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Trade pattern reversal: The role of technological change, factor accumulation and government intervention

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During the last decades, some Asian countries have experienced a substantial change of their production structure leading to a change of their trade patterns. Whereas formerly high-tech goods were imported, they now constitute an important fraction of the countries' exports. Possible reasons for such a situation to arise are investigated in a model combining Ricardian and Heckscher-Ohlin effects. In addition the role of government intervention on factor accumulation and technological change is studied. A reversal of trade patterns can be endogenously explained by these features.

1. Introduction

Since the end of World War II some East Asian countries have undergone a rapid economic change. The Four Tigers and Japan are leading in average per-capita GDP growth in the period of 1960-1985, which, to a large extent, is attributable to a physical and human capital accumulation process, taking place at a significantly faster pace than in other countries[1]. As a consequence, some countries which formerly were exporters of labour intensive goods have witnessed a change of their trade relations and are now high-tech exporters. Empirical evidence is reported e.g. for Japan by Grossman (1990) and Balassa and Noland (1989). Few theoretical investigations are undertaken of how to explain the mechanisms of such a reversal of trade relations, except Grossman and Helpman (1991a, ch. 7) and Grossman (1990). In these models, however, a change of trade patterns is explained by an *exogenous* change in the endowment of human capital which does not allow to study the mechanisms, how such a change in endowment takes place, and, most importantly, why it takes place at a faster pace than in other countries. It is the scope of this paper to provide a first attempt to study more in detail, how trade patterns can be reversed.

In having the experience of Japan or the Four Tigers in mind, it seems necessary to take both technological progress as well as differences in factor endowment into consideration. Consequently, trade patterns will be determined by the interaction of Ricardian as well as Heckscher-Ohlin effects. The next section presents a two-country world with endogenous technological change and human capital accumulation, thus combining two "engines of growth": Human capital accumulation as in Lucas (1988) and a public knowledge stock as in Grossman and Helpman (1991a). The third section studies the stability properties of the models,

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1 For a comprehensive study of the "East Asian miracle", cf. World Bank (1993).

drawing a distinction between three cases: a world in which either Ricardian or Heckscher-Ohlin features dominate and a world where both effects interact. The subsequent section then gives sufficient conditions for a reversal of trade patterns. In the fifth section, the influence of government intervention on the evolution of the economies is studied. We allow both for subsidies for the R&D activity and measures which increase the human capital accumulation capacity. Again, conditions for a reversal of trade patterns are derived under the different cases. Finally, the sixth section completes the findings on trade pattern reversal and discusses, under which conditions an initially poor country (measured in terms of per capita consumption) can increase its share in world consumption.

2. A two-country world

The theoretical growth literature has put forward many different explanations for long-run positive growth rates. One distinguishes models based on technological progress (Solow, 1956), learning-by-doing (Romer, 1986), human capital accumulation (Lucas, 1988), public stocks of knowledge (Romer, 1990; Grossman and Helpman, 1990), public infrastructure (Barro, 1990) or linearities in the production function (Romer, 1987; Rebelo, 1991). Trying to capture essential features of a catching-up process of East Asian countries, however, requires to allow for several endogenous changes, both of technology and of stocks of production factors. Differences in technology could be modelled the easiest by assuming different productivity factors, which, however, would not allow to discuss an endogenous change of technology. Thus we model the state of the technology by the depth of division of labour, and use the specification proposed by Ethier (1982), which is the analogue to the Dixit and Stiglitz (1977) formulation in consumption theory. With this specification, technological progress is equivalent to an increase of the number of intermediate goods.
The world economy we consider trades in final goods, but does not allow for international capital movements and trade in intermediate goods. For convenience, we assume that both countries are incompletely specialized after opening up to trade.

Consumption side
For modelling the consumption side, we choose the Lucas (1988) framework extended to two consumption goods. Since the scope of the model is not to study the effects of disconnectedness between individuals or cohorts (Blanchard, 1985; Weil, 1987; Buiter, 1988) we assume a bequest motive à la Barro (1974) and thus let members of household *i*, living in country *j*, maximize at time *t* the discounted stream of future utility with an infinite horizon which can be expressed by

$$U^i(t) = \int_t^\infty \exp\left[-\rho(\tau - t)\right] u^i(\tau) d\tau, \qquad (1)$$

where the time preference rate is given by ρ. Instantaneous felicity u^i results from the consumption of the final goods X^i and Z^i. It is expressed in the log form as $u^i(\tau) = \ln\left[X^{i\sigma} Z^{i1-\sigma}\right]$, where $0 < \sigma < 1$. Static utility maximization implies standard demand functions for final goods, depending on consumer *i*'s total expenditure $E^i(t)$ and on prices p_X and p_Z,

$$X^i = \frac{\sigma}{p_X} E^i(t), \qquad Z^i = \frac{1-\sigma}{p_Z} E^i(t). \tag{2}$$

Now reinsert these static optimality conditions into (1) and maximize the modified utility function with respect to the share of human capital allocated to production $\mu^i(t)$ and total expenditure $E^i(t)$, subject to a flow budget constraint and a human capital accumulation constraint which read

$$E^i(t) + \dot{A}^i(t) = r^j(t)A^i(t) + \phi\iota^i(t) + w_L^j(t) + w_H^j(t)\mu^i(t)H^i(t), \tag{3}$$

$$\dot{H}^i(t) = H^i(t)\delta\left(1-\mu^i(t)\right). \tag{4}$$

The first constraint equates expenditures and the change of the value of assets, household i holds, $\dot{A}^i(t)$, to asset, R&D, labour and human capital income, $r^j(t)A^i(t)$, $\phi\iota^i$, $w_L^j(t)$ and $w_H^j(t)\mu^i(t)H^i(t)$, where each household owns one unit of labour and allocates a share $\mu^i(t) \in]0,1]$ of human capital to production[2]. The second constraint relates the remaining human capital share $1-\mu^i(t)$ to the human capital accumulation process. Since preferences (ρ) and human capital accumulation capabilities (δ) are by assumption identical for each household and the interest rate $r^j(t)$ and factor rewards $w_H^j(t)$ are exogenously given, every household of country j behaves the same. Thus the outcomes of the individual maximization problems can be easily aggregated to determine the behaviour of the economy as a whole. As optimality conditions we find (with the time index suppressed)

$$\frac{\dot{E}^j}{E^j} = r^j - \rho, \tag{5}$$

$$\frac{\dot{w}_H^j}{w_H^j} = r^j - \delta, \tag{6}$$

and as aggregate accumulation constraints

$$E^j + \dot{A}^j = r^j A^j + \phi\iota^j + w_L^j L^j + w_H^j \mu^j H^j, \tag{7}$$

$$\dot{H}^j = H^j\delta\left(1-\mu^j\right). \tag{8}$$

Some remarks can already be made with respect to the outcome of the model, given these equations. Provided that interest rates equalize, by (5) each country will experience the same growth of aggregate expenditure, a result emphasized by Rebelo (1992). This means that, given internationally perfectly mobile financial capital, the rate of growth of consumption per capita is completely independent of the production side (in contrast to, of course, the *level* of consumption per capita). Internationally completely mobile capital, however, is not a necessary condition for "consumption growth rate equalization": Assume that the endowment of both countries is such that neither country specializes; then factor prices equalize and, by equation (6), interest rates equalize – even with internationally immobile capital – which, again, implies identical growth rates of consumption per capita.

In addition to the dynamic optimality conditions and constraints, aggregate static demand functions for final goods have to be added. They are given by

$$X^D = \frac{\sigma}{p_X}\left(E^A + E^B\right), \qquad Z^D = \frac{1-\sigma}{p_Z}\left(E^A + E^B\right).$$

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[2] The source of R&D income amounting to $\phi\iota^i$ on the individual and $\phi\iota^j$ on the economy level will become clear when discussing the R&D process.

Production side

Two final homogenous goods X and Z are produced, using human capital and labour, given a certain technology level. Since technology changes over time, the specification of how technology influences the productivity of human capital and labour, is crucial for the outcome of the model. Two extreme examples help to illustrate the possibilities: Either technological progress increases the productivity of human capital and labour to the same extent (as e.g. in Wälde, 1993 or in Grossman and Helpman, 1991a, ch. 6) or only one factor enjoys higher productivity, leaving the other factor unaffected. In a two-country world, the first specification would not be very interesting: The technological level of a country would simply determine the country's income level, but would not exert any influence on trade patterns. Consequently, Ricardian effects would be missing in the model and a simultaneous study of technological change and factor accumulation would not give further insight. Therefore, we take here the second approach and assume that the technological level increases the productivity of labour only[3]. The basic structure of the production side of the economy is thus given by three activities: Production of two final homogenous goods and of differentiated middle goods.

The production of the consumption goods follows standard constant-returns-to-scale production functions with two inputs, human capital H and a set of intermediate goods i, represented by an index K,

$$Z^j = Z_0^j \left(H_Z^j\right)^{\alpha} \left(K_Z^j\right)^{1-\alpha}, \tag{9}$$

$$X^j = X_0^j \left(H_X^j\right)^{\beta} \left(K_X^j\right)^{1-\beta}. \tag{10}$$

Without loss of generality, the output elasticity of X with respect to H is higher than the output elasticity of Z with respect to H: $\beta > \alpha$. Perfect competition ensures zero-profits which can be expressed by equality of prices and unit costs. Further, since consumption goods are perfect substitutes, independently of their country of origin, free trade equalizes goods prices and unit costs must satisfy, as long as both countries are incompletely specialized

$$p_Z = \left(w_H^j\right)^{\alpha} \left(w_K^j\right)^{1-\alpha},$$

$$p_X = \left(w_H^j\right)^{\beta} \left(w_K^j\right)^{1-\beta},$$

where the price index w_K reflects the prices of intermediates necessary to obtain one unit of K. As is well-known, these pricing conditions imply factor price equalization in w_H and w_K, given the absence of factor intensity reversal.

The production process in the consumption good sectors specified in (9) and (10) has to be understood such that firms employ human capital in order to assemble intermediate goods i to obtain final output X or Z. The index K is given by, similar to Ethier (1982),

$$K^j = \left(\int_0^{n^j} \left(y^i\right)^{\theta} di \right)^{1/\theta}, \qquad 0 < \theta < 1. \tag{11}$$

This index increases in the (measure of the) number n^j of intermediate goods i, available in country j, which allows to represent technological progress in the

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[3] There is a further - technical - necessity why technological progress has to be restricted to labour only: The rate of innovation in models à la Grossman and Helpman is an increasing function of a country's factor endowment. Thus, if the stock of human capital influenced technological progress, we would obtain ever increasing rates of growth which is hard to be justified on an empirical basis.

production of X and Z by an increase in the range of intermediate goods. The higher n^j, the higher the degree of specialization and the higher output of consumption goods.

Intermediate goods i are produced in a monopolistic competition setting. Firms producing either X or Z then choose among intermediate goods optimally by minimizing total costs $\int_0^{n^j} p_y^i y^i di$ with respect to y^i subject to (11). This gives demand for each variety of the form

$$y^i = \left(p_y^i\right)^{-1/(1-\theta)} K^j \left(\int_0^{n^j} \left(p_y^i\right)^{-\theta/(1-\theta)} di \right)^{-1/\theta}. \tag{12}$$

The production of varieties of the middle goods consists of two steps. Due to infinite patent protection, each producer of intermediate goods, as the first step, has to develop its own blueprint for producing a variety. This R&D process is described by a simple cost function which reads for firm i

$c_R^i = \phi / K n^i$.

This equation basically says that the development of a new blueprint requires the spending of some fixed costs ϕ (a similar specification was used by Barro and Sala-i-Martin, 1992) divided by the total non-appropriable stock of public knowledge accessible for firm i. Clearly, this is the Romer (1990) idea of technological spillovers which results from the R&D process as a by-product to the development of new blueprints, an idea used extensively by Grossman and Helpman (1991a).

It is well-known, that those spillovers resulting from R&D can result in positive long-run innovation rates. Since in the present model, technological change relies on that mechanism, a further, very important assumption about *inter*national spillovers has to be made in a context of a two-country world. Information which flows freely between countries leads to innovation rate equalization (Rivera-Batiz and Romer, 1991; Grossman and Helpman, 1991a, ch.7; 1991b, section 6). The balanced growth paths of models of that kind can also be shown to be locally and globally saddle path stable (Wälde, 1994). On the other hand, if knowledge diffusion is restricted to the domestic market, then the world economy is characterised by hysteresis effects (Grossman and Helpman, 1991a, ch.8; Feenstra, 1990; Devereux and Lapham, 1993). It seems unrealistic to assume that the rate of technological progress equalizes automatically across countries; thus we prefer the second specification where the public knowledge stock is restricted to the country where it was build up. This implies innovation rates which differ between countries[4].

Relating the public knowledge stock to the number of differentiated goods available in country j, restricted R&D spillovers translate into an expression for R&D costs of ϕ / n^j instead of $\phi / \left(n^A + n^B\right)$, which would be the case if we had international knowledge spillovers. Consequently, the intertemporal zero-profit condition reads

$$\phi / n^j = v^j(t) = \int_t^\infty \exp\left[-\left(R^j(\tau) - R^j(t)\right)\right] \pi^j(\tau) d\tau, \tag{13}$$

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4 Different innovation rates due to restricted knowledge flows are also obtained by Rivera-Batiz and Xie (1993). Other possibilities to model growth rates which differ between countries are, e.g., Buiter and Kletzer (1991), Alogoskoufis and van der Ploeg (1991a,b).

where profits $\pi^i = (1-\theta) p_y^j y^j = (1-\theta) \frac{w_L^j}{\theta} \frac{L^{D^j}}{n^j}$ are discounted by a cumulative factor $R^j(u) = \int_0^u r^j(s) ds$ depending on the interest rate $r^j(s)$. Note, that R&D costs as in (13) which firms have to pay, flow back to consumers and thus generate income, amounting at the economy level to $\dot{n}^j \phi / n^j = \phi \iota^j$, as included in the economy's budget constraint (7).

The production of the intermediate goods follows a simple constant-returns-to-scale production function $y^i = L^i$ and producers, facing demand function (12), set prices according to

$$p_y^j = \frac{w_L^j}{\theta}. \tag{14}$$

Since all firms of country *j* have to pay the same labour rewards w_L^j, equation (14) implies that all varieties produced in that country are equally priced at p_y^j. The expression for costs of the set of intermediate goods necessary to obtain one unit of *K* therefore takes a simple form. Inserting (12) into the cost function $\int_0^{n^j} p_y^i y^i di$ and using (14) gives costs as a function of labour rewards and the technological level of the economy, $w_K^j = \left(\int_0^{n^j} \left(p_y^j \right)^{\theta/(\theta-1)} di \right)^{-(1-\theta)/\theta} = \left(n^j \right)^{-(1-\theta)/\theta} \frac{w_L^j}{\theta}$.

Market equilibria

By equation (12), equal prices for all intermediate goods lead to equal output per variety. Thus, aggregate demand for labour can be simply expressed as a function of demand for intermediate inputs as

$$L^{D^j} = n^j y^j. \tag{15}$$

Demand for intermediate goods, in turn, depends on the total quantity of *K* used in the production process of consumption goods, since by (11)

$$K^j = \left(n^j \right)^{1/\theta} y^j = \left(n^j \right)^{(1-\theta)/\theta} Y^j.$$

Finally, since the index *K* is related to final output of consumption goods *X* and *Z* by

$$a_{KX}\left(w_H^j, w_K^j \right) X^j + a_{KZ}\left(w_H^j, w_K^j \right) Z^j = K^j, \tag{16}$$

where $a_{K\cdot}\left(w_H, w_K \right)$ are unit demand functions, the full employment condition for labour reads

$$a_{KX}\left(w_H^j, w_K^j \right) X^j + a_{KZ}\left(w_H^j, w_K^j \right) Z^j = L^j \left(n^j \right)^{(1-\theta)/\theta}. \tag{17}$$

The supply of human capital depends on the choice of consumers of how to allocate it between production and accumulation of human capital; the quantity $\mu^j H^j$ used for production is then further subdivided between production of final goods,

$$a_{HX}\left(w_H^j, w_K^j \right) X^j + a_{HZ}\left(w_H^j, w_K^j \right) Z^j = \mu^j H^j. \tag{18}$$

3. Solution of the model and dynamic properties

Reduced form

In order to derive the reduced form of the model presented above, one first solves for human capital rewards and the price index for intermediate goods with the help of clearing conditions for factor markets and goods markets and pricing conditions for final goods[5]. This gives[6]

$$w_K = \frac{c_L}{\left(n^A\right)^{\frac{1-\theta}{\theta}} L^A + \left(n^B\right)^{\frac{1-\theta}{\theta}} L^B} E, \qquad w_H = \frac{1-c_L}{\mu^A H^A + \mu^B H^B} E,$$

where c_L is a constant, representing preference and technology parameters.

Then the budget constraints are simplified by inserting factor rewards, w_K and w_H, and the capital market no-arbitrage condition which can be obtained by differentiating (13) with respect to time. It turns out that one budget constraint is always fulfilled if the other one is satisfied (which is the verification of Walras' law). This allows us to choose total expenditure as numeraire which implies, since by (6) interest rates equalize given incomplete specialization, $r = \rho$. Finally, the remaining budget constraint and the expression giving the factor reward for human capital is differentiated with respect to time and the resulting equations can be solved to yield

$$\dot{\mu}^A = f(\Omega)\left(\mu^A + \mu^B \Lambda\right) + \mu^A\left(\mu^A \delta - \rho\right), \tag{19}$$

$$\dot{\mu}^B = -f(\Omega)\left(\mu^A \Lambda^{-1} + \mu^B\right) + \mu^B\left(\mu^B \delta - \rho\right), \tag{20}$$

where, for notational convenience, we defined

$$f(\Omega) := \frac{c_L^2}{(1-c_L)} \frac{(1-\theta)^2}{\phi\theta} L^A L^B \Omega^{\frac{1-\theta}{\theta}} \left(\Omega^{\frac{1-\theta}{\theta}} L^B - L^A\right)\left(L^A + \Omega^{\frac{1-\theta}{\theta}} L^B\right)^{-3}.$$

Due to the simple R&D structure, we can derive an explicit expressions for the rate of innovation of country *j* which turns out to be

$$\iota^j = (1-\theta) \frac{\left(n^j\right)^{\frac{1-\theta}{\theta}} L^j}{\left(n^A\right)^{\frac{1-\theta}{\theta}} L^A + \left(n^B\right)^{\frac{1-\theta}{\theta}} L^B} \frac{c_L}{\phi} - \rho. \tag{21}$$

It is an increasing function of the countries productivity-adjusted labour endowment $\left(n^j\right)^{\frac{1-\theta}{\theta}} L^j$ and decreases in the fixed component of R&D costs ϕ, in consumers' time preference rate ρ and in θ. A value of θ close to unity indicates a high elasticity of substitution (given by $\varepsilon = 1/(1-\theta)$; cf. equation (11)) between any two varieties. Thus the higher θ, the less an additional variety contributes to an increase of output and the lower the innovation rate.

Analyzing further, one finds that the outcome of the model does not depend on absolute quantities of differentiated goods or human capital, but on ratios only. Thus we introduce two new variables which express the ratios of the number of varieties and of the stock of human capital between countries. They are defined as

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5 Note that factor rewards can be computed explicitly only in the case of factor prices equalization in w_H and w_K. This is given as long as both countries are incompletely specialised.

6 A detailed derivation is contained in an appendix, available from the author upon request.

$\Omega := n^B / n^A$ and $\Lambda := H^B / H^A$. Using the human capital accumulation constraint (8) and the innovation rate of country *j*, the reduced form is completed by

$$\dot{\Lambda} = \delta\left(\mu^A - \mu^B\right)\Lambda, \tag{22}$$

$$\dot{\Omega} = \left(1-\theta\right)\frac{c_L}{\phi}\frac{\Omega^{\frac{1-\theta}{\theta}} L^B - L^A}{L^A + \Omega^{\frac{1-\theta}{\theta}} L^B}\Omega. \tag{23}$$

Thus the relatively simple reduced form consists of four equations only; (19), (20), (22) and (23).

Stability analysis

In analyzing the stability of the model, we first focus our attention on human capital accumulation alone and thus exclude (relative) technological change. Then, only technological change is taken into consideration before, finally, the interaction of technological change and human capital accumulation is studied.

No relative technological change – In the present framework, no relative technological change occurs (and thus Ricardian effects vanish) if the ratio of differentiated goods equals a fixed number given by the countries' labour endowment: $\dot{\Omega} = 0 \Leftrightarrow n^B / n^A = \left(L^A / L^B\right)^{-\frac{1-\theta}{\theta}}$. In the subsequent analysis we will assume that this holds. Note, however, that the results which will turn out are not restricted to this, admittedly, rather special case, but are valid equally if e.g. technological spillovers would be excluded or the model would be specified with identical technologies right from the outset.

If $\dot{\Omega} = 0$, $f\left(\Omega\right) = 0$ and equation (23) can be neglected. The economy is on a balanced growth path, defined by (19), (20) and (22), for $\mu^i = \rho/\delta$. The stability properties of such an equilibrium are easily analyzed by graphical phase plane analysis in the $\left(\mu^A, \mu^B\right)$ space, where solid arrows indicate the direction of motion of μ^i and dotted arrows pointing upwards indicate an increase of Λ (imagine a third dimension).

[Insert figure 1 around here]

Note that both shares of human capital allocated to production, μ^A and μ^B, are control variables. If one of the countries decided to choose as initial value, right after opening up to trade, a point which does *not* satisfy $\mu^i = \rho/\delta$, then either μ^A or μ^B must eventually hit one of the boundaries $\{0,1\}$. Then the optimality condition (6) could no longer be satisfied. Consequently, provided both countries want to maximize current utility, they both choose at once the equilibrium values[7].

No human capital accumulation – This would simplify the model to such an extend that one would no longer have to worry about stability properties. The optimal share of allocation of human capital between education and production would no longer be an issue and the model would be a simplified version of the hysteresis model of Grossman and Helpman (1991a, ch.8).

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[7] It will be shown later that equilibrium values fall within the required interval $]0,1]$.

Technological change and human capital accumulation – Let us now turn to the certainly more interesting case where technological change and human capital accumulation interact. Assume that initial conditions are such that $\dot{\Omega} \geq 0 \ \forall t$ which implies that $f(\Omega) > 0$[8]. Then the innovation rate of country *B* approaches its autarky innovation rate

$$\iota^B = (1-\theta)\frac{L^B}{\Omega^{\frac{1-\theta}{\theta}} L^A + L^B}\frac{c_L}{\phi} - \rho \xrightarrow[\Omega\to\infty]{} (1-\theta)\frac{c_L}{\phi} - \rho, \tag{24}$$

and the innovation rate of country *A* becomes zero (in finite time!) since

$$\iota^A = (1-\theta)\frac{L^A}{L^A + \Omega^{\frac{1-\theta}{\theta}} L^B}\frac{c_L}{\phi} - \rho \geq 0 \Leftrightarrow \Omega^{\frac{1-\theta}{\theta}} \leq \left(\frac{1-\theta}{\phi}\frac{c_L}{\rho} - 1\right)\frac{L^A}{L^B}. \tag{25}$$

The innovation rate of country *j* was shown in (21) to be a function of its relative size (measured in effective labour endowment) in the world economy. If innovation in country *B* is higher than in the other country, its effective labour endowment $\left(n^B\right)^{\frac{1-\theta}{\theta}} L^B$ increases absolutely and relatively to the other country and, as a corollary, the relative effective size of the other country decreases. Then, however, at a certain point in time, country *A*'s effective labour endowment is too small to sustain positive innovation rates[9].

Since Ω is a monotonous function in time, we can divide equations (19) - (22) by equation (23) and thereby obtain a non-autonomous three dimensional system in Ω, which can be treated by graphical means. To this end, determine the zero-motion loci by setting the equations of motion equal to zero. This gives

$$\dot{\mu}^A \geq 0 \Leftrightarrow \mu^B \geq -\frac{\mu^A}{\Lambda}\left(\left(\mu^A\delta - \rho\right) f(\Omega)^{-1} + 1\right), \tag{26}$$

$$\dot{\mu}^B \geq 0 \Leftrightarrow \mu^A \leq \mu^B \Lambda\left(\left(\mu^B\delta - \rho\right) f(\Omega)^{-1} - 1\right), \tag{27}$$

$$\dot{\Lambda} \geq 0 \Leftrightarrow \mu^A \geq \mu^B. \tag{28}$$

Obviously, the function $f(\Omega)$ plays a crucial role for the dynamic properties of the model. Its optima are implicitly determined by the function

$$\Omega^{\frac{1-2\theta}{\theta}}\left(-L^{A^2} + 4L^A L^B \Omega^{\frac{1-\theta}{\theta}} - L^B \Omega^{2\frac{1-\theta}{\theta}}\right) = 0.$$

In the special case $\theta = 1/2$, they can be explicitly computed to be at

$$\Omega_{1,2} = \frac{L^A}{L^B} + \frac{L^B}{L^A} \pm \sqrt{\left(\frac{L^A}{L^B} + \frac{L^B}{L^A}\right)^2 - 1},$$

otherwise they lie near these values. The function $f(\Omega)$ approaches zero as Ω goes to infinity: $f(\Omega) \xrightarrow[\Omega\to\infty]{} 0$ and its value at the origin is zero: $f(0) = 0$. Since it is positive iff $\Omega \geq \left(L^A / L^B\right)^{\frac{\theta}{1-\theta}}$, a typical representation of $f(\Omega)$ can be plotted as in figure 2.

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8 Making the opposite assumption $\dot{\Omega} \leq 0 \ \forall t$ would lead to a completely symmetrical analysis where country *A* has been renamed country *B* and vice versa.

9 This is the crucial difference to models with *international* spillovers of knowledge. In this case the innovation rates of both countries would equalise and depend on the *sum* of the effective labour endowment of both countries.

[Insert figure 2 around here]

After this analysis of the technology function, we can plot the zero-motion loci for a certain $f(\Omega) = f(\overline{\Omega})$ in figure 3, taking into consideration only the loci falling broadly within the range of $\mu^j \in]0,1]$[10]. The planes are determined by equations (26), (28) and the upper branch of (27), solved for μ^B, which is given by

$$\mu^B = \frac{1}{2\delta\Lambda}\left(\Lambda\left(f(\Omega)+\rho\right) \pm \sqrt{4\delta f(\Omega)\Lambda\mu^A + \Lambda^2\left(f(\Omega)+\rho\right)^2}\right). \tag{29}$$

[Insert figure 3 around here]

How does this phase diagram change with Ω increasing further? The $\dot{\mu}^B = 0$ plane becomes flatter, until in the limit it is a horizontal line (compare equation (29) $\mu^B \xrightarrow[f(\Omega)\to 0]{} \frac{\rho}{\delta}$). The maximum of the $\dot{\mu}^A = 0$ parabola moves up such that the slopes at the μ^B intercepts become steeper and steeper until, in the limit, the lines are vertical. In order to find initial values chosen by consumers in country *j*, we subdivide figure 3 in two parts, one for Λ greater and one for Λ smaller than 2.

[Insert figure 4 around here]

Figure 4 represents the first part of figure 3 and shows the zero-motion lines for control variables μ^j and state variable Λ in the (μ^A, μ^B) plane, for $\Lambda < 2$. Again, solid lines indicate the direction of motion of control variables and dotted lines of Λ. The same relations hold with respect to zero-motion loci, as in the three-dimensional graph: An increasing Ω will increase the slope of the $\dot{\mu}^A = 0$ and flatten the $\dot{\mu}^B = 0$ line, which, in the limit, gives a phase diagram identical to figure 1. Note, however, that here the equilibrium point can never be reached since there will always be an increase of the technological gap. Which are the initial conditions the consumers of country *A* and *B* will choose? By analogy to the phase diagram with no technological progress, we take the intersection point of the $\dot{\mu}^A = 0$ and $\dot{\mu}^B = 0$ line, denoted by *S*, as a candidate for a solution. On this point all equations of motion are satisfied, thus the optimality conditions for both countries are fulfilled. Further, this point approaches the long-run equilibrium point which lies on the $\dot{\Lambda} = 0$ line. An analogue argument to the decision problem with no technological change excludes initial values other than point *S*: Eventually one of the boundaries for μ^j would be reached and one optimality condition would be violated. Since the point S lies within the area $0 < \mu^A, \mu^B \leq 1$, the optimal initial values constitute an interior solution.

A similar argument can be invoked for the area $\Lambda > 2$. In this case, the $\dot{\mu}^i = 0$ lines do not intersect. The initial conditions for μ^A and μ^B are then chosen such that they will reach the intersection point of the $\dot{\mu}^i = 0$ lines with Λ growing large and $f(\Omega)$ approaching zero. Thus, independently of initial conditions of state variables Λ and Ω, the preceding analysis has shown that the system has well-defined

[10] This graph was drawn by using Mathematica (cf. Wolfram, 1991).

equilibrium trajectories. For the subsequent analysis, it should be retained that an increasing Ω implies that μ^B is always greater than μ^A.

4. Endogenously changing trade patterns

As a benchmark case, consider two economies with no technological change. Then the decision problem of consumers consists in choosing the optimal repartition of human capital between production and education. The last section (cf. figure 1) has shown that this leads to an immediate jump on a balanced growth path where both countries allocated the share $\mu^j = \rho/\delta$ to the production sector. Human capital is then accumulated in both countries at the same rate $\delta - \rho$ and the countries' relative factor richness does not change over time[11]. Clearly, in such a situation, initial trade patterns will be observed ever after. If, on the other hand, capital accumulation ceases and the evolution of the economies is characterised by technological change only, then the technologically leading country will continuously expand its technological leadership. In our case of internationally immobile capital, interest rates would not be equalized and the share of the technologically leading country in world expenditure would approach unity.
Let us now turn to the interaction of accumulation of human capital and technological change. In the absence of government intervention and given fixed preferences, the evolution of technology is completely determined by variables, out of the range of consumers' influence. Once a country is technologically leading, it will forever expand its leadership. Its innovation rate will approach its autarky innovation rate from below, whereas the other country's innovation rate will become zero in finite time (compare equation (24) and (25)). The accumulation of human capital, however, is determined by an intentional choice of the consumers: If a country is technologically backward (again, say, country *A*), it will allocate a lower share of human capital to production ($\mu^A < \mu^B$) and thus experience a higher human capital accumulation rate, as was shown in the last section. Intuitively, this becomes clear from looking at its budget constraint, which can be expressed by

$$\overline{E}^A = c_L \frac{1}{1+\Omega^{(1-\theta)/\theta} L^B/L^A} + (1-c_L)\frac{1}{1+\Lambda \mu^B/\mu^A}.$$

Total expenditure of country *A* is given by the constant amount $\overline{E}^A$. It can neither in- nor decrease over time, since otherwise optimality condition (5) would be violated. Since the ratio of intermediate goods Ω increases, the first part of the budget constraint falls and, consequently, its second term must increase which can be achieved only by decreasing the human capital ratio $\Lambda = H^B/H^A$. Thus the technologically backward country *A* increases human capital more rapidly than country *B* and vice versa: The country which is technologically leading accumulates human capital at a lower rate than the other country.

Under which circumstances can a change of trade patterns take place? Our assumption that both countries remain incompletely specialized when they open up to trade amounts to requiring that the countries' factor endowments are not too dissimilar, a condition which is familiar from static Heckscher-Ohlin models. Since

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[11] Of course, in order that growth rates are positive we need $\rho < \delta$. If this holds then $\mu^j < 1$.

preferences do not differ between countries, an export of the human capital intensive good X is equivalent to an higher output ratio $\xi := X/Z$. This in turn depends exclusively on factor endowment since it can be shown that

$$\frac{X^A}{Z^A} > \frac{X^B}{Z^B} \Leftrightarrow \frac{\mu^A H^A}{\left(n^A\right)^{\frac{1-\theta}{\theta}} L^A} > \frac{\mu^B H^B}{\left(n^B\right)^{\frac{1-\theta}{\theta}} L^B} \Leftrightarrow \frac{L^B}{L^A}\Omega^{\frac{1-\theta}{\theta}} > \frac{\mu^B}{\mu^A}\Lambda .$$

Thus trade patterns change only if the relative endowment of the economies changes. In our context, where country B is technologically more advanced and its innovation rate is higher than the innovation rate of country A, country A accumulates human capital faster than country B. As a consequence, the dynamic equilibrium is characterised by an increasing Ω and a decreasing Λ. Thus, if the initial conditions are such that country B is human capital richer than country A, then country B will initially export the human capital intensive good. After some time, however, it will be technologically advanced to such an extend and its endowment with human capital has fallen relatively to the human capital stock of the other country that it will export the technology intensive good. A change of trade patterns has occurred.

A change of relative factor richness can also occur if human capital accumulation does not take place. An argument for a reversal of trade patterns can then be invoked following the same lines as above. Contrarily, however, interest rates would not be equalized and the technologically leading country would expand its share in world expenditure.

5. Government intervention

The last section has shown that changing trade patterns can occur in this model only if a technologically leading country initially has a relatively low stock of human capital. Such an evolution, however, does not describe the experience of, e.g., Japan in a satisfying way since it would imply that all countries which experienced a fundamental change of their economic structure during the last decades were, when beginning to trade with industrialized countries, already technologically leading. Therefore, in this section we turn to government interventions and study how they can influence the evolution of an economy. It should be stressed, that this is a purely positive attempt to re-draw observations and the question, whether these interventions are to be favoured from a welfare point of view, is not a matter of concern here, although it is at least of equal importance and should be at the core of future investigations.

Two natural possibilities come into mind, how a government can intervene: Either research activities are subsidized or the accumulation of human capital is encouraged. We discuss both of these possibilities separately.

R&D subsidies

Many factors influence research and development in an economy; consequently, the government can choose among many possibilities, how to encourage research activities. In the following we will discuss the influence of subsidies to R&D (as e.g. Grossman and Helpman, 1991a, ch. 8).

Costs to produce one new variety are given by $c_R = \phi / n^j$. Let the government commit itself to subsidize each production of a new variety by paying a share s^j of total development costs. Given an innovation rate of ι^j the government's

expenditure at every moment is given by $G^j = s^j \phi \iota^j$. These expenditures are financed by lump-sum taxes T^j which changes the aggregate budget constraint of country *j* to

$$E^j + \dot{A}^j = r^j A^j + \phi \iota^j + w_L^j L^j + w_H^j \mu^j H^j - T^j. \tag{30}$$

Due to this subsidy, R&D costs on the *firm* level diminish to $c_R = \phi\left(1 - s^j\right)\Big/ n^j$ and the free-entry condition becomes $\phi\left(1 - s^j\right)\Big/ n^j = v^j(t)$. Since factor rewards and allocation decisions of firms are not influenced by this modification, factor price equalization in w_K and w_H still holds (given incomplete specialization) and the same rearrangements and conclusions as in the model without intervention can be undertaken. What changes, however, is the evolution of the technological gap between countries. After modifying equation (23) to include R&D subsidies, the condition whether country *B* increases its technological advance over country *A* does not only depend on current technological differences (represented by the ratio Ω of differentiated goods in country *B* and *A*) but also on R&D subsidies and reads

$$\dot{\Omega} \geq 0 \Leftrightarrow \frac{\Omega^{\frac{1-\theta}{\theta}} L^B}{1 - s^B} \geq \frac{L^A}{1 - s^A}.$$

Let us continue our analysis with the assumption of country *B* being more technologically advanced. If both countries renounce from subsidizing R&D then the technological lead over country *A* would ever increase, thus country *A* might want to intervene[12]. Consider the limiting case that country *A* simply wants to avert falling further behind. Then it chooses (let country *B* be passive, $s^B = 0$) s^A such that

$$s^A = 1 - \frac{L^A}{\Omega^{\frac{1-\theta}{\theta}} L^B}. \tag{31}$$

From this equation interesting implications can be derived. Assume a certain technological lead of country *B*, which translates into a certain $\Omega = \overline{\Omega} > 1$, assuming $L^A = L^B$ for simplicity. Then the share of R&D costs to be paid by country *A* are unambiguously determined by (31). This share, however, might be too high when opening up to trade and approaches unity if the technological gap increases. Thus, in this model it seems advisable, if the technological level is a matter of concern, not to intervene too late.

What happens if s^A is chosen above the critical level which keeps the technological difference at a constant level? Then Ω begins to fall and the technological gap narrows. Thus s^A can be reduced over time to vanish where

$$n^{B\frac{1-\theta}{\theta}} L^B = n^{A\frac{1-\theta}{\theta}} L^A.$$

Combining these results with the analysis undertaken in the last section, it is clear, that these interventions can result in changing trade patterns: Assume that country *A* is technologically backward for some time. It will then be certainly be more human capital rich than country *A*. If it subsidizes R&D to such an extent that it becomes the technologically leading country, it will accumulate human capital at a lower rate than country *B*. Provided that this technological lead is extended over

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12 As mentioned before, the scope of the model is *not* to study normative issues. In this case we would run into difficulties since both countries could subsidize R&D to increase growth rates to infinity, due to the specification of the R&D process.

time, trade patterns will change from exporting the human capital intensive good to exporting the high-tech good.

Subsidies to human capital accumulation

Again, one can imagine different ways, how the government could try to foster the accumulation of human capital. A natural suggestion could be to subsidize factor rewards for human capital. One could think that higher rewards for human capital should lead to an increasing share of human capital allocated to the education sector. The individuals' optimizing problem, however, leads to the same optimal behaviour as with no factor reward subsidies. Thus, the aggregate behaviour of the economy does not change and such a policy would fail. An intervention which does influence human capital accumulation rates, the one we will investigate here, consists in changing the productivity of the education sector which is represented by the parameter δ in human capital accumulation constraint (8). The basic assumption in the following is that this productivity can be increased by investment of the government into the education sector. We will undertake a formal analysis for a two-country world with human capital accumulation *and* technological change. Section 6 will then discuss what happens if technological progress is neglected.

Productivity of the education sector is expressed by $\delta\left(1+d^j\right)$, given that the government decides to support the education sector by subsidies d^j. The human capital accumulation constraint (8) then becomes

$$\dot{H}^j = H^j\delta\left(1+d^j\right)\left(1-\mu^j\right),$$

and the optimality condition concerning the allocation of human capital to the production sector (6) changes to

$$\frac{\dot{w}_H^j}{w_H^j} = r^j - \delta\left(1+d^j\right). \tag{32}$$

Factor rewards can still be explicitly computed and we obtain factor price equalization in w_K and w_H. Now, however, factor price equalization does *not* imply interest rate equalization, since education subsidies might differ between countries. Consequently, in contrast to (21), the innovation rate now depends on country *j*'s interest rate r^j

$$\iota^j = (1-\theta)\frac{\left(n^j\right)^{\frac{1-\theta}{\theta}} L^j}{\left(n^A\right)^{\frac{1-\theta}{\theta}} L^A + \left(n^B\right)^{\frac{1-\theta}{\theta}} L^B}\frac{c_L}{\phi} - r^j. \tag{33}$$

What is further needed to discuss the evolution of the two-country world is a relation between interest rates which can be derived from equation (32), by noting that $w_H^A = w_H^B$, as

$$r^A = r^B + \delta\left(1+d^A\right) - \delta\left(1+d^B\right) = r^B + \delta\left(d^A - d^B\right),$$

and the budget constraint of one country which can be obtained by following the same steps as in the last section. One finds for country *A*

$$E^A = c_L\frac{L^A}{L^A + \Omega^{\frac{1-\theta}{\theta}} L^B} + (1-c_L)\frac{\mu^A}{\mu^A + \mu^B\Lambda}.$$

Further take into consideration the ratio of human capital endowments and the technological advancement variable Ω. Their timepaths are determined by

$$\dot{\Lambda}/\Lambda = \delta\left(d^B\left(1-\mu^B\right)-d^A\left(1-\mu^A\right)+\mu^A-\mu^B\right),$$

$$\dot{\Omega}/\Omega = (1-\theta)\frac{c_L}{\phi}E\left(\frac{\Omega^{\frac{1-\theta}{\theta}}L^B-L^A}{L^A+\Omega^{\frac{1-\theta}{\theta}}L^B}+\delta\left(d^A-d^B\right)\right).$$

Finally, again, we choose total expenditure as numeraire. Now, however, since interest rates are not equalized (provided that education efforts differ: $d^A \neq d^B$), this choice does not imply that expenditure of each country is constant. We now have

$$E^B = 1-E^A, \qquad \frac{\dot{E}^A}{E^A} = r^A-\rho, \qquad \frac{\dot{E}^B}{E^B} = r^A-\delta\left(d^A-d^B\right)-\rho.$$

It can easily be seen that the system behaves just as the two-country world where governments are absent, if both governments undertake the same steps to increase the productivity of their education sector $\left(d^A = d^B\right)$. Now assume that the technologically backward country *A* undertakes a higher education effort than country *B*. Then its interest rate is higher $r^A > r^B$ which implies that its share in world expenditure increases. Higher interest rates, however, have a dampening effect on the innovation rate since future profits are discounted at a higher rate, which reduces the present value of a firm. Consequently, R&D costs must not be as high as with lower interest rates and innovation, by (33), is dampened. As a consequence the widening of the technological gap to the other country is speeded up.

On the other hand, if the technologically leading country undertakes education productivity enhancing activities, it can grasp an ever increasing share of world expenditure and at the same time does not loose its technological lead. The effects on trade patterns are obvious: If an increase of the productivity in the education sector leads to a change of the technological leadership, eventually, also trade patterns will change.

6. Conclusion and Do countries catch up?

There are several mechanisms which can lead to a change of trade patterns. Figure 5 distinguishes between laissez-faire situations in the first column and government intervention in the second. The different variants of the model are in rows one to three. First, trade patterns can reverse if the technologically leading country is initially human capital abundant but, after some time, technological progress overcompensates for human capital abundance. This is represented in the figure by "Initial conditions" and was discussed at the end of section 4. Second, various government interventions can change trade patterns, too. We have seen in section 5, that both R&D subsidies, as well as investment into education can have that effect in a world with both technological change and factor accumulation.

What remains to be discussed is the influence of trade and government intervention on convergence. In figure 5, the possibility of catching up or forging ahead, terms taken from Abramovitz (1986), is indicated by an upwards pointing arrow ↗. Catching up/ forging ahead is understood in this context as an increase of a country's share in total world expenditure. Analogously, falling behind is indicated by an downwards pointing arrow ↘. The third arrow is then self-explaining.

[Insert figure 5 around here]

Section 4 mentioned already, that in a world characterised only by technological change, strong hysteresis effects can be observed and a technologically leading country will preserve and extent forever its technological lead and will gain an ever increasing share of world expenditure. In such a situation government intervention can stop this forging ahead and can induce a catching up process of the technologically backward country. The analysis of government investment in education (section 5) has shown that a country which reaches a higher accumulation rate than its trading partner will increase its share in total world expenditure. Of course, the same result turns out if technological change is neglected. Intervention into R&D activities, as well as no interventions at all, will, in the presence of human capital accumulation, leave expenditure ratios unchanged since interest rates equalize as long as factor prices are equalized.
Finally, a concluding remark concerning preferences. Who says that time preference rates are identical between countries? Neglect technological change for simplicity; then a more patient country (lower ρ) will increase its share in world expenditure. This shift of preferences could perfectly explain the economic success of some Asian countries.

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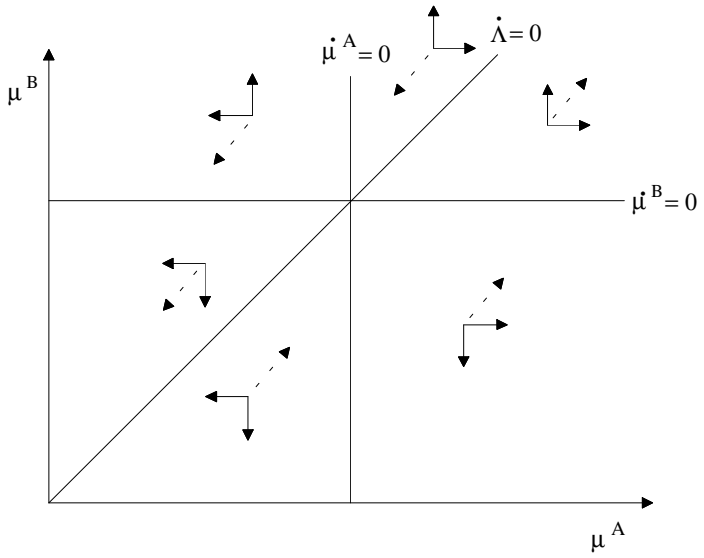

Figure 1: The decision problem with no technological change

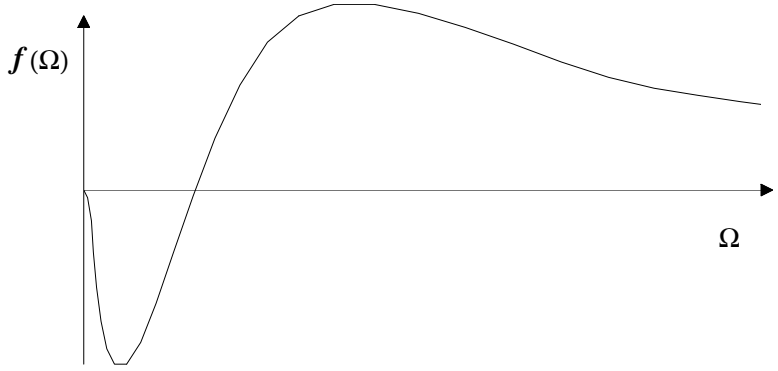

Figure 2: The technology function $f(\Omega)$

Here you should find a graph of a three dimensional phase diagram. Unfortunately, I did not manage to include it here. Please send an email to

kwaelde@econ-theory.uni-kiel.d400.de

with Subject: TPR 3D

and your fax number in the body. I will then immediately fax you the graph.

Or send a fax to Klaus Wälde, Institut für Theoretische Volkswirtschaftslehre
+ 49 / 4 31 / 880 20 72

Sorry for any inconveniences.

Figure 3: A two-country world with technological change

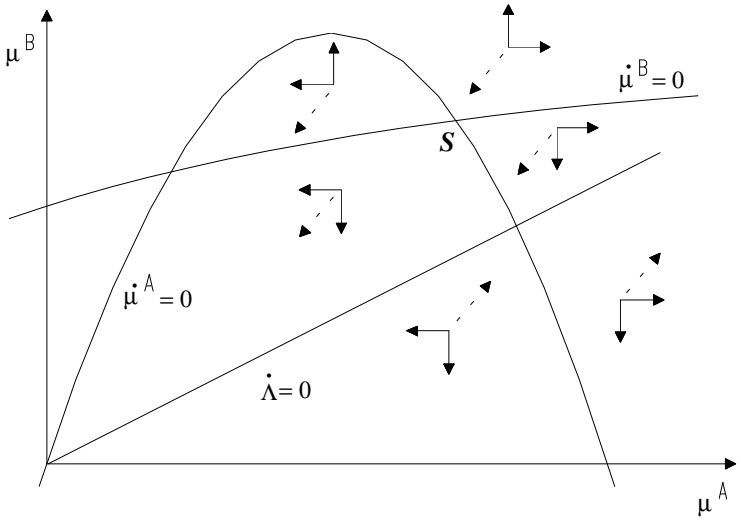

Figure 4: The decision problem with technological change

	Free trade	Government intervention on	
		R&D	Human capital accumulation
Human capital accumulation	No (Section 4) →		Yes ↗
Technological change	Initial conditions (Section 4) ↘ (Section 4)	Yes ↗	
Technological change and human capital accumulation	Initial conditions (Section 4) →	Yes (Section 5) →	Yes (Section 5) ↗ (Section 5)

Trade pattern reversal
↗ Catching up ↘ Falling behind

Figure 5: Trade pattern reversal, catching up and falling behind