Refutable Implications of the Heckscher-Ohlin Model

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Abstract

Previous empirical tests have found that, contrary to the conclusions of the Heckscher-Ohlin model, the factor composition of traded goods fails to reveal relative factor abundance rankings. Using a nonparametric approach, this paper discusses the refutability of two of the assumptions of the HO model: that countries have identical homothetic preferences and identical constant returns to scale production functions. We find that, for two countries, the assumption on preferences cannot be refuted with expenditure data alone. However, the assumption on technologies is refutable even when some factor prices (such as the rental rate on capital) are unobserved. Finally, we consider the refutability of Deardorff's (1982) more general HO model, the main result of which is that the value of net factor exports at (intrinsically unobservable) autarky factor prices is negative. We show that, in the 2×2 case, this model can be refuted using just observed data, i.e. data from the observed situation under trade.

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1 Introduction

The main conclusion of the Heckscher-Ohlin-Vanek (HOV) model is that, through trade in commodities, countries export factors they are relatively abundant in, and import scarce inputs. Further, ranking factors by net factor exports reveals their relative abundance. Previous empirical tests have tended to reject this conclusion. For example, Maskus (1985) finds that factor abundance rankings implied by U.S. trade in 1972 do not correspond to the rankings in terms of endowment shares. Bowen, Leamer and Sveikauskas (1987) test the HOV model under different assumptions about preferences, technology, and measurement errors, and also conclude that trade does not reveal relative factor abundance. Staiger (1988) finds evidence of misspecification of the HOV model, and concludes that endowments affect trade in important ways not captured by the HOV relationship.

The conclusions of the HOV model are driven by four sets of assumptions—identical homothetic preferences across countries, identical constant returns to scale production functions across countries, perfect competition in commodity and factor markets, and equal commodity and factor prices across countries[1]. Commodity and factor prices are, in fact, not equal across countries, as a result of both trade barriers (such as tariffs) and domestic policies such as taxes and subsidies. Since the failure of any one of the assumptions may invalidate the conclusions of the model, it is natural to ask whether there is any support for the remaining assumptions, or for more general versions of the HOV model that do not rely on commodity and factor price equalization.

This paper discusses nonparametric tests of the assumptions of identical homothetic preferences and identical constant returns to scale production functions across countries. One reason for being interested in these questions is the lowering of trade barriers in pockets across the globe (such as in Europe and North America), which will lead to commodity prices across certain countries being roughly similar. Predicting trade patterns in such a world depends on the accuracy of the assumptions used by the modeller. We also investigate the refutability of Deardorff's (1982) model, the main result of which is that, for each country, the value of net factor exports at (unobservable) autarky factor prices is negative.

The mathematical principle underlying the tests discussed is the Tarski-Seidenberg Theorem (Tarski, 1951, and Seidenberg, 1954), which states that, given a system of polynomial inequalities $\Gamma(x, y) \leq 0$ in observed (x) and unobserved (y) variables, there exist values

[1] If there exactly as many goods as factors, factor price equalization can be derived from the other assumptions.

for the unobserved variables such that the system of inequalities is satisfied *if and only if* the observed variables, x, satisfy a corresponding system of inequalities, $\Sigma(x) \leq 0$. An early application of this principle in economics was in the area of revealed preference, with Afriat (1967) deriving conditions on observed prices and quantities consumed that implied the existence of unobserved utility levels that could have come from a well-behaved utility function.

Extending Afriat's (1967) work, Varian (1982,1983,1984) proposed conditions under which there exist well-behaved homothetic utility functions or constant returns to scale production functions consistent with observed data on prices and quantities of goods and factors. Again, no functional form assumptions were made. Brown and Matzkin (1993) further extended this approach to general equilibrium, when quantities consumed by individual agents are unobservable. Snyder (1994), also in a general equilibrium framework, derives conditions for a more general class of utility functions, and also examines the testability of Pareto optimality.

Two important features about this approach are that, firstly, *no* parametric assumptions need to be made about the utility (or production, as the case may be) function, and secondly, the conditions on observed data exhaust *all* testable implications of the model. Clearly, to reject the model, it is sufficient to reject any implication of the model. However, if the observed data satisfy the proposed tests, no other tests on the same set of data can reject the model.

Varian's tests for homothetic preferences depend on the observability of both prices and quantities consumed. Country level consumption data tends to be in the form of expenditures (since the data generally refer to aggregate commodities), usually with price indices (relative to some base year) for each aggregate commodity. In Section 2 of this paper, we show that, for two countries, if, on even one commodity, only expenditure data are available, the hypothesis of a common homothetic utility function across the two countries is irrefutable.

However, if across the two countries, we observe relative price and quantity ratio (i.e. the ratio of price in country 1 to price in country 2), Varian's test can be implemented. This is the case with the Kravis, Heston, and Summers (1982) data set, which quantities prices and quantities (relative to U.S. levels) for aggregate commodities across 34 countries. This data set was used by Hunter and Markusen (1988), who rejected the hypothesis of a common homothetic utility function across all 34 countries. However, Hunter and Markusen tested for a specific functional form of the utility function (one that gives rise to linear expenditure systems). The nonparametric approach avoids this pitfall. Further, it can be used on as few as two countries, unlike the regression approach of Hunter and Markusen. For example, it may be used to test the hypothesis that countries that are geographically close have the same homothetic preferences. To illustrate this, we apply this test to the pairs (U.S. and India), (India and Malawi), and (U.S. and India), and find that *no* homothetic utility function is consistent with the data from U.S. and India. However, for the pairs (India and Malawi) and (U.S. and India), the data are consistent with common homothetic utility functions.

We examine tests on technologies in Section 3. Even when some factor prices are unobserved[2], we find restrictions that observed quantities of factors and the prices that are observed must satisfy in order to be consistent with common constant returns to scale production functions across two countries. We show that Helpman's (1984) post-trade restrictions on data are necessary, but not sufficient, for the existence common constant returns to scale production functions across two countries.

In Section 4, we consider a generalized Heckscher-Ohlin model, due to Deardorff (1984). The main result of this model is that, under the maintained assumptions, the value of net factor exports at autarky factor prices is negative. Since autarky factor prices are intrinsically unobservable, it may appear that this model cannot be refuted. For the 2 goods, 2 factors case, we show that the model can be refuted using only data from the (observed) trade situation.

2 Testing the Assumptions on Preferences

Suppose that at some point of time, we have (cross-sectional) data for K countries and N commodities. For each country k, we observe y^k (the N-vector of commodity outputs), and p^k (the N-vector of commodity prices). The test proposed by Varian (1983) may then be used to check whether the data support the assumption that all countries have identical, homothetic utility functions.

Theorem (Varian, 1983, Theorem 2)
Suppose that we observe (p^k, y^k) for $k = 1, \ldots, K$. Then the data are consistent with utility maximization given a monotonic, concave, homothetic utility function if and only if

$$(p^a y^b)(p^b y^c) \ldots (p^d y^a) \quad \geq \quad (p^a y^a) \ldots (p^d y^d) \tag{1}$$

[2] For example, the return to capital or land may not be observed.

for all choices of indices $a, b, c, \ldots, d \in [1, \ldots, K]$.

Varian proves the "if" part of this theorem by actually constructing a utility function with the required properties. Therefore, when (1) holds, we can find a homothetic utility function consistent with the data.

This test requires that both prices and quantities consumed be observed. Country level data generally entails some level of aggregation, and is often in expenditure terms. The question we pose is: are expenditure data sufficient to refute the homotheticity hypothesis?

Suppose, then, that for $N_1 \geq 1$ commodities, we observe only aggregate expenditures $\bar{y}_i^k$, where the subscript denotes the commodity and the superscript the country. For all other $N - N_1$ commodities, both prices and quantities are observed. Suppose further that $K = 2$, i.e. there are only two countries. In this context, we cannot refute the hypothesis that the countries have identical homothetic preferences.

Theorem 2.1 *Suppose that we observe* $(p_j^k, y_j^k, \bar{y}_i^k)$ *for* $k = 1, 2$, $i = 1, \ldots, N_1$, *and* $j = N_1 + 1, \ldots, N$, *with some* $\bar{y}_i^k > 0$ *for* $k = 1, 2$. *Then there always exists a monotonic, concave, homothetic utility function consistent with the data.*

PROOF

We prove the theorem for the case of $N_1 = 1$. The case $N_1 > 1$ follows readily from this by just fixing p_i^k, y_i^k such that $p_i^k y_i^k = \bar{y}_i^k$ for $k = 1, 2$ and $i = 1, \ldots, N_1 - 1$.

Suppose, then, that we have prices and quantities for $N - 1$ commodities and just expenditures for the N^{th} commodity.

Define

$$
\begin{aligned}
A^k &= \sum_{i=1}^{N} p_i^k y_i^k \quad \text{for } k = 1, 2, \\
B^1 &= \sum_{i=1}^{N-1} p_i^1 y_i^2 \\
B^2 &= \sum_{i=1}^{N-1} p_i^2 y_i^1
\end{aligned}
$$

Then, by Varian (1983) Theorem 2, the data are consistent with a utility function satsifying the requisite properties if and only if there exist p_N^k, y_N^k for $k = 1, 2$ such that:

$$
(B^1 + p_N^1 y_N^2)(B^2 + p_N^2 y_N^1) \geq A^1 A^2 \quad (2)
$$

$$
p_N^k y_N^k = \bar{y}_N^k \quad \text{for } k = 1, 2 \quad (3)
$$

If

$$B^1B^2 + \bar{y}_N^1\bar{y}_N^2 \geq A^1A^2$$

any choice of p_N^k, y_N^k that satisfies $p_N^k y_N^k = \bar{y}_N^k$ for $k = 1, 2$ will work.

Suppose, therefore, that

$$B^1B^2 + \bar{y}_N^1\bar{y}_N^2 < A^1A^2$$

Fix $p_N^1 = q$, and set $y_N^2 = z$, where

$$z \geq \frac{A^1A^2 - B^1B^2 - \bar{y}_N^1\bar{y}_N^2}{qB^2}$$

Now set

$$y_N^1 = \frac{\bar{y}_N^1}{q}$$
$$p_N^2 = \frac{\bar{y}_N^2}{z}$$

Clearly, these choices satisfy (2) and (3). ∎

In practice, national level input-output data often contain price information in the form of price indices, so that for each commodity aggregate, the price in the current year relative to the price in some base year is observed. This is still not enough, since the relative prices across commodities (i.e. of one commodity to another) remain unobserved.

Let q_i^k, for $i = 1, \ldots, N$ indicate the price index associated with good i in country k, relative to the price of the same good in the same country in some previous year. Let λ_i^k denote this unobserved base price[3].

Theorem 2.2 *Suppose that we observe* $(p_j^k, y_j^k, \bar{y}_i^k, q_i^k)$ *for* $k = 1, 2,\ i = 1, \ldots, N_1$, *and* $j = N_1 + 1, \ldots, N$, *with some* $\bar{y}_i^k > 0$ *for* $k = 1, 2$. *Then there always exists a monotonic, concave, homothetic utility function consistent with the data.*

PROOF

Again, we prove the theorem for the case $N_1 = 1$. Let the N^{th} good be the good for which only expenditure data and a price index are available. Let $B^1 = \sum_{i=1}^{N-1} p_i^1 y_i^2$, $B^2 = \sum_{i=1}^{N-1} p_i^2 y_i^1$, and $A^k = \sum_{i=1}^{N} p_i^k y_i^k$ for $k = 1, 2$. Then, (1) reduces to

$$(B^1 + \lambda_N^1 q_N^1 y_N^2)(B^2 + \lambda_N^2 q_N^2 y_N^1) \geq A^1A^2 \tag{4}$$

[3] If both the base price and the price index are observed, the current price can be computed, and (1) is implementable.

As with Theorem (2.1), if $B^1B^2 + \bar{y}^1\bar{y}^2 \geq A^1A^2$, any choice of λ_N^k, y_N^k such that $\lambda_N^k q_N^k y_N^k = \bar{y}_N^k$ leads to (4) being satisfied. Suppose, then, that

$$B^1B^2 + \bar{y}^1\bar{y}^2 < A^1A^2$$

Fix $\lambda_N^1 = \mu$, and set $y_N^2 = z$, where

$$z \geq \frac{A^1A^2 - B^1B^2 - \bar{y}_N^1\bar{y}_N^2}{\mu q_N^1 B^2}$$

Now set $y_N^1 = \bar{y}_N^2/\mu q_N^1$ and $\lambda_N^2 = \bar{y}_N^2/q_N^2 y_N^2$. Clearly, these choices lead to (4) being satisfied.

■

Note that the two theorems above hold if there is just one commodity for which we have only expenditure and price index data[4]. Further, it is clear from the proof of Theorem 2.2 that having a quantity index in addition to a price index for commodities on which only expenditure data are available is of no benefit. In equation (4) above, interpreting q_N^k as the product of a price index in country k and a quantity index in country ℓ creates no difficulty whatsoever.

Additional observations (i.e. data from additional countries) increase both the number of unknowns and the number of inequalities in the system. In general, if there are q commodities for which only aggregate expenditures are observed, and we have data from r countries, we have a system with $2qr$ unknowns, qr equations, and $\sum_{j=2}^{r} \frac{r!}{j!(r-j)!}$ inequalities. In priniciple, the Tarski-Seidenberg algorithm (outlined in the proof of the Tarski-Seidenberg theorem, by Tarski, 1951, and Seidenberg, 1954) could be used to reduce this system of nonlinear inequalities to an equivalent system in the observables. However, this algorithm is doubly exponential in the unobservables. There is no known practical method for effecting this reduction.

The implication of these theorems is that, for two countries, we cannot test the assumptions on preferences made by the Heckscher-Ohlin model with expenditure data alone. There are at least three alternatives available. One is to try and bound relative or absolute prices. Notice that the proof assumes that we are free to pick prices and quantities on the i^{th} commodity. Every bound obtained increases the number of inequalities in the system. This creates no theoretical difficulty, but may make it harder to determine whether or not there is a feasible solution.

[4]Typically, national level consumption data on most commodities is in expenditure form.

Secondly, we could make additional assumptions and test a joint hypothesis. A famous example of this is, of course, the HOV model itself.

Thirdly, one could try and find data on prices and quantities separately. This data is not easily available. Any test must be at a highly disaggregated level (since aggregation is necessarily by value) and will therefore involve hundreds of commodities, if not more. This will further mean that several sources will have to be consulted to gather complete data. For example, the *Statistical Abstract of the United States* contains price and quantity data for the U.S.A. for several non-manufacturing commodities. Obtaining manufacturing data may involve consulting several specialized industry bulletins.

Finally, we consider the data provided by Kravis, Heston, and Summers (1982), who do provide comparable prices for commodity aggregates, for a sample of 34 countries, and for the year 1975. Both prices and quantities of commodities across countries are provided relative to their levels in the U.S. However, commodity prices in the U.S. are not quoted separately. In other words, we can compare prices across countries for the same commodity, but not across commodities for the same country. Total expenditure on each commodity category for each country is also shown.

Hunter and Markusen (1988) test for identical homothetic preferences across countries, using this data set. They perform their test by assuming a linear expenditure system, derived from a utility function of the form

$$U(c) = \prod_{i=1}^{N} (c_i - \bar{c}_i)^{\beta_i}$$

and estimate it with and without restrictions imposed by homotheticity. They find that there are strong grounds for rejecting homotheticity of preferences across countries, in favour of common but non-homothetic preferences. However, their test clearly fails to rule out any other functional form for the utility function. Further, like any regression-based analysis, it cannot be carried out with a small sample of countries[5].

In this situation, we observe all we need to directly check (1). The data for country k include $(p_i^k y_i^k, \alpha_i^k, \beta_i^k)$, where

$$\alpha_i^k = \frac{p_i^k}{p_i^{US}}$$

$$\beta_i^k = \frac{y_i^k}{y_i^{US}}$$

[5] The data contains 34 observations, and the regression estimated has 12 coefficents.

Define $z_i = p_i^{US}\, y_i^{US}$. It is easy to see that there exists a monotonic, concave, homothetic utility function consistent with the data if and only if

$$(\sum_{i=1}^{N} \alpha_i^a \beta_i^b\, z_i)\ldots(\sum_{i=1}^{N} \alpha_i^c \beta_i^a\, z_i) \quad \geq \quad (\sum_{i=1}^{N} \alpha_i^a \beta_i^a\, z_i)\ldots(\sum_{i=1}^{N} \alpha_i^c \beta_i^c\, z_i) \tag{5}$$

for all choices of indices $a, b, \ldots, c \in [1, \ldots, K]$.

Unlike a regression-based method, this test can be carried out with data from as few as two countries (i.e. just two observations). It is therefore possible to check whether subsets of countries have common homothetic preferences. To illustrate this method, we used the Kravis, Heston and Summers (1982) data set to test whether the U.S.A., India and Malawi could have had common homothetic preferences. The details of the test are reported in appendix A.1. The results indicate that there exist homothetic utility functions consistent with the data from the U.S.A. and Malawi, and India and Malawi, but not the U.S.A. and India.

We next ask whether there exist *any* well-behaved (i.e. continuous, monotonic, concave) utility functions consistent with the data from the three countries. This test is reported in Appendix A.2. We find that we cannot refute the hypothesis that the three countries had a common well-behaved, non-homothetic utility function.

3 Testing the Assumptions on Technologies

Suppose that we observe data on prices and quantities, and on M factors of production, i.e. for each country k we observe p_i^k, y_i^k, w_i^k (the M-vector of factor prices in the i^{th} sector) and x_i^k (the M-vector of factor quantities used to produce the i^{th} commodity). Suppose further that producers in each sector are competitive profit maximizers. This implies that, regardless of the price paid by consumers in each country, the producers of country k always receive the same price for their output[6]. Varian (1984) has proposed a test for the assumption of a common, constant returns to scale production function across all countries. Again, given data that satisfy the conditions of his theorem, we can actually construct a production function with the requisite properties.

Theorem (Varian, 1984, Theorem 6)
Suppose that for each country $k = 1, \ldots, K$ and for some commodity i, we observe $(p_i^k, y_i^k,$

[6] In other words, the benefits of price differentials are captured by governments, say in the form of tariffs.

w_i^k, x_i^k). *Then the data are consistent with profit maximization given a monotonic, concave, CRS production function if and only if*

$$0 = p_i^k y_i^k - w_i^k x_i^k \geq p_i^k y_i^j - w_i^k x_i^j \tag{6}$$

for all $k \in \{1, \ldots, K\}$ *and for all* $j \neq k$.

In the case where only commodity expenditures are observed, but not prices or quantities, the condition above can be further simplified. The following lemma shows that, if there are two countries and only commodity expenditures are observed, the inequality in (6) reduces to a condition analogous to the one for well-behaved homothetic preference functions.

Lemma 3.1 *Suppose that* $K = 2$*, and that for each country* k *and some commodity* i*, we observe* $(\bar{y}_i^k, w_i^k, x_i^k)$. *Then the data are consistent with profit maximization given a monotonic, concave, CRS production function if and only if*
(i) $p_i^k y_i^k = w_i^k x_i^k$, *for* $k = 1, 2$, *and*
(ii) $(w_i^1 x_i^2)(w_i^2 x_i^1) \geq (w_i^1 x_i^1)(w_i^2 x_i^2)$

PROOF

From Varian (1984) Theorem 6, we need that there exist p^1, p^2, y^1, y^2 such that (dropping the $_i$ subscript for convenience)

$$p^1 y^1 - w^1 x^1 = 0 \tag{7}$$
$$p^1 y^2 - w^1 x^2 \leq 0 \tag{8}$$
$$p^2 y^2 - w^2 x^2 = 0 \tag{9}$$
$$p^2 y^1 - w^2 x^1 \leq 0 \tag{10}$$
$$p^1 y^1 = \bar{y}^1 \tag{11}$$
$$p^2 y^2 = \bar{y}^2 \tag{12}$$

(7) and (9) are restated as condition (i) of the lemma.

Transfer the observables (w, x) to the right-hand side of the first four equations above, and then divide (7) by (8) and (10) by (9) to get

$$\frac{w^1 x^1}{w^1 x^2} \leq \frac{y^1}{y^2} \leq \frac{w^2 x^1}{w^2 x^2} \tag{13}$$

To see sufficiency of this condition, consider the first inequality in (13) and multiply the RHS above and below by p^1. (7) now implies (8). To get (10), consider the second inequality in (13) and multiply the LHS above and below by p^2.

Condition (ii) of the lemma follows immediately from (13). ■

In general, factors used to produce a commodity will include primary factors (labor, capital, land) and other (intermediate) commodities. However, in principle, input-output data can be used to solve out for the outputs as a function of the primary inputs alone.

Brecher and Choudhri (1992) implement this test with production data for several commodities from the U.S. and Canada, and find support for the hypothesis that each commodity is produced with a common, CRS production function. The basic test they propose is

$$\frac{w^1x^2}{p^2y^2}\frac{w^2x^1}{p^1y^1} \geq \frac{w^1x^1}{p^1y^1}\frac{w^2x^2}{p^2y^2} = 1 \qquad (14)$$

which is equivalent to condition (ii) of Lemma 3.1 above.

Even this test is difficult to implement due to lack of data (the rental rate on capital is not directly observable). Brecher and Choudhri propose different methods for inferring the rental rate. One is to assume perfect competition in commodity markets, and that capital is the only factor with an unobserved price. Since perfect competition implies that returns to factors exhaust the value of the output, the rental rate can be inferred (given the value of the capital stock). Another is to use the long-term interest rate as a proxy. Neither is entirely satisfactory, but there are yet further complications.

Firstly, there may be more than one kind of capital, or other primary inputs (factors or commodities) with unobserved prices. Brecher and Choudhri use data on two kinds of capital (equipment and structures), whereas Maskus (1985) examines a third, human capital. The returns to different kinds of capital may well differ, rendering estimation of each rental rate impossible. Secondly, Brecher and Choudhri mention that observed wage rates differ across industries within the same country. This implies that prices of other factors may also exhibit inter-sectoral differences, so that it is impossible to use a single proxy variable to approximate them. Brecher and Choudhri circumvent these problem by using different methods to infer the rental rate on capital, and assuming a price of one for primary commodity inputs with unobserved prices.

In this context, if we observed commodity expenditures, quantities of all factors and prices of all but one, the assumptions of constant returns to scale and competitive, frictionless markets allow us to compute the single unobserved price (since the value of the output must equal the total cost of production). This would enable the above test to be carried out.

However, even if there is more than one factor whose return is observed, we can derive a test for CRS production functions. In general, suppose that[7] we observe quantities of all M factors, and prices of just $M_1 < M$. Denote the observed prices and quantities of these M_1 factors by w^k, x^k, and the unobserved prices and observed quantities of the remaining factors by v^k, z^k. Suppose further that there is data for just two countries. Then the following theorem provides a necessary and sufficient condition for the data to be consistent with a well-behaved CRS production function.

Theorem 3.2 *Suppose that $K = 2$, and that, for the i^{th} commodity, we observe $(p^k, y^k, w^k, x^k, z^k)$ for $k = 1, 2$, where $w^k, x^k \in R_+^{M_1}$, with $M_1 < M$, and $z^k \in R_{++}^{M-M_1}$. Then the data is consistent with profit maximization given some monotonic, concave, CRS production function if and only if:*
(i) $p^k y^k \geq w^k x^k$, and
(ii) $(p^k y^k - w^k x^k) \max_{j \in \{M_1+1,\ldots,M\}} \left(\frac{z_j^\ell}{z_j^k}\right) \geq p^\ell y^k - w^\ell x^k \quad$ for $k = 1, 2$ and $\ell \neq k$.

PROOF

Consider the problem

$$\min_{v^1} \quad p^1 y^2 - w^1 x^2 - v^1 z^2 \tag{15}$$

$$\text{subject to} \quad p^1 y^1 - w^1 x^1 - v^1 z^1 = 0 \tag{16}$$

Clearly, the minimum value of (15) is attained by setting

$$v_k^1 = \begin{cases} \frac{p^1 y^1 - w^1 x^1}{z_k^1} & \text{if } k = \text{ Arg max}_{j \in \{M_1+1,\ldots,M\}} \frac{z_j^2}{z_j^1} \\ 0 & \text{otherwise} \end{cases}$$

Now we just need the minimized value of (15) to be non-positive, from which part (ii) of the Lemma readily follows. ■

Finally, suppose now that for some commodity i, in each of two countries all we observe is the expenditure on the commodity (but not price or quantity), quantities of all M factors, and prices of $M_1 < M$ factors. Further, define

$$\alpha^k = \max_{j \in \{M_1+1,\ldots,M\}} (z_j^\ell / z_j^k) \quad \text{for } k = 1, 2 \text{ and } \ell \neq k$$

[7] Again, for some commodity i. The $_i$ subscript is dropped for convenience.

The hypothesis of a CRS production function is still refutable, as the next theorem shows.

Theorem 3.3 *Suppose that* $K = 2$*, and that, for the* i^{th} *commodity, we observe* $(\bar{y}^k, w^k, x^k, z^k)$ *for k=1,2, where* $w^k, x^k \in R_{+}^{M_1}$*, with* $M_1 < M$*, and* $z^k \in R_{++}^{M-M_1}$*. Then there exists a monotonic, concave, CRS production function consistent with the data if and only if:*
(i) $\bar{y}^k \geq w^k x^k$ *for* $k = 1, 2$*, and*
(ii) $(w^1 x^2 + \alpha_1(\bar{y}^1 - w^1 x^1))(w^2 x^1 + \alpha_2(\bar{y}^2 - w^2 x^2)) \geq \bar{y}^1 \bar{y}^2$

PROOF

Such a production function exists if and only if there exist v^k such that part (ii) of Lemma 3.1 above is satisfied. In this case part (ii) of Lemma 3.1 is written as

$$(w^1 x^2 + v^1 z^2)(w^2 x^1 + v^2 z^1) \geq \bar{y}^1 \bar{y}^2 \tag{17}$$

Consider the terms on the left-hand side. Since the data from each country must also satisfy (16), it is clear that the maximum value of $(v^k z^\ell)$ is attained by setting

$$v_j^k = \begin{cases} \frac{\bar{y}^k - w^k x^k}{z_j^k} & \text{if } j = \text{ Arg max } \frac{z_j^\ell}{z_j^k} \\ 0 & \text{otherwise} \end{cases}$$

leading to part (ii) of the theorem. ∎

Helpman's (1984) post-trade restrictions on data can be seen to be necessary but not sufficient conditions for the existence of common CRS production functions for each commodity. For two countries, k and ℓ, Helpman's condition is stated as

$$(w^\ell - w^k) \sum_{i=1}^{N} (T_i^{\ell k} a_i^k - T_i^{k\ell} a_i^\ell) \geq 0 \tag{18}$$

where a_i^k is the cost-minimizing quantity of inputs necessary to produce one unit of commodity i in country k, and $T_i^{\ell k}$ is the gross exports of commodity i from k to ℓ. If commodity i is exported from ℓ to k, $T_i^{k\ell} > 0$ and $T_i^{\ell k} = 0$.

This condition is necessary for the existence of common CRS production functions across the two countries. To see this, note that under CRS, $a_i^k = x_i^k / y_i^k$, so that from equation (6) we have $p_i^k = w^k a_i^k$ and $p_i^\ell \leq w^\ell a_i^k$. Now, setting $p_i^k = p_i^\ell$ yields $(w^\ell - w^k) a_i^k \geq 0$. Similarly, we have $(w^k - w^\ell) a_i^\ell \geq 0$. Now, multiplying the first inequality by $T_i^{\ell k}$ and the second by $T_i^{k\ell}$, and then summing across all commodities yields Helpman's result.

This result fails to be sufficient on two counts. Firstly, sufficiency requires that $p_i^k = w^k a_i^k$, and, secondly, the result must hold for each commodity, and not just in the aggregate. Since the result boils down to a test of CRS production functions for each commodity, the results of this section go through completely in terms of deriving necessary and sufficient conditions for this model to be consistent with the data in the presence of unobserved variables.

4 A Generalized Heckscher-Ohlin Model

In this section, we consider the model of Deardorff (1982), who relaxes two of the three assumptions of the basic HOV model. There is no assumption on commodity or factor price equalization, and the only restriction on preferences is that they satisfy WARP (the Weak Axiom of Revealed Preference). On the production side, constant returns to scale are assumed for each good, along with frictionless domestic markets and competitive behavior[8]. For the main theorem of the paper, technology is not assumed to be identical across countries[9]. This theorem states that $w^a S < 0$, where w^a is a vector of factor prices for the country under autarky, and S_j represents the country's net exports (positive or negative) of factor j, as embodied in the commodities traded by the country.

Autarky factor prices inversely reflect factor abundance in the sense of Ohlin, so that this result says that countries tend to export their more abundant factors (those with lower prices) and import their less abundant ones. In the absence of joint production, and assuming CRS production functions, S can be directly computed from observed data for one country alone, by imputing factors to goods on the basis of domestic factors of production[10]. Autarky factor prices, however, are intrinsically unobservable, and are a natural candidate for elimination from the system of inequalities that defines this model.

To illustrate the refutable implications of this model, we restrict the analysis to the 2×2 case (i.e. 2 commodities and 2 factors). Further, we assume that only the commodities, and not the factors, are directly traded.

Suppose, then, that over a single time period, for some country, we observe p^t, the vector of commodity prices under trade, y^p, the produced vector of goods, y^c, the consumed vector of goods, w^t, the vector of factor prices, and $\{x^t_{ij}\}_{i,j=1,2}$, the quantities of factors used

[8] The lack of commodity or factor price equalization, therefore, is assumed to arise due to trade policies alone.

[9] However, identical technologies are assumed for the two corollaries that follow.

[10] As computed by this method, S satisfies Deardorff's Assumption 11, which is needed to prove his main theorem, but not his Assumption 12, which is used only for the corollaries.

in the production of each good[11]. The net factor exports vector, S is then computed as:

$$S_j = \sum_{i=1}^{2} x_{ij}(y_i^p - y_i^c) \quad \text{for } j = 1,2 \tag{19}$$

Let p^a, y^a, w^a denote the unobserved commodity prices, the produced (and consumed) commodity bundle, and factor prices under autarky. Following Deardorff, we assume that both the observed trade quantities and the unobserved autarky quantities imply full employment of factors. For each country, we can now write down a system of polynomial inequalities in the observed and unobserved variables that describes the assumptions and conclusion of the model:

$$\sum_{i=1}^{2} x_{ij}^a = \sum_{i=1}^{2} x_{ij}^t \quad \text{for } j = 1,2 \tag{20}$$

$$p_i^t y_i^p - \sum_{j=1}^{2} w_j^t x_{ij}^t = 0 \quad \text{for } i = 1,2 \tag{21}$$

$$p_i^t y_i^a - \sum_{j=1}^{2} w_j^t x_{ij}^a \leq 0 \quad \text{for } i = 1,2 \tag{22}$$

$$p_i^a y_i^a - \sum_{j=1}^{2} w_j^a x_{ij}^a = 0 \quad \text{for } i = 1,2 \tag{23}$$

$$p_i^a y_i^p - \sum_{j=1}^{2} w_j^a x_{ij}^t \leq 0 \quad \text{for } i = 1,2 \tag{24}$$

$$p^t y^c \geq p^t y^p \tag{25}$$

$$p^a y^c > p^a y^a \tag{26}$$

$$w^a S < 0 \tag{27}$$

Equation (20) above merely states that there is full employment of factors under both trade and autarky (consistent with a monotonic production function). (21)-(24) are necessary and sufficient for the existence of monotonic, concave CRS production functions for each good. (25) is an assumption of the model (it is a natural assumption; importantly, for our purposes, it is a restriction only on observed data).

Note that (20)-(24) together imply

$$p^t y^p \geq p^t y^a \tag{28}$$

$$p^a y^a \geq p^a y^p \tag{29}$$

[11] Note that, as mentioned in earlier sections, this is more than is readily observed with available data.

Now, (28) and (25) lead to $p^t y^c \geq p^t y^a$. The bundle consumed under autarky is therefore affordable at trade prices, so that WARP (necessary and sufficient for the existence of a monotonic, concave utility function, as shown by Varian, 1982) now implies (26).

Notice that (27) in this case holds if and only if

$$\frac{w_1^a}{w_2^a} S_1 + S_2 < 0 \tag{30}$$

Hence, the (unobserved) factor price ratio under autarky, $\frac{w_1^a}{w_2^a}$ is of immediate interest. We approach the problem of deriving restrictions on observables such that the above system of equations is satisfied by first ignoring (27), and trying to determine restrictions on $\frac{w_1 f^a}{w_2^a}$. Once we have those, we can then re-impose (27).

First, we define some restrictions that we will need observed data to satisfy. Of these, OC 2 and OC 3 below merely offer notational convenience: OC 2 defines good 1 to be the import good, and OC 3 states that good 1 is relatively intensive in factor 2. OC 4 is necessary for the existence of CRS production functions. OC 1 states that the country has balanced trade at its own prices. The natural assumption here is $p^f y^p = p^f y^c$, where p^f represents world prices for commodities. OC 1 will hold for a country that either practices free trade or has a uniform tariff on all commodities.

Definition 4.1 The following are referred to as *Observable Conditions*:

OC 1. $p^t y^p = p^t y^c$.

OC 2. $y_1^c > y_1^p$ and $y_2^c < y_2^p$.

OC 3. $\frac{x_{11}^t}{x_{12}^t} < \frac{x_1}{x_2} < \frac{x_{21}^t}{x_{22}^t}$.

OC 4. $p_i^t y_i^p = \sum_{j=1}^2 w_i^t x_{ij}^t$ for $i = 1, 2$.

Definition 4.2 An economy is a *Heckscher-Ohlin Production Economy* (HOPE) if it has a monotonic, concave utility function and monotonic, concave CRS production functions for each good.

Lemma 4.3 *Suppose that, for some country, the observed data* (p^t, y^p, y^c) *satisfy OC 1 and OC 2. Then the economy can be a HOPE only if (i)* $y_1^a \geq y_1^p$ *and (ii)* $\frac{p_1^a}{p_2^a} > \frac{p_1^t}{p_2^t}$.

PROOF

$p^t y^p = p^t y^c$ implies that $\frac{p_1^t}{p_2^t} = \frac{y_2^p - y_2^c}{y_1^c - y_1^p}$. Further, from (29) and (26), we have $p^a y^c > p^a y^p$, so that $\frac{p_1^a}{p_2^a} > \frac{y_2^p - y_2^c}{y_1^c - y_1^p}$, leading to part (ii) of the Lemma.

Now, (28) and (29) further imply that

$$\frac{p_1^t}{p_2^t}\,(y_1^a - y_1^p) \;\leq\; y_2^p - y_2^a \tag{31}$$

$$\frac{p_1^a}{p_2^a}\,(y_1^a - y_1^p) \;\geq\; y_2^p - y_2^a \tag{32}$$

We therefore have $(\frac{p_1^a}{p_2^a} - \frac{p_1^t}{p_2^t})(y_1^a - y_1^p) \geq 0$, and, given part (ii), part (i) of the Lemma now follows. ■

This intuition is expressed in Figure 1 below, which exhibits the budget line under trade. (28) implies that y^a lies inside the triangle AOB. Further, the budget line is steeper under autarky than trade, and, along with (29), this further restricts the autarky bundle y^a to lie in the shaded region.

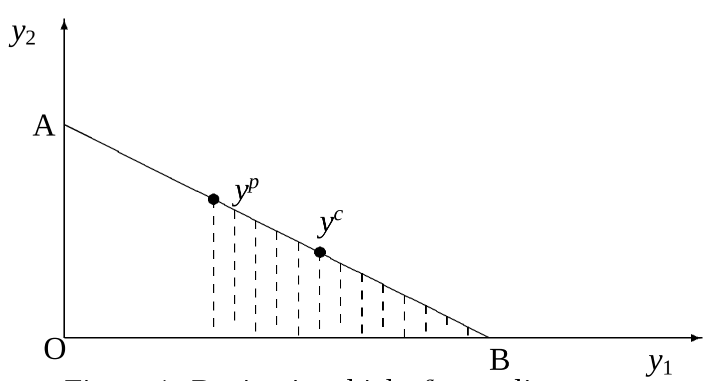

Figure 1: Region in which y^a must lie

The next theorem provides a necessary and sufficient condition for autarky factor prices to be consistent with a HOPE.

Theorem 4.4 *Suppose that for some country the observed data* $(p^t, y^p, y^c, w^t, \{x_{ij}^t\}_{i,j=1,2})$ *satisfy conditions OC 1-4. Then, for given autarky factor prices* w^a*, there exist commodity* *prices p^a and quantities y^a, and factor quantities $\{x^a_{ij}\}_{i,j=1,2}$ such that all observed and unobserved values are consistent with a HOPE if and only if $\frac{w^a_1}{w^a_2} < \frac{w^t_1}{w^t_2}$.*

PROOF

"Only if" part:

From Lemma 4.3, it must be the case that $\frac{p^a_1}{p^a_2} > \frac{p^t_1}{p^t_2}$. The rest now follows from a standard Stolper-Samuelson argument. Consider the unit output isoquant for each good, and let z_{ij} denote the optimal quantities of factors along these isoquants, given $\frac{w_1}{w_2}$. We have

$$\frac{p_1}{p_2} = \frac{z_{11} + (w_2/w_1)z_{12}}{z_{21} + (w_2/w_1)z_{22}} \tag{33}$$

so that, at given $\{z_{ij}\}_{i,j=1,2}$, $\frac{w_2}{w_1}$ is an increasing function of $\frac{p_1}{p_2}$.

"If" part:

Suppose that $\frac{w^a_1}{w^a_2} < \frac{w^t_1}{w^t_2}$. We will choose $p^a, y^a, x^a_{11}, x^a_{21}, x^a_{12}, x^a_{22}$ such that (20)-(26) are satisfied.

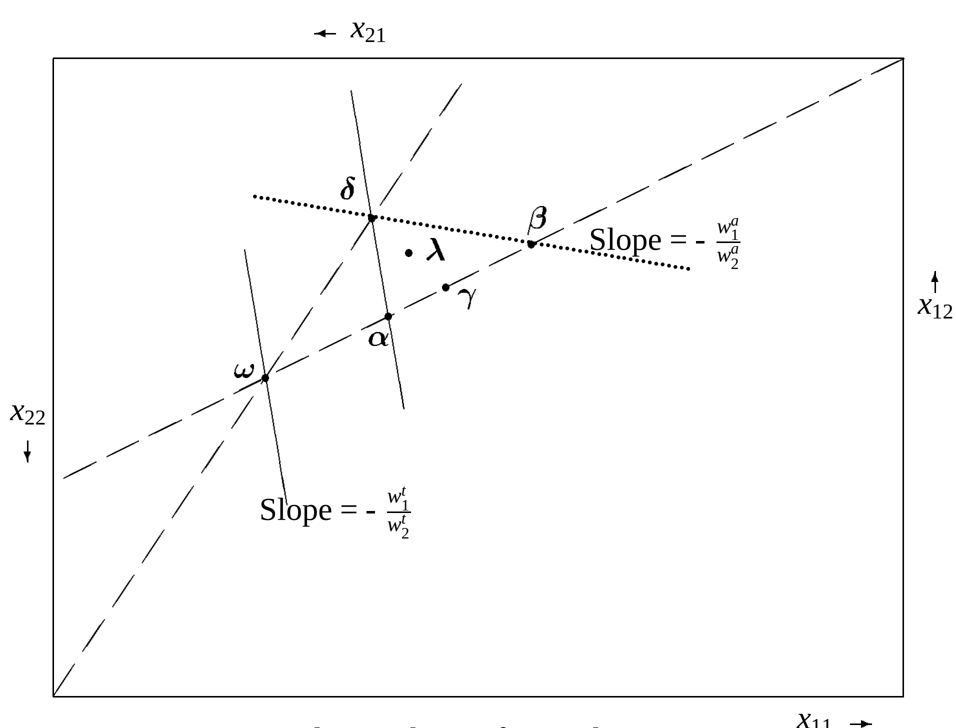

Figure 2: Edgeworth Box for Production

Consider the Edgeworth box for production, depicted in Figure 2. Let ω denote the production point under trade. Choose any y_1^a such that $y_1^p < y_1^a < y_1^c$. Let $\delta = (\delta_1, \delta_2) = \frac{y_1^a}{y_1^p}(x_{11}^t, x_{12}^t)$. Now, define $\alpha, \beta \in R_{++}^2$ such that

$$\begin{aligned} (w^t\alpha = w^t\delta) \quad &\text{and} \quad (\frac{x_1 - \alpha_1}{x_2 - \alpha_2} = \frac{x_{21}^t}{x_{22}^t}) \\ (w^a\beta = w^a\delta) \quad &\text{and} \quad (\frac{x_1 - \beta_1}{x_2 - \beta_2} = \frac{x_{21}^t}{x_{22}^t}\} \end{aligned}$$

Notice that $\alpha_1 < \beta_1$ and $\alpha_2 < \beta_2$, so that $w^t\beta > w^t\alpha$. Choose $\rho \in (0, 1)$ and define $\gamma = \rho\alpha + (1 - \rho)\beta$. Set

$$y_2^a = y_2^p \frac{x_1 - \gamma_1}{x_1 - x_{21}^t} = y_2^p \frac{x_2 - \gamma_2}{x_2 - x_{22}^t}$$

Finally, choose $\mu \in (0, 1)$, and define $\lambda = \mu\delta + (1 - \mu)\gamma$. λ is a candidate production point under autarky.

These choices obviously satisfy (20), (21) and (23) above. To see that (22) is satisfied, notice that

$$p_1^t y_1^a = (\frac{y_1^a}{y_1^p})(p_1^t y_1^p) = (\frac{y_1^a}{y_1^p})(w_1^t x_{11}^t + w_2^t x_{12}^t) = w^t\alpha$$

But, since $w^t\beta > w^t\alpha$, we have $w^t\lambda > w^t\alpha$, proving that (22) holds for good 1. A similar argument shows that it holds for good 2 as well.

Next, consider (24) for good 1. From the definition of p_1^a, we have

$$\begin{aligned} p_1^a y_1^p - w_1^a x_{11}^t - w_2^a x_{12}^t < 0 \quad &\Longleftrightarrow \quad (\frac{y_1^p}{y_1^a})(w_1^a x_{11}^a + w_2^a x_{12}^a) < w_1^a x_{11}^t + w_2^a x_{12}^t \\ &\Longleftrightarrow \quad w^a\lambda < w^a\delta \end{aligned}$$

where the last inequality follows from the construction of λ. Again, a similar argument proves the case for good 2.

Finally, (22) and (24), together with $y_1^a > y_1^p$, imply that $\frac{p_1^a}{p_2^a} > \frac{p_1^t}{p_2^t}$, which along with $p^t y^c > p^t y^a$ and $y_1^c > y_1^a$ implies (26). ■

Now, we are in a position to re-impose (27). We say that a set of observed data $(p^t, y^p, y^c, w^t, \{x_{ij}^t\}_{i,j=1,2})$ is consistent with the generalized Heckscher-Ohlin model of Deardorff if there exists some HOPE that is both consistent with the observed data and satsifies (27).

Definition 4.5 The observed data $(p^t, y^p, y^c, w^t, \{x_{ij}^t\}_{i,j=1,2})$ are *GHO-consistent* if there exists some HOPE such that:
(i) all observed and unobserved values are consistent with that HOPE, and
(ii) $w^a S < 0$.

Theorem 4.6 *Suppose that for some country the observed data* $(p^t, y^p, y^c, w^t, \{x_{ij}^t\}_{i,j=1,2})$ *satisfy conditions OC 1-4. Then the data are GHO-consistent if and only if at least one of the following two conditions holds:*
(i) $S_2 < 0$ *or (ii)* $w^t S < 0$.

PROOF

"If" part:

Suppose that $S_2 < 0$. Then either $S_1 < 0$, in which case we are done, or $S_1 \geq 0$, in which case there exists an $\epsilon > 0$ such that

$$\delta S_1 + S_2 < 0 \quad \text{for all } \delta < \epsilon$$

Choose $\frac{w_1^a}{w_2^a} \in (0, \min(\frac{w_1^t}{w_2^t}, \epsilon))$. The rest now follows from Theorem 4.4.

Suppose now that $S_2 \geq 0$ but $w^t S < 0$. This can happen only if $S_1 < 0$. Further $w^t S < 0 \iff \frac{w_1^t}{w_2^t} S_1 + S_2 < 0$. Then there exists a $\rho \in (0, \frac{w_1^t}{w_2^t})$ such that $\rho S_1 + S_2 < 0$. Choose $\frac{w_1^a}{w_2^a} = \rho$; once again, the rest follows from Theorem 4.4.

"Only if" part:

Suppose that neither (i) nor (ii) holds; i.e. $S_2 \geq 0$ and $w^t S \geq 0$. Since $\frac{w_1^a}{w_2^a} < \frac{w_1^t}{w_2^t}$, we have $w^a S \geq 0$. ∎

The observed data are therefore consistent with the model if and only if at least one of the following hold: the country is a net importer of the factor that the import good is relatively intensive in, or the value of net factor exports at factor prices under trade is negative. Each of these reflects a more traditional interpretation of the HOV model.

Finally, we consider the case where $p^t y^c > p^t y^p$. This is the normal case when there is a tariff on the import good. The country trades on the world markets at world prices, p^f, so that balanced trade implies that $p^f y^p = p^f y^c$. In this case, significantly weaker restrictions on observed data are derived. The intuition here is that the observed situation under tariffs allows us to make an inference about the unobserved situation under free trade, from which a further inference about autarky must be drawn.

Definition 4.7 The observed data $y^p, y^c, p^f, \{x_{ij}\}_{i,j=1,2}$ satisfy *Observable Condition 5* (OC 5) if :
(i) $p^f y^p = p^f y^c$,
(ii) $p^t y^c > p^t y^p$.

Let w^f denote the factor prices, y^{fp} the production bundle, and y^{fc} the consumption bundle of the economy in a hypothetical free trade situation.

Lemma 4.8 *Suppose that the observed data* $(p^f, p^t, y^p, y^c, w^t)$ *satisfy OC 2-5. Then, given* w^f*, there exist* y^{fp}, y^{fc} *such that all observed and unobserved values are consistent with some HOPE if and only if* $\frac{w_1^f}{w_2^f} > \frac{w_1^t}{w_2^t}$.

PROOF

Here $\frac{p_1^f}{p_2^f} < \frac{p_1^t}{p_2^t}$. It is straightforward to modify the proof of Theorem 4.4 to this case. ■

Lemma 4.9 *Suppose that the observed data* $(p^f, p^t, y^p, y^c, w^t)$ *satisfy OC 2-5. Then, for all* $w^a \in R^2_{++}$*, there exist* $w^f, y^{fp}, y^{fc}, p^a, y^a$ *such that all observed and unobserved values are consistent with some HOPE.*

PROOF

As in the proof of Theorem 4.4, given a $w^a \in R^2_{++}$, choose a production point under autarky, and p^a, y^a such that (20)-(26) are satisifed. Now choose any w^f, y_1^f such that

$$\frac{w_1^f}{w_2^f} > \max\left\{\frac{w_1^t}{w_2^t}, \frac{w_1^a}{w_2^a}\right\}$$
$$y_1^f < \min\{y_1^t, y_1^a\}$$

Now, as in the proof of Theorem 4.4, choose a production point under free trade, such that $(p^f, y^{fp}, \{x_{ij}^f\}_{i,j=1,2})$ and $(p^t, y^p, \{x_{ij}^t\}_{i,j=1,2})$ are consistent with CRS production functions for each good. Clearly, it is possible to choose y^{fc} such that $p^f y^{fc} = p^f y^{fp}$ and the bundles y^c, y^a, y^{fc} satisfy WARP pairwise.

It is now straightforward to check that the production points under autarky and free trade chosen above satisfy the additional inequalities

$$p_i^f y_i^a - \sum_{j=1}^{2} w_i^f x_{ij}^a \leq 0 \quad \text{for } i = 1, 2$$
$$p_i^a y_i^f - \sum_{j=1}^{2} w_i^a x_{ij}^f \leq 0 \quad \text{for } i = 1, 2$$

Hence, all observed and unobserved values are consistent with some HOPE. ∎

The next theorem describes necessary and sufficient conditions for the data to be consistent with this model under the new set of assumptions about observed data. These conditions are, of course, significantly weaker than those of Theorem 4.6; in this case, the model is rejected if and only if a country is a net exporter of both factors.

Theorem 4.10 *Suppose that the observed data* $(p^f, p^t, y^p, y^c, w^t, \{x^t_{ij}\}_{i,j=1,2})$ *satsify conditions OC 2-5. Then the data are GHO-consistent if and only if* $\min\{S_1, S_2\} < 0$.

PROOF

The "only if" part is obvious. Consider the "if" part. By Lemma 4.9, w^a could lie anywhere in R^2_{++} to be consistent with some HOPE. Clearly, w^a can be chosen to satisfy the inequality $w^a S < 0$ whenever at least one of S_1, S_2 is strictly negative. ∎

5 Conclusion

Previous tests of the HOV model have tended to reject the conclusions of the model. Effectively, these tests amount to a joint hypothesis test of all the assumptions of the model, and the failure of any one assumption may render the conclusions invalid. Commodity and factor prices are known to differ across countries. This paper discusses nonparametric tests of two of the remaining assumptions of the model—the assumptions of identical homothetic preferences and identical constant returns to scale production fucntions across countries, as well as the refutability of Deardorff's version of the HOV model, which does not assume commodity or factor price equalization, and which asserts that net factor exports, valued at autarky factor prices, have a negative value.

The nonparametric tests we investigate are based on earlier work on revealed preference and revealed cost theory by Afriat (1967) and Varian (1982,1983,1984). Importantly, this technique leads to restrictions that are necessary and sufficient for observed data to be consistent with a proposed model, and that therefore exhaust *all* refutable implications of a given set of data.

With respect to consumption data, we find that it is impossible to refute the hypothesis that two countries have identical homothetic utility functions unless data on prices and quantities of all goods are available, at least in the form of ratios across countries. It is not sufficient to have, on even *one* commodity, only expenditure data and price or quantity indices for each country.

We illustrate the use of the nonparametric test for a common homothetic utility function using the Kravis, Heston and Summers (1982) data set. We find that there does not exist a homothetic utility function consistent with the data from India and the U.S. However, there do exist such functions for the U.S. and Malawi, and Malawi and India. The nonparametric approach contrasts strongly with the assumed functional form approach of Hunter and Markusen (1988), who use the same data set and reject the assumption of a common homothetic utility function across all 34 countries in the sample. An additional virtue of the nonparametric approach is that it can be used on as few as two countries.

On the production side, we investigate the refutability of the assumption of an identical constant returns to scale production function for some good across two countries, under different assumptions about what data are observed. If all factor prices and factor quantities and the total value of the good produced are observed, but the price and quantity of the good remain unobserved, the only restriction on the observed data is analogous to that imposed by any homothetic production function. We further show that even when some factor prices are unobserved, the constant returns to scale assumption imposes substantive restrictions on factor quantities and the observed factor prices. These restrictions are weaker the greater the number of factors with unobserved prices.

Finally, we illustrate how refutable propositions may be derived in the more general Heckscher-Ohlin model of Deardorff (1982), whose result states that the value of net factor exports at (clearly unobservable) autarky factor prices is negative. We show that, in the case of 2 goods and 2 factors, and assuming balanced trade (which is a restriction only on observed data), observed data on commodity and factor prices and quantities (from the situation under trade) are consistent with the model if and only if either (i) the country in question is a net importer of the factor that the imported good is intensive in, or (ii) the value of net factor exports at factor prices under trade is negative.

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Category	U.S. (dollars)	India (rupees)	Malawi (kwacha)
Food, beverages, tobacco	818.24	565.34	55.543
Clothing, footwear	336.33	66.32	6.704
Gross rent, fuel	903.84	53.80	5.916
House furnishings, operations	343.14	24.43	9.000
Medical care	653.69	28.40	1.834
Transport, communications	692.17	57.20	7.194
Recreation, education	783.74	46.48	6.267
Other expenditures	628.47	39.74	3.206

Table 1: Expenditures by Commodity Category (in Local Currency)

Appendix

A Testing for Common Preferences—Malawi, India, U.S.A.

A.1 Homothetic Preferences

The Kravis, Heston, and Summers data set reports, for 34 countries and several commodity categories, the per capita expenditure on that commodity category, a purchasing power parity (PPP) index relative to the U.S., and a per capita quantity index relative to the U.S. All data are for the year 1975. The PPP index can be converted into a relative price index by dividing by the exchange rate. However, for our purposes, there is no need to do so.

Notice that (5), the inequality we wish to test, is invariant to the scale of the price and quantity indices, α_i^k and β_i^k. Hence, we can directly use the PPP index numbers to represent the α values, and similarly the per capita quantity indices (rather than aggregate national consumption) to represent the β values.

For this illustration, we examined the eight major commodity categories reported by Kravis, Heston, and Summers under consumption. The data we use, therefore, do not consider the consumption versus saving decision.

The raw data are shown in Tables 1 and 2, with the results of the calculations performed for the test are shown in Table 3.

The calculations indicate that we can reject the hypothesis that the U.S. and India had the same monotonic, concave, homothetic utility function. For the pairs U.S. and Malawi, and Malawi and India, this hypothesis cannot be rejected.

Finally, we use the method of Varian (1982, Theorem 2) to construct a homothetic utility function consistent with the data from the U.S. and Malawi. Let the superscript M

Category	U.S.		India		Malawi	
	PPP	Quantity	PPP	Quantity	PPP	Quantity
Food, beverages, tobacco	1.00	1.00	3.78	0.183	0.373	0.182
Clothing, footwear	1.00	1.00	4.54	0.043	0.606	0.033
Gross rent, fuel	1.00	1.00	1.85	0.032	0.672	0.010
House furnishings, operations	1.00	1.00	3.53	0.020	0.508	0.052
Medical care	1.00	1.00	1.05	0.041	0.112	0.025
Transport, communications	1.00	1.00	2.59	0.032	1.020	0.010
Recreation, education	1.00	1.00	0.69	0.086	0.128	0.062
Other expenditures	1.00	1.00	1.83	0.035	0.391	0.013

Table 2: PPP and Quantity Indices

Country k	Country ℓ	$(p^k y^\ell)(p^\ell y^k)$	$(p^k y^k)(p^\ell y^\ell)$
U.S.	Malawi	645,263	493,590
India	Malawi	101,187	84,348
U.S	India	3,950,008	4,549,288

Table 3: Calculations for Homotheticity Test

denote values for Malawi and U those for the U.S. Define

$$\begin{aligned} V^M &= \min\{1, \frac{p^U y^M}{p^U y^U}, \frac{p^M y^U}{p^M y^M}\frac{p^U y^M}{p^U y^U}\} \\ &= 0.052 \\ V^U &= \min\{1, \frac{p^M y^U}{p^M y^M}, \frac{p^U y^M}{p^U y^U}\frac{p^M y^U}{p^M y^M}\} \\ &= 1 \end{aligned}$$

To define the utility function, we further need to know commodity prices in the U.S., p^U. Since that information is not reported in the Kravis, Heston, and Summers data set, we can pick any values of p_i^U, y_i^U consistent with the reported expenditures. Now, define $U(y)$ as

$$U(y) = \min\{V^M \frac{\sum_{i=1}^N \alpha_i^M p_i^U y_i}{p^M y^M},\ V^U \frac{p^U y}{p^U y^U}\} \tag{34}$$

This function is monotonic, concave, homothetic, and consistent with the observed data for Malawi and the U.S.

A.2 Any Common Preferences

Since the data from the U.S.A., India, and Malawi are not consistent with a common homothetic utility function, we next test for whether they are consistent with *any* well-

$p^U y^U$	5,159.62
$p^I y^I$	881.71
$p^M y^M$	95.66
$p^U y^I$	338.38
$p^U y^M$	267.08
$p^I y^U$	11,673.24
$p^I y^M$	776.17
$p^M y^U$	2,415.99
$p^M y^I$	130.37

Table 4: Calculations for Common Utility Function

behaved (i.e. continuous, concave, monotonic) utility function.

We say that bundle 1 is revealed preferred to bundle 2 ($y^1 \succeq^p y^2$) if

$$p^1 y^1 \geq p^1 y^2 \tag{35}$$

By Afriat's Theorem (Afriat, 1967) the data are consistent in this manner if and only if the Generalized Axiom of Revealed Preference (see Varian, 1982) is satisfied, i.e. if and only if

$$x^i \succeq^p x^j \ldots \succeq^p x^\ell \Longrightarrow \text{ not } x^\ell \succeq^p x^i \tag{36}$$

Notice that the relationship (36) is invariant to the scale of the prices or quantities. Hence, we can again directly use the price and quantity indices provided by Kravis, Heston and Summers.

Let $p^k = \alpha^k p^{US}$ for $k = U, I, M$, where U, I, M represents the U.S.A., India, and Malawi respectively. Similarly, let $y^k = \beta^k y^{US}$ for $k = U, I, M$. The following table indicates the revealed preferred relationships.

The calculations show that $y^U \succeq^p y^I \succeq^p y^M$. Hence, there does exist a well-behaved common utility function consistent with the data from the three countries. Following Varian (1982), one such utility function is given by:

$$\begin{aligned}
U^U &= 1 \\
\lambda^U &= 1 \\
U^I &= \min\{U^U + \lambda^U p^U(y^I - y^U, U^U\} = -4,820.24 \\
\lambda^I &= \max\{(U^U - U^I)/p^I(y^U - y^I), 1\} = 0.4468 \\
U^M &= \min_{x=U,I} \min\{U^x + \lambda^x p^x(y^x - y^M), U^x\} = -4,891.54
\end{aligned}$$

$$
\begin{aligned}
\lambda^M &= \max_{x=U,I} \max\{(U^x - U^M)/p^M(y^x - y^M), 1\} = 1 \\
U(y) &= \min_{x=U,I,M} \{U^x + \lambda^x p^x (y - y^x)\}
\end{aligned}
$$