REMOVAL OF PROTECTIONISM, FOREIGN INVESTMENT AND WELFARE IN A MODEL OF INFORMAL SECTOR

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Abstract: The paper develops a three-sector general equilibrium model with two informal sectors with complete mobility of labour between these sectors and with a positive relationship between wage income and labour's efficiency to show that the results relating to foreign capital inflow and removal of protectionism may be counterintuitive to the conventional wisdom. The paper is also devoted to explain why some developing countries implement tariff reforms very slowly compared to others, even after formally choosing free trade as their development strategies, in a more general fashion than the existing tariff-jumping theory.

JEL classification: F10, F13, F21, O17.

Keywords: Foreign capital inflow, tariff reduction, mobility of labour, wage efficiency hypothesis, tariff-jumping theory.
REMOVAL OF PROTECTIONISM, FOREIGN INVESTMENT AND WELFARE IN A MODEL OF INFORMAL SECTOR

1. Introduction:

Until recently, the less developed countries followed a stringent trade policy and adopted an inward-oriented strategy, making use of discriminating policies like tariffs, quotas, restricting free inflow of foreign capital and import of commodities. Only since the conclusion of the multilateral agreement and the formation of the World Trade Organization (WTO) in the Uruguay round of discussions there have been revolutionary changes in liberalizing international trade across countries whether developed or developing. Liberalization involves both inflow of foreign capital as well as reduction of protection of domestic industries and integrating the domestic market with the world market.

It has been observed that some developing countries, notably the non-OECD countries, are relatively slow in carrying out tariff reforms compared to other countries, although they have opted for the policy of free trade as their development strategy and have been able to attract substantial amount of Foreign Direct Investment (FDI) during the last decade. The explanation is provided by the tariff-jumping theory\(^1\) that suggests a positive correlation between the amount of FDI in a country and the tariff rate imposed by it. There is no doubt that the major driving force behind FDI by the multinational corporations (MNCs) in the developing countries is the higher rate of return on their capital in these countries vis-à-vis the international market. Countries with protected domestic markets are likely to attract foreign investment\(^2\), but only for the purpose of jumping the tariff walls and reaping a good harvest by serving their markets directly. On the contrary, reductions of import tariffs imply larger volumes of imports, lower rates of return to capital and smaller amounts of FDI in these countries.

While many developing countries undertake tariff reforms slowly and yearn for foreign capital, the effects of inflow of foreign capital in such economies are, in general, discouraging according to both trade and development theorists. Brecher and Alejandro (1977) have analyzed the welfare effects of foreign capital inflow in a two-commodity, two-factor full employment model; and Khan (1982) has considered a mobile capital Harris-Todaro model with urban unemployment. The important result, common to both is the following: inflow of foreign capital with full repatriation of its earnings is necessarily immiserizing if the import-competing sector is capital-intensive and is protected by a tariff. However, in the absence of any tariff, foreign capital inflow with full repatriation of its earnings does not affect welfare. Here welfare is defined as a positive function of national income.
In the literature, the Brecher-Alejandro proposition has also been re-examined in terms of three-sector models. The third sector may either be a duty-free zone (DFZ) (sometimes called foreign enclave) as in the works of Beladi and Marjit (1992a, 1992b) or it may be an urban informal sector as in the works of Grinols (1991) and Chandra and Khan (1993). Beladi and Marjit (1992a) have shown that with full repatriation of foreign capital income, growth in the foreign capital can lead to immiserization in the presence of tariff-distortion even if the foreign capital is employed in the export sector. This generalizes the main result in the existing literature, which primarily focuses on foreign capital movement in the protected sector of the economy.

Thus according to the conventional view, inflow of foreign capital with full repatriation of foreign capital income and in the presence of tariff protection leads to deterioration in the welfare of a small open economy. This is based upon the presumption that it cuts back the volume of trade further for a small open economy and moves it further away from the free trade situation, which is the optimal policy. An increase in tariff protection of the import-competing sector is also welfare reducing for the same reason.

However, Grinols (1991) in terms of a three-sector specific factor model with an urban informal sector and Harris-Todaro setting has questioned the validity of the Brecher-Alejandro proposition. In Grinols (1991) inflow of foreign capital in the presence of a capital intensive and tariff-protected import-competing sector is not necessarily immiserizing. This is because of an increase in the return to the sector specific input, which may outweigh the increased cost of tariff protection resulting from an expansion of the protected sector. Chandra and Khan (1993) have presented a paper on the three-sector general equilibrium analysis, which offers a set of models based upon a few alternative concepts of the informal sector to investigate the desirability of inflow of foreign capital. They have found that the Brecher-Alejandro proposition holds in general if the urban sector (formal plus informal) is more capital intensive than the rural sector.

Both Grinols (1991) and Chandra and Khan (1993) are based on the Harris-Todaro (1970) framework. Unfortunately, the labour allocation mechanism in a Harris-Todaro type model is not very realistic. Migration of workers from the rural to the urban sectors takes place so long as the expected urban wage rate, which is the weighted average of the urban formal and informal wage rates, is greater than the actual rural wage rate. If migration is cost-less, the informal wage rate lies below both the unionized wage rate of the formal sector and the rural sector wage rate in migration equilibrium. In a static model like this where migration involves no apparent costs and given the security of jobs of the workers employed in the formal sector owing to the presence of trade unions, there is no satisfactory answer to why some of those who are unable to find employment in the urban formal sector do not return to the rural sector and ultimately lead to the equalization of the wage rates.
between these two informal sectors. In other words, the lack of complete labour mobility between the two informal sectors of the economy is not tenable from a theoretical point of view. Secondly, like most of the static trade models the labour endowment is measured in physical units and therefore treated as exogenously given. Thus the role of the Wage Efficiency Hypothesis, as developed by Leibenstein (1957), Mirrlees (1975) and Bliss and Stern (1978), in determining the labour endowment has been ignored. The basic idea of this hypothesis is that the efficiency of a worker is positively related to the wage rate he receives. In a developing economy where a lion's share of the total labour force lives below the poverty line one simply cannot leave out the possibility of the changes in labour's efficiency and hence in the labour endowment of the economy in efficiency unit owing to changes in wage incomes.

The central objective of the present paper is two-fold. First, it shows that in a production structure appropriate for a developing economy there may be cases where one is able to derive results relating to foreign capital inflow and reduction in import tariff, which are counterintuitive to the conventional wisdom. We have developed a three-sector general equilibrium model with two informal sectors where there is complete mobility of labour between these two sectors and assumed a positive relationship between wage income and labour's efficiency. In this scenario, we explore the possibilities of welfare improvement with simultaneous increases in labour and capital endowments of the economy where the latter is caused by inflow of foreign capital while the former is the result of a consequent positive effect on labour's efficiency of an increase in the labour incomes arising from the reallocation of labour among the different sectors of the economy. We shall show that in the presence of labour market distortions, foreign capital inflow may be desirable both in the presence and absence of tariff protection due to its favourable impact on welfare. This result cannot be found in earlier papers in the Harris-Todaro framework with labour market distortions. Again, quite contrary to the popular belief that reduction of tariff leads to an increase in the volume of trade, thereby improving welfare, we have cited the possibility of welfare deterioration even with slashed tariff rate. Secondly, the paper is devoted to explain why some developing countries implement tariff reforms very slowly compared to others, even after formally choosing free trade as their development strategy in a more general fashion than the existing tariff-jumping theory. The tariff-jumping argument is valid only if foreign capital enters into the import-Competing sector of the host country. On the contrary, the present paper shows that the positive correlation between tariff rates and foreign capital flows prevails even if foreign capital comes into the export sector of a developing economy. Moreover, the paper reveals that some countries may endeavour tariff reforms slowly not only because these deter inflow of foreign capital but also because tariff reductions may directly lead to deterioration of the welfare of these economies by lowering domestic factor incomes and thus reducing the efficiency of labour.
2. The Model:

We consider a small open economy, with three sectors – two informal and one formal where all the sectors operate at close vicinity. One of the two informal sectors (sector 1) produces an agricultural commodity using capital of type 1 and labour. The other informal sector (sector 2) produces a manufacturing product using capital of type 2 and labour. The formal sector also produces a manufacturing commodity with the help of capital of type 2 and labour. So capital of type 2 is mobile between the two manufacturing sectors and labour is mobile among all the three sectors. But capital of type 1 is specific to sector 1. Let us now assume that labour in the formal sector earns a contractual wage, $\bar{W}$, while the wage rate in the two informal sectors, $W$, is market determined. Throughout the paper we shall assume that the formal manufacturing sector is more capital (of type 2) intensive than the informal manufacturing sector in value terms i.e. $\lambda_{L3} \lambda_{K2} < \lambda_{L2} \lambda_{K3}$, where $\lambda_{ji}$ is the proportion of the $j^{th}$ factor employed in the $i^{th}$ sector for $j = L, K$ and $i = 2, 3$. Owing to our small open economy assumption we consider all the three product prices to be given internationally. We assume that the formal sector is the import-competing sector of the economy and is protected by a tariff. Production functions exhibit constant returns to scale with diminishing marginal productivity to each factor. The three inputs, labour and the two types of capital, are fully employed. The endowment of labor in physical units is given. The efficiency of the representative worker, $h$, is a positive function of his consumption. All the workers are assumed to possess identical efficiency functions. Thus, there is possibility of changes in labour endowment measured in efficiency units.

The following symbols will be used in the formal presentation of the model:

- $a_{Li}$ = labour-output ratio in the $i^{th}$ sector, $i = 1, 2, 3$;
- $a_{Ki}$ = capital-output ratio in the $i^{th}$ sector, $i = 1, 2, 3$;
- $P_i$ = world price of the $i^{th}$ commodity, $i = 1, 2, 3$;
- $t$ = ad-valorem rate of tariff on the import of commodity 3;
- $r_j$ = return to capital of type $j$, $j = 1, 2$;
- $h$ = efficiency of the representative worker;
- $W$ = wage rate (per efficiency unit) in the two informal sectors;
- $\bar{W}$ = institutionally given wage rate (per efficiency unit) in the formal sector;
- $L$ = labour endowment in physical unit (normalized to unity);
- $K_j$ = stock of capital of type $j$, $j = 1, 2$;
- $\theta_{ji}$ = distributive share of the $j^{th}$ input in the $i^{th}$ industry;
\lambda_{ji} = \text{proportion of the } j^{\text{th}} \text{ input employed in the } i^{\text{th}} \text{ sector, } i = 1, 2, 3; \text{ and, } j = L, K_1, K_2;

"^\wedge" = \text{proportionate change.}

The general equilibrium is represented by the set of following equations.

\begin{align*}
& a_{L1} W + a_{K1} r_1 = P_1 \\
& a_{L2} W + a_{K2} r_2 = P_2 \\
& a_{L3} W + a_{K3} r_2 = P_2 (1+t) \\
& a_{K1} X_1 = K_1 \\
& a_{K2} X_2 + a_{K3} X_3 = K_2
\end{align*}

Now assuming that the Wage Efficiency Hypothesis holds and after normalizing the labour endowment in physical unit to unity, we may write the efficiency of the representative worker, \( h \), as a positive function of the average wage. Thus,

\[ h = h(w); \quad h' > 0 \]

where \( w \) is the average wage of the workers in the economy and is given by

\[ (a_{L1} X_1 + a_{L2} X_2)W + a_{L3} X_3 W = w \]

The reasons for \( h'(w) \) to be positive are explained as follows. Let us first assume that the consumption efficiency hypothesis of Leibenstein (1957) is valid. It states that the nutritional efficiency of the worker varies positively with his level of consumption. We assume all households to be identical so that the proportion of intersectoral labour allocation of each household will be identical to that of the entire economy. So the average wage rate of the workers in the economy is the same as the wage income per member in the representative household. We also assume that the representative household consumes its entire wage income and that consumption is equally distributed among its members. Then each worker consumes an amount equal to the average wage rate in the economy; and using the consumption efficiency hypothesis we can then explain the positive relationship between \( h \) and \( w \).

It should be noted that the validity of the consumption efficiency hypothesis is subject to a lot of criticisms. This is valid only when the level of consumption is very low. However, this hypothesis has been widely used in the theoretical literature on development economics.

The full-employment of labour in efficiency unit implies the following.

\[ a_{L1} X_1 + a_{L2} X_2 + a_{L3} X_3 = h((a_{L1} X_1 + a_{L2} X_2)W + a_{L3} X_3 W) \]
There are eight endogenous variables in the system: \( W, r_1, r_2, X_1, X_2, X_3, h \) and \( w \). The system possesses the decomposition property. So the factor prices are determined independently of the output system. As \( W \) is given, \( r_2 \) is found from equation (3). Given \( r_2 \), equation (2) can be solved to get \( W \). Once \( W \) is known, the equilibrium value of \( r_1 \) is determined from (1). If factor prices are known the factor coefficients, \( a_{ij} \), are also known. Equations (4), (5) and (8) can be solved to get the product-mix. The average wage, \( w \) is obtained from (7). Finally, the efficiency of each worker, \( h \), is found from equation (6).

Before going to comparative statics, it is important to mention that our measure of welfare in this small open economy is national income measured at world prices, \( Y \), which is expressed as follows.

\[
Y = W (a_{l1}X_1 + a_{l2}X_2) + \bar{W}a_{l3}X_3 + r_1K_1 + r_2K_2 - tP_3X_3
\]  

(9)

In equation (9), \( W(a_{l1}X_1 + a_{l2}X_2) \) gives the total wage income in the two informal sectors of the economy. \( \bar{W}a_{l3}X_3 \) is the amount of the wage income of the labourers employed in the formal sector. \( r_1K_1 \) and \( r_2K_2 \) are the rental incomes from capital of types 1 and 2, respectively. Finally, \( tP_3X_3 \) measures the cost of tariff protection of the import-competing sector. **It should be pointed out that even if one measures welfare using a quasi-concave social welfare function, which captures both the demand side and supply side distortions, the qualitative results of the paper remain unaffected.**

### 2.1 Effects of Foreign Capital Inflow and Tariff Reduction on Welfare

According to the conventional wisdom inflow of foreign capital in a developing economy leads to deterioration in its welfare while a reduction in tariff protection is welfare improving. However, in this paper we are interested to reanalyze the impact of foreign capital inflow (of either type) and / or a reduction in import tariff on the welfare of a small open economy in the presence of perfect labour mobility between the two informal sectors and endogenous determination of labour efficiency.

Total differentials of (1), (2) and (3) and use of envelope conditions yield

\[
\theta_{l1} \hat{W} + \theta_{k1} \hat{r}_1 = 0 \tag{1.1}
\]

\[
\theta_{l2} \hat{W} + \theta_{k2} \hat{r}_2 = 0 \tag{2.1}
\]

\[
\theta_{k3} \hat{r}_2 = T \hat{t} \tag{3.1}
\]

where \( T = (t/1+t) \). From (3.1), (2.1) and (1.1) it respectively follows that
\[ \hat{r}_2 = \frac{T \hat{i}}{\theta_{K3}}; \]
\[ \hat{W} = -\left( T \frac{\theta_{K2}}{\theta_{L2}} \theta_{K3} \right) \hat{i}; \text{ and,} \]
\[ \hat{r}_1 = \left( T \frac{\theta_{L1}}{\theta_{K1}} \frac{\theta_{K3}}{\theta_{L2}} \theta_{K3} \right) \hat{i} \]  \hspace{1cm} (10)

From (10) it follows that \( \hat{r}_2, \hat{r}_1 < 0 \) if \( \hat{i} < 0 \). So a reduction of import tariff lowers the rates of return of both types of capital. If inflow of foreign capital and the rate of return on capital in the host country are related positively the following proposition follows immediately.

**PROPOSITION 1:** Tariff reforms may act as deterrent to foreign capital inflow either when it enters into the import-competing sector or into the export sector of a developing economy.

It should be noted that in the present paper a positive correlation between inflow of foreign capital and tariff rate holds even if foreign capital enters into the export sector of a developing economy. The existing *tariff-jumping theory* fails to explain the relationship in this situation.

To derive the effects of foreign capital inflow and tariff reforms on the output composition, after differentiating equation (5) and using (10) one can obtain \(^4\) the following expression.

\[ \lambda_{K2} \hat{X}_2 + \lambda_{K3} \hat{X}_3 = T (\lambda_{K2} \sigma_2 + \lambda_{K3} \theta_{L3} \sigma_3) \hat{i} / \theta_{K3} + \hat{K}_2 \]  \hspace{1cm} (11)

Again differentiation of (4) and (8) and use of (10) yield \(^5\)

\[ \lambda_{L2} (1 - Wh') \hat{X}_2 + \lambda_{L3} (1 - Wh') \hat{X}_3 \]
\[ = -\left( T \frac{\theta_{K2}}{\theta_{L2}} \theta_{K3} \right) (\left( (\lambda_{L1} \sigma_1 / \theta_{K1}) + \lambda_{L2} \sigma_2 \right) (1 - Wh') + h' W (\lambda_{L1} + \lambda_{L2}) \]
\[ + \lambda_{L3} (1 - Wh') \sigma_3 (\theta_{L2} \theta_{K3} / \theta_{K2}) \hat{i} - \lambda_{L3} (1 - Wh') \hat{K}_1 \]  \hspace{1cm} (12)

Solving (11) and (12) by Cramer's rule one gets the following \(^6\).

\[ \hat{X}_2 = \frac{1}{\hat{p}_{h'}} \left[ \lambda_{K3} \lambda_{L1} (Wh' - 1) \hat{K}_1 - (1 - Wh') \lambda_{L3} \hat{K}_2 - \{ A \lambda_{K3} + B \lambda_{L3} (1 - Wh') \} \hat{i} \right] \]  \hspace{1cm} (13)

and,

\[ \hat{X}_3 = \frac{1}{\hat{p}_{h'}} \left[ (1 - Wh') \lambda_{L3} \hat{K}_2 + (1 - Wh') \lambda_{L1} \lambda_{K2} \hat{K}_1 + \{ B \lambda_{L2} (1 - Wh') + A \lambda_{K2} \} \hat{i} \right] \]  \hspace{1cm} (14)

where, \( A = \left( T \frac{\theta_{K2}}{\theta_{L2}} \theta_{K3} \right) (\lambda_{L1} \sigma_1 / \theta_{K1} + \lambda_{L2} \sigma_2) (1 - Wh') + h' W (\lambda_{L1} + \lambda_{L2}) \]
\[ + \lambda_{L3} (1 - Wh') \sigma_3 (\theta_{L2} \theta_{K3} / \theta_{K2}) \);
\[ B = (T / \theta_{K2})(\lambda_{K2}\sigma_2 + \lambda_{K3}\theta_{I3}\sigma_3) > 0; \text{ and,} \]
\[ \|\lambda_h\| = (1 - W'h')\lambda_{L2}\lambda_{K3} - (1 - Wh')\lambda_{L3}\lambda_{K2} \]  

(15)

We now state and prove the following proposition.

**PROPOSITION 2**: There is possibility of welfare improvement owing to foreign capital inflow in either of the two broad sectors of the economy in the presence of tariff if \( W'h' \geq 1 \) and \( \|\lambda_h\| > 0 \). In the absence of tariff, foreign capital is welfare improving iff \( \|\lambda_h\| > 0 \).

Proof: Suppose that foreign capital flows to the informal agricultural sector. Thus \( K_1 \) increases with \( K_2 \) and \( t \) remaining unchanged, i.e. \( \hat{K}_1 > 0 \) and \( \hat{K}_2 = \hat{t} = 0 \).

Differentiating (9) with respect to \( K_1 \) one can obtain the following expression.\(^7\)

\[
(dY/dK_1) = \left(h\lambda_{L1}\lambda_{L3}\lambda_{K2} / \|\lambda_h\| K_1\right)(W - W) - \left(tP_3.X_3 / K_1\right)\lambda_{L1}\lambda_{K2} \]  

(16)

Now since \( W > W' \), \( (dX_3 / dK_1) < 0 \) and \( (dY/dK_1) > 0 \) if (i) \( W'h' \geq 1 \) and (ii) \( \|\lambda_h\| > 0 \). Note that if \( \|\lambda_h\| > 0 \), \( (dY/dK_1) > 0 \) even in the absence of tariff protection.

Let us now consider the case where owing to foreign capital inflow \( K_2 \) increases but \( K_1 \) and \( t \) remain unchanged. i.e. \( \hat{K}_2 > 0 \), \( \hat{K}_1 = \hat{t} = 0 \).

Differentiating (9) with respect to \( K_2 \) we can derive the following.

\[
(dY/dK_2) = Wa_{L2}(dX_2 / dK_2) + W\lambda_{L3}(dX_3 / dK_2) - tP_3(dX_3 / dK_2) \]

With the help of (13) and (14) it may be rewritten as

\[
(dY/dK_2) = Wa_{L2}(1 - W'h')(\lambda_{L3}X_2 / \|\lambda_h\|K_2) + W\lambda_{L3}(1 - Wh')(\lambda_{L2}X_3 / \|\lambda_h\|K_2)
\]

\[ - \left(tP_3X_3 / K_2\right)\lambda_{L2} \]

\[ = (\lambda_{L2}\lambda_{L3}h / \|\lambda_h\|K_2).\left(W - W'\right) - \left(tP_3X_3 / K_2\right)\lambda_{L2}\left(1 - W'h'\right)\lambda_{L2} \]  

(17)

Here also \( (dY/dK_2) > 0 \) if (i) \( W'h' \geq 1 \) and (ii) \( \|\lambda_h\| > 0 \). Again \( (dY/dK_2) > 0 \) even in the absence of tariff protection iff \( \|\lambda_h\| > 0 \). Q.E.D.
Both equations (16) and (17) can be interpreted in terms of the labour-reallocation effect and the output (of the formal sector) effect. Owing to foreign capital inflow of either type, labour reallocation among the three sectors of the economy takes place affecting the average wage income of the workers. For example if the highest wage-paying formal sector expands at the cost of a lower wage-paying informal sector, the average wage income of the workers, which also happens to be their aggregate wage income (as the labour endowment in physical unit has been normalized to unity) increases and as a consequence the welfare of the economy measured by the national income at world prices also goes up. This may be called the labour-reallocation effect, which is captured by the first term in the right-hand side of either (16) or (17). Again, inflow of foreign capital of either type produces a change in the output composition of the economy. As the import-competing formal sector is protected by a tariff its expansion or reduction also raises or lowers the distortionary costs of protection and hence the welfare. We may call it the output (of the formal sector) effect and is captured by the second term in the right-hand side of each (16) and (17).

Let us now interpret the two sufficient conditions under which \( (dY/dK_1) \) and \( (dY/dK_2) \) are positive. As the efficiency of each worker, \( h \), is a positive function of the average wage income of the workers, \( w \), an increase in employment of labour (in efficiency unit) in any sector given the employment levels in other sectors raises aggregate and hence average wage income of the workers, which in turn leads to an increase in labour endowment in efficiency unit. For example, if one additional efficiency unit of the labour input is employed in sector 2, the average wage rate, \( w \), increases by \( W \) that in turn raises the labour endowment (in efficiency unit) by the amount, \( h'.W \). Thus instead of one unit, the availability of labour to the other two sectors actually decreases by only \( (1 - h'.W) \) efficiency unit. Now if \( h'.W > (\leq) 1, (1 - h'.W) < (\geq) 0 \). In other words, \( h'.W > (\leq) 1 \) implies that the employment of labour by one additional efficiency unit in sector 2 (or in sector 1) raises (leaves unchanged) the availability of labour (in efficiency unit) to the rest of the economy.

On the other hand, \( |\lambda_h| = (1 - W.h').\lambda_{4,2} h_{K3} - (1 - W.h').\lambda_{4,3} h_{K2} > 0 \) implies that \( \lambda_{K3} / (1 - W.h').\lambda_{L3} > \lambda_{K2} / (1 - W.h').\lambda_{L2} \) i.e. the formal manufacturing sector (i.e. sector 3) is more capital-intensive vis-à-vis the informal manufacturing sector (i.e. sector 2) even when the effect of changes in average wage income on the labour endowment (in efficiency unit) is taken into account.

Now depending upon the different values of \( W.h' \) and \( W.h' \) one can conceive of the following three cases.
Case I: $W.h'.> W.h'. > 1$ and $|\lambda_h| > 0$ holds. In this case from (14) it follows that $\hat{X}_3 < 0$ when $\hat{K}_1, \hat{K}_2 > 0$. Then from (16) and (17), $(dY/dK_1), (dY/dK_2) > 0$ trivially follow.

Case II: $W.h'. > 1 > W.h'$. It means that $(1 - W.h') < 0 < (1 - W.h')$. So $|\lambda_h| > 0$ and $\hat{X}_3 > 0$. However, $(dY/dK_1)$ and $(dY/dK_2)$ can be still positive if the labour reallocation effects are stronger than the respective output effects (of the formal sector).

Case III: $1 > W.h'. W.h'$. Thus $0 < (1 - W.h') < (1 - W.h')$. As sector 3 is more capital intensive than sector 2, we have $|\lambda_h| > 0$. From (14) it then follows that $\hat{X}_3 > 0$ when $\hat{K}_1, \hat{K}_2 > 0$. In this case also $(dY/dK_1)$ and $(dY/dK_2)$ can be positive.

Proposition 2 can be intuitively explained as follows. Since the system possesses the decomposition property factor prices and hence factor coefficient remain unaltered owing to foreign capital inflow of either type. Thus if foreign capital is of type 1 the informal agricultural sector (sector 1) expands. This requires more labour, which is to be released by the other sectors. Now as the informal manufacturing sector (sector 2) is more labour intensive than the formal manufacturing sector (sector 3) with respect to capital of type 2, the latter expands while the former contracts due to Rybczynski effect. On the contrary, if foreign capital is of type 2, owing to foreign capital inflow the formal sector expands and the informal manufacturing sector contracts. As $\overline{W} > W$ owing to reallocation of labour among different sectors, the average wage income rises which in turn raises the efficiency of the workers and hence the labour endowment of the economy in efficiency unit. This produces another Rybczynski effect, which works in the opposite direction of the previous one. The net effect would be a contraction of the formal sector if (i) $W.h'. > 1$ and (ii) $|\lambda_h| > 0$. This would produce a favourable effect on the welfare since the volumes of trade would now be higher and hence the protectionary cost of tariff would be lower. There would be another positive effect on the economy's welfare due to an increase in aggregate wage income that results from a reallocation of labour among the different sectors of the economy. The latter effect would exist because of the presence of imperfections in the labour market for the formal sector.

Thus in the presence of imperfections in the formal sector labor market, foreign capital inflow in either of the two broad sectors of the economy is welfare improving even in the absence of any protectionary policy, which is the consequence of reallocation of employment among the different sectors of the economy. One cannot obtain such a result in a Harris-Todaro type of model despite the presence of imperfections in the labour market for the formal manufacturing sector. This is due to an interesting property implied by the Harris-Todaro labour migration framework. In such a framework the average wage of the workers in the economy is the rural wage rate and hence there does not exist
any possibility of an increase in the total wage income resulting from any reallocation of employment unless the rural wage rate rises. Beladi, Marjit and Frasca (1998) have discussed this point in details in the context of analyzing the impact of inflow of foreign capital on national income in a small open economy.

A pertinent question in this context is which type of capital (type 1 or type 2) is more desirable to make foreign direct investment improve welfare. To provide an answer to this question after subtracting (17) from (16) we get the following expression.

\[
[(dY/dK_1) - (dY/dK_2)] = [h.λ_{L3}((W - W)/λ_1)].[(λ_{L4}/λ_{K1}) - (λ_{L2}/λ_{K2})] - tP_3.[(dX_3/dK_1 -
(dX_3/dK_2))]
\]

Using (14) it can be easily rewritten as

\[
[(dY/dK_1) - (dY/dK_2)] = [h.λ_{L3}((W - W)/λ_1)].[(λ_{L4}/λ_{K1}) - (λ_{L2}/λ_{K2})]
- [tP_3.(1 - W h')/λ_1].[(λ_{L4}/λ_{K1}) - (λ_{L2}/λ_{K2})] \quad \text{(note that } λ_{K1} = 1) 
\]

\[
= [λ_{L2}/λ_1].h.λ_{L3}((W - W) - tP_3.X_1.(1 - W h')].[(λ_{K2}/λ_{L2})/(λ_{K1}/λ_{L1})]
- (K_1/K_2) \quad \text{(18)}
\]

From (18) it follows that \((dY/dK_1) > (dY/dK_2)\) if (i) \(|λ_1| > 0\); (ii) \(W h' ≤ 1\); and, (iii) \((λ_{K2}/λ_{L2})/(λ_{K1}/λ_{L1}) > (K_1/K_2)\). This establishes the following proposition.

**Proposition 3:** For improving the welfare of the economy inflow of foreign capital of type 1 is preferable to inflow of capital of type 2 if (i) \(|λ_1| > 0\); (ii) \(W h' ≤ 1\); and, (iii) \((λ_{K2}/λ_{L2})/(λ_{K1}/λ_{L1}) > (K_1/K_2)\).

According to proposition 3 under the above three sufficient conditions inflow of foreign capital into the export sector of the economy is more desirable compared to the alternative situation where it flows into the two manufacturing sectors including the import-competing sector.

We are now interested to analyze the effect of tariff reduction on the import of the formal sector's product upon the welfare of the economy. Totally differentiating equation (9) and after using (4), (10), (13) and (14) one can derive the following expression.
\[ Y. \hat{Y} = \left( h.T. \hat{t} / \theta_{k2}.\theta_{k3}.\theta_{k1} \right) \left[ W.\theta_{k2}.\{ \lambda_{l1}.(\sigma_1 - \theta_{k1}) + \lambda_{l2}.\theta_{k1}.(\sigma_2 - 1) \} + \overline{W}.\lambda_{l3}.\sigma_3.\theta_{l2}.\theta_{k3}.\theta_{k1} \right] \\
+ \left( T/\theta_{k3} \right) \left[ (r_1.K_1.\theta_{l1}.\theta_{k2}/\theta_{k1}.\theta_{l2}) + r_2.K_2 \right] \hat{t} \\
+ \hat{t}.(h / \lambda_h) \left[ B.\lambda_{l2}.\{ \lambda_{l3}.(\overline{W} - W) - (tP_3.X_3/h).\left( 1 - Wh' \right) \} + A.\{ \lambda_{k2}.(\overline{W}.\lambda_{l3} - (tP_3X_3/h)) \\
- W.\lambda_{l2}.\lambda_{k3} \} - (tP_3X_3.\left| \lambda_h \right| / h) \right] \] (19)

From (19) it is evident that

\[ \hat{Y} < 0 \text{ when } \hat{t} < 0 \text{ under the following sufficient conditions:} \]

(i) \( C \geq 0 \), where \( C = [W.\theta_{k2}.\{ \lambda_{l1}.(\sigma_1 - \theta_{k1}) + \lambda_{l2}.\theta_{k1}.(\sigma_2 - 1) \} + \overline{W}.\lambda_{l3}.\sigma_3.\theta_{l2}.\theta_{k3}.\theta_{k1}] \)

(ii) \( \left| \lambda_h \right| > 0 \); and,

(iii) \( D \geq 0 \), where \( D = [B.\lambda_{l2}.\{ \lambda_{l3}.(\overline{W} - W) - (tP_3.X_3/h).\left( 1 - Wh' \right) \} + A.\{ \lambda_{k2}.(\overline{W}.\lambda_{l3} - (tP_3X_3/h)) \\
- W.\lambda_{l2}.\lambda_{k3} \} - (tP_3X_3.\left| \lambda_h \right| / h) \]

So we can now establish the following proposition.

**PROPOSITION 4:** Reduction of tariff protection may have an adverse impact on welfare if (i) \( C \geq 0 \), (ii) \( \left| \lambda_h \right| > 0 \); and (iii) \( D \geq 0 \).

We should note that if the production functions of sectors 1 and 2 are of the Cobb-Douglas type condition (i) stated in proposition 4 is automatically satisfied as \( \sigma_1, \sigma_2 = 1 \). In that case \( \hat{Y} < 0 \) when \( \hat{t} < 0 \) if (i) \( \left| \lambda_h \right| > 0 \); and, (ii) \( D \geq 0 \). So we have the following corollary.

**COROLLARY 1:** If the production functions of sectors 1 and 2 are of the Cobb-Douglas type, Proposition 4 holds if (i) \( \left| \lambda_h \right| > 0 \); and if (ii) \( D \geq 0 \).

Proposition 4 may be explained as follows. A reduction of import tariff, \( t \), affects the welfare of the economy by affecting the aggregate factor income and the cost of tariff protection of the import-competing sector. As \( t \) is lowered, the rate of return on capital of type 2, \( r_2 \), falls, which in turn raises the informal sector wage rate, \( W \), to satisfy the zero profitability condition in sector 2. As \( W \) rises the rate of return on capital of type 1, \( r_1 \), falls. The wage-rental ratios in all the three sectors rise forcing the producers to adopt more capital-intensive techniques of production. As sector 1 uses a
specific factor an increase in $a_{K1}$ implies a fall in $X_1$. On the other hand, sectors 2 and 3 both use capital of type 2. Given $X_2$ and $X_3$ adoption of more capital intensive techniques means a shortage of capital of type 2 leading to a contraction of sector 3 and an expansion of sector 2 as sector 3 is more capital intensive vis-à-vis sector 2 in value terms. As the labour-output ratios in all these three sectors fall both sectors 1 and 3 would employ less labour (in efficiency unit) while sector 2 will now use more than before. However, the two informal sectors together would now absorb more labour. As the formal sector wage rate, $\bar{W}$, is greater than the informal sector wage rate, $W$, total wage income of the workers would have surely fallen if $W$ were unchanged. But as $W$ has increased it may go in either way. However, total wage income would still fall under the sufficient conditions: (i) $C \geq 0$; (ii) $|\lambda_h| > 0$; (iii) $E \geq 0$ where $E = [\lambda_{L3} \lambda_{L2} B (\bar{W} - W) + A (\bar{W} \lambda_{L3} \lambda_{K2} - W \lambda_{L2} \lambda_{K3})]$; and, (iv) either $C$ or $E$ is non-zero. Aggregate rental income on capital unambiguously falls as rates of return on both types of capital fall. On the contrary, if the formal sector contracts the cost of tariff protection falls, which works favourably on welfare. However, the net effect of all the above factors would be a deterioration in welfare under the sufficient conditions: (i) $C \geq 0$; (ii) $|\lambda_h| > 0$; and, (iii) $D \geq 0$.

Combining propositions 2 and 4 we may write the following proposition.

**PROPOSITION 5:** Sufficient foreign investments are necessary along with reduction in tariff rates for welfare improvement.

It follows from proposition 4 that a slash in the tariff rate may have adverse welfare effects. However, even with no tariff protection, foreign capital inflow may improve welfare as shown in proposition 2. Hence if an economy strives for liberalization by opening up avenues conducive to freer international trade, withdrawal of protection is an effective policy, provided it is supplemented by a massive foreign investment to overcome the bottlenecks of the inefficient and vulnerable unprotected firms.

3. **Concluding Remarks:**

Developing countries have been vigorously implementing trade liberalization policies for the last decade and a half. However, many developing countries are implementing their tariff reforms at slower rates than the average rate, although these are successful in attracting substantial amount of foreign direct investment (FDI). The present paper provides explanation as to why they are walking behind others with respect to the pace of tariff reforms, more satisfactorily than the existing tariff-jumping theory. It shows that foreign capital inflow may be desirable from the welfare perspective of the developing countries if there exist two distortions rather than one in the markets: a commodity market distortion in the form of tariff and a labour market distortion in the form of unionized wage,
and also if the efficiency of labour is positively related to the average wage in the economy. Besides, the paper has pointed out that a reduction of tariff may act as a deterrent to FDI even if it flows into the export sector of the economy. Moreover, tariff reduction also lowers welfare directly by reducing the rental income from domestic capital stock as well as the labour income through reallocation of labour among the different sectors of the economy.

However, we need not be too much worried about the static effects of implementing free trade, which are likely to prove costly for the developing countries. In the absence of protection, the inefficiency of the inward-oriented firms will be exposed, and their survival will be difficult unless they adjust radically to the new environment. However, they expect to benefit from the new opportunities generated by the dynamic effects of free trade. Indeed, estimates indicate that further reforms coupled with more domestic and foreign investment could make the benefits of free trade outweigh its costs and actually increase welfare. Lifting protection is expected to make firms behave more efficiently and adapt to international requirements. The dampening effects of removal of protectionism on FDI can be overcome by other liberalization policies like simplification of repatriations laws, reduction of obligatory export requirements, opening up of hitherto unexposed sectors to the MNCs etc. These policies, if successfully undertaken, are expected to stimulate FDI, which is crucial. FDI in the export sector shoots up exports and in the import-competing sector reduces total imports. Expansion of the export sector is expected to increase employment, since in the LDCs this sector is generally the more labour-intensive one. The end result will depend on the net outcome of destruction and creation effects of tariff reduction and stimulation of foreign investment.

**Footnotes:**

1  See for example, Massimo (1992) and Noriyuki (1990) for details.
2  Although the supply of foreign capital in an economy is positively related to the rate of return on capital in the host country the actual amount of foreign capital that is allowed to go into a developing economy in many cases is directly regulated by its government. In the process of liberalization the governments of these countries are allowing more and more foreign capital to enter into their economies. See Marjit (1994) in this context.
3  In a static general equilibrium model like this the application of the Wage Efficiency Hypothesis (WEH), one may think, is somewhat restrictive since it assumes an instantaneous relationship between the wage income (and hence consumption) and the worker's efficiency. It is more plausible to consider that the level of consumption of a worker in a particular period influences his nutritional efficiency more in the future than in the current period. However, the existing literature (see for example, Mirrlees (1975), Stiglitz (1976), Bliss and Stern (1978), Dasgupta and Ray (1986)) on WEH considers a one-period world and hence assumes an instantaneous
adjustment between the level of consumption of the worker and his efficiency. Besides, in a
developing economy where the majority of the labour force lives below the poverty line and
earns wage income less than or just equal to the subsistence level, there must be some impacts of
changes in labour incomes on their efficiency even in the same period.

- See appendix I for detailed derivations.
- For detailed derivation see appendix I.
- This has been derived in appendix I.
- This has been derived in details in appendix II.
- This proposition has been developed following the suggestions of the anonymous referee of this
  journal to whom we are grateful.
- For detailed derivation see appendix III.

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**APPENDIX I:**

Differentiating (4) one gets

\[ \dot{X}_1 = -\hat{a}_{K1} + \dot{K}_1 = -\theta_{L1} \sigma_1 (\hat{W} - \hat{r}_1) + \dot{K}_1 \]

or, \[ \dot{X}_1 = (\theta_{L1} \sigma_1 T \theta_{K2} / \theta_{K1} \theta_{L2} \theta_{K3}) \dot{t} + \dot{K}_1 \] \hspace{1cm} (A.1)

Differentiation of (5) gives

\[ \lambda_{K2} \dot{X}_2 + \lambda_{K3} \dot{X}_3 = -\lambda_{K2} \hat{a}_{K2} - \lambda_{K3} \hat{a}_{K3} + \dot{K}_2 \]

\[ = -\lambda_{K2} \theta_{L2} \sigma_2 (\hat{W} - \hat{r}_2) + \lambda_{K3} \theta_{L3} \sigma_3 \hat{r}_2 + \dot{K}_2 \]

Use of (10) yields

\[ \lambda_{K2} \dot{X}_2 + \lambda_{K3} \dot{X}_3 = T (\lambda_{K2} \sigma_2 + \lambda_{K3} \theta_{L3} \sigma_3) \dot{t} / \theta_{K3} + \dot{K}_2 \] \hspace{1cm} (11)

Differentiation of (8) gives

\[ \lambda_{L1} \dot{X}_1 + \lambda_{L2} \dot{X}_2 + \lambda_{L3} \dot{X}_3 = -\lambda_{L1} \hat{a}_{L1} - \lambda_{L2} \hat{a}_{L2} - \lambda_{L3} \hat{a}_{L3} + h \left[ W \lambda_{L1} (\dot{X}_1 + \hat{a}_{L1} + \hat{W}) + W \lambda_{L2} (\dot{X}_2 + \hat{X}_2) + W \lambda_{L3} (\hat{a}_{L3} + \hat{X}_3) \right] \]
or, \( \lambda_{l_1} \hat{X}_1 (1 - W h^\prime) + \lambda_{l_2} \hat{X}_2 (1 - W h^\prime) + \lambda_{l_3} \hat{X}_3 (1 - W h^\prime) \)

\[
= - \lambda_{l_1} \hat{a}_{l_1} (1 - W h^\prime) - \lambda_{l_2} \hat{a}_{l_2} (1 - W h^\prime) - \lambda_{l_3} \hat{a}_{l_3} (1 - W h^\prime) + h W \hat{W} (\lambda_{l_1} + \lambda_{l_2})
\]

\[
= \lambda_{l_1} (1 - W h^\prime) \sigma_1 \theta_{K_1} (\hat{W} - \hat{r}_1) + \lambda_{l_2} (1 - W h^\prime) \sigma_2 \theta_{K_2} (\hat{W} - \hat{r}_2) - \lambda_{l_3} (1 - W h^\prime) \sigma_3 \theta_{K_3} \hat{r}_2
\]

\[
+ h W \hat{W} (\lambda_{l_1} + \lambda_{l_2})
\]

\[
= - \lambda_{l_1} (1 - W h^\prime) \sigma_1 (T \theta_{K_2} / \theta_{l_2} \theta_{K_3}) \hat{r}_1 - \lambda_{l_2} (1 - W h^\prime) \sigma_2 (T \theta_{K_2} / \theta_{l_2} \theta_{K_3}) \hat{r}_2
\]

\[
- \lambda_{l_3} (1 - \lambda h^\prime) \sigma_3 T \hat{r} - h W (\lambda_{l_1} + \lambda_{l_2}) (T \theta_{K_2} / \theta_{l_2} \theta_{K_3}) \hat{r}
\]

\[
= -(T \theta_{K_2} / \theta_{l_2} \theta_{K_3}) \hat{r} [(\lambda_{l_1} \sigma_1 + \lambda_{l_2} \sigma_2) (1 - W h^\prime) + h W (\lambda_{l_1} + \lambda_{l_2})]
\]

\[
+ \lambda_{l_3} (1 - \lambda h^\prime) \sigma_3 (\theta_{l_2} \theta_{K_3} / \theta_{K_2})
\]

[Note that \( \hat{a}_{l_1} = - \sigma_1 \theta_{K_1} (\hat{W} - \hat{r}_1); \hat{a}_{l_2} = - \sigma_2 \theta_{K_2} (\hat{W} - \hat{r}_2); \hat{a}_{l_3} = \sigma_3 \theta_{K_3}; \hat{r}_2 \) and \( \sigma_i \) is the elasticity of substitution between capital and labour in the \( i \)th sector, \( i = 1, 2, 3 \).]

Now, after using (A.1) it becomes

\[
\lambda_{l_2} (1 - W h^\prime) \hat{X}_2 + \lambda_{l_3} (1 - \lambda h^\prime) \hat{X}_3
\]

\[
= -(T \theta_{K_2} / \theta_{l_2} \theta_{K_3}) [(\lambda_{l_1} \sigma_1 / \theta_{K_1}) + \lambda_{l_2} \sigma_2] (1 - W h^\prime) - h W (\lambda_{l_1} + \lambda_{l_2})
\]

\[
+ \lambda_{l_3} (1 - \lambda h^\prime) \sigma_3 (\theta_{l_2} \theta_{K_3} / \theta_{K_2}) - \lambda_{l_1} (1 - W h^\prime) \hat{K}_1
\]

After putting (12) and (11) in matrix notation one gets

\[
\begin{bmatrix}
\lambda_{l_2} (1 - W h^\prime) & \lambda_{l_3} (1 - W h^\prime) \\
\lambda_{l_2} & \lambda_{l_3}
\end{bmatrix}
\begin{bmatrix}
\hat{X}_2 \\
\hat{X}_3
\end{bmatrix}
= \begin{bmatrix}
-A \hat{r} - \lambda_{l_1} (1 - W h^\prime) \hat{K}_1 \\
B \hat{r} + \hat{K}_2
\end{bmatrix}
\]

(A.2)

where, \( A = (T \theta_{K_2} / \theta_{l_2} \theta_{K_3}) [(\lambda_{l_1} \sigma_1 / \theta_{K_1}) + \lambda_{l_2} \sigma_2] (1 - W h^\prime) - h W (\lambda_{l_1} + \lambda_{l_2})
\]

\[
+ \lambda_{l_3} (1 - \lambda h^\prime) \sigma_3 (\theta_{l_2} \theta_{K_3} / \theta_{K_2})
\]

and, \( B = (T / \theta_{K_3}) (\lambda_{l_2} \sigma_2 + \lambda_{l_3} \theta_{l_3} \sigma_3) > 0 \).

Solving (A.2) by Cramer's rule one gets

\[
\hat{X}_2 = \frac{1}{|P_h|} [\lambda_{l_3} \lambda_{l_1} (W h^\prime - 1) \hat{K}_1 - (1 - W h^\prime) \lambda_{l_3} \hat{K}_2 - \{A \lambda_{K_3} + B \lambda_{l_3} (1 - W h^\prime)\} \hat{r}]
\]

(13)

and,
\[
\hat{X}_3 = \frac{1}{\lambda_h} [(1 - Wh') \lambda_{L2} \hat{K}_2 + (1 - Wh') \lambda_{L1} \lambda_{K2} \hat{K}_1 + \{B \lambda_{L2} (l - Wh') + A \lambda_{K2}\} \hat{t}] 
\] (14)

where, \(\lambda_h = (1 - Wh') \lambda_{L2} \lambda_{K3} - (1 - Wh') \lambda_{L3} \lambda_{K2}\) (15)

APPENDIX II:

Differentiating (9) with respect to \(K_1\) one obtains
\[
(dY/dK_1) = W \{a_{L1} (dX_1 / dK_1) + a_{L2} (dX_2 / dK_1)\} + \bar{W}a_{L3} (dX_3 / dK_1) - tP_3 (dX_3 / dK_1)
\]
Using (4) and (13) the above expression can be rewritten as
\[
(dY/dK_1) = W[(a_{L1} / a_{K1}) + a_{L2} \lambda_{K3} \lambda_{L1} (Wh' - 1) \frac{X_2}{[\lambda_h K_1]} + \bar{W}a_{L3} (1 - Wh') \frac{\lambda_{L1} \lambda_{K2} X_3}{[\lambda_h K_1]} - tP_3 (dX_3 / dK_1)
\]
\[
= W(a_{L1} / a_{K1}) + \frac{\lambda_{L1} (Wh' - 1)}{[\lambda_h K_1]} W(\lambda_{L2} \lambda_{K3} - \bar{W} \lambda_{K2} \lambda_{L3}) - tP_3 (dX_3 / dK_1)
\]
\[
= \frac{1}{K_1 [\lambda_h] \lambda_{K1} X_1 \{(1 - Wh') \lambda_{L2} \lambda_{K3} - (1 - Wh') \lambda_{L3} \lambda_{K2}\}
\]
\[
+ \lambda_{L1} (Wh' - 1) \lambda_{L2} \lambda_{K3} - \bar{W} \lambda_{K2} \lambda_{L3}) - tP_3 (dX_3 / dK_1)
\]
\[
= \frac{\lambda_{L1} \lambda_{L2} \lambda_{K2}}{[\lambda_h K_1]} W(Wh' - 1) \lambda_{L3} \lambda_{K2} - \bar{W} (Wh' - 1) \lambda_{K2} \lambda_{L3}) - tP_3 (dX_3 / dK_1)
\]

or, \((dY/dK_1) = \frac{\lambda_{L1} \lambda_{L2} \lambda_{K2}}{[\lambda_h K_1]} (\bar{W} - W) - tP_3 (dX_3 / dK_1)
\]

Now using (14) the above expression becomes
\[
(dY/dK_1) = \frac{\lambda_{L1} \lambda_{L2} \lambda_{K2}}{[\lambda_h K_1]} (\bar{W} - W) - (tP_3 X_3 / K_1) (1 - Wh') \lambda_{L1} \lambda_{K2}\) (16)

APPENDIX III:

Differentiating equation (9) the following expression can be obtained.
\[
dY = dW (a_{L1} X_1 + a_{L2} X_2) + W (a_{L1} dX_1 + X_1 da_{L1} + a_{L2} dX_2 + X_2 da_{L2}) + \bar{W} (a_{L3} dX_3 + X_3 da_{L3})
\]
\[
+ K_1 dr_1 + K_2 dr_2 - tP_3 dX_3 - P_3 X_3 dt
\]
So

\[ Y. \hat{Y} = [\hat{W}.a_{l1}.X_1 + a_{l2}.X_2] + \hat{W}.a_{l1}.X_1.(\hat{X}_1 + \hat{a}_{l1}) + \hat{W}.a_{l2}.X_2.(\hat{X}_2 + \hat{a}_{l2}) \]

\[ + \hat{W}.a_{l3}.X_3.(\hat{X}_3 + \hat{a}_{l3}) + r_1.K_1.\hat{r}_1 + r_2.K_2.\hat{r}_2 - tP_3.X_3.\hat{X}_3 - tP_3X_3 \hat{t} \]

\[ = [W.a_{l1}.X_1.(\hat{W} + \hat{a}_{l1}) + W.a_{l2}.X_2.(\hat{W} + \hat{a}_{l2}) + W.a_{l3}.X_3.(\hat{W} + \hat{a}_{l3})] \]

\[ + r_1.K_1.\hat{r}_1 + r_2.K_2.\hat{r}_2 - tP_3.X_3.\hat{X}_3 - tP_3X_3 \hat{t} \]  

(A.3)

Now using (4), (10), (13) and (14)

\[ (\hat{W} + \hat{a}_{l1}) = (T.\theta_{k2}/\theta_{l2}.\theta_{k3}).[\sigma_1/(\theta_{k2}h).\hat{W}].(1/\lambda h).\hat{t} \]  

(A.4.1)

\[ (\hat{W} + \hat{a}_{l2}) = (T.\theta_{k2}/\theta_{l2}.\theta_{k3}).[\sigma_2/(\theta_{k2}h).\hat{W}].(1/\lambda h).\hat{t} \]  

(A.4.2)

and, \( (\hat{W} + \hat{a}_{l3}) = \sigma_3.T.\hat{t} + \{A.\lambda_{k2} + B.\lambda_{k2}[(1-hh').\hat{W}].(1/\lambda h).\hat{t} \]  

(A.4.3)

Using (A.4.1), (A.4.2) and (A.4.3) we may write

\[ [W.a_{l1}.X_1.(\hat{W} + \hat{a}_{l1}) + W.a_{l2}.X_2.(\hat{W} + \hat{a}_{l2}) + W.a_{l3}.X_3.(\hat{W} + \hat{a}_{l3})] \]

\[ = h.T.\hat{t}.[(\lambda_{l1}.\theta_{k2}.W/\theta_{l2}.\theta_{k3}.\theta_{k1}).(\sigma_1 - \theta_{k1}) + (W.\lambda_{l2}.\theta_{k2}/\theta_{l2}.\theta_{k3}).(\sigma_2 - 1) + \hat{W}.\lambda_{l3}.\sigma_3] \]

\[ + \hat{t}.(h/|\lambda h|).[(\lambda_{l3}.\lambda_{l2}.B.(\hat{W} - W) + A.(\hat{W}.\lambda_{l3} - W.\lambda_{l3}.\lambda_{k3})] \]

(A.5)

With the help of (A.3) and (A.5) and after putting the values of \( \hat{r}_1 \) and \( \hat{r}_2 \) from (10) we write

\[ Y. \hat{Y} = (h.T./\theta_{l2}.\theta_{k3}.\theta_{k1}).[W.\theta_{k2}.(\lambda_{l1}.(\sigma_1 - \theta_{k1}) + \lambda_{l2}.\theta_{k1}.(\sigma_2 - 1)) + \hat{W}.\lambda_{l3}.\sigma_3] \]

\[ + \hat{t}.(h/|\lambda h|).[(\lambda_{l3}.\lambda_{l2}.B.(\hat{W} - W) + A.(\hat{W}.\lambda_{l3} - W.\lambda_{l3}.\lambda_{k3})] \]

(A.6)

\[ + (T/\theta_{k3}).[(r_1.K_1.\theta_{l1}.\theta_{k2}/\theta_{k1}.\theta_{l2}) + r_2.K_2].\hat{t} - tP_3.X_3/|\lambda h|).[B.\lambda_{l2}.(1-WW') + A.\lambda_{k2}].\hat{t} \]

\[ - tP_3X_3.\hat{t} \]
\[
= (h.T.\hat{t}/\theta_{K3}\theta_{K1})[W.\theta_{K2}.\{\lambda_{L1}.(\sigma_1 - \theta_{K1}) + \lambda_{L2}.\theta_{K1}.(\sigma_2 - 1)\} + W.\lambda_{L3}.\sigma_3.\theta_{K2}.\theta_{K1}] \\
+ (T/\theta_{K3})[(r_{K1}\theta_{L1}\theta_{K2}/\theta_{K1}\theta_{L2}) + (r_{K2}].\hat{t} \\
+ \hat{t}.(h/|\lambda_h|)[B.\lambda_{L2}.\{\lambda_{L3}.(W - W) - (tP_3.X_3/h).(1-Wh')\} + A.\lambda_{K3}.(W.\lambda_{L3} - (tP_3.X_3/h))] \\
- W.\lambda_{L2}\lambda_{K3} - (tP_3.X_3/\lambda_h/h)] \tag{19}
\]

References:

