Discussion Paper

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Factor endowment, impatience and trade patterns in a small open economy of endogenous growth

by

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Discussion Paper 97, 6/1993, revised 3/1994

JEL Classification: F12, O31

Keywords: International trade, Endogenous growth, Trade patterns, Time preference, Heckscher-Ohlin model

Paper presented at the Econometric Society European Meeting 1993, Uppsala, Sweden and the Annual Meeting 1993 of the Gesellschaft für Wirtschafts- und Sozialwissenschaften - Verein für Socialpolitik -, Münster, Germany. Any comments, remarks or suggestions will be greatly appreciated. Correspondence should be send to:

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Financial support by the state of Schleswig-Holstein is greatly appreciated.

Factor endowment, impatience and trade patterns in a small open economy of endogenous growth

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The prediction of trade patterns is usually based on physical factor abundance. In a context of endogenous growth, however, physical factor richness does not always predict trade patterns in a reliable way. In a world where countries differ only in their factor endowment trade patterns can easily be such that a physically capital rich country exports the labour intensive good. Further, if the consumers' time preference rates are allowed to vary between countries, they influence trade patterns, too, which then do not reflect physical factor abundance for fairly general conditions.

1. Introduction

The pure theory of international trade names various determinants of trade patterns: Apart from differences in technology and preferences or trade due to transportation costs, economies of scale and factor endowments are probably the most prominent ones. The latter is the unique determinant in Heckscher-Ohlin models of two homogenous goods and two factors where the physically relatively capital rich country exports the capital intensive good. Economies of scale can be found in models of intra- and inter-industrial trade (Helpman, 1981; Lawrence and Spiller, 1983; Helpman and Krugman, 1985, ch.7) where the same pattern of trade emerges, except for the fact that now there is a *net* exporter and a *net* importer of the differentiated product, provided that incomplete specialization prevails under free trade. Analogue findings can be provided in a context of endogenous growth, caused by technological spillovers, as was shown by Grossman and Helpman (1991, ch. 7).
Factor richness in these models is measured in terms of physical factor endowment and each country exports the good which makes relative intensive use of the factor which is physically relatively abundant in that country. One condition that this quantity version of the Heckscher-Ohlin theorem holds is that demand functions for final goods are internationally identical. If they differ to such an extend that demand outweighs physical factor abundance (let e.g. the capital rich country have a strong preference for the capital intensive good), the price version of the Heckscher-Ohlin theorem (cf. e.g. Ethier, 1984, p. 141) would have to be applied in order to obtain a correct prediction of trade patterns. Factor richness would then have to be measured in terms of autarky factor rewards.

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Financial support by the state of Schleswig-Holstein is greatly appreciated.
I would like to thank Richard Frensch, Horst Herberg and participants of the Econometric Society European Meeting 1993 for their comments on an earlier version of this paper. Of course, I alone am responsible for any remaining errors.

In the model presented below we derive two further reasons why the trade pattern of a country no longer reflects its physical factor richness. By doing this we introduce a further determinant of trade patterns which so far has attracted less attention in the literature. The analysis will be undertaken in a model of a small open economy of endogenous growth since this allows an easier proof of the results which would not change in a two-country world.

The organization and major findings of the paper is as follows: The next section presents a small open economy model with two perfectly competitive sectors producing two final homogeneous goods, one monopolistic sector manufacturing intermediate goods and one R&D sector. International capital flows or trade in intermediate goods are ruled out and the economy is assumed to trade in final goods only. The model is similar to Grossman and Helpman (1991, ch. 6) but uses more general production functions. The next section also discusses conditions for a balanced growth path with incomplete specialization. If the economy is too small to sustain positive long-run growth, incomplete specialization requires that the endowment point of the economy lies within a cone of diversification given by world market prices for final goods and production technologies. This condition is well-known from static Heckscher-Ohlin models. For a growing economy, however, the conditions for incomplete specialization is a function of the domestic growth rate and therefore differs considerably from standard results. It is e.g. perfectly possible that one economy is completely specialized though it has an identical endowment ratio capital to labour as another economy which is incompletely specialized. The third section analyzes the impact of factor endowment on trade patterns. We will show that a physically capital rich country which is incompletely specialized when trading can become an exporter of the labour intensive good though capital is its physically relatively abundant factor. This result does not require any differences between the countries apart from factor endowment[1]. It implies that even between countries with identical preferences and identical human capital to labour ratio, inter-industrial trade will be observed. In the fourth section we first investigate the impact of a so far neglected determinant of trade patterns – the time preference rate – and then let the effects caused by differing time preference rates interact with the effects due to factor endowment. It turns out that a country with a fairly impatient population has a tendency to export the human capital intensive good. The interaction of impatience[2] and factor endowment effects then extents the result that a capital rich country exports the labour intensive good to a broader range of parameter values and thereby generalizes these findings. Having shown that the quantity version of the Heckscher-Ohlin theorem can no longer be applied to obtain a correct prediction of trade patterns, the subsequent section will show that the results are in accordance with standard trade theory if the price version of the Heckscher-Ohlin theorem is used to measure capital richness. We find that in all situations where the quantity version fails each country has indeed a relative comparative advantage in the good it exports if it is

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[1] An alternative reason for such an effect was suggested in a recent paper by Dinopoulos et. al. (1993). By allowing for international trade in assets, they show that trade patterns depend on asset-adjusted endowment rather than physical endowment alone. International capital flows, however, are not necessary for such a situation as will be shown here.

[2] (High) time preference rate, (high) individual discount factor and high impatience will be used synonymously throughout the paper.

measured in autarky factor rewards. A final section concludes the findings and gives an outlook.

2. A small open economy

Production side

The economy in consideration is small with respect to the economic environment in which it operates. It is endowed with two primary factors and produces two final goods *X* and *Z*, which are traded at exogenously given world market prices p_X and p_Z. Both goods are produced under perfect competition using the two primary factors human capital *H* and labour *L*, as well as a set of intermediate goods *i* as inputs. Letting $D_j,\ j = X, Z$ represent an index of the intermediate goods used in sector *j*, the production function can be expressed by

$$X = X_0 H_Z^{\beta} D_Z^{\eta} L_Z^{1-\eta-\beta}, \quad 0 < \beta, \eta, 1-\eta-\beta < 1, \tag{1}$$

for the good *X* and by

$$Z = Z_0 H_Z^{\alpha} D_Z^{\eta} L_Z^{1-\eta-\alpha}, \quad 0 < \alpha, \eta, 1-\eta-\alpha < 1, \tag{2}$$

for good *Z*. The output elasticities are assumed to be such ($\beta > \alpha$) that the production of the high-tech good *X* makes more intensively use of human capital than the food industry *Z*.

From this constant-returns-to-scale production functions unit cost functions, which must equal prices p_Z and p_X, respectively[3], can be directly derived. With a suitable choice of X_0 and Z_0 they are given by $c_X(\cdot) = w_H^{\beta} p_D^{\eta} w_L^{1-\eta-\beta}$ and $c_Z(\cdot) = w_H^{\alpha} p_D^{\eta} w_L^{1-\eta-\alpha}$ where $w_j,\ j = H, L$ are the factor rewards for human capital and labour and p_D is the price of the set of intermediate goods used in production. By applying Shephard's Lemma, unit demand functions for labour, the input index *D* and human capital can be computed. This gives for good *X*, with reinserting p_X,

$$b_{HX} = \beta \frac{p_X}{w_H}, \qquad b_{DX} = \eta \frac{p_X}{p_D}, \qquad b_{LX} = (1-\eta-\beta) \frac{p_X}{w_L},$$

and for good *Z*

$$b_{HZ} = \alpha \frac{p_Z}{w_H}, \qquad b_{DZ} = \eta \frac{p_Z}{p_D}, \qquad b_{LZ} = (1-\eta-\alpha) \frac{p_Z}{w_L}.$$

With the unit demand functions b_{DX} and b_{DZ} for the input index *D*, we can readily express total demand for *D* as a function of the quantity of *X* and *Z* supplied as

$$D^D = \eta \frac{p_X}{p_D} X^S + \eta \frac{p_Z}{p_D} Z^S. \tag{3}$$

What we do not yet know, however, is, how this input index is specified. We adopt the well-known Dixit-Stiglitz (1977) specification which was introduced — in a similar formulation — into production theory by Ethier (1982). It reads

$$D^S = \left(\int_0^n \left(y^i\right)^{\theta} di \right)^{1/\theta}, \quad 0 < \theta < 1, \tag{4}$$

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[3] Of course, this holds only if both final goods are produced. Later on in this section we will give conditions for such a situation.

where n is the (measure of the) number of non-tradable intermediate goods i available domestically. How will producers in sector X and Z assemble intermediate goods in order to obtain the required quantity of D? The optimal choice results from minimizing total costs $\int_0^n p_y^i y^i di$ with respect to y^i subject to (4). This gives demand for each variety of the form

$$y^i = \left(p_y^i\right)^{-\varepsilon} D^S \left(\int_0^n \left(p_y^i\right)^{1-\varepsilon} di\right)^{-\varepsilon/(\varepsilon-1)} \qquad (5)$$

and a unit cost function, which equals the price p_D both final good sectors have to pay for one unit of D and which reads

$$p_D = c_D\left(p_y^i\right) = \left(\int_0^n \left(p_y^i\right)^{1-\varepsilon}\right)^{1/(1-\varepsilon)},$$

where $\varepsilon = 1/(1-\theta)$ is the elasticity of substitution between any two intermediate goods.

Obviously, before intermediate goods can be used in the production process of final goods, they must be produced. The production function for each intermediate good is $y^i = y_0^i H_{y^i}^{\gamma} L_{y^i}^{1-\gamma}$, $0 < \gamma < 1$. Each producer i faces her "own" demand function given by (5) and maximizes profits accordingly. The resulting profit maximizing price is

$$p_y = \frac{w_H^{\gamma} w_L^{1-\gamma}}{\theta}, \qquad (6)$$

which implies equal pricing of all varieties since the production costs do not vary between the single firms of sector Y.
Aggregating all firms of this sector, total demand for primary factors depending on factor rewards and aggregate output Y^S is given by

$$L_Y^D = nb_{LY}\left(w_L, w_H\right)y^S = (1-\gamma)\left(w_H/w_L\right)^{\gamma} Y^S, \qquad (7)$$

$$H_Y^D = nb_{HY}\left(w_L, w_H\right)y^S = \gamma\left(w_L/w_H\right)^{1-\gamma} Y^S. \qquad (8)$$

Prior to the production of a variety of the differentiated good, firms have to develop a new blueprint which introduces increasing returns to scale for the firms in sector Y. The R&D process itself requires human capital and labour plus knowledge Kn as production factors and is more human capital intensive than the manufacturing activities. The production function is assumed to exhibit constant-returns-to-scale in human capital and labour but increasing-returns-to-scale in all three factors. As a by-product of the development of new blueprints, non-appropriable knowledge itself is increased (Romer, 1990) which is assumed to be immediately available for the creation of new varieties. Formally, the production function for blueprints is $\dot{n} = f_R\left(H_R, L_R\right)Kn$. Knowledge which disseminates only locally[4] (within the country where the new blueprint was developed), is assumed to increase linearly in the measure n for the number of varieties produced and consumed domestically. Thus with a suitable choice of units Kn can be set equal to n. As is well known, it is this technological external effect which leads to positive long-run growth rates. Under these assumptions, the demand functions for factors per growth-rate unit of $g := \dot{n}/n$ are given by $a_{jR}\left(w_L, w_H\right)$, $j = L, H$.

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[4] This assumption could be relaxed (which would not change the basic results) by allowing for an international flow of knowledge with decreasing returns to scale in foreign knowledge.

In analogy to static models, free entry requires the absence of pure profits. Thus, the present value *v* of the future profit stream resulting from the development of a new variety must equal its development costs $c_R(w_L, w_H)/n$ which can be derived from the corresponding production function. Thus, the free-entry condition yields

$$v = c_R(w_L, w_H)/n, \tag{9}$$

with the present value *v* defined as

$$v(t) = \int_t^\infty \exp\left[-\left(R(\tau) - R(t)\right)\right]\pi(\tau)d\tau, \tag{10}$$

where profits $\pi(\tau)$ are discounted by a cumulative factor $R(u) = \int_0^u r(s)ds$ depending on the interest rate $r(s)$.

Consumption side

The consumption side can be treated very briefly. Since the economy is small with respect to the rest of the world, agents can buy and sell any quantities they judge as optimal at world market prices. Thus, we do not have to worry about demand functions for *X* and *Z*. What we have to take into consideration, however, is how consumers in the home economy allocate expenditure *E* over time. This is given by (see, e.g., Grossman and Helpman, 1989, p. 1265)

$$\dot{E}/E = r - \rho,$$

where *r* is the interest rate at time *t* and ρ is the constant individual time preference rate.

Market equilibria

Let us now turn to the specification of market equilibria. As already mentioned, the adoption of a small open economy framework allows to neglect equilibrium conditions for final goods; therefore only the market for the differentiated intermediate goods and the factor markets have to be taken into account.

In specifying market equilibria for middle goods, which will be expressed in terms of the index *D*, we use the fact that all varieties are equally priced; this simplifies the expression for the supplied quantity of *D* (4) to

$$D^S = n^{1/\theta} y^S = n^{(1-\theta)/\theta} Y^S. \tag{11}$$

Thus, equilibrium on the market for middle goods reads (cf. (3))

$$n^{(1-\theta)/\theta} Y^S = \eta \frac{p_X}{p_D} X^S + \eta \frac{p_Z}{p_D} Z^S. \tag{12}$$

The factor market clearing conditions can be obtained by equating aggregated demand of the four sectors to households' inelastic supply of factors,

$$\begin{aligned} a_{LR}(w_L, w_H)g + b_{LY}(w_L, w_H)Y^S + b_{LX}(p_X, w_L)X^S + b_{LZ}(p_Z, w_L)Z^S &= L, \\ a_{HR}(w_L, w_H)g + b_{HY}(w_L, w_H)Y^S + b_{HX}(p_X, w_L)X^S + b_{HZ}(p_Z, w_L)Z^S &= H. \end{aligned} \tag{13}$$

Growth rate, wages and output

The specification of the model as given above implies (cf. appendix) that new intermediate goods are introduced at a rate of

$$g = \eta(1-\theta)\frac{w_L L + w_H H}{c_R(w_L, w_H)} - \left(1 - \eta(1-\theta)\right)\rho, \tag{14}$$

under the assumption of internationally immobile financial capital. Since world market prices are constant and no adjustment costs are included in the model, there are no adjustment dynamics; therefore, immediately after trade opens up, both factor rewards and (therefore linear homogenous) $c_R(w_L, w_H)$ increase at the rate of $\eta((1-\theta)/\theta)g$ which implies that g is constant. Now define composite "productivity-adjusted" (Grossman and Helpman, 1991, p. 150) unit demand functions $a_{ij}(\cdot)$, $i = L, H$, $j = X, Z$, which stand for the total quantity of factors (both for assembling the final good and for producing the middle product) needed for the production of one unit of output. Then it can be shown that the factor market clearing condition (13) can be expressed by[5]

$$\begin{aligned} a_{LR}g(\rho) + a_{LX}X + a_{LZ}Z &= L, \\ a_{HR}g(\rho) + a_{HX}X + a_{HZ}Z &= H, \end{aligned} \qquad (15)$$

where productivity adjusted unit demand functions $a_{ij}(\cdot)$ and modified output X and Z are constant. X and Z are transformations of X^S and Z^S given by $X := X^S \exp[-\eta((1-\theta)/\theta)g(\rho)t]$ and analogously for Z. This transformation is a technically simplifying step and allows us to undertake comparative statics with ρ, L and H as exogenous variables. It should be borne in mind, however, that the output of both final goods X^S and Z^S grows at a constant rate of $\eta((1-\theta)/\theta)g(\rho)$, as well as factor rewards do, reflecting the increase of productivity due to technological progress.

So far nothing has been said about the value the growth rate takes or about X and Z. The growth rate is positive, if the country is not too small given its preferences, technology and world market prices. In figure 1 the line denoted by $g = 0$ gives the loci which divides the factor endowment space into an area where innovation takes place and one where the growth rate is zero[6]. Parallel to this zero-growth line, which can be found by setting $g = 0$ in equation (14), other iso-growth lines can be drawn. The more the distance between the two lines increases, the higher the growth rate the second line represents. An example of an iso-growth line with positive growth is the line $g = g_1$. The area delimited by the axes and the zero-growth line is characterized by zero growth.

[insert figure 1 around here]

The conditions for incomplete specialization $(X > 0, Z > 0)$ can be related to the conditions in the traditional Heckscher-Ohlin model: Provided that the factor endowment lies within a certain cone of diversification the country produces both the high-tech good and food. In contrast to other Heckscher-Ohlin models, however, the

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[5] Since in the following we will stress the importance of the time preference rate, the growth rate is written as $g(\rho)$.

[6] The growth rate cannot become negative, since this would either require an allocation of a negative quantity of factors to the R&D sector or a reduction of the number of middle goods. Since the former does not make sense and the latter is not possible by definition, growth must always be non-negative.

cone of diversification changes as the growth rate changes, hence a close relationship between the conditions for X and Z to be positive and the growth rate.

Consider an endowment represented by the point N_1. In this case the country's growth rate is zero and the factor market clearing conditions (15) become

$$\begin{bmatrix} a_{LX} & a_{LZ} \\ a_{HX} & a_{HZ} \end{bmatrix} \begin{bmatrix} X \\ Z \end{bmatrix} = \begin{bmatrix} L \\ H \end{bmatrix},$$

just as in a two good, two factor model. Letting $h_j := a_{Hj} / a_{Lj}$, $j = X, Z$ define the human capital intensity of the production process, the boundaries of this cone of diversification are given by $h_Z L < H < h_X L$. In figure 1 the cone is delimited by the lines starting from the origin and denoted by h_Z and h_X.

Now assume that the endowment of the home economy is such that the growth rate is positive; to be concrete, take a point which lies on the line $g = g_1$. Then by equation (14) the value of the growth rate is unambiguously determined and the factors allocated to R&D are given by the vector $(a_{LR}, a_{HR}) g(\rho)$ which is depicted in the figure by R_2. Now by rewriting the factor market clearing conditions as

$$\begin{bmatrix} a_{LX} & a_{LZ} \\ a_{HX} & a_{HZ} \end{bmatrix} \begin{bmatrix} X \\ Z \end{bmatrix} = \begin{bmatrix} L - a_{LR} g(\rho) \\ H - a_{HR} g(\rho) \end{bmatrix}, \tag{16}$$

it is obvious that the boundaries of the cone of diversification whose origin lies now at R_2 have the same slopes as the one starting from the origin O. If the endowment point lies inside (e.g. N_2) then the allocation of factors to the manufacturing sectors can be derived in the usual way by drawing a parallel line to h_X starting from N_2; $R_2 Z_2$ is then the quantity of factors allocated to sector Z and $Z_2 N_2$ is the quantity of factors allocated to sector *X*. If the point lies outside the cone of diversification (e.g. N_3), the country specializes completely in the production of one good (for N_3 of the food good *Z*).

Apart from factor endowment, the growth rate is further influenced by world market prices, the time preference rate ρ and the technological parameter η and θ. The impact of prices is ambiguous, we know however, that g decreases both in θ and ρ and increases in η. Whereas the interpretation for η and ρ is obvious, the parameter θ deserves further attention: A decrease in θ implies a decrease in the elasticity of substitution $\varepsilon = 1/(1-\theta)$ between two varieties which in turn expresses a higher marginal gain from an additional variety (the derivative of (11) with respect to n decreases in θ). Consequently the lower the elasticity of substitution the higher are prices (cf. (6)) charged by firms producing the intermediate good. Since higher prices go along with higher current profits, the incentive for firms to develop new varieties will be higher, thus the probability of g to be positive is higher.

Since we are interested in a situation where the country produces both goods and grows at a positive rate of g, we assume in the following that the country's endowment is given by a point such as N_2.

3. Factor endowment and trade patterns

In the model presented above, at least two factors can be made out which influence the country's trade pattern: The time preference rate and the factor endowment. Throughout the following analysis it is assumed that they constitute the only difference between the small open economy and its trading partners. This implies that demand functions for final goods are internationally identical which allows to infer trade patterns of an economy from its output.

Whereas factor endowment is probably *the* determinant of trade pattern in the literature, the role of impatience has so far attracted less attention[7]. Before we are going to analyze the impact of factor endowment on trade patterns in this section a remark on (the absence of) Ricardian features of the model: Due to the assumption of limited (i.e. only country wide) dissemination of knowledge, the technology will differ between countries: Take, say, the production function for good Z (neglect constants), insert the expression for the input index given by (11) and insert the production function for Y. One finds $Z = n^{\eta\frac{1-\theta}{\theta}} H^{\alpha+\gamma} L^{2-\eta-\alpha-\gamma}$ which means that the output of Z grows at a rate of $\eta((1-\theta)/\theta)g(\rho)$ if the allocation of factors to this sector is constant. The production function for X can be rewritten analogously as $X = n^{\eta\frac{1-\theta}{\theta}} H^{\beta+\gamma} L^{2-\eta-\beta-\gamma}$. Since the output of X grows at the same rate as the output of Z, the relative productivities at home and abroad are the same[8]. Thus, though the absolute productivity differs between the home economy and the rest of the world (the economies might even overtake each other), Ricardian effects play no role in this setting.

Factor endowment

Let us now discuss trade patterns when the home economy differs from its economic environment only in its factor endowment. If the economy was endowed as indicated by N_1 (cf. figure 1), the influence of factor endowment on trade patterns could be studied by standard trade theorems. The Heckscher-Ohlin Theorem would tell us that an increase of the factor endowment ratio H/L leads to an increase of the output ratio X/Z. As a consequence, a shift of N_1 on the line ON_1 (not drawn) would not change the output ratio and thus neither the trade pattern. In economies with positive growth rates (e.g. factor endowment point N_2), however, a relationship between output ratio and factor endowment ratio can not be as easily established. Analogy would suggest that moving N_2 on the line R_2N_2 (not drawn) would not change the output ratio. This, however, does not hold since a, say, upward shift implies an increase of the growth rate which in turn requires a higher quantity of factors allocated to R&D. As a

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7 An accompanying paper (Wälde, 1994a) investigates the impact of time preference rates on factor-price equalization and trade patterns in a two country world with global technological spillovers.

8 If we had assumed that intermediate goods have an unequal effect on the productivity of factors in the production of the final goods (in our specification the output elasticity of the input index is identical in both production functions), then different growth rates would imply an endogenous change of relative productivity. However, a balanced growth path analysis would then be ruled out (compare Grossman and Helpman, 1991, p. 146).

consequence, R_2 (and with R_2 the cone of diversification) would shift upwards. Therefore, in order to study the relationship between trade patterns and factor endowment, we first have to find out how the factor endowment has to change such that the output ratio does *not* change.

Figure 2 reproduces the factor endowment point in consideration, the origin of its cone of diversification and the iso-growth line it lies on (compare figure 1). In addition the factor endowment ratio is drawn as the dotted line ON_2 and an iso-output ratio line ξ_2 is shown which gives the loci where all countries with positive (but different) growth rates and identical output ratio lie on. This line can be found as follows: Solving the factor market clearing conditions (15) for X and Z and dividing results in the output ratio equation

$$\frac{X}{Z} = \frac{g(\rho)a_{LR}a_{LZ}(h_R - h_Z) + a_{HZ}L - a_{LZ}H}{g(\rho)a_{LR}a_{LX}(h_X - h_R) - a_{HX}L + a_{LX}H}.$$

Setting $X/Z = \xi$, inserting the expression for the growth rate (14) and solving for H gives the loci where the output-ratio does not change.

[insert figure 2 around here]

Without going too far into details (cf. appendix) it is worthwhile to mention that the intersection point of this line with the *H*-axis lies below the origin which is the reason why it intersects the line R_2N_2 (not drawn) and the endowment ratio line ON_2 from below.

In looking how factors are re-allocated if N_2 shifts upwards, it becomes clear why a shift on R_2N_2 is not enough to keep the output ratio constant: A fraction of the additional quantity the economy gets by shifting N_2 will go to the R&D sector which, by assumption, is the most human capital intensive one. Thus, the more the economy shifts on the line R_2N_2, the more the ratio of human capital to labour, provided for the sectors X and Z decreases which excludes a constant output ratio. Therefore, some more human capital must be added in order to keep that ratio constant which then allows to keep the output ratio unchanged. Consequently the slope of the iso-output ratio line must be larger than the slope of the line R_2N_2.

Now consider the following

<u>Lemma</u>

The output ratio X/Z increases, if the factor endowment ratio H/L increases on an iso-growth line, given that the three activities of R&D and manufacturing X and Z can be uniquely ranked by factor intensity h_i such that $h_R > h_X > h_Z$.

$$\left.\frac{\partial\,(X/Z)}{\partial\,(H/L)}\right|_{g=const.} > 0 \text{ if } h_R > h_X > h_Z.$$

<u>Proof:</u> An increase of H/L on an iso-growth line requires an increase of H and a simultaneous decrease of L. Since $g(\rho)$ is constant, factor market clearing conditions (15) are better written in the form of (16). By dividing the increase of H/L into two steps with one factor held constant in each step, the application of the Rybczynski theorem completes the proof. ◊

This allows us to complete our investigation of the influence of factor endowment on trade patterns. Assume a country whose endowment is given by N_2 and another country whose endowment is given by N_4. Then the output ratio of country 2 is identical to the output ratio of an (imaginary) country whose endowment is given by N_2' since they share an iso-output line. Country 2', however, can be placed on the same iso-growth line as country 4 and therefore by the above lemma we know that the output ratio of country 4 is higher than the output ratio of country 2. Thus the more a country lies to the left of ξ_2, the higher its output ratio.

Given this situation, it should be obvious why a country relatively abundant in one factor does no longer necessarily export the good making relatively intensive use of this factor, even if countries differ only in their factor endowment. Compare endowment point N_2 and N_5. Prediction by physical factor abundance would state an export of the capital intensive good by country 5 and an import of that good by country 2, because country 5 is physically relatively capital rich. Country 5, however, experiences a higher growth rate which can be achieved only be reducing the human capital abundance of the manufacturing sectors below the one of country 2. Therefore country 2 exports the human capital intensive good and country 5 exports the labour intensive good. This has proven proposition 1:

<u>Proposition 1</u>
A country which is physically capital rich will export the labour intensive good if its human capital intensive R&D sector employs a high share of human capital and thereby leaves a labour rich factor endowment for the production sectors of the economy.

Please note that this implies that countries trade with each other, even if they share a common factor endowment ratio, though there are no differences in the countries apart from the absolute size of factor endowment.
The next section will generalize the result that a physically capital rich country exports the labour intensive good which was shown to hold here for a relatively small part of the factor endowment space. The same result can be obtained if we let interact factor endowment and discount factor as determinants of trade patterns.

4. Time preference, factor endowment and trade patterns

In the last section we discussed trade patterns as determined by factor endowment which is one determinant of the trade pattern of the economy. This determinant adds to the influence time preference rates exert. In the present section, we first exhibit the effect discount factors have on trade patterns (which we will call "impatience-effect") in a world economy in which all determinants of trade patterns other than the difference in the discount factor are ignored. Then we study the interaction of differences in time preference and factor endowment.

Time preference

The "impatience-effect" is demonstrated for the balanced growth path with a positive growth rate of a small open economy with incomplete specialization. In this case the factor market clearing conditions are given by (15). Assume a change of the individual discount factor. The effects are summarized in proposition 2.

Proposition 2

With constant prices and a unique ranking of factor intensities, an increase of the discount factor of the representative consumer leads (a) to a lower growth rate and (b) to a relative increase of the output of the high-tech product. With internationally identical demand functions for final goods (b) is equivalent to saying that the "impatient country" has a tendency to export the human capital intensive good.

(a) $g'(\rho) < 0,$

(b) $\dfrac{\partial (X/Z)}{\partial \rho} > 0$ if $h_R > h_X > h_Z$.

Proof: (cf. appendix)

Figure 3 illustrates this proposition. The box *OLNH* represents the given total factor endowment of our small open economy. *OH* equals total human capital and *OL* total labour force. The rays OR_l, $R_l X_l$ and $X_l N$ represent the allocation of factors to R&D, the high-tech sector *X* and the food industry *Z*, respectively, for a

[insert figure 3 around here]

country with a low discount rate. If the same country had a higher discount rate (increasing impatience) allocations would change to OR_h, $R_h X_h$ and $X_h N$. Fewer resources would be allocated to R&D and thus, the growth rate would be lower. The allocation to the production of the high-tech good increases, which leads to an increase of the output ratio X/Z. Note that the human capital intensity of the different activities, which is reflected by the slope of the rays, does not change since it depends solely on exogenously given world market prices.

The result that the country with a higher (lower) growth rate exports the labour (capital) intensive good could, at first sight, be astonishing. Reformulating the factor market clearing condition and splitting the derivative $\partial(X/Z)/\partial\rho$, however, gives good insight into the economic mechanisms. Write the factor market clearing condition as

$$a_{LX}X + a_{LZ}Z = L - a_{LR}g(\rho),$$
$$a_{HX}X + a_{HZ}Z = H - a_{HR}g(\rho),$$

and split the derivative into

$$\frac{\partial (X/Z)}{\partial \rho} = \frac{\partial (X/Z)}{\partial (H_{XZ}/L_{XZ})} \frac{\partial (H_{XZ}/L_{XZ})}{\partial \rho},$$

where H_{XZ} and L_{XZ} is the total allocation of human capital and labour in the *X* and the *Z* sector, respectively $(H_{XZ} = H_X + H_Z, L_{XZ} = L_X + L_Z)$. The first term is the change of the output ratio with respect to the change of the factor endowment after subtracting the factors allocated to R&D. This is the Heckscher-Ohlin theorem. The second derivative gives the change of the factor endowments available for the manufacturing activities with respect to a change of the discount rate. Both derivatives are positive if $h_R > h_X > h_Z$.

This splitting of the derivative allows to give an economic interpretation: An increase of the discount factor, by reducing the R&D activity (implying a lower growth rate), increases the factor endowment ratio of the rest of the economy, which is due to the fact that the R&D sector is (by assumption) the most human capital intensive one. The rest of the economy then produces relatively more of the high-tech product than before. Obviously, this implies that two countries which differ in their time preference rate (cf. figure 3) and take world market prices as given (assume they are both small) will trade with each other though they have identical endowment. In contrast to static models, where trade between countries with identical endowment trade requires internationally different demand functions for final goods, the result here is solely due to differences in the time preference rate.

Interaction of time preference rate and factor endowment

We are now in the position to study the interaction between the determinants of trade patterns. Assume an economy (cf. figure 1) endowed with human capital and labour as given by N_2 and whose time preference rate ρ increases. This shifts the zero-growth line to the right which decreases the distance to the line $g = g_1$. Thus the growth rate g_1, with factor endowments unchanged, falls, which implies a decrease of the quantity of factors allocated to R&D and a shift of R_2 towards the origin. Consequently the vector $Z_2 N_2$, reflecting the factors used in sector *X*, increases whereas at the same time the length of the vector $R_2 Z_2$ decreases. Now change the factor endowment ratio by moving the endowment point N_2 (for simplicity on the iso-growth line) towards N_3. Clearly, this has the opposite effect with respect to the output ratio X/Z. The trade pattern which will finally result therefore depends on which effect is more important compared to the factor endowment and discount rate of the trading partner. We can easily imagine, however, that the "impatience-effect" becomes stronger than the factor endowment effect and thus a physically relatively capital rich country becomes an importer of the capital intensive good and an exporter of the labour intensive good.

5. The price version of the Heckscher-Ohlin theorem

The ambiguity concerning trade patterns and physical factor richness due to different growth rates leads to a failure of the quantity version of the Heckscher-Ohlin theorem. We will now show that all of these findings can be nicely integrated in the pure theory of international trade by evoking, as trade theorists might already have conjectured, the price-version of the Heckscher-Ohlin theorem (cf. e.g. Ethier, 1984, p. 141).

First consider economies with identical discount rates which differ only in their factor endowment. In two good, two factor "traditional" Heckscher-Ohlin models a shift of an economy's endowment which does not change the economy's capital to labour ratio does not change its output ratio. Expressing factor abundance in terms of autarky factor-price ratio ω also yields that a shift of the factor endowment point of the economy which does not change the economy's capital to labour ratio does not change ω. Thus shifting on the output ratio conserving line is equivalent to keeping the autarky price ratio constant. Drawing an analogy to the model presented here[9] a shift on the output ratio conserving line ξ_2 in figure 2 implies a constant autarky wage ratio $\omega := w_H / w_L$. It can easily be shown that an increase of H (and a decrease of L) decreases ω, hence a decrease of the factor endowment ratio parallel to the L-axis leads to higher ω. Therefore shifting N_2 on the line ξ_2 does not change ω; shifting it further to N_5 increases the autarky wage ratio, and consequently, under free trade country 5 exports the labour intensive good and country 2 imports it.

Now consider two countries which differ only in their discount factors. In order to predict trade patterns in a correct way, ω must fall in ρ. In fact, we can show:

Proposition 3

In autarky the country with higher patience (lower time preference rate) has a higher factor reward ratio w_H / w_L and thus is economically human capital poor. Consequently, under free trade, it will export the labour intensive good.

$$\frac{\partial (w_H / w_L)}{\partial \rho} < 0 \text{ if } h_R > h_x > h_Z .$$

Proof: (cf. appendix)

This result has an easy economic interpretation. If ρ decreases, the consumers' preferences shift from consumption today to consumption in the future which induces increased R&D. Since the R&D sector is the most human capital intensive one, human capital becomes relatively more scarce, and thus, the wage ratio increases.

Finally consider the interaction of factor endowment and discount factor. In the last section the measure for capital richness was the physical endowment ratio H/L. H/L reflects only partially the determinants of the output ratio $\xi = X/Z$ which depends on the time preference rate and the endowment of human capital and labour: $\xi = \xi(\rho, H, L)$. Thus it is not enough to know the ratio of production factors in order to predict trade patterns. Moreover, it is in any case possible to "cheat" by choosing an appropriate ρ. In contrast to the physical endowment ratio, the autarky factor reward ratio w_H / w_L itself is a function of all factors which influence the output ratio: $\omega = \omega(\rho, H, L)$. Since it decreases in ρ and it behaves in the same way as the output ratio with respect to factor endowment (as was shown by analogue reasoning above), it can be used as a correct predictor of trade patterns.

6. Conclusion

In a small open economy model of endogenous growth we have shown that in contrast to standard trade theory results, the production structure of an economy does not depend on its relative factor endowment but on its absolute size, measured in physical factor endowment. The basic engine of growth is a domestic public stock of knowledge which is used by and a by-product of the R&D sector. Due to the absence

9 Unfortunately, we have so far been unable to proof this formally.

of (or imperfect) international flows of knowledge a larger economy will experience a higher rate of growth. Higher growth rates require a higher allocation of factors in the R&D sector which by assumption uses human capital most intensively. Thus, more human capital than labour is withdrawn from the manufacturing part of the economy which will therefore be left with a lower human capital to labour ratio. In a trade perspective this means that an economy with a higher growth rate has a tendency to specialize in the production of the labour intensive good. Comparing factor endowment ratios alone does therefore no longer form a solid basis for predicting trade patterns. In a world where factor endowments are the only determinant of trade patterns, a physically capital rich country can therefore easily become an exporter of the labour intensive good.

Taking the time preference rate into consideration, we found that it influences trade patterns by its effect on the growth rate, too. The more impatient a country, the lower its growth rate; less factors are employed in the R&D sector and the trading sectors have a more human capital rich endowment at their disposal. Thus, a country with a high time preference rate has a tendency to export the human capital intensive good. As a corollary one finds that two countries with identical factor endowment and identical demand function for final goods will trade with each other in final goods if their time preference rates differ.

Combining the impatience effect and the factor endowment effect can again lead to a failure of the quantity version of the Heckscher-Ohlin theorem and make a physically capital rich country an exporter of the labour intensive good if the mechanism factor endowment and time preference rates exert on the growth rate makes the production sectors relatively labour rich.

Given this result, at first sight contradictory to standard theorems of the pure theory of international trade, we have finally shown that the price version of the Heckscher-Ohlin theorem, where factor richness is measured in terms of autarky factor rewards, can indeed explain the outcome. The higher the growth rate of an economy the higher relative demand by the R&D sector for human capital and the higher the autarky factor reward ratio $\omega := w_H / w_L$ since this factor is most intensively used in the R&D process. Thus, in contrast to the quantity version of the Heckscher-Ohlin theorem, the price version does correctly predict trade patterns.

One assumption of the model deserves further attention: Since knowledge was assumed to disseminate only locally, growth rates can differ between countries which is one stylized fact required for growth models. To this respect the model is in line with Feenstra (1990) or Grossman and Helpman (1991, ch. 9.3). This assumption allows to model endogenously the fact that some countries' consumption per capita rises faster as other countries' does and might even overtake. Another empirical finding, the change of trade patterns as observed e.g. for the Japanese economy by Balassa and Noland (1989) or Grossman (1990), can be found in this model only by an exogenous change of the individual time preference rate. An extension of this model (Wälde, 1994b) to a two-country world including human capital accumulation shows which circumstances lead to an endogenous change of trade patterns.

Appendix[10]

Solution of the model[11]

Growth rate of factor rewards

Free trade implies pricing equations $p_Z = c_Z(\cdot) = w_H^{\alpha} p_D^{\eta} w_L^{1-\eta-\alpha}$ and $p_X = c_X(\cdot) = w_H^{\beta} p_D^{\eta} w_L^{1-\eta-\beta}$. Since all intermediates are equally priced, we have

$$p_D = n^{-(1-\theta)/\theta} p_y. \tag{A.1}$$

Deriving the pricing equations with respect to time results in, by using equation (6) and (A.1) and the definition $g := \dot{n}/n$,

$$(\alpha+\gamma\eta)\,\dot{w}_H/w_H + (1-\eta\gamma-\alpha)\,\dot{w}_L/w_L = \eta((1-\theta)/\theta)g,$$

$$(\beta+\gamma\eta)\,\dot{w}_H/w_H + (1-\eta\gamma-\beta)\,\dot{w}_L/w_L = \eta((1-\theta)/\theta)g.$$

Solving yields that both factor rewards grow at the rate of $\eta((1-\theta)/\theta)g$.

Factor market clearing conditions

Using (12), unit demand functions, (A.1) and the definitions (*t* stands for time)

$$\begin{aligned}
a_{LX} &:= p_X\left(b_{LY}(\cdot)\eta/p_y + (1-\eta-\beta)/w_L\right)\exp\left[\eta((1-\theta)/\theta)gt\right],\\
a_{HX} &:= p_X\left(b_{HY}(\cdot)\eta/p_y + \beta/w_H\right)\exp\left[\eta((1-\theta)/\theta)gt\right],\\
a_{LZ} &:= p_Z\left(b_{LY}(\cdot)\eta/p_y + (1-\eta-\alpha)/w_L\right)\exp\left[\eta((1-\theta)/\theta)gt\right],\\
a_{HZ} &:= p_Z\left(b_{HY}(\cdot)\eta/p_y + \alpha/w_H\right)\exp\left[\eta((1-\theta)/\theta)gt\right],
\end{aligned} \tag{A.2}$$

$$X := X^S \exp\left[-\eta\frac{1-\theta}{\theta}gt\right], \qquad Z := Z^S \exp\left[-\eta\frac{1-\theta}{\theta}gt\right],$$

factor market clearing conditions (13) can be written in the form of (15). With constant final goods prices, all unit input functions (p_y increases at $\eta((1-\theta)/\theta)g$, too, cf. (6)), the growth rate and (transformed) output are constant.

Growth rate g

Differentiating (10) with respect to time, inserting profits of sector *Y*, which can be computed with (11), (3) and (A.1) as $\pi = (1-\theta)\eta\left(p_X X^S + p_Z Z^S\right)/n$, gives

$$\dot{v}/v = r - \pi/v = r - (1-\theta)\eta\left(p_X X^S + p_Z Z^S\right)/c_R(w_L, w_H). \tag{A.3}$$

Since we exclude international capital flows, trade balance equilibrium requires

$$E = p_X X^S + p_Z Z^S. \tag{A.4}$$

Thus total expenditure grows at the rate of $\eta((1-\theta)/\theta)g$ and interest rates are

$$r = \eta((1-\theta)/\theta)g + \rho. \tag{A.5}$$

Finally differentiating (9) with respect to time gives (remember that $g := \dot{n}/n$)

$$\dot{v}/v = g\left(\eta((1-\theta)/\theta) - 1\right). \tag{A.6}$$

(A.3) to (A.6) can be rearranged to $g = (1-\theta)\eta\left(E/c_R(w_L, w_H)\right) - \rho.$ (A.7)

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10 A more detailed derivation of the results and additional derivations are contained in an appendix which is available from the author upon request.

11 The solution is largely inspired by Grossman and Helpman (1991, ch. 6).

Now insert (A.2) with unit demand functions into (15), multiply the equations by w_L and w_H, respectively, and sum up. With equations (6) and (A.4) we find

$$\left(w_L a_{LR}(\cdot)+w_H a_{HR}(\cdot)\right)g+\left(1-\eta(1-\theta)\right)E=w_L L+w_H H. \qquad \text{(A.8)}$$

Together with (A.7) we find (14). Writing g as $g(\rho)=c_1 L+c_2 H-c_3$, (A.9)
the constants are $c_1=\eta(1-\theta)(1-\delta)/a_{LR}$, $c_2=\eta(1-\theta)\delta/a_{HR}$, $c_3=\left(1-\eta(1-\theta)\right)\rho$.

Iso-output ratio line

The output ratio is given by

$$X/Z=\left(g(\rho)a_1+a_{HZ}L-a_{LZ}H\right)/\left(g(\rho)a_2-a_{HX}L+a_{LX}H\right), \qquad \text{(A.10)}$$

where $a_1=a_{HR}a_{LZ}-a_{LR}a_{HZ}$ and $a_2=a_{LR}a_{HX}-a_{HR}a_{LX}$. Setting $\xi:=X/Z$, inserting (A. 9) and rearranging gives $-d_1 L+d_2 H+d_3=0$, with positive constant coefficients d_i, $i=1,2,3$ defined as $d_1=-\left(c_1(a_2\xi-a_1)-(a_{HX}\xi+a_{HZ})\right)$, $d_2=c_2(a_2\xi-a_1)+(a_{LX}\xi+a_{LZ})$ and $d_3=-c_3(a_2\xi-a_1)$. Solving the iso-output ratio equation for *H*, gives $H=-d_3/d_2+d_1/d_2\,L$ which implies that the graph of this function intersects the *H*-axis at a negative value of *H* if $g>0$.

Proof of proposition 2

(a) The growth rate is given by (14). With constant prices and constant endowment we have $g'(\rho)=-\left(1-\eta(1-\theta)\right)<0$. ◊

(b) The output ratio is given by (A.10). Deriving yields

$$\partial(X/Z)/\partial\rho=g'(\rho)\left\{H(a_1a_{LX}+a_2a_{LZ})-L(a_1a_{HX}+a_2a_{HZ})L\right\}/(\cdot)^2.$$

Reinserting a_1 and a_2 and rearranging gives

$\partial(X/Z)/\partial\rho=g'(\rho)(h_X-h_Z)a_{LX}a_{LZ}a_{LR}(H-Lh_R)/(\cdot)^2$. This is positive since $h_X>h_Z$, $g'(\rho)<0$ and $h_R>H/L$ because of $h_R>h_X>h_Z$. ◊

Proof of proposition 3

The specification of a closed economy differs from the one of a small open economy only in the fact that now prices are determined by goods market clearing conditions. Insert them, middle goods market clearing condition (12) and unit demand functions into (13). Use (A.1), (6), (A.7) and choose *E* as numeraire. This gives

$$(1-\delta)\left(\frac{w_H}{w_L}\right)^{\delta}\left(\frac{(1-\theta)\eta}{w_L^{\delta}w_H^{1-\delta}}-\rho\right)+\left((1-\gamma)\eta\theta+1-\eta-\beta\right)\frac{p_X}{w_L}X^S+$$
$$+\left((1-\gamma)\eta\theta+1-\eta-\alpha\right)\frac{p_Z}{w_L}Z^S=L$$

$$\delta\left(\frac{w_L}{w_H}\right)^{1-\delta}\left(\frac{(1-\theta)\eta}{w_L^{\delta}w_H^{1-\delta}}-\rho\right)+(\gamma\eta\theta+\beta)\frac{p_X}{w_H}X^S+(\gamma\eta\theta+\alpha)\frac{p_z}{w_H}Z^S=H.$$

With Cobb-Douglas utility functions which imply $p_X X^S=\sigma$ and $p_Z Z^S=1-\sigma$ and by multiplying the equations by $(-w_L)$ and $(-w_H)$, respectively, we obtain

$$g_1(\cdot):=-\bar{c}_1+(1-\delta)\rho\,c_R(\cdot)+w_L L=0,$$
$$g_2(\cdot):=-\bar{c}_2+\delta\rho c_R(\cdot)+w_H H=0, \qquad \bar{c}_1,\bar{c}_2=\text{const.} \qquad \text{(A.11)}$$

By using the implicit function theorem we find $\partial w_L / \partial \rho < 0$ and $\partial w_H / \partial \rho < 0$. The change of the ratio of factor rewards with respect to a change of ρ is given by

$$\frac{\partial (w_H / w_L)}{\partial \rho} = \frac{((\partial w_H)/(\partial \rho))w_L - ((\partial w_L)/(\partial \rho))w_H}{w_L^2} < 0$$ if (note that $\partial w_L / \partial \rho < 0$)

$$\frac{\partial w_H / \partial \rho}{\partial w_L / \partial \rho} > \frac{w_H}{w_L} \Leftrightarrow \frac{H}{L} < \frac{\delta}{1-\delta} \frac{w_L}{w_H} = h_R$$. With $h_R > h_x > h_Z$ this is fulfilled. ◊

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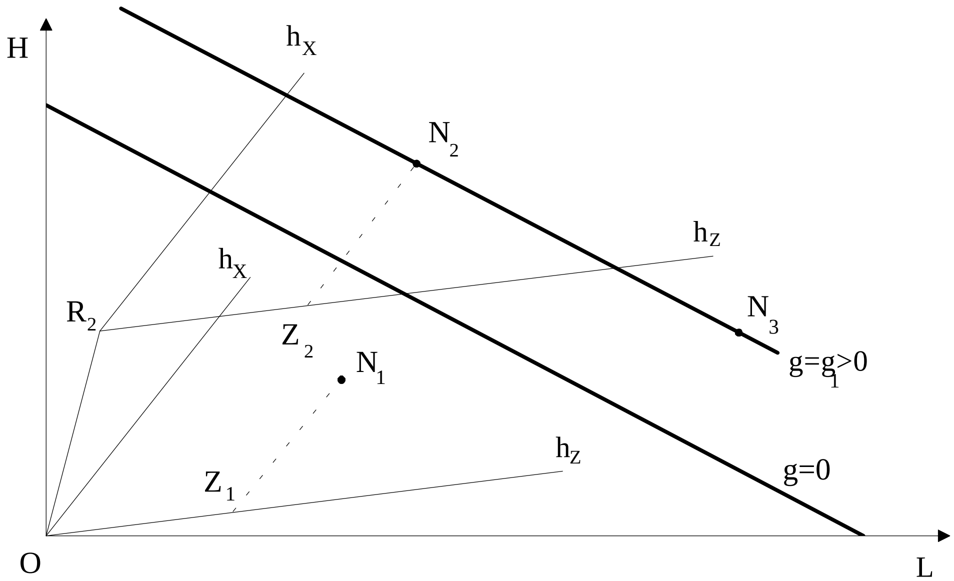

Figure 1: Cone of diversification of the small open economy

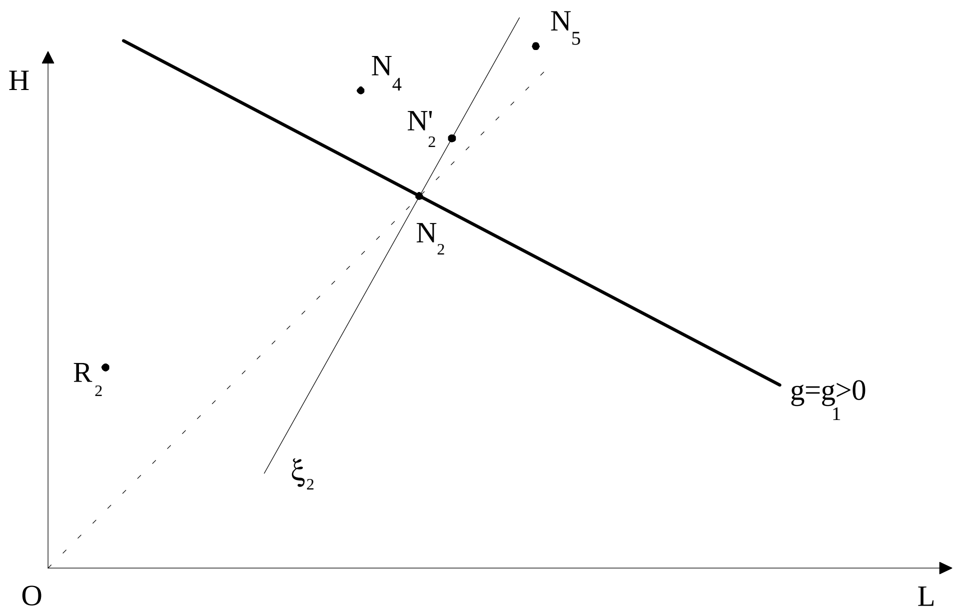

Figure 2: Iso-growth and iso-output ratio line

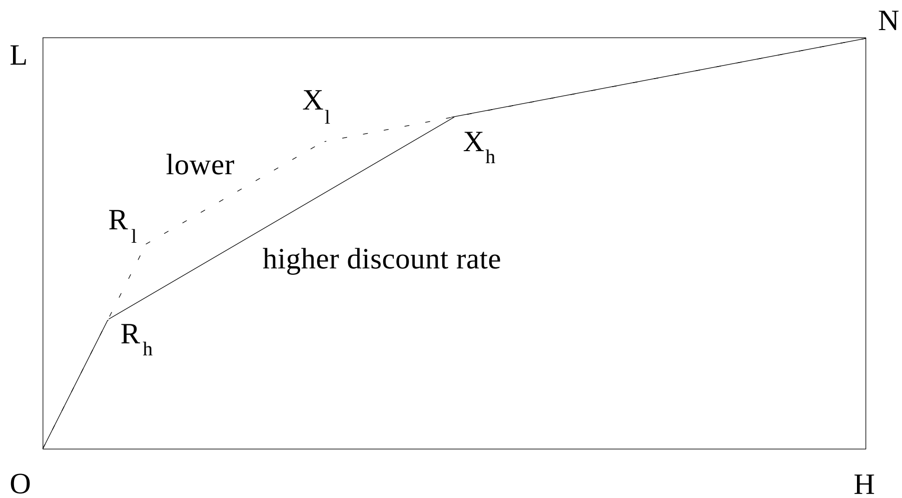

Figure 3: The allocation of factors for different time preference rates in a small open economy