Some Deadweight Losses from the Minimum Wage: The Cases of Full and Partial Compliance *

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Abstract

This paper highlights the social costs from non-price rationing of the labour force due to the minimum wage. By short-circuiting the ability of low reservation-wage workers to underbid high-reservation wage workers, the minimum wage interferes with the market's basic function of grouping the lowest cost workers with the highest productivity firms. The present paper models the deadweight loss that society bears when high reservation-cost workers displace low reservation-cost workers. When firms can evade part or all of the minimum wage, an extra deadweight loss arises. Firms with high evasive ability but low productivity may displace firms with low evasive ability but high productivity. *Keywords: Minimum wage, informal sector, deadweight loss J.E.L.* classification: D61, J30.

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I. Introduction

Administer a Rorschach free-association test to an economist and the expression "minimum wage" will make him or her shout "employment effects." This is natural. The economic literature on minimum wages speaks of little else. The literature gives minor attention to the deadweight losses from the minimum wage. The present paper suggests that paying attention to these losses leads us to new insights about how labour markets work under government regulations, and the costs at which these regulations work.

The basic insight of the present paper is that the minimum wage produces a deadweight loss from non-price rationing that may rival in size the traditional triangle loss associated with price-controls. The chance of earning the minimum wage attracts workers whose reservation wages exceed the free market wage. These high reservation wage workers have the same chance of finding work as low reservation wage workers because the minimum-wage short-circuits the ability of the low reservation wage workers to compete on price. When a high-cost worker displaces a low-cost worker, society suffers a social loss. I show how to calculate this loss.

Any discussion of the deadweight losses from the minimum-wage must pay attention to non-compliance by firms. Squire and Narueput (1997) are forceful about this point. Their work focused on the transactions costs firms pay to evade the minimum wage and the enforcement costs governments incur. The present paper looks to a different source of inefficiency: non-compliance produces a social loss by allowing some firms with good evasive skills but low outputs to put out of business firms with high outputs but poor evasive skills. Far from easing the social losses due to the minimum wage, non-compliance may make these losses worse. These insights rest on the present paper's novel assumption that non-compliance may be partial and that firms differ in their abilities to evade the minimum wage. To date the literature on the general equilibrium consequences of minimum wage evasion has assumed that compliance is complete or non-existent. I assume a continuum of evasive decisions by firms because I believe this accords better with reality. The payoff to making this assumption is that it throws light on a previously unexplored social cost of the minimum wage. The price of getting to this insight is that the assumption of a continuum of non-compliance by firms complicates the modeling of equilibrium.

I proceed by first describing a benchmark labour market in which there is no evasion. Then I examine the case where employers evade the minimum wage to varying degrees and present simulations of deadweight losses, and employment effects from the minimum wage. I also discuss the evidence on minimum wage evasion and suggest the countries to which I believe my theoretical results are applicable. In spite of focusing on the social losses of the minimum wage, the object of this paper is not to discuss the worth of the minimum wage as an instrument of government policy. Far from providing a basis for either rejecting or accepting the minimum wage, the present paper suggests that we have much to learn about the interaction of government regulations with the labour market.

II. Labour market without evasion

This section lays out a basic model of equilibrium in the labour market. The goal is to establish a benchmark free market wage and employment level against which results from later modifications to the model can be compared.

Consider an industry with an infinity of firms. The size or "measure" of firms is F. The measure can be thought of as the size of the industry and is the continuous analogue of a finite, discrete number of firms. Each firm draws its particular productive ability mp from a uniform distribution over the range $[0, MP]$. Each firm produces a fixed, infinitesimal level of output $mp \times df$ which sells for a dollar per unit. The df term emphasizes the atomistic nature of firms. I assume each firm hires only one worker for analytical convenience. Such an assumption is of no substantive consequence provided one is not interested in capturing the effects of employer size on wages. Faced with a market wage of w the firm will demand a unit of labour if $mp \geq w$. For example, firm 5 may draw $mp = 8$ and firm 6 may draw $mp = 2$. If the wage is $2.10 firm 5 will produce and firm 6 will not produce. The proportion of firms demanding labour is $Pr(mp \geq w)$. With F firms, this means that the industry demand for labour $L_d(w)$ is:

$$
\begin{aligned}
L_d(w) &= F \times Pr(mp \geq w) && (1)\\
&= F\left(1 - \int_0^w \frac{1}{MP}\right) && (2)\\
&= F\left(1 - \frac{w}{MP}\right) && (3)
\end{aligned}
$$

Labour demand is not stochastic, as the use of a distribution function might suggest. The distribution function of firms serves simply to calculate the proportion of firms who will hire workers at any given wage. Some features of equilibrium we will see are stochastic in the sense that the identity of certain producers and workers is uncertain at any given wage. Total outputs and labour supplies however will retain their non-stochastic nature.

There is an infinity of labourers with measure N who each supplies a fixed amount of labour. The worker draws his or her particular reservation wage w_r from a uniform distribution on the interval $[0, W_r]$. The proportion a labourers that chooses to work is $Pr(w_r \leq w)$. Since the measure of workers is N, aggregate labour supply $L_s(w)$ is

$$\begin{aligned} L_s(w) &= N \times Pr(w_r \leq w) && (4) \\ &= N \int_0^w \frac{1}{W_r} && (5) \\ &= \frac{N}{W_r} w && (6) \end{aligned}$$

The equilibrium free market wage w_{free} comes from setting labour demand and supply equal to each other. It is simple to show that in equilibrium this wage is

$$w_{free} = \frac{MP \times W_r}{N \times MP + F \times W_r} F \tag{7}$$

See Figure 1 for an illustration of these two curves, with their slopes and intercepts labeled.

Now impose a minimum wage on this market and on all other markets where it binds. The price floor creates deadweight loss in two ways. The first and best known loss is the traditional triangle deadweight loss. By restricting employment, the minimum wage drives a wedge between what the marginal employer is willing to pay and what the marginal worker is willing to be paid. The sum of such wedges between employment under a minimum wage, $L_d(W_{min})$, and employment in the free market, $L_d(W_{free})$, is the triangle loss. We need not worry that unemployed workers find work in other markets, because these too are under a minimum wage. The best these workers can do is enjoy the alternate uses of their time, perhaps in domestic production, given by the height of the supply curve.

The triangle loss just described is calculated by first calculating producer output between these the two levels of employment $[L_d(W_{min}), L_d(W_{free})]$. The maximum price any particular producer is willing to pay is his output from hiring labour mp. From equation (1) it is easy to see that the reservation price (the height of the demand curve) at some quantity of labour demanded L is $mp = MP(1 - L/F)$. So lost producer output due to the minimum wage is

$$\int_{L_d(W_{min})}^{L_d(W_{free})} MP\left(1 - \frac{L}{F}\right) dL = MP\left[L_d(W_{free}) - L_d(W_{min}) - \frac{1}{2F}(L_d(W_{free})^2 - L_d(W_{min})^2)\right] \tag{8}$$

The personal cost to workers of producing this output is, similarly, the area under the supply curve between the same range of labour supplies:

$$\int_{L_d(W_{min})}^{L_d(W_{free})} \frac{W_r}{N} L dL = \frac{W_r}{2N}[L_d(W_{free})^2 - L_d(W_{min})^2] \tag{9}$$

Subtracting saved worker costs (9) from lost output (8) gives the triangle loss. This loss is illustrated in Figure 1 as area A.

There is another deadweight loss from the minimum wage that has been neglected in the literature: the extra social cost of inducing workers with high reservation wages to work. To see this, consider that workers lucky enough to find employment are not necessarily those with the lowest reservation wages in the range $[L_s(0), L_d(W_{min})]$ in Figure 1. Rather, willing job candidates flock from the broader range of reservation wages defined by $[L_s(0), L_s(W_{min})]$ in Figure 1. Each one of these workers has an equal chance of getting a minimum wage job because none can compete with each other on wages. An equal chance for workers uniformly distributed in the range of reservation wages $[0, W_{min}]$ means that the average reservation wage of a worker seeking a minimum wage job will be $W_{min}/2$. The cost of total labour provided under the minimum wage will then be $(W_{min}/2) \times L_d(W_{min})$. This cost exceeds the minimum possible cost of supplying $L_d(W_{min})$. The minimum cost is the area under the labour supply curve between the indicated ranges:

$$\int_{0}^{L_d(W_{min})} \frac{W_r}{N} L dL = \frac{W_r}{2N} L_d(W_{min})^2 \tag{10}$$

Subtracting this minimum cost from the actual cost of providing $L_d(W_{min})$ units of labour gives the deadweight loss of the minimum wage due to improper selection of high reservation wage workers:

$$selection \quad loss = L_d(W_{min})\frac{W_{min}}{2} - \frac{W_r}{2N} L_d(W_{min})^2 \tag{11}$$

This loss is illustrated in Figure 1 as the area B. The concavity of the loss equation (11) indicates that as the minimum wage rises and labour employed falls, selection loss will rise, and then fall. Two opposing forces are at work. At low minimum wages some jobs are lost and this would tend to mitigate selection loss as the fewer workers there are, the less such loss there can be. The opposing force that pulls overall selection loss upwards is that as the minimum wage rises, higher cost workers are enticed into the job search and the chance that any one low cost worker is displaced by a higher cost worker increases. This means that the selection loss *per worker* increases. The balance of opposing tendencies between selection loss per worker and overall number of workers determines whether selection loss rises or falls with the minimum wage. It is easy to verify from (11) that below some critical level of labour demand elasticity, overall selection loss rises with the minimum wage.

An example can give a feel for the non-price rationing loss (area B). Consider a minimum wage of $10. A farm boy has a reservation wage of $8 an hour which represents in part the value of chores he can provide around the farm to help feed his family. If the farm boy gets the minimum wage job, the economic surplus created will be at least $2. A city boy with the same tastes for leisure as the farm boy has reservation wage of $2 and hour which represents in part the limited value he can bring to his family from working around the house. If the city boy gets the minimum wage job he will create a surplus of at least $8. If both have an equal chance of getting the job, the expected cost of labour is $.5\times\$8+.5\times\$2=\$5$. This is $3 more than the cost if the city boy were guaranteed the minimum wage job. This $3 excess is the rationing loss from the minimum wage. The minimum wage may improperly select a high cost worker where a lower cost worker could serve. I speak of expected costs in this example because there are only two workers. In the present paper's model there is an infinity of workers and so there is no expectation or uncertainty to deal with. To my knowledge the only researcher to have spoken of how jobs are arbitrarily allotted under a minimum wage is King (1974). He discusses how the lottery to get high paying jobs influences the payoffs to workers, but pays no attention to the deadweight loss from non-price rationing.

Table 1, column (4) shows the rationing loss, simulated for increasing levels of the minimum wage. The parameters of demand and supply are $N = 1000$, $F = 1000$, $W_r = 20$, $MP = 20$. Equilibrium wage in an unregulated market would be $10 and the number of jobs would be 500. When a minimum wage is imposed, the number of jobs can be read from the labour demand function (column 2). Triangle deadweight loss (column 5) rises with the minimum wage, as we would expect from the well-known properties of this form of loss. The loss from non-price rationing (column 4) at first rises, but then falls with the minimum wage. Column (7) shows that at low levels of the minimum wage, *selection loss can exceed triangle loss.*

Readers may wonder where go the efficient workers who lose the lottery for jobs. Can they not transfer their skills to another labour market, displace high cost workers there, and so make up for some of the social losses of the market they left? This secondary recapture of social gains need not concern us because I have assumed the minimum wage is imposed on all markets. The efficient workers who do not have jobs here have no higher chance of capturing jobs in other markets because those markets also choose workers by lottery. If these other markets were not covered by the minimum wage then the social loss from improper selection would not be as large as the improper selection loss implied by equations (9) and (10). The supply curve would then have to be interpreted as the value of workers in alternate labour market employment.

What would happen to the improper selection loss if workers could spend resources equally effectively to raise their probability of employment? The payoffs to getting a minimum wage job, $W_{min} - w_r$, are greater for low reservation wage workers than for high reservation wage workers. Workers with low reservation wages would have an incentive to spend more money raising their probabilities than workers with high reservation wages. These efforts to raise the probability of getting a job would undo the improper selection just described. In undoing the improper selection, workers would expend resources. These resources are the costs of "rent-seeking" and depend on the technology for transforming effort into probability of getting a job. Lott (1990) has explored the social cost of this sort of rent-seeking in a setting where workers with low reservation wages sink resources into keeping their minimum wage jobs. Later, these workers develop high reservation wages but cannot be bribed away by low reservation unemployed workers, because their investment in securing the job cannot be transferred, and compensated. Lott does not devote attention to the deadweight losses from the improper selection described here.

III. Evasion by Firms

The previous example of deadweight losses due to improper selection of high reservation cost workers is applicable only to countries where compliance with the minimum wage is perfect. Of which countries can this be said? Ashenfelter and Smith (1979) found that in the early 1970's only 65% of US firms were compliant with the minimum wage law. In their view "While substantial, compliance with the minimum wage law is anything but complete. This implies that the most useful future analyses of the effects of this law will incorporate a thorough analysis of the compliance issue. Indeed the failure to do so can lead (and has led) economists to public policy predictions that are simply silly." Card (1992) found that in California, 46% of workers directly affected by the minimum wage were paid less than the minimum wage. In a survey of research papers from various international labour organizations, Squire and Narueput (1997) report that in Mexico in 1988 as many as 66% of women in certain sectors were paid less than the minimum wage. In Morocco more than 50% of firms paid their unskilled workers less than the minimum wage. There also seems to be a variable pattern of evasion. Some firms evade a greater part of the minimum wage than others, and some firms are more likely to be evaders than others. Ashenfelter and Smith (1978) found that firms hiring a greater proportion of women than others were less likely than others to violate the law. Non-compliance is a phenomenon to be reckoned with by the researcher interested in the social costs of the minimum wage.

The above indications on the pattern of minimum wage evasion are a guide to modeling deadweight losses. As in the previous section a firm can be thought of as drawing its particular productivity mp from a range $[0, MP]$. The new assumption is that the firm can evade paying a part e of the minimum wage, so that the minimum price it will be able to offer willing workers for their services is $W_{min} - e$. The firm draws its evasive ability from a uniform distribution over the range $[0, E]$ where, by necessity, $E \leq W_{min}$. For example, a particular firm may draw $mp = 10$ and $e = 5$. If the minimum wage is $14, then the firm can make the worker an offer of as little as $9. Different firms may draw different values of e and mp.

The variable evasion parameter e may strike the reader as having fallen from the sky. Where in the jungle of literature on labour economics has such a specimen been spotted? General equilibrium treatments of the underground economy, such as those of Harris and Todaro (1970), treat evasion as all or nothing (implicitly e is either zero or one in their model). The innovation in this paper is to treat non-compliance with the minimum wage as something that can vary across employers, so that e varies on a continuum between $[0, E]$. The evasion parameter e is shorthand for the detailed inquiries into what motivates individual firms to evade the minimum wage. The most prominent among these are Ashenfelter and Smith (1979), Grenier (1982), Chang and Erlich (1985), Kim and Yoo (1989), Squire and Narueput (1997). They study the forces that drive a firm to evade the minimum wage. Their analyses are in the tradition of the Allingham and Sandmo (1972) optimal tax evasion literature. My analysis does not ask why some firms evade. I assume that all firms evade to differing degrees and consider the consequences for equilibrium and social cost. Kim and Yoo (1989), Rauch (1991), Fortin et al. (1997), Squire and Narueput (1997) consider general equilibrium in a market where firms evade, but assume that either firms evade the minimum wage completely or not at all, and as a result are unable to focus on the deadweight loss consequences from the improper selection of high-cost firms over low-cost firms. The present paper differs from theirs in that it assumes that no two firms have the same evasive abilities. This heterogeneity in evasive skills drives the unconventional equilibrium and deadweight loss results that follow.

How reasonable is the assumption that evasive abilities are uniformly distributed over firms? As mentioned earlier studies of developed countries show large amounts of non-compliance but also show large amounts of compliance. This suggests that a distribution bunched at the low end of e is more realistic than a uniform distribution. For developing countries the issue is not so clear. I stick with the uniform distribution assumption in part because it may have some bearing on developing countries and in part because my objective is to make the point that non-compliance produces a deadweight loss due to improper selection of firms. Using a more complicated distribution would create important technical problems in modeling that would distract attention from this point. Even if it is not accurate for a developed country, the uniform distribution can still be of practical interest. The deadweight loss results will be more extreme using the uniform distribution assumption than using the assumption of a distribution grouped around the low end of e. This means my results will provide an upper bound on the deadweight loss from improper selection of firms.

1. Demand

Consider the aggregate demand curve for labour. Demand at any particular underground wage W_u in this market is the number of firms times the proportion of firms for whom output exceeds what they pay for labour $Pr(mp \geq W_u)$ and the proportion of this proportion which is able to evade the minimum wage sufficiently $Pr(W_{min} - e \leq W_u)$:

$$L_d(W_u) = F \times Pr(mp \geq W_u)Pr(W_{min} - e \leq W_u) \tag{12}$$

The functional form is a simply multiple of probabilities because of the assumption that productive and evasive abilities are uncorrelated. Noting that $Pr(W_{min} - e \leq W_u) = Pr(e \geq W_{min} - W_u)$, labour demand can be reduced to

$$L_d(W_u) = F\left(1 - \int_0^{W_u} \frac{1}{MP} dmp\right)\left(1 - \int_0^{W_{min}-W_u} \frac{1}{E} de\right) \tag{13}$$

$$= \frac{F}{E \times MP}(E - W_{min} + W_u)(MP - W_u) \tag{14}$$

For $W_u > W_{min}$ i.e. a non-binding minimum wage, equation (14) reduces to equation (3), labour demand under no evasion.

2. The Punctured Supply Curve

Equation (14) will come in handy briefly, but first note that there cannot be just one underground wage W_u^* in equilibrium. Imagine gathering all firms and workers in the same room and telling them that only one underground wage will hold. An auctioneer then starts announcing wages and stops where demand is equal to supply. The problem with this auction is that after the hammer falls, firms that have high potential output but insufficient evasive ability to bid down to the unique equilibrium wage must, by the way I have constructed evasive and productive abilities, have unsatisfied demand. These frustrated firms are bound to advertise their presence to workers so as to get out of the one-wage auction. Workers will not even bother to participate in the auction. Instead $L_s(W_{min})$ of them will flock to firms with the poorest evasive abilities and the highest salaries ($e = 0$ and $mp \geq W_{min}$). The highest wages being offered will be the minimum wage (at higher wages than this, evasive ability is irrelevant and firms are able to bargain with workers for lower wages).

The proportion of firms offering W_{min} will be the proportion whose output is greater than the minimum wage $Pr(mp \geq W_{min})$ times the frequency $f(e)$ of firms having precisely no evasive abilities ($e = 0$), which is $f(e) = 1/E$, in a narrow band de (note that f is the same for any level of avoidance because avoidance is uniformly distributed, a fact that I use later in section 2). Call this proportion $\alpha(e)$. The total number of workers to get jobs at these top paying firms with zero evasive ability ($e = 0$) is $F\alpha(0)$:

$$F\alpha(e) = F \times Pr(mp \geq W_{min}) f(e) de \tag{15}$$

$$= F\left(1 - \frac{W_{min}}{MP}\right)\frac{1}{E}de \tag{16}$$

Firms with high productivity but who are stuck with poor evasive abilities are not reticent about signaling their type to workers. If they do not advertise their desire to pay higher wages they will not be able to break out of one-wage underground auction for workers. Without workers these high-productivity, low evasive ability firms go out of business.

The remaining workers flock to firms with slightly better evasive abilities ϵ to earn an underground market wage of $W_{min} - \epsilon$. To figure out how many such workers offer their services we have to note that holes have been punched into the supply curve at even intervals between reservation wages of zero to W_{min}. The weight of these holes in the supply curve is the ratio of workers who found jobs with firms paying W_{min}, namely $F\alpha(0)$, to the number of workers who offered their services $L_s(W_{min})$. So labour supply is now

$$\Lambda_1(W_{min} - \epsilon) = \left[1 - \frac{F\alpha(0)}{L_s(W_{min})}\right] L_s(W_{min} - \epsilon) \tag{17}$$

Note that the new "punctured" supply curve is the regular supply curve, multiplied by a factor less than one which accounts for the attrition of workers along the length of reservation wages between $[0, W_{min}]$ who managed to get a job with high paying, zero evasive ability firms. I call the new "punctured" supply curve after this first iteration Λ_1. The subscript is very important to understand. Further iterations raise the subscript and change the functional form of supply. This means that Λ_1 and Λ_2 are different. Their relation lies in the recursiveness that follows from the exercise of calculating supply at ever greater increments of ϵ until the epsilons sum to e. The final iteration gives labour supply at $W_{min} - e$.

Next, an excess supply of workers flock to firms offering 2ϵ less than the minimum wage. How many? Once again we have to note that some of the workers who earlier flocked to firms offering $W_{min} - \epsilon$ were successful at finding a job. The fraction of such successes was $F\alpha(\epsilon)/\Lambda_1(W_{min} - \epsilon)$ of the supply curve $\Lambda_1(W_{min} - \epsilon)$. It is by this fraction that we must multiply what was left of the supply curve after workers flocked to firms offering W_{min}, to give us the proportion of the supply curve that has been punctured away by the time workers offer their services at $W_{min} - 2\epsilon$. This second iteration of the punctured supply curve has the following form:

$$\Lambda_2(W_{min} - 2\epsilon) = \left[1 - \frac{F\alpha(\epsilon)}{\Lambda_1(W_{min} - \epsilon)}\right] \Lambda_1(W_{min} - 2\epsilon) \tag{18}$$

After n iterations, punctured labour supply is

$$\Lambda_n(W_{min} - n\epsilon) = \left[1 - \frac{F\alpha((n-1)\epsilon)}{\Lambda_{n-1}(W_{min} - (n-1)\epsilon)}\right] \Lambda_{n-1}(W_{min} - n\epsilon) \tag{19}$$

Supply at some underground wage $W_u = W_{min} - e$ is the limit of the above term as n tends to infinity. This produces a hybrid of what we would ordinarily consider the supply equation of labour, with a factor that accounts for the attrition of workers of different reservation wages being plucked randomly from the range of workers willing to offer their services.

The fact that the minimum wage appears in both supply and demand equations illustrates that when firms can avoid all or part of the minimum wage, the minimum wage becomes "woven" into the demand curve as one of its parameters. With evasion the demand curve is not as we usually understand it because the ordering from left to right is not only based on productivities, but on a combination of productivities and an ability to avoid the minimum wage. This is what leads to a deadweight loss from improper selection of low output firms over high output firms. The same holds for the punctured supply curve. As the underground wage falls, workers of both high and low reservation costs who did not manage to get a job with high-paying, evasively inept firms, are mixed together along the upper reaches of the supply curve. This mixing is what gives rise to the deadweight losses from improper selection of high cost workers and low productivity firms.

2. Equilibrium Range of Wages

Equilibrium can be pinned down by the critical underground wage W_u^* (or identically at the critical sum of epsilons subtracted from the minimum wage) where there is no further excess supply. This is the wage at which punctured supply (the workers left in the market after their lucky colleagues have found employment with high-paying, low-evasive ability firms) suffices to satisfy the demand of firms who are both able to pay the wage, due to their evasive ability, and willing to pay it, due to their productivity. Put differently, calculating the equilibrium is a simple matter of equating the demand curve $L_d(W_u)$ expressed in equation (14) to the punctured supply curve $\Lambda_s(W_u)$ that falls out of the above iterations. The equilibrium will have the feature that there is a range of wages $(W_u^*, W_{min}]$ offered by different firms, and then one wage W_u^* offered by the remainder of firms. This remainder is the group who have among them the highest evasive talents and are left to bid against one another once the high productivity, evasively inept firms have all been satisfied by the excess labour that flocked to them. Total employment is calculated as $L_d(W_u^*)$ (the sum of firms who each pay the same wage) plus $H(W^*)$, the sum of high paying firms who faced an excess supply of workers and paid within the equilibrium range of wages $({W_u}^*, W_{min}]$. This sum of high paying firms is found by first identifying how many high paying firms with defective evasive skills there are at each wage level w between $({W_u}^*, W_{min}]$. At any wage level the number of such firms is the measure of firms F multiplied by the proportion who have a productivity higher than the wage $Pr(mp \geq w)$ and the proportion of this proportion who are too evasively inept to avoid paying less than that particular underground wage $f(w) = 1/E$. We then sum these firms over the range of underground wages $({W_u}^*, W_{min}]$ to get $H(W_u^*)$:

$$\begin{aligned} H(W_u^*) &= F \int_{W_u^*}^{W_{min}} Pr(mp \geq w) f(w) dw && (20) \\ &= F \int_{W_u^*}^{W_{min}} \left(1 - \frac{w}{MP}\right) \frac{1}{E} dw && (21) \\ &= \left(W_{min} - \frac{W_{min}^2}{2MP} - W_u^* + \frac{W_u^{*2}}{2MP}\right) \frac{F}{E} && (22) \end{aligned}$$

The above can be recognized as the measure of firms F times the integral of α with the limits of integration recast into wages instead of evasive abilities. The result is identical in either case. Provided that demand and supply meet, total employment at the equilibrium underground wage is $L_d(W_u^*) + H(W_u^*)$. If the minimum wage is very high relative to evasive abilities, there will be no equilibrium wage at which demand and supply cross, only an equilibrium lower bound $W_{min} - E$ to the above integral, at which all firms who participate in the market will have been faced by an excess supply of labour. In that case, employment is simply $H(W_{min} - E)$. These points are illustrated in the examples that follow. Figure 2 illustrates these concepts schematically.

3. Two Examples of Equilibrium

There is no obvious solution to the recursion relation for punctured supply. The appendix shows that a lower bound to punctured supply at some wage w is simply the ordinary "unpunctured" supply at that wage less the number of workers who have been hired by the firms who paid within the range $[w, W_{min}]$. I explain there why this lower bound produces slight underestimates of equilibrium employment. It is these underestimates, detailed in Table 2, I use in the remaining discussion.

Figure 3 looks at the equilibria that fall out of minimum wages of $11 and $18 (the supply curve used is the underestimate of supply described in the appendix). The demand curve in Figure 3 is taken from equations (3) and (14) with the same parameters used in the previous sections. Recall that without a minimum wage, market equilibrium would produce a wage of $10 and employment of 500. We can see that demand is backward bending. The downward sloping part of the curve at wages above W_{min} is the piece of the "ordinary" demand curve above the minimum wage, as given by equation (3). The upward sloping part of the demand curve comes from equation (14) and incorporates the fact that below the minimum wage some firms can evade. W now becomes W_u. At lower levels of W_u there are two opposing forces. Firms with low productivity are enticed to demand labour. This would tend to raise demand. But firms with high productivity who cannot evade well are forced out of the market because they are willing but not able to offer the underground wage. This second effect dominates the first and gives the demand curve in this zone its downward slope. We normally understand a demand equation to be an ordering of firms according to their *willingness* to pay. The lower part of the demand equation in Figure 3 is a mix of willingness to pay and *ability* to pay. This is what gives the lower part of the curve its upward slope. At a minimum wage of $11 (recall that the free market wage is $10), demand and supply meet and as the Figure 3(a) shows, the lower bound of wages W_u^* is $9.97. Equilibrium labour employed in this case is found by plugging $9.97 into the demand function for labour, $L_d(9.97) = 449$, and adding to this the number of firms $H(9.97) = 49$ who were constrained to pay higher wages spanning from this lower bound to W_{min}. The result is an employment level of 498. This total cannot be read off the diagram because, as explained, the crossing of supply and demand determine only the cutoff wage W_u^*, not the employment level.

When $W_{min} = 18$ the shape of both supply and demand curves changes. The change reflects that the minimum wage is a parameter woven not just into the supply curve, but also into the demand curve. In Figure 3(b), where $W_{min} = 18$, supply and demand no longer meet. The rise in the minimum wage has created a situation where no lowering of the market wage will reduce the excess of labour supply over labour demand. In this case employment is just $H(W_{min} - E) = 350$, which cannot be read from the graph. What takes some getting used to is that there is an equilibrium *even though demand and supply do not cross*. All firms in this case face an excess supply of workers at the wages they are able to offer. In columns (1) and (2) of Table 2 we see a sequence of minimum wages, and the lower bounds of wages these imply. The lowest of these lower bounds is $7.95 (see column 2) and occurs when the minimum wage is $17.50. For any minimum wage higher than this, demand and supply do not meet, which explains why at $W_{min} = 18$ the curves separate.

One of the striking things about equilibrium is that it takes place over a range of wages. There is an equilibrium price dispersion that has nothing to do with search, or uncertainty about the parameters of demand and supply. Prices are dispersed because the differing evasive abilities of firms interact with the minimum wage in such a way as to give workers some ability to act as price discriminating monopolists. An equilibrium of this sort has a surprising consequence for the incomes of employed workers. As Table 2 shows, provided evasive talents are large enough (so that $W_{min} - E$ is less than the free market wage), the equilibrium range of wages $[W_u^*, W_{min}]$ dips below the free market wage. In other words, some workers "lucky" enough to find a job may discover they are working for wages below what they would be earning in a free market. The intuition is that workers lucky enough to crowd into high-paying jobs are drawn from a broader range of reservation wages than would be the case without the minimum wage. Some workers in high paying jobs will be from the high end of this broadened band of reservation wages. This crowds low reservation wage workers into the contest for lower paying jobs where they cause a glut. The glut pushes wages for the remaining workers downward, and below what wages would be in a free market. Note that this is similar to the Harris-Todaro (1970) result that some workers will be made worse off by the minimum wage because employment falls in the covered sector, workers shift to the uncovered sector, bidding the wage down there.

4. General simulations

Figure 4 graphs employment under different minimum wages in two cases: when firms evade, and when no one evades. The Figure is based on data in Tables 1 and 2. What strikes the eye about Figure 4 is that at low minimum wages, employment falls little when firms evade, compared with how much employment would fall without evasion. The small employment effect of the minimum wage is in line with empirical findings by Card (1992), and Katz and Krueger (1992). The result holds because at low minimum wages, there is a sufficient number of firms with the correct combination of a relatively high output and relatively poor evasive ability. This combination allows many workers to find employment by the process of price discrimination described earlier. The large number of jobs open to price discrimination dampens the job-killing effect of the minimum wage. At higher minimum wages, there are too few firms that have the necessary high output to make price discrimination much of a brake against unemployment. When firms evade, a rise in the minimum wage shrinks the pool of firms with outputs high enough to exceed the wages they will have to pay. This simultaneously shrinks the pool of firms with potentially strong evasive abilities.

Column (2) of Table 2 shows that the lower bound of discriminatory wages W_u^* decreases as the minimum wage rises. To see why the lower bound of wages decreases, note first that a higher minimum reduces the number of firms able to offer low wages. This would tend to put upward pressure on the lower equilibrium bound of wages. The rise in the minimum wage also has a contrary effect. It swells the number of workers seeking employment, by enticing workers with previously excessive reservation wages into the hunt for jobs. It so happens that this latter effect dominates in the model I have derived. As a result the lower equilibrium bound of wages declines with a rise in the minimum wage. Rauch (1991) has found a similar result in his modeling of formal and informal markets. In his model a rise in the formal sector wage squeezes workers into the informal sector where they bid wages down. He does not model a continuum of evasive abilities, but rather, assumes that firms either evade the minimum wage completely or evade it not at all. My results from modeling a continuum of evasive abilities show that the *range* of underground wages stretches downwards, to a point, as the minimum wage increases. The familial resemblance of my result to Rauch's is reassurance that the present model is not too far off from what exists in the literature.

5. Improper Selection Costs

The low drop in employment from the minimum wage comes at a deadweight loss from improper selection of firms and workers. Recall that in the case when there was no evasion, the minimum wage led to the selection of workers with too high a reservation cost. This problem persists in the present case of evasion by firms. The workers who get the limited number of jobs once again may not be those with the lowest reservation wages. There is another source of improper selection to account for. The firms who end up producing are not chosen entirely because of their efficiency. Evasive ability is a criterion of survival. This means that some high cost firms will displace low cost firms.

To prepare the reader for the formal results about deadweight loss that follow it helps to take an extreme, and elementary example of improper selection in the labour market. Consider again Figure 1, which portrays ordinary supply and demand curves for labour. The regular result is that equilibrium is the place where the two curves meet---at an employment level of 500 (recall that there are 1000 firms and 1000 workers). Output in such a market could be greater than this. Suppose the firm with the highest output were matched to the worker with the highest reservation wage. Match the firm with the next highest output to the worker with the next highest reservation wage. Keep doing this and what you get is that all 1000 workers are employed and all 1000 firms produce. Employment is high, but the surplus produced by this market is zero. Matching of this sort violates what markets are supposed to do, namely, to maximize economic surplus. Surplus is maximized by grouping people who value highly the good or service in question with people who can supply it efficiently. The matching scheme I have just outlined produces a lot of output but makes a mess of grouping people efficiently. This example is extreme because it rests on a matching mechanism that ensures workers with the highest reservation wages get the choicest jobs and that low productivity firms are as likely to thrive as high productivity firms. This of course was not the case in the model I developed in the previous section. There, some efficient firms still managed to be matched to low reservation wage workers, so mismatching did not go haywire to the point where all surplus was dissipated and everyone earned a different wage.

To calculate the deadweight loss from the minimum wage when firms evade let us start with the improper selection of high cost firms above low cost firms. We want to calculate the actual amount produced by firms and compare it to the amount that would be produced with the same labour if there were no improper selection. The actual amount produced has two components. First there is the amount produced by firms to whom an excess supply of workers flock. At each wage level w between $(W_u^*, W_{min}]$ there are firms in the range $[w, MP]$ who may produce, as highlighted in equation (22). The proportion of all such firms whose evasive abilities is precisely w is $(1/E)dmp$. The output of these firms is:

$$F\int_{w}^{MP} mp\frac{1}{MP}\frac{1}{E}dmp = F\frac{MP^2 - w^2}{2E \times MP} \tag{23}$$

The above is a small part of the total number of firms finding themselves being offered services by a surplus of workers. These firms pay wages falling in the range $(W_u^*, W_{min}]$, so that their total output is the integral of the above equation:

$$\int_{W_u^*}^{W_{min}} F\frac{MP^2 - w^2}{2E \times MP}dw = \frac{F}{2E \times MP}[MP^2(W_{min} - W_u^*) - (1/3)(W_{min}^3 - W_u^{*3})] \tag{24}$$

The second component of total output comes from those firms who have an excess demand for workers. All of these firms have outputs greater than W_u^* and some range as high as MP. Their combined output is summed over this range and weighted by the proportion $Pr(W_{min} - e \leq W_u^*)$ whose evasive abilities exceed W_u^*, :

$$F\int_{W_u^*}^{MP} mp\frac{1}{MP}\frac{E - W_{min} + W_u^*}{E}dmp = \frac{F}{2E \times MP}(E - W_{min} + W_u^*)(MP^2 - W_u^{*2}) \tag{25}$$

The above two equations are added to get actual output. The output that would have been produced using the same number of workers L^* is the integral of the height of the demand curve over the range $[0, L^*]$. It is a simple matter to show that this comes to $MP \times L^*[1 - L^*/(2F)]$. The difference between actual and lowest cost output appears in column (6) of Table 2. This is the deadweight loss from the improper selection of high cost firms above low cost firms.

The cost of the improper selection of high reservation wage workers over low reservation wage workers is calculated in a similar manner. First sum the reservation wages of those working. Then compare this to the sum of reservation wages for a similar level of employment in an unregulated market. The sum of reservation wages of those working has two components. Some of the workers who flocked in excess to high paying firms will have found work. At each level of wage w there are, as explained earlier in reference to equation (16), $F \times Pr(mp \geq w) f(W_{min} - w)$ such firms. The average reservation wage of workers flocking to these firms will be simply $w/2$. The sum of such reservation wages is then taken over all firms who find themselves beset by an excess of workers. Recall that these firms paid wages in the range $(W_u^*, W_{min}]$:

$$\int_{W_u^*}^{W_{min}} F \times Pr(mp \geq w) f(W_{min} - w) \frac{w}{2} \tag{26}$$

$$= \int_{W_u^*}^{W_{min}} F \left(1 - \frac{w}{MP}\right) \frac{1}{E} \frac{w}{2} \tag{27}$$

$$= \frac{F}{2E} \left(\frac{W_{min}^2 - W_u^{*2}}{2} - \frac{W_{min}^3 - W_u^{*3}}{3MP} \right) \tag{28}$$

The second component of cost are the reservation wages of those employed by firms who together all bid W_u^* and together employ L_u units of labour:

$$L_u \int_0^{W_u^*} \frac{1}{W_u^*} w = L_u \frac{W_u^*}{2} \tag{29}$$

The two above components are summed to get total reservation cost. From this cost we then subtract the minimum cost of employing the same number of workers as represented in the above two equations. This cost was derived earlier and is given in equation (10). The resulting difference is the social loss from the improper selection of high reservation cost workers. Columns (6) and (7) of Table 2 show the selection costs generated by the improper selection of firms and workers.

Column (8) shows that the ratio of selection loss to triangle loss can be quite large at low levels of the minimum wage. The ratio is much larger than in the case of no evasion. With evasion, fewer jobs are lost at low levels of the minimum wage, meaning there is less triangle loss than for a similar wage in the case of no evasion. With evasion there is an extra source of selection loss: the improper selection of high cost firms. These factors combine to give the high ratios of selection to triangle loss at low minimum wages.

Another way of comparing social losses in the case of evasion and no evasion is to look at the ratio of deadweight loss to employment. Figure 5 graphs this ratio for the employment levels generated both in Tables 1 and 2. The graph suggests that for any given level of employment, the total deadweight loss under evasion is *larger* than with no evasion. This is no surprise. With evasion there is an additional source of improper selection (that of firms). Usually the underground economy is viewed as a source of economic efficiency. As Frey (1989) writes "One of the major benefits [of the unobserved economy] is often considered to be the fact that it is one of the most productive sectors of the economy, without which the population would be materially much worse off." The results here suggest that problems may arise from improper selection of firms and workers due to interaction of the minimum wage with the underground economy.

V. Empirical Implications

From an econometric point of view, minimum wage evasion poses daunting challenges. All of a sudden, the minimum wage becomes a parameter woven into the functional form of both supply and demand equations. It is beyond the scope of the present paper to present a fully-developed econometric model of labour market equilibrium with such a weighty modification to established econometric norms. But something can be said about the distribution of underground wages under a minimum wage with evasion and some data can be invoked to provide indirect support of my model's assumptions. What falls out of the equilibrium modeling of previous sections is that a minimum wage with evasion gives rise to an equilibrium distribution of jobs over a *range* of underground wages. This distribution has nothing to do with uncertainty or search. When the minimum wage rises, this underground wage distribution becomes more spread out over the higher end of underground wages and more concentrated at lower ends. To see this I present Figures 6(a) and 6(b) which are drawn from the simulations in Table 2. Figure 6(a) shows how many workers are employed at different underground wages for a minimum wage of $12.5. As described in detail earlier, there is a range of high paying firms who pay between W_u^* and W_{min}. Then there are the low paying firms all bunched at W_u^*. When the minimum wage rises to $15, the job distribution becomes even more spread out over the high end of wages and more concentrated over the low end of wages. That is, non-compliance becomes more severe, if one judges non-compliance by the number and degree to which workers are paid below the minimum wage. Is there any indication of such a change in job distributions in the empirical literature? Card (1992) studied the rise in 1988 of California's minimum wage from $3.35 an hour to $4.25 an hour. He found "a sharp decline in the percentage of California workers earning between $3.35 and $4.24 per hour" after the minimum wage was introduced. He goes on that "this change was accompanied by a sharp increase in the percentage of workers reporting exactly $4.25 per hour ... In contrast to the effect on the fraction earning $3.25-4.24 per hour, the increase in the minimum wage had virtually no effect on the fraction earning less than $3.35 per hour. This stability implies that the subminimum wage work force ... increased after the effective date of the new law. Using Ashenfelter and Smith's (1979) notion of a noncompliance rate, 31% of all workers with wage rates less than or equal to the minimum wage in 1987 earned less than the minimum. With the rise in the minimum to $4.25, the non-compliance rose to 46%." Card was not looking for the effects on the distribution of underground jobs I have suggested could arise, but his results are consistent with the notions put forth in the present paper. He found a worsening of non-compliance and a strong stability in the fraction of highly sub-minimum wage jobs even after the rise in the minimum wage. Another finding of Card's is that when the minimum wage was $3.35 per hour, the average wage paid was $2.64 with very little spread. If one looks at Figures 6(a) and 6(b), one notices a very tight concentration of jobs around the lower end of underground wages, as Card's data suggest existed in California.

While these conjectures are tantalizing, a full testing of the notions in the present paper would require that labour market equilibrium be derived under alternate assumptions about the distribution of reservation wages, firm productive abilities, and firm evasive abilities. My assumptions were that these variables were uniformly and independently distributed. Alternate assumptions such as those of a normal or skewed distribution, would give different functional forms for supply and demand for labour, and would give rise to varying likelihood ratio tests of the difference between observed job distributions over a range of underground wages and hypothesized distributions. I have not pursued modeling labour market equilibrium under these alternate assumptions for reasons of tractability. These alternate assumptions give rise to complicated closed form solutions which added little to the central theoretical points I wished to make in the present paper. These points are that the minimum wage can give rise to an improper selection of high reservation wage workers, and that when evasion by firms is considered, the minimum wage may give rise to an improper selection of high cost firms. If a critique is to be made of the uniformity assumption it is that it might understate the degree to which a minimum wage can reduce employment. A normal distribution for productive talents and reservation wages could have given supply and demand curves with greater elasticity in their central portions. These however are not distinctions which detract from the theme of the present paper.

What is the empirical relevance of the present theoretical model? There are two ways to answer this. The first comes by looking at Figure 7 which shows the percentage of output lost per worker due to improper selection (this includes output lost directly in the industry due to improper selection of high cost firms and output lost indirectly due to improper selection of high reservation wage workers) and the percentage of output lost due to triangle loss. These percentages were derived from Table 2. Against these percentages I graph the percentage loss in employment due to the minimum wage. The range of minimum wages I take is between $10 and $17 and this percentage loss in jobs is again derived from Table 2. At low levels of employment loss, improper selection dominates the triangle loss effects of a minimum wage. Under what seems the extreme assumption (in light of a range of studies too broad to cite here but of which Neumark and Wascher [1992] might be taken as representative) that the minimum wage leads to a 20% employment loss, the selection costs come out to roughly 14% of output. At the other extreme, as represented by Card (1992) of an employment loss close to zero, we still find selection losses of close to 2% of output. In calculations not shown here, I have varied the range of parameters of demand and supply but there has been no first order change in magnitudes of selection reported above. If one accepts some of the general premises of the model in the present paper as valid, then selection losses may not seem huge but are nonetheless a cost to be reckoned with in discussions of minimum wages, even when those wages lead to almost no change in employment.

To what sorts of economies are the present results applicable? Recall that "the present results" encompass two sorts of economies. The first part of the paper showed that there could be a deadweight loss from improper selection of workers even in an economy where compliance is perfect. The second part of the paper showed that when firms do not comply, there can also be an added social loss from improper selection of high cost firms. The question then is, to which countries are these two parts of the paper relevant? Squire and Narueput (1997) conclude that "Non-compliance occurs in a variety of countries and is significant even among industrialized countries." By significant they meant that in markets affected by the minimum wage non-compliance can be significant. For example, according to Card's (1992) analysis of California, the non-compliance rate was as high as 46%. Squire and Narueput add however that non-compliance is a relevant issue for only about 2% of the entire US labour force. Among this labour force it is illegal immigrants who are likeliest to be the targets of non-compliance. North and Houstoun (1976) report a ratio of .59 for the relative wages of illegal aliens to similar legal workers in the U.S. Bailey (1985) has similar findings. One could then think of the present paper's results on improper selection loss with full compliance as being more directly applicable to legal workers in industrialized countries. Illegal workers in industrialized countries and workers in third world informal sectors might be better modeled with the second part of the present paper, where non-compliance was the issue. Squire and Narueput, citing internal World Bank studies suggest that up to 16% of men in Mexico's large informal sector are the targets of minimum wage non-compliance. Regardless of which case one considers, non-compliance or compliance, the deadweight losses from the minimum wage predict by the present model are similarly worth.

VI. Policy Implications

Whenever a new deadweight loss is identified, we have to ask ourselves whether government or market institutions have not already moved to prevent the loss. Unions exercise monopoly power to raise their wages. They may also choose their members so that these members have low reservation wages. This preserves the surplus from high wages, and may be the union's way of combating needless deadweight losses from improper selection. Even if unions do their part to keep down the deadweight losses from controlled wages we must ask whether some government actions can help mitigate the problem. The US seems to have hit upon a solution by exempting certain categories of workers from being paid the full-minimum wage. These are usually young people living at home with their parents. These are likely to be individuals with low alternate values of their time and so are likely to have low reservation wages. Exempting them from part of the minimum wage makes them attractive to hire and helps them jump the queue ahead of higher reservation wage workers not exempted from the minimum wage. Unfortunately, micro-management of any part of the economy has proved a frustrating and ambiguous exercise for governments and I am reluctant to see such an intervention as a major policy implication of the present paper. A more fruitful application of the results presented here would be to use the present deadweight loss calculations to compare different methods of redistributing income. The minimum wage is, in all policy discussion, held as an income redistribution policy. But is it the best policy? Might the deadweight losses from taxation not be lower than those from the minimum wage, and as such the costs of redistributing through the income tax system cheaper than the deadweight losses of redistributing through wage control? Such questions have not been asked in the public finance or labour economics literature. The theoretical apparatus developed in the present paper points to a means by which the optimal mix of redistribution through the tax system and wage controls might be calculated.

VII. Conclusion

This paper has presented an analysis of some deadweight losses that might arise from the minimum wage. When there is perfect compliance with the minimum wage, there is a deadweight loss due to improper selection of high reservation wage workers. The minimum wage attracts these workers to the labour market and allows them to vie for jobs with lower reservation wage workers. These lower reservation wage workers cannot compete on the margin of price and the labour market becomes a lottery in which some high reservation workers may displace low reservation wage workers. The difference in reservation wages of the actual and displaced workers is a measure of the costs of improper selection that might arise from the lottery for jobs induced by the minimum wage. When firms can avoid all or part of the minimum wage, there is an improper selection of high cost firms with good evasive skills who manage to crowd out low cost firms with poor evasive skills. Two interesting results to emerge from the case when firms evade are that:

- *Employment effects:* Under the most general assumptions about evasive abilities, the minimum wage at low levels has minimal effect on employment. This result is obtained without appeal to monopsony behavior by the firms who demand labour. Employment is resistant to the minimum wage because in the presence of firms with differing abilities to evade, the minimum wage allows workers to act as if they were part of a price discriminating monopolistic collective.

- *Income change*: When employers evade, market equilibrium is not represented by a single wage, but by a *range* of wages. Price dispersion has nothing to do with search costs or imperfect information about demand and supply parameters. Some wages in the range lie below what wages would be in a free market. Workers who find employment in the lower end of wages will find that their incomes have dropped from what they were before the arrival of the minimum wage.

The central policy implication of the model has been to draw attention to the fact that there may be a deadweight loss from the minimum wage and that this loss must be calculated and compared to the deadweight loss from alternate government instruments for redistributing money.

APPENDIX

The purpose of this appendix is to derive a lower bound to the exact expression for labour supply and to develop a technique for more closely approximating labour supply than this lower bound allows. The need to work with such bounds comes out of equation (19) in the main body of the text:

$$\Lambda_n(W_{min} - n\epsilon) = \left[1 - \frac{F\alpha((n-1)\epsilon)}{\Lambda_{n-1}(W_{min} - (n-1)\epsilon)}\right] \Lambda_{n-1}(W_{min} - n\epsilon) \tag{30}$$

Let us take the first four terms of this expression:

$$\Lambda_0(W_{min}) = L_s(0) \tag{31}$$

$$\Lambda_1(W_{min}-\epsilon) = \left[1 - \frac{\alpha(1)F}{\Lambda_0(W_{min})}\right]\Lambda_0(W_{min}-\epsilon) \tag{32}$$

$$= \Lambda_0(W_{min}-\epsilon) - \alpha(1)F\frac{\Lambda_0(W_{min}-\epsilon)}{\Lambda_0(W_{min})} \tag{33}$$

$$\Lambda_2(W_{min}-\epsilon) = \left[1 - \frac{\alpha(2)F}{\Lambda_1(W_{min}-\epsilon)}\right]\Lambda_1(W_{min}-2\epsilon) \tag{34}$$

$$= \Lambda_1(W_{min}-2\epsilon) - \alpha(2)F\frac{\Lambda_1(W_{min}-2\epsilon)}{\Lambda_1(W_{min}-\epsilon)} \tag{35}$$

$$\Lambda_3(W_{min}-\epsilon) = \left[1 - \frac{\alpha(3)F}{\Lambda_2(W_{min}-2\epsilon)}\right]\Lambda_2(W_{min}-3\epsilon) \tag{36}$$

$$= \Lambda_2(W_{min}-3\epsilon) - \alpha(3)F\frac{\Lambda_2(W_{min}-3\epsilon)}{\Lambda_2(W_{min}-2\epsilon)} \tag{37}$$

The ratio of Λ's on the right hand side of the above expressions is less than one. If we were to replace this ratio by one, then each of the above expressions would be an underestimate of supply. With this modification the above could be written out as

$$\Lambda_0(W_{min}) = L_s(W_{min}) \tag{38}$$

$$\Lambda_1(W_{min}-\epsilon) = \Lambda_0(W_{min}-\epsilon) - \alpha(0)F \tag{39}$$

$$\Lambda_2(W_{min}-2\epsilon) = \Lambda_1(W_{min}-2\epsilon) - \alpha(1)F \tag{40}$$

$$\Lambda_3(W_{min}-3\epsilon) = \Lambda_2(W_{min}-3\epsilon) - \alpha(2)F \tag{41}$$

Repeated substitutions show that

$$\Lambda_3(W_{min}-3\epsilon) = L_s(W_{min}-3\epsilon) - \alpha(0)F - \alpha(1)F - \alpha(2)F - \alpha(3)F \tag{42}$$

If we iterated this process infinitely until the epsilons summed to some non- infinitesimal number e', we would find the underestimate of the punctured supply curve to be

$$\Lambda(W_{min}-e') \geq L_s(W_{min}-e') - F\int_{W_{min}-e'}^{0}\alpha(e)de \tag{43}$$

The final term in the above expression is simply the amount of labour employed by firms who are the subjects of price-discrimination. In the text an operator $H(W_{min}-e')$ was derived for the amount of labour hired by these firms. What this says is that our underestimate of the punctured supply curve is the unpunctured supply curve, less the number of workers who represent the sum of punctures at some underground wage $w = W_{min} - e$.

$$\Lambda(w) \geq L_s(w) - H(w) \tag{44}$$

The critical w at which supply and demand cross is defined in the main text as W_u^* and is calculated by equating the above expression with demand as defined by equation (20) and solving for the wage which is the lower bound of wages offered by firms to whom an excess of workers flock. By using this underestimate of supply we underestimate the degree to which workers will be able to price discriminate against firms. By limiting this effect we can expect the above approximation of the supply curve to show the minimum wage having a more adverse effect on jobs than it really does.

Another way of estimating labour supply is to treat the epsilon terms as discrete units. If I want to estimate labour supply at $W_{min} - e$ I can divide e into 40 parts Δe and take 40 iterations of the equation (42). This is easily done with a spreadsheet. The results produced (not shown here but available, with spreadsheet, on request) by this technique produce an equilibrium labour supply of two or three workers above the lower bound estimate of labour supply. For some minimum wages employment is above the free market level. This result cannot be taken to mean that the model shows that minimum wages can raise employment. Raising the iterations reduces the equilibrium labour supply, and it is possible that with an infinite number of iterations, equilibrium labour supply would be less than the free market level.

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TABLE 1

Deadweight Loss and Employment Effects of Minimum Wage W_{min} Without Evasion

(1) W_{min}	(2) $L_d(W_{min})$	(3) $L_s(W_{min})$	(4) Selection loss	(5) Triangle loss	(6) Total deadweight loss (4)+(5)	(7) Ratio selection loss to triangle loss (4)÷(5)
10	500	500	0	0	0	
11	450	550	450	50	500	9.00
12	400	600	800	200	1,000	4.00
13	350	650	1,050	450	1,500	2.33
14	300	700	1,200	800	2,000	1.50
15	250	750	1,250	1,250	2,500	1.00
16	200	800	1,200	1,800	3,000	0.67
17	150	850	1,050	2,450	3,500	0.43
18	100	900	800	3,200	4,000	0.25
19	50	950	450	4,050	4,500	0.11
20	0	1,000	0	5,000	5,000	

Note: The parameters of supply and demand used in the above simulations are N = 1000, F = 1000, W_r = 20, MP = 20. The wage that would prevail in an unregulated market is $10 and the number of jobs created would be 500. Blanks indicate a calculation is not applicable.

TABLE 2

Deadweight Loss and Employment Effects of Minimum Wage W_{min} When Firms Evade

(1) W_{min}	(2) W_u (see note)	(3) Jobs offered by high paying firms ($H(W_u^*)$ in equation 22)	(4) Jobs offered by low paying firms (see equation 14)	(5) Total employment (3)+(4)	(6) Firm selection loss	(7) Worker selection loss	(8) Triangle deadweight loss	(9) Ratio selection to triangle loss (6)+(7) ÷(8)
10.00	10.00	500.00	na	500.00	na	na	na	
10.50	9.99	474.98	24.70	499.68	0.10	3.10	0.00	3,067.14
11.00	9.97	449.86	48.82	498.68	0.83	12.30	0.03	733.51
11.50	9.94	424.52	72.43	496.95	2.80	27.47	0.19	311.27
12.00	9.89	398.82	95.60	494.43	6.60	48.50	0.62	166.81
12.50	9.82	372.60	118.43	491.03	12.81	75.33	1.61	101.47
13.00	9.73	345.64	141.02	486.66	21.93	107.91	3.56	66.80
13.50	9.62	317.72	163.50	481.23	34.41	146.29	7.05	46.38
14.00	9.49	288.55	186.05	474.60	50.53	190.59	12.91	33.45
14.50	9.33	257.75	208.87	466.62	70.43	241.09	22.28	24.80
15.00	9.14	224.87	232.23	457.11	93.92	298.22	36.80	18.76
15.50	8.92	189.33	256.49	445.82	120.38	362.71	58.71	14.41
16.00	8.65	150.35	282.11	432.46	148.52	435.68	91.25	11.18
16.50	8.33	106.89	309.72	416.61	175.92	518.92	139.08	8.73
17.00	7.95	57.48	340.24	397.72	198.34	615.22	209.21	6.83
17.50	7.50	na	375.00	375.00	208.33	729.17	312.50	5.33
18.00	8.00	na	350.00	350.00	208.33	841.67	450.00	4.20
19.00	9.00	na	300.00	300.00	208.33	991.67	800.00	2.74
20.00	10.00	na	250.00	250.00	208.33	1,041.67	1,250.00	1.83
21.00	11.00	na	200.00	200.00	208.33	991.67	1,800.00	1.22
22.00	12.00	na	150.00	150.00	208.33	841.67	2,450.00	0.77
23.00	13.00	na	100.00	100.00	208.33	591.67	3,200.00	0.43
24.00	14.00	na	50.00	50.00	208.33	241.67	4,050.00	0.17
25.00	15.00	na	na	na	na	na	5,000.00	

Note: The parameters of supply and demand used in the above simulations are $N = 1000$, $F = 1000$, $W_r = 20$, $MP = 20$. Past a minimum wage of $17.50 there is no longer an equilibrium lower wage bound W_u which firms are *willing* to offer. For higher minimum wages than $17.50 all firms face an excess supply of workers. What then becomes relevant in determining how many firms will offer jobs is the lower bound of possible wages they are *able* to offer, namely $W_{min} - E$. This is what appears in column (2) of the above table for values of $W_{min} \geq 17.50$.

Figure 1: Deadweight Loss and Employment Effects of the M
Wage: No Evasion

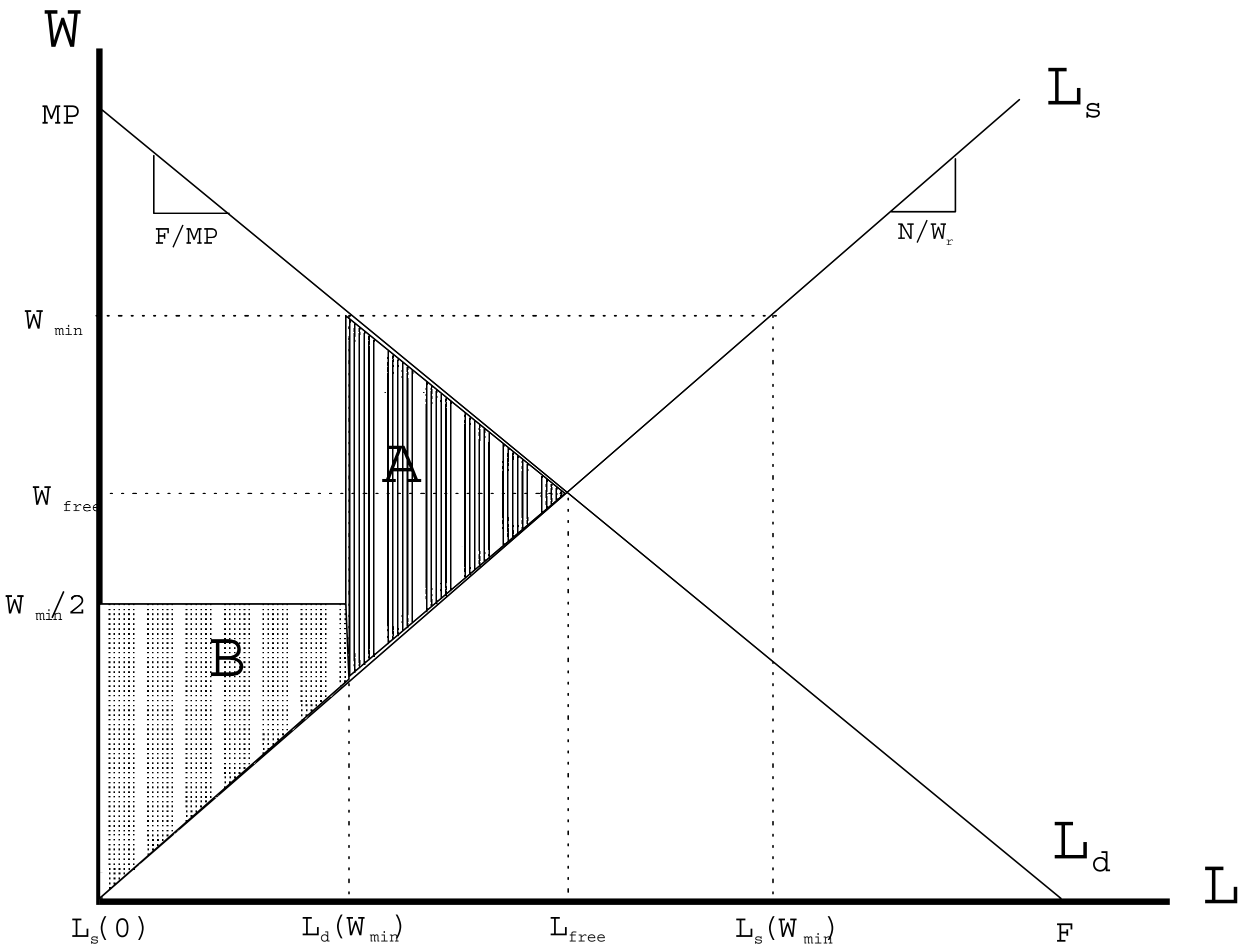


Figure 2: Range of wages

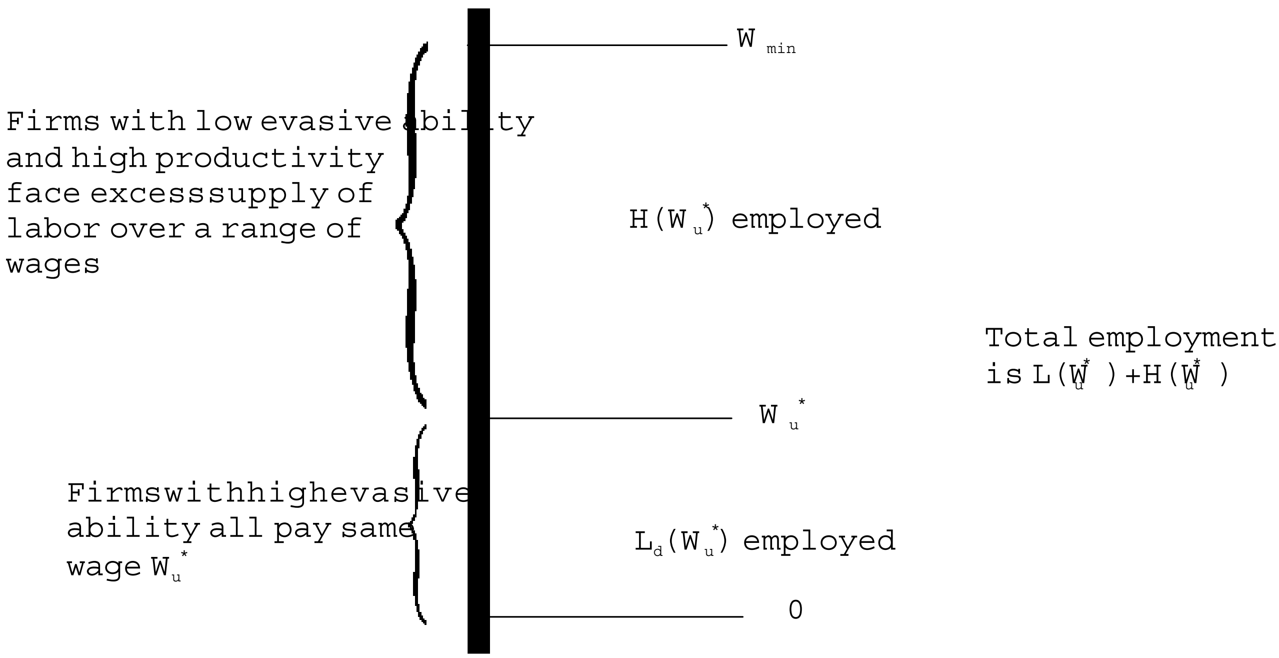


Figure 3 (a) : Demand and Supply when W_{min} =11

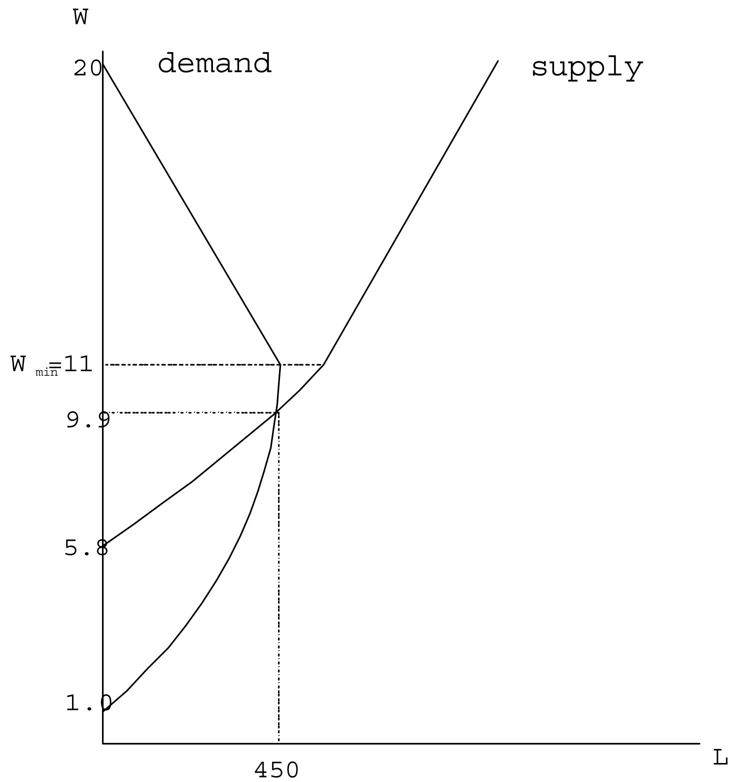
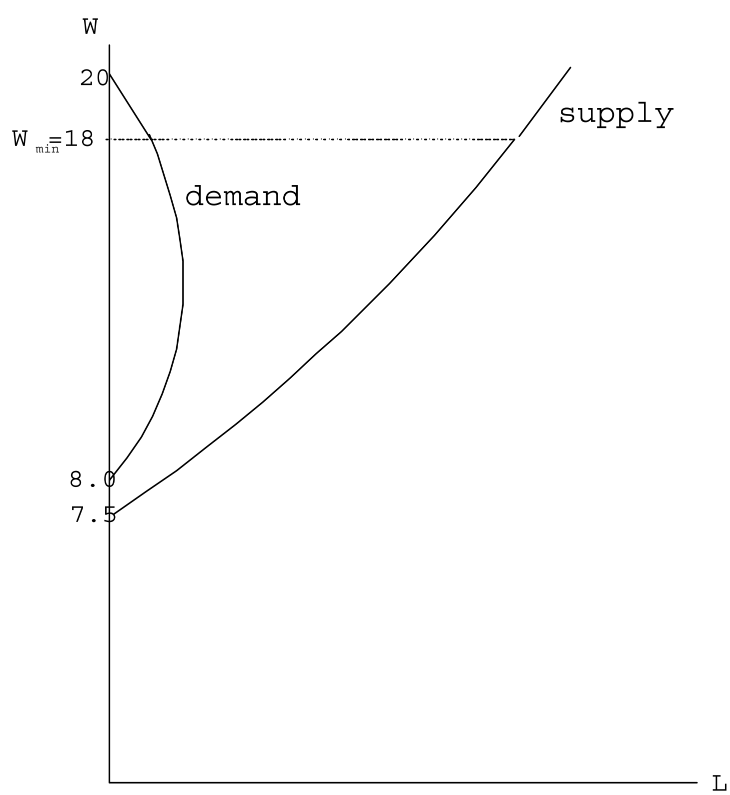


Figure 3 (b) : Demand and Supply when W_{min} =18



Note: The curves in the above figures are not schematic drawings, but precise renderings generated from closed form solutions derived in the main body of the text.

Figure 4: Employment of Effects of Minimum

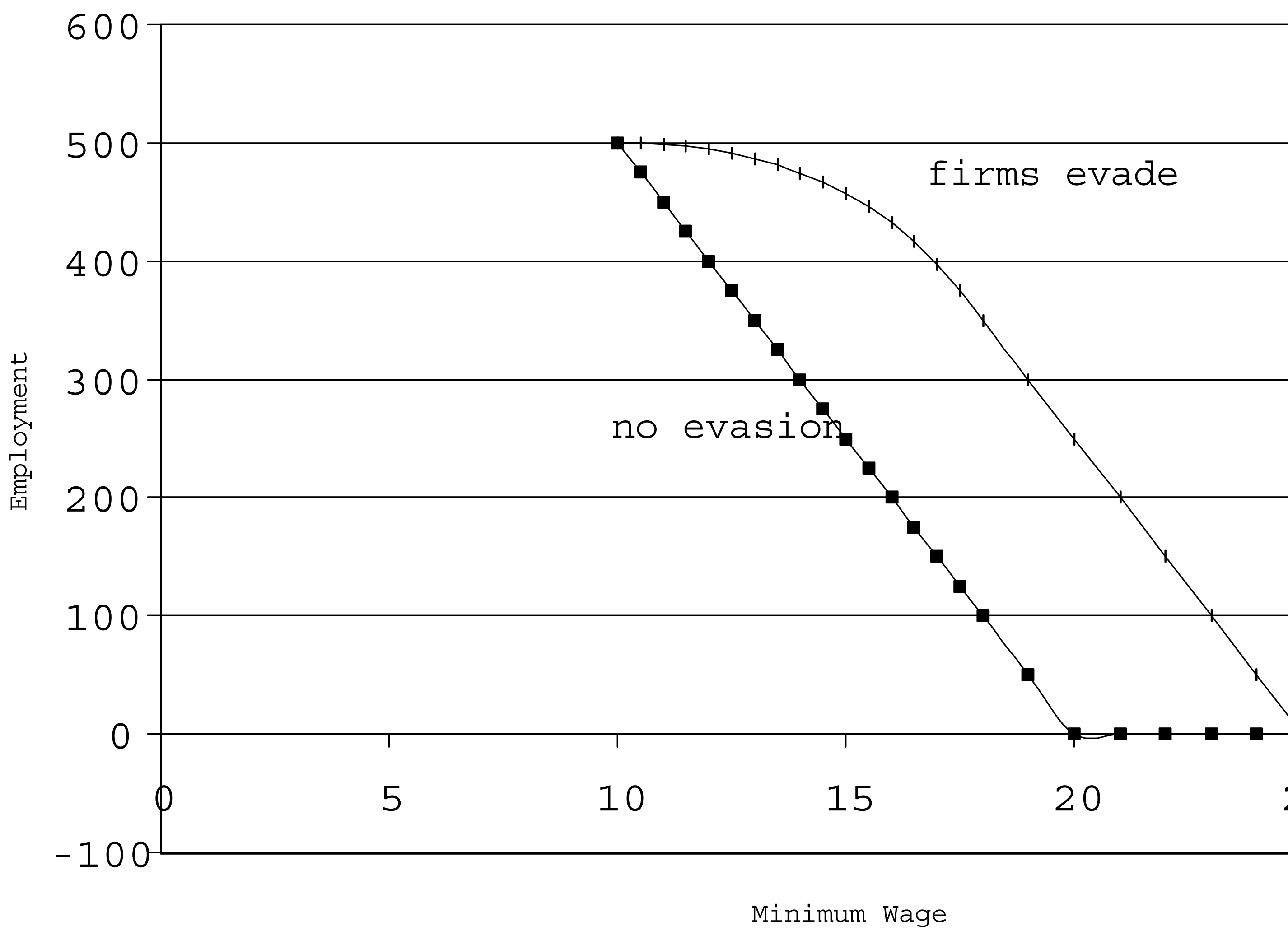



Figure 5(a): Distribution of jobs over
underground wages when Wmin=12
number of jobs
140
120
100
80
60
40
20
0
9.00
9.05
9.10
9.15
9.20
9.25
9.30
9.35
9.40
9.45
9.50
9.55
9.60
9.65
9.70
9.75
underground wage

Figure 5(b): Distribution of jobs over the equilibrium wages when Wmin=15

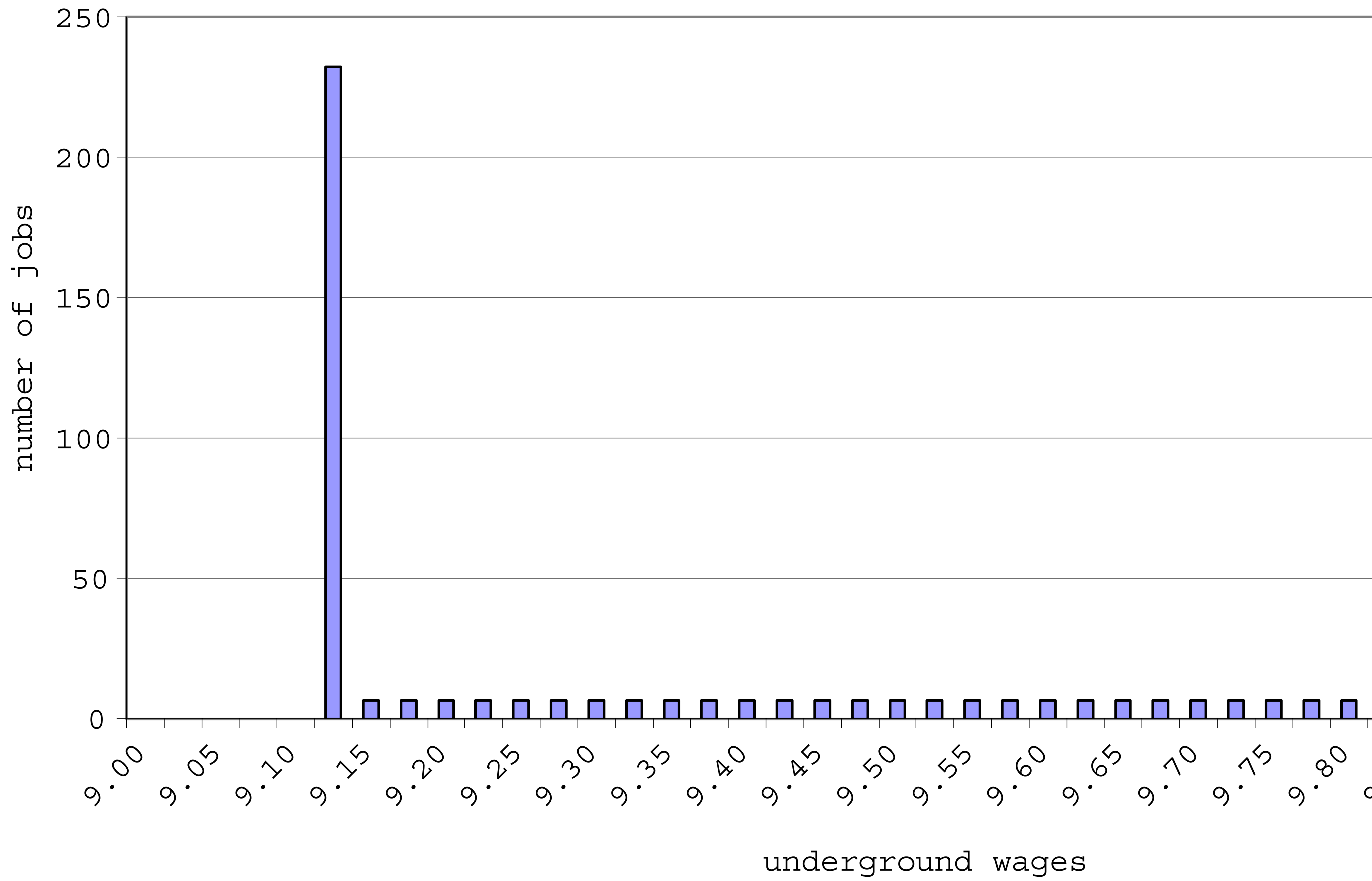


Figure 6(a): Distribution of jobs over the equilil
wages when Wmin=12.5

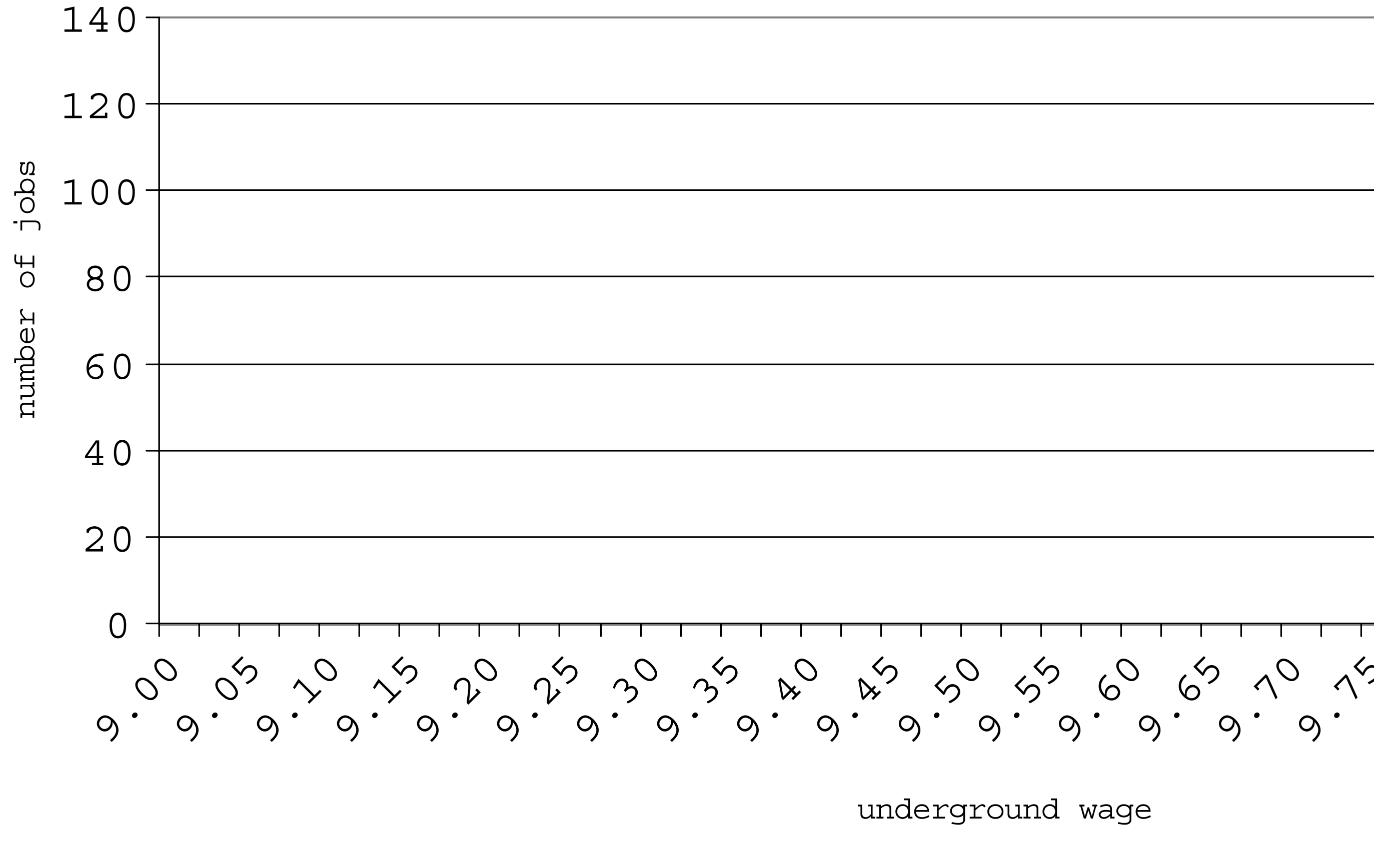
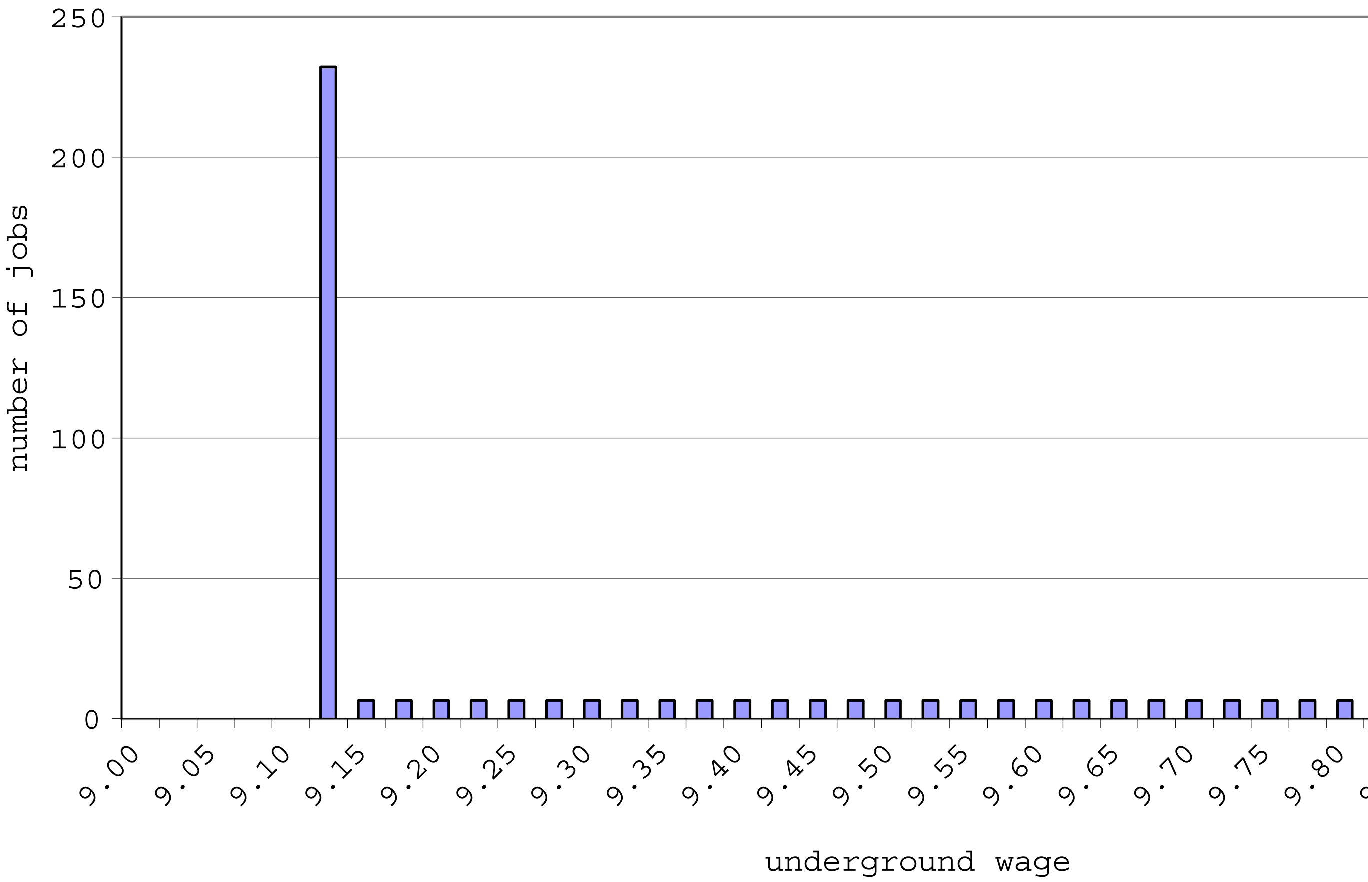


Figure 6(b): Distribution of jobs over the equilibrium
when Wmin=15



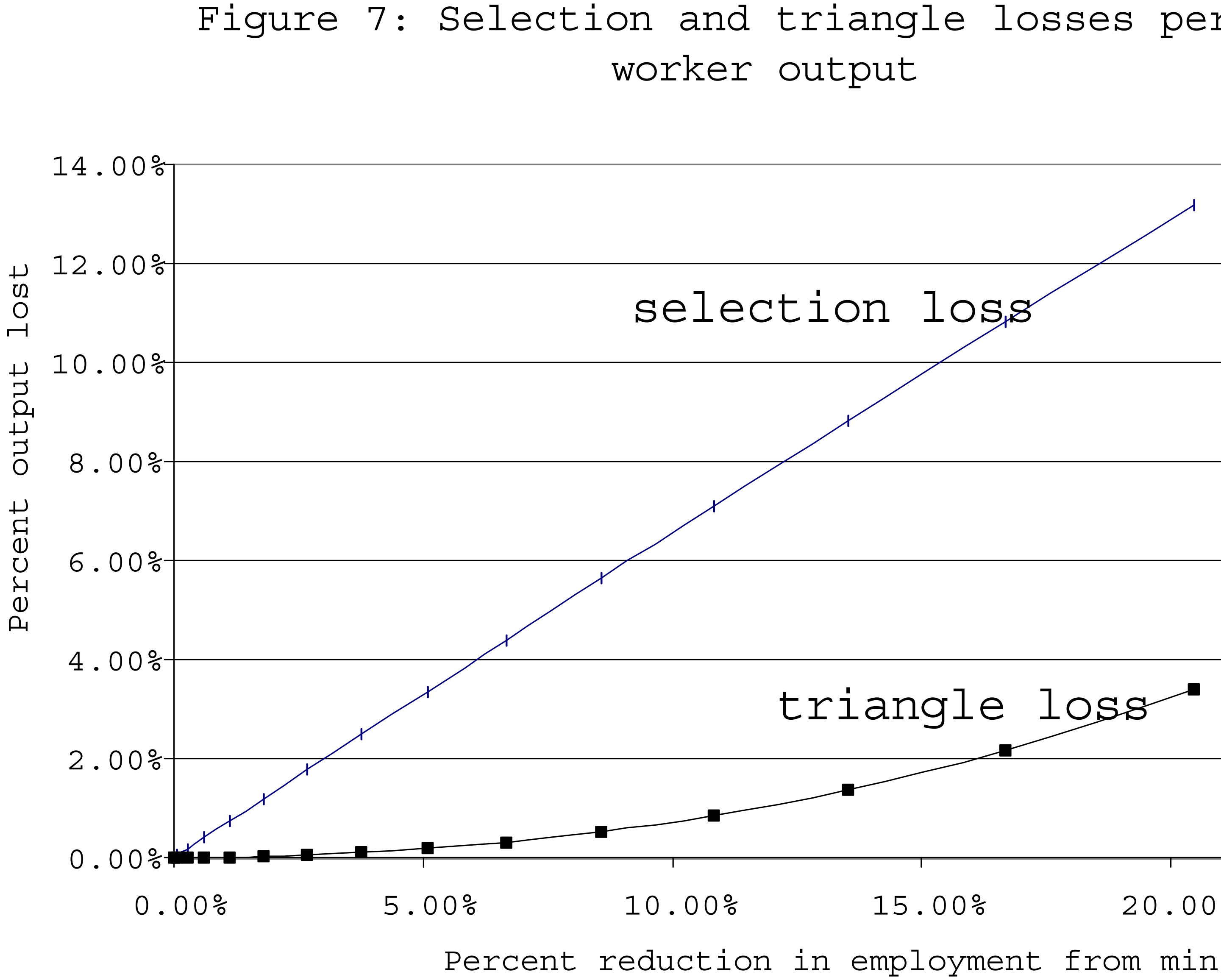


Figure 7: Selection and triangle losses per
worker output
14.00%
12.00%
10.00%
8.00%
6.00%
4.00%
2.00%
0.00%
Percent output lost
selection loss
triangle loss
0.00%
5.00%
10.00%
15.00%
20.00
Percent reduction in employment from min