Trade Liberalization and Heterogeneous Rates of Time Preference across Countries: A Possibility of Trade Deficits with China

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Abstract

Strategies for trade liberalization in developing countries when time preference rates are heterogeneous across countries are examined in the context of endogenous growth. The paper concludes that the best strategy for a developing country with the higher rate of time preference is generally the strategy of free trade with wielding market power if the country is large enough to wield market power, because all the optimality conditions are satisfied and markets are not distorted. By this strategy, the country generally accumulates current account surpluses, which implies a possibility that China implicitly has taken this strategy.

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I. INTRODUCTION

The trade liberalization in developing countries has been actively studied in the last decade. A belief that trade liberalization promotes growth has been widely held by economists because conventional models predict that openness raises the steady state level of income, and many empirical studies support this conjecture although there are many econometric difficulties to establish an empirical link between trade liberalization and economic performance. Winters (2004) concludes after surveying the recent literature on this issue, e.g. Easterly and Levine (2001) and Dollar and Kraay (2004), that the weight of evidence is quite clearly in the direction that openness enhances growth. Hence, on the whole the conjecture that trade liberalization promotes growth seems true. However, the actual processes of growth through trade liberalization do not seem so clear-cut. For example, if preferences of households are heterogeneous across countries, the link between trade liberalization and economic performance is not as simple as the case of the identical preferences across countries. Owing to some disturbing factors, the actual processes of growth initiated by trade liberalization may not proceed on a straight course but be amalgamation of complex processes. The paper studies these complex processes of growth initiated by trade liberalization in developing countries by examining economic performances of developing countries with and without trade liberalization.

Among many possible disturbing factors, the paper directs its attention to heterogeneity in time preference rate, i.e. a phenomenon that poor countries have the higher rate of time preference, and examines the relation between the higher rate of time preference and strategies for trade liberalization in developing countries on the basis of the framework of endogenous growth. It has been typically argued that people in poor countries have the higher rate of time preference. Importance of this factor is stressed particularly in the literature of environmental economics. Lawrance (1991) concludes that time preference rates have a strong negative correlation with labor income. Cuesta et al. (1997) concludes that there is some evidence of declining discount rates with increasing income based on empirical research in Costa Rica and a review of 14 other empirical studies.[1] Mink (1993) suggests that an inherently short time-horizon of the poor produces environmental degradation.[2]

The reason why the paper focus on the higher rate of time preference in developing countries is that heterogeneity in time preference rates across countries makes international transactions very complex and it will play a very important role when trade liberalization policies are executed in developing countries. Becker (1980) argues that the heterogeneous rate of time preference results in a misery consequence for people with higher rate of time preference such that the whole capital is owned by the most patient household, and thus if the market of a country with the higher rate of time preference is fully opened, it is conjectured that the county loses ownership of the whole capital.

The paper commences its analysis starting from the fact that people in poor countries have the higher rate of time preference, and thus it is not examined in the paper why the poor has the higher rate of time preference. The paper merely examines theoretical consequences of trade liberalization when the rates of time preference are heterogeneous across countries. In addition, in the model in the paper, the factors that generate the large difference of per capita income across countries are put together and described by a single parameter like Parente and Prescott (2000). When examining economic performances of developing countries, the large difference of per capita income across countries can not be ignored and without considering it the effects of trade liberalization on economic performances in developing countries will not be properly evaluated. Prescott (1998) argues that the neoclassical growth model accounts for differences

[1] The arguments over the reason why the poor has the higher rate of time preference are inconclusive. Pender (1996) concludes that credit constraints are the main reason, and some argue that they have the higher rates of time preference because they are poor.

[2] The notion that the poor has the higher rate of time preference is implicitly argued in the broader literature of sustainable development. See e.g. World Bank (1992).

across countries only if total factor productivity differs across countries, and that there are barriers to adopting technologies in developing countries. Another view on this issue is that there is a mechanism that makes some developing countries fall into "poverty traps." Galor (1996) and Deardorff (2001) show that a neoclassical growth model provides an explanation for a "poverty trap," "club convergence," or "twin peaks" in terms of specialization and international trade because the model with diverse initial endowments across countries results in multiple steady states. To abstract these factors, it is assumed in the paper that the combined effects of these factors can be expressed by a single parameter. This parameter is basically same as "the efficiency component of TFP" in Parente and Prescott (2000). Using this parameter, an endogenous growth model is constructed to extract the effects of higher time preference rate in a developing country when the market is opened.

Based on the endogenous growth model, strategies for a developing country with the higher rate of time preference to deal with trade liberalization are examined. One strategy is the protection of trade that has been taken in many countries, and another strategy is the free trade. In addition, the paper examines a third strategy: the strategy of free trade with wielding market power. This strategy is based on the arguments in Sorger (2002) and Ghiglino (2002) to solve the problem raised by Becker (1980). They argue that if a country wields her market power, the results in Becker (1980) does not hold anymore.

The results are previewed as follows. (i) When a developing country is large enough and can wield market power, the best strategy for the developing country is generally the strategy of free trade with wielding market power, because only this strategy can achieve all the optimality conditions and does not distort markets at the same time. This strategy may provide insights into the recent trade behavior of China whose economy may be large enough to wield market power. The large bilateral current account deficit of the U.S. with China has been persisting. The model in the paper predicts that the current account deficit of the U.S. with China will be observed generally if the rate of time preference in China is relatively higher than that in the U.S. and if

China is wielding market power. (ii) When a developing country is not large enough and can not wield market power, it is very difficult to say which strategy is the best because it is impossible for the developing country to wield market power. Nevertheless, the strategy for the developing country may be judged not only from the economic but the political point of view, because the point is whether the situation that the whole capital is owned by foreigners is politically acceptable. (iii) There is a third way if small developing countries with similar preferences can cooperate and integrate their economies. The integrated economy may be large enough to wield combined market power.

There may be one criticism to the analysis in the paper that if the higher rate of time preference is a result of being poor, the heterogeneity in time preference rate will disappear eventually when economies of developing countries grows rapidly owing to opening markets and thus examining the link between trade liberalization and heterogeneous impatience across countries is meaningless. It might be really meaningless if the large difference between developing and developed countries is solely caused by trade protection. However, even if markets are opened fully, the large difference between them will remain owing to barriers to technology or a mechanism of "poverty trap," and thus even if markets are perfectly opened the heterogeneity in time preference rate remains almost as same as before opening markets because the remaining large difference of per capita income makes the heterogeneity of time preference rate remain. Hence examining the link between trade liberalization and heterogeneous impatience across countries does not seem meaningless.

The paper is organized as follows. In section II, a two-country endogenous growth model in which international transactions and heterogeneous time preference rates are incorporated is built, and the basic nature of the model is examined. In section III, three strategies for a developing country with the higher rate of time preference, i.e. the strategy of free trade without wielding market power, the strategy of trade protection, and the strategy of free trade with wielding market power, are examined. In section IV, the three strategies are compared with regard to optimality, market distortion, the level of output, long-run growth rates, and the balance on current account, and the best strategy for the country is examined. Finally some concluding remarks are offered in section V.

II. THE MODEL

1. The basic model

In most endogenous growth models, the rate of time preference is one of the important parameters that determine steady state growth rates. In this sense, many types of endogenous growth models in which international transactions are incorporated may be used for the sake of the analysis in the paper and may lead to the same conclusions. From among various endogenous growth models, however, the paper chooses a model that is examined in Harashima (2004), because this model has the advantage of being free from both scale effects and the influence of population growth.[3] This advantage seems very important when examining economies of both developing and developed countries simultaneously because these countries have very different demographic features.

The production function is assumed to be $Y_t = F(A_t, K_t, L_t)$, where Y_t (≥ 0) is outputs, K_t (≥ 0) is capital inputs, L_t (≥ 0) is labor inputs, and A_t (≥ 0) is knowledge/technology/idea inputs in period t. The model is based on the following assumptions.

Assumptions:

(A1) The accumulation of capital and knowledge/technology/idea is $\dot{K}_t = Y_t - C_t - v\dot{A}_t - \delta K_t$, where $v(>0)$ is a constant and a unit of K_t and $\frac{1}{v}$ of a unit of A_t are produced using the same

[3] See e.g. Jones (1995), Aghion and Howitt (1998), and Peretto and Smulders (2002).

amounts of inputs, and δ is the rate of depreciation.[4]

(A2) Every firm is identical and has the same size, and for any period, $m = \frac{M_t^{\rho}}{L_t} = \text{constant}$ where M_t is the number of firms and $\rho(>1)$ is a constant.

(A3) $\frac{\partial Y_t}{\partial K_t} = \frac{1}{M_t^{\rho}} \frac{\partial Y_t}{\partial(vA_t)}$ and thus $\frac{\partial y_t}{\partial k_t} = \frac{1}{mv}\frac{\partial y_t}{\partial A_t}$.

Assumption (A1) is standard one in the literature of endogenous growth. Assumption (A2) simply assumes that the number of population and the number of firms in an economy are positively related, which seems intuitively natural. In assumption (A3), the paper assumes that returns on investing in K_t and investing in A_t for a firm are kept equal. In addition, it is also assumed in (A3) that a firm that invents a new technology can not obtain all the returns on investing in A_t. This means that investing in A_t increases Y_t but returns of an individual firm that invests in A_t is only a fraction of the increase of Y_t such that $\frac{1}{M_t^{\rho}}\frac{\partial Y_t}{\partial(vA_t)} = \frac{1}{mL_t}\frac{\partial Y_t}{\partial(vA_t)}$. The reason why only a fraction of the increase in Y_t the returns of an individual firm is, is uncompensated knowledge spillovers to other firms.

More specifically, the production function is assumed to have the following functional form: $Y_t = F(A_t, K_t, L_t) = A_t^{\alpha} f(K_t, L_t)$, where $\alpha(0 < \alpha < 1)$ is a constant. Let $y_t = \frac{Y_t}{L_t}$, $k_t = \frac{K_t}{L_t}$, $c_t = \frac{C_t}{L_t}$ and $n_t = \frac{\dot{L}_t}{L_t}$ and assume that $f(K_t, L_t)$ is homogenous of degree one. Thereby $y_t = A_t^{\alpha} f(k_t)$, and $\dot{k}_t = y_t - c_t - \frac{v\dot{A}_t}{L_t} - n_t k_t - \delta k_t$. By assumptions (A2) and (A3),

[4] Hence, like Jones' (1995) non-scale model, A_t, as well as K_t, is produced less as A_t and L_t increase if the usual production function of homogeneous of degree one is assumed.

$A_t = \dfrac{\alpha f(k_t)}{m v f'(k_t)}$ because $\dfrac{\partial y_t}{m v \partial A_t} = \dfrac{\partial y_t}{\partial k_t} \Leftrightarrow \dfrac{\alpha}{mv} A_t^{\alpha-1} f(k_t) = A_t^{\alpha} f'(k_t)$. Since $A_t = \dfrac{\alpha f}{mvf'}$, then $y_t = A_t^{\alpha} f = \left(\dfrac{\alpha}{mv}\right)^{\alpha} \dfrac{f^{1+\alpha}}{f'^{\alpha}}$ and $\dot{A}_t = \dfrac{\alpha}{mv} \dot{k}_t \left(1 - \dfrac{f f''}{f'^2}\right)$.

2. The model in open economies

For simplicity, it is assumed that there are only two countries, i.e., country 1 and country 2 and the parameters as well as population in the two countries are identical except the rate of time preference. For simplicity the growth rate of population is assumed to be zero, i.e., $n_t = 0$. Let the rate of time preference in the country 1 be θ_1 and that in the country 2 be θ_2 where $\theta_1 < \theta_2$. Goods and services and capital are freely traded but labor is immobilized in each country. The production function in the country 1 is $y_{1t} = A_{1t}^{\alpha} f(k_{1t})$, and that in the country 2 is $y_{2t} = A_{2t}^{\alpha} f(k_{2t})$ where y_{it} is outputs, k_{it} is capital inputs and A_{it} is knowledge/technology/idea inputs $(i = 1,2)$ in each country. In the paper, only the case of Harrod neutral technological progress such that $y_{it} = A_{it}^{\alpha} k_{it}^{1-\alpha}$ and thus $Y_{it} = K_{it}^{1-\alpha} (A_{it} L_t)^{\alpha}$ $(i = 1,2)$ is examined.[5]

Because it is presumed that the country 1 is a developed country and the country 2 is a developing country, the production of technologies is assumed to be different between two countries as follows;

Assumptions:

(A4) Only the country 1 produces new knowledge/technology/idea inputs, i.e. $\dot{A}_{2t} = 0$, and the country 2 makes use of knowledge spillovers from the country 1.

(A5) The level of technology in the country 2 is always lower by $q(0 < q < 1)$ than that in the

[5] As is well known, only Harrod neutral technological progress matches the stylized facts presented by Kaldor (1961).

country 1, i.e. $A_{2t} = qA_{1t}$, owing to some kinds of obstacles that prevent adopting higher technologies. The parameter q is assumed to be constant.

The parameter q represents the combined effects of factors that generate the large difference of per capita income across countries. Various factors have been argued over driving forces that generate the large difference. Prescott (1998) argues that the neoclassical growth model accounts for differences across countries only if total factor productivity differs across countries, and that there are barriers to adopting technologies in developing countries. Many economists support this view basically and argue that for some reasons e.g. bad social infrastructure, bad institutions, or resistances to new technologies, the total factor productivity in developing countries is lower than that in developed countries. For example, Hall and Jones (1999) conclude that "social infrastructure," that is an indicator of government anti-diversion policies, explains much of the differences across countries in output per worker. Another important view on this issue is that there is a mechanism that makes some developing countries fall into "poverty traps." Quah (1996) argues that there are "twin peaks" in the empirical distribution of national per capita incomes. Galor (1996) and Deardorff (2001) show that a neoclassical growth model provides an explanation for a "poverty trap," "club convergence," or "twin peaks" in terms of specialization and international trade because the model with diverse initial endowments across countries results in multiple steady states.

The parameter q represents the combined effects of these factors and is basically same as "the efficiency component of TFP" in Parente and Prescott (2000). However, the paper does not ask questions what truly q is and why q is not unity, but simply assumes that q is less than one, constant and thus independent of the rate of time preference. That is, while Parente and Prescott (2000) examines the characteristics of the efficiency component of TFP assuming a homogeneous rate of time preference, the paper examines instead the characteristics of heterogeneous rates of time preference assuming the constant efficiency component of TFP.

Since both countries are free open economies, returns on investments in both countries are kept equal through international arbitration such that $\frac{\partial y_{1t}}{\partial k_{1t}} = \frac{1}{mv}\frac{\partial(y_{1t}+y_{2t})}{\partial A_{1t}} = \frac{\partial y_{2t}}{\partial k_{2t}}$. An increase in A_t enhances outputs in both countries because of knowledge spillovers and thus returns on investing in A_{1t} is described as $\frac{1}{mv}\frac{\partial(y_{1t}+y_{2t})}{\partial A_{1t}}$. Because the equation $\frac{\partial y_{1t}}{\partial k_{1t}} = \frac{\partial y_{2t}}{\partial k_{2t}}$ is always held through international arbitration and because $A_{2t} = qA_{1t}$, the following equations are held: $qk_{1t} = k_{2t}$ and $qy_{1t} = y_{2t}$. Thereby, $A_{1t} = \frac{(1+q)\alpha f(k_{1t})}{mvf'(k_{1t})}$.

Here, the balance of payments is introduced in the model. The balance on current account in the country 1 is τ_t and the balance on current account in the country 2 is $-\tau_t$. For the time being, it is assumed that each country can not control the sequence of τ_t and thus τ_t is treated as an exogenous variable in each country. Later in the paper, the model is modified to ones with alternative mechanisms of determination of the sequence of τ_t.

The optimization problem in the country 1 is;

Max $E_0\int_0^\infty u(c_{1t})\exp(-\theta_1 t)dt$,

subject to

(1) $\dot{k}_{1t} = y_{1t} + \left(\frac{\partial y_{2t}}{\partial k_{2t}} - \delta\right)\int_0^t \tau_s ds - \tau_t - c_{1t} - \frac{v\dot{A}_{1t}}{L_t} - \delta k_{1t}$,

and the optimization problem in the country 2 is;

Max $E_0\int_0^\infty u(c_{2t})\exp(-\theta_2 t)dt$,

subject to

(2) $\dot{k}_{2t} = y_{2t} - \left(\frac{\partial y_{1t}}{\partial k_{1t}} - \delta\right)\int_0^t \tau_s ds + \tau_t - c_{2t} - \delta k_{2t}$,

where $u_{it}(i=1,2)$ is the utility function in each country, L_t is the population, and $\dot{A}_{1t}$ is the increase of A_{1t} by investments in R&D. The accumulated current account balance $\int_0^t \tau_s ds$ mirrors international capital flows owing to current account imbalances, i.e. a country with current account surpluses invest them in the other country. Since $\frac{\partial y_{1t}}{\partial k_{1t}}\left(=\frac{\partial y_{2t}}{\partial k_{2t}}\right)$ are returns on investments, $\left(\frac{\partial y_{1t}}{\partial k_{1t}}-\delta\right)\int_0^t \tau_s ds$ and $\left(\frac{\partial y_{2t}}{\partial k_{2t}}-\delta\right)\int_0^t \tau_s ds$ represent international income receipts on assets or income payments on assets. Hence, $\tau_t-\left(\frac{\partial y_{2t}}{\partial k_{2t}}-\delta\right)\int_0^t \tau_s ds$ is the balance on goods and services in the country 1, and $\left(\frac{\partial y_{1t}}{\partial k_{1t}}-\delta\right)\int_0^t \tau_s ds-\tau_t$ is the balance on goods and services in the country 2. Equations (1) and (2) implicitly assume that at $t = 0$ each country does not have any foreign asset.

Because the production function is Harrod neutral such that $y_{it} = A_{it}^{\alpha} k_{it}^{\ 1-\alpha}$ and thus $Y_{it} = K_{it}^{\ 1-\alpha}(A_{it}L_t)^{\alpha}$, and because $A_{1t} = \frac{(1+q)\alpha f(k_{1t})}{mvf'(k_{1t})}$ and $f = k_{1t}^{1-\alpha}$, then $A_{1t} = \frac{(1+q)\alpha}{mv(1-\alpha)}k_{1t}$, $\frac{f f''}{f'^2} = -\frac{\alpha}{1-\alpha}$ and $\frac{\partial y_{1t}}{\partial k_{1t}} = (1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}$. Since $\frac{\partial y_{1t}}{\partial k_{1t}} = \frac{\partial y_{2t}}{\partial k_{2t}}$, the accumulation of capital in the country 1 proceeds by

$$\dot{k}_{1t} = y_{1t} + \left(\frac{\partial y_{2t}}{\partial k_{2t}}-\delta\right)\int_0^t \tau_s ds - \tau_t - c_{1t} - \frac{v\dot{A}_{1t}}{L_t} - \delta k_{1t}$$

$$= (1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}\frac{f^{1+\alpha}}{f'^{\alpha}} + \left(\frac{\partial y_{1t}}{\partial k_{1t}}-\delta\right)\int_0^t \tau_s ds - \tau_t - c_{1t} - \frac{(1+q)\alpha}{mL_t}\dot{k}_{1t}\left(1-\frac{f f''}{f'^2}\right) - \delta k_{1t}.$$ Hence,

$$\dot{k}_{1t} = \frac{(1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}\frac{f^{1+\alpha}}{f'^{\alpha}} + \left[(1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\int_0^t \tau_s ds - \tau_t - c_{1t} - \delta k_{1t}}{1+\frac{(1+q)\alpha}{mL_t}\left(1-\frac{f f''}{f'^2}\right)}$$

$$=\frac{mL_t(1-\alpha)}{mL_t(1-\alpha)+(1+q)\alpha}\left\{\left[(1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta\right]k_{1t}+\left[(1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\int_0^t\tau_s ds-\tau_t-c_{1t}\right\}.$$

The optimization problem in the country 1 therefore can be rewritten as

Max $E_0\int_0^{\infty}u(c_{1t})\exp(-\theta_1 t)dt$,

subject to

$$\dot{k}_{1t}=\frac{mL_t(1-\alpha)}{mL_t(1-\alpha)+(1+q)\alpha}\left\{\left[(1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta\right]k_{1t}+\left[(1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\int_0^t\tau_s ds-\tau_t-c_{1t}\right\}.$$

Let Hamiltonian H_1 be

$$H_1=u(c_{1t})\exp(-\theta_1 t)$$
$$+\lambda_{1t}\frac{mL_t(1-\alpha)}{mL_t(1-\alpha)+(1+q)\alpha}\left\{\left[(1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta\right]k_{1t}+\left[(1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\int_0^t\tau_s ds-\tau_t-c_{1t}\right\}$$

where λ_{1t} is a costate variable, thus the optimality conditions for the country 1 are

(3) $\dfrac{\partial u(c_{1t})}{\partial c_{1t}}\exp(-\theta_1 t)=\dfrac{[mL_t(1-\alpha)+(1+q)\alpha]}{mL_t(1-\alpha)}\lambda_{1t}$,

(4) $\dot{\lambda}_{1t}=-\dfrac{\partial H_1}{\partial k_{1t}}$,

(5) $\dot{k}_{1t}=\dfrac{mL_t(1-\alpha)}{mL_t(1-\alpha)+(1+q)\alpha}\left\{\left[(1+q)^{\alpha}\left(\dfrac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta\right]k_{1t}+\left[(1+q)^{\alpha}\left(\dfrac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\int_0^t\tau_s ds-\tau_t-c_{1t}\right\}$,

(6) $\lim_{t\to\infty}\lambda_{1t}k_{1t}=0$.

Since $A_{2t}=qA_{1t}$ and thus since $qk_{1t}=k_{2t}$ and $qy_{1t}=y_{2t}$, the accumulation of capital proceeds in the country 2 by

$$\dot{k}_{2t}=y_{2t}-\left(\frac{\partial y_{1t}}{\partial k_{1t}}-\delta\right)\int_0^t\tau_s ds+\tau_t-c_{2t}-\delta k_{2t}$$

$$=\left[(1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta\right]k_{2t}-\left[(1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\int_0^t\tau_s ds+\tau_t-c_{2t}.$$

Hence, similarly, the optimization problem in the country 2 can be rewritten as

Max $E_0 \int_0^{\infty} u(c_{2t}) \exp(-\theta_2 t) dt$,

subject to

$$\dot{k}_{2t} = \left[(1+q)^{\alpha} \left(\frac{\alpha}{mv} \right)^{\alpha} (1-\alpha)^{-\alpha} - \delta \right] k_{2t} - \left[(1+q)^{\alpha} \left(\frac{\alpha}{mv} \right)^{\alpha} (1-\alpha)^{1-\alpha} - \delta \right] \int_0^t \tau_s ds + \tau_t - c_{2t} .$$

Let Hamiltonian H_2 be

$$H_2 = u(c_{2t}) \exp(-\theta_2 t) + \lambda_{2t} \left\{ \left[(1+q)^{\alpha} \left(\frac{\alpha}{mv} \right)^{\alpha} (1-\alpha)^{-\alpha} - \delta \right] k_{2t} - \left[(1+q)^{\alpha} \left(\frac{\alpha}{mv} \right)^{\alpha} (1-\alpha)^{1-\alpha} - \delta \right] \int_0^t \tau_s ds + \tau_t - c_{2t} \right\}$$

where λ_{2t} is a costate variable, and thus the optimality conditions for the country 2 are

(7) $\dfrac{\partial u(c_{2t})}{\partial c_{2t}} \exp(-\theta_2 t) = \lambda_{2t}$,

(8) $\dot{\lambda}_{2t} = -\dfrac{\partial H_2}{\partial k_{2t}}$,

(9) $\dot{k}_{2t} = \left[(1+q)^{\alpha} \left(\frac{\alpha}{mv} \right)^{\alpha} (1-\alpha)^{-\alpha} - \delta \right] k_{2t} - \left[(1+q)^{\alpha} \left(\frac{\alpha}{mv} \right)^{\alpha} (1-\alpha)^{1-\alpha} - \delta \right] \int_0^t \tau_s ds + \tau_t - c_{2t}$,

(10) $\lim_{t \to \infty} \lambda_{2t} k_{2t} = 0$.

3. The basic nature of the model

Before examining the strategy for trade liberalization in the model, the basic nature of the model is examined. To begin with, the transversality conditions are examined. Since the problem of scale effects in endogenous growth models is not a focal point in the paper, it is assumed for simplicity that L_t is sufficiently large and thus $\dfrac{mL_t(1-\alpha)}{mL_t(1-\alpha)+(1+q)\alpha} = 1$ hereafter.

Lemma 1: The transversality conditions (6) $\lim_{t \to \infty} \lambda_{1t} k_{1t} = 0$ and (10) $\lim_{t \to \infty} \lambda_{2t} k_{2t} = 0$ are not satisfied if and only if

$$\lim_{t\to\infty}\left\{\left(\frac{\partial\tau_t}{\partial k_{1t}}-\frac{\tau_t}{k_{1t}}\right)-\left[(1+q)^\alpha\left(\frac{\alpha}{mv}\right)^\alpha(1-\alpha)^{1-\alpha}-\delta\right]\left[\frac{\partial\left(\int_0^t\tau_s ds\right)}{\partial k_{1t}}-\frac{\int_0^t\tau_s ds}{k_{1t}}\right]-\frac{c_{1t}}{k_{1t}}\right\}\geq 0 \text{ or}$$

$$\lim_{t\to\infty}\left\{\left(\frac{\tau_t}{k_{2t}}-\frac{\partial\tau_t}{\partial k_{2t}}\right)-\left[(1+q)^\alpha\left(\frac{\alpha}{mv}\right)^\alpha(1-\alpha)^{1-\alpha}-\delta\right]\left[\frac{\int_0^t\tau_s ds}{k_{2t}}-\frac{\partial\left(\int_0^t\tau_s ds\right)}{\partial k_{2t}}\right]-\frac{c_{2t}}{k_{2t}}\right\}\geq 0 \cdot$$

Proof: See Appendix 1.

By lemma 1, an important nature of the model is shown in the following lemma.

Lemma 2: If and only if $\lim_{t\to\infty}\frac{\dot{c}_{1t}}{c_{1t}}=\lim_{t\to\infty}\frac{\dot{c}_{2t}}{c_{2t}}=\lim_{t\to\infty}\frac{\dot{k}_{1t}}{k_{1t}}=\lim_{t\to\infty}\frac{\dot{k}_{2t}}{k_{2t}}=$ constant, all the optimality conditions are satisfied.

Proof: See Appendix 2.

III. THREE STRATEGIES

1. The strategy of free trade

The strategy of trade liberalization for a developing country is examined in the following sections based on the model built above. To begin with, the strategy of free trade is examined. Taking lemma 2 into consideration, it is highly likely that rational households in both country 1 and country 2 behave so as to achieve a steady state growth path such that $\lim_{t\to\infty}\frac{\dot{c}_{1t}}{c_{1t}}=\lim_{t\to\infty}\frac{\dot{c}_{2t}}{c_{2t}}=\lim_{t\to\infty}\frac{\dot{k}_{1t}}{k_{1t}}=\lim_{t\to\infty}\frac{\dot{k}_{2t}}{k_{2t}}=$ constant. However, we must consider, beforehand, how the sequence of τ_t is determined. In the well-known paper of Becker (1980), it is proved that if households are purely price takers, the most patient household owns all wealth in the conventional Ramsey models if households have heterogeneous rates of time preference. Ghiglino (2002) predicts that it is likely that under appropriate assumptions the results in Becker (1980) still hold in endogenous growth models. Farmer and Lahiri (2004) show that in general, balanced growth equilibria do not exist in a multi-agent economy except for the special case where all agents have the same constant rate of time preference. Their results hold basically in this model, which is shown in the following proposition.

Proposition 1: If each country sets τ_t without regarding the other countries optimality, then if and only if $\frac{\dot{c}_{2t}}{c_{2t}} = \frac{\dot{\tau}_t}{\tau_t} = \lim_{t\to\infty} \frac{\dot{k}_{2t}}{k_{2t}}$, all the optimality conditions for the representative households in the country 2 can be satisfied.

Proof: See Appendix 3.

At first glance, proposition 1 appears to provide a possibility that the country 2 can escape the misery situation predicted in Becker (1980) if condition $\frac{\dot{c}_{2t}}{c_{2t}} = \frac{\dot{\tau}_t}{\tau_t} = \lim_{t\to\infty} \frac{\dot{k}_{2t}}{k_{2t}}$ is satisfied. However, it is extremely difficult that the condition $\frac{\dot{c}_{2t}}{c_{2t}} = \frac{\dot{\tau}_t}{\tau_t}$ is satisfied because $\frac{\dot{c}_{2t}}{c_{2t}}$ and $\frac{\dot{\tau}_t}{\tau_t}$ are exogenously and independently given as shown in the proof, i.e., $\frac{\dot{c}_{2t}}{c_{2t}} = \frac{(1+q)^\alpha \left(\frac{\alpha}{mv}\right)^\alpha (1-\alpha)^{-\alpha} - \delta - \theta_2}{\varepsilon}$ and $\frac{\dot{\tau}_t}{\tau_t} = (1+q)^\alpha \left(\frac{\alpha}{mv}\right)^\alpha (1-\alpha)^{1-\alpha} - \delta$. Only in extremely lucky cases with the combination of exogenous parameters that satisfies the knife-edge condition

$\left(1+q\right)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}\left(1-\alpha\right)^{-\alpha}\left[1-\varepsilon\left(1-\alpha\right)\right]-\left(1-\varepsilon\right)\delta-\theta_2=0$, the condition $\frac{\dot{c}_{2t}}{c_{2t}}=\frac{\dot{\tau}_t}{\tau_t}$ is satisfied, which will be exceedingly rare. As a result, proposition 1 indicates that virtually all the optimality conditions for a representative household in the country 2 can not be satisfied simultaneously. This result corresponds to the well-known result of Ramsey models with exogenous technologies and heterogeneous households shown in Becker (1980).[6] In addition, the following corollary shows that the whole capital in the country 2 will be virtually owned by foreigners in the long-run, which is same as the main conclusion in Becker (1980).

Corollary 1: If each country sets τ_t without regarding the other countries optimality, then if $\left(1+q\right)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}\left(1-\alpha\right)^{-\alpha}\left[1-\varepsilon\left(1-\alpha\right)\right]-\left(1-\varepsilon\right)\delta-\theta_2<0$, the country 2 can not own capital in the long run.

Proof: See Appendix 4.

Because the degree of relative risk aversion ε is generally considered to be much larger than unity, the condition $\left(1+q\right)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}\left(1-\alpha\right)^{-\alpha}\left[1-\varepsilon\left(1-\alpha\right)\right]-\left(1-\varepsilon\right)\delta-\theta_2<0$ may be satisfied usually.[7] Hence people in a country with the higher rate of time preference virtually can not own capital in the long run.[8] As a result, the consequences of this strategy suggest the necessity of the

[6] It is easily shown by modifying the proof of proposition 1 that if there is no heterogeneity in time preference rate and thus if $\theta_1=\theta_2$, the optimal growth path requires that the both countries hold $\tau_t=0$, i.e. the balance on current account should be balanced at any time.

[7] As Lucas (1987) argues, the degree of relative risk aversion in the U.S. may be much higher than 1 but less than 20.

[8] It should be noted that even though the households in the country 2 possess no capital, the capital stock in the

second best strategy other than the strategy of free trade.

Before going to other strategies, the growth rate and the balance on current account when this strategy is taken are examined. Because the growth rates in both country 1 and 2 are determined by the technology progress generated by investments in A_{1t} in the country 1, i.e. $\frac{\dot{A}_{1t}}{A_{1t}}$ and because the steady state growth rate in country 1 is $\lim_{t\to\infty}\frac{\dot{c}_{1t}}{c_{1t}} = \lim_{t\to\infty}\frac{\dot{k}_{1t}}{k_{1t}} = \lim_{t\to\infty}\frac{\dot{y}_{1t}}{y_{1t}} = \lim_{t\to\infty}\frac{\dot{A}_{1t}}{A_{1t}}$,

then the growth rate in the country 2 is $\lim_{t\to\infty}\frac{\dot{y}_{2t}}{y_{2t}} = \lim_{t\to\infty}\frac{\dot{y}_{1t}}{y_{1t}} = \frac{(1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - \delta - \theta_1}{\varepsilon}$.[9] As for the balance on current account, by (Step 2) in the proof of proposition 1, $\tau_t > 0$ if the country 2 takes this strategy, i.e. trade deficits in the country 2.

2. The strategy of trade protection

Because the strategy of free trade is not optimal for the country 2, the country 2 may take some measures to avoid the unacceptable situation that emerges as a consequence of this strategy. A natural choice may be the protection of trade. To examine the strategy of trade protection, the model needs to be modified to allow the situation in case of trade protection and thus the following assumptions are added.

Assumptions: If the country 2 takes the strategy of trade protection,

(A6) the balance on current account τ_t and the inflow of capital are kept zero at any time by

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country 2 is still kept to be $k_{2t} = qk_{1t}$ and thus $y_{2t} = qy_{1t}$. Point is that all the capital in the country 2 is owned by foreigners.

[9] The growth rate is obtained by the relation $\lim_{t\to\infty}\frac{\dot{c}_{1t}}{c_{1t}} = \lim_{t\to\infty}\frac{\dot{c}_{2t}}{c_{2t}}$ and equations (19) and (20) in Appendix. See Harashima (2004, 2005) for more detailed proofs.

the measures to protect trade in the country 2,

(A7) the returns on investing in A_{1t} in the country 1 are described as $\frac{1}{mv}\frac{\partial y_{1t}}{\partial A_{1t}}$,

(A8) the country 2 can utilize fully technology spillovers from the country 1 without compensation.

The assumption (A6) symbolizes the measures of trade protection because, if the trade is not protected, the country 2 suffers permanent trade deficits that are shown in proposition 1, and in this sense, the measures to protect trade will be ones that make trade deficits decrease significantly. The assumptions (A7) and (A8) are based on the conjecture that if the trade is protected, firms in the country 1 can not fully obtain the returns on investing in R&D from the country 2, because, in countries that protect trade, the protection of patent may be also insufficient.[10] Under these assumptions, both countries grow as if they are economically independent, i.e. the country 1 grows endogenously and the country 2 grows by exogenously given technology shocks that are knowledge spillovers from the country 1. As a result, the returns on investments are not necessarily identical in both countries.

The optimization problem in the country 1 therefore can be rewritten as

Max $E_0\int_0^\infty u(c_{1t})\exp(-\theta_1 t)dt$,

subject to

$$\dot{k}_{1t} = \left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - \delta\right]k_{1t} - c_{1t} .$$

Let Hamiltonian H_1 be

[10] These assumptions may appear extreme, but they seem to abstract the situation of trade protection sufficiently and clarify well the difference between the strategies of free trade and trade protection. Even if more complicated assumptions are introduced, it will not change essential results in the paper but only makes analyses less tractable.

$$H_1 = u(c_{1t})\exp(-\theta_1 t) + \lambda_{1t}\left\{\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - \delta\right]k_{1t} - c_{1t}\right\}$$

where λ_{1t} is a costate variable, thus the optimality conditions for the country 1 are

(11) $\dfrac{\partial u(c_{1t})}{\partial c_{1t}}\exp(-\theta_1 t) = \lambda_{1t}$,

(12) $\dot{\lambda}_{1t} = -\dfrac{\partial H_1}{\partial k_{1t}}$,

(13) $\dot{k}_{1t} = \left[\left(\dfrac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - \delta\right]k_{1t} - c_{1t}$,

(14) $\lim_{t\to\infty}\lambda_{1t}k_{1t} = 0$.

For the country 2, it is not necessary to hold $\dfrac{\partial y_{1t}}{\partial k_{1t}} = \dfrac{\partial y_{2t}}{\partial k_{2t}}$ any more owing to the protection of trade. Because activities of technology progress are carried out only in the country 1, the optimality problem in the country 2 is same as the Ramsey model with exogenous technology progress. Since $A_{2t} = qA_{1t}$, then the accumulation of capital proceeds in the country 2 by $\dot{k}_{2t} = y_{2t} - c_{2t} - \delta k_{2t} = (qA_{1t})^{\alpha} k_{2t}^{1-\alpha} - c_{2t} - \delta k_{2t}$ where the technology A_{1t} is an exogenous variable. The optimization problem in the country 2 therefore can be rewritten as

Max $E_0\int_0^{\infty} u(c_{2t})\exp(-\theta_2 t)dt$,

subject to

$\dot{k}_{2t} = (qA_{1t})^{\alpha} k_{2t}^{1-\alpha} - c_{2t} - \delta k_{2t}$.

Let Hamiltonian H_2 be

$$H_2 = u(c_{2t})\exp(-\theta_2 t) + \lambda_{2t}\left[(qA_{1t})^{\alpha} k_{2t}^{1-\alpha} - c_{2t} - \delta k_{2t}\right]$$

where λ_{2t} is a costate variable, thus the optimality conditions for the country 2 are

(15) $\dfrac{\partial u(c_{2t})}{\partial c_{2t}}\exp(-\theta_2 t) = \lambda_{2t}$,

(16) $\dot{\lambda}_{2t} = -\dfrac{\partial H_2}{\partial k_{2t}}$,

(17) $\dot{k}_{2t} = (qA_{1t})^{\alpha} k_{2t}^{1-\alpha} - c_{2t} - \delta k_{2t}$,

(18) $\lim_{t\to\infty} \lambda_{2t} k_{2t} = 0$.

Because the technology A_{1t} is an exogenous variable in this optimality problem for the country 2, $\dfrac{\partial y_{2t}}{\partial k_{2t}} = \theta_2 + \delta$ at steady state by the optimality conditions (15), (16) and (17). The technology progress in the country 1 is assumed to be non-stochastic and thus smooth, and therefore the equation $\dfrac{\partial y_{2t}}{\partial k_{2t}} = \theta_2 + \delta$ holds at any time in the country 2. By this equation, an important feature regarding the ratio of output in the country 2 to that in the country 1 is shown in the following proposition.

Proposition 2: When the country 2 takes the strategy of trade protection, if $\theta_2 > \left(\dfrac{\alpha}{mv}\right)(1-\alpha)^{1-\alpha} - \delta$, then $y_{2t} < qy_{1t}$.

Proof: See Appendix 5.

This result has two important implications. One is that there is a possibility of $y_{2t} > qy_{1t}$ if $\theta_2 < \left(\dfrac{\alpha}{mv}\right)(1-\alpha)^{1-\alpha} - \delta$, i.e. if θ_2 is not so high. Thereby, in this case, the output in the country 2 may grow faster initially if the trade is newly protected than that before the trade is protected. The other is that if θ_2 is higher than a critical point $\left(\dfrac{\alpha}{mv}\right)(1-\alpha)^{1-\alpha} - \delta$, the ratio of output in the country 2 is lower than that in the country 1. Here, the coefficient of negative impact of trade protection Π is defined as $\Pi = \left[\frac{\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}}{\theta_2 + \delta} \right]^{\frac{1-\alpha}{\alpha}}$. If $\Pi = 0$ the output is completely reduced by the protection of trade, and if $\Pi = 1$ the output is not reduced by the protection of trade. It is self-evident that $\frac{d\Pi}{d\theta_2} < 0$ and $\lim_{\theta_2 \to \infty} \Pi = 0$.

Next, the growth rate in this modified model is examined. Because the growth rates in both country 1 and 2 are determined by the technology progress generated by investments in A_{1t} in the country 1, i.e. $\frac{\dot{A}_{1t}}{A_{1t}}$ and because the steady state growth rate in country 1 is $\frac{\dot{c}_{1t}}{c_{1t}} = \frac{\dot{k}_{1t}}{k_{1t}} = \frac{\dot{y}_{1t}}{y_{1t}} = \frac{\dot{A}_{1t}}{A_{1t}}$, then the growth rate in this modified model is

$$\frac{\dot{y}_{2t}}{y_{2t}} = \frac{\dot{y}_{1t}}{y_{1t}} = \frac{\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - \delta - \theta_1}{\varepsilon}.$$ [11]

3. The strategy of free trade with wielding market power

The strategy of free trade does not achieve optimality but the strategy of trade protection distorts markets. Is there a third way for a developing country? Sorger (2002) shows that if a government levies a progressive income tax, or if there are few households of each type and thus they are not simple price takers but play a Nash equilibrium, the results shown in Becker (1980) do not hold anymore. Ghiglino (2002) argues that the latter case in Sorger (2002) can be interpreted as a model of international trade with a common market simply by associating each household's type to a country with a national central planner or a representative household.

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[11] The growth rate is obtained by setting $\tau_t = 0$ in equation (19) in Appendix. See Harashima (2004, 2005) for more detailed proofs.

Based on the arguments in Sorger (2002) and Ghiglino (2002), in the model of two non-small countries with heterogeneous households in the paper, it is possible to assume that each representative household in the two countries play a Nash equilibrium with regard to the sequence of τ_t in the optimization problems described in the previous section. As Sorger (2002) argues, if a household in a country behave as a member of a large group of households and know demand functions in markets, the households can wield market power. As a citizen of a national, a household may behave considering e.g. the sentiment of "Buy American." This kind of nationalistic behavior may have been widely observed in many countries, and may be interpreted as reflecting the behavior of household as a member of a large group of households, i.e. households belong and are loyal to a national, a representative household associated with whom is playing a Nash equilibrium.

If a developing country is large enough like e.g. China or India, it may be possible to wield market power against developed countries.[12] In this situation, i.e. if both country 1 and 2 have market powers each other, it will be possible to assume that households in both countries select a sequence of τ_t and set the initial consumptions so as to achieve a growth path that satisfies all the optimality conditions, because, by lemma 2, if and only if $\lim_{t\to\infty}\frac{\dot{c}_{1t}}{c_{1t}} = \lim_{t\to\infty}\frac{\dot{c}_{2t}}{c_{2t}} = \lim_{t\to\infty}\frac{\dot{k}_{1t}}{k_{1t}} = \lim_{t\to\infty}\frac{\dot{k}_{2t}}{k_{2t}} =$ constant all the optimality conditions in each country are satisfied, which is a condition for a Nash equilibrium.

According to the above arguments, a situation that representative households in both countries have market powers is examined as an alternative strategy for the country 2. It is assumed that for the initial capital stocks $k_{10} = k_{20}$ and knowledge/technology/idea A_0,

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[12] If a developing country is not large enough, even combined households in the country are viewed as an atomic household and merely a price taker by developed countries, and thus the developed countries will not consider the optimality of households in the developing country.

households in both countries select a sequence of τ_t and set the initial consumptions so as to achieve a growth path that satisfies all the optimality conditions, i.e. a growth path of $\lim_{t\to\infty}\frac{\dot{c}_{1t}}{c_{1t}}=\lim_{t\to\infty}\frac{\dot{c}_{2t}}{c_{2t}}=\lim_{t\to\infty}\frac{\dot{k}_{1t}}{k_{1t}}=\lim_{t\to\infty}\frac{\dot{k}_{2t}}{k_{2t}}=$ constant, while firms in both countries adjust k_t so as to achieve $\frac{\partial y_{1t}}{\partial k_{1t}}=\frac{1}{mv}\frac{\partial(y_{1t}+y_{2t})}{\partial A_{1t}}=\frac{\partial y_{2t}}{\partial k_{2t}}$. In this case, it is easily proved by lemma 2 that

$$\lim_{t\to\infty}\frac{\dot{\tau}_t}{\tau_t}=\lim_{t\to\infty}\frac{\frac{d\int_0^t \tau_s ds}{dt}}{\int_0^t \tau_s ds}=\lim_{t\to\infty}\frac{\dot{c}_{1t}}{c_{1t}}=\lim_{t\to\infty}\frac{\dot{c}_{2t}}{c_{2t}}=\lim_{t\to\infty}\frac{\dot{k}_{1t}}{k_{1t}}=\lim_{t\to\infty}\frac{\dot{k}_{2t}}{k_{2t}}=\lim_{t\to\infty}\frac{\dot{y}_{1t}}{y_{1t}}=\lim_{t\to\infty}\frac{\dot{y}_{2t}}{y_{2t}}=\lim_{t\to\infty}\frac{\dot{A}_{1t}}{A_{1t}}=\text{ a positive constant.}$$[13]

What should be uncovered firstly is in which country the balance on current account shows deficits. The following proposition uncovers that, contrary to the strategy of free trade without wielding market power, in many cases the balance on current account in the country 2 shows surpluses and in reverse that in the country 1 shows deficits if the strategy of free trade with wielding market power is taken.

Proposition 3: Suppose that representative households in both countries have market powers. If $\frac{\theta_1+\theta_2}{2}>[1-\varepsilon(1-\alpha)](1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-(1-\varepsilon)\delta$, the balance on current account in the country 1 shows deficits permanently such that $\lim_{t\to\infty}\frac{\tau_t}{y_{1t}}=\text{a negative constant}$, and that in the country 2 shows surpluses permanently such that $-\lim_{t\to\infty}\frac{\tau_t}{y_{2t}}=\text{a positive constant}$.

Proof: See Appendix 6.

[13] See Harashima (2004, 2005) for more detailed explanations.

Because the degree of relative risk aversion ε is generally considered to be much larger than unity, the condition $\frac{\theta_1+\theta_2}{2} > [1-\varepsilon(1-\alpha)](1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - (1-\varepsilon)\delta$ may be satisfied usually.[14] If it is satisfied, the capital in the country 2 is owned by people in the country 2 permanently because the balance on current account in country 2 shows surpluses permanently. Since both $\lim_{t\to\infty}\frac{\tau_t}{y_{1t}}$ and $\lim_{t\to\infty}\frac{\tau_t}{y_{2t}}$ are constant, the ratio of the balance on current account stabilizes in the long-run, i.e. the balance on current account does not explode. Nevertheless, to conclude more strictly that the balance on current account does not explode, it is necessary to show that a supplementary condition $\lim_{t\to\infty}\frac{\left|\int_0^t \tau_s ds\right|}{k_{1t}} < 1$ is satisfied. The following corollary shows in what case the condition $\lim_{t\to\infty}\frac{\left|\int_0^t \tau_s ds\right|}{k_{1t}} < 1$ is satisfied.

Corollary 2: If $-(\theta_1-\theta_2)\varepsilon < \left(1+\frac{1}{q}\right)\left|(1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}[1-(1-\alpha)\varepsilon] - (1-\varepsilon)\delta - \frac{\theta_1+\theta_2}{2}\right|$, then $\lim_{t\to\infty}\frac{\left|\int_0^t \tau_s ds\right|}{k_{1t}} < 1$.

Proof: See Appendix 7.

Finally, like the other strategies, the growth rate when this strategy is taken is examined. Because the steady state growth rate in the country 1 and 2 is

[14] As Lucas (1987) argues, the degree of relative risk aversion in the U.S. may be much higher than 1 but less than 20.

$$\lim_{t\to\infty}\frac{\dot{\tau}_t}{\tau_t}=\lim_{t\to\infty}\frac{\frac{d\int_0^t \tau_s ds}{dt}}{\int_0^t \tau_s ds}=\lim_{t\to\infty}\frac{\dot{c}_{1t}}{c_{1t}}=\lim_{t\to\infty}\frac{\dot{c}_{2t}}{c_{2t}}=\lim_{t\to\infty}\frac{\dot{k}_{1t}}{k_{1t}}=\lim_{t\to\infty}\frac{\dot{k}_{2t}}{k_{2t}}=\lim_{t\to\infty}\frac{\dot{y}_{1t}}{y_{1t}}=\lim_{t\to\infty}\frac{\dot{y}_{2t}}{y_{2t}}=\lim_{t\to\infty}\frac{\dot{A}_{1t}}{A_{1t}}$$

, then the growth rate when the strategy with wielding market power is taken is

$$\lim_{t\to\infty}\frac{\dot{y}_{2t}}{y_{2t}}=\lim_{t\to\infty}\frac{\dot{y}_{1t}}{y_{1t}}=\lim_{t\to\infty}\frac{\dot{c}_{2t}}{c_{2t}}=\lim_{t\to\infty}\frac{\dot{c}_{1t}}{c_{1t}}=\lim_{t\to\infty}\frac{\frac{\dot{c}_{1t}}{c_{1t}}+\frac{\dot{c}_{2t}}{c_{2t}}}{2}=\frac{(1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta-\frac{\theta_1+\theta_2}{2}}{\varepsilon}.$$

IV. DISCUSSION

1. The comparison of strategies

The three strategies for a developing country examined above, i.e. the strategy of free trade without wielding market power, the strategy of trade protection, and the strategy of free trade with wielding market power, bring quite different consequences. The following is the comparison of these consequences (See Table).

(i) The optimality conditions for the country 2

- The strategy of free trade without wielding market power

  Not satisfied

- The strategy of trade protection

  Satisfied

- The strategy of free trade with wielding market power

  Satisfied

By proposition 1, if the strategy of free trade without wielding market power is taken, households in the country 2 can not achieve the optimality conditions.

(ii) Market distortion

- The strategy of free trade without wielding market power

  Not distorted

- The strategy of trade protection

  Distorted

- The strategy of free trade with wielding market power

  Not distorted

Protecting trade is a typical type of market distortion.

(iii) Outputs

- The strategy of free trade without wielding market power

  $y_{2t} = qy_{1t}$

- The strategy of trade protection

$$y_{2t} = qy_{1t}\left[\frac{\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}}{\theta_2 + \delta}\right]^{\frac{1-\alpha}{\alpha}}, \text{ and}$$

$$y_{2t} < qy_{1t} \text{ if } \theta_2 > \left(\frac{\alpha}{mv}\right)(1-\alpha)^{1-\alpha} - \delta$$

$$y_{2t} = qy_{1t} \text{ if } \theta_2 = \left(\frac{\alpha}{mv}\right)(1-\alpha)^{1-\alpha} - \delta$$

$$y_{2t} > qy_{1t} \text{ if } \theta_2 < \left(\frac{\alpha}{mv}\right)(1-\alpha)^{1-\alpha} - \delta$$

- The strategy of free trade with wielding market power

  $y_{2t} = qy_{1t}$

By proposition 2, the ratio of output in the country 2 to that in the country 1 when the strategy of trade protection is taken is lower than that when the strategies of free trade both with and without wielding market power are taken, if $\theta_2 < \left(\frac{\alpha}{mv}\right)(1-\alpha)^{1-\alpha} - \delta$.

(iv) Long-run growth rates of output

- The strategy of free trade without wielding market power

$$\lim_{t\to\infty}\frac{\dot{y}_{2t}}{y_{2t}} = \lim_{t\to\infty}\frac{\dot{y}_{1t}}{y_{1t}} = \frac{(1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha} - \delta - \theta_1}{\varepsilon}$$

- The strategy of trade protection

$$\frac{\dot{y}_{2t}}{y_{2t}} = \frac{\dot{y}_{1t}}{y_{1t}} = \frac{\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha} - \delta - \theta_1}{\varepsilon}$$

- The strategy of free trade with wielding market power

$$\lim_{t\to\infty}\frac{\dot{y}_{2t}}{y_{2t}} = \lim_{t\to\infty}\frac{\dot{y}_{1t}}{y_{1t}} = \frac{(1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha} - \delta - \frac{\theta_1+\theta_2}{2}}{\varepsilon}$$

The highest rate of growth is achieved when the strategy of free trade without wielding market power is taken. If $\left[(1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha} - \delta - \frac{\theta_1+\theta_2}{2}\right] - \left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha} - \delta - \theta_1\right] > 0$, and thus if $2\left[(1+q)^{\alpha} - 1\right]\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha} > \theta_2 - \theta_1$, the growth rate when the strategy of free trade with wielding market power is taken is higher than that when the strategy of trade protection is taken, and thus the larger the parameter q is, and the lower the rate of time preference in the country 2 is, the higher the probability that the growth rate is higher when the strategy of free trade with wielding market power is taken is.

(v) The balance on current account

- The strategy of free trade without wielding market power

  Deficits

- The strategy of trade protection

  Balanced

- The strategy of free trade with wielding market power

  Surpluses if $\frac{\theta_1 + \theta_2}{2} > [1 - \varepsilon(1-\alpha)](1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - (1-\varepsilon)\delta$

  Balanced if $\frac{\theta_1 + \theta_2}{2} = [1 - \varepsilon(1-\alpha)](1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - (1-\varepsilon)\delta$

  Deficits if $\frac{\theta_1 + \theta_2}{2} < [1 - \varepsilon(1-\alpha)](1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - (1-\varepsilon)\delta$

There features are shown in proposition 1, assumption (A6) and proposition 3.

2. The best strategy

According to the above comparisons, strategies for a developing country that has the relatively higher rate of time preference are evaluated.

2.1 The country 2 is large enough and can wield market power.

In this case, the country 2 has all the three options: the free trade without wielding market power, the protection of trade, and the free trade with wielding market power. Among them, only the strategy of free trade with wielding market power achieves all the optimality conditions and does not distort markets at the same time. The strategy of free trade without wielding market power can not achieve all the optimality conditions simultaneously and the strategy of trade protection distorts markets. In this sense, the best strategy for the country 2 will be the strategy of free trade with wielding market power.

Although the optimality conditions are not satisfied, the strategy of free trade without wielding market power shows the highest long-run growth rate and thus the highest long-run level of output. From this point of view, if households in the country 2 do not care about the optimality conditions, they may choose the strategy of free trade without wielding market power. Nevertheless, this presumption that households in the country 2 do not care about the optimality conditions means that households in the country 2 do not behave rationally, and thus this presumption makes no sense if rationality is the most fundamental principle for agents.

On the other hand, the growth rate when the strategy of free trade with wielding market power is taken is the least one if $2\left[(1+q)^{\alpha}-1\right]\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}<\theta_2-\theta_1$. Hence, in this case, the levels of output and consumption when this strategy is taken are the least in the long run among the three strategies. In this sense, the country 2 that can wield market power may choose the strategy of trade protection, even though this strategy makes markets be distorted. The smaller the parameter q is, and the higher the rate of time preference in the country 2 is, the more easily the condition $2\left[(1+q)^{\alpha}-1\right]\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}<\theta_2-\theta_1$ is satisfied. Hence, if a developing country has very low q and very high θ_2, she may tend to choose the strategy of trade protection, even though she can wield market power.

As a whole, if the country 2 is large enough and does not have so low q nor so high θ_2, and if the households in the country behave rationally, the best strategy is the strategy of free trade with wielding market power. This strategy may provide insights into the recent trade behavior of China whose economy may be large enough to wield market power. The large bilateral current account deficit of the U.S. with China has been persisting and is a big political issue between the U.S. and China. The reason why the large bilateral current account deficit of the U.S. with China has been persisting has been debated actively and many argue that the problem is China's currency manipulation. Probably China's currency manipulation has truly distorted markets significantly and may explain a large part of the deficit of the .U.S. with China, but current account imbalances are basically complex phenomena and thus some other ingredients may also have influence to some extent. The model in the paper shows a possibility of another element that if the rate of time preference in China is high compared with the U.S. and if China is wielding market power and satisfies the condition $\frac{\theta_1+\theta_2}{2} > [1-\varepsilon(1-\alpha)](1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-(1-\varepsilon)\delta$, the balance on current account in China shows surpluses permanently as a result of rational behavior in both countries. Although it is merely a theoretical possibility and may not be so important compared to China's currency manipulation, this possibility may be worth pursuing considering the importance of this issue.

2.2 The country 2 is not large enough and can not wield market power.

In this case, the country 2 has only two options: one is the strategy of free trade without wielding market power and the other is the strategy of trade protection, because the country 2 can not wield market power. However, it is impossible for both strategies to achieve all the optimality conditions and not to distort markets at the same time. In this sense, both strategies are not satisfactory for households in the country 2.

Even apart from criterions of optimality and market distortion, neither strategy seems to have a decisive advantage. If the strategy of free trade without wielding market power is selected, the ratio of output to that in the country 1 and the long-run growth rate is higher than those when the strategy of trade protection is taken, but in the long-run the whole capital in the country 2 is owned by foreigners. If the strategy of the trade protection is selected, the whole capital is owned by people in the country 2 forever, but because the long-run growth rate when this strategy is taken is lower than that when the strategy of free trade without wielding market power is taken, the levels of output and consumption in the country 2 are far lower than those in the country 1 and than those when the strategy of free trade without wielding market power is taken in the long-run, and these gaps continues to widen forever.

One way to evaluate the two strategies is to simply compare the expected utilities without considering whether these consumption streams satisfy optimality conditions. However, it is very difficult to say which strategy provides the higher expected utility for households in the country 2, since it is not easy to tract consumption streams analytically. Intuitively, however, $|\tau_t|$, i.e. the absolute values of the balance on current account that equals the inflow of capital from the country 1 to the country 2 in each period, may not be so large compared to the output, and if so, the expected utility when the strategy of free trade without wielding market power is taken will be higher than that when the strategy of trade protection is taken, because the output in the near future when the strategy of free trade without wielding market power is taken will be roughly identical to the consumption and thus the consumption in the near future when the strategy of free trade without wielding market power is taken will be higher than that when the strategy of trade protection is taken.

In addition, because $\frac{d\Pi}{d\theta_2} < 0$ and $\lim_{\theta_2 \to \infty} \Pi = 0$, as θ_2 becomes higher, the negative impacts of trade protection becomes larger. In these cases, the damage of trade protection in the near future will be much larger and thus the expected utility when the strategy of trade protection is taken may be smaller than that when the strategy of free trade without wielding market power is taken. Hence the higher the rate of time preference θ_2 is, the higher the probability that the expected utility when the strategy of free trade without wielding market power is taken is higher may be.

On the other hand, by proposition 2, $y_{2t} > q y_{1t}$ if $\theta_2 < \left(\frac{\alpha}{mv}\right)(1-\alpha)^{1-\alpha} - \delta$ and thus there is a possibility that if the trade is newly protected the output in the country 2 may be higher and grow faster initially than that before the trade is protected. In this case, the consumption in the near future will be also higher than that that is possible if markets remain open. Taking these effects into consideration, the expected utility may be higher if the trade is newly protected than that before the trade is not protected if $\theta_2 < \left(\frac{\alpha}{mv}\right)(1-\alpha)^{1-\alpha} - \delta$.

As a whole, unless a developing country has the very high rate of time preference, it is not easy to say which strategy provides the higher expected utility. Nevertheless, the criterion of the expected utility dose not have overwhelming power because the optimality conditions are not satisfied if the strategy of free trade without wielding market power is taken. The crucial point seems rather how the situation that the whole capital is owned by foreigners is evaluated when the government of the country 2 makes the decision which strategy should be taken. This problem for the government may not be solved by purely an economical criterion, i.e. the maximization of expected utilities, but its solution may be judged partially from the political point of view. Politically a government may not accept the situation that the whole capital is owned by foreigners because the government may consider "national economic security" or the pride of the nation that may be hurt by "economical occupation" by foreigners. Anyway, it may be a hard choice for the country 2.

2.3 The third way

Is there no other way for a developing country with the higher rate of time preference if she has no market power? A possible alternative way is that small countries with similar preferences cooperate, integrate their economies and wield a combined market power. If these countries can successfully integrate their economies, this integrated economy may be large enough to wield market power, and thus can choose the strategy of free trade with wielding market power, which is basically better than the other strategies for the developing countries.

FTA among these countries may be a way to integrate their economies, although many political problems still need to be overcome.

3. The interaction between q and the strategies

Finally, the interaction between q and the strategies is considered briefly. The model in the paper concentrates on heterogeneity of time preference rates across countries and abstracts other factors by introducing a parameter q, and implicitly assumes that q and the strategies are independent each others. However, the value of the parameter q may not be independent of the choice of strategy and may be changed by strategies. For example, if the strategy of trade protection is taken, the competition in domestic markets may be somewhat restricted, e.g. domestic markets are dominated by a few domestic companies, and thus technology inflows from developed countries may be much more obstructed. As a result, the strategy of trade protection may have much larger negative impacts on output.

V. CONCLUDING REMARKS

In general the conjecture that trade liberalization promotes growth seems true. However, the actual processes of growth initiated by trade liberalization do not seem so clear-cut. The paper studies complex processes of growth through trade liberalization in developing countries. It has been typically argued that people in poor countries have the higher rate of time preference, and the paper directs its attention to this fact and examines the relation between the higher rate of time preference and strategies of trade liberalization for developing countries on the basis of the framework of endogenous growth.

The paper starts its analysis from the fact that poor countries have the higher rate of time preference, and thus it is not examined in the paper why the poor has the higher rate of time preference. In addition, the combined effects of factors that generate the large difference of per capita income across countries, e.g. barriers to technology adoption, are put together and expressed by a single parameter in the model. Based on a two-country endogenous growth model, strategies for a developing country with higher rate of time preference to deal with trade liberalization are examined. The results are summed up as follows.

(i) When a developing country is large enough and can wield market power, the best strategy for the developing country is generally the strategy of free trade with wielding market power because only this strategy can achieve all the optimality conditions and does not distort markets at the same time.

(ii) When a developing country is not large enough and can not wield market power, it is very difficult to say which strategy is the best for the developing country because it is impossible for the developing country to wield market power. Nevertheless, the strategy for the developing country may be judged not only from the economic but political point of view because a government in the country must decide whether the situation that the whole capital is owned by foreigners is politically acceptable.

(iii) There is a third way if small developing countries with similar preferences can cooperate and integrate their economies. The integrated economy may be large enough to wield combined market power. FTA among these countries may be a way to integrate their economies, although many political problems still need to be overcome.

The strategy of free trade with wielding market power that is generally the best strategy for a developing country when the developing country is large enough may provide insights into the recent trade behavior of China whose economy may be large enough to wield market power. The large bilateral current account deficit of the U.S. with China has been persisting. The model in the paper predicts that the current account deficit of the U.S. with China will be generally observed if the rate of time preference in China is relatively higher than that in the U.S. and if China is wielding market power. It is merely a theoretical possibility and may not have important influence compared to China's currency manipulation. Many empirical researches are necessary to conclude that this mechanism has really worked and has had influence to some extent. However, considering the importance of this issue, this possibility may be worth pursuing.

APPENDIX

1. Proof of lemma 1

(Step 1) By the optimality condition (5),

$$\frac{\dot{k}_{1t}}{k_{1t}} = \left[(1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - \delta\right] + \left[(1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha} - \delta\right]\frac{\int_0^t \tau_s ds}{k_{1t}} - \frac{\tau_t + c_{1t}}{k_{1t}}.$$

On the other hand, by the optimality condition (4),

$$\frac{\dot{\lambda}_{1t}}{\lambda_{1t}} = -\left\{\left[(1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - \delta\right] + \left[(1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha} - \delta\right]\frac{\partial\left(\int_0^t \tau_s ds\right)}{\partial k_{1t}} - \frac{\partial \tau_t}{\partial k_{1t}}\right\}.$$

Here, $\lim_{t\to\infty}\left(\frac{\dot{\lambda}_{1t}}{\lambda_{1t}} + \frac{\dot{k}_{1t}}{k_{1t}}\right)$

$$= -\lim_{t\to\infty}\left\{\left[(1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - \delta\right] + \left[(1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha} - \delta\right]\frac{\partial\left(\int_0^t \tau_s ds\right)}{\partial k_{1t}} - \frac{\partial \tau_t}{\partial k_{1t}}\right\}$$

$$+ \lim_{t\to\infty}\left\{\left[(1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - \delta\right] + \left[(1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha} - \delta\right]\frac{\int_0^t \tau_s ds}{k_{1t}} - \frac{\tau_t + c_{1t}}{k_{1t}}\right\}$$

$$= \lim_{t\to\infty}\left\{\left(\frac{\partial \tau_t}{\partial k_{1t}} - \frac{\tau_t}{k_{1t}}\right) - \left[(1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha} - \delta\right]\left[\frac{\partial\left(\int_0^t \tau_s ds\right)}{\partial k_{1t}} - \frac{\int_0^t \tau_s ds}{k_{1t}}\right] - \frac{c_{1t}}{k_{1t}}\right\}.$$

Thereby, if $\lim_{t\to\infty}\left\{\left(\frac{\partial \tau_t}{\partial k_{1t}} - \frac{\tau_t}{k_{1t}}\right) - \left[(1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha} - \delta\right]\left[\frac{\partial\left(\int_0^t \tau_s ds\right)}{\partial k_{1t}} - \frac{\int_0^t \tau_s ds}{k_{1t}}\right] - \frac{c_{1t}}{k_{1t}}\right\} < 0$, then

$\frac{\dot{\lambda}_{1t}}{\lambda_{1t}} + \frac{\dot{k}_{1t}}{k_{1t}} < 0$.

(Step 2) By the optimality condition (9),

$$\frac{\dot{k}_{2t}}{k_{2t}} = \left[(1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - \delta\right] - \left[(1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha} - \delta\right]\frac{\int_0^t \tau_s ds}{k_{2t}} + \frac{\tau_t - c_{2t}}{k_{2t}}.$$

On the other hand, by the optimality condition (8),

$$\frac{\dot{\lambda}_{2t}}{\lambda_{2t}} = -\left\{\left[(1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - \delta\right] - \left[(1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha} - \delta\right]\frac{\partial\left(\int_0^t \tau_s ds\right)}{\partial k_{2t}} + \frac{\partial \tau_t}{\partial k_{2t}}\right\}.$$

Here, $\lim_{t\to\infty}\left(\frac{\dot{\lambda}_{2t}}{\lambda_{2t}} + \frac{\dot{k}_{2t}}{k_{2t}}\right)$

$$= -\lim_{t\to\infty}\left\{\left[(1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - \delta\right] - \left[(1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha} - \delta\right]\frac{\partial\left(\int_0^t \tau_s ds\right)}{\partial k_{2t}} + \frac{\partial \tau_t}{\partial k_{2t}}\right\}$$

$$+ \lim_{t\to\infty}\left\{\left[(1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - \delta\right] - \left[(1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha} - \delta\right]\frac{\int_0^t \tau_s ds}{k_{2t}} + \frac{\tau_t - c_{2t}}{k_{2t}}\right\}$$

$$= \lim_{t\to\infty}\left\{\left(\frac{\tau_t}{k_{2t}} - \frac{\partial \tau_t}{\partial k_{2t}}\right) - \left[(1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha} - \delta\right]\left[\frac{\int_0^t \tau_s ds}{k_{2t}} - \frac{\partial\left(\int_0^t \tau_s ds\right)}{\partial k_{2t}}\right] - \frac{c_{2t}}{k_{2t}}\right\}.$$

Thereby, if $\lim_{t\to\infty}\left\{\left(\frac{\tau_t}{k_{2t}} - \frac{\partial \tau_t}{\partial k_{2t}}\right) - \left[(1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha} - \delta\right]\left[\frac{\int_0^t \tau_s ds}{k_{2t}} - \frac{\partial\left(\int_0^t \tau_s ds\right)}{\partial k_{2t}}\right] - \frac{c_{2t}}{k_{2t}}\right\} < 0$,

then $\lim_{t\to\infty}\left(\frac{\dot{\lambda}_{2t}}{\lambda_{2t}} + \frac{\dot{k}_{2t}}{k_{2t}}\right) < 0$.

Hence, the transversality conditions (6) $\lim_{t\to\infty}\lambda_{1t}k_{1t} = 0$ and (10) $\lim_{t\to\infty}\lambda_{2t}k_{2t} = 0$ are not satisfied if and only if

$$\lim_{t\to\infty}\left\{\left(\frac{\partial \tau_t}{\partial k_{1t}} - \frac{\tau_t}{k_{1t}}\right) - \left[(1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha} - \delta\right]\left[\frac{\partial\left(\int_0^t \tau_s ds\right)}{\partial k_{1t}} - \frac{\int_0^t \tau_s ds}{k_{1t}}\right] - \frac{c_{1t}}{k_{1t}}\right\} \geq 0 \text{ or}$$

$$\lim_{t\to\infty}\left\{\left(\frac{\tau_t}{k_{2t}}-\frac{\partial\tau_t}{\partial k_{2t}}\right)-\left[(1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\left[\frac{\int_0^t\tau_s ds}{k_{2t}}-\frac{\partial\left(\int_0^t\tau_s ds\right)}{\partial k_{2t}}\right]-\frac{c_{2t}}{k_{2t}}\right\}\geq 0\cdot$$

Q.E.D.

2. Proof of lemma 2

(Step 1) By the optimality conditions (3), (4) and (5),

$$\text{(19)}\quad \lim_{t\to\infty}\frac{\dot{c}_{1t}}{c_{1t}}=\frac{\left[(1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta\right]+\left[(1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\lim\limits_{t\to\infty}\frac{\partial\left(\int_0^t\tau_s ds\right)}{\partial k_{1t}}-\lim\limits_{t\to\infty}\frac{\partial\tau_t}{\partial k_{1t}}-\theta_1}{\varepsilon}.$$

Similarly, by the optimality conditions (7), (8) and (9),

$$\text{(20)}\quad \lim_{t\to\infty}\frac{\dot{c}_{2t}}{c_{2t}}=\frac{\left[(1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta\right]-\left[(1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\lim\limits_{t\to\infty}\frac{\partial\left(\int_0^t\tau_s ds\right)}{\partial k_{2t}}+\lim\limits_{t\to\infty}\frac{\partial\tau_t}{\partial k_{2t}}-\theta_2}{\varepsilon}.$$

In addition,

$$\text{(21)}\quad \lim_{t\to\infty}\frac{\dot{k}_{1t}}{k_{1t}}=\left[(1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta\right]+\left[(1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\lim_{t\to\infty}\frac{\int_0^t\tau_s ds}{k_{1t}}-\lim_{t\to\infty}\frac{\tau_t+c_{1t}}{k_{1t}}$$

and

$$\text{(22)}\quad \lim_{t\to\infty}\frac{\dot{k}_{2t}}{k_{2t}}=\left[(1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta\right]-\left[(1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\lim_{t\to\infty}\frac{\int_0^t\tau_s ds}{k_{2t}}+\lim_{t\to\infty}\frac{\tau_t-c_{2t}}{k_{2t}}.$$

(Step 2) By equations (1) and (2), $c_{1t}-c_{2t}=2\left[(1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}\int_0^t\tau_s ds-\tau_t\right]$. Hence, if $\lim\limits_{t\to\infty}\frac{\dot{c}_{1t}}{c_{1t}}=\lim\limits_{t\to\infty}\frac{\dot{c}_{2t}}{c_{2t}}=$ a positive constant, then $\lim\limits_{t\to\infty}\frac{\dot{c}_{1t}}{c_{1t}}=\lim\limits_{t\to\infty}\frac{\dot{c}_{2t}}{c_{2t}}=\lim\limits_{t\to\infty}\frac{\dot{\tau}_t}{\tau_t}=\lim\limits_{t\to\infty}\frac{\frac{d\left(\int_0^t\tau_s ds\right)}{dt}}{\int_0^t\tau_s ds}$. Thus, by lemma 1 the transversality conditions (6) and (10) are satisfied, and also all the other optimality conditions are satisfied.

However, if $\lim_{t\to\infty}\frac{\dot{c}_{1t}}{c_{1t}} \neq \lim_{t\to\infty}\frac{\dot{c}_{2t}}{c_{2t}}$, then $\lim_{t\to\infty}\frac{\dot{\tau}_t}{\tau_t} \neq \lim_{t\to\infty}\frac{\frac{d\left(\int_0^t \tau_s ds\right)}{dt}}{\int_0^t \tau_s ds}$. Thus by lemma 1, for both countries to satisfy the transverality conditions, it is necessary that $\lim_{t\to\infty}\frac{c_{1t}}{k_{1t}} = \infty$ or $\lim_{t\to\infty}\frac{\dot{c}_{2t}}{k_{2t}} = \infty$, which violates the optimality conditions (5) or (9). If $\lim_{t\to\infty}\frac{\dot{c}_{1t}}{c_{1t}} \neq \lim_{t\to\infty}\frac{\dot{k}_{1t}}{k_{1t}}$ or $\lim_{t\to\infty}\frac{\dot{c}_{2t}}{c_{2t}} \neq \lim_{t\to\infty}\frac{\dot{k}_{2t}}{k_{2t}}$, the transversality conditions (6) or (10), or the optimality conditions (5) or (9) is violated.

As a result, if and only if $\lim_{t\to\infty}\frac{\dot{c}_{1t}}{c_{1t}} = \lim_{t\to\infty}\frac{\dot{c}_{2t}}{c_{2t}} = \lim_{t\to\infty}\frac{\dot{k}_{1t}}{k_{1t}} = \lim_{t\to\infty}\frac{\dot{k}_{2t}}{k_{2t}} =$ constant, all the optimality conditions are satisfied.

Q.E.D.

3. Proof of proposition 1

(Step 1) In this case, τ_t can be seen as a control variable for each country in each country's optimization problem. Hence, the optimality condition

$$(23)\quad \left[(1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha} - \delta\right]\frac{\partial\left(\int_0^t \tau_s ds\right)}{\partial \tau_t} = 1$$

is added to the optimality conditions for the country 1, and the optimality condition

$$(24)\quad \left[(1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha} - \delta\right]\frac{\partial\left(\int_0^t \tau_s ds\right)}{\partial \tau_t} = 1$$

is added to the optimality conditions for the country 2. The optimality conditions (23) and (24) are identical, and by them, $\left[(1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha} - \delta\right]\frac{\partial\left(\int_0^t \tau_s ds\right)}{\partial k_{1t}} - \frac{\partial \tau_t}{\partial k_{1t}} = 0$ and thus the optimal consumption growth rates are $\frac{\dot{c}_{1t}}{c_{1t}} = \frac{(1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - \delta - \theta_1}{\varepsilon}$ and $\frac{\dot{c}_{2t}}{c_{2t}} = \frac{(1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - \delta - \theta_2}{\varepsilon}$. Thereby $\frac{\dot{c}_{1t}}{c_{1t}} > \frac{\dot{c}_{2t}}{c_{2t}}$.

On the other hand, by the optimality conditions (23) and (24),

$$\left[(1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha} - \delta\right]\frac{\partial\left(\int_0^t \tau_s ds\right)}{\partial \tau_t} = \left[(1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha} - \delta\right]\frac{\frac{\partial\left(\int_0^t \tau_s ds\right)}{\partial t}}{\frac{\partial \tau_t}{\partial t}} = 1$$ and thus

$$\frac{\dot{\tau}_t}{\tau_t} = (1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha} - \delta .$$

(Step 2) For the country 1, it is necessary to hold $\frac{\dot{c}_{1t}}{c_{1t}} = \lim_{t\to\infty}\frac{\dot{k}_{1t}}{k_{1t}}$ for the optimality conditions (3), (4), (5) and (6) to be satisfied, and for the country 2, it is necessary to hold $\frac{\dot{c}_{2t}}{c_{2t}} = \lim_{t\to\infty}\frac{\dot{k}_{2t}}{k_{2t}}$ for the optimality conditions (7), (8), (9) and (10) to be satisfied.

Here, because the equations such that $qk_{1t} = k_{2t}$, $qy_{1t} = y_{2t}$ are kept by firms at any time, the households in the country 2 must set the higher initial consumption level than that in the country 1 times q by importing goods and services from the country 1 to hold $\frac{\dot{c}_{1t}}{c_{1t}} > \frac{\dot{c}_{2t}}{c_{2t}}$. Therefore it must be that $\tau_t > 0$ and $\frac{\dot{c}_{1t}}{c_{1t}} \geq \frac{\dot{k}_{1t}}{k_{1t}} = \frac{\dot{k}_{2t}}{k_{2t}} \geq \frac{\dot{c}_{2t}}{c_{2t}}$.

(Step 3) There are three cases: i) $\frac{\dot{\tau}_t}{\tau_t} > \lim_{t\to\infty}\frac{\dot{k}_{1t}}{k_{1t}} = \lim_{t\to\infty}\frac{\dot{k}_{2t}}{k_{2t}}$, ii) $\frac{\dot{\tau}_t}{\tau_t} = \lim_{t\to\infty}\frac{\dot{k}_{1t}}{k_{1t}} = \lim_{t\to\infty}\frac{\dot{k}_{2t}}{k_{2t}}$, and iii) $\frac{\dot{\tau}_t}{\tau_t} < \lim_{t\to\infty}\frac{\dot{k}_{1t}}{k_{1t}} = \lim_{t\to\infty}\frac{\dot{k}_{2t}}{k_{2t}}$.

In case of i), eventually $\lim_{t\to\infty}\frac{\left|\int_0^t \tau_s ds\right|}{k_{2t}} > 1$ and thus the optimality condition (7) or (24) is violated. In case of iii), because $\frac{\dot{c}_{1t}}{c_{1t}} \ge \frac{\dot{k}_{1t}}{k_{1t}} = \frac{\dot{k}_{2t}}{k_{2t}} \ge \frac{\dot{c}_{2t}}{c_{2t}}$, then $\frac{\dot{c}_{1t}}{c_{1t}} > \frac{\dot{\tau}_t}{\tau_t}$, and thus the ratio of τ_t to c_{1t} diminishes to zero as time passes and the balance on current account becomes negligible for the country 1. In this situation the country 1 selects a path such that $\frac{\dot{c}_{1t}}{c_{1t}} = \lim_{t\to\infty}\frac{\dot{k}_{1t}}{k_{1t}}$, thereby it is not possible for the country 2 to hold $\lim_{t\to\infty}\frac{\dot{c}_{1t}}{c_{1t}} > \lim_{t\to\infty}\frac{\dot{c}_{2t}}{c_{2t}}$. In case of ii), because the country 2 must hold $\frac{\dot{c}_{2t}}{c_{2t}} = \lim_{t\to\infty}\frac{\dot{k}_{2t}}{k_{2t}}$ for the optimality conditions (7), (8), (9) and (10) to be satisfied, only and only if $\frac{\dot{c}_{2t}}{c_{2t}} = \frac{\dot{\tau}_t}{\tau_t} = \lim_{t\to\infty}\frac{\dot{k}_{2t}}{k_{2t}}$, it is possible for the country 2 to satisfy all the optimality conditions by setting appropriate initial values of c_{10} and τ_0.

Q.E.D.

4. Proof of corollary 1

By (Step 2) in the proof of proposition 1, it is necessary for the country 2 that $\frac{\dot{c}_{2t}}{c_{2t}} = \lim_{t\to\infty}\frac{\dot{k}_{2t}}{k_{2t}}$. Since $\frac{\dot{c}_{2t}}{c_{2t}} = \frac{(1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - \delta - \theta_2}{\varepsilon}$ and $\frac{\dot{\tau}_t}{\tau_t} = (1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha} - \delta$, if $(1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}[1-\varepsilon(1-\alpha)] - (1-\varepsilon)\delta - \theta_2 < 0$, then $\frac{\dot{\tau}_t}{\tau_t} > \frac{\dot{c}_{2t}}{c_{2t}} = \lim_{t\to\infty}\frac{\dot{k}_{2t}}{k_{2t}}$. If $\frac{\dot{\tau}_t}{\tau_t} > \lim_{t\to\infty}\frac{\dot{k}_{2t}}{k_{2t}}$, then the whole capital must be owned by foreigners in the long-run because τ_t is an increase of capital owned by foreigners in period *t* by assumption.

Q.E.D.

5. Proof of proposition 2

Since $A_{2t} = qA_{1t}$ and $A_{1t} = \dfrac{\alpha}{mv(1-\alpha)} k_{1t}$, then at the steady state

$$\frac{\partial y_{2t}}{\partial k_{2t}} = (1-\alpha)\left(\frac{qA_{1t}}{k_{2t}}\right)^{\alpha} = \left[\frac{qk_{1t}}{k_{2t}}\frac{\alpha}{mv}\right]^{\alpha}(1-\alpha)^{1-\alpha} = \theta_2 + \delta \quad \text{and thus}$$

$$k_{2t} = qk_{1t}\left[\frac{\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}}{\theta_2 + \delta}\right]^{\frac{1}{\alpha}} \quad \text{and}$$

$$y_{2t} = (qA_{1t})^{\alpha} k_{2t}^{1-\alpha} = qA_{1t}^{\alpha} k_{1t}^{1-\alpha}\left[\frac{\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}}{\theta_2 + \delta}\right]^{\frac{1-\alpha}{\alpha}} = qy_{1t}\left[\frac{\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}}{\theta_2 + \delta}\right]^{\frac{1-\alpha}{\alpha}}.$$

Hence, if $\theta_2 > \left(\dfrac{\alpha}{mv}\right)(1-\alpha)^{1-\alpha} - \delta$, then $y_{2t} < qy_{1t}$.

Q.E.D.

6. Proof of proposition 3

(Step 1) Because current account imbalances grow at the same rate with output, consumption, or capital eventually, the ratio of the balance on current account to output approaches to a unique finite constant value.

(Step 2) Because $\lim_{t\to\infty}\dfrac{\dot{c}_{1t}}{c_{1t}} = \lim_{t\to\infty}\dfrac{\dot{c}_{2t}}{c_{2t}}$, then by equations (19) and (20),

$$\left[(1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha} - \delta\right]\left[\lim_{t\to\infty}\frac{\partial\left(\int_0^t \tau_s ds\right)}{\partial k_{1t}} + \lim_{t\to\infty}\frac{\partial\left(\int_0^t \tau_s ds\right)}{\partial k_{2t}}\right] - \left(\lim_{t\to\infty}\frac{\partial \tau_t}{\partial k_{1t}} + \lim_{t\to\infty}\frac{\partial \tau_t}{\partial k_{2t}}\right)$$

$$= \left[(1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha} - \delta\right]\left[\lim_{t\to\infty}\frac{\partial\left(\int_0^t \tau_s ds\right)}{\partial k_{1t}} + \frac{1}{q}\lim_{t\to\infty}\frac{\partial\left(\int_0^t \tau_s ds\right)}{\partial k_{1t}}\right] - \left(\lim_{t\to\infty}\frac{\partial \tau_t}{\partial k_{1t}} + \frac{1}{q}\lim_{t\to\infty}\frac{\partial \tau_t}{\partial k_{1t}}\right)$$

$$=\left(1+\frac{1}{q}\right)\lim_{t\to\infty}\left\{\left[(1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\frac{\partial\left(\int_0^t \tau_s ds\right)}{\partial k_{1t}}-\frac{\partial \tau_t}{\partial k_{1t}}\right\}$$

$$\left(=\left(1+\frac{1}{q}\right)\lim_{t\to\infty}\left\{\left[(1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\frac{\partial\left(\int_0^t \tau_s ds\right)}{\partial k_{2t}}-\frac{\partial \tau_t}{\partial k_{2t}}\right\}\right)=\theta_1-\theta_2$$

. Hence,

$$\lim_{t\to\infty}\left\{\frac{\partial \tau_t}{\partial k_{1t}}-\left[(1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\frac{\partial\left(\int_0^t \tau_s ds\right)}{\partial k_{1t}}\right\}=-\frac{\theta_1-\theta_2}{\left(1+\frac{1}{q}\right)}$$

= a positive constant.

(Step 3) Because $\lim_{t\to\infty}\frac{\dot{\tau}_t}{\tau_t}=\lim_{t\to\infty}\frac{\frac{\partial\left(\int_0^t \tau_s ds\right)}{\partial t}}{\int_0^t \tau_s ds}$ = a positive constant, for sufficiently large $t=t'$,

$\tau_t=\tau_{t'}\exp\left(\frac{\dot{\tau}_t}{\tau_t}t\right)$ and thus $\lim_{t\to\infty}\frac{\int_0^t \tau_s ds}{\tau_t}=\frac{1}{\lim_{t\to\infty}\frac{\dot{\tau}_t}{\tau_t}}=\frac{1}{\lim_{t\to\infty}\frac{\frac{\partial\left(\int_0^t \tau_s ds\right)}{\partial t}}{\int_0^t \tau_s ds}}$ = a positive constant.

Since $\lim_{t\to\infty}\frac{\dot{c}_{1t}}{c_{1t}}=\lim_{t\to\infty}\frac{\dot{k}_{1t}}{k_{1t}}=\lim_{t\to\infty}\frac{\dot{\tau}_t}{\tau_t}=\lim_{t\to\infty}\frac{\frac{\partial\left(\int_0^t \tau_s ds\right)}{\partial t}}{\int_0^t \tau_s ds}$,

$$\text{(25)}\quad \lim_{t\to\infty}\left\{\frac{\partial \tau_t}{\partial k_{1t}}-\left[(1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\frac{\partial\left(\int_0^t \tau_s ds\right)}{\partial k_{1t}}\right\}$$

$$=\lim_{t\to\infty}\left\{\frac{\frac{\dot{\tau}_t}{\tau_t}}{\frac{\dot{k}_{1t}}{k_t}}\frac{\tau_t}{k_{1t}}-\left[(1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\frac{\frac{\frac{\partial\left(\int_0^t \tau_s ds\right)}{\partial t}}{\int_0^t \tau_s ds}}{\frac{\dot{k}_{1t}}{k_t}}\frac{\int_0^t \tau_s ds}{k_{1t}}\right\}$$

$$= \lim_{t\to\infty}\left\{\frac{\tau_t}{k_{1t}} - \left[(1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha} - \delta\right]\frac{\int_0^t \tau_s ds}{k_{1t}}\right\} = \lim_{t\to\infty}\frac{\tau_t}{k_{1t}}\left[1 - \frac{(1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha} - \delta}{\frac{\dot{\tau}_{1t}}{\tau_{1t}}}\right]$$

$$= \lim_{t\to\infty}\frac{\tau_t}{k_{1t}}\left[1 - \frac{(1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha} - \delta}{\frac{\dot{c}_{1t}}{c_{1t}}}\right] = -\frac{\theta_1 - \theta_2}{\left(1+\frac{1}{q}\right)} = \text{a positive constant.}$$

Here, $\lim_{t\to\infty}\frac{\dot{c}_{1t}}{c_{1t}} = \lim_{t\to\infty}\frac{\dot{c}_{2t}}{c_{2t}} = \lim_{t\to\infty}\frac{\frac{\dot{c}_{1t}}{c_{1t}} + \frac{\dot{c}_{2t}}{c_{2t}}}{2} = \frac{(1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - \delta - \frac{\theta_1+\theta_2}{2}}{\varepsilon}$. Thereby, if

$$\frac{(1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha} - \delta}{\frac{(1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - \delta - \frac{\theta_1+\theta_2}{2}}{\varepsilon}} > 1$$ and thus if

$\frac{\theta_1+\theta_2}{2} > [1-\varepsilon(1-\alpha)](1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - (1-\varepsilon)\delta$, then

$\lim_{t\to\infty}\left[1 - \frac{(1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha} - \delta}{\frac{\dot{c}_{1t}}{c_{1t}}}\right] < 0$. Hence, because $\lim_{t\to\infty}\frac{\tau_t}{k_{1t}}\left[1 - \frac{(1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha} - \delta}{\frac{\dot{c}_{1t}}{c_{1t}}}\right]$

= a positive constant, if $\frac{\theta_1+\theta_2}{2} > [1-\varepsilon(1-\alpha)](1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - (1-\varepsilon)\delta$, then $\lim_{t\to\infty}\frac{\tau_t}{k_{1t}} =$ a negative constant, and thus $\lim_{t\to\infty}\frac{\tau_t}{y_{1t}} =$ a negative constant and $-\lim_{t\to\infty}\frac{\tau_t}{y_{2t}} =$ a positive constant.

Q.E.D.

7. Proof of corollary 2

(Step 1) Because $\lim_{t\to\infty}\frac{\dot{\tau}_t}{\tau_t} = \lim_{t\to\infty}\frac{\frac{d\int_0^t \tau_s ds}{dt}}{\int_0^t \tau_s ds} = \lim_{t\to\infty}\frac{\dot{c}_{1t}}{c_{1t}} = \lim_{t\to\infty}\frac{\dot{c}_{2t}}{c_{2t}} = \lim_{t\to\infty}\frac{\dot{k}_{1t}}{k_{1t}} = \lim_{t\to\infty}\frac{\dot{k}_{2t}}{k_{2t}}$ and

$$\lim_{t\to\infty}\frac{\dot{c}_{1t}}{c_{1t}}=\lim_{t\to\infty}\frac{\dot{c}_{2t}}{c_{2t}}=\lim_{t\to\infty}\frac{\frac{\dot{c}_{1t}}{c_{1t}}+\frac{\dot{c}_{2t}}{c_{2t}}}{2}=\frac{(1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta-\frac{\theta_1+\theta_2}{2}}{\varepsilon}$$, then

$$\lim_{t\to\infty}\frac{\dot{\tau}_t}{\tau_t}=\lim_{t\to\infty}\frac{\dot{k}_{1t}}{k_{1t}}=\lim_{t\to\infty}\frac{\dot{k}_{2t}}{k_{2t}}=\frac{(1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta-\frac{\theta_1+\theta_2}{2}}{\varepsilon}.$$

(Step 2) By equations (25),

$$\lim_{t\to\infty}\frac{\tau_t}{k_{1t}}\left[1-\frac{(1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta}{\frac{\dot{c}_{1t}}{c_{1t}}}\right]=-\frac{\theta_1-\theta_2}{\left(1+\frac{1}{q}\right)}.$$

Because $\lim_{t\to\infty}\frac{\frac{\dot{\tau}_t}{\tau_t}\int_0^t\tau_s ds}{\tau_t}=1$ and $\lim_{t\to\infty}\frac{\dot{\tau}_t}{\tau_t}=\lim_{t\to\infty}\frac{\dot{c}_{1t}}{c_{1t}}$,

$$\lim_{t\to\infty}\frac{\tau_t}{k_{1t}}\left[1-\frac{(1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta}{\frac{\dot{c}_{1t}}{c_{1t}}}\right]=\lim_{t\to\infty}\frac{\frac{\dot{\tau}_t}{\tau_t}\int_0^t\tau_s ds}{k_{1t}}\left[1-\frac{(1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta}{\frac{\dot{\tau}_t}{\tau_t}}\right]$$

$$=\lim_{t\to\infty}\frac{\int_0^t\tau_s ds}{k_{1t}}\left[\frac{\dot{\tau}_t}{\tau_t}-(1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}+\delta\right]=-\frac{\theta_1-\theta_2}{\left(1+\frac{1}{q}\right)}.$$

Since $\lim_{t\to\infty}\frac{\dot{\tau}_t}{\tau_t}=\lim_{t\to\infty}\frac{\frac{\dot{c}_{2t}}{c_{2t}}+\frac{\dot{c}_{2t}}{c_{2t}}}{2}=\frac{(1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta-\frac{\theta_1+\theta_2}{2}}{\varepsilon}$ by (Step 1),

$$\lim_{t\to\infty}\frac{\left|\int_0^t\tau_s ds\right|}{k_{1t}}=\frac{-\frac{\theta_1-\theta_2}{\left(1+\frac{1}{q}\right)}}{\left|\lim_{t\to\infty}\frac{\dot{\tau}_t}{\tau_t}-(1+q)^{\alpha}\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}+\delta\right|}$$

$$
= \frac{-\dfrac{\theta_1-\theta_2}{\left(1+\dfrac{1}{q}\right)}}{\left|\dfrac{(1+q)^{\alpha}\left(\dfrac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta-\dfrac{\theta_1+\theta_2}{2}}{\varepsilon}-\left[(1+q)^{\alpha}\left(\dfrac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\right|}
$$

$$
= -\frac{(\theta_1-\theta_2)\varepsilon}{\left(1+\dfrac{1}{q}\right)\left|(1+q)^{\alpha}\left(\dfrac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}\left[1-(1-\alpha)\varepsilon\right]-(1-\varepsilon)\delta-\dfrac{\theta_1+\theta_2}{2}\right|}.
$$

Hence, if $-(\theta_1-\theta_2)\varepsilon < \left(1+\dfrac{1}{q}\right)\left|(1+q)^{\alpha}\left(\dfrac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}\left[1-(1-\alpha)\varepsilon\right]-(1-\varepsilon)\delta-\dfrac{\theta_1+\theta_2}{2}\right|$,

then $\lim\limits_{t\to\infty}\dfrac{\left|\int_0^t \tau_s\,ds\right|}{k_{1t}} < 1$.

Q.E.D.

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Table: Summary of the three strategies for the country 2

	Free trade without wielding market power	Trade protection	Free trade with wielding market power
-The optimality conditions	Not satisfied	Satisfied	Satisfied
-The market distortion	Not distorted	Distorted	Not distorted
-The ratio of output to that in the country 1	q if $\theta_2 > \left(\frac{\alpha}{m\,v}\right)(1-\alpha)^{1-\alpha} - \delta$	$q >$	q
	q if $\theta_2 < \left(\frac{\alpha}{m\,v}\right)(1-\alpha)^{1-\alpha} - \delta$	$q <$	q
-The long-run growth rate	Highest if $2\left[(1+q)^{\alpha} - 1\right]\left(\frac{\alpha}{m\,v}\right)^{\alpha}(1-\alpha)^{1-\alpha} > \theta_2 - \theta_1$	Lowest	Middle
	Highest if $2\left[(1+q)^{\alpha} - 1\right]\left(\frac{\alpha}{m\,v}\right)^{\alpha}(1-\alpha)^{1-\alpha} < \theta_2 - \theta_1$	Middle	Lowest
-The balance on current Account	Deficits if $\frac{\theta_1 + \theta_2}{2} > \left[1 - \varepsilon(1-\alpha)\right](1+q)^{\alpha}\left(\frac{\alpha}{m\,v}\right)^{\alpha}(1-\alpha)^{-\alpha} - (1-\varepsilon)\delta$	Balanced	Surpluses
	Deficits if $\frac{\theta_1 + \theta_2}{2} < \left[1 - \varepsilon(1-\alpha)\right](1+q)^{\alpha}\left(\frac{\alpha}{m\,v}\right)^{\alpha}(1-\alpha)^{-\alpha} - (1-\varepsilon)\delta$	Balanced	Deficits