Undesirable Goods, Illegal Trade and Taxation*

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Abstract

This paper considers the problem of a government whose objective is to reduce total consumption of a socially undesirable good using taxation and enforcement. Consumers have access to an illegal sector offering a perfect substitute to official goods and whose size is determined by competitive entry. When the official price is lower than the optimal monopoly price of an illegal seller, legal purchases will depend only on the cost parameters of the illegal sector. Then, if demand is inelastic, entry is important or the profit margin of illegal sellers is low, there may be a global minimum in consumption such that the good is legally sold. The optimal pricing decision of an official monopoly who may or may not sell to the illegal sector is also examined.

JEL classification: C71; D42; D62; H26

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1. Introduction

When public regulations or taxation impair mutually-beneficial exchanges of goods and services, illegal markets may appear and allow participants to reap private benefits from unrealized gains from trade. Private gains achieved in a relatively unconstrained but illegal environment may overcome the risk of legal liability. Effectively, high and persistent distortions to the market typically induce a large underground economy. Schneider and Ernste (2000) document that the underground economy, in its broadest definition, represents up to 30 percent of GDP in such countries as Greece, Italy, Belgium, Spain and Portugal and up to 10 percent in the U.S., Switzerland and Austria. An analysis of the aggregate consumption of goods traded illegally must take into account the presence of illegal markets.

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The existence of alternative means of exchange is particularly acute in the case of goods that are highly taxed or illegal. Such public constraints are often associated to goods deemed to be socially undesirable, for example in the presence of large consumption externalities or habit formation. There are many well-known examples of such goods: tobacco and alcohol are legal but typically exhibit high levels of taxation, services such as prostitution and gambling[1] are legal or tolerated only in certain countries, activities such as drug-trafficking[2] are prohibited in most countries. For simplicity, hard drugs will be used as a generic example.

This paper presents a novel framework in order to discuss the effect on aggregate consumption of a (controlled) liberalization of certain markets. Specifically, enforcement takes the form of a participation cost. Entry in the illegal sector responds endogenously to the return over this cost. Competition by an official seller will generally reduce the returns to illegal sellers and thus curb the availability of illegal drugs. The model allows to characterize how total purchases, legal and illegal, will respond to changes in official prices, taxation or enforcement.

Although there is an extensive literature that is concerned with the optimal taxation of regular goods, there is less work on the appropriate legislation for the drug market. Several recent papers have put forward that, based on economic principles, pure prohibition should rarely be optimal (Becker, Murphy and Grossman 2004) and excessive enforcement may be expensive and could potentially increase the social burden associated to drug use (MacCoun and Reuter 2001, Miron 2004). There is now an open political debate about the desirable level of taxation on legal goods such as tobacco and alcohol and whether certain drugs should be legalized[3]. From the standpoint of optimal taxation, a controlled liberalization of the drug market should benefit agents with a high willingness to pay as well as simultaneously raise tax revenues with no distortionary effect[4].

An important difficulty when trying to design an optimal legislation for the drug market is to select the correct welfare criterion. Unlike for more common goods, open consumption of drugs raises important social and political concerns. In this respect, aggregation tools that are used in many settings (such as for example social surplus) may be less easily applicable in the context of drugs[5]. The analysis presented here follows a different approach in that the optimal taxation is analyzed given an objective to decrease aggregate consumption. The paper presents a benchmark model to discuss the impact of different drug policies on aggregate consumption in the special case of arbitrarily large consumption externalities.

[1] Although these examples are restricted to "sin" services, there may be other politically sensitive goods such as euthanasia, abortion and firearms. However, these examples have distinctive features that are not considered here.

[2] Other examples of prohibited goods include offensive media and forced labor. More generally, any form of criminal activity may be seen as an example of a good with a (possibly very large) externality.

[3] See for example MacCoun and Reuter (1997) on the legalization of marijuana, Caulkins (1993) and Richardson (1992) on the effect of drug policies, Miron and Zwiebel (1991) on the consumption of alcohol during the prohibition; Desimone (2001) investigates the relationship between drug prices and crime, Eadington (1999) analyzes the market for gambling services, Caputo and Ostrom (1996) examine an optimal dynamic liberalization policy under a cost-benefit approach, Miron and Zwiebel (1995) argue in favor of relatively free drug markets.

[4] These revenues are significant: Caputo and Ostrom (1994) estimate that legalizing the marijuana market could generate between $2.55 and $9.09 billion in additional tax revenues in the U.S.; Miron (2003) estimates that it would save $120.6 million per year in enforcement and yield $16.9 million in tax revenues in the state of Massachussetts.

[5] Formally, there are two problems that may appear when considering the social surplus as a measure of aggregate welfare with drugs: the utility function of consumers may not be linear in remaining goods, the social criterion function may overweight the preferences of particular agents.

The purpose of the paper is to discuss the determination of aggregate consumption of undesirable goods in a model with endogenous participation of illegal sellers and price competition. This problem is related to Becker et al. (2004) who analyze the optimal legislation for hard drugs. The model differs from their approach in that the illegal sector is modelled as an entry game where the number of participants may be affected by prices in the official market.

The model is more closely related to entry games with imperfect coordination (Shapiro and Dixit 1986, Bolton 1990). As in these papers, it is assumed that there is a participation cost (representing here a risk of legal liability) and that potential entrants in the illegal market, who may be under scrutiny by authorities, may not coordinate their decisions using a centralized mechanism, public signals or sequential moves. Imperfect coordination among participants to the illegal market induces, ex-post, excess or insufficient entry in the illegal market with a probability that may depend on official prices.

The question considered here is also related to a large literature on international smuggling and evadable indirect taxation as in, among many others, Bhagwati and Hansen (1973), Thursby, Jensen and Thursby (1991) and Lovely (1994). These models often exhibit both an official and an illegal sector. Further, aggregate consumption may or may not increase in response to an increase in taxation. This effect is driven by the fact that the existence of illegal sellers may soften imperfect competition in the official market and thus ambiguously affect supply. In comparison, here, changes in taxation affect entry in the illegal market even if there is perfect competition in the official market[6]. The model presented here offers an alternative rationale to account for the dependence of the illegal market on taxation and enforcement.

Finally, several papers investigate why, from the perspective of individual welfare, there should be taxation on addictive goods (Becker and Murphy 1988, Gruber and Mullainathan 2002). Although having a theory that jointly considers enforcement and addiction may be of significant interest, this important issue is not addressed here. The problem of the paper is exclusively the role of taxation and enforcement on aggregate consumption and not whether an optimal policy may be recovered as a function of individual preferences.

The paper is organized as follows. In Section 2, the model is presented, outlining the production technology, the competition, the government choice and the equilibrium concept. Then, the determination of supply in the illegal sector is discussed and its dependence on the official price is derived explicitly. In Section 3, the model is examined under the assumption that the government may set the official price. The response of aggregate and official demand to changes in the legislation is decomposed into several economically meaningful variables. In Section 4, the model is considered under the alternative assumption that the government sets taxes but a profit-maximizing monopoly sets the official price.

[6]There are other important differences. First, much of the literature on international smuggling focuses on Cournot competition which may be less intuitively appealing than price competition as a model for decentralized (street) drug vendors or illegal producers with little initial capacity investment and, as it focuses on common goods, assumes no participation cost. Second, many results originate from the fact that introducing a probability of detection affects the production technology in the illegal market and thus, in essence, creates a second (often more concave) production technology.

2. The Model

2.1. Main Assumptions

The assumptions of the model are stated next.

A.1. There is a production technology with constant returns to scale that may be used to produce drugs at a constant marginal cost $\gamma_v > 0$.

The good may be produced domestically in many small-sized facilities or purchased from a large economy, or a concave production function may be replicated at no cost. Two sectors may supply the good using this production technology: an official and an illegal sector. Legal producers supply the good in the official market.

A.2. There is a unit tax τ for each good sold officially. Sellers in the illegal market evade the tax; however, they incur an additional unit cost γ_b.

In the model, γ_b corresponds to any additional cost in connection to the illegal sale of the good. For example, it represents in reduced-form a dissimulation cost or a probability to be detected that depends on the number of units sold[7]. Consumers may also discount goods purchased illegally; in this case, the price in the illegal sector is defined net of this subjective discount and γ_b is defined as the sum of the variable cost and the consumer discount.

A.3. Sellers in the illegal sector incur a participation cost $\Gamma > 0$ which induces (ex-ante) increasing returns to scale.

The participation cost represents the initial investment to create an illegal trade and incorporates a cost of legal liability that is not tied to quantities sold but only to the act of selling drugs. Since legal liability enters both terms[8] γ_b and Γ, let us define Γ as "fixed" enforcement in that it represents how legal liability may deter entry by shifting upwards the cost function of illegal sellers. The government may also choose other forms of enforcement that may lead to an increase in γ_b, for example by increasing police coverage. This second form of enforcement is defined as "variable" enforcement in that it affects the marginal cost of illegal sellers.

In the rest of the paper, $\overline{\gamma}$ (resp. $\underline{\gamma}$) denotes the net marginal cost of the official (resp. illegal) seller including tax τ (resp. additional cost γ_b).

A.4. Goods sold legally and illegally are perfect substitutes.

[7] Unlike in the international smuggling litterature, the model does not put the focus on how the probability to be detected affects the production technology of illegal entrants and thus may have effects on the aggregate production technology. It is assumed that, given this probability of detection, the technology remains with constant returns to scale. As is discussed later on, this assumption is made for convenience and, although it is important for the analytical results, it is not central for the model itself.

[8] In the model, the fact that the government may seize illegally sold goods is not considered explicitly; however, such policies are equivalent to an increase in Γ (if detection occurs before production cost), γ_b (if detection occurs after production cost and seized merchandize is sold on the market) or both Γ and γ_b (if detection occurs after production cost and seized merchandize is not sold in the market).

The purpose of this assumption is to keep the focus on the structure of illegal markets and not on product heterogeneity. A violation of this assumption weakens the tight relationship between official and illegal sales and quantitatively increases the effectiveness of taxation.

A.5. The demand $D(p)$ is a twice continuously-differenciable decreasing function from $\mathbb{R}^+$ to $\mathbb{R}^+$. Further, it is assumed that for any $C > 0$, $-D'(p)/D(p)$ and $1/(p-C)$ cross only once.

The price elasticity of demand is written $\epsilon(p)$. Denote p^* (resp. p^{**}), the optimal monopoly price[9] for a producer with constant marginal cost $\overline{\gamma}$ (resp. $\underline{\gamma}$); $\hat{p}$ is defined as the current price for the good in the official market.

The timeline of the game is as follows.

Stage 1: The government may set the tax rate τ. For now, the enforcement level is supposed to be exogenous and only comparative statics involving Γ and γ_v are considered.

Stage 2: The official price $\hat{p}$ is set, either by price competition among many official producers (Section 3) or, at the other extreme, by a single risk-neutral profit-maximizing monopoly (Section 4).

Stage 3: A large number of risk-neutral potential entrants simultaneously decide whether or not to participate in the illegal sector.

Stage 4: Illegal sellers compete on prices given the presence of an official seller offering a price $\hat{p}$.

In order to close the model, the following additional assumptions are made.

A.6. There is no first-mover advantage among the pool of potential entrants so that the solution is restricted to the unique symmetric (mixed-strategy) Nash equilibrium.

A.7. All consumers purchase from the seller offering the most attractive price.

If there is no entrant (ex-post), there will be no illegal market and the official price will prevail. If there is a single entrant, the unique entrant achieves monopoly profit under the constraint of selling above the official price $\hat{p}$. If there is more than one entrant, entrants must sell at marginal cost $\underline{\gamma}$ and achieve zero profit. Price competition is presented as a reference point for the analysis and possible violations of the assumptions of the model are discussed later on.

2.2. Entry in the Illegal Sector

Let us assume first that $\hat{p}$ is set directly by the government. For example, if there are many official suppliers competing on prices, the official price will be equal to $\overline{\gamma}$ and may be set

[9]Formally, p^* (resp. p^{**}) is the unique solution to $1/(p^* - \overline{\gamma}) = -D'(p^*)/D(p^*)$ (resp. $1/(p^{**} - \underline{\gamma}) = -D'(p^{**})/D(p^{**})$).

directly by an appropriate[10] tax τ. Alternatively, the official price of the good may be directly regulated by law[11]. Assume that Γ is such that the illegal sector is potentially profitable so that $\Pi \equiv D(\min(p^{**}, \hat{p}))(\min(p^{**}, \hat{p}) - \underline{\gamma}) > \Gamma$; Π denotes the maximum profit gross of fixed enforcement that can be achieved illegally. When $\hat{p}$ (or τ) is not arbitrarily large, the government legalizes official sales of the good.

Common wisdom would suggest that, under price competition, an increase in the official price may increase illegal sales but not enough to match the decrease of purchases in the official market. For example, provided that there is always one illegal seller, the effective price is $\min(\hat{p}, p^{**})$: an illegal monopoly[12] may only increase prices when $\hat{p}$ increases. On the other haand, if there is always more than one illegal seller, the price in the illegal market remains $\underline{\gamma}$, unchanged. In both cases, an increase in official prices will decrease aggregate consumption. However, these arguments rely on the assumption that entry is exogenous. However, an increase in the official price may also increase the profit that can be achieved by illegal entrants and thus may increase entry. The market will be crowded more often thus leading to low prices more frequently. Solving for the symmetric mixed Nash equilibrium of the game allows to capture endogenous participation.

Proposition 1. *For any n finite, there exists a unique (mixed-strategy) symmetric equilibrium with an initial pool of n potential entrants. Denote N_n, the corresponding equilibrium number of entrants. When n is large, N_n converges in distribution to a Poisson random variable with parameter $\zeta \equiv \ln(\Pi/\Gamma)$.*

Proof. Assume a pool of $(n+1)$ potential entrants in the illegal sector and denote $r(n)$ the equilibrium probability of not entering. Any potential entrant must be indifferent between entry and exit.

$$r(n)^n(\Pi - \Gamma) + (1 - r(n)^n)(-\Gamma) = 0$$

Solving for $r(n)$, there exists a unique symmetric mixed-strategy equilibrium such that every potential entrant enters with probability $1 - (\Gamma/\Pi)^{1/n}$. It follows that N_n must be distributed as a Binomial random variable $\mathcal{B}(n+1, 1-(\Gamma/\Pi)^{1/n})$. Taking the limit when n goes to infinity yields a Poisson distribution with parameter $\lim n(1 - r(n)) = \ln(\Pi/\Gamma)$ (by the law of rare events). □

Proposition 1 characterizes the number of entrants in the illegal sector, and thus the prevailing prices in the market, as a function of the potential profitability.

The model is restricted to symmetric equilibria with a large number of potential entrants. It should be noted that Proposition 1 is presented as an approximation of the symmetric equilibrium when the number of potential numbers is large but remains finite, not as a Nash equilibrium of the game with a countable number of entrants. In fact, it can be verified that, with a countable number of entrants, there is no Nash equilibrium that prescribes non-zero probability

[10] This interpretation of tax will be considered in this section, so that a change in taxation will be equivalent to a change in official prices.

[11] In the case of tobacco, legislation often takes the form of a minimum markup over production cost. For example, in the U.S., 39 states regulate the minimum price of cigarettes and only 18 states do not. Although firms can sell above this lower bound, a substantial share of cigarette sales is usually sold at minimum price. On the other hand, alcohol is rarely regulated directly by prices.

[12] As is noted later, this is an equilibrium outcome in the game; however, this equilibrium is asymmetric in that this illegal monopoly participates with probability one.

of entry for all potential entrants[13]. However, in the countable case, there exists a sequence of (asymmetric) Nash equilibria such that the number of entrants converges to the random variable featured here.

There are also many asymmetric equilibria in the game, including all pure-strategy equilibria such that one potential entrant participates with probability one and all other potential entrants do not participate. Selecting these equilibria would suggest that some entrants may signal participation. However, such entrants would plausibly attract attention from legal authorities and eventually be driven out of the market.

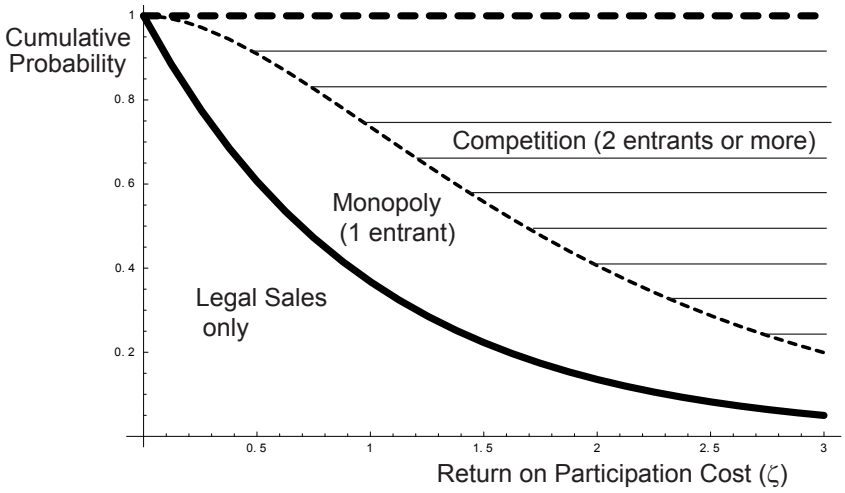

Figure 1. Profitability and Entry

In the model, entrants are induced to participate by the potential profitability of the illegal sector when no other entrant is present. However, the corresponding profit also depends on prevailing prices in the official sector. More precisely, the average number of entrants is the log of the gross return on the fixed cost, ζ, of a monopoly in the illegal market. When the number of entrants is close to one, it follows from a Taylor expansion that the expected number of entrants in the market is equal to $(\Pi - \Gamma)/\Gamma$, the net return of an illegal monopoly on fixed enforcement. The distribution is plotted in Figure 1. For very low levels of potential profitability, entry is very low. As the potential profitability rises, supply from the illegal sector becomes more frequent. However, since entry is relatively costly, the probability of monopolistic supply rises faster than the probability of competitive supply. Consequently, coordination problems are initially low and thus the probability of official supply decreases very fast. As the profitability increases above a certain threshold, coordination problems become more severe and the probability of monopolistic illegal supply decreases.

It is now possible to derive how official price changes may also affect total consumption. In the case of a large number of potential entrants, the total number of entrants[14] is denoted

[13]To prove this, define $(p_i)_{i=1}^{\infty}$, a particular strategy profile in the countable case where p_i is the probability that entrant i enters. Select $j_1 \neq j_2$, then both entrants must get zero profit conditional on entry. It follows immediately that $p_{j_1} = p_{j_2}$ and the equilibrium must be symmetric. But this would imply that there is always an infinite number of entrants or no entrant.

[14]In practice, official and illegal sellers may coexist; this does not necessarily contradicts the ex-post outcome in the model: each realization of D_∞ corresponds to a particular occurrence of the entry game in time and space

N_∞ and the corresponding equilibrium consumption (resp. trading price) is denoted D_∞ (resp. P_∞) in the case of a large number of entrants.

3. Regulated Official Price

3.1. Low Official Price

Assume first that $\hat{p} \leq p^{**}$. This situation may prevail whenever the government does not actively promote high prices, if the additional variable cost of illegal sellers is high or if the demand curve is very inelastic. Proposition 2 follows immediately from Proposition 1.

Proposition 2. *Suppose that $\hat{p} \leq p^{**}$, then:*

$$\mathbb{E}(D_\infty) = D(\underline{\gamma}) - k(\hat{p})(D(\underline{\gamma}) - D(\hat{p})) \tag{1}$$

where: $k(\hat{p}) \equiv \frac{\Gamma}{\Pi} + \frac{\Gamma}{\Pi \ln(\Pi/\Gamma)}$ is the probability to observe either one or no illegal seller.

Illegal sellers are constrained to sell above the legal price $\hat{p}$ even if their monopoly program dictates a higher price p^{**}. Notice that when official prices are low, a change in fixed and variable enforcement will always reduce aggregate consumption since $k(\hat{p})$ is decreasing in Γ. More enforcement is always desirable when there is an official market with low prices since it deters entry. In contrast, increasing official prices will not always decrease consumption.

$$\frac{\partial \mathbb{E}(D_\infty)}{\partial p} = k(\hat{p})D'(\hat{p}) - k'(\hat{p})(D(\underline{\gamma}) - D(\hat{p})) \tag{2}$$

Equation (2) illustrates how official price changes may turn out to have an ambiguous effect on aggregate consumption. Note that by (A.5.), $k'(\hat{p})$ must be negative. An increase in official prices increases the probability to observe perfect competition in the illegal sector. As a result, low prices $\underline{\gamma}$ occur more frequently. The effect on consumption must then be decomposed into a decrease in official consumption and an increase in the supply originating from illegal sellers. The first part in Equation (2) corresponds to the direct effect of higher prices on consumption and is always negative. However, it is scaled down by the probability of occurrence of official prices as the effective price. The second part corresponds to changes in the attractiveness of the illegal market to potential entrants. Changes in official prices induce an endogenous increase of the probability of competitive illegal supply at a lower price. This endogenous change of the expected number of entrants will dominate the effect on official demand when Equation (3) is verified.

$$\frac{D(\underline{\gamma})}{D(\hat{p})} \geq \frac{D(\hat{p}) - D'(\hat{p})(\hat{p} - \underline{\gamma})/\zeta}{D(\hat{p}) + D'(\hat{p})(\hat{p} - \underline{\gamma})} \tag{3}$$

On the left-hand side, the presence of illegal sellers imply an expansion of consumption by a maximum factor of $D(\underline{\gamma})/D(\hat{p})$. When very low prices induce very high demand, increasing

so that aggregate observations correspond to the first moment of the random variables studied here rather than a single realization. However, possible substitution effects between these realizations are not explored in the model.

the probability of competitive ex-post illegal supply will sharply increase expected consumption. In the right-hand side, the effect of price changes on $k(\hat{p})$ is decomposed into an efficiency effect (in the numerator) and a profit effect (in the denominator). The numerator incorporates the average number of entrants ζ and captures the effect of potential competition. The denominator can be rewritten as the change in monopoly profits resulting from a marginal price change at $\hat{p}$. In equilibrium, these profits are the driving force that induces entry.

Denote $m(\hat{p}) \equiv (\hat{p} - \underline{\gamma})/\hat{p}$, the margin of the illegal seller at official price $\hat{p}$. Rewriting the right-hand side of Equation (3) as $(1 + \epsilon(\hat{p})m(\hat{p})/\zeta)/(1 - \epsilon(\hat{p})m(\hat{p}))$, the result can be interpreted in terms of the characteristics of illegal sellers and the demand curve.

Corollary 1. *Aggregate demand is increasing with official prices if:*

(i) The demand curve is very inelastic.

(ii) The expected number of entrants in the illegal sector is high.

(iii) The unit margin of illegal sellers is low.

When $-D'(\hat{p})/D(\hat{p})$ is small, Corollary 1 can be made even more apparent.

$$\frac{D(\underline{\gamma})}{D(\hat{p})} \geq m(\hat{p})\epsilon(\hat{p})(1 + \frac{1}{\zeta}) \tag{4}$$

Markets with more entrants and lower unit margin reduce the ability of an illegal monopoly to generate additional profit. Similarly, an inelastic demand at $\hat{p}$ implies that the cost of the official price change is particularly large. These effects reduce the attractiveness of participation for illegal sellers. Figure 2 illustrates these results. The expected demand will be a combination of the demand given a competitive illegal market $D(\underline{\gamma})$ and the demand given a monopolistic illegal market or no illegal entrant, $D(\hat{p})$. The set of possible locations for the expected price and expected consumption is represented as a dotted segment. Policies may either shift the boundaries of this segment or only shift the outcome in the interior of the segment. In Figure 2, it is shown first that an increase in official price and variable enforcement shift the boundaries of the segment toward less consumption. Variable enforcement, in addition, shifts the equilibrium outcome in the segment toward the legal consumption level. The ambiguity arises from the fact that taxation also shifts the outcome in the segment toward the price occurring in a competitive illegal market.

Define the official supply D_o as the expected demand serviced by official sellers. In practice, it may be difficult to control for illegal sales or various biases that may appear in consumption surveys. In the model, biases that may arise from omitting supply from the illegal sector may be very important.

$$D_o(\hat{p}) = \frac{\Gamma}{\hat{p} - \underline{\gamma}} \tag{5}$$

Competitive illegal markets absorb characteristics due to the original demand curves so that official purchases do not depend on the demand curve. The aggregate demand curve $D(p)$ is not identified from the official demand. This extreme result is due to the very strong assumptions that were made in this model, i.e. perfect substitution, price competition and constant returns to scale. Here, official purchases may be very loosely tied to the aggregate consumption and

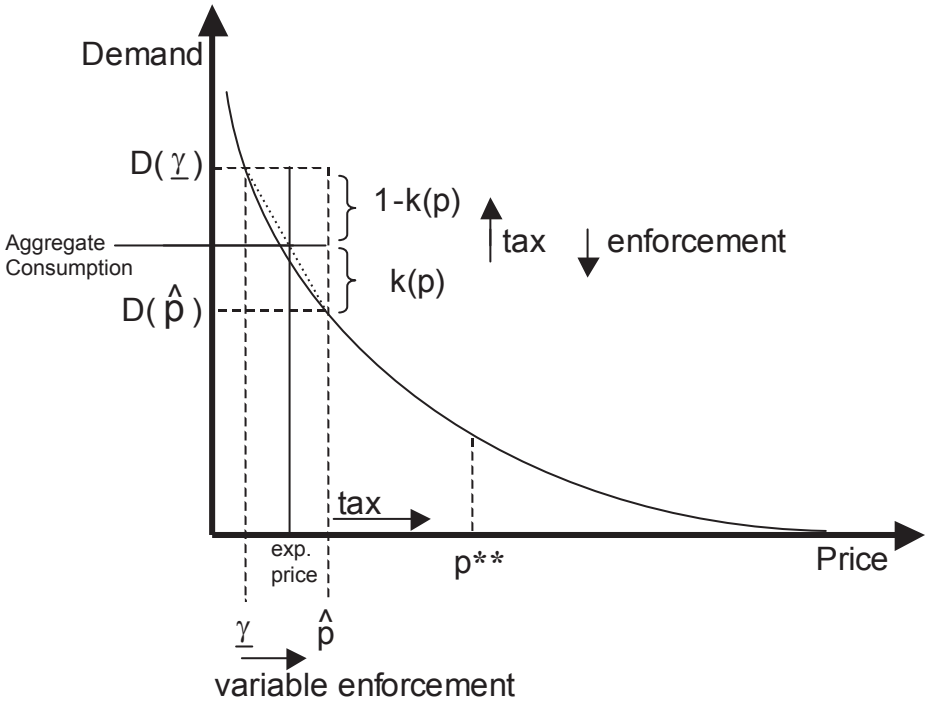

Figure 2. Aggregate Consumption with Low Official Prices

will always decrease sharply in response to a change in official prices although consumption decreases much slower and may even increase.

3.2. High Official Price

Now consider the second case, when $\hat{p} \geq p^{**}$. If countries set high prices in order to deter consumption, the pricing solution of a monopoly in the illegal sector may become interior. Then, given the opportunity, illegal sellers will choose p^{**} as the trading price.

Proposition 3. *Suppose that* $p^{**} < \hat{p}$,

$$\mathbb{E}(D_\infty) = D(\hat{p})\frac{\Gamma}{\Pi} + D(p^{**})\frac{\Gamma \ln(\Pi/\Gamma)}{\Pi} + D(\underline{\gamma})(1 - k(p^{**})) \tag{6}$$

The profit of an illegal monopoly and the corresponding supply are independent from $\hat{p}$. Additional price hikes effectively reduce consumption since they have no effect on monopoly profit. Further, variable enforcement, as in the previous case, reduces entry.

$$\frac{\partial \mathbb{E}(D_\infty)}{\partial \Gamma} = D(\hat{p}) - D(p^{**}) - \zeta(D(p^{**}) - D(\underline{\gamma}) \tag{7}$$

Equation (7) shows that an increase in fixed enforcement, as with low prices, will always reduce consumption. The effectiveness of fixed enforcement will be greater when the number of entrants is large or when demands prevailing under competition differ substantially from demand given monopoly or demand at the official price.

Notations:

(i) $\kappa(p) \equiv \frac{D(p^{**})/D'(p^{**})}{D(p)/D'(p)}$, the ratio of relative changes in demand for an absolute price change.

(ii) $\eta(p) \equiv D(p)/D(p^{**})$, the relative ex-post size of the official market to the size of the market of an illegal monopoly.

(iii) $\mu \equiv \Gamma/(p^{**} - \underline{\gamma})$, the unit breakeven point of the illegal seller.

The response of aggregate consumption to an official price change is characterized next.

$$\frac{\partial \mathbb{E}(D_\infty)}{\partial p} = -\frac{\mu\kappa(\hat{p})\mu\eta(\hat{p})}{\Gamma} \tag{8}$$

Although an increase in official price will always reduce aggregate demand, the cost parameters of the illegal sector remain crucial to understand to what extent. The decrease in total consumption is proportional to the inverse of the participation cost. High fixed cost imply that an illegal monopoly will have to maintain a high asset turnover and therefore total quantities will be less sensitive to variations in official prices.

In addition, the variation of total demand is decomposed into two effects: the relative sensitivity of demand to price changes, $\kappa(p)$, and the relative size of the legal sector to the illegal sector, $\eta(p)$. Since the total variation is expressed in units, $\kappa(p)$ and $\eta(p)$ must be multiplied by the relevant number of units affected by these variations. It is made explicit that this number is exactly the unit breakeven of an illegal seller.

Corollary 2. *Under the conditions of Proposition 3:*

$$D_o(\hat{p}) = \mu\eta(\hat{p}) \tag{9}$$

When the official price $\hat{p}$ is above p^{**}, the property that illegal markets absorb characteristics of the demand curve breaks down as a result of the strategic pricing of the illegal monopoly. Total official demand is decomposed as the product of the relative size of each market and the unit breakeven. Official demand is now equal to the official demand without strategic pricing, magnified by a term corresponding to the price sensitivity of demand at $\hat{p}$. The official demand can be decomposed as a constant share $\mu/D(p^{**})$ of its potential size without the presence of the illegal sector, $D(\hat{p})$.

The results are illustrated in Figure 3. The set of possible expected prices and aggregate consumptions is now represented as a dotted triangle. As in the previous case, different policies can be represented as either a shift of the triangle's vertices or a shift in the interior outcome in this triangle. An increase in enforcement will shift this outcome in the triangle toward less supply. Further, variable enforcement will also decrease $D(\underline{C}_v)$ and thus shift the triangle toward less consumption. However, an increase in official prices (or taxation) no longer induces effects on entry and thus will not induce a shift in the interior of the triangle; it does however shift the dotted triangle toward less consumption.

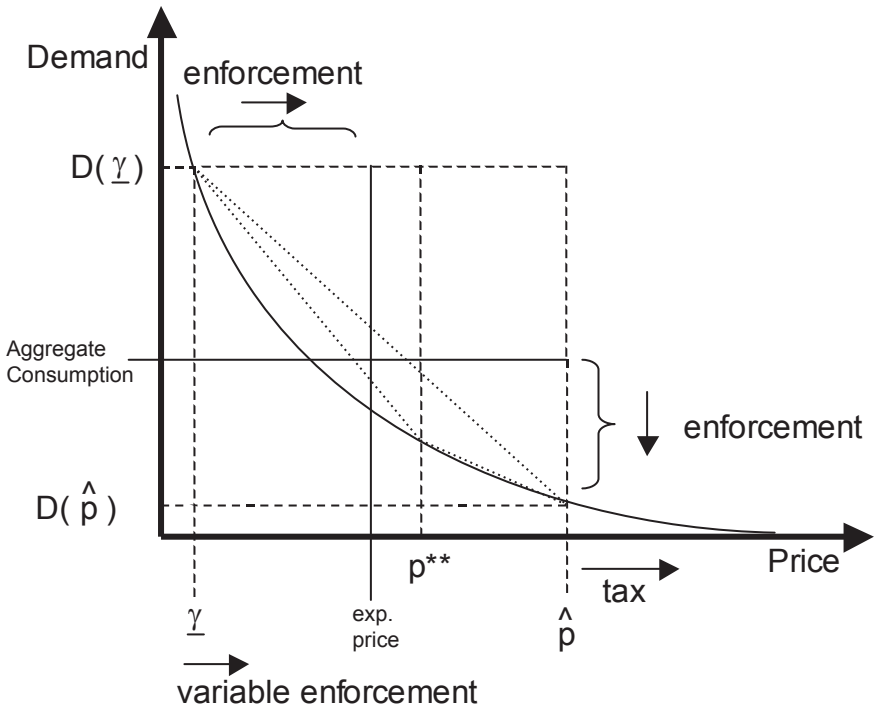

Figure 3. Aggregate Consumption with High Official Prices

3.3. Discussion and Robustness

For a given level of enforcement (γ_b, Γ), prohibition does not always yield a global minimum in consumption[15]. Enforcement and taxation appear here as complements. The second result that is presented here is that the aggregate demand curve may decrease very slowly in the presence of an illegal market. The official demand, on the other hand, will appear to decrease very fast for low levels of taxation but then will decrease very slowly. Several simplifying assumptions that were used in the model need however to be addressed.

Two assumptions are central for this model: the existence of a participation cost for each entrant in the illegal sector and imperfect coordination among entrants.

In the model, the participation cost (A.3.) Γ represents the cost of setting up an illegal trade or a fixed cost of legal liability during this initial stage. For example, in a heavily regulated industry, producing or importing the good illegally may require capital, labor and time. Further, acquiring the initial client base may also be difficult. Formally, a participation cost due to legal liability will exist as long as the probability of detection does not converge to zero as quantities sold become small. If this assumption is violated, the number of entrants in the illegal sector will always be very large and thus, the aggregate demand will not depend on the official price if the illegal sector exists.

Next, the solution of the game is restricted to symmetric Nash equilibrium with a large number of entrants (A.6.). Other potential solution candidates include asymmetric equilibria where only a small finite number of entrants enter with an equal probability. One may think for

[15]Numerically, there are reasonable demand curves that generate an interior price $\hat{p}$ that minimizes consumption, a local minimum that yields more aggregate consumption than prohibition or an aggregate demand curve that is always decreasing. However, it is fair to say that for most common demand curves, total demand is very flat when $\hat{p} > p^{**}$.

example that illegal trade is centralized by organized crime organizations whose role, among other things, is to avoid excess participation. Having such organizations will in general weaken the argument presented here. The underlying assumption here is that organized crime, if it exists, is constrained to decentralized competition due to the vigilance of authorities.

Second, constant returns to scale are assumed in the legal and illegal sector (A.1.). This assumption is made to separate endogenous participation from changes in the production technology (and thus competitive environment) induced by the legislation. The analysis carries over to the case of a concave production technology after entry has occurred. For example, concavity may appear if the probability to be detected depends on quantities sold and there is no other source of increasing returns. Two effects will be at play given this alternative assumption. First, a concave production technology reduces the ability of illegal sellers to capture high levels of demand and sell for a low price. Second, it increases the probability of entry for a given monopoly profit. Depending on the trade-off between these two effects, the results that prevail under constant returns to scale may be magnified or weakened.

Third, in the final stage of the game (i.e. after entry of illegal sellers), price competition is assumed (A.7.). Under pure price competition, the profit of an entrant will be zero or equal to the (constrained) monopoly profit. Other forms of competition including collusion, imperfect price competition or Cournot competition will yield a less stylized prediction. The existence of other forms of competition is not incompatible with the model and would generally increase entry as with a concave production function. However, other forms of competition may also change the equilibrium quantities sold by illegal sellers. For example, with Cournot competition, illegal sellers would reduce their sales and thus the effect on aggregate consumption of entry would be decreased. On the other hand, with collusion, quantities sold are unchanged but entry increases. However, an analytical characterization of the effect of taxation and enforcement under these other forms of competition is not intuitively apparent and is far beyond the scope of this paper.

Fourth, a joint change in enforcement and tax can always minimize consumption globally. However, unlike taxation, enforcement requires costly implementation and feasible choices may be bounded. Allowing for comparative statics at an exogenous level of enforcement, the model captures to what extent an increase in enforcement may be, locally, desirable. Further, the model can be readily adapted to find an optimal policy that solves the trade-off between the cost of enforcement and the social benefit of lower consumption.

The next section considers an extension of the result to the case of a particular form of imperfect competition in the official sector, in that there is an official profit-maximizing monopoly that may endogenously set a price different from $\overline{\gamma}$.

4. Unregulated Official Price

4.1. Monopoly Pricing

Until now, the official price $\hat{p}$ was assumed to be directly regulated. Taxation, therefore, indirectly sets the level of the official price. Suppose now that the government sets the tax τ but a monopoly sets the official retail price $\hat{p}$. For example, liberalization may be proposed via a profit-maximizing private firm. The presence of a monopoly implies imperfect competition

in the official sector and thus, illegal sellers will be appear not only to evade tax but also to sell below the prices of this legal monopoly.

Two cases are considered. The illegal sector is said to exhibit (pure) "competition" motives if the marginal cost of illegal sellers is greater than the marginal cost of the official monopoly, i.e. $\tau < \gamma_b$. In this case, illegal entrants will enter not because of a taxation hedge but because of a market imperfection in the legal sector. In the other case, the illegal sector is said to exhibit "tax-evasion" motives as illegal entrants also acquire a tax break over the official monopoly. The following Lemma is standard and given without proof.

Lemma 1. *$p^{**} \leq p^*$ if and only if $\gamma_b \leq \tau$.*

It is assumed that the official monopoly must set the official price before entry has occurred. The official monopoly may be a large firm whose price is less flexible than short-term, decentralized illegal vendors. The official monopoly maximizes profit with rational expectations on the presence of illegal sellers. The next proposition solves for the optimal price. Define as $\underline{p}$, the maximum official price such that the illegal sector is profitable (i.e. such that $\Pi = \Gamma$).

Proposition 4. *If $\gamma_b \leq \tau$ then $\hat{p} = p^*$, else $\hat{p} = \min(\underline{p}, p^*)$.*

Proof. The profit of the official monopoly for a price $\hat{p}$ is denoted $\Pi_o(\hat{p})$. Suppose first that $\gamma_b \leq \tau$ i.e. (Lemma 1) $p^{**} \leq p^*$. For any $\hat{p} \in (\underline{p}, p^{**}]$,

$$\Pi_o(\hat{p}) = \Gamma(\hat{p} - \overline{\gamma})/(\hat{p} - \underline{\gamma})$$

Π_o is increasing on $[\underline{p}, p^{**}]$. Suppose now that $\hat{p} \geq p^{**}$, the first-order condition for the program of the monopoly is:

$$-D_o'(\hat{p})/D_o(\hat{p}) = 1/(\hat{p} - \overline{\gamma})$$

So that by Corollary 2,

$$-D'(\hat{p})/D(\hat{p}) = 1/(\hat{p} - \overline{\gamma})$$

And finally, the official monopoly chooses $\hat{p} = p^*$.

Suppose now that $\gamma_b > \tau$ i.e. (Lemma 1) $p^{**} \geq p^*$. For any $\hat{p} \in (\underline{p}, p^{**}]$, Π_o is now decreasing on $(\underline{p}, p^{**}]$. Similarly, if $\hat{p} > p^{**}$, the first-order condition for the official monopoly is the same as in the problem without the illegal sector and since $p^* < p^{**}$, there is no solution to the monopoly pricing problem with a price greater than p^{**}. Therefore, the official monopoly chooses $\hat{p} = \min(\underline{p}, p^*)$. □

In the case of competition motives, the official monopoly will choose a price that is sufficiently low to shut down the illegal sector even if it implies pricing below the optimal monopoly price p^*. The official monopoly has a high margin on each unit sold so that an outflow of demand to the illegal sector has a substantial opportunity cost. Further, in Equation (5), the official demand decreases very fast for an official price change. Taking into consideration both effects, the official monopoly will always prefer to choose an optimal price below the price $\underline{p}$ that allows the illegal sector to survive. This choice also maximizes the productive efficiency gross of tax transfers.

In the case of tax-evasion motives, the official monopoly will no longer capture a sufficient share of the value of the good and thus will prefer to increase the unit margin although high prices will also shift demand to the illegal sector. Then, when taxes are sufficiently high, the

official monopoly will revert to its optimal monopoly price. Therefore, high taxes encourage the official monopoly to accomodate[16], rather than fight, illegal entry.

Corollary 3. *Aggregate consumption is weakly decreasing with the tax τ. Further, it is constant in τ if $\tau < \gamma_b$ and $p^* > \underline{p}$.*

In the presence of an official monopoly, the coexistence of a low official price ($\hat{p} < p^{**}$) and an illegal sector is impossible given the strategic pricing decision of the official monopoly. There are three possible pricing regions. In the first region, the monopoly price of the official seller, p^*, is low and allows the official monopoly to achieve the best possible outcome while simultaneously detering entry by illegal sellers. As the model simplifies to the standard monopoly pricing problem, additional taxation will generally decrease quantities sold. In the second region, the monopoly is constrained to limit-pricing $\underline{p}$, so that the official price no longer depends on taxation. In this region, taxation will be ineffective as the price is determined by strategic deterrence. Then, a change in taxation is equivalent to a fixed tax on the official monopoly[17]. In the last region, for greater levels of taxation, the official monopoly becomes unable to deter entry and reverts to its optimal monopoly price p^*; it now bears the shift of demand to the illegal sector. Note also that an official monopoly will never choose an official price that minimizes aggregate consumption even if this price is interior and taxation is set appropriately. However, an official monopoly will set low prices to shut down the illegal sector if it can.

4.2. Endogenous Supply Prices with Competition Motives

Suppose now that the illegal sector may not produce the good but may purchase it from an official monopoly that is not properly monitored. Equivalently, one may assume there a monopoly in a different country supplies both the official monopoly and the illegal sector and may extract all the surplus of the official monopoly via fixed transfers.

It is assumed now that the illegal seller must purchase from the official monopoly at a marginal price $(M_v + \gamma_v)q$, where M_v is a variable markup. Further, the official monopoly may also set a fixed markup M_f that is paid by the illegal seller at Stage 3. For simplicity, the official monopoly does not incur any risk of legal liability when supplying illegal sellers. For example, it may difficult to control foreign firms supplying the domestic illegal market through international[18] trading flows. The assumption of a fixed markup is made to try to separate the effect of standard double-marginalization from the role of endogenous supply by the official monopoly. It is weakened in certain cases. The optimal choice of the markups for the monopoly is denoted M_v^* and M_f^*.

The expected profit of the official monopoly is denoted $\Pi_o(p, M_v, M_f)$. Note that since the illegal seller must pay a variable markup, p^{**} will no longer be the optimal monopoly price for

[16] An alternative interpretation, outside the scope of this paper, is that the revenue-maximizing tax rate given competitive illegal markets is such that $\tau = \min(\underline{p}, p^*) - \gamma_v$. Higher enforcement also implies more tax and higher revenues.

[17] A different interpretation is that taxes, in this region, do not induce any distortion since the monopoly is constrained by the illegal market.

[18] As evidence of such practices, accused of oversupplying cigarettes to a number of low-tax countries, Philip Morris recently settled a long-standing dispute with the European Union for $1.25 billion. Although the settlement is substantial in absolute terms, such events are rare and possibly small in comparison to illegal international trade flows.

illegal sellers. Denote $\overline{p}^{**}$ the solution to the monopoly pricing equation for the illegal seller with variable cost $(\underline{\gamma} + M_v)$. Making the arguments explicit, $\overline{\Pi}(\gamma_b, \Gamma)$ is defined as the profit of an illegal monopoly, now including the fixed participation cost Γ. Whenever $\overline{\Pi}(\gamma_b, \Gamma) > 0$, the illegal sector is potentially profitable. Extending Proposition 1 by continuity[19], it is assumed that when $\overline{\Pi}(\gamma_b, \Gamma) = 0$, there is no entrant with probability one.

$$\overline{\Pi}(\gamma_b, \Gamma) \equiv D(\min(\overline{p}^{**}, \hat{p}))(\min(\overline{p}^{**}, \hat{p}) - \gamma_v - \gamma_b - M_f) - \Gamma - M_f \tag{10}$$

The following Propositions follow readily from Proposition 4.

Proposition 5. *Assume that the official monopoly may maximize profits over either M_f, M_v or both then $M_f^* = M_v^* = 0$ is not optimal.*

The argument underlying the proposition is straightforward: the monopoly will either try to extract consumer surplus from illegal resellers or prevent them to reduce competition. As a follow-up, it is necessary to ask if a self-interested monopoly may optimally increase (M_f, M_v) to shut down the illegal sector.

Proposition 6. *Assume that the official monopoly may maximize profits over either M_f, M_v or both then if $\gamma_b > \tau$, the official monopoly must set $\overline{\Pi}(\gamma_b, \Gamma) \leq 0$.*

Whenever the official monopoly may produce at cheaper prices than illegal sellers, as in the previous case, it will never select markup prices that are compatible with an illegal market. Therefore, for low level of taxation, illegal sellers will not appear and thus higher taxation, by its gradual effect on p^*, also yields lower total consumption.

4.3. Endogenous Supply Prices with Tax-Evasion Motives

It will be useful to write the profit of the official monopoly $\Pi_o(p, M_v, M_f)$ conditional on the existence of an illegal sector:

$\Pi_o(p, M_v, M_f) \equiv$

$$(p - \gamma_v)D(p) + \zeta\mu M_v + (1 - e^{-\zeta} - \zeta e^{-\zeta})M_v D(\underline{\gamma} + M_v) + \zeta M_f \tag{11}$$

where μ (resp. ζ) is the unit breakeven (resp. log of gross return on capital) including the markups.

The parameters μ and ζ correspond to the structural parameters of the illegal sector and are endogenously determined by M_v and M_f, as well as p when $\overline{p}^{**} \geq \hat{p}$.

Proposition 7 analyzes the optimal pricing strategy of the official monopoly when γ_b becomes smaller than τ.

Proposition 7. *If $\gamma_b < \tau$, then: $\exists \overline{\Lambda} > 0$ such that $\Gamma \geq \overline{\Lambda}$ if and only if $\overline{\Pi}(\gamma_b, \Gamma) \leq 0$.*

Proof. See Appendix. □

[19] This extension is natural when Proposition 1 is viewed as an approximation since in this case, there exists a symmetric Nash equilibrium in the countable case such that no potential entrant participates.

As a result of tax-evasion motives, a given level of legal liability may fail to prevent the formation of the illegal sector when taxation increases. Whenever $\Gamma < \overline{\Lambda}$, the illegal sector will exist. However, the pricing decisions of the official monopoly remain unknown at this point.

It is shown next that in the case of low official prices ($\overline{p}^{**} > p$), the optimal markups can be characterized further. Alternatively, one may suppose that illegal resellers, in particular if small and decentralized, have little information about demand curves and use official prices as a focal point for their monopoly prices. In fact, by a natural change of variables, the choice of the official monopoly, $(\hat{p}, M_f, M_v)$ can be simplified to a choice of the structural parameters of the illegal market (ζ, μ, M_v). Denote (μ^*, ζ^*) the parameters corresponding to the optimal choice of the monopoly.

Deciding over the cost parameters of the illegal sector is equivalent to setting directly the markup prices. An official monopoly might be seen as exclusively managing the illegal sector rather than optimizing its own sales in the presence of another seller.

The official monopoly directly regulates the number of entrants and the volume traded on the illegal market at the expense of binding the illegal monopoly to suboptimal prices. Suppose that the official monopoly can set M_v and M_f and the optimal pricing choice induces $\Gamma < \overline{\Lambda}$. The official monopoly will always abide by the following optimality conditions:

$$\frac{1}{\epsilon(\gamma_v + \gamma_b + M_v^*)} = \frac{M_v^*}{\gamma_v + \gamma_b + M_v^*} \tag{12}$$

$$\frac{1}{\epsilon(\hat{p})} = \frac{(\hat{p} - \gamma_v - \tau - (\gamma_b + \tau)\zeta^*/(\zeta^* + 1))}{\hat{p}} \tag{13}$$

$$\zeta^* = \frac{-1/\epsilon(\hat{p}) + (\hat{p} - \gamma_v - \gamma_b)/\hat{p} - \Gamma/\hat{p}\mu^*}{1/\epsilon(\hat{p}) - M_v^* D(\gamma_v + \gamma_b + M_v^*)/\hat{p}D(\hat{p})} \tag{14}$$

Equations (12), (13), (14) correspond to the first-order condition with respect to the variable markup, the average number of entrant and the breakeven point of illegal sellers.

From Equation (12), the monopoly will set the markup on the illegal sector according to the standard monopolistic pricing equation evaluated at the competitive illegal supply price. When allowing for an illegal sector, the official monopoly will set the variable markup in order to implement monopoly pricing in the presence of competition. There are no distortions to the monopolistic pricing equation in the presence of competition. The pricing equation for official prices given by Equation (14) is now different from the monopoly pricing equation. The official monopoly will incorporate partially the tax-evasion discount $(\gamma_b - \tau)$ in the pricing decision. Given that there is only one official monopoly but an average of ζ^* illegal entrants, only a ratio of $\zeta^*/(\zeta^* + 1)$ is incorporated in the pricing decision. Equation (14) determines the average number of entrants that allows to capture the benefits of tax-evasion without inducing excessive participation cost.

Equation (12) also implies that M_v^* and therefore the illegal competitive supply will not depend on the tax rate. If additional tax increases the number of entrants chosen by the monopoly, it may therefore also increase total consumption. This feature will be weakened when the official monopoly is constrained to $M_f^* = 0$.

When $\overline{p}^{**} < \hat{p}$, the previous change of variables does not apply as official prices have no influence on the illegal sector and M_v now determines $\overline{p}^{**}$ as well. The choice of ζ can still be substituted for the choice of M_f.

The official monopoly is now able to set an official price without constraining an illegal monopoly to suboptimal prices. It is readily verified that the optimal official price will be p^*. There are no distortions to the monopoly pricing decision in the official sector. Since M_v now determines both $D(\gamma_v+\gamma_b+M_v)$ and $D(\overline{p}^{**})$, the official monopoly will take both demands into account in order to set M_v. Therefore, unlike in the previous case, there may be distortions to the monopoly pricing decisions in the case of illegal supply. There are three effects at play: high cost from taxation with official supply, double marginalization arising from the lack of control over the price of the illegal monopoly and excess entry with a competitive illegal sector. When the last term is small compared to the other terms, for example if Γ is small, $\tau - \gamma_b$ is large or $\epsilon(\hat{p})$ is large, the official monopoly may increase total output.

5. Concluding Remarks

This paper has focused on participation cost and (symmetric) competition between a large number of entrants induced by active vigilance by authorities as one reason accounting for the formation of the illegal sector. The main finding of the paper is that taxation and enforcement are complements; further, official purchases may proxy very imperfectly for total consumption. An official monopoly will generally shut down the illegal sector when taxation is low but may accomodate or even benefit from it with higher levels of taxation.

There are many other reasons for considering legalization or prohibition that are not considered in the model and thus the results presented here cannot be considered in isolation. Several arguments, although they are not incompatible with the framework presented here, should be taken into account. First, some of the stylized assumptions made here may be violated. Second, the cost of enforcement and prohibition may also take the form of a marginalization of drug-addicts and participants to the underground economy. Third, the availability of certain drugs may induce a "transfer" to more deadly drugs. Although, the response to such a concern may be to minimize aggregate consumption, prohibition may also be a public signal that a drug has severe health hazards.

Appendix

Proof of Proposition 7.

In order to prove this result, define the following functions: $\xi(\gamma_b) \equiv \text{Inf}(\Gamma/\overline{\Pi}(\gamma_b,\Gamma) \leq 0)$ and $\chi(\Gamma) \equiv \text{Inf}(\gamma_b/\overline{\Pi}(\gamma_b,\Gamma) \leq 0)$.
The next lemma proves that these functions are well-behaved, so that there is a well-defined frontier in the fixed and variable cost such that the illegal sector does not exist.

Lemma 2. *$\xi(\gamma_b)$ (resp. $\chi(\Gamma)$) verifies $\Pi(\gamma_b, \xi(\gamma_b)) \leq 0$ (resp. $\Pi(\chi(\Gamma),\Gamma) \leq 0$).*

Proof. Clearly, the set of possible values for (p, M_v, M_f) can be restricted to a ball $B(0,\overline{C})$ without loss of generality. From Berge's Maximum Theorem, since $\Pi_o(\tilde{p}, M_v, M_f)$ is continuous, $(\hat{p}, M_v^*, M_f^*)$, are compact-valued, upper-hemicontinuous functions of γ_b and Γ. Therefore, for $\overline{C}$ sufficiently large, $\{\Gamma/\overline{\Pi}(\gamma_b,\Gamma) \leq 0\} \cap \{\Gamma \leq \overline{C}\} \cap \{\gamma_b \leq \overline{C}\}\}$ is a compact set. Therefore, by Weierstrass Theorem, $\chi(\Gamma) \in \{\Gamma/\Pi(\gamma_b,\Gamma) \leq 0\}$ (resp. $\xi(\gamma_b) \in \{\gamma_b/\Pi(\gamma_b,\Gamma) \leq 0\}$) and from continuity, $\xi(\gamma_b)$ (resp. $\chi(\Gamma)$) verifies $\Pi(\gamma_b, \xi(\gamma_b)) \leq 0$ (resp. $\Pi(\chi(\Gamma),\Gamma) \leq 0$). □

The proposition is now proved in two steps: when the monopoly may set M_f (a) or only M_v (b).
(a) Assume the monopoly may set M_f. Consider $\Gamma > \xi(\gamma_b)$ and suppose that at this cost level, the illegal sector exists, and denote M_f^Γ the corresponding fixed payment. Now consider setting: $(M_v^\Gamma, M_f^\Gamma + (\Gamma - \xi(\gamma_b))$ at $\xi(\gamma_b)$. Then, necessarily:

$$\Pi_o(\xi(\gamma_b), p^\Gamma, M_v^\Gamma, M_f^\Gamma + (\Gamma - \xi(\gamma_b)), M_v^\Gamma)$$

$$> \Pi_o(\Gamma, p^\Gamma, M_v^\Gamma, M_f^\Gamma + (\Gamma - \xi(\gamma_b)), M_v^\Gamma) \geq (p^* - \tau - \gamma_v))D(p^*)$$

This contradicts the optimality of the non-existence of the illegal sector at $\xi(\gamma_b)$. Now suppose by contradiction that even when Γ is zero, there is no illegal sector. Then, for $\Gamma = 0$, the optimal strategy must prescribe no illegal sector. Now consider allowing for illegal sellers with strictly positive probability in that: $M_f/\Pi = 1 - \epsilon$. Then for ϵ small, there will be no entrant with probability $1 - \epsilon$ and an entrant with probability ϵ. The official monopoly will therefore increase its profit by: $\epsilon((1-\epsilon)(\overline{p}^{**} - \gamma_v - \gamma_b)D(\overline{p}^{**}) - (\hat{p} - \gamma_v - \tau)D(\hat{p})) > 0$.

(b) Assume now the monopoly is constrained to $M_f = 0$. Consider $\gamma_b > \chi(\Gamma)$ and denote M_v^Γ the corresponding variable payment. Now consider setting: $M_v^{\gamma_b} + ((\gamma_b + \tau) - \chi(\Gamma))$. Then, as in (a), it is easily checked that the official monopoly will achieve a strictly greater profit in $\chi(\Gamma)$ than in γ_b, which is itself greater than the profit without the illegal sector, thus yielding the same contradiction. Now verify that if $\Gamma = 0$, there will always be competition between illegal sellers and therefore the official monopoly may set the markup M_v to achieve monopoly profits at variable cost $\gamma_v + \gamma_b < \gamma_v + \tau$.

To conclude, $\overline{\Lambda}$ is defined as $\xi(\gamma_b)$ from Lemma 2. □

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