The Competitive Effects of Price-Floors

V. Bhaskar*

University of St. Andrews

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Abstract

We analyze the effects of a legally-binding price floor using Hotelling's model of locational competition. A moderate price-floor destroys the maximal differentiation equilibrium of d'Aspremont et. al., by allowing firms to compete more aggressively for market share. Minimum differentiation results, with lower equilibrium prices. A low price floor results in mulitiple equilibria - both minimum and maximum differentiation are possible.

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*Dept. of Economics, University of St. Andrews, St. Andrews KY16 9AL, UK. Email: V.Bhaskar@st-and.ac.uk. I thank John Beath and Kai-Uwe Kühn for helpful suggestions.

1. Introduction

A *price-floor* is a price which firms may not legally undercut, imposed either by the government or, as a vertical restraint, by the supplier upon retailers. Public debate on the effects of price floors has been sparked off by recent UK developments such as the collapse of the net-book agreement and price cuts in pharmaceuticals by the supermarket Asda. This debate emphasizes the effects of a price-floor upon product variety and the degree of product differentiation, as well as the number of firms in the industry via exit/entry. The focus of this paper is on the first aspect, the effect of variety choice, although our analysis also has implications for the second. In line with this focus, we employ the natural model for this purpose, viz. Hotelling's model of locational competition. Our paradoxical finding is that price floor can *reduce* equilibrium prices. Our argument relies upon the effect of the price floor upon firms' location decisions. We consider a model where two firms compete for consumers who are distributed on the unit interval, and have quadratic transport costs. In the absence of a price-floor, the model results in maximal differentiation - the firms locate at the two endpoints of the interval, in order to relax price competition. The resulting price equals the transport cost parameter, t (normalizing marginal costs to zero). Consider now a price floor, at any price greater than t/2. The price floor destroys the maximal differentiation equilibrium. By reducing price competition, the price floor allows firms to compete more aggressively for market share via location. This makes an extreme locations unviable. The only equilibrium is when both firms locate at the midpoint of the interval, and price at the price-floor.

If the price-floor is at an intermediate level (i.e. below t/2 but not too low), the maximal differentiation equilibrium remains viable. Nevertheless, there is also an equilibrium with minimal differentiation, where firms price at the floor. Although firms earn more profits in the equilibrium with maximal differentiation, we argue that coordination problems may make the minimum differentiation equilibrium more likely. Finally, if the price-floor falls even lower, maximum differentiation is ensured in any pure strategy equilibrium.

The organization of the remainder of this paper is as follows. Section 2 sets out the basic analysis, setting out the conditions under which maximum differentiation and minimum differentiation emerge as equilibria. Section 3 discusses questions of equilibrium selection and welfare, and also the possibility of free entry. Section 4 analyzes the case of price-ceilings, and finds that a price-ceiling reduces differentiation. The final section concludes.

2. Analysis

Our model is the standard Hotelling model of locational competition between two firms for customers located on the line (see eg. Tirole, [6]). The marginal cost of the product is normalized to zero. A unit measure of consumers is uniformly distributed on [0,1]. Transport costs are quadratic, and are given by td^2, where d is the distance travelled. Each consumer consumes at most one unit of the product and has a reservation utility v which is "sufficiently high" so she always buys one unit ($v > 5t/4$). We consider the two-stage game, where firms choose location in the first stage, and prices in the second stage, having observed location choices. d'Aspremont et.al. [1] focused on pure strategy equilibria and showed that maximal differentiation obtains in any pure strategy equilibrium. More precisely, the two stage game has a pair of pure strategy equilibria: an equilibrium where firm 1 locates at 0 and firm 2 at 1, and a second equilibrium where firm 1 locates at 1 and firm 2 at 0. The equilibria are equivalent in the sense that prices and profits of the two firms are equal in both. The basic reason for this maximal differentiation result is that firms locate far apart in order to mitigate price competition. The resulting equilibrium price is t, with profits per firm of t/2.

We now proceed to an analysis of the effects of a price-floor. The analysis focuses upon the following extensive form, although we shall also, at various points, consider some alternative sequences of moves. In stage 0, the price floor $\bar{p}$ is announced. In stage 1 firms choose locations, and in stage 2 firms choose prices which are greater than or equal to $\bar{p}$. Consider the second (pricing) stage of this game, and assume without loss of generality that firm 1 has located at $a \in [0, 1]$

and firm 2 has located at $1 - b \in [0,1]$, where $a \leq 1 - b$. Hence the distance between the firms is $1 - a - b$. Let p_1 and p_2 be the prices charged by firms 1 and 2 respectively. Their respective demands are given by

$$D_1(p_1, p_2) = a + \left(\frac{1 - a - b}{2}\right) + \left(\frac{p_2 - p_1}{2t(1 - a - b)}\right) \tag{2.1}$$

$$D_2(p_1, p_2) = b + \left(\frac{1 - a - b}{2}\right) + \left(\frac{p_1 - p_2}{2t(1 - a - b)}\right) \tag{2.2}$$

The profits of the two firms are given by $p_i D_i(p_1, p_2), i = 1, 2$. In the absence of any price floor, each firm would maximize profits with respect to own price, which yields the unconstrained optimal price, $\tilde{p}_i$, as a function of p_j, and the locations:

$$\tilde{p}_1 = \left(\frac{t(1 + a - b)(1 - a - b)}{2}\right) + \left(\frac{p_2}{2}\right) \tag{2.3}$$

$$\tilde{p}_2 = \left(\frac{t(1 + b - a)(1 - a - b)}{2}\right) + \left(\frac{p_1}{2}\right) \tag{2.4}$$

The constraint imposed by the price floor implies that the each firm i's reaction function, for $i = 1, 2, j = 3 - i$, is given by

$$\hat{p}_i(p_j) = \min\{\tilde{p}_i(p_j), \bar{p}\} \tag{2.5}$$

A Nash equilibrium of the pricing stage is a pair of prices (p_1^*, p_2^*) such that both firms are on their reaction functions. Since Nash equilibrium prices depend upon the chosen locations and upon the price floor, we write these as $p_1^*(a, b, \bar{p})$ and $p_2^*(a, b, \bar{p})$. We say that the price-floor *binds on firm i* if the $\tilde{p}_i(p_j^*) < \bar{p}$. Let Δ_i denote the dummy variable which takes value 0 if the price-floor binds on firm i, and 1 otherwise. Observe that (2.3) and (2.4) show that each firm's unconstrained optimal price is an affine function of its rival's price, with a positive intercept and slope one-half. The intercept, for each firm, depends upon the locations, a and $1 - b$. Note that the price-floor may be binding for two, one or none of the firms. Further, inspection of the intercept terms reveals that if $a < b$, then firm 1 has an incentive to price lower than firm 2. Hence if the price floor binds on only one firm, it must be the firm with the smaller "captive" market, i.e. in this case firm 1. Formally, if $a < b$, then $\Delta_2 = 0 \Rightarrow \Delta_1 = 0$. Finally, observe that the intercept term for both firms tends to zero as the firms get closer and $1 - a - b$ tends to zero. Hence for any $\bar{p} > 0$, the price-floor will be binding on both firms if $1 - a - b$ is sufficiently small.

Consider now the first (location) stage of the game. The two firms choose locations simultaneously, with firm 1 chooses location a, and firm 2 chooses location $1 - b$, knowing that prices in the second stage will depend upon location choices, and be given by $p_1^*(a, b, \bar{p})$ and $p_2^*(a, b, \bar{p})$ respectively. This determines demands and profits. The critical factor here is the dependence of equilibrium prices in the second stage upon locations. Hence the profits of each firm as a function of locations are given by

$$\pi_1(a,b) = p_1^*(a,b,\bar{p})D_1[p_1^*(a,b,\bar{p}), p_2^*(a,b,\bar{p}), a, b] \tag{2.6}$$

$$\pi_2(a,b) = p_2^*(a,b,\bar{p})D_2[p_1^*(a,b,\bar{p}), p_2^*(a,b,\bar{p}), a, b] \tag{2.7}$$

A subgame perfect equilibrium in locations is a pair (a^*, b^*) such that a^* maximizes $\pi_1(a, b^*)$ and b^* maximizes $\pi_2(a^*, b)$. d'Aspremont et. al. [1] show that in the absence of a price floor (or equivalently, when the price-floor is zero) the in any pure strategy equilibrium in locations, the firms maximally differentiate their products. For example, one such equilibrium is the pair $(0, 1)$ where firm 1 locates at 0 and firm 2 at 1. If firm 1 increases a (i.e. moves towards the center of the market) it faces two distinct effects on its profits. The direct effect is to increase its market demand and profits, and the strategic effect is to reduce the price charged by firm 2, which reduces its demand. Maximal differentiation arises since the strategic effect exceeds the direct effect.

2.1. Local Optimality

We shall call a pair of locations $(\hat{a}, \hat{b})$ *locally optimal* if $\pi_1(a, \hat{b})$ has a local maximum at $(\hat{a}, \hat{b})$ and $\pi_2(\hat{a}, b)$ has a local maximum at $(\hat{a}, \hat{b})$. Local optimality is necessary but not sufficient for a subgame perfect equilibrium. Our analysis here is partly instrumental, in order to characterize subgame perfect equilibrium in locations. Nevertheless, it may also have some independent interest, if location decisions are subject to inertia, as we discuss later.

We now show that any location configuration which is locally optimal must exhibit either maximum differentiation or minimum differentiation, where by minimum differentiation we mean the configuration where both firms locate at the centre of the market, i.e at 1/2. Consider the payoff to *marginal* locational changes in the presence of a price-floor. Let $a < 1 - b$ so that firm 1 is located to the left of firm 2, and consider the derivative of firm $1's$ profits with respect to a. Observe that we can ignore the term $\frac{\partial \pi_1}{\partial p_1} \frac{dp_1^*}{da}$ since this equals zero - either the price-floor is binding on firm 1 in which case $\frac{dp_1^*}{da} = 0$) or $\frac{\partial \pi_1}{\partial p_1} = 0$ (the envelope theorem). Hence the derivative of firm 1's profits with respect to a can be written as:[1]

$$\frac{d\pi_1(a,b)}{da} = p_1^*(a,b,\bar{p}) \left[\frac{\partial D_1}{\partial a} + \frac{\partial D_1}{\partial p_2} \Delta_2 \left(\frac{\partial \tilde{p}_2}{\partial a} + \frac{\partial \tilde{p}_2}{\partial p_1} \frac{dp_1^*}{da} \right) \right] \tag{2.8}$$

Since $p_1^* > 0$ if $a < 1 - b$, the sign of (2.8) depends upon the total effect on D_1, which consists of the terms in square brackets. The first term is always positive

[1] The profit function is not differentiable at values of (a, b) at which either Δ_1 or Δ_2 have a discontinuity, i.e. where a firm experiences a regime switch from a binding to a non-binding price-floor. However, at such points left and right hand derivatives always exist, and further, the expressions for these correspond to those of the derivatives in the corresponding regimes. This complication plays no role here, but is relevant when we discuss the case of a price-ceiling in section 4.

- $\partial D_1/\partial a > 0$, since the firm always gains market share by moving towards the middle of the market. This immediately implies that if the price-floor binds on firm 2, so that $\Delta_2 = 0$, (2.8) is positive and firm 1's profits are locally increasing as it reduces differentiation.

$$\left.\frac{d\pi_1}{da}\right|_{\Delta_2=0} = p_1^* \left(\frac{1}{2} + \frac{(p_2^* - p_1^*)}{2t(1-a-b)^2}\right) > 0 \tag{2.9}$$

This immediately implies that if the price-floor is non-trivial, so that $\bar{p} > 0$, then minimum differentiation is always locally optimal, since the price-floor is always binding when both firms locate sufficiently close together..

Suppose that the price-floor does not bind on firm 2. In this case the strategic effect of an increase in a is to reduce firm $2's$ price, thereby reducing D_1. This strategic effect can be decomposed into two distinct effect, which are set out in the two terms within curved brackets. An increase in a, by reducing product differentiation, induces firm to reduce price directly. Second, this also induces firm 1 to reduce its own price, p_1^*, which causes firm 2 to also reduce price given the strategic complementarity. [2] Hence the size of the strategic effect also depends upon whether firm 1 is the price-floor is binding for firm 1 or not, and is larger in the latter case. Consider first the case where the price-floor binds for neither firm.

[2] Although the direct effect of the reduction in firm 1's price upon its own profits is zero, by the envelope theorem, and may hence be ignored, the indirect effect via the induced change in firm 2's equilibrium price cannot be ignored. It is this effect that this last term captures.

As d'Aspremont et. al. [1] have shown, in this case the strategic effect outweighs the direct effect, so that the firm loses by reducing differentiation.

$$\left.\frac{d\pi_1}{da}\right|_{\Delta_2=1,\Delta_1=1} = \frac{-(1+3a+b)p_1^*}{6(1-a-b)} < 0 \tag{2.10}$$

This implies that if $\bar{p} < t$, the equilibrium price in the absence of a price-floor, then maximum differentiation is locally optimal.

Consider now the intermediate case where the price-floor binds on firm 1 but does not bind on firm 2, which is only possible if $a < b$. Hence $p_1^*(a,b,\bar{p}) = \bar{p}$, and $p_2^*(a,b,\bar{p}) = \tilde{p}_2(a,b,\bar{p})$. Substituting these prices in firm 1's demand function, we obtain

$$D_1 = \frac{(1+a-b)}{2} + \frac{(1-a+b)}{4} - \frac{\bar{p}}{2t(1-a-b)} \tag{2.11}$$

Differentiating, we obtain

$$\left.\frac{dD_1}{da}\right|_{\Delta_1=0,\Delta_2=1} = \frac{1}{4} - \frac{\bar{p}}{2t(1-a-b)^2} \tag{2.12}$$

Since the price-floor binds on firm 1 we must have

$$\bar{p} \geq \frac{[(1-b)^2 - a^2]}{2} + \frac{\tilde{p}_2}{2} \tag{2.13}$$

Substituting this inequality in (2.12) and using (2.4) yields the inequality

$$\left.\frac{dD_1}{da}\right|_{\Delta_1=0,\Delta_2=1} \leq -\frac{1}{4} - \frac{(b+2a)}{3(1-a-b)} < 0 \qquad (2.14)$$

Since firm 1's profits are given by $\bar{p}D_1$, the derivative of profits with respect to a is also negative.

This result is interesting in its own right, since it shows that the strategic effect of a change in location upon firm 2's price is sufficiently large to offset the direct effect in (2.9), even when there is no effect via the impact of firm 1's own price upon firm 2's optimal price.

We now show at any locally optimal locational configuration must exhibit either minimum or maximum differentiation. Let (a, b) be an arbitrary locational configuration. If $a = 1 - b$ so that both firms are located at the same point, then both must equal 1/2, since otherwise either firm could increase its market share and profits by a small shift towards the longer end of the market. Without loss of generality we may let $a < 1 - b$. If the price-floor binds on either firm, then this firm can increase its profits by moving towards the other. If the price-floor binds on neither firm, then by (2.10) either firm can increase its profits by moving away from its rival. We summarize this discussion in the following proposition.

Proposition 2.1. *Any locally optimal locational configuration exhibits either maximum differentiation or minimum differentiation. Minimum differentiation is always locally optimal if* $\bar{p} > 0$. *Maximum differentiation is locally optimal if*

$\bar{p} < t$, the equilibrium price when there is no price-floor.

The above proposition implies that if the price-floor lies in the interval $(0, t)$, then *both* minimum and maximum differentiation are locally optimal. However, they are not necessarily subgame perfect equilibria in our basic game since they may be destablized by *large* deviations. This arises since with a price-floor, the profits of a firm is no longer a quasi-concave function of its location decision. However, if firms' cannot alter their product variety easily, locally optimal configurations may well be globally optimal. To see this, consider a infinite-horizon game, where in each period τ consists of two-stages. In stage 1, each firm chooses location $l_{i\tau}$, and incurs a quadratic adjustment cost $c(l_{it} - l_{i\tau-1})^2$,where $l_{i\tau-1}$ is the firm's location in the previous period. In stage 2 firms choose prices. Firms seek to maximize the discounted sum of profits, where profits equal revenues minus adjustment costs. Initial locations (l_{i0}) are given arbitrarily. A *Markov-perfect* equilibrium in this game is a subgame perfect equilibrium where each firm's location decision in each period, $l_{i\tau}$, depends only upon its previous location, $l_{i\tau-1}$, and where its pricing decision only depends upon current location. We conjecture that in any Markov-perfect equilibrium, locations and prices will converge to either the minimum or maximum differentiation outcomes (i.e. the associated locations and prices). Further, if the adjustment cost parameter, c, is sufficiently large, both minimum differentiation and maximum differentiation outcomes can be approached given the appropriate initial conditions. However, verification of this conjecture is beyond the scope of this paper.

We now return to our basic two-stage game, to explore the global stability of both maximum and minimum differentiation configurations in turn.

2.2. Maximum Differentiation

Proposition 1 implies that if the price-floor is greater than t, the equilibrium price in the absence of a price-floor, then maximum differentiation is not locally optimal and hence cannot be a subgame perfect equilibrium. We see now that the maximum differentiation equilibrium is destroyed even when the price-floor is "not binding", i.e. is substantially lower than t, equilibrium price in the absence of the price-floor.

Proposition 2.2. *Maximum differentiation is an equilibrium if and only if* $\bar{p} \leq t/2$.

Proof. Let $\bar{p} > t/2$. If firm 2 locates at 1, and firm 1 locates at 0, the equilibrium price is t, and firm 1 earns profits of t/2. If firm 1 locates at 1-ϵ, where ϵ is small, the price floor comes into operation and firm 2 must charge at least $\bar{p}$. If firm 1 also charges $\bar{p}$, its demand will be 1 - $\epsilon/2$, and profits are $\bar{p}$(1-$\epsilon/2$). Hence in any pricing equilibrium firm 1 earns at least $\bar{p}(1 - \epsilon/2)$. If $\bar{p} > t/2$, then there exists ϵ sufficiently small such that $\bar{p}(1 - \epsilon/2) > t/2$, where $t/2$ is the profit that the firm earns by locating at 0.

We show now that maximal differentiation remains an equilibrium if $\bar{p} \leq t/2$. To see this, consider firm 1 and observe that it profits are locally decreasing in a as long as the price-floor does not bind upon firm 2, and are locally increasing in a only when the price-floor binds upon firm 2, which occurs when a is sufficiently close to 1. If $a = 1-\epsilon$, then $p_1^*(1-\epsilon, 1, \bar{p}) \to \bar{p}$ as $\epsilon \to 0$, so that firm 1's profits are increasing in ϵ and tend to $\bar{p}$(1-ϵ/2) as $\epsilon \to 0$. This is strictly less than t/2, the profits at the maximal differentiation locations, so that maximal differentiation remains an equilibrium. ■

The maximum differentiation outcome is not an equilibrium with a price-floor since a firm which locates at the end of the market becomes vulnerable to aggressive locational behavior by its rival. Interestingly, this aggressive behavior is not incremental but large. Indeed, if the price-floor is between t and $t/2$, then proposition 1 tells us that a maximum differentiation is locally optimal, and can only be destabilized by large deviations.

The logic of proposition 2 does not hinge upon our assumption that consumers are uniformly distributed. Consider an arbitrary symmetric density which produces the maximum differentiation result in the absence of a price floor, and let p* be the resulting equilibrium price. Any price floor greater than p*/2 will destroy this equilibrium by the same argument as in the proposition.

2.3. Minimum Differentiation

We now consider the conditions under which minimum differentiation emerges as an equilibrium outcome. Since minimum differentiation is always locally optimal, it follows that it can only be destabilized by large deviations. Under minimum differentiation, both firms locate at $1/2$ and earn profits of $\bar{p}/2$. Consider a deviation by firm 1 to the point $a = 0$. If the price-floor does not bind on both firms, we can solve equations (2.3) and (2.4) for the equilibrium prices when $a = 0$ and $b = 1/2$. This yields

$$p_1^*(0, 1/2, \bar{p}) = 5t/12 \text{ if } \bar{p} \leq 5t/12 \tag{2.15}$$

$$p_2^*(0, 1/2, \bar{p}) = 3t/4 \text{ if } \bar{p} \leq 5t/12. \tag{2.16}$$

The profits of firm 1 at this configuration equal $25t/144$. Minimum differentiation can only be an equilibrium if the corresponding profits are no less than this. Hence we must have $\bar{p}/2 \geq 25t/144$ or $\bar{p} \geq 25t/72$. We have therefore established that if $\bar{p} < 25t/72$, minimum differentiation cannot be an equilibrium.

We now show that the condition $\bar{p} \geq 25t/72$ is sufficient for minimum differentiation to be an equilibrium. Let $a < 1/2$ be an arbitrary location for firm 1, and let $b = 1/2$. Clearly the price-floor cannot bind for firm 2 if this deviation from 1/2 is to be profitable for firm 1. However, if $\Delta_2 = 0$, then (2.12) and (2.10) imply that firm 1's profits are locally decreasing in a, and hence firm 1's profits are greater if it chooses location 0. However, we have established that if $\bar{p} \geq 25t/72$, locating at 1/2 is better than locating at 0, and hence firm 1 cannot increase its profits by locating at a.

We have therefore proved the following proposition.

Proposition 2.3. *Minimum differentiation is an equilibrium outcome if and only if* $p \geq 25t/72$.

Propositions 2 and 3 imply that one uniquely has maximal differentiation if the price floor is very low and minimum differentiation if the price floor is sufficiently high. At an intermediate range - $\bar{p} \in$[25t/72, t/2], one has multiple equilibria - both minimum and maximum differentiation are possible.

The stability of the minimum differentiation equilibrium in the presence of a price-floor seems fairly robust, in the sense that it should also obtain in a wide variety of specifications. For example, if transport costs are linear instead of being quadratic, minimum differentiation should be an equilibrium, and is indeed likely to be a an equilibrium for a larger range of values of $\bar{p}$. Further, our results seem to be robust to the possibility that the price-floor is announced *after* firms choose location rather than before, as we have assumed. In this case the price-floor can depend upon the location choices, and would be described by an exogenously given function $\bar{p}(a,b)$. If $\bar{p}(a,b)$ is always greater than $t/2$, this suffices to ensure that maximum differentiation is not an equilibrium, while if it is always greater than 25t/72, minimum differentiation is an equilibrium.

3. Implications and Extensions

We now consider the implications of our analysis, considering questions of equilibrium selection in the presence of multiple equilibria and the welfare implications of a price-floor. We also consider the implications of a price-floor for entry.

3.1. Multiple Equilibria

The relation between the price-floor and pure strategy equilibrium price are set out in Fig.1. If the price floor is low ($\bar{p} < 25t/72$) then the equilibrium price is t. If the price-floor is high ($\bar{p} > t/2$), then the equilibrium price is $\bar{p}$. If the price-floor is above $t/2$ but below t, then the price-floor actually reduces equilibrium price. If the price-floor is at an intermediate level (25t/72$\leq \bar{p} \leq t/2$), then one has multiple equilibria, with both $\bar{p}$ and t as equilibrium prices.

We now consider the issue of equilibrium selection in the situation where the equilibrium price is not unique. Clearly the firms are better off in the maximum differentiation equilibrium since equilibrium and profits are greater. However, this is not a sufficient reason for assuming that this equilibrium is likely to prevail.

Indeed, one can argue that the minimum differentiation equilibrium is more likely to be selected. The reason for this is that the maximum differentiation equilibrium involves an additional coordination problem between firms, since there is a *pair* of such equilibria - an equilibrium where firm 1 locates at 0 and firm 2 at 1, and an equilibrium where firm 1 locates at 1 and firm 2 at 0. Firms must be able to coordinate, since otherwise they could both end up at the same location. More formally, neither of the maximum differentiation equilibria satisfies the Harsanyi-Selten [5] criterion of *symmetry invariance.* In a symmetric game, any symmetry invariant equilibrium must be symmetric, in the sense that the strategies of the two players must be identical. Neither of the maximum differentiation equilibria satisfy this condition.[3] On the other hand, the minimum differentiation equilibrium is symmetric and symmetry-invariant. In this equilibrium, firms need not overcome a coordination problem akin to that in the maximum differentiation case.

3.2. Welfare

We briefly consider the welfare implications of a price-floor. If $\bar{p}$ lies between t and t/2, then consumers are clearly better off as the equilibrium price is lower than in the absence of a price-floor. However, if we take the usual utilitarian welfare

[3]Bester et. al. [3] also make this point in the context of the model without a price-floor, and show that there is a unique symmetric (and hence symmetry-invariant) equilibrium in mixed strategies.

criterion of producer surplus + consumer surplus, there is no difference in welfare. This arises for two reasons - first, all consumers have a high enough valuation and always buy one unit of the good, and second, the uniform distribution of consumers implies that aggregate transport costs with minimum differentiation equal those with maximum differentiation. However, one can argue that if one slightly modifies either of these assumptions, a price-floor is likely to be welfare improving. The most plausible reason for this is that a price-floor reduces the divergence between price and marginal costs.

Consider first a model where at each location there are two types of consumers, with valuations v_H and v_L respectively, where $v_H > 5t/4 > v_L$. The two types of consumers have measure 1 and μ_L respectively. Observe that as $\mu_L \to 0$, this model approaches the model that we have analyzed in this paper, and will have qualitatively similar equilibria. In the absence of a price-floor, maximum differentiation will ensue and the equilibrium price will be close to t. Hence some low valuation consumers will not purchase the product. A price-floor which reduces the equilibrium price will be welfare-improving since it allows these consumers to purchase the product.

Minimum differentiation and maximum differentiation are also no longer welfare equivalent if the distribution of consumers is not uniform. Consider a small perturbation of the uniform density[4], so that once again equilibria are close to those analyzed in this paper. If this density is unimodal, then transport costs will be lower at minimum differentiation as compared to maximum differentiation. This may be a second reason for arguing that a price-floor increases welfare, if one thinks that the market is likely to be uni-modal. However, this reason is weaker than the first one discussed earlier, since neither minimum nor maximum differentiation comes close to minimizing transport costs. As we shall see later, a price-ceiling is likely to perform better from this point of view.

3.3. Entry

Although our focus is on variety choice rather than entry (Salop's circle model is probably a more flexible vehicle for this purpose), our model has implications for entry as well. Consider a the following three-stage game, where the price floor is announced in stage 0. In stage 1 two firms must choose whether to enter or not, where the decision to enter entails a sunk cost F. In stage 2 the firms which have decided to enter choose locations, and then chose prices in stage three. At each stage, firms are informed of the choices made in previous stages. Our analysis implies that a price floor can reduce entry into the industry. To see this, let $F < t/2$, so that in the absence of the price floor both firms will enter and

[4] I.e., let the density be a continuous function whose value at any point in $[0, 1]$ is within ϵ distance of 1.

maximally differentiate their products, earning profits of $t/2 - F$. Consider now a price floor $\bar{p} > t/2$. This ensures that the unique equilibrium outcome if both firms enter has both firms locating at the mid-point of the interval and earning profits of $\bar{p}/2 - F$. Hence if $\bar{p} < t$, and $F > \bar{p}/2$, the firms will make losses in the event that both enter. Consequently, in any pure strategy subgame perfect equilibrium, only one firm can enter. Moreover, since this firm will be a monopolist, it charges the monopoly price, which corresponds to the consumers' reservation value, v. Since we have assumed that $v > 5t/4$, this implies that in this instance a price-floor increases equilibrium prices. The mechanism whereby this comes about is of course quite different from the usual mechanism - in our model, a price-floor increases prices with free-entry precisely because it reduces prices in the event that both firms enter the market.

4. Price-Ceiling

We consider now the effects of a price ceiling. Let $\breve{p}$ be the price-ceiling, so that firms must charge prices no greater than $\breve{p}$. We shall see that much of our analysis is easily extended to this context, although the substantive results are quite different. In particular the equations (2.1-2.4) for demands and optimal unconstrained price remain unaffected, but the optimal price is now given by

$$\hat{p}_i(p_j) = \max\{\tilde{p}_i(p_j), \breve{p}\} \tag{4.1}$$

The price-ceiling *binds* on firm i if $\tilde{p}_i(p_j) > \breve{p}$, and as before, let the dummy variable $\Delta_i = 0$ in this case. Obviously the expressions for profits etc. are unaffected. Let $p_1^*(a, b, \breve{p})$ and $p_2^*(a, b, \breve{p})$ denote Nash equilibrium prices given locations and the price-ceiling.

The analysis of section 2.1, and in particular, the expressions for the derivative of profits in the various regimes, i.e. the equations (2.9),(2.10),and (2.12) still continue to apply. In addition, we also have to consider what happens at values of (a, b) where Δ_1 or Δ_2 have a discontinuity, i.e. where we have a regime change. At such points the profit function is not differentiable, but its right-hand and left-hand derivatives are given by the above expressions. This is made more precise below.

We shall now show that for any price ceiling there is an equilibrium where firms locate sufficiently far apart so that each firm's optimal price equals the price-ceiling. In this equilibrium each firm locates equidistant from the centre, i.e. $a = b$. Since we can exchange the locations of the firms, there is obviously a pair of such equilibria; however, there is no other pure strategy equilibrium. Given any price-ceiling $\breve{p} < t$, let $z(\breve{p})$ be the value of z which solves the equation

$$\check{p} = p_1^*(z, z, t) \tag{4.2}$$

In other words, if $a = b = z(\check{p})$ and the price-ceiling is irrelevant, the equilibrium price of firm 1 (and of firm 2) will be $\check{p}$. It is easy to see that such a solution exists, since the right-hand side of (4.2) equals t when $z = 0$, and equals 0 when $z = 1/2$. Since the Nash equilibrium price varies continuously with z, a solution to (4.2) always exists. Further, since the Nash equilibrium price is strictly decreasing in z, this solution is unique. To verify that this locational configuration ($a = b = z(\check{p})$) is an equilibrium with a price-ceiling of $\check{p}$), observe that the right hand derivative of firm 1's profits with respect to a at this point is given by (2.10), and is hence negative, so that the firm has no incentive to increase a. On the other hand, since the price-ceiling will bind on firm 2 if the firm reduces a, the left-hand derivative of firm 1's profits with respect to a is given by (2.9), and hence the firm has no incentive to reduce a. This proves local optimality. Further, since the derivative of profits with respect to a is given by (2.10) for $a > z(\check{p})$ and by (2.9) for $a < z(\check{p})$, we also have global optimality, i.e. the locational configuration is a subgame perfect equilibrium.

We now show that there cannot be any other subgame perfect equilibrium (other than the one with the roles of the firms reversed). Let a, b be an arbitrary locational configuration. If the price-ceiling binds on any firm, then the other firm has an incentive to move closer. However, firm j is pricing below the price-ceiling, (i.e. $p_j^*(a,b,\breve{p}) < \breve{p}$), then firm i's profits are decreasing in a, and hence firm i has an incentive to move further away. Hence in any equilibrium in locations, the firms unconstrained optimal prices must equal the price-ceiling.

We summarize this discussion in the following proposition.

Proposition 4.1. *A price-ceiling induces firms to locate exactly so far apart so that the resulting equilibrium prices equal the price-ceiling and the ceiling does not bind on either firm.*

Since a price-ceiling makes for intermediate locations, it can clearly be welfare improving. To see this consider the problem of a social planner who seeks to maximize the sum of consumer and producers' surplus. Since all consumers always buy the product, this program is equivalent to the problem of minimizing the sum of consumers' transport costs, which can be achieved by directing one firm to locate at 1/4 and the other at 3/4 (i.e. $a = b = 1/4$). If the product variety decisions of the firms are not publicly verifiable, as is likely if one has a less literal interpretation of "location", then this directive cannot be enforced. Nevertheless, the planner can achieve the same result if prices are verifiable, by imposing a price-ceiling which induces the corresponding location choices.

A price-ceiling may also be optimally used as a vertical restraint, by a supplier upon retailers. Consider a monopoly supplier of a durable good, who licenses retailers to supply complementary services. If the monopolist can use the pricing of the durable good to extract consumer surplus and franchise fees to mop up the profits of the retailers, then the monopolist's problem is very similar to that of the social planner. Hence a price-ceiling can be used by the monopolist to ensure that the variety choice of retailers is optimal.

5. Concluding Comments

A price floor relaxes price competition, allowing firms to compete more aggressively via other variables such as location. In our model, the resulting change in locational patterns intensifies price competition, making equilibrium prices lower than in the absence of a price floor. It may be useful to interpret this result using Fudenberg and Tirole's [4] taxonomy. The standard maximum differentiation result is example of the "puppy-dog" strategy - firms locate far from the centre of the market in order to stimulate favorable pricing behavior by their opponent. However, a price-floor makes this strategy unviable, since a puppy-dog is vulnerable to an aggressive "top-dog" who seeks to corner the market.The price-floor reduces the cost to a top-dog strategy by bounding the extent to which aggressive behavior reduces prices.

The idea that restraints upon price competition result in intensified competition in other dimensions is not a new one. For example, the originator of the kinked demand curve theory, Paul Sweezy, argued that modern oligopoly was characterized by an absence of price-competition, and argued that this resulted in intensified competition via advertising (see Baran and Sweezy [2]). This idea has been formalized in the context of variety choice by Zhang [7]. Zhang considers the basic Hotelling model, as in this paper, but allows firm the possibility of choosing a "price-matching policy", before prices are finally chosen. Zhang focuses on subgame perfect equilibria which satisfy an iterative dominance criterion and shows that this has both firms locating at the midpoint, adopting price-matching policies and choosing the monopoly price.[5] The twist in our paper, as compared to this idea, is the fact that intensified locational competition finally results in lower prices! While superficially counter-intuitive, the intuition behind this result is quite straightforward. By increasing the price in some subgames, a price-floor ensures that the subgame where both firms maximally differentiate is never reached, hence ensuring lower equilibrium prices.

Our analysis of the effects of a price-floor has relevance to a number of other contexts. The analysis of the effects of anti-dumping legislation in the context of international trade is an obvious example. A more challenging extension would be to consider contexts where, unlike this paper, the price-floor is endogenously

[5]This formalization raises the question, what happens if firms can choose other types of policies which relate the price paid by the consumer to both the announced prices, such as a committment to undercut rivals. The answer to this question (and hence the robustness of Zhang's result) is not clear since iterative dominance arguments are very sensitive to addition of other strategies.

chosen. A repeated oligopoly where firms choose both price and product variety, and seek to collude on price, is an example. A proper treatment of such issues also requires incorporation of the reasons, possibly information based, why such asymmetry arises. We leave such extensions for future work.

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