Explaining Cross–Supplies

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Abstract

Cross–supplies describe the phenomenon that two or more firms in the same industry supply each other with their final products. A prominent example is the cooperation in the European flat glass industry, which was recently criticized by the European Commission. In a simple model we try to explain what incentives firms may have to use cross–supplies (instead of producing the goods themselves) and what welfare effects cross–supplies have if they are used. Contrary to the ruling of the European Commission we find that cross–supplies are welfare improving whenever they are employed. Furthermore, for a large range of parameters, they are even benefiting consumers.

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1 Introduction

On December 8, 1988, the European Commission imposed fines totalling more than 13 million ECU on three Italian glass producers,[1] claiming a violation of Article 85 of the Treaty of Rome. The Commission alleged that the three firms participated over a period of five years in a scheme of mutual cross–supplies with the purpose of dividing up the market for flat glass. This claim was substantiated by a long history of incidences in which firms supplied each other with the final product (flat glass of some specification), which could have been produced by the receiver of these supplies itself. There were instances in which one firm unilaterally supplied the others without in turn receiving cross–supplies of a different product (one–sided cross–supplies), and there were instances in which firms mutually supplied each other with flat glass of some specification.

While the existence of these cross–supplies is undisputed, their anti–competitive effects are. In fact, the European Court of Justice in 1992 overruled the Commission's decision with regard to cross–supplies mainly on the grounds that the Commission succeeded neither in proving the existence of an "institutionalized system for glass exchange" nor in proving the firms' intent to divide up the market.

We take this case as an opportunity to think about cross–supplies in general. In particular, we are interested in two questions. (1) What do firms have to gain from cross–supplies? And (2), if cross–supplies are used, what are the welfare effects?

Cross–supplies occur more frequently than one thinks but most of the time, they are less obvious than in the flat glass market. For example, network owners in telecommunications often rent out network capacity to their direct competitors. Consider the case of two phone companies. Instead of both operating a transatlantic and a transpacific cable, they might each operate only one of these cables and rent capacity on the other cable.[2]

Another example for cross–supplies can be found in the trucking business, where it is quite common that companies rent space in a competitor's

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[1] They were Fabbrica Pisana SpA, Societa Italiana Vetro SIV SpA, and Vernante Pennitalia SpA.

[2] Here, cross–supplies take place between network operators. Frequently, however, network operators are vertically integrated with phone companies, which calls for an analysis of vertically integrated structures (see Salinger, 1988; or Schrader and Martin, 1995, for approaches in this direction).

truck especially on otherwise empty return trips.

It might seem strange that firms which are originally in a symmetric position use cross–supplies. After all, both firms must gain from this transaction. There are several stories that can be told, but most explanations for the existence of cross–supplies will be based on either fixed costs (or, more generally, increasing returns to scale) or on capacity constraints coupled with uncertainty about production or demand conditions.[3] We will focus here on the first explanation, namely, a model with two firms, two goods with possibly interdependent demand (i.e. the goods can be either complements or substitutes), certainty about demand and supply conditions, and constant marginal cost of production. The key ingredients are fixed costs for the production of each good.

Our model has three stages. First, firms decide simultaneously whether to offer cross–supplies to the other firm or not. On the second stage, firms decide whether to accept those cross–supply offers. We assume that firms do not purchase goods from the other firm unless they intend to sell them on the final product market (e.g. because storing and disposal costs are too high). Finally, on the third stage, firms decide how much to produce in addition to what they have already received through cross–supplies. Since cross–supplies cannot be stored, this is equivalent to deciding how much to sell on the final product market.

An interesting aspect of this model is that upon accepting cross–supplies the receiver becomes a Stackelberg leader in that market since the supplier knows exactly how much the receiver will sell on the market, and reacts accordingly.[4] Anticipating the optimal reaction the receiver of cross–supplies will take it into account when ordering the good from the producer. This is only half of the story, however, as the producer, in turn, can influence the demand for cross–supplies by setting the price for them.

There are two main results of the paper. The first is that equilibria in which cross–supplies are used by the firms exist for a broad range of parameter constellations. In fact, in these cases the only undominated equilibria are those in which cross–supplies are used. Moreover, there are different types of cross–supply equilibria. On the one hand there are equilibria in which one firm produces both products and supplies the other firm with them. On the other hand there are equilibria in which each firm produces one of the goods and purchases the remaining good from the other firm.

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[3] For a model based on capacity constraints see Schenk (1996).

[4] This follows due to our assumption that the receiver of cross–supplies cannot sell *less* on the market. It is easy to see that a firm would never want to sell *more* than the Stackelberg leader output.

This finding corresponds well with what was observed in the flat glass market. The different time periods in which various arrangements of one–sided or both–sided cross–supplies were observed could therefore be explained by switches between different equilibrium regimes.

The second result of the paper concerns the welfare aspects of cross–supplies. Given the European Commission's attempt to prohibit firms from using cross–supplies, this is the aspect we are most interested in. Since cross–supplies are obviously used only when they are of advantage to both firms, the question is only how cross–supplies affect consumers.

We find that the size of the welfare effects depends on the type of equilibria as well as on the type of the two goods (whether they are complements or substitutes). However, we show that, compared to the normal Cournot outcome, cross–supplies unambiguously raise welfare whenever they are used. In fact, for a large range of parameters even consumers benefit from cross–supplies.

The paper is structured as follows. In the next section the model along with our assumptions is presented. Since the game is solved by backward induction, we calculate in Section 3 the quantity decisions of firms on the last step. Section 4 treats the special case of independent markets, where the analysis is much simpler but already gives the flavor of the results of the more general model. Section 5 then follows up with the characterization of equilibria in the general model. The key welfare results are presented in Section 6. Finally, conclusion an policy recommendations are contained in Section 7.

2 The Model

We consider a very simple model that despite its simplicity allows for the existence of cross–supplies. There are two (possibly interdependent) markets for goods x and y, respectively. Two firms denoted by $i = 1, 2$ choose whether to produce and/or sell these goods.

The timing is as follows.

1. Both firms decide simultaneously whether to set up production of x and/or y. These decisions are commonly observable. There are fixed set up costs for the production of each good. If firms decide to produce one or both of the goods, they can also make a binding price offer for supplying the other firm with these goods.

2. Given these price offers each firm decides simultaneously on the amounts

of x and y to buy from the other firm or — if dissatisfied with these prices — whether to set up production for the goods itself. Again, these decisions are commonly observable.

3. Both firms simultaneously decide how much to produce and how much to sell on the market.

This game may have multiple (subgame perfect) equilibria which we classify as follows.

Cournot equilibrium with entry in stage 1

This equilibrium is the standard quantity setting Cournot equilibrium in both markets. Both firms set up production of both goods in stage 1. No cross–supply offers are being made..

Cournot equilibrium with entry in stage 2

In this equilibrium no firm enters in stage 1 and no cross–supplies are being offered. Both firms set up production of both goods on stage 2 and supply the Cournot quantities.

One–sided cross–supply equilibria

One firm (say firm 1) produces both goods. The other firm does not produce either good. Firm 1 cross–supplies firm 2 with both goods.

Semi cross–supply equilibria

One firm (say firm 2) produces only one of the goods. Firm 1 produces both goods and supplies firm 2 with the good firm 2 does not produce.

Both–sided cross–supply equilibria

Each firm produces one of the goods and supplies the other firm with the good that this firm does not produce.

2.1 Assumptions

To keep the analysis as simple as possible we make the following assumptions. Inverse demand functions for x and y are linear.

$$p_x = 1 - (x_1 + x_2) + c(y_1 + y_2), \;\; p_y = 1 - (y_1 + y_2) + c(x_1 + x_2),$$

with $-1 \leq c < 1$. Thus, for $c > 0$ ($c < 0$) both goods are complements (substitutes), and for $c = 0$ they are independent.

The cost structure is also as simple as possible. We assume that there are constant marginal cost of production, which we normalize to zero. The set up costs k for each product are sunk. In order to guarantee that in the absence of cross–supplies firms will produce both, x and y, we further assume that the fixed cost k are not too large,

$$k \leq k^m := \min\left\{\tfrac{1+c}{9(1-c)}, \tfrac{1}{2(1-c)} - \tfrac{1}{(2-c)^2} - \tfrac{5}{36}\right\}.$$

3 Quantity decisions on the last step

Since the game defined in the previous section is one of perfect information, we solve it by backward induction. For this, we first have to calculate the optimal quantities on the last step for given entry decisions and given cross–supply prices.

Cournot quantities
Given that both firms have decided to produce both goods, x and y, a regular Cournot competition results with equilibrium quantities and profits of $(i = 1, 2)$

$$x_i^c = y_i^c = \frac{1}{3-3c}$$
$$\pi_i^c = \frac{2}{9(1-c)} - 2k.$$

Next, we consider the quantity decisions for cases in which cross–supplies are being used. As pointed out in the Introduction the receiver of cross–supplies becomes the Stackelberg leader with respect to the quantity decisions since the supplier can anticipate the quantity the receiver will put on the market.

One–sided cross–supplies
Suppose firm 1 sets up production of x and y whereas firm 2 does not. However, firm 2 may purchase x and y at cross–supply prices s_x and s_y, respectively. Then, profits are given by

$$\pi_1^{or} = p_x x_1 + p_y y_1 + s_x x_2 + s_y y_2 - 2k, \; \pi_2^{os} = p_x x_2 + p_y y_2 - s_x x_2 - s_y y_2.$$

Purchasing the quantities x_2 and y_2 makes firm 2 the Stackelberg leader in these markets. Firm 1's reaction functions are then $x_1^{or} = \frac{1-x_2+cx_2}{2(1-c)}$ and $y_1^{or} = \frac{1-y_2+cy_2}{2(1-c)}$. Taking these quantities into account firm 2 will optimally buy from firm 1 the following quantities x_2^{os} and y_2^{os}:[5]

$$x_2^{os} = \frac{1+c-2s_x-2cs_y}{2(1-c^2)}, \; y_2^{os} = \frac{1+c-2s_y-2cs_x}{2(1-c^2)}.$$

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[5] To explain our notation the first letter of a superscript, either o or s or b, stands for 'one–sided', 'semi–cross' and 'both–sided', respectively. The second letter denotes r as in 'reaction function' of a firm who plays Stackelberg follower or Cournot, and s for 'Stackelberg leader'.

Semi cross–supplies

Suppose now that both firms produce x but firm 2 decides to purchase good y from firm 1. In this case profits are

$$\pi_1^{sr} = p_x x_1 + p_y y_1 + s_y y_2 - 2k, \ \pi_2^{ss} = p_x x_2 + p_y y_2 - s_y y_2 - k.$$

Given firm 2's demand for cross–supplies, y_2, the optimal quantities on the last step are

$$x_1^{sr} = \frac{2 + c - 3cy_2 - 3c^2 y_2}{6(1-c)}, \ y_1^{sr} = \frac{1 - y_2 - cy_2}{2(1-c)}, \ x_2^{sr} = \frac{1}{3} + cy_2, \qquad (1)$$

which yields an optimal quantity y_2^{ss} of

$$y_2^{ss} = \frac{1 + c - 2s_y}{2(1-c^2)}.$$

Both–sided cross supplies

In this case each firm produces one of the goods itself and buys the second good from the other firm. We assume throughout that firm 1 produces good y and firm 2 produces good x. Profits are given by

$$\pi_1^{br} = p_x x_1 + p_y y_1 + s_y y_2 - s_x x_1 - k, \ \pi_2^{bs} = p_x x_2 + p_y y_2 + s_x x_1 - s_y y_2 - k.$$

By purchasing good x (y) firm 1 (2) becomes the Stackelberg leader in the market for x (y). Knowing what they delivered to the other firm the reaction functions of firm 1 and 2 are, respectively

$$y_1^{br} = \frac{2 + c + 3cx_1 - 2y_2 + 2c^2 y_2}{4 - c^2}, \ x_2^{br} = \frac{2 + c + 3cy_2 - 2x_1 + 2c^2 x_1}{4 - c^2}.$$

Given these reaction functions firms will optimally purchase the following quantities from the other firm

$$\begin{aligned} x_1^{bs} &= \frac{4 + 2c - c^2 + c^3 + 4cs_y - c^3 s_y - 8s_x + 2c^2 s_x}{8 - 7c^2 - c^4} \\ y_2^{bs} &= \frac{4 + 2c - c^2 + c^3 + 4cs_x - c^3 s_x - 8s_y + 2c^2 s_y}{8 - 7c^2 - c^4}. \end{aligned}$$

We have now fully specified the quantity decisions of the firms for given cross–supply prices and entry decisions. In the next two sections we can specify the conditions under which each type of equilibrium exists.

4 The case of independent markets

We will treat the case of independent demand functions ($c = 0$) in some detail since it is easy to follow and yet exemplifies most of the intuitive results of the general case. The advantage is that we can consider the two markets separately.

Consider first the Cournot equilibrium, in which both firms set up production in stage 1. Equilibrium quantities x_i^c, y_i^c and profits π_i^c ($i = 1, 2$) are

$$x_i^c = y_i^c = \frac{1}{3},\ \pi_i^c = \frac{2}{9} - 2k$$

It is easy to see that this equilibrium is (weakly) dominated as one could always wait for possible advantageous cross–supply offers and - if they do not arrive - enter in the second stage. We will henceforth consider only equilibria in undominated strategies.

In the case of independent markets the one–sided and both–sided cross–supply equilibria are essentially the same. Without loss of generality we will take up the one–sided case here. Suppose one firm (say firm 1) enters the markets for x and y and offers to cross–supply firm 2, which did not enter in the first stage, with both goods. Using the quantity decisions calculated in the last section we get the reduced form profit function π_2^{os} of firm 2

$$\pi_2^{os} = \frac{1}{4} - \frac{1}{2}s_x + \frac{1}{2}s_x^2 - \frac{1}{2}s_y + \frac{1}{2}s_y^2.$$

In order for this to be an equilibrium firm 2 must not have an incentive to enter the market in the second stage. Therefore, firm 1 must set cross–supply prices such that the profit of firm 2 with cross–supplies is at least as large as the Cournot profit, i.e. $\pi_2^{os} \geq \pi_2^c$. Since the markets are independent and firm 1 must keep firm 2 out of both markets, it is clearly optimal to set $s_x = s_y$. Using the price s_x that satisfies $\pi_2^{os} \geq \pi_2^c$ with equality and substituting the resulting upper bound for s_x into firm 1's profit function yields

$$\pi_1^{or} = \frac{1}{6} + k.$$

Obviously, firm 1 as the supplier must make at least as much with cross–supplies as without, i.e. we have the further constraint that $\pi_1^{or} \geq \pi_1^c$. It follows immediately that an equilibrium with cross–supplies exists only for $k \geq \frac{1}{54}$.

Now suppose that none of the firms has set up production on the first stage and consequently no cross–supply offers were made. Again we have a regular Cournot game in two markets simultaneously with equilibrium quantities of $\frac{1}{3}$ and profits of $\frac{2}{9} - 2k$. Given the analysis above this equilibrium requires that no firm has an incentive to enter on the first stage and make a successful cross–supply offer. Hence, the Cournot equilibrium with entry in stage 2 exists for values of k in the interval $[0, \frac{1}{54}]$. Note that in contrast to the Cournot equilibrium with entry in stage 1, this equilibrium is not weakly dominated. Collecting these results we have

Proposition 1 *In the case of independent markets for* $\frac{1}{54} < k \leq \frac{1}{9}$ *the only undominated equilibria are equilibria with cross–supplies. For* $0 \leq k < \frac{1}{54}$ *the Cournot equilibrium with entry in stage 2 is the unique undominated equilibrium.*

5 The case of complementary and substitutional demands

If x and y are complements ($c > 0$) or substitutes ($c < 0$) the analysis is more complex but also more interesting as the two markets are not independent of each other anymore. As before for an equilibrium with cross–supplies two conditions have to be satisfied. First, it must not be worthwhile for the receiver of cross–supplies to produce itself instead (which would result in Cournot competition in that market). Second, the cross–supplier must earn at least as much with cross–supplies as without them.

5.1 The one–sided cross–supply equilibrium

In this equilibrium one firm (say firm 1) sets up production for both goods and makes price offers for cross–supplies to the other firm. Firm 2 does not produce itself but accepts the cross–supply offers from firm 1 for both goods. In order for this to be an equilibrium firm 1 must offer the cross–supplies at prices such that firm 2 does not find it profitable to produce one or both of the goods itself. After receiving the price offers firm 2 has the following opportunities.

1. Firm 2 does not produce itself but accepts cross–supplies of both goods.
2. Firm 2 produces one of the goods itself (say x) and receives cross–supplies in y.

3. Firm 2 produces both goods. No cross–supplies are used. A Cournot outcome results.

Taking $s = s_x = s_y$ firm 2 will find it unprofitable to produce both goods itself if

$$2\,(p_y - s)\,y_2^{os} \geq \frac{2}{9(1-c)} - 2k, \tag{2}$$

that is, if its profit as the receiver of cross–supplies is larger than the profit it would make if it deviated from the equilibrium by producing both goods itself. Let $\sigma_1(k,c) = s_x = s_y$ be the price such that constraint (2) is satisfied with equality, that is

$$\sigma_1(k,c) := \frac{1}{2} - \frac{1}{3}\sqrt{1 - 9k + 9ck}. \tag{3}$$

Thus, at cross–supply prices $\sigma_1(k,c) = s_x = s_y$ firm 2 is indifferent between accepting the cross–supply offer and rejecting it.

If both prices s_x and s_y are above $\sigma_1(k,c)$, then firm 2 would have an incentive to produce both goods itself. Otherwise, firm 2 will not produce *both* goods but there is still the possibility that it might want to produce just one of the goods. Therefore, we need the following additional constraint that makes this deviation unprofitable.

$$2(p_y - s)y_2^{os} \geq p_x x_2^{sr} + (p_y - s)y_2^{ss} - k. \tag{4}$$

Let $\sigma_2(k,c) = s_x = s_y$ be the price such that (4) is satisfied with equality. To determine which of these two constraints is binding we first define k^v as the k, such that firm 2's profit from producing just one of the goods equals the profit from receiving cross–supplies in both goods, i.e.

$$p_x x_2^{sr} + (p_y - \sigma_1)y_2^{ss} - k^v = 2(p_y - \sigma_1)y_2^{os} \equiv \frac{2}{9(1-c)} - 2k. \tag{5}$$

We claim that there exists exactly one such k^v. To verify this one can show using equation (3) and the definitions of y_2^{ss} and x_2^{sr} that $p_x y_2^{sr} + (p_y - \sigma_1)y_2^{ss} - \frac{2}{9(1-c)} + k$ is strictly monotone in k for $c \neq 0$ and assumes zero exactly once.

Lemma *For $c < 0$ and $k \leq k^v$ or $c > 0$ and $k \geq k^v$ the binding constraint to prevent firm 2 from deviating is (2).*

For $c > 0$ and $k \leq k^v$ or $c < 0$ and $k \geq k^v$ the binding constraint to prevent firm 2 from deviating is (4).

Proof Differentiating the identity on the right hand side of (5) yields $\frac{\partial \sigma_1}{\partial k} = \frac{1}{y_2^{os}}$. Using this fact the derivative of the left hand side of (5) with respect to k becomes

$$\frac{\partial[p_x x_2^{sr} + (p_y - \sigma_1) y_2^{ss} - k^v]}{\partial k} = -\frac{y_2^{ss}}{y_2^{os}} - 1.$$

Since the derivative of the right hand side of (5) is -2 we see that the left hand side is steeper than the right hand side if and only if $y_2^{ss} - y_2^{os} < 0$. Using the definitions of y_2^{ss} and y_2^{os} it is easy to show that $y_2^{ss} - y_2^{os} = \frac{c}{1-c^2}\sigma_1$. Hence, for $c < 0$ and $s_x = s_y = \sigma_1(k, c)$ firm 2 will not produce just one of the goods if $k < k^v$ and vice versa. That is, for $c < 0\,(c > 0)$ and $k \leq k^v$ $(k \geq k^v)$ the binding constraint is (2). On the other hand, if $c > 0\,(c < 0)$ and $k \leq k^v$ $(k \geq k^v)$ the binding constraint is (4). ■

We can now define

$$\sigma^{os}(k, c) := \min\{\sigma_1(k, c), \sigma_2(k, c)\}$$

as the highest price such that firm 2 would not deviate from the one–sided cross–supply equilibrium.

Next, we consider possible deviations from equilibrium by firm 1. If firm 1 tried to charge a higher price for the cross–supplies than $\sigma^{os}(k, c)$, firm 2 would not accept. Charging a lower price would only reduce firm 1's profit. Thus, the only deviation we have to consider is for firm 1 to forgo cross–supplies (or equivalently, charge such a high price that firm 2 would not accept). One constraint for the equilibrium to exist is therefore that firm 1 makes a higher profit than what it would make when not offering the cross–supplies, i.e.

$$p_x x_1^{or} + p_y y_1^{or} + \sigma^{os}(k, c)\, x_2^{os} + \sigma^{os}(k, c)\, y_2^{os} \geq \frac{2}{9(1-c)}. \tag{6}$$

Furthermore, firm 1 must not be able to increase its profit by offering cross–supplies in just one of the goods (say y), which requires that

$$p_x x_1^{or} + p_y y_1^{or} + \sigma^{os}(k, c)\, x_2^{os} + \sigma^{os}(k, c)\, y_2^{os} \geq p_x x_1^{sr} + p_y y_1^{sr} + \rho(k, c) y_2^{ss}, \tag{7}$$

where $\rho(k, c)$ is defined as the price such that

$$p_x x_2^{sr} + (p_y - \rho(k, c))\, y_2^{ss} - k = \frac{2}{9(1-c)} - 2k. \tag{8}$$

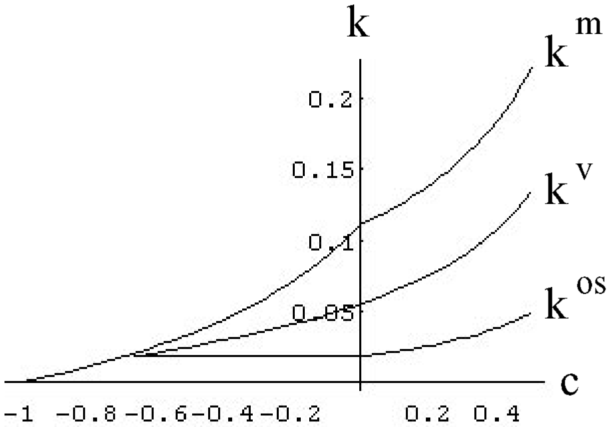

Figure 1:

That is, $\rho(k,c)$ is the highest price such that firm 2 would not reject the cross–supply offer for y (in the case where firm 1 only makes the offer for y). After substituting for $\sigma^{os}(k,c)$ and $\rho(k,c)$ in (6) and (7) it can be seen that (6) is the binding constraint for $c>0$ and (7) for $c<0$.

Now, we can implicitly define $k^{os}(c)$ in a piecewise manner. For $c>0$ it is the minimum fixed cost such that (6) is satisfied with equality.

$$p_x x_1^{or} + p_y y_1^{or} + \sigma^{os}(k^{os}(c),c)\, x_2^{os} + \sigma^{os}(k^{os}(c),c)\, y_2^{os} = \frac{2}{9(1-c)}.$$

Similarly, for $c<0$, $k^{os}(c)$ is the minimum fixed cost such that (7) is satisfied with equality. That is, for $k>k^{os}(c)$ firm 1 would not deviate from equilibrium. Collecting these results we have

Proposition 2 *$\forall k$, $k^{os}(c) \leq k \leq k^m$ an equilibrium with one–sided cross–supplies exists. The cross–supply prices are given by $s_x = s_y = \sigma^{os}(k,c)$.*

Graphically we can display the set of parameters for which an equilibrium with one–sided cross–supplies exists in a c,k–diagram as the area between k^m and k^{os} in Figure 1.

While the profit of firm 1 with cross–supplies is strictly higher than in the Cournot case for all $k>k^{os}$, firm 2 can be held down to the Cournot profit only if $c<0$ and $k \leq k^v$ or $c>0$ and $k \geq k^v$. On the other hand, if $c>0$ and $k \leq k^v$ or $c<0$ and $k \geq k^v$ we have $\sigma^{os}(k,c)=\sigma_2(k,c)$ implying that the profits of both firms are higher than in the Cournot case.

5.2 The semi cross–supply equilibrium

As in the equilibrium described in the last section a semi cross–supply equilibrium is characterized by firm 1 setting up production for both goods initially and firm 2 waiting for possible cross–supply offers. In contrast to the last equilibrium firm 1 makes an offer only for one of the goods (say y) which induces firm 2 to produce good x itself on the second step. It is obvious that the semi cross and the one–sided cross–supply equilibria can coexist for given parameter values only if they yield the same payoff for firm 1. Assume to the contrary that for some parameter values both types of equilibria exist but do not yield the same payoff. It is then always possible to deviate from the equilibrium with lower payoff by simply (not) offering to supply the other firm with one of the goods, which yields a contradiction.

To specify the range of parameters for which a semi cross–supply equilibrium exists let us consider again all possible deviations. In particular, firm 1 could either not offer cross–supplies at all or offer cross–supplies in both goods. The first deviation is unprofitable (compare equation 6) if

$$p_x x_1^{sr} + p_y y_1^{sr} + \rho(k,c) y_2^{ss} \geq \frac{2}{9(1-c)}, \tag{9}$$

the second is unprofitable if

$$p_x x_1^{sr} + p_y y_1^{sr} + \rho(k,c) y_2^{ss} \geq p_x x_1^{or} + p_y y_1^{or} + \sigma^{os}(k,c)\, x_2^{os} + \sigma^{os}(k,c)\, y_2^{os}, \tag{10}$$

where $\rho(k,c)$ is defined as in (8). Note that (10) is just (7) with reversed inequality sign. Substituting for $\rho(k,c)$ and $\sigma^{os}(k,c)$ one can prove that (10) is satisfied only if $c \leq 0$ and if $k < k^{os}(c)$, where $k^{os}(c)$ as above is the k for which (10) holds with equality. Additionally, constraint (9) has to be fulfilled. Let $k^{ss}(c)$ be the k such that (9) is satisfied with equality. We can summarize the results in

Proposition 3 *A semi cross–supply equilibrium exists if and only if* $c \leq 0$ *and* $k^{ss}(c) \leq k \leq k^{os}(c)$.

The set of parameters for which a semi cross–supply equilibrium exists corresponds to the area enclosed by k^{ss}, k^{m}, and k^{os} in Figure 2.

5.3 The both–sided cross–supply equilibrium

In this class of equilibria both firms produce one of the goods and purchase the second good from the other firm (in the following we will assume that

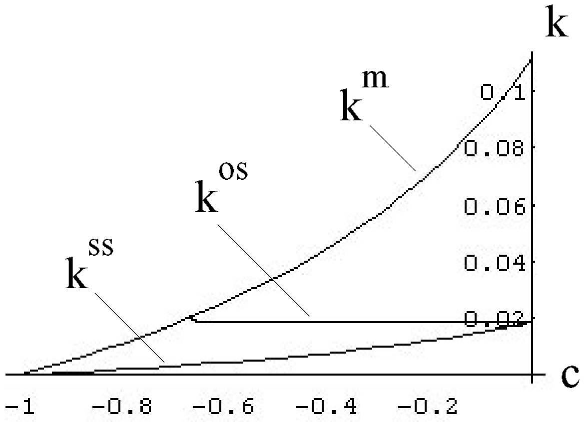

Figure 2:

firm 1 produces y and firm 2 produces x). It is therefore the only equilibrium that has some symmetry properties in the sense that the profits of both firms are equal in equilibrium.

To characterize the set of parameters for which a both–sided cross–supply equilibrium exists we consider again all possible deviations. In particular, firms could deviate on stage one (before hearing any cross–supply prices of the other firm) or on stage two. Consider first the case that cross–supply offers have been made. The only possible deviation for firm 1 would be to reject the cross–supply offer and produce the good x itself on the second step. For this deviation to be unprofitable it must hold that

$$(p_x - s_x)\,x_1^{bs} + p_y y_1^{br} + s_y y_2^{bs} - k \geq p_x x_1^* + p_y y_1^* + s_y y_2^{bs} - 2k, \tag{11}$$

where y_2^{bs} is the equilibrium quantity (note that firm 2 cannot react to firm 1's deviation) and $x_1^* = \frac{2+c-3cy_2^{bs}-3c^2y_2^{bs}}{6(1-c)}$ and $y_1^* = \frac{1-y_2^{bs}-cy_2^{bs}}{2(1-c)}$ are the best responses by firm 1 to y_2^{bs} (cf. equation 1). Implicitly (11) defines an upper bound for the cross–supply prices. For prices higher than that firms would rather produce both goods themselves. In a both–sided cross–supply equilibrium the prices will always assume this upper bound since profits of both firms are increasing in the cross–supply prices. Let $\sigma^{bs}(k, c) = s_x = s_y$ be the price that satisfies (11) with equality.

Next consider deviations on the first step (i.e. before the cross–supply prices have been announced). Three different kinds of deviations are possible (let firm 1 be the deviator).

1. Firm 1 sets up production for both goods, does not make a cross–supply offer and rejects any offers by firm 2. Since firm 2 would then also produce both goods, the Cournot quantities result.
2. Firm 1 refrains from making a cross–supply offer in y and does not produce x. Firm 2 supplies firm 1 with x and produces y on the second step itself, which results in the semi cross–supply quantities.
3. Firm 1 sets up production for both goods, makes a cross–supply offer for y (which firm 2 accepts) and rejects any offers by firm 2.

The first possible deviation yields the following constraint.

$$(p_x - s_x)\, x_1^{bs} + p_y y_1^{br} + s_y y_2^{bs} - k \geq \frac{2}{9(1-c)} - 2k. \tag{12}$$

In order to make the second deviation unprofitable it must hold that

$$(p_x - s_x)\, x_1^{bs} + p_y y_1^{br} + s_y y_2^{bs} - k \geq (p_x - s_x) x_1^{ss} + p_y y_1^{sr} - k. \tag{13}$$

And finally, the third deviation yields

$$(p_x - s_x)\, x_1^{bs} + p_y y_1^{br} + s_y y_2^{bs} - k \geq p_x x_1^{sr} + p_y y_1^{sr} + \rho y_2^{ss} - 2k, \tag{14}$$

where $\rho(k,c)$ is again defined as the price such that

$$p_x x_2^{sr} + (p_y - \rho(k,c))\, y_2^{ss} - k \equiv \frac{2}{9(1-c)} - 2k.$$

It can be shown that (12), (13) and (14) each define a lower bound for the cross–supply prices. Since profits are increasing in those prices and (11) defines an upper bound, a both–sided cross–supply equilibrium exists only if this upper bound $\sigma^{bs}(k,c) = s_x = s_y$ satisfies (12), (13) and (14). Solving for the set of parameters which satisfy the four constraints involves some straightforward but tedious calculations. We restrict ourselves therefore to a graphical analysis (see Figure 3).

For $c < 0$ it can be shown that (11) and (14) are incompatible. Thus, a both–sided cross–supply equilibrium does not exist for substitutes. The reason for this is the following. It is easy to check that the profit of a Stackelberg leader reacts more strongly to shifts in the demand curve than the Cournot profit. Therefore, if demand shifts down, the profit of a Stackelberg leader decreases by more than that of a Cournot duopolist. With independent markets ($c = 0$) the two profits are just the same.[6] Thus, if ceteris paribus c is lowered, demand decreases and the profit of the Stackelberg leader decreases relatively more. Consequently, the receiver of cross–supplies (in this case, firm 1 as the receiver of x) will deviate from the equilibrium by rejecting the cross–supply offer, which results in a Cournot equilibrium in the market for x. Hence, constraint (14) cannot be satisfied whenever (11) holds.

For $c \geq 0$ and low k the relevant lower bound is (13) yielding the curve $k^{bs}(c)$ in Figure 3. Summarizing we get

Proposition 4 *A both–sided cross–supply equilibrium exists if* $c \geq 0$ *and* $k^{bs}(c) \leq k \leq k^m(c)$.

That is, the equilibrium, in which firms are in a symmetric position exist only if the goods are complements or independent. Since $k^{bs}(c) < k^{os}(c)$ for $c > 0$, the region for which both–sided equilibria exist with complements is larger than the region in which one–sided equilibria exist.

After excluding (weakly) dominated equilibria we have isolated a unique equilibrium (up to permutation of players or goods) for all parameter constellations except for $c \geq 0$ and $k^{os} \leq k \leq k^m$, where both, one–sided and both–sided equilibria are possible. One may wonder whether equilibrium selection arguments as in Harsanyi and Selten (1988) would allow us to pick one of these equilibria as the more plausible one. However, both equilibria are strict and a payoff–dominance relation holds only for region A in Figure 3. In that region both firms receive higher profits in the both–sided equilibrium than in the one–sided equilibrium. For all remaining parameters the supplier of cross–supplies in the one–sided equilibrium gets more and the receiver of cross–supplies gets less than in the both–sided equilibrium.

6 Welfare aspects

As usual welfare is measured as the sum of profits and consumer surplus $W = \pi_1 + \pi_2 + CS$. Consumer surplus, CS, is calculated as the line integral

$$\int_{p^*}^{p^{\max}} [x(p) + y(p)]\, dp,$$

where $p^* := (p_x, p_y)$ and $p^{\max} = (1, 1)$.[7]

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[6] Recall that the receiver of cross–supplies is the Stackelberg leader in that market. In Section 4 we have shown that the Stackelberg leader is being held down to the Cournot profit ($\pi_2^{os} = \pi_2^c$).

[7] The order of integration plays no role since $\frac{\partial x}{\partial p_y} = \frac{\partial y}{\partial p_x}$ (see Auerbach, 1985).

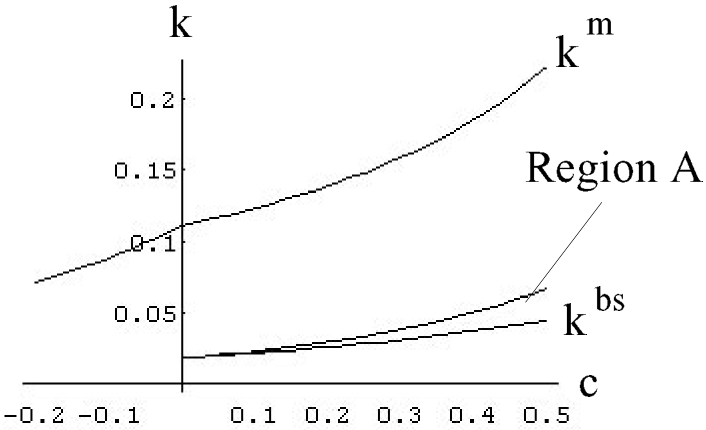

Figure 3:

6.1 The case of independent markets

Again we consider the case of independent demand functions ($c = 0$) first. For the Cournot equilibrium consumer surplus is easily calculated as being

$$CS^c = \tfrac{4}{9}.$$

Since the profit of each firm is $\pi_i = \frac{2}{9} - 2k$, total welfare is

$$W^c = \pi_1^c + \pi_2^c + CS^c = \tfrac{8}{9} - 4k.$$

In all equilibria with cross–supplies consumer surplus is

$$CS^s = \left(\tfrac{1}{2} + \tfrac{1}{6}\sqrt{2 - 18k}\right)^2.$$

Profits of the receiver of cross–supplies (say firm 2) are just as in the Cournot case. However, profits of the supplier are $\pi_1^{os} = \frac{1}{6} + k$ and therefore higher than in the Cournot equilibrium for all k such that a cross–supply equilibrium exists. Total welfare with cross–supplies is then

$$W^s = \tfrac{25}{36} - \tfrac{1}{2}k + \tfrac{1}{6}\sqrt{2 - 18k}.$$

We have thus established that cross–supplies are actually welfare increasing in the range of parameters for which such equilibria exist ($\frac{1}{54} \leq k \leq \frac{1}{9}$). Even

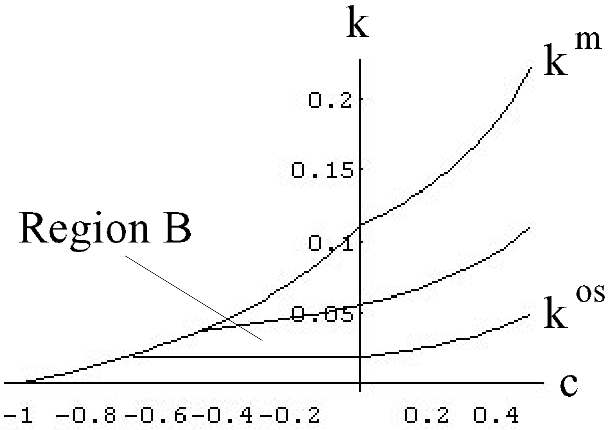

Figure 4:

consumers benefit from cross–supplies if fixed costs are small as $CS^s > CS^c$ for all k such that $\frac{1}{54} \leq k \leq \frac{1}{18}$.

The reason for this somewhat surprising result is that with Stackelberg competition total quantities are higher than with Cournot competition for small k. This can easily be seen by comparing the Cournot quantities $x^c = x_1^c + x_2^c = \frac{2}{3}$ with the Stackelberg quantities $x_1^{or} + x_2^{os} = \left(\frac{1}{2} - \frac{1}{6}\sqrt{2-18k}\right) + \frac{1}{3}\sqrt{2-18k}$. Thus, for $k < \frac{1}{18}$ quantities are higher in each market with cross–supplies. Total welfare increases even more since fixed costs are saved in addition.

6.2 The case of complementary or substitutional demands

6.2.1 One–sided cross–supply equilibria

Again we restrict ourselves to a graphical presentation of our results since the algebraic expressions are not very informative. Figure 4 shows the comparison between welfare and consumer surplus in the one–sided cross–supply equilibrium and the Cournot equilibrium. For all parameter combinations for which the one–sided cross–supply equilibrium exists ($k^{os} \leq k \leq k^m$) total welfare is higher with cross–supplies. Additionally, in region B even consumer surplus is higher.

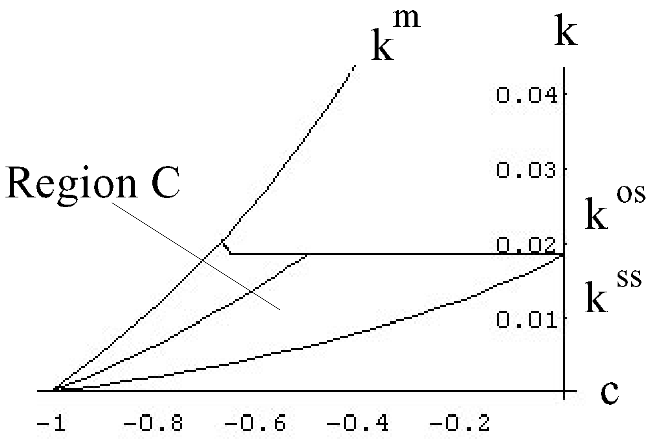

Figure 5:

6.2.2 Semi cross–supply equilibria

Recall that semi cross–supply equilibria exist only if the goods are substitutes. Welfare with cross–supplies is higher than in the Cournot equilibrium. Furthermore, consumers are better off with cross–supplies in the region C in Figure 5.

6.2.3 Both–sided cross–supply equilibria

Equilibria in which both firms supply each other were shown to exist only for complements. Again, welfare is higher with both–sided cross–supplies than in the Cournot equilibrium for all parameter values for which cross–supplies are profitable for the participating firms. Furthermore, region D in Figure 6 shows all (k, c) combinations for which even consumer surplus increases when cross–supplies are being used.

Comparing both–sided with one–sided equilibria it can be shown that welfare is higher with one–sided cross–supplies if and only if the receiver of the cross–supplies is not held down to the Cournot profit, i.e. when fixed costs are not too high.

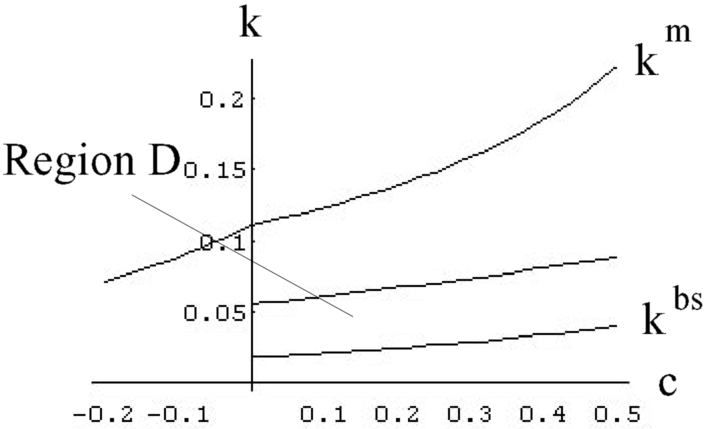

Figure 6:

7 Conclusion and policy recommendations

The main motivation for this paper was to evaluate the welfare effects of cross–supplies in light of the European Commission's ruling that prohibited glass producers from using cross–supplies. Based on Article 85 of the Treaty of Rome the Commission argued that cross–supplies are anti–competitive as they allegedly lead to a division of markets among firms. In a simple 3-stage duopoly model with two goods we find no support for this position.

In contrast, we find that cross–supplies unambiguously raise welfare whenever they are used. Furthermore, we find that for a large range of parameters even consumer surplus increases with cross–supplies. There are two reasons for the increase in total welfare. First, fixed cost of production can be saved when not all firms produce every good. Second, we argue that cross–supplies lead naturally to Stackelberg competition, in which total quantities are higher than in Cournot competition for a large range of parameters. Higher quantities lead to lower prices and hence consumers are better off.

In a static framework, thus, there appears to be no reason to prohibit cross–supplies since when they are used, they are beneficial and when they are not beneficial, they are not used. However, one needs to be careful when deriving policy conclusions from a purely static framework. It is well known that issues like market entry and its effect on, for example, innovation activity appear in a different light when considered in a dynamic framework. Since the European Commission argued essentially in a static framework, we think that the decision of the European Court of Justice to overrule the Commission's decision was justified. However, further work is needed to evaluate the welfare effects of cross–supplies in a dynamic framework.

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