x\exp(-\beta x))=m-(m-\beta s^2)\exp(-\beta m+\frac{\beta^2 s^2}{2})$$

Appendix 2

Proof of Lemma 2

Let consider

(8) $$\exp(-\beta x)=\exp\left[-a\beta\frac{b-x}{b-a}-b\beta\frac{x-a}{b-a}\right]$$

which by convexity could be expressed as:

(9) $$\exp(-\beta x)\leq\frac{b-x}{b-a}\exp(-a\beta)+\frac{x-a}{b-a}\exp(-b\beta)$$

For positive values of x the expression (9) could be transformed in:

(10) $$x\exp(-\beta x)\leq x\left[\frac{b-x}{b-a}\exp(-a\beta)+\frac{x-a}{b-a}\exp(-b\beta)\right]$$

(11) $$E\left[x\exp(-\beta x)\right]\leq-\frac{b-E(x)}{b-a}\exp(-a\beta)-\frac{E(x)-a}{b-a}\exp(-b\beta)$$

(12) $$E\left[x(1-\exp(-\beta x))\right]\geq E(x)-\frac{E\left[x(b-x)\right]}{b-a}\exp(-a\beta)-\frac{E\left[x(x-a)\right]}{b-a}\exp(-b\beta)$$

which could be transformed as

(13)
$$E\left[x(1-\exp(-\beta x))\right]\geq E(x)-E(x)\frac{b\exp(-\beta a)-a\exp(-\beta b)}{b-a}-E(x^2)\frac{\exp(-\beta b)-\exp(-\beta a)}{b-a}$$

If $0 \leq x \leq 1$ then

(14) $$E[x(1-\exp(-\beta x))] \geq E(x^2)(1-\exp(-\beta))$$

Likewise for negative x I'll have:

(15) $$x\exp(-\beta x) \geq x\left[\frac{b-x}{b-a}\exp(-a\beta)+\frac{x-a}{b-a}\exp(-b\beta)\right]$$

(16) $$E[x\exp(-\beta x)] \leq \frac{b-E(x)}{b-a}\exp(-a\beta)+\frac{E(x)-a}{b-a}\exp(-b\beta)$$

(17) $$E[x(1-\exp(-\beta x))] \geq E(x)-\frac{E[x(b-x)]}{b-a}\exp(-a\beta)-\frac{E[x(x-a)]}{b-a}\exp(-b\beta)$$

which could be transformed as

(18)

$$E[x(1-\exp(-\beta x))] \leq E(x)-E(x)\frac{b\exp(-\beta a)-a\exp(-\beta b)}{b-a}-E(x^2)\frac{\exp(-\beta b)-\exp(-\beta a)}{b-a}$$

If $-1 \leq x \leq 0$ then

(19) $$E[x(1-\exp(-\beta x))] \leq E(x^2)(\exp(\beta)-1)$$

therefore

(20) $$[\exp(\beta)-1]E(x^2) \geq E[x(1-\exp(-\beta x))] \geq [1-\exp(-\beta)]E(x^2)$$