Local Trade Networks and Spatially Persistent Unemployment [⋆]

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Abstract

This paper studies the effects of local trade networks on the spatial distribution of employment in a Cooper and John (1988) type model with effective demand externalities between trade partners. It is shown that, if labor can be hired in continuous quantities, then the long run spatial distribution of employment is uniform, and independent of any trade network topology. When labor has binary support, however, local trade networks are found to generate spatial unemployment clusters which can persist indefinitely.

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1 Introduction

Markets are systems in which large numbers of interacting agents operate in parallel. In a standard Arrow-Debreu framework, trade between these agents is typically assumed to be 'global'. That is, conditional on equilibrium prices, the buyers and sellers of a given good are indifferent as to whom to trade with, hence each agent has an equal probability of trading with any other agent in the economy.

The assumption of global trade plays an important role in macroeconomics because it allows us to use laws of large numbers, which helps to solve important aggregation problems. In real world markets, however, there is no doubt that the assumption of global trade is violated. In spite of substantial reductions in transportation costs, trade volumes still tend to decrease with distance even when one controls for spatial correlations in income.[1] Due to imperfect information and incomplete contracts, agents who

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[1] See, e.g., Bergstrand (1985) and Frankel (1998).

are unknown to each other tend to be less likely to trade. Because of legal constraints, agents cannot choose to live and work wherever they want. And even if they could, transportation costs, cultural differences or language barriers may keep them from moving, or may bias them towards consumption of home-produced goods and services.

In the past two decades, a large literature has emerged that has identified many of the reasons why agents may be unable or unwilling to trade with each other.[2] However, not much research has been done that has focussed on the consequences, rather than the causes, of non-global trade. This chapter makes a start with such an analysis, and studies the effects that local trade networks can have on the spatial distribution of unemployment.

One of the most striking features of spatial unemployment distributions in large cities is that high levels of unemployment tend to be clustered together in geographically contiguous areas.[3] Such spatial correlations in unemployment have often been explained by ‘neighborhood effects’, which can take many forms. For example, neighbors could serve as role models for each other (Wilson 1997); may exert mutual ‘peer pressure’ on each other (Crane 1991); or may provide each other with information on available job openings (Montgomery 1991, Topa 2001). As a consequence, agents are more likely to find a job if more of their neighbors are employed, while they are more likely to be

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[2] See, e.g., Williamson (1985) on asset specificity, Fukuyama (1995) on trust, Kranton (1996) on search costs, or Fafchamps (1998) on reputation.

[3] E.g, on the basis of Census tracts for the city of Chicago, Conley and Topa (2001) find that “there is a strong positive and statistically significant degree of spatial dependence in the distribution of raw unemployment rates”.

unemployed the higher the unemployment rate among their neighbors.

Without denying the importance of the above explanations, this chapter shows that a similar 'neighborhood effect' could result from the existence of local trade networks. In particular, when local trade networks are characterized by effective demand externalities, Keynesian-type multiplier effects can occur that may cause unemployment to persist not just in time but also in space. While this is certainly not the only explanation for spatially persistent unemployment, one reason why these 'spatial multiplier effects' are worth studying is that they can help explain spatial correlations in unemployment not just at the level of cities but also at larger levels of aggregation.

The setup of this chapter is as follows. First, section 2 develops a Cooper and John (1988) type model with strategic complementarities in employment, i.e., the demand for labor by a given firm increases with the labor demand by other firms. Intuitively, these complementarities result from the fact that, the more workers are employed by a given firm, the more income these workers have to spend, which increases the effective demand for the goods produced by their trade partners, and hence leads to an increase in the trade partners' demand for labor.

Section 3 discusses the equilibrium properties of the model under various assumptions. It is shown that, if labor can be hired in continuous quantities, then for any trade network topology, the long run distribution of employment is uniform, i.e., there are no spatial correlations in employment. Section 4, however, finds that, under binary labor support, local trade networks can lead to spatial unemployment clusters that can persist indefinitely. As section 5 shows, this is true even in the presence of noise, in which case local trade networks in combination with a strong 'home bias' in trade can still generate spatially persistent unemployment. Section 6 concludes this chapter.

2 The Model

We consider an economy that is situated on a two-dimensional lattice $S \subseteq \mathbb{Z}^2$. Each site of the lattice is inhabited by one (representative) worker and one (representative) firm, who are indexed by their coordinates $x = (i, j)$.[4] There are three goods in the economy: a labor commodity η_t, a tradable market good q_t, and a nontradable 'home-made' good h_t. All goods are nonstorable. Labor is hired at the beginning of each period, and is used to produce the other two goods, which are sold at the end of each period. Notation is as follows:

$\eta_t^d(x)$	demand for labor to be employed during period t at site x
$\eta_t^s(x)$	supply of labor to be employed during period t at site x
$\eta_t(x)$	labor employed during period t at site x
η_t	employment configuration during period t
$\hat{\eta}_t$	economy-wide employment rate during period t

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[4] Alternatively, each worker-firm pair could be thought of as a single representative agent, e.g., a social planner. This interpretation will be used in section 3.3. For now, however, we will treat the worker and the firm as separate entities, possibly representing a unit mass of workers and a unit mass of firms.

$q_t^d(x)$	demand for market goods at the end of period t at site x
$q_t^s(x)$	supply of market goods at the end of period t at site x
$q_t(x)$	market goods sold at the end of period t at site x
$c_t(x)$	market goods consumed during period t at site x
$h_t(x)$	home-made goods produced and consumed during period t at site x

2.1 Trade Structure

The trade structure of the economy is represented as a matrix $\mathbf{P}$ with elements $p(x, y)$, denoting the fraction of income spent by worker x on the goods produced by firm y. A trade network N_x is then defined as the set of trade partners of x, i.e., the set of workers y who have a positive demand for the market goods produced by x:

$$N_x = \{y : p(y, x) > 0\}. \tag{1}$$

To simplify the analysis, $\mathbf{P}$ is taken as exogenous, i.e., the selection of trade partners is not explicitly modeled.[5] Initially, we only impose two restrictions on $\mathbf{P}$. First of all, we assume that, for all x, $p(y, x) > 0$ for some $y \neq x$, i.e., each agent is able to trade

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[5] A simple way to 'derive' $\mathbf{P}$ would be to think of each firm as producing a different good, and have workers maximize a Cobb-Douglas utility function, the coefficients of which represent the equilibrium fractions of income that each worker would like to spend on each good. A more interesting way to endogenize the trade network structure would be to let the selection of trade partners be determined by differentials in price levels and transportation costs. We leave this as a suggestion for future work.

with at least one other agent. Secondly, it is assumed that, due to transportation or travel costs, the volume of trade between x and y is decreasing with their distance, i.e., $p(x, y) = \psi(\| x - y \|) \in [0, 1]$, where $\| x - y \|$ denotes the Euclidean metric, and $\psi(\cdot)$ is monotonically decreasing. Since $\psi(\cdot)$ is the same for all agents, this implies that $p(x, y) = p(y, x)$, i.e., $\mathbf{P}$ is symmetric. Moreover, since the agents are located on a lattice, the two assumptions combined imply that each agent is able to trade with at least her four nearest neighbors (since each nearest neighbor is equidistant).

While these are the only restrictions on $\mathbf{P}$ that will be made for now, starting from section 4 we will focus on two specific trade structures, which are defined as follows:

- Global trade networks: $N_x = S$
- Local trade networks: $N_x = \{y : \| x - y \| \leq r\}$.

If trade networks are global, each agent is able to trade directly with any other agent. In local trade networks, on the other hand, trade takes place only between 'nearest neighbors', i.e., agents that are within some finite Euclidean distance r of each other. This distance r will be referred to as the 'range' of the trade network.

2.2 Endowments and Technology

Each worker is endowed with one homogeneous unit of labor time per period, of which $\eta_t(x)$ is allocated to market production, while the remaining time $1 - \eta_t(x)$ is devoted to home production. The production processes for both activities are constant returns to scale, but market production is assumed to have a higher marginal productivity. Normalizing the marginal productivity of market production to one, this gives a pair of production functions for each x:

$$q_t(x) = \eta_t(x) \tag{2}$$

$$h_t(x) = \sigma(1 - \eta_t(x)), \tag{3}$$

where $\sigma < 1$ indicates the marginal productivity of home production.

In this section and section 3, the model is analyzed under the assumption that agents can choose to work any amount of time they want, i.e., $\eta_t(x) \in [0, 1]$. Later on, sections 4 and 5 will focus on the binary case where $\eta_t(x) \in \{0, 1\}$, i.e., during any given period, a given worker is involved either in market production or in home-production.

2.3 Workers

While home-made goods take more time to produce than market goods, from the perspective of consumption they will be considered perfect substitutes. That is, we assume that there is no essential difference in quality between, say, an apple grown in a worker's own garden and an apple bought at the market.[6] This implies that each worker's pref-

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[6] Alternatively, we could assume that home-made goods and market goods are perfect substitutes at a rate different from unity, for instance, home-made goods could be assumed to generate twice as much utility as goods purchased in the market. However, this would not change any of the results.

erences can be represented by a utility function of the form

$$u_t(x) = \phi(h_t(x) + c_t(x)), \tag{4}$$

where $\phi(\cdot)$ is a monotonically increasing function, $h_t(x)$ denotes the amount of home-made goods, and $c_t(x)$ denotes the amount of consumption goods purchased in the market.

Since home-made goods are nontradable, market goods can only be purchased in exchange for income earned in the market which, in real terms, is simply the real wage $w_t(x)$ times the amount of labor sold to the firm, $\eta_t(x)$. To simplify the analysis, it is assumed that borrowing or saving is impossible, and that the firm is owned by the worker, i.e., the firm's profits $\pi_t(x)$ are transferred to the worker at the end of each period in the form of dividends.[7] At the beginning of each period, then, each worker solves the following problem:

$$\begin{aligned} \max_{h_t(x), c_t(x)} \quad & \phi(h_t(x) + c_t(x)) \\ \text{s.t.} \quad & c_t(x) \leq w_t(x)\eta_t(x) + \pi_t(x) \\ & h_t(x) \leq \sigma(1 - \eta_t(x)). \end{aligned} \tag{5}$$

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[7] Since the worker takes these profits as given, this has no effect on the worker's decisions.

This gives the following labor supply curve:

$$\eta_t^s(x) \begin{cases} = 0 & \text{if } w_t(x) < \sigma \\ \in [0,1] & \text{if } w_t(x) = \sigma \\ = 1 & \text{if } w_t(x) > \sigma. \end{cases} \tag{6}$$

2.4 Firms

At the end of each period, each firm chooses the amount of labor it wishes to hire in the next period, in order to maximize expected profits:

$$\begin{aligned} \max_{\eta_t(x)} \quad & E_{t-1}\pi_t(x) = E_{t-1}q_t(x) - w_t(x)\eta_t(x) \\ \text{s.t.} \quad & q_t^s(x) \leq \eta_t(x) \\ & q_t(x) = \min\{q_t^d(x), q_t^s(x)\}, \end{aligned} \tag{7}$$

where E_{t-1} is the subjective expectations operator, conditional on information at the end of period $t-1$.

The first constraint says that the supply of market goods must be within the production possibility set. This constraint will bind for any solution to the profit maximization problem, since it is always optimal to produce as much as possible given the amount of labor that is hired. The second constraint says that firms cannot sell as much as they want, implying that firms may be 'quantity constrained'. Assuming that the trade structure **P** is common knowledge, the firm at x should expect the demand for its goods to be

$$E_{t-1}q_t^d(x) = \sum_{y \in S} p(y,x) E_{t-1}c_t(y), \tag{8}$$

where $E_{t-1}c_t(y)$ denotes the expected total amount of consumption goods purchased by y, a fraction $p(y,x)$ of which are purchased from x. Substituting equation (8) and the binding constraint $y_t^s(x) = \eta_t(x)$ into the firm's objective function gives the unconstrained maximization problem:

$$\max_{\eta_t(x)} \quad \min \left\{ \sum_{y \in S} p(y,x) E_{t-1}c_t(y), \eta_t(x) \right\} - w_t(x)\eta_t(x). \tag{9}$$

To find the solution to this problem, first note that, if $w_t(x) > 1$, expected profits are always negative, hence in this case profits are maximized at $\eta_t(x) = 0$. Secondly, whenever $w_t(x) < 1$, it is always optimal to set $\eta_t(x) = \sum_y p(y,x)E_{t-1}c_t(y)$. This gives the following labor demand curve:

$$\eta_t^d(x) \begin{cases} = \sum_y p(y,x) E_{t-1}c_t(y) & \text{if } w_t(x) < 1 \\ \in \{0, \sum_y p(y,x) E_{t-1}c_t(y)\} & \text{if } w_t(x) = 1 \\ = 0 & \text{if } w_t(x) > 1. \end{cases} \tag{10}$$

2.5 Labor Market Equilibrium

Definition 1 *A* ***labor market equilibrium*** *of this model is a set of allocations* $\eta_t(x)$, $q_t(x)$, $c_t(x)$, *and* $h_t(x)$, *a trade structure* $\mathbf{P}$, *real wages* $w_t(x)$, *and profits* $\pi_t(x)$, *such that the labor market at site* $x \in S$ *is in equilibrium. This requires that:*

(i) given expectations $E_{t-1}c_t(y)$, *trade structure* $\mathbf{P}$, *and wages* $w_t(x)$, *the firm at* x *chooses* $\eta_t^d(x)$ *and* $q_t^s(x)$ *so as to solve its profit maximization problem;*

(ii) taking profits $\pi_t(x)$ *and wages* $w_t(x)$ *as given, the worker at* x *chooses* $\eta_t^s(x)$, $c_t(x)$, *and* $h_t(x)$ *so as to solve her utility maximization problem;*

(iii) the labor market clears: $\eta_t^d(x) = \eta_t^s(x) = \eta_t$.

Proposition 1 (Labor Market Equilibrium)

If all labor markets are in equilibrium, then it must be true that, for all $x, y \in S$,

$$\eta_t(x) = \sum_{y \in S} p(y, x) E_{t-1}\eta_t(y). \tag{11}$$

Proof: If all labor markets are in equilibrium, then the profits of each firm y equal $\pi_t(y) = \eta_t(y) - \eta_t(y)w_t(x)$, and so total consumption at site y equals

$$c_t(y) = w_t(y)\eta_t(y) + \pi_t(y) = \eta_t(y). \tag{12}$$

Substituting this into the labor demand function for firm x gives:

$$\eta_t^d(x) \begin{cases} = \sum_y p(y,x)E_{t-1}\eta_t(y) & \text{if } w_t(x) < 1 \\ \in \{0, \sum_y p(y,x)E_{t-1}\eta_t(y)\} & \text{if } w_t(x) = 1 \\ = 0 & \text{if } w_t(x) > 1. \end{cases} \tag{13}$$

To show that any labor market equilibrium must satisfy $\eta_t(x) = \sum_y p(y,x)E_{t-1}\eta_t(y)$ for all $x, y \in S$, we need to check five possible cases:

i. Suppose $w_t(x) \in [0, \sigma\rangle$. Then $\eta_t^s(x) = 0$, whereas $\eta_t^d(x) = \sum_y p(y,x)E_{t-1}\eta_t(y)$. This can be an equilibrium when $E_{t-1}\eta_t(y) = 0$ for all y.

ii. Suppose $w_t(x) = \sigma$. Then $\eta_t^s(x) \in [0,1]$, whereas $\eta_t^d(x) = \sum_y p(y,x)E_{t-1}\eta_t(y)$. This can be an equilibrium for any $E_{t-1}\eta_t(y) \in [0,1]$.

iii. Suppose $w_t(x) \in \langle\sigma, 1\rangle$. Then $\eta_t^s(x) = 1$, whereas $\eta_t^d(x) = \sum_y p(y,x)E_{t-1}\eta_t(y)$. This can be an equilibrium when $E_{t-1}\eta_t(y) = 1$ for all y.

iv. Suppose $w_t(x) = 1$. Then $\eta_t^s(x) = 1$, whereas $\eta_t^d(x) \in \{0, \sum_y p(y,x)E_{t-1}\eta_t(y)\}$. This can be an equilibrium when $\eta_t^d(x) \neq 0$ and $E_{t-1}\eta_t(y) = 1$ for all y.

v. Suppose $w_t(x) > 1$. Then $\eta_t^s(x) = 1$, whereas $\eta_t^d(x) = 0$. Obviously, this cannot be an equilibrium.

Since $\eta_t^d(x) = 0$ can never be an equilibrium, any labor market equilibrium must satisfy

$$\eta_t^s(x) = \eta_t^d(x) = \sum_y p(y,x) E_{t-1}\eta_t(y) = \eta_t^*(x) \tag{14}$$

An example of such a labor market equilibrium is plotted in figure 1. □

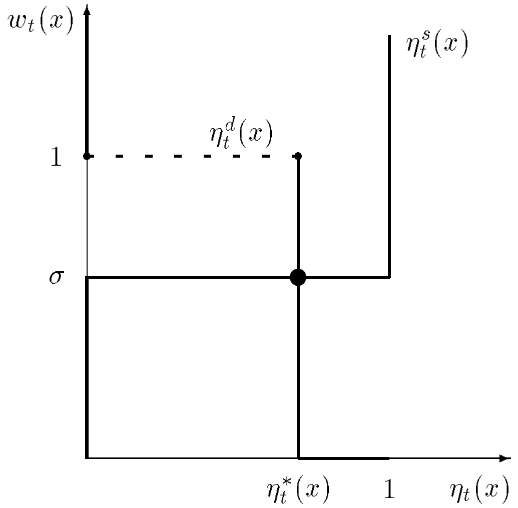

Figure 1: Labor Market Equilibrium.

3 Equilibrium Properties

The previous section showed that, when all labor markets are in equilibrium, the employment distribution must satisfy

$$\eta_t(x) = \sum_{y \in S} p(y,x) E_{t-1}\eta_t(y), \qquad \forall x, y \in S. \tag{15}$$

In game theoretic terms, this equation can be interpreted as a 'best response function', i.e., it denotes the optimal 'response' by x to the expected choices of others, $E_{t-1}\eta_t(y)$. Since this best response function is positively sloped, there exist 'strategic complementarities' in employment (Cooper and John 1988). That is, if firm x expects the employment rate at any site $y \in N_x$ to increase, then this creates a higher expected demand for the goods produced by x, which in turn increases the demand for labor in x.

It is important to note, however, that even when all labor markets are in equilibrium, this does not yet imply that expectations are correct, i.e., it is not necessarily a Nash equilibrium of the model. Moreover, it also does not imply that all goods markets clear, i.e., it is not necessarily a general equilibrium. Interestingly, a Nash equilibrium in this model is equivalent to a general equilibrium: when firms can correctly predict the demand for their goods, they will be able to avoid any surpluses or shortages, hence goods markets will clear. For simplicity, we will refer to this situation simply as an 'equilibrium' of the model. This gives the following definition:

Definition 2 *An* ***equilibrium*** *of this model is a set of allocations* $\eta_t(x)$*,* $q_t(x)$*,* $c_t(x)$*, and* $h_t(x)$*, a trade structure* $\mathbf{P}$*, real wages* $w_t(x)$*, and profits* $\pi_t(x)$*, such that:*

(i) all labor markets are in equilibrium;

(ii) all goods markets are in equilibrium.

In this section we will analyze the properties of this equilibrium for general trade structures $\mathbf{P}$, finite S, and specific assumptions on expectations formation. As a benchmark case, section 3.1 first analyzes the equilibrium properties under the assumption of *rational expectations*, i.e., the assumption that subjective expectations equal mathematical expectations. Since the model is deterministic so far, this implies perfect foresight:[8]

$$E_{t-1}\eta_t(y) = E\eta_t(y) = \eta_t(y).$$

Under this assumption, the model is shown to be essentially static, in that there are no temporal correlations between the equilibrium employment levels in different periods. Section 3.2 will discuss a more interesting, dynamic version of the model, in which firms are assumed to have *adaptive expectations*:

$$E_{t-1}\eta_t(y) = \eta_{t-1}(y).$$

Under this assumption, the distribution of employment follows a Markov process, which

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[8] In sections 4 and 5, stochastic versions of the model are analyzed, in which case rational expectations are no longer equivalent to perfect foresight.

is shown to converge to a rational expectations equilibrium.

3.1 Equilibrium Properties under Rational Expectations

Definition 3 *A* ***symmetric equilibrium*** *is an equilibrium for which*

$$\eta_t(x) = \eta_t^*, \quad \forall x \in S. \tag{16}$$

Proposition 2 (Rational Expectations Equilibria)

Let $\eta_t(x) \in [0.1]$, let firms have rational expectations, and let S be finite. Then,

a. any $\eta_t^* \in [0,1]$ constitutes a symmetric equilibrium;

b. there exist no asymmetric equilibria.

Proof: Under rational expectations, the equilibrium demand for labor is given by

$$\eta_t(x) = \sum_{y \in S} p(y,x)\eta_t(y), \quad \forall x, y, t. \tag{17}$$

Now define $\mu_t(x) \equiv \frac{\eta_t(x)}{\sum_y \eta_t(y)}$. This gives as the relative employment distribution a system of $|S|$ linear equations in $|S|$ unknowns, which in matrix notation can be written as

$$\mu_t = \mathbf{P}'\mu_t, \tag{18}$$

where μ_t is a row vector with elements $\mu_t(x)$, and $\mathbf{P}'$ denotes the transpose of $\mathbf{P}$.

Since both the rows and the columns of $\mathbf{P}$ sum to one, this is a doubly stochastic matrix, which is known to have the uniform distribution as its invariant measure. To see this, suppose that $\mu_t(x) = |S|^{-1}$ for all $x \in S$. Then we must have $\eta_t(x) = \eta_t^*$ for all $x \in S$, where $\eta_t^* = |S|^{-1} \sum_x \eta_t(x)$ denotes the economy-wide employment rate. Substituting this into the labor demand function for x gives $\eta_t(x) = \sum_y p(y,x)\eta_t^* = \eta_t^*$, which confirms that any $\eta_t^* \in [0,1]$ constitutes a symmetric equilibrium.

To prove (b), note that in equation (18), $\mathbf{P}$ can be interpreted as a transition matrix, and μ_t as a probability vector.[9] The μ_t^* that solves equation (18) is an invariant distribution, which can be interpreted as a steady state distribution of the Markov chain $\mu_t = \mathbf{P}'\mu_{t-1}$. A state of this Markov chain is a site x, hence its state space equals the set of sites $S \subset \mathbb{Z}^2$.

It is well known that, if a Markov chain is irreducible and positive recurrent, its invariant distribution is unique.[10] In order for the state space S to be irreducible, $\mathbf{P}$ must be such that it is possible for each worker to trade either directly or indirectly with any other worker in the economy, so that there exists no closed economies, i.e., no subset of agents that only trade with each other.[11] This condition is satisfied under the given assumptions on $\mathbf{P}$, which imply that each x is able to trade with at least its four

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[9] In this interpretation, $p(y,x)$ denotes the probability that y's income is transferred to x, and μ_t is the relative distribution of income, i.e., $\mu_t(x)$ is the probability that all income is located at site x.

[10] E.g., Hoel et al. (1972: 64), theorem 5.

[11] Another way of saying this is that S must be a 'connected graph'. Formally, a graph G consists of a nonempty set $V(G)$ of vertices, and a list $E(G)$ of unordered pairs of these vertices, called edges. In our case vertices are called 'sites' and an edge of the form xy indicates that x and y are able to trade with each other, i.e., $p(x,y) = p(y,x) > 0$. In a connected graph, any site can be reached from any other site by a path consisting of only a finite number of edges (trades).

nearest neighbors (see section 2.1). If all workers are able to trade with their nearest neighbors then, indirectly, each worker is able to trade with any other worker, hence S is irreducible.

Next we need to show that all states in S are positive recurrent, i.e., if we consider some initial income owned by x, then the mean return time of this income to x must be finite. Under the assumption that S is finite, there must be at least one positive recurrent x, since it is impossible for all initial income never to return to anyone. Moreover, since S is irreducible, the fact that at least one x must be positive recurrent implies that all states in S are positive recurrent.[12] Given irreducibility and positive recurrence, we can conclude that the uniform distribution constitutes the unique steady state distribution of $\mathbf{P}$, hence asymmetric equilibria cannot exist. $\square$

To summarize, proposition 2 implies that, when firms have rational expectations, each worker must have the same level of employment at each moment in time along each equilibrium path, i.e., spatial correlations in employment cannot exist. Moreover, since the employment levels in different periods are mutually independent, this model cannot explain temporal correlations in employment rates either. Along each equilibrium path, the economy is in equilibrium at all times, but it is possible that each period a new equilibrium is selected.

These conclusions are in contrast with the fact that we do observe high serial cor-

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[12] E.g., Hoel et al. (1972: 61-2), theorems 2 and 3.

relations (even 'unit roots') in a typical employment or GDP time series.[13] But they are also disconcerting from a theoretical point of view. What is missing is a theory of equilibrium selection, and an explanation of how it is possible that a large number of agents could possibly coordinate on any given equilibrium.

As evolutionary game theorists have pointed out,[14] the strict rationality assumptions that might be reasonable consistency requirements in one-shot two player games become quite untenable when applied to dynamic games with a large number of players that repeatedly encounter similar situations. In such cases, of which this model is an instance, it might be more reasonable to assume that agents have adaptive expectations, i.e., they base their expectations of others' behavior on the past. The next section will explore the implications of this assumption, which will introduce a dynamic element into the model.

3.2 Equilibrium Properties under Adaptive Expectations

Proposition 3 (Adaptive Expectations Equilibria)

Let $\eta_t(x) \in [0.1]$, let firms have adaptive expectations, and let S be finite. Then,

a. any steady state of the adaptive expectations model is a rational expectations equilibrium;

b. when $p(x,x) > 0$ for some x, $\lim_{t\to\infty} \eta_t(x) = \hat{\eta}_0, \forall x$.

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[13] Nelson and Plosser (1982); Blanchard and Summers (1986).

[14] E.g., Samuelson (1997).

Proof: Under adaptive expectations, the amount of labor employed by a given firm is a weighted average of the amounts of labor employed by its trade partners in the last period:

$$\eta_t(x) = \sum_{y \in S} p(y, x)\eta_{t-1}(y). \tag{19}$$

Again defining $\mu_t(x) \equiv \frac{\eta_t(x)}{\sum_y \eta_t(y)}$, the temporal evolution of the employment distribution is given by a Markov chain:

$$\mu_t = \mathbf{P}'\mu_{t-1}, \quad \forall t. \tag{20}$$

A steady state distribution of this chain must satisfy

$$\mu^* = \mathbf{P}'\mu^*, \tag{21}$$

and therefore, by proposition 2, must be a rational expectations equilibrium.

To prove (b), first observe that, by proposition 2, the steady state distribution is unique. Uniqueness, however, is not yet a sufficient condition for convergence. In order for the vector μ_t to converge to the steady state distribution μ^*, we must ensure that the Markov chain is *aperiodic.* This means that there should not exist a period $d > 1$ such that

$$\mu_t = \mathbf{P}'\mu_{t-d}, \forall t. \tag{22}$$

Given that the Markov chain is irreducible, a sufficient condition for it to be aperiodic is that $p(x, x) > 0$ for some $x \in S$, i.e. it should be possible for at least one worker to consume her own market goods. By the ergodic theorem for Markov chains, if an irreducible and positive recurrent Markov chain is aperiodic, it is *ergodic*, which implies that the chain will converge to its unique uniform steady state distribution.

While the system is thus ergodic in terms of the relative steady state distribution of employment, it is non-ergodic in absolute terms, i.e., different initial conditions imply different steady state levels of employment. To see this, note that equation 19 implies that the total number of employed agents never changes:

$$\sum_{x \in S} \eta_t(x) = \sum_{x \in S} \sum_{y \in S} p(y, x) \eta_{t-1}(y) = \sum_{y \in S} \eta_{t-1}(y), \qquad \forall x, y, t. \tag{23}$$

and therefore the average amount of employment in the economy, $\hat{\eta}_t = |S|^{-1} \sum_x \eta_t(x)$, must be the same at any t. This implies that, in any steady state of this model, $\eta_t(x) = \hat{\eta}_t = \hat{\eta}_0$ for all $x \in S$. □

3.3 Efficiency of Equilibria

In order to judge whether the equilibria (or steady states) of the model are efficient, this section will analyze a 'planner' version of the model, in which each site is inhabited by a single representative agent (e.g., a social planner) who maximizes the worker's utility, subject to the constraints faced earlier by both the worker and the firm:[15]

$$\begin{aligned} \max_{h_t(x),c_t(x)} \quad & \phi(h_t(x) + c_t(x)) \\ \text{s.t.} \quad & h_t(x) \leq \sigma(1 - \eta_t(x)) \\ & q_t^s(x) \leq \eta_t(x) \\ & q_t^d(x) = \sum_{y \in S} p(y,x) E_{t-1} q_t(y) \\ & c_t(x) \leq \min\{q_t^s(x), q_t^d(x)\}. \end{aligned} \tag{24}$$

The first and second constraints are the production functions for home-made and market goods, respectively. The third constraint states, as before, that the demand for market goods produced at x depends on the network structure $\mathbf{P}$ and the output produced at sites y. Finally, the fourth constraint follows from the assumption that home-made goods are nontradable, which implies that the consumption of market goods cannot exceed the production of market goods.

Since all constraints will bind at any solution to the planner's problem, they can be substituted into the objective function, which yields the following maximization

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[15] Note that a solution to this planner's problem is efficient only in the sense that it maximizes the utility of a representative agent at a given site, given the behavior of other representative agents at all other sites. The only efficient solution for the economy as a whole is obviously for all agents to be employed at all times.

problem:

$$\max_{\eta_t(x)} \quad \phi\left((1-\eta_t(x))\sigma + \min\{\eta_t(x), \sum_{y\in S} p(y,x)E_{t-1}\eta_t(y)\}\right). \tag{25}$$

The solution to this problem gives the following 'best response correspondence':

$$\eta_t(x) \quad \begin{cases} = 0 & \text{if } \sum_y p(y,x)E_{t-1}\eta_t(y) < \sigma \\ \in \{0, \sum_y p(y,x)E_{t-1}\eta_t(y)\} & \text{if } \sum_y p(y,x)E_{t-1}\eta_t(y) = \sigma \\ = \sum_y p(y,x)E_{t-1}\eta_t(y) & \text{if } \sum_y p(y,x)E_{t-1}\eta_t(y) > \sigma. \end{cases} \tag{26}$$

Proposition 4 (Efficient Rational Expectations Equilibria)

Let $\eta_t(x) \in [0.1]$, let firms have rational expectations, and let S be finite. Then,

a. any $\eta_t^* \in \{0\} \cup [\sigma, 1]$ constitutes an efficient symmetric equilibrium;

b. there exist no asymmetric equilibria.

Proof: In a symmetric equilibrium, $\eta_t(y) = \eta_t^*, \forall y \in S$. Substituting this into equation (26) gives

$$\eta_t(x) \quad \begin{cases} = 0 & \text{if } \eta_t^* < \sigma \\ \in \{0, \hat{\eta}_t\} & \text{if } \eta_t^* = \sigma \\ = \hat{\eta}_t & \text{if } \eta_t^* > \sigma. \end{cases} \tag{27}$$

This best response correspondence is plotted in figure 2. A symmetric equilibrium must satisfy $\eta_t(x) = \eta_t^*$ and therefore must lie on the 45-degree line. As figure 2 illustrates, this implies that the set of symmetric equilibria is given by $\eta_t^* \in \{0\} \cup [\sigma, 1]$.

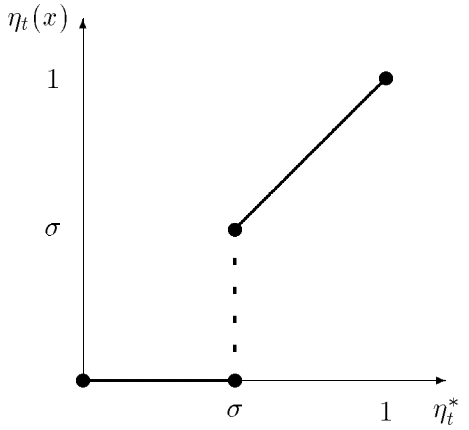

Figure 2: Best Response Correspondence for the Planner's Problem

To prove (b), first consider the case $\sum_y p(y,x)\eta_t(y) \geq \sigma$. By (26), the best response for a planner at site x in this case is $\eta_t(x) = \sum_y p(y,x)\eta_t(y)$.[16] By proposition 2(b), this equation cannot have asymmetric equilibria under the given assumptions.

Next consider the case $\sum_y p(y,x)\eta_t(y) < \sigma$, which gives as a best response $\eta_t(x) = 0$. The condition $\sum_y p(y,x)\eta_t(y) < \sigma$ requires that there is at least one $y : p(y,x) > 0$ for which $\eta_t(y) < \sigma$. However, the best response correspondence (26) implies that $\eta_t(y) \notin \langle 0, \sigma \rangle$. Therefore, $\sum_y p(y,x)\eta_t(y) < \sigma$ if and only if there is at least one $y : p(y,x) > 0$ for which $\eta_t(y) = 0$. But if unemployment is induced as a best response whenever x has at least one trade partner y who is unemployed, then all other trade partners of y must also have unemployment as their best response, and so must all trade

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[16] While for $\sum_y p(y,x)\eta_t(y) = \sigma$, there exists another best response, this is irrelevant for the existence of Nash equilibria.

partners of those trade partners, etc. Since $\mathbf{P}$ is irreducible, this means that if one agent is unemployed, all agents must be unemployed. As before, the condition that S is finite guarantees uniqueness of this equilibrium. $\square$

Proposition 5 (Efficient Adaptive Expectations Equilibria)

Let $\eta_t(x) \in [0,1]$, let firms have adaptive expectations, and let S be finite. Then,

a. any steady state of the adaptive expectations model is a rational expectations equilibrium;

b. when $p(x,x) > 0$ for some x, the distribution of employment converges to a rational expectations equilibrium.

Proof: The proof is parallel to that of proposition 3. First of all, for the same reasons as before, each rational expectations equilibrium constitutes a steady state of the adaptive expectations model. Secondly, the restrictions on $\mathbf{P}$ guarantee irreducibility, and the fact that S is finite guarantees positive recurrence, thus implying uniqueness of the steady state distribution. Finally, the condition $p(x,x) > 0$ for some x guarantees aperiodicity, which implies convergence. $\square$

Proposition 5 implies that the set of equilibria of the planner's problem is a subset of the set of market equilibria. This suggests that the remaining set of market equilibria is inefficient, i.e., those equilibria with $\eta_t^* \in \langle 0, \sigma \rangle, \forall x$. The reason for this market imperfection is that workers, as opposed to planners, take profits as given, and do not realize that $\eta_t(x) = w_t(x)\eta_t(x) + \pi_t(x)$. Therefore, they do not realize that $\eta_t(x) \in \langle 0, \sigma \rangle$ implies that $w_t(x)\eta_t(x) + \pi_t(x) < \sigma$, i.e. the marginal utility of working for the market is less than the marginal utility of working at home. A social planner knows this, and therefore will never set $\eta_t(x) \in \langle 0, \sigma \rangle$.

4 Employment Games

The previous section has shown that, if labor support is continuous, it is impossible for spatial correlations in employment to exist in the long run, independently of the structure of trade networks. In this section, we will analyze the model under the assumption that $\eta_t(x) \in \{0, 1\}$, i.e., during any given period, a given worker is either employed (full time involved in market production) or unemployed (full time involved in home-production). As will be shown below, this assumption turns the model into a simple coordination game that can be interpreted as a spin system, i.e., an interacting particle system with state space $\{0, 1\}^S$.

As before, the equilibrium properties of this model will be analyzed for both rational and adaptive expectations. In this case, rational expectations imply that agents know the probability with which each agent becomes employed.

$$E_{t-1}\eta_t(y) = E\eta_t(y) = p_{y,t}, \tag{28}$$

where $p_{y,t} = \Pr(\eta_t(y) = 1)$. Note that this is not the same as perfect foresight, since knowing the probability with which an agent becomes employed is generally not sufficient to predict the actual employment state of that agent (unless, of course, the agent is employed with probability zero or one). Under adaptive expectations we have, as before,

$$E_{t-1}\eta_t(y) = \eta_{t-1}(y). \tag{29}$$

Since in general, $\eta_{t-1}(y) \neq p_{y,t-1}$, it is now not guaranteed anymore that a rational expectations equilibrium constitutes a steady state of the adaptive expectations model. As is shown in section 4.3, this is the case only when trade networks are global and the number of agents is infinite, so that the law of large numbers can be applied. When trade networks are local, however, the behavior of the model changes quite radically and, in particular, can lead to spatially persistent unemployment.

4.1 Maximization Problem and Best Responses

In order to eliminate inefficient equilibria, we continue to consider the 'planner' version of the model, in which each site of the lattice is inhabited by a single agent who maximizes the following expected utility function:

$$\max_{\eta_t(x)} \quad E_{t-1}u_t(x) = \phi\left((1-\eta_t(x))\sigma + \min\{\eta_t(x), \sum_{y \in S} p(y,x) E_{t-1}\eta_t(y)\} \right). \tag{30}$$

The main difference with the planner's problem discussed in the previous section is that here, $\eta_t(x) \in \{0, 1\}$, i.e., there are only two possible employment states: $\eta_t(x) = 0$ and $\eta_t(x) = 1$. Moreover, we will assume here that agents can choose to be employed with a certain probability $p_{x,t} = \Pr(\eta_t(x) = 1)$, i.e., in game theoretic terms, they can decide to play a 'mixed strategy'.

Assuming for simplicity that $\phi(\cdot)$ is the identity function, the levels of expected utility associated with each employment state, unemployment or employment, can be expressed, respectively, as:

$$E_{t-1}u_t^0(x) = \sigma \tag{31}$$

$$E_{t-1}u_t^1(x) = \sum_{y \in S} p(y, x) E_{t-1}\eta_t(y) \tag{32}$$

As before, the expected utility associated with unemployment, $E_{t-1}u_t^0(x)$, equals σ no matter how expectations are formed or what other agents are doing. The expected utility associated with full employment on the other hand, $E_{t-1}u_t^1(x)$, equals the expected proportion of employed workers among the trade partners of agent x.

In binary space, the fraction $p(y, x)$ of income spent by y at x can be equivalently interpreted as the probability that agent x will be 'matched' with agent y, who then has the opportunity to trade its market goods with x. Assuming that x has an equal chance to be matched with each $y \in N_x$, this implies the following restriction on $\mathbf{P}$:

$$p(y,x) = \begin{cases} 0 & \text{if } y \notin N_x \\ |N_x|^{-1} & \text{if } y \in N_x, \end{cases} \tag{33}$$

where $N_x = \{y : p(y,x) > 0\}$ denotes x's trade network, and $|N_x|$ denotes the cardinality of the set N_x, i.e., the number of trade partners of x. Under this assumption, we can write:

$$\sum_{y \in S} p(y,x)\eta_t(y) = |N_x|^{-1} \sum_{y \in N_x} \eta_t(y) \equiv \eta_t(\hat{y}) \tag{34}$$

Using this notation, the solution to the utility maximization problem yields the following best response correspondence:

$$\Pr(\eta_t(x) = 1) \quad \begin{cases} = 0 & \text{if } E_{t-1}\eta_t(\hat{y}) < \sigma \\ \in [0,1] & \text{if } E_{t-1}\eta_t(\hat{y}) = \sigma \\ = 1 & \text{if } E_{t-1}\eta_t(\hat{y}) > \sigma. \end{cases} \tag{35}$$

The binary model can now be alternatively interpreted as stating that each period, each agent x is matched with a random trade partner, denoted by $\hat{y}$, after which the two agents play an "Employment Game", the payoff matrix of which is represented in table 1. Again, the utility associated with unemployment equals σ irrespective of the employment state of the random trade partner $\hat{y}$. The utility associated with employ-

	$\eta_t(\hat{y}) = 0$	$\eta_t(\hat{y}) = 1$
$\eta_t(x) = 0$	σ, σ	$\sigma, 0$
$\eta_t(x) = 1$	$0, \sigma$	$1, 1$

Table 1: The Employment Game

ment, on the other hand, equals zero if the random trade partner is unemployed, and equals one when the random trade partner is employed. In addition, we can consider the possibility that the trade partner with which x is matched plays a 'mixed strategy', i.e., is employed with some probability $p_{\hat{y},t}$. Naturally, $p_{\hat{y},t} = \eta_t(\hat{y})$, i.e, the probability that the randomly chosen trade partner $\hat{y}$ is unemployed equals the average proportion of employed agents in N_x.

Interestingly, as Blume (1997) has noted, this interpretation of mixed strategies as the 'average play' in a population coincides with that by Nash himself (1950: 21-23):

> It is unnecessary to assume that the participants have full knowledge of the total structure of the game, or the ability and inclination to go through any complex reasoning process. But the participants are supposed to accumulate empirical information on the relative advantages of the various pure strategies at their disposal ...
>
> We assume that there is a population (in the sense of statistics) of participants for each position of the game. Let us also assume that the 'average playing' of the game involves n participants selected at random from the n populations, and that there is a stable average frequency with which each pure strategy is employed by the 'average member' of the appropriate population. ... Thus the assumption we made in this 'mass-action' interpretation leads to the conclusion that the mixed strategies representing the average behavior in each of the populations form an equilibrium point.

4.2 Equilibrium Properties under Rational Expectations

Definition 4 *A* ***symmetric Nash equilibrium*** *is an equilibrium for which*

$$p_{x,t} = p_t^*, \quad \forall x \in S, \tag{36}$$

and will be denoted by the equilibrium strategy profile $\{p_t^*\}$*.*

Proposition 6 (Rational Expectations Equilibria)

Let $\eta_t(y) \in \{0, 1\}$, let $S \subseteq \mathbb{Z}^2$, and let agents have rational expectations. Then,

a. there exist three symmetric Nash equilibria: $\{0\}$, $\{\sigma\}$, and $\{1\}$;

b. there exist no asymmetric Nash equilibria.

Proof: In a symmetric Nash equilibrium, $p_{\hat{y},t} = p_{y,t} = p_t^*$ for all $\hat{y}, y$. Under rational expectations, this implies that $E_{t-1}\eta_t(\hat{y}) = p_t^*$. Substituting this into the best response correspondence gives:

$$p_{x,t} \begin{cases} = 0 & \text{if } p_t^* < \sigma \\ \in [0,1] & \text{if } p_t^* = \sigma \\ = 1 & \text{if } p_t^* > \sigma \end{cases} \tag{37}$$

This correspondence is plotted in figure 3. In a symmetric equilibrium we must also have $p_{x,t} = p_t^*$, hence any symmetric equilibrium must lie on the 45-degree line. As figure 3 illustrates, this implies that there exist three symmetric Nash equilibria: $\{0\}$,

$\{\sigma\}$, and $\{1\}$.

To prove (b), observe that the only way in which asymmetric Nash equilibria could exist is if it were possible for different closed subsets of agents to settle on different symmetric Nash equilibria. As shown before, the restrictions on $\mathbf{P}$ guarantee that S is irreducible and the fact that S is finite guarantees that all states in S are positive recurrent. This implies that there are no subsets of agents who only trade with each other, thus excluding the possibility of asymmetric equilibria. $\square$

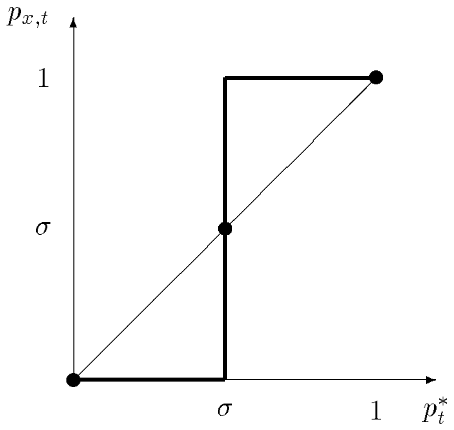

Figure 3: Best Response Correspondence for the Binary Model

4.3 Equilibrium Properties under Adaptive Expectations

Proposition 7 (Binary Adaptive Expectations Equilibria, Global Trade)

Let $\eta_t(y) \in \{0, 1\}$, let agents have adaptive expectations, and let trade networks be global: $N_x = S$. Then,

a. when S is countably infinite, the three rational expectations Nash equilibria constitute the unique steady states of the adaptive expectations model. However, the mixed Nash equilibrium constitutes an unstable steady state, so that:

$$\lim_{t\to\infty} \eta_t = \begin{cases} \{0\} & \text{if } \hat{\eta}_0 < \sigma \\ \{\sigma\} & \text{if } \hat{\eta}_0 = \sigma \\ \{1\} & \text{if } \hat{\eta}_0 > \sigma \end{cases} \tag{38}$$

b. when S is finite, the two pure Nash equilibria constitute the unique steady states, and the distribution of employment converges to one of these pure Nash equilibria.

Proof: Under global trade, each agent has an equal probability of trading with any other agent in the economy, hence $E_{t-1}\eta_t(\hat{y}) = E_{t-1}\hat{\eta}_t$. Under adaptive expectations, this implies $E_{t-1}\eta_t(\hat{y}) = \hat{\eta}_{t-1}$, which gives the following best response correspondence:

$$p_{x,t} \begin{cases} = 0 & \text{if } \hat{\eta}_{t-1} < \sigma \\ \in [0,1] & \text{if } \hat{\eta}_{t-1} = \sigma \\ = 1 & \text{if } \hat{\eta}_{t-1} > \sigma \end{cases} \tag{39}$$

Since, under global trade, all agents have the same best response correspondence, they all have the same probability p_t^* of being employed at any given time. The law of large numbers in this case implies that

$$\lim_{|S|\to\infty} \hat{\eta}_t = p_t^*. \tag{40}$$

When S is countably infinite, this gives $\hat{\eta}_{t-1} = p_{t-1}^*$. Substituting this into equation (39) yields a best response correspondence of the same form as equation (37), except p_t^* is replaced by p_{t-1}^*. In any steady state, however, we must have $p_{t-1}^* = p_t^*$, hence any rational expectations equilibrium constitutes a steady state. Uniqueness of these steady states is guaranteed by the fact that trade is global, which implies that there cannot exist closed subsets of agents that coordinate on different steady states.[17]

To see that the mixed strategy is an unstable steady state, observe that a configuration $\{\sigma\}$, in which all agents are employed with probability σ, can occur only if $\hat{\eta}_0 = \sigma$, and even then, it is only one of many best responses. However, when the initial fraction of employed agents equals $\hat{\eta}_0 = \sigma + \epsilon$, where ϵ is an arbitrarily small negative or positive number, the best response of all agents will be 0 or 1, respectively, hence the economy converges to $\{0\}$ or $\{1\}$ in one period.

Part (b) follows from the fact that, when the number of agents is finite, the law of large numbers cannot be applied. In this case, even when all agents start out playing the mixed equilibrium strategy $p_t^* = \sigma$, due to random chance the resulting average employment rate $\hat{\eta}_t$ will sooner or later be either slightly below or slightly above σ,

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[17] Note that, in this case, we cannot rely on the assumption that S is finite to prove uniqueness, since we needed S to be infinite in order to apply the law or large numbers.

in which case all agents will respond, not with σ, but with 0 or 1, respectively. This implies that the mixed Nash equilibrium cannot be a steady state of the finite model, and that this model must therefore converge to a steady state in which either all agents are employed or all agents are unemployed.[18] $\Box$

Proposition 8 (Binary Adaptive Expectations Equilibria, Local Trade) Let $\eta_t(y) \in \{0, 1\}$, let agents have adaptive expectations,let S be infinite, and let trade networks be local: $N_x = \{y : \| x - y \| \leq r\}$, where $r < \infty$. Then,

a. the two pure Nash equilibria $\{0\}$ and $\{1\}$ constitute the unique symmetric steady states;

b. when $\sigma < 0.5(1 - \frac{1}{|N_x|})$, the employment distribution converges to $\{1\}$, i.e., a symmetric steady state in which all agents are employed;

c. when $\sigma > 0.5(1 + \frac{1}{|N_x|})$, the employment distribution converges to $\{0\}$, i.e., a symmetric steady state in which all agents are unemployed;

d. when $\sigma = 0.5$, the employment distribution converges to an asymmetric steady state.

Proof: It can be easily checked that the pure Nash equilibria $\{0\}$ and $\{1\}$ constitute steady states of the adaptive expectations model even when trade networks are local.

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[18] To which steady state it will converge depends on the initial fraction of employed agents $\hat{\eta}_0$, as well as on the value of σ. For large enough S, the central limit theorem implies that the employment rate after one period, $\hat{\eta}_1$, is distributed approximately normal with mean $\hat{\eta}_0$ and variance $|S|^{-1}\hat{\eta}_0(1 - \hat{\eta}_0)$. Thus, for example, for $\sigma = 0.5$ and $\hat{\eta}_0 = 0.5$, $\Pr(\hat{\eta}_1 = 0.5 \mid \hat{\eta}_0 = 0.5)$ is negligible, and so the probability of converging to $\{1\}$ equals $\Pr(\hat{\eta}_2 = 1) = \Pr(\hat{\eta}_1 > 0.5 \mid \hat{\eta}_0 = 0.5) = 0.5$.

The fact that the mixed Nash equilibrium, or any other 'mixed' symmetric equilibria for that matter, cannot constitute a steady state follows directly from proposition 7 in that, when trade networks are local, $|N_x|$ is finite, hence the law of large numbers cannot be applied.

To prove part (b), let us consider the example of $r = 1$, in which case the number of agents in the trade network equals $|N_x| = 5$: the four nearest neighbors of x plus x itself. [19] In this case, $\sigma < 0.5(1 - \frac{1}{|N_x|})$ means $\sigma < 2/5$. For example, suppose that $sigma \in [0, 1/5\rangle$, so that agents will be employed if at least one out of five trade partners are employed (and will be unemployed only if zero out of five are employed). This rule implies that, even if there is only one employed agent at $t = 0$, this agent will continue to be employed forever, and moreover, all her trade partners will become employed at $t = 1$, the trade partners of whom will become employed at $t = 2$, and so on until the economy has reached a steady state of full employment. When $\sigma \in [1/5, 2/5\rangle$, the same convergence to full employment will occur as long as the initial distribution contains at least one trade network in which at least two out of five agents are employed.

Next, consider a larger trade network, e.g., a network with range $r = 4$. In this case, a necessary condition to converge to full employment is $\sigma < 20/41$, i.e., convergence to full employment will take place as long as the initial distribution contains at least one trade network in which at least 19 out of 41 agents are employed. [20] While this

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[19] Such networks are sometimes referred to as 'common border networks' or 'Von Neumann neighborhoods'. The idea that x can be her own trade partner simply means that it is possible for x to consume part of her own market output.

[20] The formula for the number of agents in a trade network with range r is $|N_x| = 1 + 4\sum_{i=1}^{r} i$.

requirement obviously becomes more difficult to satisfy as the range of the trade network increases, on an infinite lattice such a trade network will always exist somewhere, and it will continue to grow until it takes over the entire lattice.

The proof of part (c) is parallel to that of part (b). As for part (d), we note that, for $\sigma = 0.5$, the model is equivalent to the so-called 'majority vote model', which says that agents will change their state if and only if a majority of their neighbors have the opposite state. In one-dimensional space ($S \subseteq \mathbb{Z}$), this model has been shown to 'fixate' almost surely (Fisch and Gravner 1995, proposition 6.2). This means that $\lim_{t\to\infty} \eta_t(x)$ exists for all x, i.e., each agent changes her employment state only finitely many times, which in our terminology is equivalent to saying that the economy reaches an asymmetric steady state. While no rigorous proofs appear to exist for two-dimensional space,[21] our numerical experiments strongly suggest that the same result holds in two dimensions. The results of these experiments are reported below. □

4.4 Numerical Experiments

In order to study the behavior of the binary model under local trade, this model was simulated on the WinCA software developed by David Griffeath and Robert Fisch.[22]

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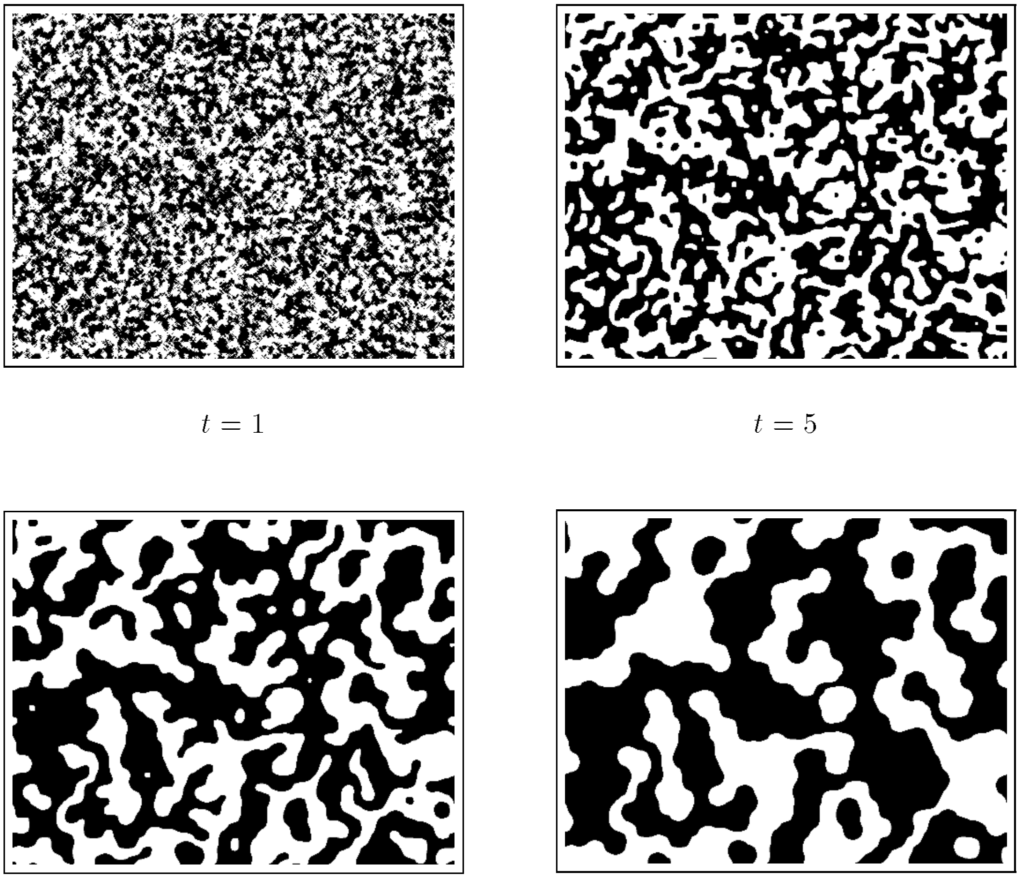
[21] However, see Durrett and Steif (1993) for properties of the 'threshold voter model' (i.e., the majority vote model with $\sigma \neq 0.5$) in higher dimensions. According to this model, agents change their state if at least a fraction σ of their neighbors (not including themselves) have the opposite state. While this appears to be very similar to the present model, the two models are not equivalent when $\sigma \neq 0.5$; moreover, most of the results are for continuous time, hence they do not apply to this case.

[22] This software is freely available at `http://psoup.math.wisc.edu/sink.html`. Those interested in replicating the numerical experiments discussed in this section may contact the author for precise information on the setup of the experiments.

For a better visualization of the lattice, each site was assumed to correspond with the center of a cell in a 512 by 400 'matrix' (square tessellation of the plane). Representing the states 'employed' and 'unemployed' by the colors black and white, respectively, each cell of this matrix was then painted the color of its corresponding site in the lattice. To ensure that each agent had an equal number of neighbors, the right and left edges of the matrix were connected, and so were the upper and lower edges, thus turning the economy into a 'torus'. Various numerical experiments were carried out, starting from different random initial configurations η_0, and varying the network range r between one and four. Figure 4 illustrates the typical evolution of the 'majority vote' model with $\sigma = 0.5$, started from a random initial configuration $\eta_0 = \{0.5\}$. The network range chosen for this experiment was $r = 4$ (i.e, $|N_x| = 41$) but very similar behavior could be observed for other ranges.

After only one update, the model can already be seen to self-organize into clusters of employed and unemployed agents. These clusters then grow and evolve by what is known as 'curvature-driven surface tension clustering', which means that clusters of unemployment that are completely surrounded by employment clusters gradually disappear, and *vice versa.* The reason why this occurs is that the boundaries between the clusters approximate (as $r \to \infty$) a fundamental nonlinear partial differential equation called 'motion by mean curvature' (Evans, Soner, and Souganidis (1992)). This means that, along any curved boundary between a cluster of employed agents and a cluster of unemployed agents, the cluster with the highest frequency grows at a rate which



$t = 1$ $t = 5$

$t = 20$ $t > 180$

Figure 4: Clustering of the Binary Model with $\sigma = 0.5$, $r = 4$, and $\eta_0 = \{0.5\}$.

decreases with the curvature of the boundary.

Since the system is ultimately discrete, the clustering process comes to a halt when the boundaries of the clusters achieve uniformly small curvature. That is, after a certain time t, the boundaries of the clusters become flat enough so that the grow rate of the clusters is reduced to zero. At this point the system fixates, i.e., it reaches an asymmetric steady state. In the particular experiment portrayed in figure 4, the asymmetric steady state was reached around $t = 180$.

Similar experiments were carried out for smaller and larger network ranges. Asymmetric steady states were reached in each case, but they contained increasingly larger spatial clusters as trade networks became more 'global'. This can be explained by the fact that, as the network range r increases, it becomes more difficult to reach flat boundaries, hence the clustering process will continue for a longer time before grinding to a halt. If r is increased while holding the total number of agents constant, the clusters will at some point of course become big enough so as to cover the entire economy. On an infinite lattice, however, this will never be the case.[23]

Figure 5 illustrates what happens when the model is started from an initial distribution $\{p_0\}$ with $p_0 \neq \{0.5\}$. As long as p_0 is still 'close enough' to 0.5 relative to r, asymmetric steady states continue to exist, although the steady state employment clusters become significantly smaller in size. The more p_0 differs from $\{0.5\}$, and the larger r relative to S, the more likely it is that the economy will converge to a symmetric steady state. As figure 5 shows, for example, this already happens for $\eta_0 = 0.45$ and $r = 4$. On an infinite lattice, however, the configuration will remain asymmetric since, no matter how large r, stable clusters will be able to form somewhere.

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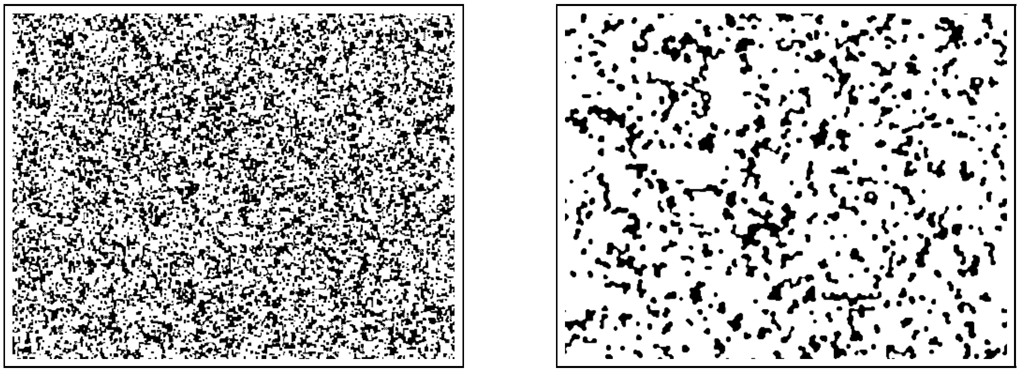
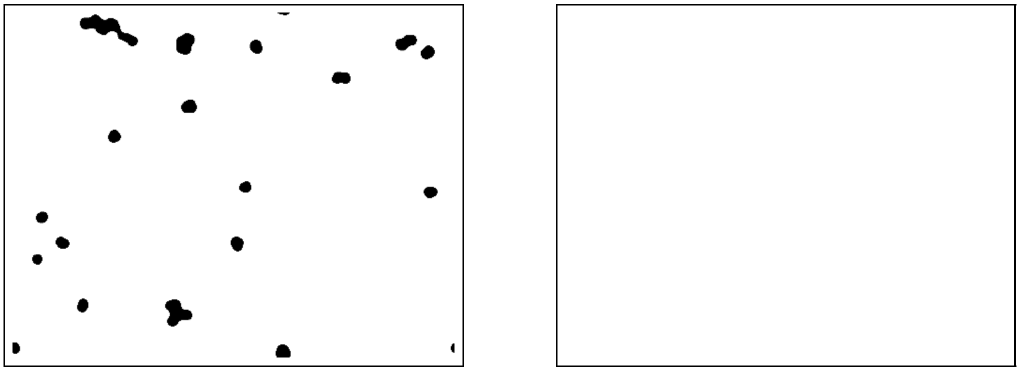
[23] One might expect that, if $r \to \infty$, while scaling the size of the lattice such that it grows faster than the size of the neighborhoods, the discrete dynamics will at some point approximate a continuous flow well enough so that the model will cluster indefinitely, i.e., it will converge to $\{0\}$ or $\{1\}$. However, Griffeath and Gravner (personal communication) have shown that this is not the case. They claim that, while the probability of converging to either $\{0\}$ or $\{1\}$ does increase with r, in the limit this probability is not equal to unity, if this limit is taken over both r and S. Griffeath (2000) notes that, instead, this probability converges "to a quantity from critical percolation theory predicted by the celebrated Conformal Invariance Conjecture of R. P. Langlands."



$r = 1,\ t > 10$ $r = 2,\ t > 50$



$r = 3,\ t > 90$ $r = 4,\ t > 100$

Figure 5: Asymmetric Steady States of the Binary Model with $\sigma = 0.5$ and $\eta_0 = \{0.45\}$.

5 Noisy Employment Games

As was shown in the previous section, it is possible for spatial unemployment clusters to persist in the binary model with local trade networks and adaptive expectations. In some sense, however, these asymmetric steady states are somewhat artificial, as they result entirely from the fact that the economy is located in discrete space.[24] One way to prevent the economy from getting stuck in such artificial asymmetric steady states is to introduce some noise into the system. This section does just that, by allowing for uncertainty with respect to the marginal productivity of home production, σ, which will now be considered to vary both in space and in time. In particular, it will be assumed that

$$\sigma_t(x) = 0.5 + \mu\epsilon_t(x), \tag{41}$$

where $\epsilon_t(x)$ is a logistically distributed random utility term, and μ measures the impact of this random term on agent's decisions.

While the logistic distribution is chosen mainly for its simplicity, what is interesting about this distribution is that it is quite commonly used in econometric discrete choice models of labor market participation (e.g., Heckman and Willis 1977). Moreover, under this assumption the model closely resembles an interacting particle system

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[24] Indeed, Griffeath (1994) has shown that in continuous space, the model with $\sigma = 0.5$ clusters indefinitely, i.e. it never fixates and therefore converges to one of the symmetric steady states $\{0\}$ and $\{1\}$.

called the Ising model. Originally developed in the area of statistical mechanics, this model was introduced to economics by Blume (1993) and Brock (1993), the latter of whom developed a connection between Ising-type models and discrete choice modeling in econometrics. This connection was further elaborated by Brock and Durlauf (2001a,b), who have shown that it is possible to empirically estimate the parameters of Ising-type discrete choice models with social or local interactions. While empirical estimation is beyond the scope of the present chapter, an interesting direction for future work would be to follow Glaeser, Sacerdote, and Scheinkman (1996) and Topa (2001), who have developed empirical procedures to estimate the strength of social interactions and local spillovers in crime and unemployment, respectively.

5.1 Maximization Problem and Best Reponses

When $\sigma_t(x) = 0.5 + \mu\epsilon_t(x)$, each agent solves the following maximization problem:

$$\max_{\eta_t(x)} \quad \phi\left((1-\eta_t(x))[0.5+\mu\epsilon_t(x)] + \min\{\eta_t(x), \sum_{y\in S} p(y,x)E_{t-1}\eta_t(y)\} \right). \tag{42}$$

Given that $\epsilon_t(x)$ is logistically distributed, this yields the following best response function:

$$\begin{aligned}\Pr(\eta_t(x)=1) &= \Pr(E_{t-1}\eta_t(\hat{y}) > 0.5 + \mu\epsilon_t(x)) \\ &= \Pr(\epsilon_t(x) < \frac{1}{\mu}\left(E_{t-1}\eta_t(\hat{y}) - 0.5\right)) \\ &= \left(1 + e^{-2\beta(E_{t-1}\eta_t(\hat{y})-0.5)}\right)^{-1},\end{aligned} \tag{43}$$

where $\beta = \frac{1}{2\mu}$ is chosen so as to allow for a better comparison between this model, where $\eta_t(x) \in \{0,1\}$, and the Brock-Durlauf model, where the choice variable is $\omega_t(x) \in \{-1,+1\}$.[25]

Following Brock and Durlauf (1995), we can interpret β as the 'intensity of choice'. That is, as β increases, agents' choices become less disturbed by noise, i.e., become less sensitive to $\epsilon_t(x)$. This implies that, as $\beta \to \infty$, the best response correspondence reduces to that of the deterministic model with $\sigma = 0.5$ discussed in section 4.[26]

5.2 Equilibrium Properties under Rational Expectations

Proposition 9 (Binary Rational Expectations Equilibria)

Let $\eta_t(y) \in \{0,1\}$, let $\sigma_t(x) = 0.5 + \mu\epsilon_t(x)$, and let agents have rational expectations. Then,

a. for $\beta < 2$, there exists a unique Nash equilibrium, denoted by $\{0.5\}$;

b. for $\beta > 2$, there exist two additional Nash equilibria: an equilibrium where the majority of agents is employed, denoted by $\{p^*_+\}$, and an equilibrium where the majority of agents is unemployed, denoted by $\{p^*_-\}$.

Proof: Under rational expectations, $E_{t-1}\eta_t(\hat{y}) = p_{\hat{y},t}$. In a symmetric Nash equilibrium,

---

[25] In order to go from a $\{0,1\}$ state space to a $\{-1,+1\}$ state space, a transformation $\omega_t(x) = 2(\eta_t(x) - 0.5)$ is needed.

[26] The only difference is that, in the deterministic model, the probability of being employed in case of indifference could be anywhere in the domain $[0,1]$, whereas in this case this probability equals 0.5. However, for local trade networks this does not matter since the number of trade partners is always odd, hence the average employment rate in a trade network can never be identically equal to 0.5.

$p_{\hat{y},t} = p_{y,t} = p_t^*$ for all $\hat{y}, y$. Substituting this into the best response function gives:

$$p_{x,t} = \left(1 + e^{-2\beta[p_t^* - 0.5]}\right)^{-1} \tag{44}$$

This equation is plotted in figure 6 for several values of β. In a symmetric equilibrium it must be the case that $p_{x,t} = p_t^*$, hence any such equilibrium must lie on the 45-degree line.

Figure 6 shows that for low values of β, such as $\beta = 0$ and $\beta = 1$, a unique Nash equilibrium exists, which is given by the strategy profile $\{0.5\}$. In this equilibrium, the distribution of employment is random, in the sense that, each period, each agent has 50-50 chance of being employed ($p_t^* = 0.5$). Substituting this into equation (44) confirms that this constitutes an equilibrium. For larger values of β, such as $\beta = 3$, the best response function can be seen to intersect the 45-degree line in three places, corresponding to the three equilibria $\{0.5\}$, $\{p_-^*\}$, and $\{p_+^*\}$.

In order to prove that the transition between one and three equilibria does indeed take place at $\beta = 2$, we need to calculate the slope of the best response function evaluated at $p_t^* = 0.5$. When this slope is less than unity (i.e., is less than the slope of the 45-degree line), the best response function intersects the 45-degree line only once, and hence the model has a unique Nash equilibrium. When the slope of the best response function exceeds unity at $p_t^* = 0.5$, figure 6 shows that the model has three Nash equilibria. The slope of the best response function is given by

$$\frac{\partial p_{x,t}}{\partial p_t^*} = 2\beta \left(1 + e^{-2\beta[p_t^* - 0.5]}\right)^{-2} \tag{45}$$

Evaluated at $p_t^* = 0.5$, this gives:

$$\left.\frac{\partial p_{x,t}}{\partial p_t^*}\right|_{0.5} = \frac{\beta}{2}, \tag{46}$$

which confirms that the transition between one and three equilibria does indeed take place at $\beta = 2$. □

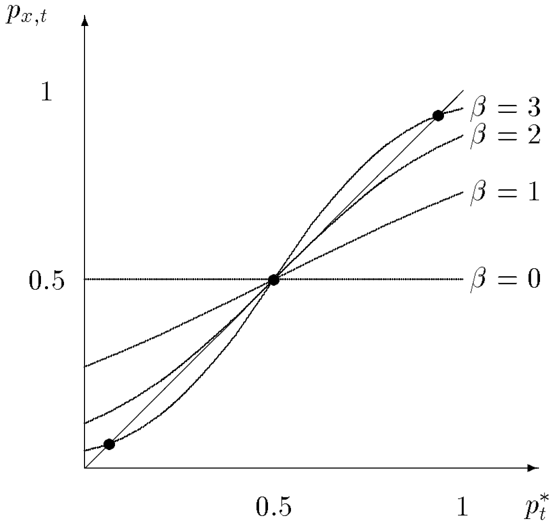

Figure 6: Best Response Functions for the Binary Model with Noise

5.3 Equilibrium Properties under Adaptive Expectations

Proposition 10 (Binary Adaptive Expectations Equilibria, Global Trade)

Let $\eta_t(y) \in \{0, 1\}$, let $\sigma_t(x) = 0.5 + \mu\epsilon_t(x)$, let agents have adaptive expectations, let S be countably infinite, and let trade networks be global. Then,

a. when $\beta < 2$, the distribution of employment converges to the Nash equilibrium $\{0.5\}$, i.e., $\lim_{t\to\infty} \eta_t = \{0.5\}$.

b. when $\beta > 2$,

$$\lim_{t\to\infty} \eta_t = \begin{cases} \{p^*_-\} & \text{if } \hat{\eta}_0 < 0.5 \\ \{0.5\} & \text{if } \hat{\eta}_0 = 0.5 \\ \{p^*_+\} & \text{if } \hat{\eta}_0 > 0.5 \end{cases} \tag{47}$$

Proof: Under global trade and adaptive expectations, $E_{t-1}\eta_t(\hat{y}) = \hat{\eta}_{t-1}$. This implies that all agents have the same best response function, hence any equilibrium must be symmetric. When S is infinite, the law of large numbers implies that $\hat{\eta}_t = p_{x,t}$. Substituting this into the best response function yields the following difference equation for the economy-wide employment rate:

$$\hat{\eta}_t = \left(1 + e^{-2\beta[\hat{\eta}_{t-1} - 0.5]}\right)^{-1}. \tag{48}$$

Plotting this equation generates a figure similar to figure 6, with $\hat{\eta}_{t-1}$ on the horizontal and $\hat{\eta}_t$ on the vertical axis. In any steady state, $\hat{\eta}_{t-1} = \hat{\eta}_t$ for all t, hence the steady states must lie on the 45-degree line, and must coincide with the rational expectations equilibria. This implies that, for $\beta < 2$ there exists a unique steady state distribution with $\hat{\eta}^* = p^* = 0.5$, while for $\beta > 2$ two additional steady states exist, $\{p^*_-\}$ and $\{p^*_+\}$, which in this case may be alternatively denoted by $\hat{\eta}^*_-$ and $\hat{\eta}^*_+$, respectively.

It was shown in proposition 9 that for $\beta > 2$, the slope of the best response function at the 'random' steady state exceeds unity. This implies that this steady state is locally unstable, while the other two steady states are locally stable. To see this, assume that we start with an initial distribution such that $\hat{\eta}_0 = 0.5^+$, i.e., $\hat{\eta}_0$ just above 0.5. By equation (48), $\hat{\eta}_t > \hat{\eta}_{t-1}$ for all $t > 1$, so the sequence is monotonically increasing. Since $\hat{\eta}_t$ is bounded, the sequence must converge to some limit which, by proposition 9, must be $\hat{\eta}^*_+$, since there are no other steady state solutions $\hat{\eta}^* > 0.5$. This shows that $\hat{\eta}^*_+$ is locally stable from below. A similar argument can be used to show that, when we start with $\hat{\eta}_0 = 1$, the steady state $\hat{\eta}^*_+$ is locally stable from above; that $\hat{\eta}^*_-$ is locally stable both from above and from below, and that, for $\beta < 2$, $\hat{\eta}^* = 0.5$ is locally and globally stable.[27] $\square$

5.4 Accounting for the Home Bias in Trade

So far, it was assumed that all agents have an equal chance of trading with any other trade partner, including with themselves (i.e., they are able to consume part of their own market output). There are reasons to believe, however, that $p(x, x) > p(y, x)$ in

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[27] This proof is based on Brock and Durlauf (1995), proposition 4.

real world trade networks, especially when we think about international trade, where more trade typically takes place within rather than between countries. A dramatic demonstration of such 'home bias in trade' was provided by McCallum (1995), who estimated that trade among individual Canadian provinces was 20 times greater than trade between individual Canadian provinces and individual U.S. states. While later work has tempered McCallum's estimates, in a recent overview of the literature Obstfeld and Rogoff (2000) concluded that "Overall, a balanced interpretation of the literature is that countries do exhibit a considerable degree of home bias in trade, but the bias is not as extreme as McCallum's original estimates suggested."

In order to account for a home bias in trade, the present model can be generalized by assuming that agents are able to consume some fraction $p(x,x) = \alpha$ of the market goods produced in their own region. A trade network structure with home bias can then be redefined as a matrix $\mathbf{P}$ with elements

$$p(y,x) = \begin{cases} \alpha & \text{if } y = x; \\ (|N_x| - 1)^{-1}(1-\alpha) & \text{if } y \neq x. \end{cases} \tag{49}$$

Proposition 11 (Binary Adaptive Expectations Equilibria, Home Bias) Let $\eta_t(y) \in \{0,1\}$, let $\sigma_t(x) = 0.5 + \mu\epsilon_t(x)$, let agents have adaptive expectations, let S be countably infinite, and let there be a home bias $p(x,x) = \alpha$. Then, there exists a threshold β^*, which depends on α, such that

a. when $\beta < \beta^*(\alpha)$, the distribution of employment converges to the random Nash equilibrium $\{0.5\}$.

b. when $\beta > \beta^*(\alpha)$, the distribution of employment converges to either $\{p^*_+\}$ or $\{p^*_-\}$;

c. $\beta^*(\alpha)$ is increasing in α.

Proof: We will prove this proposition only for the case of global trade networks on an infinite lattice. However, as section 5.5 reports, extensive computational experiments strongly suggest that it holds for local trade networks on finite lattices as well, and that for $\beta = \beta^*(\alpha)$, asymmetric steady states may exist in the local case.

Under global interactions with home bias $p(x,x) = \alpha$, the best response function can be written as:

$$\Pr(\eta_t(x) = 1) = \left(1 + e^{-2\beta[\alpha\eta_{t-1}(x)+(1-\alpha)\hat{\eta}_{t-1}-0.5]}\right)^{-1} \tag{50}$$

This implies that agents who were employed in the previous period are more likely to become employed again:

$$\Pr(\eta_t(x) = 1 \mid \eta_{t-1}(x) = 1) \geq \Pr(\eta_t(x) = 1 \mid \eta_{t-1}(x) = 0). \tag{51}$$

When S is infinite, the law of large numbers implies that $\hat{\eta}_t = \Pr(\eta_t(x) = 1)$. Letting $\hat{\eta}_{t-1}$ and $(1 - \hat{\eta}_{t-1})$ denote the fraction of agents who were respectively employed and unemployed last period, we obtain the following expression for the evolution of the average employment rate:

$$
\begin{aligned}
\hat{\eta}_t &= \hat{\eta}_{t-1} \cdot \Pr(\eta_t(x) = 1 \mid \eta_{t-1}(x) = 1) \quad + \quad (1 - \hat{\eta}_{t-1}) \cdot \Pr(\eta_t(x) = 1 \mid \eta_{t-1}(x) = 0) \\
&= \hat{\eta}_{t-1} \left(1 + e^{-2\beta[\alpha + (1-\alpha)\hat{\eta}_{t-1} - 0.5]}\right)^{-1} + \quad (1 - \hat{\eta}_{t-1}) \left(1 + e^{-2\beta[(1-\alpha)\hat{\eta}_{t-1} - 0.5]}\right)^{-1}.
\end{aligned}
\tag{52}
$$

Following proposition 10, the transition from a unique 'random' steady state to a situation with one unstable and two stable steady states occurs when:

$$
\begin{aligned}
\left.\frac{\partial \hat{\eta}_t}{\partial \hat{\eta}_{t-1}}\right|_{0.5} &= \left(1 + e^{-\alpha\beta}\right)^{-1} + \left(1 + e^{-\alpha\beta}\right)^{-2} \beta(1-\alpha)e^{-\alpha\beta} \\
&+ \left(1 + e^{\alpha\beta}\right)^{-1} + \left(1 + e^{\alpha\beta}\right)^{-2} \beta(1-\alpha)e^{\alpha\beta} \quad \geq 1,
\end{aligned}
\tag{53}
$$

which reduces to

$$
\beta(1-\alpha) \geq 1 + e^{-\alpha\beta}. \tag{54}
$$

This inequality confirms that, as α increases, larger values of β are required in order for the random steady state to become unstable, i.e., $\beta^*(\alpha)$ is increasing in α. $\Box$

5.5 Numerical Experiments

In order to study whether proposition 11 holds for local trade networks as well, we carried out an extensive number of numerical experiments on the Cellular Automata Machine (CAM8) at the Santa Fe Institute in New Mexico, which allowed us to study the evolution of the model on a 512 by 512 array at speeds of several hundred updates per second.

Each experiment was started from a random initial distribution $\eta_0 = \{0.5\}$, for 'common border trade networks' with range $r = 1$, i.e., $N_x = \{y : |y - x| \leq 1\}$.[28] In order to account for a home bias in trade, the trade network structure was modified as follows:

$$p(y,x) = \begin{cases} \alpha & \text{if } \| y - x \| = 0 \\ \frac{1}{4}(1-\alpha) & \text{if } \| y - x \| = 1 \\ 0 & \text{if } \| y - x \| > 1 \end{cases} \tag{55}$$

We first studied the case $\alpha = 0.2$, for which each agent has an equal probability of trading within her own region as with any of the agents from the four neighboring regions. The results are shown in figure 7, and suggest that the critical value of β in the model with local trade and $\alpha = 0.2$ lies around $\beta \sim 3$, which is significantly larger than the critical value $\beta^*(0.2) \sim 2.075$ under global trade. For β as large as 2.8, that is, the economy still has a steady state distribution of employment that is close to random noise. The fact that it is not entirely random is because of the local trade network structure, due to which agents who live in a neighborhood with a high unemployment rate are more likely to become unemployed themselves. However, this effect is not strong

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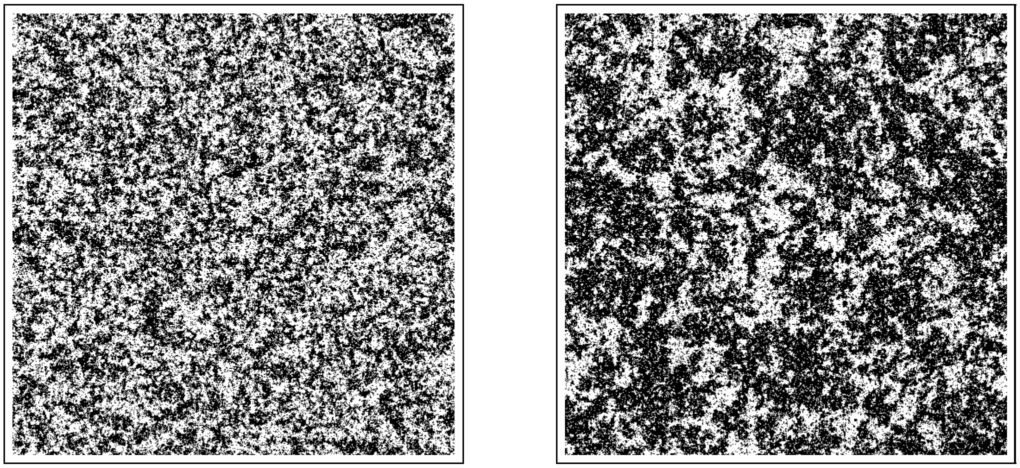
[28] Although this is obviously an extreme assumption, it is interesting to note that common borders are often found to be highly significant in explaining trade patterns (e.g., Balassa 1986).

enough for large unemployment clusters to evolve.

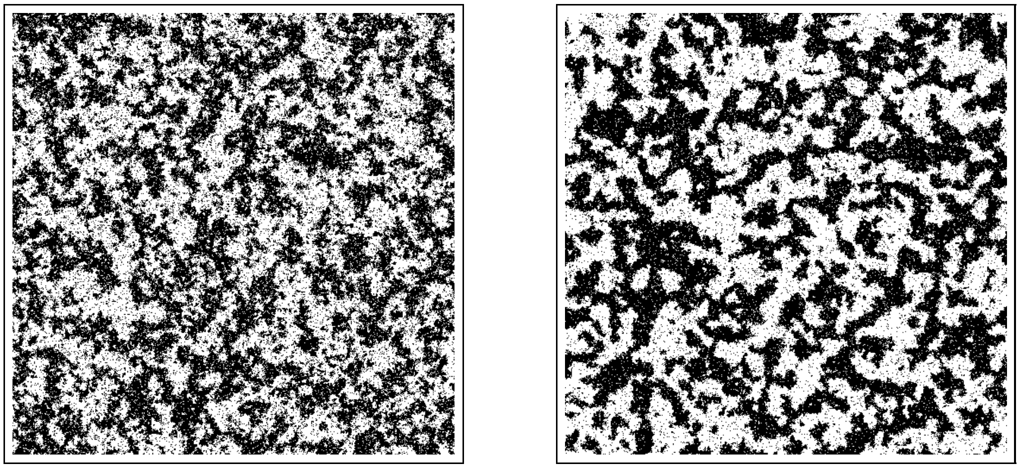
For $\beta = 3.1$, however, high-employment and high-unemployment clusters start to form quickly, and continue to grow until the economy converges to a symmetric steady state in which either nearly all agents are employed or nearly all agents are unemployed. This is even more clear for $\beta = 3.5$, as is illustrated in figure 8. As in the case without noise, this model evolves by curvature-driven surface tension clustering: if one type of cluster completely surrounds the other type, the latter cannot survive. However, this time the boundaries between the clusters remain smooth due to noise, which guarantees that the model clusters indefinitely, until it has reached a symmetric distribution in which all agents have an equal probability to be unemployed.

It is interesting to focus on what happens around the critical value $\beta = \beta^*(0.2)$, i.e., $\beta \sim 3$. While it is somewhat difficult to observe, there appears to exist a separate regime here, which neither converges to a random steady state distribution nor to a steady state distribution in which a majority of the agents is either employed or unemployed. To confirm this hypothesis, further numerical experiments were carried out for different combinations of α and β. These experiments indeed suggested that, for any level of α, exists a $\beta^*(\alpha)$ such that, for $\beta = \beta^*(\alpha)$, the economy is on the borderline between a 'random' and a 'clustering' regime. When α is relatively small, as in the $\alpha = 0.2$ case, this borderline regime is hard to distinguish from the random regime, since the clusters that persist are quite small. When α is too large, on the other hand, the clusters are quite big, so that it becomes hard to distinguish them from the clusters in the clustering



$\beta = 2.8, t = 30,000$

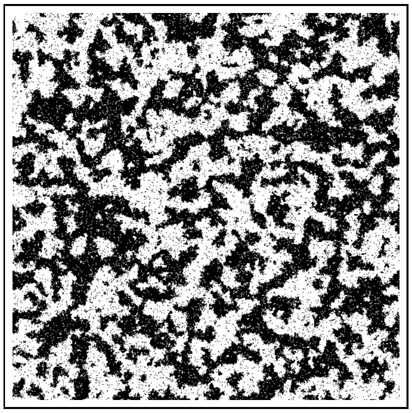
$\beta = 3, t = 50,000$



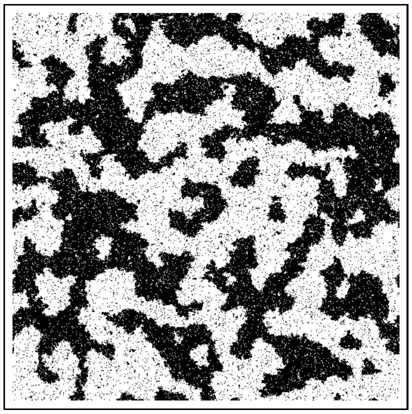
$\beta = 3.1, t = 100$

$\beta = 3.5, t = 100$

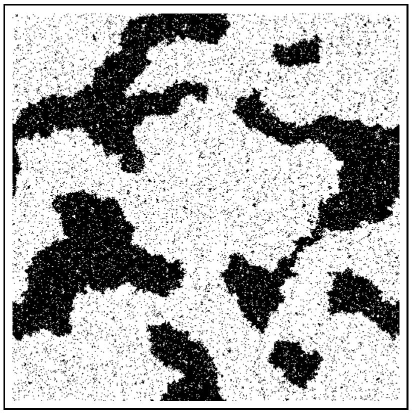
Figure 7: Behavior of the Binary Model with Noise for $\alpha = 0.2$ and different values of β.



$t = 100$



$t = 500$



$t = 3000$

Figure 8: Surface Tension Clustering of the Binary Model with Noise for $\alpha = 0.2$ and $\beta = 3.5$.

regime.

For intermediate values of α, however, the borderline regime can be observed quite clearly. This is illustrated in figure 9 for $\alpha = 0.5$. Interestingly, the general pattern here is very similar to the $\alpha = 0.2$ case, while the critical value of β in this case appears to lie around 3.9. For $\beta = 3.5$, for example, the steady state distribution is approximately random, with close to zero spatial persistence. For $\beta = 4$, on the other hand, the intensity of choice is large enough so as to allow for unemployment clusters to form quite quickly, which eventually disappear by surface tension clustering. For $\beta = 3.9$, medium sized clusters emerge that are able to persist indefinitely. Similar clusters emerge for $\beta = 3.8$, but these clusters are so small that the configuration can hardly be distinguished from random noise.[29]

The results of all experiments are summarized in figure 10, on the basis of which we can draw two conclusions. First of all, the critical value $\beta^*(\alpha)$ is clearly increasing with the amount of home bias α, both for global and for local trade networks, thus confirming proposition 11. Secondly, while the two phase boundaries do have a quite similar shape, the phase boundary of the local trade model lies above that of the global trade model, i.e., the critical value of β appears to be substantially higher under local trade than it is under global trade.[30] An intuitive explanation for both conclusions is that an increase

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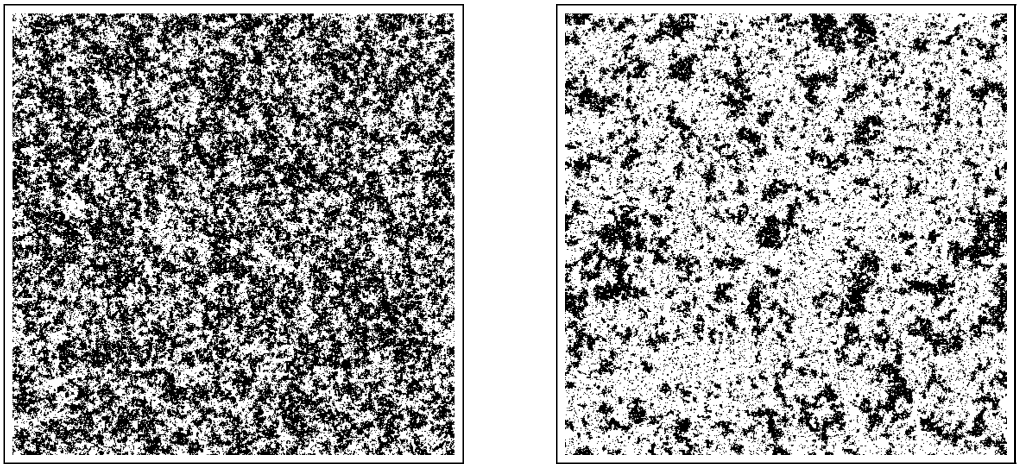
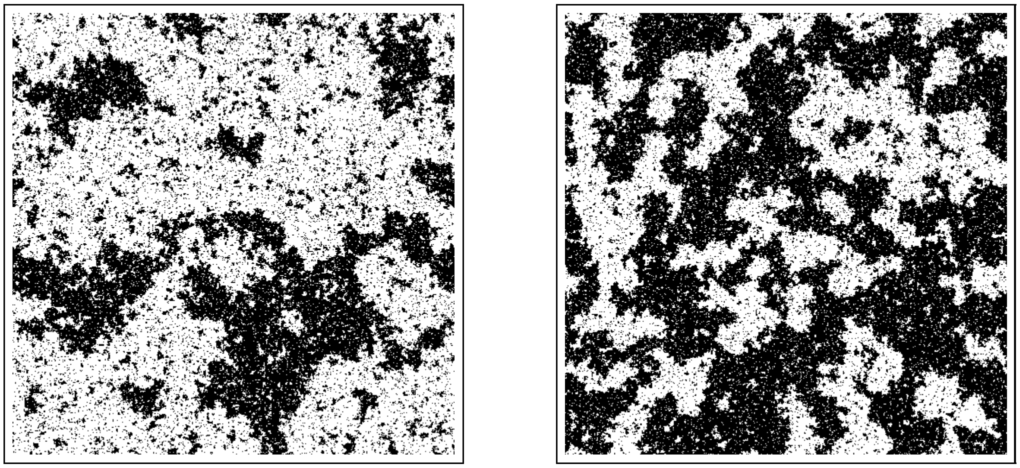
[29] The time periods of $t = 50,000$ and $t = 100,000$ indicate the number of updates after which the simulation was stopped; these times are somewhat arbitrary in that the configuration did not seem to change much either before or after this time.

[30] For the local trade model, the points close to $\alpha = 0$ are not plotted, since for $\alpha = 0$ and large β, 'checkerboard configurations' tend to become stable, which disturbs the relationship between α and β^*.



$\beta = 3.5, t = 100,000$

$\beta = 3.8, t = 100,000$



$\beta = 3.9, t = 50,000$

$\beta = 4, t = 2,000$

Figure 9: Behavior of the Binary Model with Noise for $\alpha = 0.5$ and different values of β.

in either the home bias (α) or the 'local bias' implies that agents become less likely to change their current state, hence a larger 'intensity of choice' (β) is needed in order for clustering to occur.

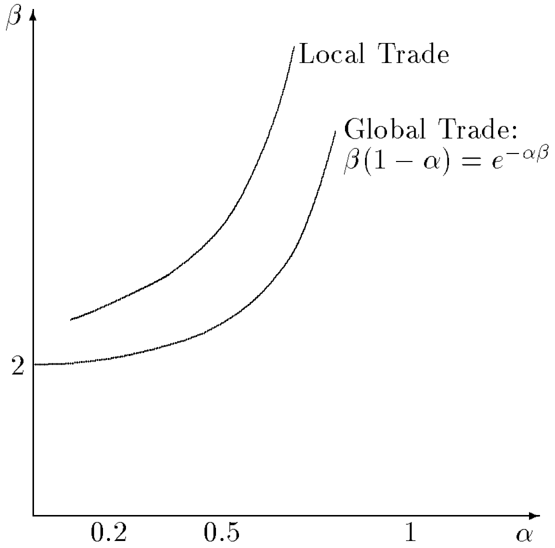


Figure 10: Estimates of $\beta^*(\alpha)$ for Local and Global Trade Networks

6 Conclusion

This chapter has studied the effects of local trade networks on spatial employment distributions in a Cooper and John (1988) type model with strategic complementarities. In section 3, it was first shown that, for general trade networks and continuous labor support, the distribution of employment converges to a symmetric equilibrium, i.e., this model could not explain spatial correlations in employment. In sections 4 and 5 , we then studied the case of binary labor support, and showed that, under various conditions, asymmetric employment distributions can persist indefinitely. While the assumption of binary support may seem extreme, we expect that similar results will hold for more general models in which labor support is discrete.

Perhaps the most interesting result of this chapter is that spatially persistent unemployment can occur even in the presence of noise. The explanation for this is that, while the probability of becoming unemployed conditional on living in a full employment trade network is very small, in the presence of noise some workers do occasionally become unemployed even if this is an 'irrational' decision on the part of firms. When a given agent becomes unemployed, this decreases the expected demand for goods produced by the trade partners of this agent, which in turn increases the probability that these trade partners will become unemployed as well. Once those trade partners become unemployed, however, the trade partners of those trade partners are more likely to do so too, etc., so that a spatial cluster of unemployed workers may start to grow within an economy that originally was at full employment.

Of course, the probability that this happens at any given location is extremely small, since workers who become unemployed by chance will tend to be hired again as soon as firms find out that there is in fact sufficient demand for their output. However, the probability that an unemployment cluster starts to grow *somewhere* within a full employment cluster decreases with the size of trade networks, and increases with the home bias in trade. This implies that when the 'intensity of choice' (β) is sufficiently large relative to the home bias to allow for unemployment clusters to form, but sufficiently small to allow for occasional 'irrational' decisions, it is possible for the process to continue indefinitely, growing clusters within clusters within clusters.

On a final note, we may consider what will happen as the world economy becomes more 'globalized', i.e., as the size of trade networks increases, while the home bias is reduced. The model predicts that, on the one hand, globalization will make it easier to reach a steady state with low unemployment, while on the other hand, it also becomes easier to converge to a situation with high unemployment. A testable implication of the model is, therefore, that business cycles will become more volatile as the world economy becomes more integrated.[31]

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[31] Note, however, that in order to test this prediction, one would need to conduct a serious identification analysis, e.g., along the lines of Krauth (2000), to separate trade network effects from spatially correlated unobservables.

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