On the Value of Preferential Trade Agreements in Multilateral Negotiations*

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Abstract: This paper explores the effects of preferential trade agreements (PTAs) on multilateral negotiations using a three-country, noncooperative bargaining model. PTAs are treated as outside options of the multilateral negotiation, with the feature that they continue to negotiate after they form. The organization of a PTA, whether into a customs union (CU) or free-trade area (FTA), is crucial. CUs benefit from the strategic commitment afforded by common external trade barriers, but this benefit is reduced by asymmetry between the CU partners and by discounting. It is also affected by externalities that any additional PTAs impose on members of the first. FTAs reduce the multilateral bargaining outcome effectively to one of simultaneous bilateral bargaining, whereas CUs result in a large share going to the country that has the first option of forming one. By way of example it is shown that, when CUs and FTAs are considered together, the distribution lies in between the pure FTA and CU outcomes, and there is no general presumption that relatively large countries will prefer a regime that permits PTAs to one that does not.

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I. Introduction

The recent expansion and proliferation of preferential trade agreements[1] (PTAs) in North America, Europe and elsewhere has led to a renewed interest in the topic of preferential trade and its implications for the world trading system. In a departure from the traditional concern about the welfare implications of preferential trade *per se*, this interest has largely focused on questions of a "systemic" nature: how does the proliferation of PTAs affect the evolution of a world trading system that was founded upon the principle of nondiscrimination,[2] as embodied in the General Agreement on Tariffs and Trade (GATT)? Some view PTAs as antithetical to the GATT, inevitably leading to a world of warring trade blocs, while others view them as supplemental, being just one more path by which global free trade can be approached.[3] While the recent success of the Uruguay Round of multilateral trade negotiations is arguably evidence for the latter view, no one really knows how PTAs, whether actual, anticipated or threatened, affect the *distribution* of gains from such negotiations. This paper shows that the potential for PTAs substantially alters the outcome of a multilateral negotiation, and depending on the type of PTAs in question, may effectively reduce the outcome to one of independent bilateral deals.

While the literature on preferential trade has always been concerned with the international distributional aspects of PTAs, little of it has addressed the issue within the context of multilateral trade negotiations. The most common approach has been to consider the effects of reducing tariffs on trade between an arbitrary pair of countries, while holding the tariffs on trade between that pair and the rest of the world (or "external" tariffs) fixed at exogenous levels. This has produced numerous, often conflicting welfare results (see, e.g., Lipsey, 1960; Lloyd, 1982; Wooton, 1986). Kemp and Wan (1976) drew attention to the restrictiveness of assuming exogenous external tariffs by demonstrating that the appropriate choice of external tariffs can eliminate any harmful effects of forming a PTA. Recent work has sought to endogenize external tariffs and in some cases the choice of PTA partners as well. The first of these was Riezman (1985) who assumed that the PTA and rest of the world set external tariffs

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[1]The term preferential trade agreement refers to any agreement between a group of countries to lower barriers (tariffs, quotas, etc.) to intra-group trade to a level below those on trade with the rest of the world.

[2]Article 1 of the GATT, also known as the Most-Favored Nation Clause.

[3]This terminology is due to Bhagwati (1990), which also argues that Article 24 of the GATT treats preferential trade as

optimally (at Nash equilibrium levels).[4] Several papers have extended Riezman's approach (e.g., Kennan and Riezman, 1990; Krugman, 1991; Richardson and Desruelle, 1993). Still others have let external tariffs be determined by international agreement (e.g., Bagwell and Staiger, 1993, 1994; Bond and Syropoulos, 1995; and Bond, Syropoulos, and Winters, 1995), but, for the most part, the idea that the choice of either external tariffs or PTA partners may be determined through a process of negotiation has been ignored. The present paper attempts to remedy this: it assumes that all trade agreements, whether global or preferential, and all terms of those agreements are determined through negotiations.[5]

The outcome of a negotiation depends crucially on the options available to the negotiators, and in a multilateral trade negotiation the presence of PTAs qualifies these options in two important ways. First, PTAs represent "outside" options (as in Binmore, Shaked and Sutton, 1989). Any group of countries can choose to liberalize its internal trade during, or even instead of, continued negotiation towards global liberalization. Second, once a country becomes a member of a PTA, its choices may be circumscribed by the terms of that agreement. Article 24 of the GATT allows two types of PTAs: customs unions (CUs) and free-trade areas (FTAs). In a CU countries adopts a common set of external trade barriers, while eliminating barriers to intra-union trade. An FTA is the same, except that external trade barriers need not be common. Thus individual FTA members remain free to negotiate separately with other countries and may even pursue further FTAs that exclude members of the first, creating "hub-and-spoke" arrangements (Wonnacott, 1990).[6] By contrast, a CU must always take the form of a bloc or coalition in which member countries liberalize trade with every other member but not with the rest of the world. Strategically these may be very different animals.

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supplemental to the multilateral approach. For an example of the antithesis view, see Thurow (1990).

[4]Riezman (1985) was highly original in offering a theory of the choice of PTA partners, using an adaptation of the core (external tariffs are set at Nash equilbrium levels and international transfers are ruled out). He showed that global free trade may not be in this core.

[5]The work most closely related to this is Caplin and Krishna (1990) and Ludema (1991), which look at the relative merits of the most-favored nation clause in the context of negotiations. The literature on sovereign debt also models international negotiations. The closest cousin of the present analysis is Fernandez and Glazer (1989), which studies debtor cartels.

[6]For this distinction between CUs and FTAs to be of practical relevance, FTA countries must apply their duties to products on the basis of origin. This is often a complicated matter, but one we shall not address.

To examine these aspects of PTAs, this paper sets out a three-country, dynamic, non-cooperative model of trade bargaining, in which PTAs are treated as outside options. However, unlike most outside option models, this model features multiple options (numerous combinations of PTAs available), and bargaining is assumed to continue even after an option is exercised. Because of this, the type of PTA adopted becomes crucial.

The paper proceeds by examining two related questions. First, what difference does the organization of a PTA (whether into a CU or an FTA) make to a pair of countries that have already agreed to enter into a PTA? The answer to this question will help to illuminate some of the reasons why countries might wish to form PTAs and provide a theoretical explanation for some stylized facts about existing preferential arrangements. It is shown that CUs are generally more effective bargainers than FTAs, because a CU's commitment to common external tariffs forces the rest of the world to propose only deals that would be unanimously acceptable to its members. In other words, the pair gains by foreclosing the outside option of signing additional PTAs that exclude members of the first. However, the bargaining advantage of a CU over an FTA is mitigated by asymmetry between the partners and by discounting. It is also affected, either positively or negatively, by any externalities that additional PTAs impose on members of the first. The CU form of bilateral organization favors country pairs that are relatively similar in size and dependent upon trade with the rest of the world, while the FTA form favors country pairs that trade mostly with each other and differ markedly in the sizes of their markets for goods produced by the rest of the world. Moreover, when the pace of multilateral negotiations is slow or uncertain, the relative advantage of being a CU is smaller.

The second question is: what is the distributional effect of simply allowing CUs or FTAs as options, regardless of whether or not these options are exercised? Clearly, the two questions are related by the fact we must understand the effects of PTAs when they occur to understand their role as options. However, this latter question has a motivation of its own, for by answering it we can ascertain which countries benefit and which lose from Article 24 or modifications thereof. It turns out that CUs and FTAs are both "relevant" in the sense that allowing them as options alters the distribution of gains from multilateral negotiations, but they alter it in different ways. Permitting FTAs causes the distribution of gains from a multilateral negotiation to resemble the outcome of a game in which each pair of countries bargains bilaterally in three independent negotiations. By contrast, permitting CUs gives a large share of the gains to whichever country is assumed to have the first option of making a CU proposal. It turns out that if each country has the same probability of moving first, then a world regime that permits only CUs may be more egalitarian (closer to a regime with no PTAs) *ex ante* but less so *ex post* than a regime allowing only FTAs. One implication is that countries accounting for a disproportionate amount of the world trade would prefer an FTA regime to a CU regime *ex ante*.

When both FTAs and CUs are permitted, much less can be said without the aid of specific examples. We construct an example in one country accounts for a disproportionate amount of world trade, while the two smaller countries are perfectly symmetric. The resulting equilibria reflect both the large country's preference for FTAs and the symmetric countries' preference for CUs. The distribution of world trade gains lies between that of the FTA regime and the CU regime. Whether or not a country prefers a regime permitting PTAs to regime with no PTAs depends subtlety on its relative size.

Section II of this paper sets out the general bargaining framework and establishes some equilibrium conditions. Section III applies the bargaining framework to a simple partial equilibrium model to illustrate the major results. Section IV relates the results of the previous sections to the Nash bargaining solution and the Shapley value. Section V concludes.

II. THE MODEL

A. Links, States and Payoffs

Three countries, $N = \{1, 2, 3\}$, meet to negotiate the reduction of tariffs (or equivalent instruments). For the moment, we suppress such details as the number of goods, the pattern of trade and the countries' domestic decision-making processes, and simply assume that they arrive at the bargaining table with utility functions defined over international "links" and transfers. A link (the terminology is due to Myerson, 1977) is an arrangement between two countries to eliminate their internal tariffs and set external tariffs at positive levels. For now, suppose that external tariffs are set according to some exogenous rule (which includes the tariff response of the third country), with one important distinction. If the link is a CU, tariffs must be common across linked countries, implying that no CU member may link with the third country unless both do. By contrast, if the link is an FTA, tariffs may differ across linked counties, and so it is assumed that each member of an FTA is free to link with the third country independently of the other.

To keep track of links, we define the variable $x_{ij} \in \{0, 1\}$, such that $x_{ij} = 0$ if countries i and j are linked, and $x_{ij} = 1$ if they are not. The vector $\mathbf{x} = (x_{12}, x_{13}, x_{23})$ denotes the pattern of links across countries. For any two patterns, $\mathbf{x}$ and $\mathbf{x}'$, let $\mathbf{xx}' = (x_{12}{\cdot}x'_{12}, x_{13}{\cdot}x'_{13}, x_{23}{\cdot}x'_{23})$. Now define $\mathbf{x}_{12} = (0,1,1)$, $\mathbf{x}_{13} = (1,0,1)$, $\mathbf{x}_{23} = (1,1,0)$. In words, $\mathbf{x}_{ij}$ denotes the pattern in which *only* countries i and j are linked. Likewise $\mathbf{x}_{ij}\mathbf{x}_{ik}$ denotes the pattern in which country i is linked with countries j and k , but j and k are not linked with each other (i is the hub and j and k are spokes). Note that such a pattern is only feasible if the links are of the FTA variety, because otherwise it would violate the common external tariff condition. To distinguish CU links, therefore, we denote them $\mathbf{x}^c{}_{ij}$. Thus, the set of possible link patterns are, $X^{\mathrm{f}} \equiv \{\mathbf{1}, \mathbf{x}_{12}, \mathbf{x}_{13}, \mathbf{x}_{23}, \mathbf{x}_{12}\mathbf{x}_{13}, \mathbf{x}_{12}\mathbf{x}_{23}, \mathbf{x}_{13}\mathbf{x}_{23}, \mathbf{0}\}$ for FTAs, and $X^{\mathrm{c}} \equiv \{\mathbf{1}, \mathbf{x}^c{}_{12}, \mathbf{x}^c{}_{13}, \mathbf{x}^c{}_{23}, \mathbf{0}\}$ for CUs, where $\mathbf{0} = \mathbf{x}_{12}\mathbf{x}_{13}\mathbf{x}_{23} = \mathbf{x}^c{}_{12}\mathbf{x}^c{}_{13}\mathbf{x}^c{}_{23}$ or world-wide free trade.

Let the flow of utility to country i from pattern $\mathbf{x}$ be denoted $w_i(\mathbf{x}) \in \Re$. The joint utility of i and j is denoted $w_{ij}(\mathbf{x}) \equiv w_i(\mathbf{x}) + w_j(\mathbf{x})$, and world utility, $w(\mathbf{x}) \equiv w_1(\mathbf{x}) + w_2(\mathbf{x}) + w_3(\mathbf{x})$. The changes in utility (surpluses) generated by a new link are denoted: $v_i(\mathbf{x}, \mathbf{x}') \equiv w_i(\mathbf{xx}') - w_i(\mathbf{x}')$, $v_{ij}(\mathbf{x},\mathbf{x}') \equiv w_{ij}(\mathbf{xx}') - w_{ij}(\mathbf{x}')$, and $v(\mathbf{x}, \mathbf{x}') \equiv w(\mathbf{xx}') - w(\mathbf{x}')$, for any two patterns $\mathbf{x}, \mathbf{x}' \in X$. When countries i and j link they generate for themselves a joint surplus of $v_{ij}(\mathbf{x}_{ij}, \mathbf{x})$ which we shall assume to be positive, but they also affect the third country k. The surplus of k generated by the ij link we refer to as an "externality," which can be either positive or negative. Further, the surplus generated by the first CU will generally differ from that of the first FTA, because the two links may have different external tariffs. To capture such considerations, define $c_{ij} \equiv w_{ij}(\mathbf{x}^c{}_{ij}) - w_{ij}(\mathbf{x}_{ij})$, and $e_k \equiv w_k(\mathbf{x}^c{}_{ij}) - w_k(\mathbf{x}_{ij})$.

In addition to forming links, countries can make international transfers of utility.[7] Let $y_{ij} \in \Re$ be the net transfer from j to i, let $\mathbf{y}$ be a vector of net transfers, and let $\mathbf{z} = (\mathbf{x}, \mathbf{y})$, which we refer to as the

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[7]In practice, transfers may take a variety of forms. Existing PTAs typically distribute gains by selecting products to be exempt from barrier reductions or governed by alternative arrangements. Examples in the NAFTA include autos, textiles, agriculture and the side-deals on environment and labor standards. The EU maintains the Common Agricultural Policy, as well as various social and regional funds, largely for the purpose of redistribution. While these transfers are often distortionary, it is convenient for modeling purposes to assume that transfers can be made costlessly, and that payoffs are linear in transfers.

"state." We assume that net transfers sum to zero across countries. The flow of utility to country i inclusive of transfers is defined as $u_i(\mathbf{z}) \equiv w_i(\mathbf{x}) + y_{ij} + y_{ik}$.

The payoff of each country is its expected present-discounted utility over an infinite, discrete time horizon. Letting E_ζ denote the expectation over all sequences of states $\zeta = \{\mathbf{z}(t)\}_{t=0}^{\infty}$, the payoff of country i is given by:

$$U_i = E_\zeta\, (1 - \delta)\sum_{t=0}^{\infty} \delta^t u_i(\mathbf{z}(t)) \qquad (1)$$

where $\delta \in (0, 1)$ is the discount factor.

B. The Bargaining Game

Each period begins with an existing state $\mathbf{z}$(t) and one country $i \in N$ being selected at random to make a proposal to the other two. The selection probability is 1/3 for each country each period.[8] A period contains two stages. In the first stage, the proposer i offers a pair of net transfers $y'_i = (y'_{ij}, y'_{ik})$ to which j and k respond simultaneously by either accepting or rejecting. If both respondents accept the offer, then all three countries link and j and k make transfers y'_{ij}, and y'_{ik}, respectively, to i for eternity. At this point the game essentially ends, as the agreement is assumed to be binding[9] and gains from linking are exhausted. If either respondent rejects the offer, the game moves to the second stage, in which proposer i makes another offer $y''_i = (y''_{ij}, y''_{ik})$, and the respondents accept or reject as before.[10] This time any respondent j accepting the offer links with the proposer and makes transfer y''_{ij}, and any respondent rejecting the offer remains unlinked and makes no new transfers to country i this period.

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[8]The rule of order in a noncooperative bargaining game is almost always the source of its most unappealing artifacts, and this model is no exception. The assumption of equal randomization is an attempt to be as neutral as possible in this regard. It turns out that very few of the results are sensitive to the rule of order for a high enough discount factor, and those that are can be generated by the straightfoward application of a cooperative solution concept (see section IV), suggesting a certain robustness not shared by other rules of order.

[9]We assume that once links are formed, they are unbreakable. This abstracts from the issue of enforcing these agreements, a topic treated by Bagwell and Staiger (1993), Bond and Syropoulos (1995) and Bond, Syropoulos, and Winters (1995).

[10]The assumption that country i is chosen to be the second-stage proposer is arbitrary. We could, for example, re-select a proposer at random for the second-stage without substantially affecting the results.

Note that by making offers which would surely be rejected (say, by requiring infinitely large transfers to the proposer), the proposer can effectively make a bilateral offer to a single country or no offer at all. At the end of the second stage, the game moves into the next period with a new state, and the process repeats itself.

The idea behind this set-up is to capture both multilateral and bilateral bargaining in one model. The first stage is the multilateral stage, where offers are made for global trade policy reform, and to be implemented, they must be unanimously accepted.[11] Barring that, the second stage allows countries to salvage some trade gains through bilateral deals. To be implemented, a bilateral offer need only be accepted by the country receiving it. The repetition of the process represents the idea that even bilateral negotiations take place within the context of ongoing multilateral negotiations.

The equilibrium concept used is that of *stationary* perfect equilibrium. A stationary perfect equilibrium is just a subgame perfect equilibrium in stationary strategies. Whereas a general pure strategy is an infinite sequence of functions prescribing a player's action as a function of all previous actions of all players (including nature), a stationary strategy conditions current actions only on the current state (so called, payoff-relevant histories). This restriction is desirable for several reasons: first, it is generally the case in *N*-player bargaining games that any outcome is possible when subgame perfection is the sole requirement (see, Sutton, 1986); second, the stationary equilibria of this model are robust to many alterations in the underlying game, such as making the time horizon long but finite (which is arguably the effect of legislatively imposed bargaining deadlines); third, stationary strategies are not very complicated and thus require little coordination. This seems a desirable property, given that the very purpose of a bargaining model is to explain how agents arrive at coordinated outcomes.[12]

Finally, two conventions are adopted to facilitate the exposition. First, we assume that whenever a proposer or respondent is indifferent between linking and not, they the break tie in favor of the link.[13]

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[11]This is a simplification. In practice, less-than-unanimous tariff-cutting proposals can be implemented provided they satisfy the most-favored-nation clause. The loss of generality is only very slight, as demonstrated in Ludema (1991).

[12]For more and deeper discussion of the desirability of stationary perfect equilibrium, see Herrero (1985) and Binmore (1986). For examples of the usefulness of stationarity in *N*-player noncooperative bargaining models, see, e.g., Rubinstein and Wolinsky (1985), Gul (1989), and Hart and Mas-Colell (1996).

[13]The convention is borrowed form Hart and Mas-Colell (1996), and, as in their model, it is a convenient not necessary

Second, in order to solve the model and make the value comparisons discussed earlier, we will need to examine the equilibria arising from the subgame beginning in each state. Let $U_i(\mathbf{z})$ be the expected value (1) in a stationary equilibrium, beginning in any state $\mathbf{z}$ prior to the selection of a proposer. It is useful to divide this value into two components as follows: $U_i(\mathbf{z}) = u_i(\mathbf{z}) + V_i(\mathbf{z})$, referring to $u_i(\mathbf{z})$ as the "immediate value" and $V_i(\mathbf{z})$ the "bargaining value" of state $\mathbf{z}$ to country *i*. This convention facilitates the analysis, but more importantly, it helps to distinguish the immediate effect of preferential agreements, about which most of the existing literature is concerned, from the bargaining effect, which the contribution of this paper.

C. *Equilibrium conditions*

This section sets out the equilibrium conditions in general terms. The discussion will focus on FTAs, though all the conditions carry over to CUs with minor modifications.

Once a country has been selected as the proposer, the equilibrium payoffs of the respondents will be the same as if they were to reject a multilateral offer and allow the game to enter the bilateral stage; likewise, the proposer will receive $w(\mathbf{0})$ minus the payoffs of the respondents. The reason is that the proposer need not offer the others any more their bilateral-stage payoffs to secure multilateral acceptance, and to offer less would provoke rejection and actually bring about the bilateral stage. Either way, the respondents get their bilateral-stage payoffs. The proposer gets the remainder of world utility, which will be greatest when the equilibrium is efficient (i.e., when the payoffs sum to $w(\mathbf{0})$). If the outcome of the bilateral stage were expected to be inefficient, then the proposer would choose to prevent that stage by making an acceptable multilateral offer. If the bilateral stage were expected to be efficient, then the proposer is indifferent between making an acceptable multilateral offer and reaching the bilateral stage. Either way, the proposer gets $w(\mathbf{0})$ minus the payoffs of the other two.

In the subgame of the bilateral stage, the proposer has essentially three options: A) make offers which are just acceptable to both respondents; B) make an offer acceptable to one but not the other; and C) make no acceptable offer. Under option A each respondent expects to receive exactly what it would get by rejecting, given that the other respondent accepts. Under option C, each expects exactly what it

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assumption.

would get by rejecting, given that the other rejects. Under option B, the accepting respondent gets the same payoff as under option C while the rejecting respondent gets the same as under A.

Which if these options the proposer will exercise depends on the expected world surplus generated by each one, less what is received by the other countries. Formally, if i is the proposer, j and k the respondents, $\mathbf{z}$ the current state, and we define $\mathbf{z}_{ij} = (\mathbf{x}_{ij}\mathbf{x}, y''_{ij}, y_{ik}, y_{jk})$, then the proposer's expected payoff from option A is $u_i(\mathbf{z}) + A_i(\mathbf{z})$, where $A_i(\mathbf{z})$ is given by,

$$A_i(\mathbf{z}) = (1 - \delta)v(\mathbf{x}_{ij}\mathbf{x}_{ik},\mathbf{x}) + \delta v(\mathbf{0},\mathbf{x}) - R_j(\mathbf{z}) - R_k(\mathbf{z}). \qquad (2)$$

and where

$$R_j(\mathbf{z}) = v_j(\mathbf{x}_{ik},\mathbf{x}) + \delta V_j(\mathbf{z}_{ik}), \text{ and } R_k(\mathbf{z}) = v_k(\mathbf{x}_{ij},\mathbf{x}) + \delta V_k(\mathbf{z}_{ij}).$$

The first two terms in A_i consist of the world surplus generated by moving to $\mathbf{x}_{ij}\mathbf{x}_{ik}\mathbf{x}$ for one period plus that from global free trade thereafter (as next period's multilateral agreement is expected to be efficient). The term $R_j(\mathbf{x})$ represents the surplus to j from rejecting, given that i and k will link, which consists of the externality from an ik link plus the bargaining value of state $\mathbf{z}_{ik}$ beginning next period. The term $R_k(\mathbf{x})$ is k's surplus from rejecting.

Under option B (making an offer acceptable to j alone) the proposer receives $u_i(\mathbf{z}) + B_{ij}(\mathbf{z})$, where,

$$B_{ij}(\mathbf{z}) = (1 - \delta)v(\mathbf{x}_{ij},\mathbf{x}) + \delta v(\mathbf{0},\mathbf{x}) - \delta V_j(\mathbf{z}) - R_k(\mathbf{z}). \qquad (3)$$

The difference between B_{ij} and A_i is that the intermediate pattern is now $\mathbf{x}_{ij}\mathbf{x}$, and j's expected surplus from rejecting is the discounted bargaining value of the current state. This is because, by rejecting, country j can freeze the state at $\mathbf{z}$ until next period. Finally, under option C, since the state does not change, i would receive its immediate payoff plus the bargaining value of the current state when bargaining resumes next period, or $u_i(\mathbf{z}) + \delta V_i(\mathbf{z})$.

Assuming country i chooses its best option, we can summarize the bargaining value of state $\mathbf{z}$ for a respondent country j, conditional on the proposer being country i:

$$V_j(\mathbf{z} \mid i) = \begin{cases} R_j(\mathbf{z}) & \text{if } A_i \text{ or } B_{ik} \;\varepsilon\; B_{ij} \text{ and } \delta V_i(\mathbf{z}) \\ \delta V_j(\mathbf{z}) & \text{if } B_{ij} \text{ or } \delta V_i(\mathbf{z}) \;\varepsilon\; A_i \text{ and } B_{ik} \end{cases} \qquad (4)$$

From (4), it is immediate that,

$$V_i(\mathbf{z} \mid i) = v(\mathbf{0},\mathbf{x}) - V_j(\mathbf{z} \mid i) - V_k(\mathbf{z} \mid i), \qquad (5)$$

and that the bargaining value of state **z** to country i, prior to the selection of a proposer is,

$$V_i(\mathbf{z}) = (1/3)[V_i(\mathbf{z} \mid 1) + V_i(\mathbf{z} \mid 2) + V_i(\mathbf{z} \mid 3)]. \qquad (6)$$

Solving for equilibria is now reduced to solving the system of equations (2) - (6), for all countries and all states **z**. However, even in this simplified form, this is not an enjoyable exercise, so it makes sense to reduce the problem further by focusing on particular types of equilibria.

Our primary focus will be on equilibria in which, although the agreement is reached in the multilateral stage, if the bilateral stage *were* reached, the proposer would choose to make as many acceptable bilateral offers as are possible given the initial pattern. This is a particularly interesting case because it implies that the potential bilateral deals completely determine to payoffs to the multilateral agreement. A sufficient condition for this behavior is, $A_i(\mathbf{z}) \bullet B_{ij}(\mathbf{z}) \bullet \delta V_i(\mathbf{z})$, for all i and j, which is equivalent to,

$$(1 - \delta)\min[v(\mathbf{x}_{ik},\mathbf{x}_{ij}\mathbf{x}), v(\mathbf{x}_{ik},\mathbf{x})] + \delta V_j(\mathbf{z}) - R_j(\mathbf{z}) \bullet 0 \qquad (7)$$

for all i, j, and k. By making offer A instead of offer B (to j), the proposer enables the surplus $v(\mathbf{x}_{ik},\mathbf{x}_{ij}\mathbf{x})$ to be realized this period rather than next, resulting in an efficiency gain of $(1 - \delta)v(\mathbf{x}_{ik},\mathbf{x}_{ij}\mathbf{x})$ which is entirely captured by the proposer. Similarly, by exercising option B (to k) instead of C, the proposer enables the surplus $v(\mathbf{x}_{ik},\mathbf{x})$ to be realized this period rather than next, for an efficiency gain of (1 -

$\delta)v(\mathbf{x}_{ik},\mathbf{x})$. The cost to the proposer of choosing A over B (to j) or of choosing B (to k) over C is $R_j(\mathbf{z}) - \delta V_j(\mathbf{z})$, which is the difference in what it must give to country j to secure acceptance.

In any equilibrium satisfying (7), the value of (6) becomes,

$$\bar{V}_i(\mathbf{z}) = (1/3)[v(\mathbf{0},\mathbf{x}) + (R_i(\mathbf{z}) - R_j(\mathbf{z})) + (R_i(\mathbf{z}) - R_k(\mathbf{z}))] \tag{8}$$

In (8) each country expects a third of the world surplus from liberalization plus some terms that can be interpreted as measures of relative bargaining power. Bargaining power depends upon how well a country does, relative to the other countries, when excluded from a bilateral agreement. Specifically, if i does better when excluded from an agreement between j and k than j does when excluded from an agreement with i and k (i.e., $R_i(\mathbf{z}) > R_j(\mathbf{z})$), then i has greater bargaining power than j, other things equal.

If (7) is satisfied for $V_j(\mathbf{z}) = \bar{V}_j(\mathbf{z})$, then (8) is an equilibrium bargaining value. If (7) is satisfied for all $V_j(\mathbf{z})$ satisfying (5), (6) and the condition $V_j(\mathbf{z} \mid i) \in \{R_j(\mathbf{z}), \delta V_j(\mathbf{z})\}$, then (8) is the unique equilibrium bargaining value. In some cases, we can establish uniqueness using the following lemma:

Lemma 1: For all $i, j, k \in N$, if $(1 - \delta)(1/3)(3 - 2\delta)\min[v(\mathbf{x}_{ik},\mathbf{x}_{ij}\mathbf{x}), v(\mathbf{x}_{ik},\mathbf{x})] + \delta\bar{V}_j(\mathbf{z}) - R_j(\mathbf{z})$ • 0, and $R_j(\mathbf{z})$ is unique, then $\bar{V}_i(\mathbf{z})$ is the unique equilibrium bargaining value for country i.

Proof in appendix.

The first condition of lemma 1 is just (7), evaluated at $V_j(\mathbf{z}) = (1/3)[v(\mathbf{0},\mathbf{x}) - R_i(\mathbf{z}) - R_k(\mathbf{z}) + 2\delta V_j(\mathbf{z})]$, which is j's bargaining value when $V_j(\mathbf{z} \mid i) = V_j(\mathbf{z} \mid k) = \delta V_j(\mathbf{z})$. The second condition, that $R_j(\mathbf{z})$ is unique, is obviously necessary for to $\bar{V}_i(\mathbf{z})$ to be unique. However, it also means that lemma 1 is of limited usefulness in states where $R_j(\mathbf{z}) = \delta V_j(\mathbf{z})$ for some j, because in such states establishing the uniqueness of $R_j(\mathbf{z})$ is equivalent establishing the uniqueness of $V_j(\mathbf{z})$ itself. However, as the analysis to follow will demonstrate, states in which $R_j(\mathbf{z})$ • $\delta V_j(\mathbf{z})$ for all j are by far the most complex, and there we will be well-served by lemma 1.

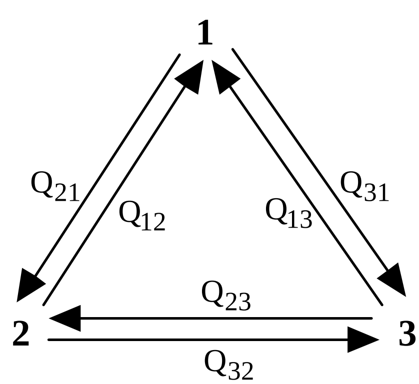
III. Application to a Simple Trade Model

To proceed much beyond the analysis of section II, we must make assumptions about the relative magnitudes of the various surpluses. There are at least two possible ways to perform this: one would be to look for the smallest set of assumptions necessary to establish equilibria and make value comparisons; another would be to write down a specific model of international trade to endogeneously determine the surpluses. We shall adopt the later approach, because what is lost in generality is more than made up for in transparency and ease of interpretation both of the economic determinants of the various factors at work and of the results in their relationship to the literature. Thus, for the remainder of the paper, we examine the implications of our bargaining game for a particular trade model, designed to reveal the main points.

A. Trade, Tariffs and Externalities

Suppose there are six goods, denoted Q_{ij} for i, j = 1, 2, 3 and i • j. denote the (dutiable) imports of country i from country j, let P_{ij} be the price of Q_{ij} in country j, and let T_{ij} denote the specific tariff levied on Q_{ij} by country i. In addition, let each country be endowed with an ample quantity of a freely-traded numeraire good. The pattern of trade is illustrated in figure 1. We assume that the supply of exports from j to i is governed by the linear supply schedule, $Q_{ij} = P_{ij}$, and that i's demand for imports from j is given by, $D_{ij} = a_{ij} - (P_{ij} + T_{ij})$. Setting $D_{ij} = Q_{ij}$ gives the equilibrium volume of trade $Q_{ij} = (1/2)(a_{ij} - T_{ij})$.

FIGURE 1



By assuming away all cross-price and income effects, we have made the model highly tractable but rather uninteresting for the study of preferential arrangements. The problem is that, without such effects, tariff changes on trade flows between any two countries have no effect on the third country, i.e., there are no externalities. If Q_{ij} were a substitute (complement) in consumption for Q_{ik}, then an increase in Q_{ij} brought on by a reduction in T_{ij} would have a negative (positive) welfare effect on suppliers of Q_{ik} in k. Likewise, if Q_{ij} were a substitute (complement) in production for Q_{kj}, then an increase in Q_{ij} would have a negative (positive) effect on consumers of Q_{kj} in k. In order to include these important effects without sacrificing the model's simplicity, we introduce externalities in a transparent but ad hoc way: let the welfare of country k (• i, j) derived from Q_{ij} be given by $(m/5)Q_{ij}^2$ where m is a parameter which may be positive or negative.[14] Thus, writing welfare as the sum of consumer surplus, producer surplus, externalities and tariff revenue gives:

$$w_i = \frac{1}{2}(Q_{ij}^2 + Q_{ik}^2 + Q_{ji}^2 + Q_{ki}^2) + \frac{m}{5}(Q_{jk}^2 + Q_{kj}^2) + T_{ij}Q_{ij} + T_{ik}Q_{ik} \qquad (9)$$

Following Kennan and Reizman (1990), we assume that payoffs of the governments correspond to (9) and, as an external tariff rule, that each government sets external tariffs so as to maximize its welfare, taking as given the tariffs of the other countries. Maximizing (9) with respect to T_{ik} gives an optimal tariff of $T^*_{ik} = (1/3)a_{ik}$. This is the tariff county i applies to imports from any country k to which it is not linked, unless i is a member of a CU.

A customs union is assumed to set a common tariff to maximize the joint welfare of its members.[15] Maximizing the joint welfare of the ij CU with respect to the common tariff T_{ck} on imports from country k gives an optimal tariff of,

$$T^*_{ck} = \frac{1 - \mu}{3} \frac{(a_{ik} + a_{jk})}{2}$$

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[14]In modelling externalities this way, we are treating them as technological rather than pecuniary externalities. Among other things, this implies that free trade is no longer globally optimal. Although undesireable, this complication does not affect the results.

[15]In our model, there is no optimal delegation of external tariff-setting, as in Gatsios and Karp (1991), because the optimal tariff of the outsider is independent of the common external tariff, due to the assumed pattern of trade.

where $\mu = 4m/(15 - 2m)$. This highlights two important properties of a CU's tariff: it is both *common* and *coordinated* across its members. That the external tariff must be common is a constraint on the CU.[16] The effect of this constraint is most evident when $m = 0$, in which case T^*_{ck} is the simple average of T^*_{ik} and T^*_{jk}. Coordination becomes relevant when $m \bullet 0$, in which case T^*_{ck} differs from average of T^*_{ik} and T^*_{jk} by the factor μ. The intuition is that, by coordinating their external tariff, i and j are able to internalize the externalities associated with their trade with k. The greater the externality the lower will be the incentive for the CU to restrict trade with k, and this will tend to lower T^*_{ck} .

The external tariff rule just described makes sense in the context of this model. If our game were augmented to include a third stage (after the bilateral negotiation stage) in which the countries simultaneously choose their external tariffs, then the stationary equilibrium of that game would be the same as under our rule. It seems reasonable that if we are going to impose rationality and stationarity in bargaining that we do so with the external tariffs as well. A similar argument can be made about the assumption that CUs coordinate, while FTAs do not. Throughout the paper we have assumed that FTAs maintain independent bargaining policies, in that they can link with other countries independently of their FTA partners, so it seems appropriate to assume the same about their external tariffs.[17]

Using these tariffs in (9) gives the result that any time two countries i and j form an FTA, they jointly gain by an amount, $v_{ij} \equiv (1/36)(a_{ij}^2 + a_{ji}^2)$, while k experiences a change in welfare equal to $r_k \equiv mv_{ij}$. Three things are worth noting here. First, by construction, these surpluses are independent of the combination of FTAs in existence at the time the FTA is formed. Thus all of the terms, $v_{ij}(\mathbf{x}_{ij}, \mathbf{1})$, $v_{ij}(\mathbf{x}_{ij}, \mathbf{x}_{jk})$, and $v_{ij}(\mathbf{x}_{ij}, \mathbf{x}_{ik}\mathbf{x}_{jk})$ are equal to v_{ij}, and all of the terms $v_k(\mathbf{x}_{ij}, \mathbf{1})$, $v_k(\mathbf{x}_{ij}, \mathbf{x}_{jk})$, and $v_k(\mathbf{x}_{ij}, \mathbf{x}_{ik}\mathbf{x}_{jk})$ are equal to r_k . Second, the gain to an FTA is increasing in the terms a_{ij} and a_{ji}, which are measures of the size of each member country's market for its partner's good. This accords with the conventional

[16]This is generally true; however, in our model we need two assumptions for the constraint to bind. First, goods imported from k by one CU member must have the same customs designation as those imported by the other CU member, even though they are supplied by separate export supply functions. Second, tariffs should be specific rather than *ad valorem*. The optimal *ad valorem* tariffs in this simple model are the same for each country, and thus if *ad valorem* tariffs were used, commonality would not be a constraint at all. However, as this does not generalize, we focus only on specific tariffs.

[17]There is one possible legal drawback to our rule. If $m < 0$, the CU tariff T^*_{ck} may be greater than one of pre-CU tariff levels of T^*_{ik} and T^*_{jk}. This would violate Article 24.

wisdom that a country's most valuable bilateral liberalization is with its largest trading partner. Third, the externality is proportional to the trade gains of the two liberalizing countries.

When two countries i and j form a CU, they jointly gain by an amount $v_{ij} + c_{ij}$, and country k gains by $r_k + e_k$, where

$$c_{ij} = \frac{1}{12}\frac{\mu^2}{(\mu + 2)}(a_{ik}^2 + a_{jk}^2) - \frac{1}{24}\frac{(1 - \mu)^2}{(\mu + 2)}(a_{ik} - a_{jk})^2$$

$$e_k = \frac{1}{72}\mu(\mu + 4)(a_{ik}^2 + a_{jk}^2) + \frac{1}{144}(1 - \mu)(\mu + 5)(a_{ik} - a_{jk})^2$$

The first term in each expression represents the immediate effect of external tariff coordination. This is always non-negative for the CU, and positive for the outside country if coordination results in the CU lowering its external tariff relative to the average FTA tariff (i.e., if $m > 0$). The coordination effect is proportional to $a_{ik}^2 + a_{jk}^2$, which can be interpreted as a measure of the PTA's monopoly power in trade (monopsony power is perhaps more accurate). The second term in each expression represents the asymmetry effect. This measures the immediate welfare loss imposed on a CU, relative to an FTA, by the constraint that the external tariff be common. This loss increases with the difference in the sizes of the CU members' markets for the third country's good. As long as $T^*_{ck} > 0$ ($\mu < 1$), the third country gains from asymmetry, because in averaging its external tariffs, the CU lowers the tariff on the larger of its markets for third-country goods.

B. The Effects of Pre-existing Preferential Trade Agreements

In this section we examine the difference that the organization of a PTA makes to a pair of countries having already agreed to enter into one. A key difference between a CU and an FTA, after being formed, is that in the continued negotiations with the rest of the world an individual CU member has no outside options, whereas an FTA member has the option of proposing (or accepting) another bilateral FTA with the third country.

1. *Free Trade Areas*

Consider the subgame beginning in a state with pattern $\mathbf{x}_{ik}\mathbf{x}_{jk}$, where country k is the hub, bilaterally linked to spoke countries i and j. The only link yet to be negotiated is between i and j, which means that, as respondent, either spoke can freeze the pattern at $\mathbf{x}_{ik}\mathbf{x}_{jk}$, regardless of the proposal. This implies that, $R_i(\mathbf{x}_{ik}\mathbf{x}_{jk}) = \delta V_i(\mathbf{x}_{ik}\mathbf{x}_{jk})$ and $R_j(\mathbf{x}_{ik}\mathbf{x}_{jk}) = \delta V_j(\mathbf{x}_{ik}\mathbf{x}_{jk})$.[18] For country k, $R_k(\mathbf{x}_{ik}\mathbf{x}_{jk}) = r_k$. Substituting these values into (8) gives,

$$\bar{V}_i(\mathbf{x}_{ik}\mathbf{x}_{jk}) = \bar{V}_j(\mathbf{x}_{ik}\mathbf{x}_{jk}) = \phi v_{ij} \qquad (10a)$$

$$\bar{V}_k(\mathbf{x}_{ik}\mathbf{x}_{jk}) = r_k + (1 - \delta)\phi v_{ij} \qquad (10b)$$

where $\phi = 1/(3-\delta)$.

Substituting (10) into (7) gives the condition $(1 + \delta\phi)v_{ij} \bullet 0$, which implies that (10) are always equilibrium bargaining values. It is also straightforward to show that this equilibrium is unique if $m \bullet (3 - 2\delta)/2\delta$. This condition is always satisfied low enough δ, and for δ near 1 it is satisfied if $m \bullet 1/2$. The reason uniqueness requires an upper bound on m is that when m is postive, spokes i and j would like to grab a share of the positive externality which their link confers on hub k . If they choose to link in the bilateral stage, they forfeit this externality. The opportunity cost of this forfiture is the discounted share of the externality they would expect receive by postponing their link to the next multilateral stage. In the equilibrium represented by (10), this opportunity cost is zero, and thus the spokes would always link in the bilateral stage instead of waiting for a discounted share of v_{ij}. An alternative equilibrium would be one in which the spokes always forgo a bilateral-stage link, in which case the equilibrium bargaining value for each country would be $(1/3)(v_{ij} + r_k)$. The opportunity cost in this equilibrium would then be $(2\delta/3)r_k$. This must be compared to the spokes' joint benefit of linking in the bilateral stage of the current period instead of waiting for the next multilateral round, which is $v_{ij} - (2\delta/3)v_{ij}$. The result is that this alternative equilibrium can only exist if $(2\delta/3)r_k > v_{ij} - (2\delta/3)v_{ij}$, or $m > (3 - 2\delta)/2\delta$.[19] In what follows, we assume $m \bullet 1/2$, so that equilibrium on any hub-and-spoke subgame is unique.

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[18]In a slight abuse of the notation, we are writing $R_i(\cdot)$ and $V_i(\cdot)$ as functions $\mathbf{x}$, supressing the vector $\mathbf{y}$. This is unimportant since the pre-existing $\mathbf{y}$ has no effect on the equilibrium bargaining values.

[19]Alternatively, simply substitute $V_h(z) = (1/3)(v_{ij} + r_k)$ for all h into (7), which becomes $(3 - 2\delta)v_{ij} \bullet 2\delta r_k$. This simplifies to

In (10), the spoke countries expect a share ϕ of their joint surplus, while the hub gets the externality as well as a share of the spokes' surplus that vanishes as δ approaches unity. To appreciate the term ϕ, it is helpful to think of the hub-and-spoke subgame as a peculiar sort of lottery. Suppose each country has a 1/3 chance of winning a dollar, but either spoke can veto the outcome and force a new drawing next period. In order to collect the dollar, therefore, the winner would have to pay the spoke(s) their discounted value of the lottery to prevent a veto. Defining value of the lottery to a spoke to be ϕ, it must that $\phi = (1/3)(1 - \delta\phi) + (2/3)\delta\phi$, simplifying to $\phi = 1/(3-\delta)$. The value to the hub is $(1/3)(1 - 2\delta\phi)$, which simplifies to $(1 - \delta)\phi$.

Next let us consider the subgame beginning with the initial pattern $\mathbf{x}_{jk}$, where countries j and k have the only FTA in place but continue to negotiate with country i for the formation of the final two links. Again consider the bilateral stage first. If country i is the proposer and chooses option A, then the pattern becomes $\mathbf{0}$. If j were to reject this offer, then k would link with i, the pattern would become $\mathbf{x}_{ik}\mathbf{x}_{jk}$, and bargaining would continue for the final link in the next period. From (10), the value to j of rejecting A is $R_j(\mathbf{x}_{jk}) = r_j + \delta\phi v_{ij}$, and likewise $R_k(\mathbf{x}_{jk}) = r_k + \delta\phi v_{ik}$. When not proposing, country i may obstruct any link, so that $R_i(\mathbf{x}_{jk}) = \delta V_i(\mathbf{x}_{jk})$. As before, substituting these values into (8) gives,

$$\bar{V}_i(\mathbf{x}_{jk}) = \phi(v_{ij} + v_{ik}) \quad (11a)$$

$$\bar{V}_j(\mathbf{x}_{jk}) = r_j + \phi v_{ij} + (1 - \delta)\phi v_{ik} \quad (11b)$$

$$\bar{V}_k(\mathbf{x}_{jk}) = r_k + \phi v_{ik} + (1 - \delta)\phi v_{ij} \quad (11c)$$

Lemma 2: Equations (11) are the unique equilibrium bargaining values for $\mathbf{x} = \mathbf{x}_{jk}$.

Proof in appendix.

The country left out of the original FTA can expect a share ϕ of each of the remaining surpluses. An FTA member receives ϕ of the surplus from its link with the third country, plus a share $(1-\delta)\phi$ of that between its original FTA partner and the third country. For δ near zero, both ϕ and $(1-\delta)\phi$ are close to 1/3, while as δ approaches unity, ϕ approaches 1/2 and $(1-\delta)\phi$ approaches zero. Intuitively, the lower is

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$(3 - 2\delta)/2\delta \cdot m$.

δ, the greater is the power of the proposer, as the responding countries are willing to give up more today to avoid waiting a period to get what they expect in the future. As δ nears zero, the proposer gets everything, and since each country is the proposer with probability 1/3, *ex ante* each country expects a third of the total surplus. As δ approaches unity, the proposer's advantage diminishes and each FTA member gets half of the surplus created by its link with the third country; since an FTA member cannot impede the link between its original FTA partner and the third country, it gets none of the surplus from this link.

2. Customs Unions

Unlike members of a FTA, members of a CU cannot sign bilateral agreements with the rest of the world independently of their partners; rather they must act as a bloc. Is this an advantage? It is often argued, for example, that the bargaining power of Europe is enhanced by the EU. Whether or not a group of countries is better off organizing itself as a CU or an FTA will be considered in this section.

An important aspect of a CU is the extent to which it can act as a unit, which in turn depends on the way in which decisions internal to the CU are resolved. The loosest form of a customs union is one in which members negotiate independently, with the restriction that any trade barrier change agreed to by a CU member must also be approved (and if so, implemented) by the other members. The tightest form is one in which the members of a CU pre-commit to a division rule internal to the union and delegate national negotiating authority to a single agent, creating effectively one big country. The loose model of a CU is the one studied here, because it enables us to isolate the commitment value of common external tariffs through comparison with the FTA case.

Consider negotiations between a CU, consisting of countries j and k, and the rest of the world, country i. Because the only alternative reachable trade state in X^c is $\mathbf{0}$, any country can prevent the state from changing, by rejecting an offer. Thus, $V_i(\mathbf{z}|j) = \delta V_i(\mathbf{z})$, for all i and j, just as in the standard Rubinstein bargaining model. Using this in (5) and (6) and assuming $v(\mathbf{0},\mathbf{x}^c{}_{jk}) > 0$, the solution is,

$$V_h(\mathbf{x}^c{}_{jk}) = (1/3)\mathrm{v}(\mathbf{0},\mathbf{x}^c{}_{jk}) = (1/3)[v_{ij} + v_{ik} + r_j + r_k - (c_{jk} + e_i)] \qquad (12)$$

for all $h \in N$.[20]

The expression (12) can be readily compared with (11), the case of the pre-existing FTA. Let the difference in joint payoff between a CU and an FTA consisting of countries j and k be $\Delta_{jk} \equiv U_{jk}(\mathbf{x}^c_{jk}) - U_{jk}(\mathbf{x}_{jk})$. Note that this measure includes the difference in immediate values. Thus,

[20]Result (5) is not as dependent on equal probability randomization as it appears. With strictly alternating (non-random) offers, (5) is the limit payoff as δ approaches unity.

$$\Delta_{jk} = [c_{jk} + V_{jk}(\mathbf{x}^c{}_{jk})] - \bar{V}_{jk}(\mathbf{x}_{jk})$$

$$= \frac{\delta\phi(v_{ij} + v_{ik})}{3} + \frac{c_{jk} - 2e_i}{3} - \frac{r_j + r_k}{3} \qquad (13)$$

The first term in (13) represents the extent to which, for a given surplus, a CU is a more effective bargaining coalition than an FTA. By precluding further bilateral deals between individual CU members and the rest of the world, a CU collectively extracts two thirds of the available world surplus, while an FTA collectively extracts $(2 - \delta)\phi$, which ranges from two thirds (as $\delta \rightarrow 0$) to one half (as $\delta \rightarrow 1$). We refer to this advantage as the "committment value" of a CU, and note that it depends positively on the volume of trade between the PTA and the rest of the world. There is, however, an important caveat: while a CU may have an advantage in bargaining over a given surplus, it does not necessarily bargain over the same surplus as does an FTA. This is reflected by the two other terms in (13).

The second term of (13) reflects that the common external tariff of a CU has the effect of removing c_{jk} and e_i from the available surplus, and hence from the bargaining table, prior to the multilateral negotiations. The CU gains by c_{jk} immediately, but had c_{jk} remained as part of the surplus, the CU would have received two thirds of it through bargaining anyway. Thus, the net gain is $(1/3)c_{jk}$. The common external tariff also gives e_i immediately to the outside country, only two thirds of which the CU would have been able to extract had e_i remained as part of the surplus. Hence, e_i produces a net loss to the CU of $(2/3)e_i$. Of course, if e_i is negative, as when m is negative and the PTA members are relatively symmetric, then the immediate effect of the common external tariff translates into an unambiguously positive bargaining effect for the CU. In other words, the long-run advantage of a CU is greater the greater is the short-run harm the common external tariff does to the rest of the world.

The intuition behind the third term in (13) is similar to that of the second but less direct. While the externalities r_j and r_k are, strictly speaking, part of the surplus regardless of the form of PTA, they are not shared equally among the countries when the PTA is an FTA. By rejecting a multilateral offer, an FTA member j knows that the other member k will link with i, and hence j's receipt of r_j is inevitable. Thus it is *as if* r_j were removed from the surplus and given to j prior to negotiations. Viewed in this way, the CU can be thought of as adding $r_j + r_k$ to the negotiable surplus, from which it receives $(2/3)(r_j + r_k)$. The net loss to a CU is therefore $(1/3)(r_j + r_k)$, which can be positive ($m > 0$) or negative ($m < 0$).

We can take this comparison further by using the definitions of v_{ij}, v_{ik}, c_{jk}, e_i, r_j and r_k:

$$\Delta_{jk} = \frac{1}{3}\left[(\delta\phi - \mathrm{m})(v_{ij} + v_{ik}) - \frac{\mu(\mu^2+3\mu+8)}{36(\mu+2)}(a_{ji}^2 + a_{ki}^2) - \frac{(1-\mu)(\mu^2+4\mu+13)}{72(\mu+2)}(a_{ji} - a_{ki})^2\right] \qquad (14)$$

Several results from (14) are evident by inspection. The value of a CU relative to an FTA is higher the higher is the discount factor and the lower is the asymmetry between PTA members. The effect on Δ_{jk} of an increase in the total surplus $v_{ij} + v_{ik}$ depends on the sign of $\delta\phi - m$. If $\delta\phi > m$, which will occur if m is small or δ is close to one, then the commitment value of the CU either dominates or works in the same direction as the externalities, and therefore any increase in total surplus benefits the CU.

The effect on Δ_{jk} of the PTA's monopoly power $a_{ji}^2 + a_{ki}^2$ depends on the sign of the externality. If $m > 0$, the coordination effect of the common external tariff ultimately harms the CU relative to an FTA, because it induces the CU to lower its external tariff and benefits the outside country so that $2e_i > c_{jk}$. If $m < 0$, the CU is relatively better off, because coordination results in an increase in the external tariff that harms the outside country. Finally, the sign of the derivative of Δ_{jk} with respect to m itself is ambiguous.

To summarize, as long as countries are patient or externalities are low, the CU form of bilateral organization favors country pairs that are relatively similar in and dependent upon trade with the rest of the world, while the FTA form favors country pairs that trade mostly with each other, yet have very different market sizes for goods produced in the rest of world. However, when the pace of multilateral negotiations is slow or uncertain, as when there is a low probability of any serious multilateral offers being tabled any time soon after the current period, the relative advantage of being a CU is smaller. These considerations may help to explain why an extremely heterogeneous continent like North America would prefer forming an FTA to a CU, particularly when the Uruguay Round appeared stalled. Also it might explain why the relatively homogeneous countries of the European Union preferred the CU form of PTA, especially in 1957 (the year of the Treaty of Rome) when the share of European trade with the rest of the world (mainly the US) was much larger than it is today.

An important feature of this model is that the two CU members continue to negotiate as two players (though the common external tariff restricts their choices). If instead we had treated the CU as being a single negotiator, it would have done worse than (13). This is because if the CU and third country each propose with probability 1/2, the expected payoff is an even split between the CU and third country. The third country gets, $V_{jk}(\mathbf{x}^c{}_{jk}) = (1/2)v(0,\mathbf{x}^c{}_{jk})$ and thus it clearly pays for a customs union not to act as a single country.[21]

C. The Effects of Preferential Trade Agreements as Outside Options

In this section we take up the more systemic question of the effect of allowing CUs and FTAs on the outcomes of multilateral trade negotiations. The "outside option principle" of Binmore, Shaked and Sutton [1989) states that an outside option will be irrelevant to the outcome of a bargaining game, unless it is better for at least one player than the outcome of a game with no outside options. While they established this for a single outside option of exogenous value, our concern is with multiple options of endogenous value, and therefore whether PTAs are relevant in this sense remains an open question. As before we shall consider FTAs first and then compare them with CUs, this time also with the benchmark, $\tilde{V}_i(\mathbf{1}) = (1/3)\mathrm{v}(\mathbf{0},\mathbf{1})$, which is the payoff each country would receive if PTAs were ruled out entirely.[22]

1. *Free Trade Areas*

Let the initial pattern be $\mathbf{x} = \mathbf{1}$ and the set of available states be X^f. Each country is in a position similar to that of country i (the country with no initial links) in section B.1. Each proposer in the bilateral stage of a period can offer two acceptable bilateral deals, one such deal or none. This gives

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[21]This accords with most non-cooperative bargaining results: a player's bargaining power hinges on its ability to place the onus of delaying agreement on the other players. A player that never proposes cannot extract any surplus. By construction, a loose CU has two proposers, a tight CU has only one. Of course, if we were to increase the probability to 2/3 that a tight CU becomes the proposer, then there would be no difference between a tight and loose CU.

[22]If PTAs were ruled out, then the only trade liberalization possible would be global free trade. Since this must be unanimously approved in negotiations, each country would receive an equal share of the total surplus.

quite a number of option permutations to consider. Fortunately, lemma 2 guarantees that $R_i(\mathbf{1}) = r_i + \delta\phi(v_{ij} + v_{ik})$ for all *i*, and thus lemma 1 can be used. Using $R_i(\mathbf{1})$ in (8) gives,

$$\bar{V}_i(\mathbf{1}) = r_i + \phi(v_{ij} + v_{ik}) + (1 - \delta)\phi v_{jk} \tag{15}$$

for all *i*. The sufficient condition for uniqueness in lemma 1 reduces to $(1/2)[(1 + \delta^2\phi^2)/\delta\phi]$ • m, which is guaranteed by *m* • 5/4.

Expression (15) establishes the relevance of FTAs as outside options for the multilateral negotiations. Note that as δ approaches unity, expression (15) becomes $\bar{V}_i(\mathbf{1}) = (1/2)(v_{ij} + v_{ik})$. Thus each bilateral surplus is spilt evenly between each pair of countries. This is same payoff that would result from three independent bilateral negotiations.

While FTAs are relevant outside options for the multilateral negotiations, whether or not a pair of countries *actually* forms an FTA matters little. From (11), the payoff of an FTA between *j* and *k* is,

$$U_{jk}(\mathbf{x}_{jk}) = r_j + r_k + v_{jk} + (2 - \delta)\phi(v_{ij} + v_{ik}), \tag{16}$$

while from (15), the payoff of the same two countries without a pre-existing FTA is,

$$U_{jk}(\mathbf{1}) = r_j + r_k + 2\phi v_{jk} + (2 - \delta)\phi(v_{ij} + v_{ik}). \tag{17}$$

The difference between (16) and (17) is $(1-\delta)\phi v_{jk}$, which is negligible for high δ. Thus countries receive almost the same equilibrium payoff whether they are members of an FTA or not. The only reason countries will sign bilateral FTAs is to salvage the short-term (one-period) gains from those links if the multilateral talks fail, and this determines the distribution of gains in the multilateral agreement.

2. *Customs Unions*

Now consider the case in which there are no pre-existing agreements and the set of possible links is X^c. The game is the same as in the FTA case, except that in the bilateral stage, if *i* and *j* both accept bilateral offers from k, then they must link not only with k (and pay y''_k) but also with each other. If only j accepts a bilateral offer from k, then the rejecting country i receives, $R^c{}_i(\mathbf{1}) = r_i + e_i + (\delta/3)v(\mathbf{0},\mathbf{x}^c{}_{jk})$, which is the gain from having j and k link plus the discounted expected value of the agreement to be reached with the jk CU next period. The relationship between this and the $R_i(\mathbf{1})$ in the FTA case is:

$$R^c{}_i(\mathbf{1}) = R_i(\mathbf{1}) + (1 - \delta)e_i - \delta\Delta_{jk} \tag{18}$$

The difference is that, when excluded from an CU, country i receives the immediate benefit of the jk common external tariff less the long-term bargaining advantage of the jk CU over the jk FTA appropriately discounted.

If every country offers bilateral CUs to both other countries in the bilateral stage of each period, each country will receive an expected value of (8), which using (18) becomes,

$$\bar{V}^c{}_i(\mathbf{1}) = \bar{V}_i(\mathbf{1}) + (1/3)[(1 - \delta)(2e_i - e_j - e_k) - \delta(2\Delta_{jk} - \Delta_{ij} - \Delta_{ik})] \tag{19}$$

To guarantee the uniqueness of $\bar{V}^c{}_i(\mathbf{z})$, lemma 1 must be modified as follows:

Lemma 3: For all $i, j, k \in N$, if $(1 - \delta)(1/3)(3 - 2\delta)\min[v(\mathbf{0},\mathbf{x}^c{}_{ij}), v(\mathbf{x}^c{}_{ik},\mathbf{1})] + \delta\bar{V}^c{}_j(\mathbf{z}) - R^c{}_j(\mathbf{z})$ • 0, then $\bar{V}^c{}_i(\mathbf{z})$ is the unique equilibrium bargaining value for country i.

Proof in appendix.

For an arbitrary discount factor, determining whether or not the condition for lemma 3 is satisfied is not an easy, or particularly worthwhile, task. However, as δ approaches one, it becomes straightforward:

Corollary: For sufficiently high δ, $V^c{}_i(\mathbf{z}) = \bar{V}^c{}_i(\mathbf{z})$, if and only if, $\Delta_{12} + \Delta_{13} + \Delta_{23}$ • 0.

Proof in appendix.

That is, all countries will offer CUs to each other in the bilateral stage, if and only if CUs are better than FTAs on average. The reason is that for high δ the efficiency differences between alternative proposals become negligible, so proposer i will choose option A if and only if $\bar{V}^c{}_j(\mathbf{z}) \bullet R^c{}_j(\mathbf{z})$ and $\bar{V}^c{}_k(\mathbf{z}) \bullet R^c{}_k(\mathbf{z})$. From (18) and (19), $\bar{V}^c{}_j(\mathbf{z}) - R^c{}_j(\mathbf{z})$ approaches $(1/3)\Sigma_{ij\in N}\Delta_{ij}$ as δ approaches one.

To compare the CU regime to an FTA regime, we can write the limit of (19) as,

$$\bar{V}^c{}_i(\mathbf{1}) = (2/3)\bar{V}_i(\mathbf{1}) + (1/3)\tilde{V}_i(\mathbf{1}) + (1/9)\{(c_{ij} - c_{jk}) + (c_{ik} - c_{jk}) - 2[(e_j - e_i) + (e_k - e_i)]\} \qquad (20)$$

The first two terms in on the right-hand side of (20) make up a weighted average of $\bar{V}_i(\mathbf{1})$ and $\tilde{V}_i(\mathbf{1})$. Thus, looking only at these terms, the CU regime is *ex ante* more egalitarian than the FTA regime, tending to draw the countries towards the mean and closer to regime without PTAs. It also implies that countries responsible for a disproportionate amount of the world surplus would *ex ante* prefer an FTA regime to a CU regime.

However, it would be too strong to argue that eliminating FTAs from Article 24 of the GATT would be a step in the direction of egalitarianism, because (20) is an *ex ante* result and its egalitarian nature is due mainly to the equal probability randomization over proposers. Once a proposer is selected, the *ex post* payoffs in a CU regime will typically be less egalitarian than an FTA regime. This is because, if CUs are better than FTAs on average, then the average proposer can extract more surplus from the respondents by threatening them with being left out of a CU than an FTA. Thus there is greater advantage to being the proposer in a CU regime.

Finally, the last term in (20) measures the relative bargaining power associated with the immediate effects of the common external tariff. If common external tariffs of the CUs involving country i generate more immediate surplus for those CUs, or inflict more immediate harm outside countries, than does the common external tariff the jk CU, then country i has more greater bargaining power. Recall from (14) that one of the determinants of the terms in (20) is asymmetry. When pair of countries differ substantially in their demand for imports from the third country, they will tend to be at bargaining disadvantage in a CU regime. The other determinant is the monopoly power of each pair, but the relevance of this depends on the sign of the externality.

What happens when $\Sigma_{ij\in N}\Delta_{ij}$ is negative? As δ approaches one, the only equilibria that survive are those in which at least one country would not offer a CU to at least one other country in the bilateral stage of negotiations. Three different types of such equilibria are possible. The first type is where one country, say 1, exercises option A, while 2 and 3 either offer a CU exclusively to 1 or offer no bilateral deal at all. The limit values in this case are, $V_1(\mathbf{1}) = R^c{}_1(\mathbf{1}) + \Sigma_{ij\in N}\Delta_{ij}$, $V_2(\mathbf{1}) = R^c{}_2(\mathbf{1})$, and $V_3(\mathbf{1}) = R^c{}_3(\mathbf{1})$. The second type of equilibrium is where two countries, say 1 and 2, make exclusive bilateral offers to each other, while 3 makes no offer at all. The limit values in this case are, $V_1(\mathbf{1}) = R^c{}_1(\mathbf{1}) + (1/2)\Sigma_{ij\in N}\Delta_{ij}$, $V_2(\mathbf{1}) = R^c{}_2(\mathbf{1}) + (1/2)\Sigma_{ij\in N}\Delta_{ij}$, and $V_3(\mathbf{1}) = R^c{}_3(\mathbf{1})$. The third possible equilibrium is that in which no country links in the bilateral stage at all. In this equilibrium, the bargaining values are just $\tilde{V}_i(\mathbf{1})$ for all i. These values are worked out in the appendix. It is not possible to rule out any of these equilibria without further assumptions.

3. Open Season

Perhaps the most natural next step in the analysis is to put FTAs and CUs together and consider a game in which the set of available patterns is $X^f \cup X^c$. Only two things can said about this game that follow directly from the analysis above: as δ approaches one, if a CU is better for each pair than an FTA ($\Delta_{ij} > 0$ for all i and j), then the unique equilibrium bargaining values are $\bar{V}^c{}_i(\mathbf{1})$; if an FTA is better for each pair than and a CU ($\Delta_{ij} < 0$ for all i and j), then the unique equilibrium bargaining values are $\bar{V}_i(\mathbf{1})$. The more interesting, though exceedingly more complex, cases are those in which some pairs prefer FTAs and others prefer CUs. Few general conclusions can be stated about such cases; however, it is possible to construct examples that convey some meaning.

Suppose $m = 0$ and δ is near one, so that the two factors determining the relative advantage of a CU are the committment value and asymmetry. Let the demand parameters be as follows: $a_{12} = a_{13} = \alpha$, and $a_{21} = a_{23} = a_{31} = a_{32} = \alpha\beta$, where $\beta \in [0, 1]$. Country 1 is can be thought of as the "big" country with demand for imports from 2 and 3 parameterized by α. Countries 2 and 3 are symmetric in their demands from imports from 1 as well as from each other. These are parameterized by $\alpha\beta$, where β measures the size 2 and 3 relative to 1. Choosing α so that $v(\mathbf{0}, \mathbf{1}) = 1$, the surpluses become,

$$v_{12} = v_{13} = \frac{1}{2}\frac{1 + \beta^2}{1 + 2\beta^2}\ , v_{23} = \frac{\beta^2}{1 + 2\beta^2}\ , \qquad (21)$$

$$c_{12} = c_{13} = \frac{-3}{8}\frac{(1 - \beta)^2}{1 + 2\beta^2}, c_{23} = 0, \qquad (22)$$

$$e_3 = e_2 = \frac{5}{8}\frac{(1 - \beta)^2}{1 + 2\beta^2},\ e_1 = 0 \qquad (23)$$

These surpluses imply,

$$\Delta_{12} = \Delta_{13} = \frac{\delta\phi}{6}\frac{1 + 3\beta^2}{1 + 2\beta^2} - \frac{13}{24}\frac{(1 - \beta)^2}{1 + 2\beta^2}\ ,\ \Delta_{23} = \frac{\delta\phi}{3}\frac{1 + \beta^2}{1 + 2\beta^2}, \qquad (24)$$

It is clear from (24) that the PTA consisting of 2 and 3 is always better off as a CU, while the other two pairs may be better-off as FTAs, if β is low enough. Let $\hat{\beta}$ denote critical value of β, such that for $\beta < \hat{\beta}$, $\Delta_{12} < 0$. It can be shown that $\hat{\beta} = [(13 - 4(13 - 3\delta\phi)^{1/2}]/(13 - 12\delta\phi)$, which for $\delta = 1$ is equal to .487.

Making use of the symmetry between 2 and 3, we make another dimensional simplification by restricting attention to symmetric equilibria, which we take to mean $V_2(\mathbf{1}) = V_3(\mathbf{1})$. This allows us to define $\theta \equiv V_2(\mathbf{1})/V_1(\mathbf{1})$, as the bargaining value of 2 (or 3) as a share of the bargaining value of 1. With $v(\mathbf{0}, \mathbf{1}) = 1$, this implies $V_1(\mathbf{1}) = 1/(1 + 2\theta)$.

The game begins with $\mathbf{x} = \mathbf{1}$. In the subgame of the bilateral-stage, each proposer now has essentially five options: A^c) propose CUs to both respondents; A) propose FTAs to both respondents; B^c) propose a CU to one respondent; B) propose an FTA to one respondent; and C) make no acceptable offer. Let the value of the best such option for proposer i be $\psi_i = \max\{A^c{}_i, A_i, B^c{}_{ij}, B_{ij}, B^c{}_{ik}, B_{ik}, \delta V_i\}$. It is possible to compute ψ_i for any pair (θ, β). Doing so enables us to partition the space of (θ, β) into six sets:

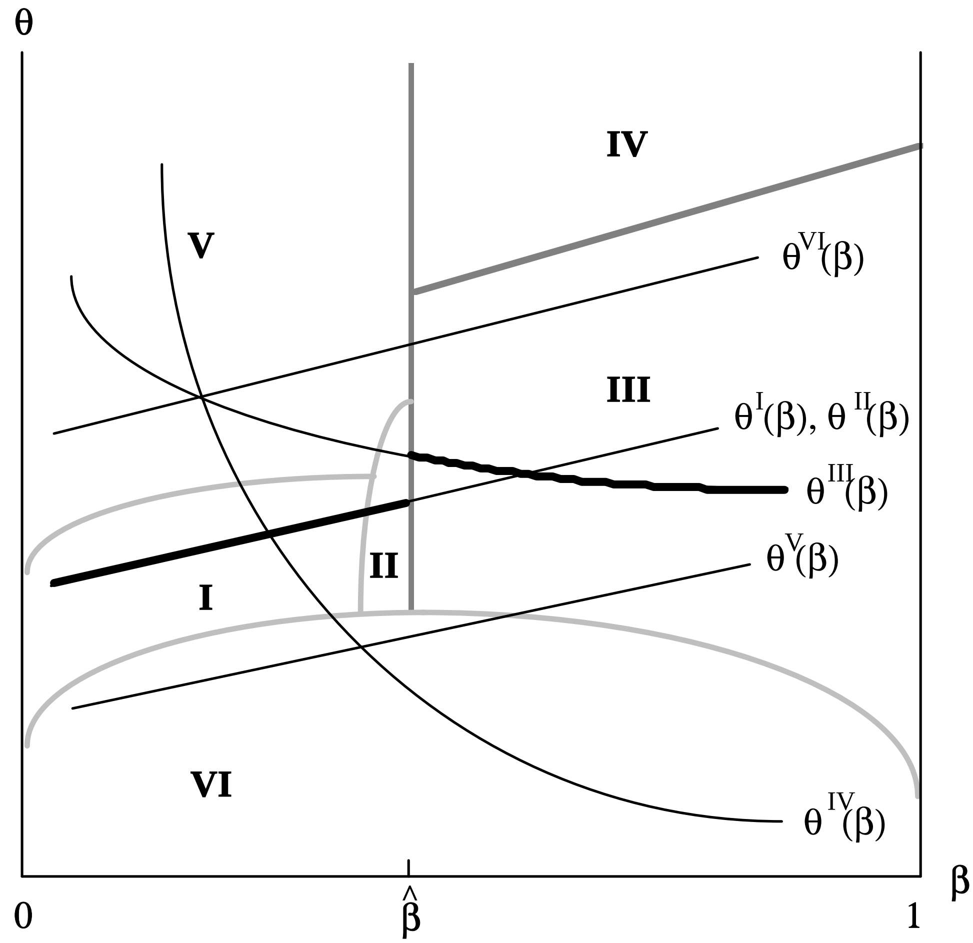
$M^I = \{\theta, \beta \mid \psi_1 = A_1,\ \psi_2 = B^c{}_{23}\}$, $M^{II} = \{\theta, \beta \mid \psi_1 = A_1,\ \psi_2 = A^c{}_2\}$

$M^{III} = \{\theta, \beta \mid \psi_1 = A^c{}_1, \psi_2 = A^c{}_2\}$, $M^{IV} = \{\theta, \beta \mid \psi_1 = A^c{}_1, \psi_2 = B^c{}_{21}\}$

$M^V = \{\theta, \beta \mid \psi_1 = A_1, \psi_2 = B_{21}\}$ and $M^{VI} = \{\theta, \beta \mid \psi_1 = \delta V_1, \psi_2 = B^c{}_{23}\}$

These sets are illustrated in Figure 2, as regions circumscribed by gray lines. Each of these regions has an intuitive explanation. Except for region VI, country 1's proposal strategy is determined by the sign of Δ_{12}. To the left of $\hat{\beta}$, country 1 proposes FTAs and to the right, CUs. In region VI, the value of θ, and hence V_2, is so low that 1 finds it optimal to give the respondents δV_2 instead of R_2. Likewise, for low θ, countries 2 and 3 prefer to give each other δV_2 instead of R_2, which they can do by each offering an exclusive bilateral CU to the other. For high θ as in regions IV and V, countries 2 and 3 prefer to give country 1 δV_1 instead of R_1, which they can do by each offering 1 an exclusive bilateral deal. Whether that deal should be a CU or an FTA depends on the sign of Δ_{12}, which is what distinguishes IV from V. Regions I, II and III feature less extreme values of θ, and so the primary determinant of ψ_2 is β. Whenever 2 and 3 propose a bilateral deal to each other, they propose a CU. The main issue, then, is whether or not to also propose a CU to 1. For β not too small, a CU with 1 is not very costly, and so 1 is proposed a CU in regions II and III. For β very small it is costly and thus 2 and 3 would rather leave 1 out, and this determines region I.

FIGURE 2



The next step in finding equilibria is to observe that for each pair (ψ_1, ψ_2) we can use equations (4), (5) and (6) to work out the corresponding bargaining values, as functions of β. This allows us to derive six functions $\theta^l(\beta)$, $l = I, II, .. VI$. These are illustrated in figure 2 as thin solid lines. For the final step, we determine equilibrium bargaining values by matching up the functions with the sets. More accurately, $\theta^l(\beta)$ is an equilibrium payoff ratio, if and only if $(\theta^l(\beta), \beta) \in M^l$. The points satisfying this condition are represented in figure 2 by thick solid lines. The equilibria lie in regions I, II, and III.

For $\beta > \hat{\beta}$, CUs are better than FTAs for each pair, and thus $V_i(\mathbf{1}) = \bar{V}^c_i(\mathbf{1})$. For $\beta < \hat{\beta}$, country 1 proposes FTAs to both 2 and 3 in the bilateral stage, while countries 2 and 3 either propose CUs to both respondents or propose an exclusive 23 CU. By virtue of the symmetry of the example, both of the proposal strategies of 2 and 3 yield the same payoffs for each country. Those payoffs are:

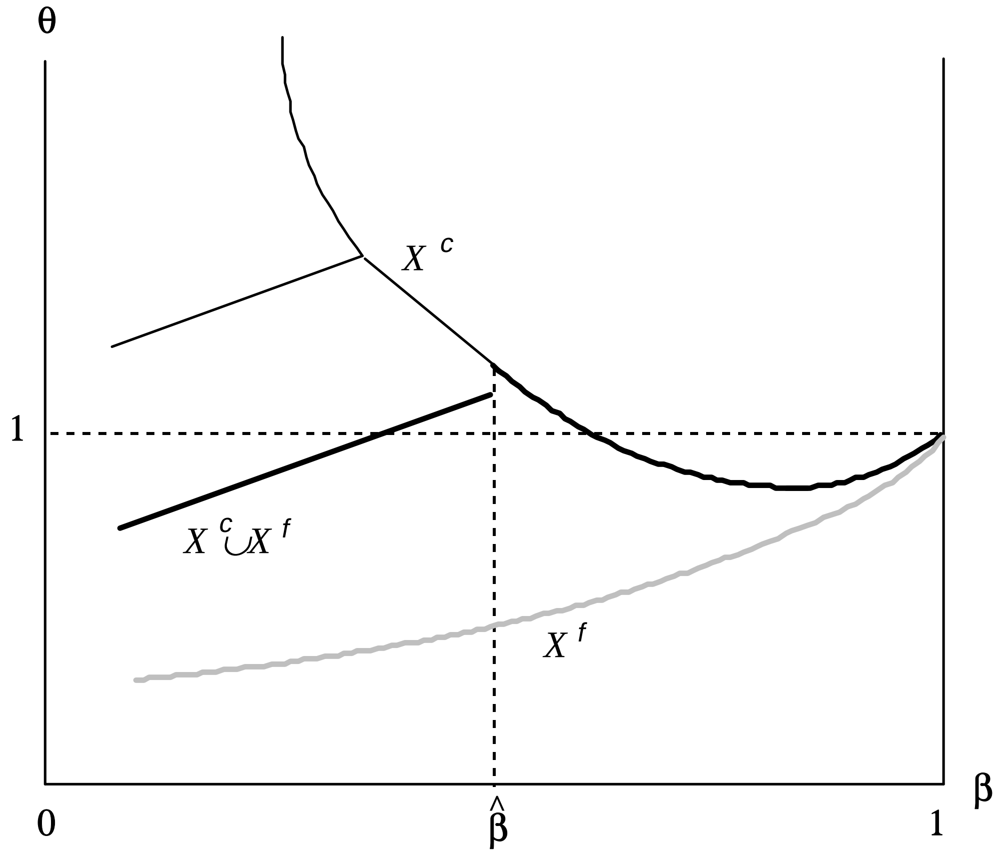
$$V_1(\mathbf{1}) = (1/3)[1 + 2(R^c_1(\mathbf{1}) - R_2(\mathbf{1}))] \quad (25a)$$

$$V_2(\mathbf{1}) = (1/3)[1 + R_2(\mathbf{1}) - R^c{}_1(\mathbf{1})] \qquad (25b)$$

We are now in a position to compare this example with the results of the two previous sections, in which we restricted X to being either X^c or X^f but not both. One technical issue that must be dealt with before making this comparison is the determination of equilibria in the CU regime when $\Sigma_{ij\in N}\Delta_{ij} < 0$ (this occurs when β is less than about .373). It turns out that the only equilibrium one can rule out is that of the third type mentioned above, the one in which no country links in the bilateral stage. The first type of equilibrium exists for .232 • β • .373 and produces a payoff ratio equal to $\theta^{IV}(\beta)$ from figure 2. The second type of equilibrium exists for all β • .373 and the corresponding payoff ratio is $\theta^{VI}(\beta)$ from figure 2. The proof of this is found in the appendix.

Piecing together the equilibria across the domain of β for the four different regimes produces figure 3. The payoff ratio in the FTA regime is $\bar{V}_2(\mathbf{1})/\bar{V}_1(\mathbf{1})$, from (19), and is depicted by the gray line, labeled X^f. The CU regime consists of $\theta^{IV}(\beta)$ and $\theta^{VI}(\beta)$ for low β and followed by $\theta^{III}(\beta)$ and is represented by thin solid lines, labeled X^c. The open regime is represented by thick solid lines, labeled $X^f \cup X^c$. Finally, the payoff ratio in a regime of no PTAs (the "closed" regime) is simply $\tilde{V}_2(\mathbf{1})/\tilde{V}_1(\mathbf{1}) = 1$. It is clear from figure 3 how the different countries would rank these regimes. The big country always prefers the FTA regime, while the small symmetric countries prefer the CU regime for small β and the closed regime for high β.

FIGURE 3



An interesting feature of this example is that there is no general presumption about which country would prefer the open regime to the closed one. For $\beta > \hat{\beta}$, $V_1(\mathbf{1}) = (1/3)[1 - 2(R^c{}_1(\mathbf{1}) - R^c{}_2(\mathbf{1}))]$ in the open regime and therefore the big country gains from the open regime whenever $R^c{}_1(\mathbf{1}) > R^c{}_2(\mathbf{1})$. For $\beta < \hat{\beta}$, (25) implies that the big country gains from the open regime whenever $R^c{}_1(\mathbf{1}) > R_2(\mathbf{1})$. The lower is β is larger is the share of the world surplus due to trade with country 1, and thus the less the big country is harmed by being left out of the 23 CU (i.e., the higher is $R^c{}_1(\mathbf{1})$). On the other hand, as β falls, it also reduces the common external tariff of any CU of which country 1 is a member, and this works to the advantage of 2 and 3 (i.e., $R^c{}_2(\mathbf{1})$ rises). As β falls, $R^c{}_2(\mathbf{1})$ rises faster than $R^c{}_1(\mathbf{1})$, working to the advantage of 2 and 3, until β falls below $\hat{\beta}$, at which point country 1 prefers FTAs and the relevant rejection value for 2 and 3 becomes $R_2(\mathbf{1})$. For $\beta < \hat{\beta}$, smaller β always benefits the big country.

IV. A Note on Cooperative Solutions

While noncooperative bargaining models have the advantage over cooperative solution concepts of explicitly representing the bargaining environment and the behavior of its agents, noncooperative models can be unwieldy. Hence, it is sometimes desirable to use a cooperative solution concept instead, provided the two approaches can be shown to be mutually consistent (see Sutton, 1986, for discussion). The problem is that most *N*-player cooperative solution concepts are ill-suited for direct application to the present context, because they rely on two components of the underlying game which do not fit our model: coalition structures (partitions of the players into disjoint coalitions) and characteristic functions (which assign to each coalition a value independent of the actions of its compliment). Because FTA links are not transitive, the set X^f cannot be adequately represented by a set of partitions (see Aumann and Myerson, 1988, Myerson, 1991). Moreover, characteristic functions do not allow for externalities.

Nonetheless, the limits of equations (10), (11), (15) and (16) can be generated by an appropriate application of the Nash bargaining solution (see Ludema, 1993). Further, if the externalities are set to zero, then following Gul (1989), the payoffs in (20) are the Shapley values of a *homomollifier* of this game in characteristic function form. If instead we were to assume that, once formed, a CU behaves as a single player, then payoffs in (20) would correspond exactly to Shapley values.

V. Conclusion

This paper has attempted to discover the effects of PTAs on multilateral negotiations, both when PTAs are in place and when they are mere options. Hopefully, it has illuminated some of the reasons why countries might wish to form PTAs and provided a theoretical explanation for some stylized facts about existing preferential arrangements. It was shown that CUs are generally more effective bargainers than FTAs, because of its commitment to common external tariffs, but numerous factors such as discounting, asymmetry and externalities can reverse this. It was also shown that simply allowing PTAs as options has a profound effect on the outcome of negotiations, even though agreements are reached at the multilateral level requiring unanimous approval of all proposals. In an FTA regime (or when all countries prefer FTAs to CUs), the outcome converges to essentially simultaneous bilateral agreements between each country pair. A CU regime (or when all countries prefer CUs to FTAs) tends to produce more egalitarian results *ex ante*, than an FTA regime, though less so *ex post*. There is much more work to be done, particularly in the open regime, where both FTAs and CUs are permitted. But even in our simple example, there is no general presumption that relatively large countries will prefer an regime that permits PTAs to one that does not.

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APPENDIX

Proof of Lemma 1:

The method of the proof will be to examine each of the admissible strategy combinations, i.e., combinations of As, Bs and Cs, and to demonstrate that, for all but one such combination, at least one of the equilibrium conditions is contradicted by the following condition:

(P1) $(1 - \delta)(1/3)(3 - 2\delta)\min[v(\mathbf{x}_{ik},\mathbf{x}_{ij}\mathbf{x}), v(\mathbf{x}_{ik},\mathbf{x})] + \delta\bar{V}_j(\mathbf{z}) - R_j(\mathbf{z}) \bullet 0$ for all $i, j, k \in$ N, $i \bullet j \bullet k$,

The only combination satisfying P1 is that in which each player chooses A. Since this combination produces bargaining value $\bar{V}_i(\mathbf{z})$ for all i, it follows that $\bar{V}_i(\mathbf{z})$ is unique if $R_j(\mathbf{z})$ is unique for all j.

<u>Case 1</u>: At least one player, say country 1, makes an acceptable bilateral-stage offer to both others, 2 and 3, i.e., country 1 chooses A.

1a) Countries 2 and 3 also choose A. A sufficient condition for this to be an equilibrium is (7) evaluated at $V_j(\mathbf{z}) = \bar{V}_j(\mathbf{z})$, which is consistent with P1, because $(1/3)(3 - 2\delta) < 1$.

1b) Countries 2 and 3 make exclusive bilateral offers to each other. For this to be an equilibrium it must be that $B_{23} > A_2$, and $B_{32} > A_3$. Summing these conditions gives,

(P2) $$(1 - \delta)[v(\mathbf{x}_{13},\mathbf{x}_{23}\mathbf{x}) + v(\mathbf{x}_{12},\mathbf{x}_{23}\mathbf{x})] + \delta V_2(\mathbf{z}) + \delta V_3(\mathbf{z}) - R_2(\mathbf{z}) - R_3(\mathbf{z}) < 0$$

Applying $B_{23} > A_2$, and $B_{32} > A_3$ to equations (4), (5) and (6) gives $V_1(\mathbf{z}) = \bar{V}_1(\mathbf{z})$, which implies $V_2(\mathbf{z}) + V_3(\mathbf{z}) = \bar{V}_2(\mathbf{z}) + \bar{V}_3(\mathbf{z})$. Substituting $\bar{V}_2(\mathbf{z}) + \bar{V}_3(\mathbf{z})$ into P2 reveals that P2 voliates P1.

For every other combination of strategies by countries 2 and 3 in case 1, it must be that, $V_1(\mathbf{z}) \in \{\bar{V}_1(\mathbf{z}), \phi[v(\mathbf{0},\mathbf{x}) + R_1(\mathbf{z}) - R_2(\mathbf{z}) - R_3(\mathbf{z})], [1/(3-2\delta)][v(\mathbf{0},\mathbf{x}) - R_2(\mathbf{z}) - R_3(\mathbf{z})]\}$ and either,

(P3) $(1 - \delta)v(\mathbf{x}_{23},\mathbf{x}_{1j}\mathbf{x}) + \delta V_1(\mathbf{z}) - R_1(\mathbf{z}) < 0$, for j = 2, 3, or

(P4) $(1 - \delta)v(\mathbf{x}_{23},\mathbf{x}) + \delta V_1(\mathbf{z}) - R_1(\mathbf{z}) < 0$.

Evidently, P3 and P4 violate P1, when $V_1(\mathbf{z}) = \bar{V}_1(\mathbf{z})$. When $V_1(\mathbf{z}) = \phi[v(\mathbf{0},\mathbf{x}) + R_1(\mathbf{z}) - R_2(\mathbf{z}) - R_3(\mathbf{z})]$, the term $\delta V_1(\mathbf{z}) - R_1(\mathbf{z})$ in P3 and P4 becomes $3\phi[\delta\bar{V}_1(\mathbf{z}) - R_1(\mathbf{z})]$, and when $V_1(\mathbf{z}) = [1/(3-2\delta)][v(\mathbf{0},\mathbf{x}) - R_2(\mathbf{z}) - R_3(\mathbf{z})]$, the term $\delta V_1(\mathbf{z}) - R_1(\mathbf{z})$ becomes $[3/(3-2\delta)][\delta\bar{V}_1(\mathbf{z}) - R_1(\mathbf{z})]$. Thus, P3 and P4 violate P1 for all possible $V_1(\mathbf{z})$ in this case.

<u>Case 2</u>: No player makes acceptable bilateral-stage offers to both others, but at least one player, say country 1, makes an exclusive bilateral-stage offer to one other player, say country 2.

2a) Countries 2 and 3 make exclusive bilateral offers to 1. This case requires, among other things, that $B_{12} > A_1$, and $B_{21} > A_2$. Solving equations (4), (5) and (6) for this case gives, $V_1(\mathbf{z}) = [1/(3\text{-}2\delta)][v(\mathbf{0},\mathbf{x}) - R_3(\mathbf{z}) - \delta V_2(\mathbf{z})]$. Thus, the condition $B_{21} > A_2$, becomes

$$(1 - \delta)v(\mathbf{x}_{23},\mathbf{x}_{12}\mathbf{x}) + \delta[1/(3 - 2\delta)][v(\mathbf{0},\mathbf{x}) - R_3(\mathbf{z}) - \delta V_2(\mathbf{z})] - R_1(\mathbf{z}) < 0, \text{ or}$$

$$(3 - 2\delta)(1 - \delta)v(\mathbf{x}_{23},\mathbf{x}_{12}\mathbf{x}) + \delta[v(\mathbf{0},\mathbf{x}) - R_2(\mathbf{z}) - R_3(\mathbf{z})] - (3 - 2\delta)R_1(\mathbf{z}) - \delta[\delta V_2(\mathbf{z}) - R_2(\mathbf{z})] < 0, \text{ or}$$

(P5) $$(3 - 2\delta)(1/3)(1 - \delta)v(\mathbf{x}_{23},\mathbf{x}_{12}\mathbf{x}) + \bar{V}_1(\mathbf{z}) - R_1(\mathbf{z}) - (\delta/3)[\delta V_2(\mathbf{z}) - R_2(\mathbf{z})] < 0$$

The sum of the first three terms in P5 is positive by P1. So for P5 to hold it must be that $\delta V_2(\mathbf{z}) - R_2(\mathbf{z}) > 0$, which contradicts the condition $B_{12} > A_1$.

2b) One player, say country 2, makes an exclusive bilateral-stage offer to 1, and country 3 makes no acceptable offer. Like case 2a this requires $B_{12} > A_1$, and $B_{21} > A_2$. Summing these conditions gives,

(P6) $$(1 - \delta)[v(\mathbf{x}_{23},\mathbf{x}_{12}\mathbf{x}) + v(\mathbf{x}_{13},\mathbf{x}_{12}\mathbf{x})] + \delta V_1(\mathbf{z}) + \delta V_2(\mathbf{z}) - R_1(\mathbf{z}) - R_2(\mathbf{z}) < 0$$

Solving equations (4), (5) and (6) for this case gives, $V_1(\mathbf{z}) = V_2(\mathbf{z}) = \phi[v(\mathbf{0},\mathbf{x}) - R_3(\mathbf{z})]$. This enables us to re-write P6 as,

$$(3 - \delta)(1/3)(1 - \delta)[v(\mathbf{x}_{23},\mathbf{x}_{12}\mathbf{x}) + v(\mathbf{x}_{13},\mathbf{x}_{12}\mathbf{x})] + \delta\bar{V}_1(\mathbf{z}) - R_1(\mathbf{z}) + \delta\bar{V}_2(\mathbf{z}) - R_2(\mathbf{z}) < 0$$

which violates P1.

2c) Country 2 makes an exclusive bilateral offer to 3, and country 3 makes an exclusive bilateral offer to 1. This requires that $B_{12} > A_1$, $B_{23} > A_2$ and $B_{31} > A_3$. Summing these conditions together gives,

(P7) $$(1 - \delta)[v(\mathbf{x}_{13},\mathbf{x}_{12}\mathbf{x}) + v(\mathbf{x}_{12},\mathbf{x}_{23}\mathbf{x}) + v(\mathbf{x}_{23},\mathbf{x}_{13}\mathbf{x})] + \delta\Sigma_{i\in N}V_i(\mathbf{z}) - \Sigma_{i\in N}R_i(\mathbf{z}) < 0$$

As all equilibria are efficient, $\Sigma_{i\in N}V_i(\mathbf{z}) = \Sigma_{i\in N}\bar{V}_i(\mathbf{z})$. This implies P7 violates P1.

For every other combination of strategies by countries 2 and 3 in case 2, it must be that $B_{ij} \bullet \delta V_i$ and $B_{ji} < \delta V_j$ for at least one pair of countries i and j. From (3) it follows that $B_{ij} - \delta V_i = B_{ji} - \delta V_j$. Thus, $B_{ij} \bullet \delta V_i$ and $B_{ji} < \delta V_j$ cannot simultaneously hold.

<u>Case 3</u>: No country makes an acceptable offer. This requires that $\delta V_i > B_{ij}$, for all i, j in N, which implies,

(P8) $$(1 - \delta)[v(\mathbf{x}_{12},\mathbf{x}) + v(\mathbf{x}_{13},\mathbf{x}) + v(\mathbf{x}_{23},\mathbf{x})] + \delta\Sigma_{i\in N}V_i(\mathbf{z}) - \Sigma_{i\in N}R_i(\mathbf{z}) < 0$$

This voliates P1 (see case 2c). QED

Proof of Lemma 2:
It follows from (7), that if the following equations are satisfied for all $V_j(\mathbf{z})$ satisfying (5), (6) and the condition $V_j(\mathbf{z} \mid \mathrm{i}) \in \{R_j(\mathbf{z}), \delta V_j(\mathbf{z})\}$, then (11) is the unique equilibrium bargaining value:

(P9) $(1 - \delta)(v_{ik} + \mathrm{r}_j) + \delta V_j(\mathbf{z}) - \mathrm{r}_j - \delta\phi v_{ij} \bullet 0$

(P10) $(1 - \delta)(v_{ij} + \mathrm{r}_k) + \delta V_k(\mathbf{z}) - \mathrm{r}_k - \delta\phi v_{ik} \bullet 0$

P9 holds iff $A_i \bullet B_{ij}$, $B_{ik} \bullet \delta V_i$, and $B_{ki} \bullet \delta V_k$. P10 holds iff $A_i \bullet B_{ik}$, $B_{ij} \bullet \delta V_i$, and $B_{ji} \bullet \delta V_j$. Thus there are only three possible equilibrium strategy combinations to consider.

Case 1: Both P9 and P10 hold. In this case, i chooses A_i, k chooses B_{ki}, and j chooses B_{ji}. The bargaining values in this equilibrium are found in equations (11). Subsituting (11) into P9 and P10 gives,

(P11) $(1 - \delta)(v_{ik} + \mathrm{r}_j) + \delta[\mathrm{r}_j + \phi v_{ij} + (1 - \delta)\phi v_{ik}] - \mathrm{r}_j - \delta\phi v_{ij} \bullet 0$

(P12) $(1 - \delta)(v_{ij} + \mathrm{r}_k) + \delta[\mathrm{r}_k + \phi v_{ik} + (1 - \delta)\phi v_{ij}] - \mathrm{r}_k - \delta\phi v_{ik} \bullet 0$

These reduce to $(1 - \delta)(1 + \delta\phi v_{ik}) \bullet 0$ and $(1 - \delta)(1 + \delta\phi v_{ij}) \bullet 0$, respectively.

Case 2: P9 holds, P10 does not. In this case, i chooses B_{ik}, k chooses B_{ki}, and j chooses δV_j, which implies: $V_j(\mathbf{z} \mid \mathrm{i}) = V_j(\mathbf{z} \mid \mathrm{k}) = R_j(\mathrm{z})$, and $V_k(\mathbf{z} \mid \mathrm{i}) = V_k(\mathbf{z} \mid \mathrm{j}) = \delta V_k(\mathbf{z})$. Using this information in equation (6) and solving gives: $V_i(\mathbf{z}) = V_k(\mathbf{z}) = \phi[v(\mathbf{0},\mathbf{x}_{jk}) - R_j]$, $V_j(\mathbf{z}) = \phi[(1-\delta)v(\mathbf{0},\mathbf{x}_{jk}) + 2R_j]$. Now if P10 fails, it must be that,

(P13) $(1 - \delta)(v_{ij} + \mathrm{r}_k) + \delta\phi[(v_{ik} + v_{ij} + \mathrm{r}_j + \mathrm{r}_k) - (\mathrm{r}_j + \delta\phi v_{ij})] - \mathrm{r}_k - \delta\phi v_{ik} < 0$

This reduces to $\dfrac{(1 - \delta) + (1 - \delta\phi)\delta\phi}{\delta(1 - \phi)} < m$, but this violates $m \bullet 1/2$.

Case 3: P1, P2 do not hold. In this case, i, j and k choose δV_i, δV_j, δV_k, repectively, which implies: $V_j(\mathbf{z} \mid \mathrm{i}) = V_j(\mathbf{z} \mid \mathrm{k}) = \delta V_j(\mathbf{z})$, and $V_k(\mathbf{z} \mid \mathrm{i}) = V_k(\mathbf{z} \mid \mathrm{j}) = \delta V_k(\mathbf{z})$. Using this information in equation (6) and solving gives: $V_i(\mathbf{z}) = V_j(\mathbf{z}) = V_k(\mathbf{z}) = (1/3)v(\mathbf{0},\mathbf{x})$. For this to be an equilbrium requires:

(P14) $(1 - \delta)(v_{ik} + \mathrm{r}_j + v_{ij} + \mathrm{r}_k) + (2\delta/3)(v_{ik} + v_{ij} + \mathrm{r}_j + \mathrm{r}_k) - \mathrm{r}_j - \mathrm{r}_k - \delta\phi v_{ij} - \delta\phi v_{ik} < 0$

This reduces to $\dfrac{(1 - 3\delta\phi^2)}{(1 - 3\phi)}$ $< m$, but this violates $m \bullet 1/2$. QED

Proof of Lemma 3:

Simply redo the proof of lemma 1, replacing $v(\mathbf{x}_{ik},\mathbf{x}_{ij}\mathbf{x})$ with $v(\mathbf{0},\mathbf{x}^c_{ij})$ and $v(\mathbf{x}_{ik},\mathbf{x})$ with $v(\mathbf{x}^c_{ik},\mathbf{1})$.

Proof of Corollary:

To establish the limit result, use (16) and (17) to write the uniqueness condition in lemma 3 as,

(P15) $$(1 - \delta)(1/3)(3 - 2\delta)\min[v(\mathbf{0},\mathbf{x}^c_{ij}), v(\mathbf{x}^c_{ik},\mathbf{1})] + \delta\bar{V}_i(\mathbf{1}) - R_i(\mathbf{1}) + \\ + (\delta/3)[(1 - \delta)(2e_i - e_j - e_k) - \delta(2\Delta_{jk} - \Delta_{ij} - \Delta_{ik})] - [(1 - \delta)e_i - \delta\Delta_{jk}] \bullet 0$$

From (8) and (15), we know that $\bar{V}_i(\mathbf{1}) - R_i(\mathbf{1}) \to 0$ as $\delta \to 1$, thus P15 $\to (1/3)[\Delta_{ij} + \Delta_{ik} - 2\Delta_{jk}] + \Delta_{jk} \bullet 0$, or $\Delta_{ij} + \Delta_{ik} + \Delta_{jk} \bullet 0$. QED

Equilibria in cases where $X = X^c$ and $\Sigma_{ij\in N}\Delta_{ij} < 0$:

As $\delta \to 1$, $V_i(\mathbf{1}|\, j) \to \min[V_i(\mathbf{1}), R^c{}_i(\mathbf{1})] = R^c{}_i(\mathbf{1}) + \pi_i$, where $\pi_i = \min[V_i(\mathbf{1}) - R^c{}_i(\mathbf{1}), 0] \bullet 0$. Thus, $V_i(\mathbf{1}) = (1/3)[v(\mathbf{0},\mathbf{1}) - R^c{}_j(\mathbf{1}) - R^c{}_k(\mathbf{1}) + R^c{}_i(\mathbf{1})] + (1/3)(2\pi_i - \pi_j - \pi_k)$, and

(P16) $$V_i(\mathbf{1}) - R^c{}_i(\mathbf{1}) = (1/3)\Sigma_{ij\in N}\Delta_{ij} + (1/3)(2\pi_i - \pi_j - \pi_k)$$

Possible equilibria fall into four cases:

<u>Case 0:</u> $\pi_i = \pi_j = \pi_k = 0$. This corresponds to the case in which each country makes acceptable bilateral-stage offers to both other countries. This case was ruled out by the corollary above.

<u>Case 1:</u> $\pi_1 < 0$, $\pi_2 = \pi_3 = 0$. In this case, $V_1(\mathbf{1}|\, j) = V_1(\mathbf{1})$ for $j = 2,3$ and $V_j(\mathbf{1}|\, k) = R^c{}_j(\mathbf{1})$ for all k, which means 2 and 3 must offer a CU exclusively to 1 or offer no bilateral deal at all, while 1 makes acceptable bilateral-stage offers to both 2 and 3. For this equilibrium to exist, it is necessary that, $V_2(\mathbf{1}) - R^c{}_2(\mathbf{1}) = (1/3)\Sigma_{ij\in N}\Delta_{ij} - (1/3)\pi_1 \bullet 0$ or $\Sigma_{ij\in N}\Delta_{ij} \bullet \pi_1$, and that $\pi_1 = (1/3)\Sigma_{ij\in N}\Delta_{ij} + (2/3)\pi_1$ or $\pi_1 = \Sigma_{ij\in N}\Delta_{ij}$. From this it follows that: $V_1(\mathbf{1}) = R^c{}_1 + \Sigma_{ij\in N}\Delta_{ij}$, and $V_j(\mathbf{1}) = R^c{}_j$, for j = 2, 3.

<u>Case 2:</u> $\pi_1 < 0$, $\pi_2 < 0$, $\pi_3 = 0$. In this case, $V_1(\mathbf{1}|\, j) = V_1(\mathbf{1})$, $V_2(\mathbf{1}|\, j) = V_2(\mathbf{1})$ and $V_3(\mathbf{1}|\, j) = R^c{}_3(\mathbf{1})$ for all j, which means that 1 and 2 make exclusive bilateral offers to each other, while 3 makes no offer at all.

For this equilibrium to exist, it is necessary that, $V_3(\mathbf{1})$ - $R^c{}_3(\mathbf{1})$ = (1/3)$\Sigma_{ij\in N}\Delta_{ij}$ - (1/3)(π_1 + π_2) • 0 or $\Sigma_{ij\in N}\Delta_{ij}$ • π_1 + π_2, that π_2 = (1/3)$\Sigma_{ij\in N}\Delta_{ij}$ + (2/3)π_2 - (1/3)π_1, and that π_1 = (1/3)$\Sigma_{ij\in N}\Delta_{ij}$ + (2/3)π_1 - (1/3)π_2. Thus, $\pi_1 = \pi_2$ = (1/2)$\Sigma_{ij\in N}\Delta_{ij}$. From this it follows that: $V_1(\mathbf{1}) = R^c{}_1$ + (1/2)$\Sigma_{ij\in N}\Delta_{ij}$, $V_2(\mathbf{1})$ = $R^c{}_2$ + (1/2)$\Sigma_{ij\in N}\Delta_{ij}$ $V_3(\mathbf{1}) = R^c{}_3$.

<u>Case 3:</u> $\pi_1 < 0$, $\pi_2 < 0$, $\pi_3 < 0$. In this case, $V_1(\mathbf{1}|\ j) = V_1(\mathbf{1})$, $V_2(\mathbf{1}|\ j) = V_2(\mathbf{1})$ and $V_3(\mathbf{1}|\ j) = V_2(\mathbf{1})$ for all j, which means no country links in the bilateral stage. For this equilibrium to exist, it is necessary that π_i = (1/3)$\Sigma_{ij\in N}\Delta_{ij}$ + (1/3)($2\pi_i - \pi_j - \pi_k$) for all i, j, k, or π_i = (1/3)$\Sigma_{ij\in N}\Delta_{ij}$ for all i. This implies $V_i(\mathbf{1}) = \tilde{V}_i(\mathbf{1})$ for all *i*.

The above discussion establishes the properties of those equilibria that are not *necessarily* ruled out as $\delta \to 1$. It does not imply that they exist for any $\delta < 1$. Next, we establish the existence of equilibria in the symmetric example of section III.C.3.

For an arbitrary discount factor, the payoffs in case 1 are: $V_1(\mathbf{1})$ = [1/(3-2δ)](1 - 2$R^c{}_2(\mathbf{1})$) and $V_2(\mathbf{1})$ = [1/(3-2δ)](1 - δ + $R^c{}_2(\mathbf{1})$). For this to be an equilibrium it must be that A_1 • max[B_{12}, δV_1], and that either B_{21} or δV_2 > max[A_2,B_{23}]. Begin by showing A_1 - B_{12} • 0. From (2) and (3) this condition is:

$$\begin{aligned} A_1 - B_{12} &= (1-\delta)v(\mathbf{0},\mathbf{x}^c{}_{12}) + \delta V_2 - R^c{}_2 \bullet 0 \\ &= (1-\delta)v(\mathbf{0},\mathbf{x}^c{}_{12}) + \delta[1/(3-2\delta)](1-\delta+R^c{}_2) - R^c{}_2 \bullet 0 \\ &= (3-2\delta)v(\mathbf{0},\mathbf{x}^c{}_{12}) + \delta - 3R^c{}_2 \bullet 0 \end{aligned}$$

Substituting $R^c{}_2 = e_2 + (\delta/3)v(\mathbf{0},\mathbf{x}^c{}_{12})$ and taking the limit as $\delta \to 1$, gives the condition: $1 - 3e_2$ • 0. Using the definition of e_2 from (23) the condition becomes, β • .232. It is straightforward to establish the conditions under which $A_1 - \delta V_1$ • 0. Using $A_1 = 1 - 2R^c{}_2$, $A_1 - \delta V_1$ = [1 - δ/(3-2δ)](1 - 2$R^c{}_2$). This is positive whenever $1 - 2R^c{}_2 > 0$, which is true for $\beta > .194$. Finally, we show that B_{21} and δV_2 > max[A_2,B_{23}]. Note that as $\delta \to 1$, both B_{21} and $\delta V_2 \to R^c{}_2$, while $A_2 = 1 - R^c{}_1 - R^c{}_2$ and $B_{23} \to 1 - R^c{}_1 - R^c{}_2$. So the limiting condition is $R^c{}_2 > 1 - R^c{}_1 - R^c{}_2$, which is equivalent to $\Sigma_{ij\in N}\Delta_{ij} < 0$. Thus, case 1 equilibria exist for all β such that β • .232 and $\Sigma_{ij\in N}\Delta_{ij} < 0$.

For an arbitrary discount factor, the payoffs in case 2 are: $V_1(\mathbf{1})$ =φ(1 - δ + 2$R^c{}_1(\mathbf{1})$) and $V_2(\mathbf{1})$ = φ(1 - $R^c{}_1(\mathbf{1})$). For this to be an equilibrium it must be that δV_1 > max[A_1, B_{12}], and B_{23} > max[A_2,B_{21},δV_2]. As $\delta \to 1$, $\delta V_1 \to R^c{}_1$, while $A_1 = 1 - 2R^c{}_2$ and $B_{12} \to 1 - 2R^c{}_2$. Thus, the limiting condition for δV_1 > max[A_1, B_{12}] is $R^c{}_1 > 1 - 2R^c{}_2$, which is equivalent to $\Sigma_{ij\in N}\Delta_{ij} < 0$. Next we show B_{23} > max[A_2,B_{21},δV_2]. We have already shown that $\delta V_1 > B_{12}$, which implies that $\delta V_2 > B_{21}$, so that we only need to show, B_{23} > max[A_2, δV_2]. As $\delta \to 1$, $B_{23} \to R^c{}_2$, while $A_2 = 1 - R^c{}_1 - R^c{}_2$. Thus, in the limit $B_{23} > A_2$ is equilvlent to $R^c{}_2 > 1 - R^c{}_1 - R^c{}_2$, or $\Sigma_{ij\in N}\Delta_{ij} < 0$. Next, consider the condition $B_{23} > \delta V_2$. Written out this is:

$$(1 - \delta)v_{23} + \delta - \delta V_2 - R^c{}_1 > \delta V_2$$
$$(1 - \delta)v_{23} + \delta > 2\delta\phi(1 - R^c{}_1) + R^c{}_1$$
$$(1 - \delta)v_{23} + (1 - \delta)\delta\phi > (1 - \delta)3\phi R^c{}_1$$
$$(3 - \delta)v_{23} + \delta > 3(\delta/3)(1 - v_{23})$$
$$(3 - \delta)v_{23} > - \delta v_{23}$$

This is true for any δ. Thus, case 2 equilibria exist for all β such that $\Sigma_{ij\in N}\Delta_{ij} < 0$.

Finally, for an arbitrary discount factor, the payoffs in case 3 are: $V_i(\mathbf{1}) = 1/3$ for all i. For this to be an equilibrium it must be, among other things, that $\delta V_1 > B^c{}_{23}$. This condition is, $\delta V_1 > (1 - \delta)v_{23} + \delta - \delta V_2 - R^c{}_1$. Using $V_i = 1/3$ and $R^c{}_1 = \delta/3(1 - v_{23})$ gives,

$$(\delta/3)(1 - v_{23}) > (1 - \delta)v_{23} + (\delta/3)$$
$$- (\delta/3)v_{23} > (1 - \delta)v_{23}$$

This is false for for any δ. Thus, case 3 equilibria do not exist.