Bertrand Competition Under Uncertainty

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Abstract

Consider a Bertrand model in which each firm may be inactive with a known probability, so the number of active firms is uncertain. This simple model has a mixed-strategy equilibrium in which industry profits are positive and decline with the number of firms, the same features which make the Cournot model attractive. Unlike in a Cournot model with similar incomplete information, Bertrand profits always increase in the probability other firms are inactive. Profits do decline more sharply than in the Cournot model, and the pattern is similar to that found by Bresnahan & Reiss (1991).

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Footnotes beginning with xxx are notes to myself for redrafting.

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1. INTRODUCTION

An enormous number of papers have been written on the Cournot and Bertrand models of oligopoly, and it is very easy to discover unpublished and uninteresting elaborations of them. Any addition to the literature needs considerable justification, which I will attempt to provide. My addition will be a Bertrand model that includes the possibility that each firm in the industry might be inactive in a particular transaction. Any reader immediately persuaded that this is interesting may skip directly to Section 2.

The dichotomy between competition in quantity and competition in price has existed for over a hundred years. Cournot (1838) proposed a model in which N firms simultaneously choose quantities and let the market determine the price. Bertrand (1883) pointed out that entirely different conclusions result if the firms choose prices simultaneously instead.

Students try to decide whether to use the Cournot or the Bertrand model based on descriptive realism, but the more sophisticated position is that both are simply models, to be used not because of simplicity and realistic properties. The Cournot model is preferred for looking at oligopoly, but not because economists think oligopolists choose quantities rather than prices. Rather, the model is easy to use and has properties we think realistic: the oligopoly industry profits are between the monopoly and perfectly competitive levels, and they fall as N increases. This makes the model a convenient building block for studying other features of industries with positive profits and varying numbers of firms. At the same time, it is well-known that the modeller must be careful not to let accidental special features of the Cournot model drive his results. If market demand is not linear, perverse results can occur, and the model is not suitable for studying mergers because it can happen that the merger of two firms reduces their joint profits rather than increasing them.[1]

The Bertrand model of competition by choice of price is descriptively more realistic, but has the perverse outcome that two firms are enough to achieve the perfectly competitive price. It is therefore often used as a building block in models where that result is desired. The model also has the unfortunate property of yielding mixed strategy equilibria when firms have standard upward-sloping marginal cost curves, the Edgeworth Paradox.[2] More moderate results can be obtained with a Bertrand model of differentiated products with constant marginal cost but such a model is harder to use as a building block.[3] Also, Kreps & Scheinkman (1983) show that a two-stage game in which firms first choose capacities and then choose prices has an outcome very similar to Cournot competition. Since Cournot competition is much simpler than the two-stage game, it seems sensible to use the Cournot

[1] See Gaudet & Salant (1991) or the papers collected in Daughety (1988) for illustration of the properties and usefulness of the Cournot model. Suzumura's 1995 book is a particularly comprehensive study, with special emphasis on the welfare aspects of free entry. Salant , Switzer & Reynolds (1983) point out the peculiar implications of the model for mergers.

[2] Nonexistence of a pure strategy equilibrium when capacities are limited in a Cournot model was discovered by Edgeworth (1893). The mixed strategy equilibrium can be found in Levitan & Shubik (1972) or Dasgupta & Maskin (1986).

[3] Hotelling (1929) is the classic reference. See p. 317ff of Rasmusen (1994) for a textbook explanation.

model instead.

Despite the considerable work on these models, little attention has been paid to the Bertrand model with incomplete information. If *N* Bertrand firms have different levels of constant marginal cost, the firm with the lowest marginal cost will serve the entire market at a price equal to the marginal cost of the next lowest firm. But what happens if firms do not know each other's costs?

In some situations one can view this as simply an auction. Auction theory is the the theory of competition in price. One difference, as

Spulber (1995) points out, is that in a normal auction the quantity to be sold is specified in advance, whereas in most markets demand is downward sloping. A bidder in an auction faces just one tradeoff: between winning and collecting a higher price. A bidder in a Bertrand game with downward-sloping demand faces an extra tradeoff: between high price with low quantity and low price with high quantity. This makes the Bertrand game with downward-sloping demand resemble an auction with risk-averse bidders.

Another difference between auctions and Bertrand competition is that one side of the market generally controls the rules of an auction. If sellers bid for the right to sell to a buyer, the buyer structures the auction rules to make the price as low as possible. Thus, articles on auction theory focus on auction design, rather than on what happens for given, suboptimal designs, though this usually must be examined to some extent on the way to finding the optimal design. In addition, the sellers in a Bertrand model can often realistically be modelled as risk-neutral, being large firms with diversified ownership, whereas risk aversion is a major complication in auction models.

One article outside the auction literature which studies price competition under asymmetric information is Spulber (1995). Spulber analyzes a market in which *N* firms with constant marginal cost and zero or positive fixed costs compete by simultaneous choice of price but do not know each other's cost curves. [4] Cost is parametrized by a single variable, which is continuously distributed. The equilibrium is in pure strategies, and the expected price is between the perfectly competitive and the monopoly levels.

The present paper will take a different approach to the same problem. I will assume that firms are either active, with marginal cost c, or inactive, with infinite marginal cost, and that firms do not know which other firms are active in a given transaction. Being inactive could have a number of interpretations— the firm has reached its capacity, it has gone out of business, it has not yet entered, it has priced grossly high by mistake, it has not discovered that it could bid on this particular transaction, it has closed down because of strike, illness, or vacation , and so forth.

[4] His model allows increasing marginal cost under the additional assumption that a firm supplies every customer that wishes to buy from it. That is optimizing behavior only under regulation or under non-increasing marginal cost, however.

The result will be a relatively simple model which has the same desirable features as the Cournot model. The oligopoly industry profits will be between the monopoly and perfectly competitive levels, and they will fall as N increases. The model is more complex than Cournot in that the equilibrium is in mixed strategies and the outcomes are expected rather than deterministic, but it is simpler in that demand can be specified as for one unit at less than a reservation price rather than as a linear function. It neither includes nor is included by the Spulber model, but it is easier to analyze. I hope, therefore, that this may be a useful building block as well as telling us something about the properties of price competition under uncertainty.

Section 2 will lay out the basic model. Section 3 analyzes simultaneous Bertrand competition by symmetric firms, and contains the main results. Section 4 compares the Bertrand and Cournot relationships between profits and concentration, with and without uncertainty. Section 5 contains an asymmetric duopoly model, in which one firm is known to be active but the other is not. Section 6 shows what happens if price choice is sequential in either the symmetric or asymmetric model. Section 7 concludes.

2. THE BASIC MODEL

Each firm i of $N+1$ risk-neutral firms submits a bid p_i to supply the customer.[5] Each firm has a marginal cost of either c, or, with probability $\theta \in (0,1)$ independent for each firm, infinity, which is incurred only if the firm wins the contract. We will say that a firm with a marginal cost of infinity is "inactive". The customer will buy one unit, at the lowest price, paying up to reservation price R. If firm i uses a mixed strategy, denote its cumulative distribution of prices by $F_i(p)$.

INTERPRETATION. The variable θ represents the probability that a given firm is inactive and does not make a bid that could possibly win the contract. As the introduction said, this could have a number of interpretations:

1. The firm has reached its capacity.
2. The firm has gone out of business.
3. The firm has not yet entered the business, though competitors thought it might have.
4. The firm priced grossly high, above R because of a mistake in judgement or mistaken information about the customer.
5. The firm has not discovered that it could bid on this particular transaction.
6. The firm has closed down temporarily because of strike, illness, or vacation.
7. The customer has not realized that he could invite the firm to bid on this transaction.

[5] xxx Consider making this N instead of N+1.

One way to view the situation being modelled is as an auction. McAfee and Macmillan (1987) analyze an auction with independent private values, possibly risk-averse bidders, and an unknown number of active bidders. They find that the number of bidders being stochastic does not matter in the optimal auction when bidders are risk neutral—the seller auctioning off an item can do as well as if the number of bidders were known. The optimal auction is most definitely not the first-price sealed-bid auction which is equivalent to the Bertrand model, however. It is, rather, an English auction, in which bidders bid sequentially, publicly, and as many times as they wish. In such an auction, it is clear that the winner will bid at the second-highest valuation of the active bidders, and it will become clear in the course of the auction who is active. In a sealed-bid auction this does not happen.[6]

Optimal or not, first-price sealed bid auctions are commonly used, and this paper's model represents them. It also represents Bertrand competition, in which a number of firms offer prices to one or more customers, without knowing how many other firms are active. Section 3 analyzes a model with the step function demand described above, while Section 4 will show the modification needed to change to linear demand.

3. THE SIMULTANEOUS SYMMETRIC BERTRAND MODEL

The first and most important model to be examined is Bertrand competition under the following clarification to the basic model.

Assumption: Firms submit bids simultaneously.

Under this assumption there

exists no equilibrium in pure strategies unless θ equals 0 or 1. If θ is zero, all firms charge c, in the ordinary Bertrand equilibrium. If θ is 1, then no firm is active. If, somehow, a firm were active anyway (which has zero probability but is conceivable), then it would charge R, the ordinary monopoly equilibrium.

What if $\theta \in (0,1)$? No set of prices of which one price is greater than c can be an equilibrium. If the two lowest prices were not equal, whichever firm has the lowest price would deviate to increase its price. If the two lowest prices were equal, one firm could reduce its price and win the entire market with certainty.

Nor is it an equilibrium for all firms to charge c. This would lead to zero profits, whereas a firm that deviated and charged R would gain profits of $R-c$ with probability θ^N.

Instead, the equilibrium must be in mixed strategies. Each firm randomizes, choosing prices of at least c and no more than R. And, since a firm can guarantee itself a positive expected profit from the pure strategy of charging R and winning with probability θ^N, it

[6] An intermediate form of auction, which has not been analyzed as far as I know, is the first-price sealed-bid auction where bidders know how many bids have been submitted, but not the size of the bids.

must be that the range of mixing is not all the way down to $p = c$, but rather is in the interval $[B, R]$, where $B > c$.

There exists a symmetric equilibrium for $N + 1 > 1$ in which

$$F(p) = \sqrt[N]{1 - \left(\frac{\theta^N}{1-\theta^N}\right)\left(\frac{R-p}{p-c}\right)}. \tag{1}$$

over the price range $[\theta^N R + (1-\theta^N)c, R]$. At both the individual firm and industry level, expected prices and profits are falling in N.[7] The rest of the section will demonstrate this.

Start by hypothesizing that such an equilibrium does exist with mixing over the range $[B, R]$.

The expected payoff to firm i from the pure strategy of $p_i = p$ is then

$$\pi_i(p) = \left[\theta^N + (1-\theta^N)(1-F(p))^N\right][p-c]. \tag{2}$$

Over the mixing range, this profit is equal for any price. If $p = R$, it must be that

$$\pi_i(R) = \theta^N [R - c], \tag{3}$$

because Firm i will win the auction only if no other firm bids (except for the infinitesimal probability that the other prices also equal R, which has probability zero in the conjectured equilibrium.) Equating (2) and

(3) yields

$$\left[\theta^N + (1-\theta^N)(1-F(p))^N\right][p-c] = \theta^N [R-c], \tag{4}$$

which can be solved to yield

$$F(p) = \sqrt[N]{1 - \left(\frac{\theta^N}{1-\theta^N}\right)\left(\frac{R-p}{p-c}\right)}. \tag{5}$$

$F(B) = 0$ by definition of B, so from (5),

$$0 = 1 - \left(\frac{\theta^N}{1-\theta^N}\right)\left(\frac{R-p}{p-c}\right), \tag{6}$$

which can be solved out to get

$$B = \theta^N R + (1-\theta^N)c \tag{7}$$

[7] A caveat: industry profits fall in N when these are defined conditionally on at least one firm being active, as will be explained below. Greater N increases the chance of at least one firm being active.

This completes the derivation of the equilibrium.

EQUILIBRIUM OUTCOMES.

The expected price a firm charges is p' such that

$$0.5 = F(p') = \sqrt[N]{1 - \left(\frac{\theta^N}{1-\theta^N}\right)\left(\frac{R-p'}{p'-c}\right)}, \tag{8}$$

which when solved out yields

$$p' = c + \frac{(R-c)\theta^N}{1-.5^N(1+\theta^N)} \tag{9}$$

Expected profit for one firm is, since it is in the industry with probability $(1-\theta)$,

$$\pi_i = (1-\theta)\theta^N(R-c). \tag{10}$$

Note that individual profit is declining in N.

Expected profit for the industry is[8]

$$\sum_{i=1}^{N+1} \pi_i = (N+1)(1-\theta)\theta^N(R-c), \tag{11}$$

Industry profit is a bit tricky. It can increase with N simply because a greater N reduces the chance that no firm is active and profits are zero.

We can back out the expected price from the expected industry profit. The probability of an industry profit of 0 is θ^{N+1}. Denote the average winning bid when at least one firm bids by p''. It is true that

$$\sum_{i=1}^{N+1} \pi_i = (N+1)(1-\theta)\theta^N(R-c) = \theta^{N+1}(0) + (1-\theta^{N+1})(p''-c). \tag{12}$$

From this we can deduce that

$$\begin{aligned} p'' &= c + \tfrac{(N+1)(1-\theta)\theta^N(R-c)}{(1-\theta^{N+1})} \\ &= c + \tfrac{(N+1)\theta^N}{(1-\theta^{N+1})}(1-\theta)(R-c) \end{aligned} \tag{13}$$

[8]Note that although the profits of the different firms are not independent, the expected profits are, so this summation is legitimate.

Expected industry profit conditioning on there being at least one active firm is then

$$p'' - c = \frac{(n+1)(1-\theta)\theta^n(R-c)}{(1-\theta^{n+1})} \tag{14}$$

To see how industry profit changes with N, note that after some algebra,

$$\frac{dp''}{dN} = \left[\frac{\theta^N(1-\theta^{N+1}+log(\theta)+Nlog(\theta))}{(\theta^{N+1}-1)^2}\right][(1-\theta)(R-c)], \tag{15}$$

a derivative which is well-defined even though only integer values of N have an economic interpretation. The sign of expression (15) is the sign of

$$1-\theta^{N+1}+log(\theta)+Nlog(\theta), \tag{16}$$

which most nearly is positive when $N=1$ and $\theta=1$. Since θ is a fraction, expression (16) falls with N, and so is greatest when $N=1$ and it equals $1-\theta^2+2log(\theta)$. Its derivative with respect to θ when $N=1$ is $-2\theta+2/\theta$, which is positive, so it is increasing in θ. But its maximum, at $N=1$ and $\theta=1$, is only $(1-1^{1+1}+log(1)+1log(1))=0$. Thus, p'' must be decreasing with N. The expected lowest price and industry profit conditional on at least one firm serving the market is falling with the number of firms.

Thus, this simple Bertrand model lacks the discontinuous behavior of the original Bertrand model. Profits are positive, but the expected price and profits decrease smoothly in the number of firms, as in the Cournot model. Note also that the Betrand model easily handles the situation where demand is given by a step-function, which creates difficulties for the Cournot model.

This model can also illustrate in a simple way the intuition for the result of McAfee and Macmillan (1987) that the seller in an auction can benefit from the risk aversion of the buyers and their lack of precise knowledge of how many bidders are active. Note that in the present model, a seller who charges below B, the lower bound of the support of the mixing distribution, will win the customer with certainty, and still earn a profit. A risk-averse seller would wish to take advantage of this, and would tend to push down the prices charged. A high price is a gamble in the hope that other firms are inactive or are themselves charging high prices, so risk aversion tends to reduce prices. This also suggests that prices might rise when firms are in financial distress. A firm near bankruptcy is often risk-loving, because its additional losses are borne by its creditors but its gains are partly kept by its owners. Such a firm should be more willing to set out a high price of R in a gamble that no other firm is active.

4. COMPARING BERTRAND AND COURNOT

To compare Bertrand and Cournot oligopoly, we cannot use a step-function demand curve, since Cournot equilibrium applies rather uncomfortably to that situation. Continue to let marginal cost be constant at c with probability $1-\theta$ and infinite with probability θ, but now assume that demand is linear, so

$$p\left(\sum_{i=1}^{N+1} q_i\right) = \alpha - \beta \sum_{i=1}^{N+1} q_i. \tag{17}$$

Let us define $q(p)$ as the demand facing a monopolist at a price of p, so

$$q(p) = \frac{\alpha}{\beta} - \frac{p}{\beta}, \tag{18}$$

and redefine R to be the monopoly price, so

$$R = \frac{\alpha + c}{2}. \tag{19}$$

(Note that $q(R) = \frac{\alpha - c}{2\beta}$.)

What I will do in this section is to compute the expected profits from Cournot and Bertrand for different levels of N, to obtain some idea of the effects of concentration in each. (The profit path, however, is no longer the same shape as the price path now, because quantity changes with price.)

BERTRAND EQUILIBRIUM

The expected payoff to firm i from the pure strategy of $p_i = p$ is

$$\pi_i(p) = \left[\theta^N + (1-\theta^N)(1-F(p))^N\right][p-c]q(p). \tag{20}$$

Over the mixing range, this profit is equal for any price. If $p = R$, it must be that individual firm profit is

$$\pi_i(R) = \theta^N [R-c] q(R). \tag{21}$$

and industry profit is

$$\sum_{i=1}^{N+1} \pi_i = (N+1)(1-\theta)\theta^N (R-c)q(R) = \theta^{N+1}(0) + (1-\theta^{N+1})\pi_{bertrand}, \tag{22}$$

where $\pi_{bertrand}$ is the industry profit conditional upon at least one firm being active. We can solve out for this to get:

$$\pi_{bertrand} = \frac{(N+1)(1-\theta)\theta^N (R-c)q(R)}{(1-\theta^{N+1})}. \tag{23}$$

COURNOT EQUILIBRIUM

Now let us compute the Cournot equilibrium. Let q^* be the Cournot output we are trying to determine. Then,

$$\begin{aligned}\pi_i(q_i) &= (1-\theta)^N[p(q_i+Nq^*)-c]q_i+\theta(1-\theta)^{N-1}[p(q_i+(N-1)q^*)-c]q_i \\ &+\theta^2(1-\theta)^{N-2}[p(q_i+(N-2)q^*)-c]q_i+\ldots \\ &= \sum_{j=0}^n \theta^j(1-\theta)^{N-j}[p(q_i+(N-j)q^*)-c]q_i\end{aligned} \tag{24}$$

The first order condition is

$$\begin{aligned}\frac{d\pi_i(q_i)}{dq_i} &= \sum_{j=0}^n \theta^j(1-\theta)^{N-j}[-\beta q_i+\alpha-\beta(q_i+(N-j)q^*)-c]=0 \\ &= \sum_{j=0}^n \theta^j(1-\theta)^{N-j}[\alpha-\beta(N-j+2)q^*-c] \\ &= \left(\sum_{j=0}^n \theta^j(1-\theta)^{N-j}\right)(\alpha-c)-\left(\sum_{j=0}^n \theta^j(1-\theta)^{N-j}(N-j+2)\right)\beta q^*,\end{aligned} \tag{25}$$

so

$$q^* = \frac{\left(\sum_{j=0}^N \theta^j(1-\theta)^{N-j}\right)(\alpha-c)}{\left(\sum_{j=0}^N \theta^j(1-\theta)^{N-j}(N-j+2)\right)\beta} \tag{26}$$

Note that if $\theta=0$, this boils down to $q^*=\frac{\alpha-c}{(N+2)\beta}$. Adding incomplete information makes no great difference in the Cournot model. If some firms might not be active, each active firm produces somewhat more than it would have otherwise.

From the quantity we can get the expected profit conditional upon there being at least one firm in the market.

$$\pi_{Cournot} = \sum_{j=1}^{N+1}\left(\frac{\theta^{N+1-j}(1-\theta)^j}{1-\theta^{N+1}}\right)[p(jq^*)-c]jq^* \tag{27}$$

Note that equation (27) is conditional upon $(N+1)*q^*$ not being so large as to drive the price to zero, which might rationally happen, since a firm would be willing to accept a price of zero occasionally as the result of all $(N+1)$ firms coincidentally being active and producing a large amount.

COMPARISONS

Profits are positive but fall with the number of firms in both the Bertrand and Cournot equilibria. The question is how fast profits fall. This is best seen with a numerical example.

Let $\alpha = 110$, $c = 100$, $\beta = 1$, and $n = [0, 7]$ for $\theta = 0$ and $\theta = .1$. Table 1 and Figures 1 throught 3 show the levels of profits. Table 1 consists of industry profits under the Bertrand and Cournot models with certainty and with $\theta = 1$, and its numbers are repeated graphically in Figure 1. Figures 2 and 3 show how the profit/concentration relationship changes for different values of θ in the two models.[9] Tables 2 and 3 pertain to the Bresnahan-Reiss empirical results, which we shall come to shortly.[10]

Consider first the Cournot model. Table 1 and Figure 1 show that a small amount of uncertainty makes little difference in the Cournot model, though, oddly enough, industry profits actually fall when the expected percentage of active firms declines. Under Cournot competition, a firm expands its output when it expects fewer rivals to be helping push down the price. Uncertainty over the number of rivals ends up increasing average output and reducing profits, a peculiar result. Figure 3 shows that this is a very delicate conclusion. Profits fall when θ rises from 0.0 to 0.1, but fall when θ rises from 0.3 to 0.9, for $N+1 > 4$, thought the reverse is true for smaller $N+1$. Conflicting forces are at work in Cournot equilibrium, and the net result is sensitive to the particular assumptions of the model.

Uncertainty is much more important in the Bertrand model, and the comparative statics are more consistent and intuitive. Table 1 and Figure 1 show that a small amount of uncertainty changes the Bertrand model in a small but crucial way, because profits do become positive and monotonic in the number of firms. The sharp fall in profits moving from monopoly to duopoly under certainty is not so unreasonable as it looks. It is extreme, but it is a limiting result as θ goes to zero, as Figure 2 illustrates. [11]

[9] In every case, expected industry profits are conditional upon at least one firm being active. When $\theta = 1$, this is to be interpreted as the probability zero (but possible) event that one firm is active and the expected number of other firms is zero.

[10] xxx Consider adding the expected proportion of firms that are active as an item of discussion. Rescale so that profits are 100 at monopoly.

[11] xxx Switch figure 2 and 3.

Figure 1: Bertrand and Cournot Profits

Figure 2: Bertrand Profits For Different Probabilities of Inactivity θ and Numbers of Firms $(N+1)$

(conditional on at least one firm being active)

Figure 3: Cournot Profits For Different Probabilities of Inactivity θ **and Numbers of Firms** $(N+1)$
(conditional on at least one firm being active)

Number of Firms $(N+1)$	1	2	3	4	5	6	7
Bertrand, $\theta = 0$	25.0	0.0	0.0	0.0	0.0	0.0	0.0
Bertrand, $\theta = 0.1$ (eq. (23))	25.0	5.6	0.9	0.1	0.02	0.003	0.0003
Cournot, $\theta = 0$	25.0	22.2	18.8	16.0	13.9	12.2	10.9
Cournot, $\theta = 0.1$ (eq. (27))	25.0	19.6	15.0	11.6	9.1	7.3	5.8

Table 1: Industry Profits for Different Concentration Levels[12]

Number of Firms $(N+1)$	1	2	3	4	5
Doctors	0.88	1.75	1.93	1.93	1.83
Dentists	0.71	1.27	1.39	1.36	1.28
Druggists	0.53	1.06	1.68	1.92	1.88
Plumbers	1.43	1.51	1.51	1.55	1.49
Tire Dealers	0.49	0.89	1.14	1.19	1.22

Table 2: Bresnahan-Reiss Entry Thresholds s_i: Original (1,000's of inhabitants)[13]

Number of Firms $(N+1)$	1	2	3	4	5
Doctors	25.0	4.3	0.0	0.0	0.0
Dentists	25.0	4.4	0.0	0.0	0.0
Druggists	25.0	15.5	4.3	0.0	0.0
Plumbers	25.0	8.3	8.3	0.0	0.0
Tire Dealers	25.0	11.3	2.7	1.0	0.0
Average	25.0	9.6	2.3	0.2	0.0

Table 3: Bresnahan-Reiss Entry Thresholds: Rescaled ($\frac{25(s_m - s_i)}{(s_m - s_1)}$)

Let us also consider the shape of the profit-concentration paths. All the curves in Figures 1 through 3 have convex shapes, if only weakly in the limiting cases, but the curvatures, and therefore the empirical implications, are different. As Figure 1 and Table 1, in particular, show, profits decline much more rapidly in Bertrand than in Cournot. For the

[12]Numerical calculations and figure-drawing used *Mathematica*. Values are rounded.

[13]Calculated from Table 5A of Bresnahan & Reiss (1991). Note that the entry of .79 in the second row of their original paper is a mistake and should be 1.09, and their Figure 4 illustrates s_i/s_5, not the s_5/s_i in the legend.

parameters chosen, industry profits fall from the monopoly level of 25 to duopoly profits of 5.6, triopoly profits of 0.9, and negligible levels thereafter. Cournot profits show a much more uniform decline as concentration falls.

Comparison of Figures 2 and 3 shows that for larger values of the inactivity probability θ the Bertrand profit path becomes flatter and the Cournot path more curved, but even at extreme values Cournot does not generate such sharp differences from the addition of one firm to the market.

For most modelling purposes, these models are building blocks, and such subtle differences in the profit-concentration path are unimportant. They are interesting, however, if one wishes to consider Bertrand and Cournot as serious oligopoly models in their own right. Empirically, then, how do profits react to the number of firms? Do they decline to zero with duopoly and then stay constant, as in the original Bertrand model? Do they decline smoothly, as either version of the Cournot model would suggest? Or do they decline rapidly, as the Bertrand model with uncertainty would suggest?

Measuring the relationship between profits and concentration is an old exercise now in some disrepute.[14] The difficulty is that the usual unit of observation has been the industry. This is natural enough, since one needs a measurement of concentration for each observation. Comparing accounting profits across industries is fraught with danger, however, since accounting profits differ from economic profits in ways that depend on the industry chosen and which are very likely to be correlated with technology, and hence with concentration. Moreover, it is not clear that the concentration-profits path is even the same across industries.

A clever recent approach to the same problem is that of Bresnahan & Reiss (1991). They took the unit of observation to be the market for a particular profuct in a particular small town, rather than for many products over the entire United States, and they looked at market size rather than directly at profits. They collected data on the size of a town and the number of dentists there, for example. If a town is very small—say, 500 people—- it will have no dentist, since a dentist incurs a fixed cost and could not make any profit there even as a monopoly. If it grows to 800 people, it will have one dentist, since the profits are enough for monopoly, but entry by a second dentist would drive them negative. If the town grows to 1,600 people, however, it may still have only one dentist— if entry by the second dentist would not just split the industry profits, but reduce them.

Bresnahan and Reiss used this approach to estimate the thresholds for entry in small markets for a number of industries. Table 2 shows these thresholds in thousands of inhabitants per firm. Table 3 rescales the same numbers to be very roughly comparable with the numerical example used earlier in this paper.[15] The rescaling is somewhat arbitrary, since

[14] xxx Find a refrence.

[15] Table 3's rescaling uses the following procedure.

Define the monopoly level of profits in an industry to be 25, and the competitive level to be 0. Assume that when s_i reaches its maximum level s_m over $[1,5]$, the competitive level of profits is reached and any further the theory of Bresnahan and Reiss is that some quasi-rents remain to cover fixed cost even when the minimum scale for entry flattens out, but it creates a comparison measure for how the intensity of competition changes with the number of firms.

What is significant is how profits flatten out, even though the choice of 0 as the flat level in Table 3 is arbitrary.[16] The empirical result that going from one firm to two is much more important than going from two to three, and that full-fledged competition kicks in very quickly matches the Bertrand model with uncertainty very well, and is inconsistent with the Cournot model.

5. THE ASYMMETRIC BERTRAND MODEL WITH TWO FIRMS

The purpose of this section of the paper is technical. It is a warning that the apparently simpler Bertrand duopoly model in which only firm might be inactive is actually more difficult. It is not the way to go to try to find a simpler building-block model.

Assumption: There are two firms. Firm 1 is always active. Firm 2 is inactive with probability θ.

Letting the firms have possibly different mixing cumulative distributions $F_i(p)$ over $[B,R]$.

The equilibrium is

$$F_1(p) = \left| \begin{array}{ll} 0 & \text{for} \quad p \leq (1-\theta)c+\theta R \\ 1-\theta\left(\frac{R-c}{p-c}\right) & \text{for} \quad p \in [(1-\theta)c+\theta R, R) \\ 1 & \text{for} \quad p \geq R \end{array} \right.$$

$$F_2(p) = \left| \begin{array}{ll} 0 & \text{for} \quad p \leq (1-\theta)c+\theta R \\ 1-\left(\frac{\theta}{1-\theta}\right)\left(\frac{R-p}{p-c}\right) & \text{for} \quad p \in [(1-\theta)c+\theta R, R] \\ 1 & \text{for} \quad p \geq R \end{array} \right.$$

The equilibrium of the asymmetric model is similar to the symmetric one of Section 3, but Firm 1's equilibrium mixing probability has an atom of probability at $p_1 = R$. This shows up very subtly in the equilibrium description, as the open-set bracket in $[(1-\theta)c+\theta R, R)$.

$F_1(R-\varepsilon) \approx 1-\theta$ for small ε, but $F(R) = 1$.

I will now justify the equilibrium. In equilibrium, the expected payoff from each pure strategy mixed over must be equal. The expected payoff to Firm 1 from the pure strategy of $p_1 = R$ is

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changes are measurement error. Apply the conversion formula $s_i^* = \frac{25(s_m-s_i)}{(s_m-s_1)}$, and Table 3 results.

[16] Add 9.1 to each entry in Table 3, and the profit at $N+1 = 5$ is 9.1, as with Cournot competition and $\theta = 0.1$ in Table 1, but the shape is still more like that of Bertrand competition.

$$\pi_1(R) = \theta(R-c) \tag{28}$$

because Firm 1 only wins then if Firm 2's cost is infinite. (With a continuous mixing distribution for Firm 2, the probability of $p_2 = R$ in a mixed strategy equals zero.)

The expected payoff to Firm 1 from the pure strategy of $p_1 = p$ is

$$\pi_1(p) = (\theta + (1-\theta)(1-F_2(p))(p-c), \tag{29}$$

since Firm 1 wins the bid if Firm 2 has infinite costs or if Firm 2's mixing has led to a price of $p_2 > p$, which has probability $(1-F_2(p))$. Equating (28) and (29) yields

$$\theta(R-c) = (\theta + (1-\theta)(1-F_2(p))(p-c), \tag{30}$$

which can be solved out for $F_2(p)$ to give

$$F_2(p) = 1 - \left(\frac{\theta}{1-\theta}\right)\left(\frac{R-p}{p-c}\right) \tag{31}$$

By definition, $F_2(B) = 0$, so

$$1 - \left(\frac{\theta}{1-\theta}\right)\left(\frac{R-B}{B-c}\right) = 0. \tag{32}$$

Solving that out yields $B = (1-\theta)c + \theta R$.

Now we do the same sort of thing starting with Firm 2's payoffs.

The expected payoff to Firm 2 from the pure strategy of $p_2 = p$ is

$$\pi_2(p) = (1-\theta)(1-F_1(p))(p-c), \tag{33}$$

since Firm 2 wins the bid if Firm 1's mixing has led to a price of $p_1 > p$, which has probability $(1-F_1(p))$. Since $F_1(B) = 0$,

$$\pi_2(p) = \pi_2(B) = (1-\theta)(1-0)(B-c), \tag{34}$$

or

$$\pi_2(p) = (1-\theta)(1-0)((1-\theta)c + \theta R - c) = (1-\theta)\theta(R-c). \tag{35}$$

Substituting for $\pi_2(p)$ in equation (33) using equation (35) yields

$$(1-\theta)\theta(R-c) = (1-\theta)(1-F_1(p))(p-c), \tag{36}$$

which when solved yields

$$F_1(p) = 1 - \theta\left(\frac{R-c}{p-c}\right). \tag{37}$$

One complication on looking at (36) is that $F_2(R) = 1-\theta$, rather than 1. Equation (36) only holds for $p < R$, and there is an atom of probability for Firm 1 at $p_1 = R$.

This establishes that the equilibrium is indeed Nash.

The expected payoffs are

$$\pi_1 = \theta(R-c), \qquad \pi_2 = (1-\theta)\theta(R-c),$$

which sum to

$$(2-\theta)\theta(R-c).$$

This sum lies between 0 and $(R-c)$, the competitive and monopoly profits, because $\theta < 1$ implies $(1-\theta)(\theta-1) < 0$, which implies that $2\theta - \theta^2 < 1$. If θ grows, then industry profits rise smoothly.

6. THE SEQUENTIAL BERTRAND MODEL

Competition when two duopolists pick quantities in sequence is known as the Stackelberg model. It is worth thinking about the Bertrand analog, which although simple is not widely understood even under certainty.

Assumption : Firms submit bids publicly in sequence from Firm 1 to Firm $(N+1)$.

First, consider what happens in the Bertrand model with no uncertainty— the special case of $\theta = 0$. There are two classes of equilibria.

In the first class of equilibrium, at least one of the first N firms chooses $p = c$, and consumers buy from firms charging $p = c$. Profits are zero, and the outcome is the same as in the simultaneous Bertrand model.

In the second class of equilibria, the first N firms choose prices in a set with minimum $p_{min} > C$ and the last firm chooses $p_{N+1} = Min\{p_{min}, R\}$. The consumers all choose to buy from firm $(N+1)$. Profits are zero for all firms except Firm $(N+1)$, who has positive profit.

The second class of equilibrium is counter-intuitive. For concreteness, consider the particular member of the class in which all firms offer the price R. None of the first N firms have any incentive to deviate. If a firm deviates to $p = c$, his profit remains zero. If a firm deviates to any price between c and R, Firm $(N+1)$ will respond with the same price and capture the market, so the deviating firm's profit remains zero. Firm $(N+1)$ clearly has no incentive to deviate. And the consumers have no incentive to deviate because all firms charge the same price.

This is, to be sure, a weak Nash equilibrium, which is why it is counterintuitive. No Nash equilibrium exists, however, in which consumers are not indifferent about where they buy, and in which more than one firm earns positive profits. This is the standard open-set problem; if consumers did not follow this behavior, the last firm would choose to undercut its lowest competitor by an infinitesimal amount and gain the entire market.[17] It seems reasonable, however, to prefer an equilibrium in which players behave symmetrically, when such an equilibrium exists, and with that assumption, the only equilibrium is the symmetric one in the first class with $p = c$ for all firms, and consumers evenly divided among them.

Next, consider the symmetric Bertrand model in which each firm is inactive with probability θ. The last active firm will undercut any previous firm in the sequence that has offered $p > c$. Thus, if any later firm in the sequence is active, the earlier firms' payoffs will all equal zero regardless of their bid. If no later firm is active, an early firm will win the market, and might as well bid $p = R$. Since if $\theta > 0$ there is a positive probability that all later firms will be inactive, every firm bids $p = R$. Consumers buy from the last firm bidding, again, to resolve the open-set problem.

The equilibrium in the sequential model is somewhat bizarre. Even a tiny amount of uncertainty reduces a continuum of equilibria to a unique equilibrium. Moreover, the most plausible level of profits rises from zero to the monopoly level.

What this illustrates is the tremendous power of open-cry auctions in revealing information. When there is no uncertainty, this does not make much difference. When there is uncertainty about the number of firms, however, the open-cry auction resolves that uncertainty, giving the last bidder, in particular, a tremendous advantage. Earlier bidders know they cannot overcome that advantage, so their only hope is that no later bidders will be active.

The sequential Bertrand model is, of course, not a typical open-cry auction, because the sequence of bidding is predetermined and each firm only gets one bid. In the classic English auction, each bidder can bid as often as he wishes. In the present context, this would result in a winning bid of $p = c$ if at least two firms are active, whatever the value of θ may be.

The caveat "if at least two firms are active", however, is important. With probability $(N+1)(1-\theta)\theta^N$, only one firm is active and the winning bid will be $p = R$. The expected industry profit is therefore $(N+1)(1-\theta)\theta^N(R-c)$, exactly the same as the profit given by equation (11) in the simultaneous game! This is the same result found in McAfee

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[17] See p. 103 of Rasmusen (1994) for a discussion of the open-set problem. Note that the first edition of that book does not contain any discussion of it.

& Macmillan (1987). From the point of view of the buyer, the English auction has the advantage of pitting bidders against each other head to head, but the disadvantage of letting a bidder know if he has no competition. As a result, the English auction has much greater risk, and a risk-averse buyer would prefer simultaneous bids.

7. CONCLUDING REMARKS

The Bertrand model with incomplete information about the number of firms is simple, but its properties are both interesting and useful. The extreme transition from monopoly to competition found in the standard Bertrand model disappears. Profits are positive, but decline with the number of firms in the industry, and decline in a way that empirical work suggests is more realistic than in the Cournot model. Expected profits also decline as the expected fraction of firms that are actively seeking business increases, in contrast to the Cournot model.

I hope that the model may be useful both as a simple description of oligopoly and as a building block for other topics in industrial organization.

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