Transitional dynamics, convergence and international capital flows in two-country models of innovation and growth

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Global stability properties of dynamic two-country models can be easily studied in the case of international capital flows and simple capital market no-arbitrage conditions. With internationally constant relative productivities, long-run balanced growth path values for factor prices will hold on any equilibrium path unless one country experiences a period of no innovation. Innovation rates converge in the case of perfect international knowledge spillovers but long-run consumption levels and trade patterns are path dependent. The trade balance of the rich country is initially positive but after some time turns into a deficit.

1. Introduction

Issues of international trade are usually analyzed in a long-run equilibrium framework where all trading economies are in a steady state and factor allocations are constant. This is especially true for investigations into dynamic effects of international trade. The usual assumption is that economies are on a long-run balanced growth path (BGP) and all findings on wages, trade patterns, consumption levels or growth rates are then restricted to this possible state of the world. The aim of this paper is to describe the evolution of two trading economies from the moment they begin to trade until they reach their common long-run BGP. We therefore do not restrict our attention to two disconnected points (autarky and trade equilibrium in a static analysis) or periods in time (autarky and trade BGP in a dynamic model) but trace the behaviour of variables at *every* point in time from autarky to a long-run trade equilibrium. Such an approach has the advantage to provide exact conditions for convergence and thereby to suggest possible policy measures to foster a catching-up. It further allows to determine some properties of the BGP, such as consumption levels or trade balances, that can not always be determined by considering the BGP alone. Since such an approach requires a global stability analysis of the model under consideration, this paper adds to a recent series of contributions on transitional dynamics in endogenous growth models. The general motivation underlying those papers is to complement an analysis of the BGP itself by a study of its stability properties since *a priori* it is not clear whether trajectories leading to a BGP exist for all possible, historically given initial conditions. Once this has been established, the task consists in determining properties of those equilibrium paths given that a real world economy is always much more likely to be in transition towards a BGP rather than on the BGP itself. Mulligan (1991) and Mulligan and Sala-i-Martin (1993) were among the first to analyze transitional dynamics in endogenous growth models.

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The basic idea is to consider *ratios* rather than levels of variables[1] which is applied by Mulligan and Sala-i-Martin (1993) to a class of models characterized by the accumulation of two factors of production. The models they consider include e.g. the Lucas (1988) model for which a numerical analysis of global stability behaviour is given. They find that for the parameter values under consideration, the long-run equilibrium is globally saddle path stable, i.e. all variables, independently of initial conditions for state variables, converge monotonically to their BGP trajectories. An analytical analysis of global stability behaviour provided by Caballé and Santos (1993) confirms the global stability results and provides additional outside-BGP findings on the *levels* of capital and human capital. A thorough analytical investigation of the local stability properties by Benhabib and Perli (1994) emphasises, however, that the BGP of the Lucas (1988) model is not necessarily saddle path stable, but can rather be reached, for certain parameter values, by a continuum of paths.[2]

Clearly, every analysis of dynamics of a model faces a trade-off between tractability and the complexity of the model. This paper definitely opts for strong results and therefore takes as a framework the simplest possible model which still has all ingredients of a typical two-country world of innovation and growth. The model which is presented in section two is a two-country version of Grossman and Helpman (1991a, ch. 3.2) that allows for international capital flows, i.e. firms are not restricted to domestic savings for financing their R&D expenditures, a fact that will turn out to be crucial for the analysis. The first analysis of out of BGP behaviour of a trade model of that kind was undertaken by Grossman and Helpman (1990) themselves, in their first paper on the impact of trade on growth. They find that an equilibrium path towards a BGP is characterized by constant relative wages and prices and that the BGP is saddle path stable. As they mention themselves these findings are subject to the condition that innovation rate in both countries are positive during the transition period and therefore conclude that their findings hold in the vicinity of a BGP only. Devereux and Lapham (1994) study the stability properties of the Rivera-Batiz and Romer (1991) model on economic integration and endogenous growth. They focus on the knowledge driven variant of Rivera-Batiz and Romer (1991) which basically is a two-country version of Romer (1990). This implies that an important assumption has to be made about international spillovers of knowledge, as pointed out by Grossman and Helpman (1991a, ch. 8). They show that knowledge spillovers which are restricted to the country where knowledge resulted as a by-product from the R&D activity imply a local instability of the BGP, i.e. innovation rates of countries will never converge. Devereux and Lapham (1994) make a similar point: Since the Rivera-Batiz and Romer model has national spillovers only, it can be shown that a BGP is a knife-edge result which holds only if knowledge stocks (i.e. number of firms active in the market) are identical prior to a trade liberalization. If this condition fails to hold, a country that has a momentary advance in terms of domestic knowledge over its trading partners will end up being the only country doing R&D. Thus the BGP with equal innovation rates in both countries is unstable. What the analysis does not provide, however, is an analysis of the model properties in the case of international knowledge spillovers where global convergence can be expected as the following analysis indicates. This is quite understandable, however, given the complexity of the model.

The main contributions of the present paper are as follows: Section three presents the basic mechanism and economic background why a simple proof of global stability analysis in this

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[1] An approach previously used e.g. in resource economics by Suzuki (1976).

[2] Segerstrom (1994) analyzes the stability properties of the quality ladders model by Grossman and Helpman (1991c).

and related models is possible and presents the proof itself. The fourth section then shows why wages and prices must be constant and, equivalently, why (the failure of) long-run factor price equalization directly translates to the entire equilibrium path. The section proceeds by highlighting the precise mechanism leading to a catching-up of a backward country and the implications for innovation rates and the half-life of convergence. Since these findings are valid for positive innovation rates in both countries during the transition period, an explicit condition for a "zero growth period" of the leading country is derived which is followed by a discussion of what happens in such a situation. The section concludes by focusing on the implications of international capital flows and stresses the path-dependence of long-run expenditure levels, a fact sometimes neglected in the analysis of growth models with international capital flows. The findings on debt accumulation during the transition period are in stark contrast to but more realistic than the prediction of a neoclassical two-country model with international capital flows. A reversal of the trade balance from a deficit to a surplus is shown to be a feature of the catching up process. A natural question to ask is then how robust these findings are to changes. Hence, section five discusses which results carry over to other models and what type of models can be analyzed in the same way as presented here. The focus is on extended versions of the present model, countries with different technologies or preferences and imperfect international knowledge spillovers. Finally, some conclusions are drawn.

2. A typical two-country world of innovation and growth

The model consists of two countries producing one differentiated final good using homogenous internationally immobile labour as input. Countries are assumed to be at or after a certain point in time denoted by t_{trade} when they start to interact actively by trading final goods and financial capital which will give rise to intra-industrial trade and interest rate equalization and passively by experiencing perfect international knowledge spillovers. Since the model is a straightforward extension of Grossman and Helpman (1991a, ch.3) to a two-country world, the description will be short. Utility of a consumer living in country i $(i = A, B)$ at time t stems from a stream of future consumption, discounted at the time preference rate ρ , and is given by $U^i(t) = \int_t^\infty \exp[-\rho(\tau - t)] u^i(\tau) d\tau$, where instantaneous utility $u^i(\tau)$ depends on consumption of varieties k of a differentiated final good x produced in both countries. Letting n^i indicate the "number" of varieties currently produced in country i, instantaneous utility amounts to $u^i(\tau) = \log\left(\int_0^{n^A+n^B} \left(x^k\right)^\alpha dk\right)^{1/\alpha}$, $0 < \alpha < 1$, where the constant elasticity of substitution between any two varieties is given by $\varepsilon = 1/(1-\alpha)$. These preferences imply that demand of country i as a whole for a variety k is a function of prices p^k, of the number of differentiated goods and of current expenditure E^i, $x^k = \left(p^k\right)^{-\varepsilon} E^i \left(\int_0^{n^A+n^B} \left(p^{k'}\right)^{1-\varepsilon} dk'\right)^{-1}$, $k \in \left[0, n^A + n^B\right]$. Optimal allocation of expenditure over time is then derived by reinserting this demand function into the intertemporal utility function and maximizing it with respect to expenditure subject to an intertemporal budget constraint. Aggregating over consumers, the consumption side of economy i as a whole is then given by an equation determining allocation of expenditure,

$$\frac{\dot{E}^i(t)}{E^i(t)} = r(t) - \rho, \tag{1}$$

where $r(t)$ is the world wide interest rate, a budget constraint for each country of the form

$$E^i(t) + \dot{A}^i(t) = r(t)A^i(t) + w^i(t)L^i, \tag{2}$$

which equates current expenditure and change of the value of asset holdings $\dot{A}^i(t)$ to asset and labour income, $r(t)A^i(t)$ and $w^i(t)L^i$, respectively, two transversality conditions, $\lim_{\tau\to\infty} \exp[-\rho\tau]\,A^i(\tau)/E^i(\tau) = 0$, and aggregate world demand for a variety produced in country *i*, given by

$$x^i = \frac{\left(p^i\right)^{-\varepsilon}}{n^A\left(p^A\right)^{1-\varepsilon} + n^B\left(p^B\right)^{1-\varepsilon}}\left(E^A + E^B\right). \tag{3}$$

In expressing the last equation, the fact has been used that, as will turn out when looking at the production side, all varieties in country *i* are equally priced.

The production side is characterized by two activities: Production of varieties for which blueprints have already been developed and development of new blueprints. The production process takes place under constant returns to scale and follows the simplest production function possible, $x^k = L_x^k$, where L_x^k stands for the quantity of labour allocated to the production of variety *k*. Since varieties are imperfect substitutes, each producer has some monopoly power and maximizes profits by charging a price

$$p^i = w^i/\alpha. \tag{4}$$

It is this equation which implies that all varieties produced in country *i* are equally priced. If the outcome of the model implies factor price equalization, then all goods in the world economy have the same price.

The development of a new blueprint requires the allocation of a certain quantity of labour and some knowledge which is regarded as a public good (Romer, 1990). The production function reads $\dot{n}^i = L_R^i Kn^i$, where Kn^i represents the knowledge stock available for firms in country i and L_R^i is the quantity of labour of country *i*, which is allocated to R&D. Knowledge results as a by-product from the R&D activity and is assumed to be proportional to the number of differentiated goods available in the world economy; this amounts to saying that knowledge flows freely between countries. By a suitable choice of units, this relationship can be expressed by $Kn^i = n = n^A + n^B$. It should be noted that this assumption is crucial for the global stability result. We will turn to this point in section 5.

In analogy to static models free entry requires the absence of pure profits. Thus the present value v^i of the future profit stream resulting from the development of a new blueprint (and the subsequent production and selling of the variety) must equal its development costs and the free-entry condition yields, given that innovation takes place,

$$v^i = c_R^i = w^i/n. \tag{5}$$

In a perfect-foresight equilibrium, the value of the firm is determined by its future profits. We obtain

$$v^i(t) = \int_t^\infty \exp\left[-\left(R(\tau) - R(t)\right)\right]\pi^i(\tau)\,d\tau, \tag{6}$$

where profits $\pi^i(\tau) = (1-\alpha)p^i x^i$ are discounted by a cumulative factor $R(u) = \int_0^u r(s)ds$ depending on the interest rate *r*(*s*). Finally, the full employment condition for the factor market requires that demand for labour of the R&D sector and of the production process equals fixed supply,

$$\dot{n}^i/n + n^i x^i = L^i. \qquad (7)$$

3. Global stability of the balanced growth path

Firm values and the reduced form

A simple analysis of global stability behaviour and thereby of wages, consumption, trade patterns and prices outside the BGP is feasible by exploiting interest rate equalization due to international capital flows and the fact that dividend rates are a decreasing function of firm values: Consumers can finance R&D activities of firms in either country. Per period returns they obtain from an investment v^i amount to the sum of capital gains plus dividends, $r\,v^i = \dot{v}^i + \pi^i$, an equation which can be obtained by differentiating (6) with respect to time. The per-period interest rate *r*, which is equalized through free international capital flows is then given by the rate of capital gains $\dot{v}^i/v^i$ plus the dividend rate π^i/v^i. This implies that dividend rates can differ only if the rate of capital gains differ and that only in opposite direction: A lower dividend rate in, say, country *A* must be compensated for by a higher rate of capital gains, otherwise interest rates would not be equalized and all financial capital would flow to country *B*. Computing the rate of change of the relative firm value as $\dot{v}^A/v^A - \dot{v}^B/v^B = \pi^B/v^B - \pi^A/v^A$, shows, however, that differing dividend rates lead to a self enforcing process if dividend rates are decreasing in firm values. The latter holds (which can be established by inserting (3), (4) and (5) into $\pi^i(\tau) = (1-\alpha)\,p^i x^i$) as long as both countries have positive innovation rates at every moment in time which is the case for all initial conditions, e.g. if the backward country is sufficiently small or the leading country sufficiently big. The rest of this and the first half of the next section assume implicitly positive innovation rates in the transition period. We will give a formal condition and a description of what happens if it does not hold at a later point. Thus, if dividend rates differed, the gap between dividend rates would increase further and further and "relative bubbles", differing firm values which are not entirely covered by different firm profits, would occur, which is a contradiction to equilibrium. Thus, we can conclude that dividend rates have to equalize not only on the BGP but everywhere on an equilibrium saddle path. Applied to our example economy, we obtain $\dot{v}^A/v^A - \dot{v}^B/v^B = 0 \Leftrightarrow \left(v^B\right)^{-\varepsilon} - \left(v^A\right)^{-\varepsilon} = 0$ and therefore firm values equalize on every equilibrium saddle path.

We will now use this result to derive a simple reduced form of the model which allows further investigation. Choosing total expenditure $E = E^A + E^B$ as numeraire and setting it equal to unity implies, due to the assumption of internationally completely mobile financial capital, that nominal interest rates are given by the time preference rate, $r = \rho$. We can then solve the factor market clearing condition (7) for $\dot{n}^i/n^i$ and combine it with aggregate demand functions (3), where (4) and (5) and $v \equiv v^A = v^B$ have been inserted. With $n \equiv n^A + n^B$ and $\pi = (1-\alpha)/n$, the reduced form reads[3]

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[3] This specification implicitly rules out an overlap of varieties – that both countries had partially produced identical varieties before opening up to trade – and excludes therefore competition between two firms for market shares for one product.

$$\frac{\dot{n}^i}{n^i} = \frac{n}{n^i} L^i - \frac{\alpha}{nv}, \tag{8}$$

$$\dot{v}/v = \rho - \pi/v. \tag{9}$$

Balanced growth path

The BGP is the only path in this economy which satisfies the transversality conditions. This requires that both countries innovate at the same rate at which firm values fall and that these rates equalize internationally. Given these conditions, the above reduced form of the model implies an innovation rate of $g = (1-\alpha)L - \alpha\rho$, $L = L^A + L^B$, which exceeds the sum of autarky innovation rates $g^i = (1-\alpha)L^i - \alpha\rho$ by $\alpha\rho$. A further property of the BGP, which relates the ratio of numbers of varieties n^A/n^B to endowment by $n^A/L^A = n^B/L^B$ makes clear why aspects of transitional dynamics are of importance in this model: This equation holds only by coincidence and everything that changes the necessary ratio of active firms puts the economy off its BGP. The value of all firms in the world economy is $V = vn^A + vn^B$ and hence the value of assets A held by consumers amounts to $V = A = (L+\rho)^{-1}$ and the value of firms in country i is $V^i = (L+\rho)^{-1} L^i/L$. Note that, as will be discussed more in detail in section 4, it is not possible to determine accumulated savings A^i of a given country by looking at the BGP alone and therefore neither consumption levels nor trade patterns.

Since the value of marginal product of labour in the R&D sector is equalized internationally factor price equalization holds which implies by the price setting behaviour (4) that good prices equalize, as well. Since further the value of firms falls at the same rate as firms enter the market, wages and prices are constant. Demand for a given variety and therefore the size of a firm can then be shown by (3) to fall at the innovation rate g.

Stability analysis of a substitute reduced form

We derive global behaviour by using a substitute reduced form reflecting the evolution of the two-country world in terms of aggregate country firm values. The idea, taken from Mulligan (1991) and Mulligan and Sala-i-Martin (1993), consists in transforming the variables of an original model, whose solution is a BGP, in such a way that the solution of the new model is a steady state (all variables are constant over time). We use as new variables the values of all firms active in country A and B, respectively,

$$V^A = n^A v, \qquad V^B = n^B v. \tag{10}$$

It is obvious that with n^A and n^B growing at g and v falling at g, the sum of firm values in either country are constant if the original model is on a BGP. Since one can also conclude from constant new variables to a BGP of the original model, the new system being in a steady state implies that the original model is on a BGP. Some rearrangements, starting from the reduced form of the original model, given by (8) and (9), then gives the substitute reduced form,

$$\dot{V}^A = \left(V^A + V^B\right)L^A + \rho V^A - \frac{V^A}{V^A + V^B},$$

$$\dot{V}^B = \left(V^A + V^B\right)L^B + \rho V^B - \frac{V^B}{V^A + V^B}, \qquad V^A, V^B > 0. \tag{S}$$

We now simply have to find the zero-motion loci and can then undertake a phase diagram analysis in the (V^A, V^B)-space. The zero-motion loci $\left(\hat{V}^A, \hat{V}^B\right)$ for $\dot{V}^A = 0$ are given by

$$\left(\hat{V}^A+\hat{V}^B\right)L^A+\rho\hat{V}^A-\frac{\hat{V}^A}{\hat{V}^A+\hat{V}^B}=0. \tag{11}$$

In order to get an idea of where these loci can be found in the positive quadrant (firm values cannot become negative), we determine the intersection point of these loci with rays starting from the origin, given by $\hat{V}^B=\theta\hat{V}^A$, where θ ranges from zero to (plus) infinity. Inserted into (11), the intersection points are at

$$(1+\theta)\hat{V}^A L^A+\rho\hat{V}^A-(1+\theta)^{-1}=0\Leftrightarrow\hat{V}^A=(1+\theta)^{-1}\left[(1+\theta)L^A+\rho\right]^{-1}.$$

Thus the zero-motion loci are given as a function of θ as

$$\left(\hat{V}^A,\hat{V}^B\right)=\left(\frac{(1+\theta)^{-1}}{(1+\theta)L^A+\rho},\frac{\theta(1+\theta)^{-1}}{(1+\theta)L^A+\rho}\right).$$

Given this notation, we see that there is one intersection point with the V^A-axis for $\theta=0$ at $\left(\left(L^A+\rho\right)^{-1},0\right)$. Letting θ increase, $\hat{V}^A$ decreases and approaches zero as θ goes to infinity. $\hat{V}^B$, however, is not monotonic in θ. Since the slope of $\hat{V}^B$ at $\theta=0$ is positive it increases first and then at a certain point must decrease since – apply L'Hôpital's rule – $\hat{V}^B$ approaches zero as θ goes to infinity. Thus we can unambiguously draw the zero-motion loci determined by (11) as in figure 1. In an analogous way the zero motion loci for $\dot{V}^B=0$ can be determined. Note, that we did not make any statements whether the maxima of the zero-motion loci (with respect to V^A and V^B, respectively) are to the left or to the right of the intersection point of zero-motion lines. This, however, is of no importance for the stability properties of the system and can thus safely be ignored.

(Insert figure 1 around here)

We can now return to our original question. Assume that two countries begin to trade with each other which are of different size. In the context of the present model, this means that the country with a larger resource base, say, country *A*, has a higher autarky innovation rate than the other country. As long as both countries do not trade, the lead of country *A* over country *B* will continuously be extended and the ratio of differentiated goods will steadily increase and deviate further and further from its trade-BGP value $n^A/n^B=L^A/L^B$. The question we want to answer is: Is it possible to find for every initial ratio of differentiated goods n^A/n^B an initial values *v* such that the two-country world finds itself on the stable saddle path an thus converges to its long-run BGP?

Since history dictates initial conditions for n^A and n^B the ratio of aggregate country firm values V^A to V^B is equally given by history and is represented in figure 1 by a dashed ray starting from the origin and denoted by *OR*. The bigger the advance of country *A* over country *B* in terms of differentiated goods, the flatter the ray *OR* and vice versa. Now the question of global stability is equivalent to asking whether we can find a suitable firm value such that initial conditions for V^A and V^B always lie on the stable saddle path (point *I*). This, however, is no problem. Varying *v* means moving up and down on *OR*, thus *v* has simply to be chosen such that initial conditions are indeed given by point *I*. Thus, independently of the initial quantities of differentiated goods in country *A* and country *B*, the world economy always converges to its long-run BGP. The system is globally saddle path stable.

4. Wages, trade patterns and consumption in the transition period

Wages, firm size and the catching-up mechanism

The last subsection has shown that, given positive innovation rates for both countries, both the transition period and the BGP are characterized by an equalization of firm values. Due to perfect international knowledge spillovers, the value of marginal product of labour in the R&D sector equalizes which implies that factor price equalization carries over to the transition period, as well. Since then all firms face identical production costs, all varieties are equally priced (4) which in turn implies by (3) that the firm size equalizes across countries. Factor and good prices are constant since the decrease of the output value of labour in the R&D sector is compensated by the increase in the productivity of labour due to the increase of knowledge available for R&D, whereas the plant size decreases by (3) as new competitors enter the market. This establishes a general finding on factor prices or prices in general: Long-run BGP properties directly carry over to the equilibrium saddle path converging towards the BGP, given the impossibility of "relative bubbles" as explained at the beginning of section 3 and internationally constant labour productivity ratios, i.e. productivity of labour must change in both countries at the same rate, which means in the present model perfect international knowledge spillovers.

These properties of the stable saddle path exclude that the catching-up process is driven by a R&D cost advantage. R&D costs are given by (5), factor rewards divided by the domestic knowledge stock. Both quantities are independent of the country, however, given free international flows of capital and knowledge. In order to explain catching-up, other reasons have therefore to be evoked why in the lagging country more factors are allocated to the R&D sector. Looking at the factor-market clearing condition (7) shows that the number of factors in the production sector increases in the number of firms active in that country. Since plant size equalizes internationally, the country that has more labour per firm (divide (7) by n^i) has more factors available for the R&D process and therefore a higher innovation rate. Since this is the case for the laggard, convergence in innovation rates takes place[4].

Innovation rates during transition

A further interesting implication of "firm value equalization" is that the world as whole is permanently on a BGP which means that the rate at which new goods are developed is constant all during and after the transition period. To see this most clearly add the budget constraints of both countries given by (2), divide by world assets $A = A^A + A^B$, insert (5), and observe that $E = 1$ and $r = \rho$, to obtain a world budget constraint which reads

$$\frac{\dot{A}}{A} = \rho + L - \frac{1}{A}. \tag{12}$$

We know from the analysis of the BGP that in the long-run the value of all firms in the world economy is given by $V = (L+\rho)^{-1}$. Since this value has to equal the value of world assets, the long-run equilibrium value of accumulated world savings A is given by that value, too. Equation (12), however, tells us, that this equilibrium value can only be reached if total wealth is constant at every moment in time and given by $A = V = (L+\rho)^{-1}$. Since the value

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[4] An immediate implication is that convergence as observed here can take place only if the R&D sector can carry out the "buffer stock" function sketched above. One condition is that labour moving from the production sector is instantaneously capable of acquiring all the skills necessary for doing the research task, which is a reasonable assumption only as a first approximation.

of world assets is $A = vn$, the value of the firms v therefore *always* falls at the same rate as the number of varieties in the world n increases. Computing the world innovation rate then shows that this can hold only if it is constant and equal to $g^W = (1-\alpha)L - \alpha\rho$. Thus, though innovation rates in countries differ during the adjustment period, new varieties are always introduced, in the world as a whole, at the constant rate of g^W.

(Insert figure 2 around here)

But how do innovation rates of countries look like? In autarky, the innovation rate of the bigger country is higher which is shown in figure 2 for the period before t_{trade} when countries open up to trade. Then, during the adjustment period, the innovation rate of country *A* and country *B* approach the BGP innovation rate asymptotically. Given the definition of V^A and V^B, the innovation rates of country *A* and *B* are

$$\frac{\dot{n}^A}{n^A} = \frac{\dot{V}^A}{V^A} - \frac{\dot{v}}{v} = \frac{\dot{V}^A}{V^A} + g \quad \text{and} \quad \frac{\dot{n}^B}{n^B} = \frac{\dot{V}^B}{V^B} - \frac{\dot{v}}{v} = \frac{\dot{V}^B}{V^B} + g,$$

respectively. Since by the stability analysis we know that V^A decreases whereas V^B increases, given that country *A* has developed more varieties relative to its labour endowment than country *B*, $\dot{V}^A / V^A$ is negative and $\dot{V}^B / V^B$ is positive. Therefore, the innovation rate of country *A* is always smaller than the long-run innovation rate *g*, whereas the innovation rate of country *B* "overshoots" and approaches its long-run value from above. A natural measure for the speed of convergence is the half-life of the distance from the BGP as given by $n^A / L^A - n^B / L^B$. Generally speaking, the half-life of a process $a(t)$ is the time $T - t$ that is required to reduce $a(t)$ by 50%, implicitly defined by $a(T) = a(t)/2$. Assuming that $a(t)$ falls exponentially at a rate of $\lambda < 0$, the half-life is given by $\ln(2)(-\lambda)^{-1}$. In the present context, we can use the constancy of world assets *A* to derive a closed-form solution for the labour market clearing conditions (8) and can then show that the distance from the BGP falls at a rate equal to the negative root of (8). Half-life of convergence is therefore given by $\ln(2)(\alpha(L+\rho))^{-1}$. Theoretically predicted convergence is therefore the faster the bigger the countries which trade with each other and the higher the time preference rate or the higher the elasticity of substitution between varieties $\varepsilon = 1/(1-\alpha)$. Interestingly, the argument is not simply the faster countries innovate, the faster they converge, since high ρ and high α reduce the world innovation rate. What should be interpreted with care is the impact of the size of countries; this term would appear differently if knowledge spillovers had not been assumed to be perfect. Consequently, it would be misleading to argue that regions of highly populated areas should converge (neither grow) faster than small trading blocs, simply because the condition of perfect knowledge spillovers (apart from other conditions) is not fulfilled.

A "zero-growth" period

So far we have assumed that innovation rates of both countries do not become negative during the transition period. If we focus on the leading country for a moment and undertake the same reasoning about its innovation rate during the adjustment period as we did for the backward country, it is clear that a low labour per firm ratio implies that factors move out of the R&D into the production sector which implies that the growth rate is the lower the lower this per firm endowment. Obviously, the interesting question is: Can there be a period of "zero growth"? Can the innovation rate $g^A(t_{trade})$ be lower as depicted in figure (2) or even become zero for some time[5]? Looking again at the factor market clearing condition (7) shows that the innovation rate equals zero if labour endowment is too small to satisfy demand by firms, $g^A = 0 \Leftrightarrow L^A < n^A x$ which is equivalent to

$$g^A = 0 \Leftrightarrow \frac{n^A}{n^B} \gtrless \frac{L^A}{L^B}\frac{L^B}{L^B - g^W}, \quad \text{if} \quad L^B - g^W \gtrless 0. \tag{13}$$

(Insert figure 3 around here)

The second term after the inequality, $L^B/(L^B - g^W)$, is illustrated in figure 3. It can be seen as a measure for how far the ratio of firms n^A/n^B may exceed its long-run BGP ratio L^A/L^B before zero growth in country *A* sets in. Since g^W equals the labour force allocated to R&D activities in the world as a whole, the possibility of a period of zero growth is based on the condition whether more researchers are active in the world as a whole than country *B*'s labour force. If this were the case, $L^B - g^W < 0$, a zero-growth period by (13) would never set in simply because even if all labour of country *B* were allocated towards catching up, this would not be enough to match the rate at which new firms enter the world economy as a whole. Clearly, such a situation can be imagined only if country *A* begins to trade with a much smaller country. If on the other hand country *B* is not too small, $L^B - g^W > 0$, a zero-growth period becomes the more likely the smaller the world innovation rate. Consider a very small g^W: If country *A* has a slight advance over country *B* and a catching up process sets in, it would become very likely that all innovation is temporarily be undertaken in country *B* and no new firms of country *A* would enter the market. Generally, the probability of a zero-growth period is reduced the bigger country *A*, the more consumers value different varieties (low α) and the more patient they are (low ρ) since this increases the world innovation rate.

The prediction of a zero innovation period for the leading country is a rather extreme one but is a logical consequence of the assumption of perfect international capital flows: If some savings of the leading country were not reinvested in the foreign country but were available domestically for R&D investments, a zero-innovation period would be excluded. We therefore sketch only briefly how a detailed analysis could be carried out and what happens during such a first phase of transition: The wage rates of trading partners are related by country *A*'s factor market clearing condition (7). Since the number of firms producing in country *A* is constant, this relation is static. Further relevant equations are then the factor market clearing equation for country *B* which can be solved for the innovation rate and an equation, derived from (5) and (6) which determines the evolution of wages in country *B* over time. We therefore have a reduced form consisting of one static equation determining w^A, and two differential equations determining n^B and w^B. Since it can be shown that innovation in country *B* must be positive and that wages in the backward country are lower than in the leading country as long as innovation stands still, this period is characterized by a convergence of wage rates and ends with factor price equalization exactly at the point where

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[5] The innovation rate cannot become negative since this would require firms to exit the market which is excluded since firms always provide positive profits. Note that we cannot find an *upper* bound for $g^A(t_{trade})$ since the closer the two countries to the BGP, the higher the innovation rate of country *A*. The upper bound for country *B* is clearly given by its factor endowment.

innovation in the leading country A resumes. We therefore have a similar result to standard trade theory, though with a completely different background: If both countries are incompletely specialized, factor prices equalize, otherwise they do not.

Consumption levels and the Balance of Payments

Let us now turn to the impact of international capital flows on consumption levels, the trade balance and the current account. Solving the budget constraint (2) of a country for $A^i(t)$ yields, by taking the transversality condition into account, the familiar budget constraint which limits the present value of future expenditure at any point in time t by the sum of wealth and the present value of future income,

$$A^i(t)+\int_t^\infty \exp\left[-\int_t^\tau r(s)ds\right]w^i(\tau)L^i d\tau=\int_t^\infty \exp\left[-\int_t^\tau r(s)ds\right]E^i(\tau)d\tau .$$

Employing the fact that on the stable saddle path interest rates, wages and expenditure levels are constant, $r=\rho$, $w^i=\overline{w}^i$, $E^i=\overline{E}^i$, asset holdings of households of country i turn out to be constant, too, $A^i=\rho^{-1}\left(\overline{E}^i-\overline{w}^i L^i\right)$. The level of consumption therefore depends entirely on the value of assets of a country after trade in financial capital is opened up, which are made up of two components: The number of firms which is entirely predetermined by the number of firms active in the economy before capital flows set in and the value of firms after trade in capital is allowed for: $A^i=v^i n^i$. Since at the moment trade in capital is allowed for, a revaluation of all firms takes place (which leads, as we have seen to an equalization of firm values) but no firm shares are internationally traded before new firm entries require financing of their R&D expenditures[6], relative wealth and therefore relative expenditure levels are a function solely of the number of active firms prior to trade in capital: If both countries, before taking up trade in capital have the same ratio of firms to labour endowment (by, say, having already enjoyed knowledge spillovers for a while) and therefore are already on their common BGP, asset holdings will be given by $A^i=(L+\rho)^{-1}L^i/L$ and the expenditure level is $E^i=L^i/L$ and, thus, consumption per capita is equalized. If, on the other hand, countries are very far away from their common BGP, expenditure per capita will never equalize due to international borrowing and lending. What we find in the latter case is the well-known consumption smoothing effect due to the possibility of international lending and borrowing[7]: The expenditure level of both countries is constant but for the backward country B on a higher level – compared to a situation with *no* international capital flows – at t_{trade} but lower in the long-run. Expenditure of the leading country, however, is initially lower but will in the long-run be higher than the no capital flow level.

Clearly, such an effect is not a unique feature of the present model but it illustrates nicely that the trade balance is nothing but a residual to intertemporal consumption and saving decisions as emphasized by the literature on current account dynamics (e.g. Obstfeld 1982, Svensson and Razin, 1983, Persson and Svensson, 1985). Further, the precise mechanism appears more plausible than in a neoclassical model with international physical capital flows where capital is a factor of production. In the latter model GDP per capita instantaneously equalizes due to a jump of debt of the poorer country, say country B, to equalize the value

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[6] I am indebted to Elhanan Helpman for discussion of this point.

[7] An identical mechanism can be observed in the absence of international capital flows when trade in goods affects interest rates and time preference rates differ internationally, as emphasized by Stiglitz (1970).

of marginal products: $Y_{K^A}\left(A^A - D^B, L^A\right) = Y_{K^B}\left(A^B + D^B, L^B\right)$. Starting from this initial level, debt will smoothly increase to reach its long-run steady state level[8] and the trade balance *TB* of the leading country is always negative[9], reflecting income from foreign wealth and slow international capital flows. It is this implausible prediction of instantaneous capital flows which led many authors to introduce explicit adjustment costs to obtain smooth behaviour (e.g. Lipton and Sachs, 1983, Matsuyama, 1987, Sen and Turnovsky, 1989). The present model does not predict instantaneous capital flows even in the absence of adjustment costs since, as argued above, allowing for capital flows leads to a revaluation of firms only and debt is zero at t_{trade}. Afterwards, debt of the backward country *B* $\left(D^B = -D^A\right)$ accumulates slowly due to the trade off between R&D and production (as in the neoclassical model the trade-off between saving and production) which can be seen by considering the identity $A^B = V^B - D^B$, observing that asset holdings of each country are constant and that the value of firms of country *B* approaches its long-run value from below. The prediction with respect to the international distribution of ownership of firms is as follows: With constant wealth A^B and values of firms falling at the world innovation rate g^W, the number of firms owned by households in country *B* increases at g^W. This implies that R&D of the "extra" firms created during the catching-up process in country *B* is entirely financed by households of country *A* and therefore ownership of those firms lies in country *A*. The implications for trade balances can be found by considering country *A*'s Balance of Payments identity, $TB^A = \dot{D}^B - r\,D^B$. Since initially foreign holdings of country *A* which is indebtedness of country *B* is zero but there is a positive flow of financial capital to country *B*, the trade balance of country *A* is positive. After some time, however, when income from foreign wealth $r\,D^B$ exceeds the flow of foreign R&D financing, the trade balance reverses and becomes negative. Hence, in this framework consumption levels are completely delinked from the trade balance. The crucial difference to a neoclassical model which is responsible for that finding is that debt accumulation starts from a zero level which stresses one condition that has to be met before capital flows set in: There must be profitable ventures to be financed which means in the present model that, as emphasized before, factors must move quickly enough from the production sector to the R&D sector.

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[8] Perfect international capital flows lead to an equalization of the per capita capital stock *k* given linearly homogenous production functions. Optimal per capita consumption c^i accumulation paths are given by $c^i = \rho\left(h + a^i\right)$ (Blanchard, 1985, with probability of death $p = 0$), where *h* is the sum of discounted future per capita labour income which is identical for both countries since interest rates and wages are equalized and a^i are per capita asset holdings which equal the difference between domestic capital stock and debt, $a^i = k - d^i$. Since $\dot{c}^i / c^i = r - \rho$, consumption of both countries increases at the same rate and $\left(\dot{h} + \dot{k} + \dot{d}^B\right) / c^A = \left(\dot{h} + \dot{k} - \dot{d}^B\right) / c^B$. Solving this for $\dot{d}^B$ shows $\dot{d}^B > 0$ since by assumption $a^A(\infty) > a^A\left(t_{trade}\right) > a^B\left(t_{trade}\right)$ and therefore $c^A > c^B$.

[9] The trade balance of a country is given by the difference of gross domestic product, consumption and domestic investment. Observing the equalization of per capita GDPs and capital stocks this reads $TB^i / L^i = y - c^i - \dot{k}$. Since $TB^B = -TB^A$, subtracting yields $TB^A / L^A + TB^A / L^B = -c^A + c^B$ which implies $TB^A < 0$.

5. Discussion of the model and possible extensions

In order to keep the analysis tractable, the results of the previous sections have been derived in a model deliberately chosen as simple as possible. The crucial finding which facilitates a proof of global stability is the fact that the dividend rate must be equalized internationally. In the present model this led to an equalization of firm values, which simplified but by no means is necessary for the proof of global stability. This convenient implication holds for other models of that type, e.g. where economies produce both a differentiated and a homogenous good with several factors of production (Grossman and Helpman, 1991a, ch. 7), where long-run growth peters out (Grossman and Helpman, 1989), or where time preference rates differ internationally (Wälde, 1994) and deriving global findings would be straightforward. In the latter case, however, the share in world expenditure of the more patient country would approach unity, whereas the other country would consume less and less. In a model where one country has a comparative advantage in R&D (Grossman and Helpman, 1990) firm values (and therefore factor prices and firm values) do not equalize, but dividend ratios would equalize on the stable saddle path. This then fixes the ratio of firm values (to a value different from unity) which leads to constant relative wages and prices all along the stable saddle path towards the BGP. A model with internationally differing technologies would not predict a constant world innovation rate g^W but a reasonable conjecture of its timepath can be made by taking the reallocation mechanism of labour into consideration which takes place at t_{trade}: The backward country shifts resources to the R&D sector whereas the leading country specializes temporarily in the production of goods. Assuming the leading country is characterized by a comparative advantage in R&D then the transition period is characterized by a lower world innovation rate which approaches its long run value from below. Such a model is also characterized by slow accumulation of foreign debt and path-dependence of long-run per-capita consumption levels. Country innovation rates would behave qualitatively in the same way as in the simple model above.

A counter-example where an easy stability analysis as sketched above is not possible is the model of endogenous product cycles by Grossman and Helpman (1991b): A two-country world is characterized by R&D activities in a technologically advanced[10] northern country and imitation of varieties developed in the north by a southern country. The values of firms equal the discounted sum of future profits; in the south firm profits are discounted at the international interest rate whereas in the north this interest rate is augmented by the probability of being imitated μ to adjust for the risk that firms stop production and distribution of dividends. Since the northern no-arbitrage condition reads $\dot{v}^N / v^N = -\pi^N / v^N + r + \mu$, the change of the ratio of firm values over time becomes a function of the imitation rate which excludes a simple stability analysis.

The analysis above focused on the case of perfect international knowledge flows, as expressed by $Kn^i = n = n^A + n^B$. This can be viewed as one benchmark case compared to the other extreme of domestic spillovers, $Kn^i = n^i$, whereas inbetween there is a continuum of cases with decreasing returns to foreign knowledge, $Kn^i = n^i + \left(n^j\right)^\varepsilon$, $\varepsilon < 1$ or time lags in the dissemination of knowledge (Grossman and Helpman, 1991a, ch.3). The case of spillovers restricted to the economy of origin was analyzed in Grossman and Helpman

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[10] In the sense of a higher productivity of labour in the R&D sector but identical productivity in the production sector.

(1991a, ch.8) where it is shown that convergence in innovation rate is excluded and R&D will be undertaken (apart from a purely coincidental case) in one country only. The case of decreasing returns to foreign knowledge or time lags can be regarded as a generalization of domestic spillovers and implications are similar: The country with a higher knowledge stock when opening up to trade will have a productivity advantage in R&D and will thereby innovate and also increase its knowledge stock faster than the other country. Even if after some time some knowledge will be disseminated internationally and the spillovers between countries are not asymmetric (i.e. the backward country can adopt all the knowledge from the leading country but not vice versa) the lead of the high knowledge country will be ever expanding and no convergence in innovation rates takes place. This means that the benchmark case of purely domestic knowledge represents all the intermediate cases of imperfect international knowledge spillovers. Note that in contrast it is not to be expected that *proportional* international knowledge spillovers, $Kn^i = n^i + \varepsilon n^j$, would lead to these strong hysteresis effects but one would rather find convergence to a common long run-innovation rate which, however, would be lower than in the case of perfect spillovers. A detailed analysis of imperfect knowledge spillovers will be the focus of future research.

6. Conclusion

This paper has studied the behaviour of an economy from the moment it opens up to trade until its long-run BGP. The framework of analysis was provided by a typical two-country model of innovation and growth with international financial capital flows. The global stability analysis which was required was shown to be possible by exploiting interest rate equalization due to perfect international capital flows and the simple structure of the capital market no-arbitrage condition. The crucial property of every stable saddle path leading to a BGP turned out to consist in an international equalization of dividend rates which can be fruitfully applied for the analysis of related models as well. Given this property, long-run factor prices directly carry over to the entire equilibrium saddle path, provided that the productivity of labour relative to the other country does not change over time which requires in the present framework perfect international knowledge spillovers. The innovation rate of the backward country overshoots its long-run BGP value, whereas innovation rates of the leading economy approach the long-run innovation rate from below. The reason for convergence lies in perfect international knowledge spillovers and a frictionless shift of resources out of the production into the R&D sector during the adjustment period caused by a higher labour per firm endowment of the backward country. Cost advances in R&D do not play any role since factor prices equalized at every moment in time. An implication of international capital flows are a non-equalization of long-run levels of consumption indicating the consumption-smoothing aspect of international borrowing and lending. The initially rich country will always have a higher consumption level than the backward country. The accumulation of foreign wealth follows in contrast to a model with directly productive capital a smooth path which implies an initially positive and then negative trade balance of the leading country. Perfect international capital flows can even have the implication that innovation in the rich country comes to a halt for some time since all investments are concentrated in the backward country. Convergence is predicted also for this case. The final section has derived results for other models with perfect international knowledge spillovers but different technologies or preferences and has argued that various types of imperfect international dissemination of knowledge exclude convergence.

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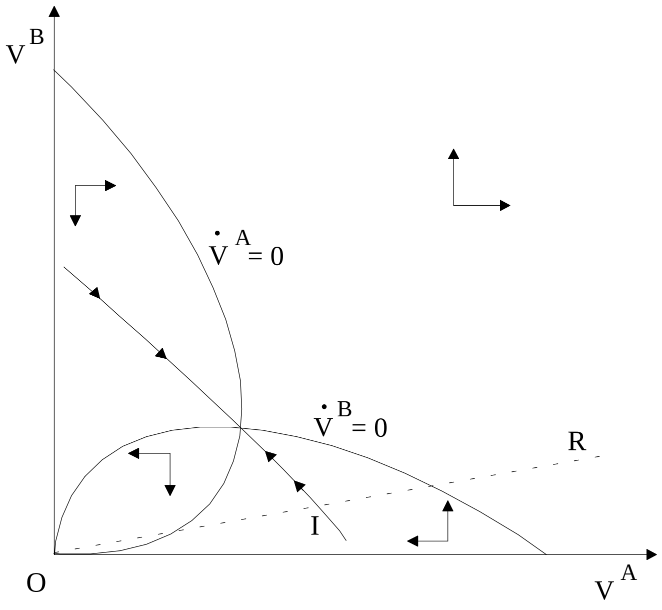

Figure 1: Global saddle path stability of the balanced growth path

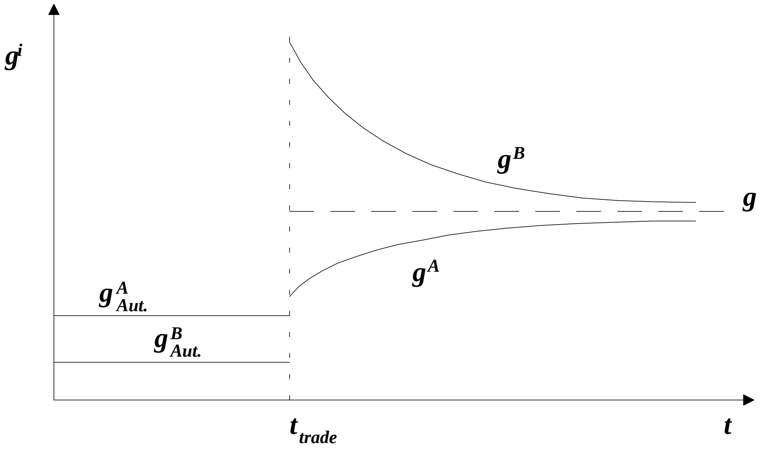

Figure 2: Innovation rates before and after opening up to trade

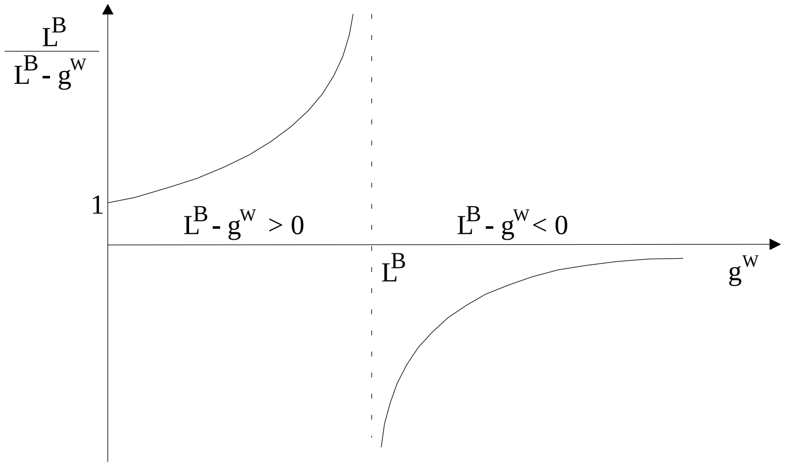

Figure 3: The zero-growth condition