Network Effects, Pricing Strategies, and Optimal Upgrade Time in Software Provision.

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Abstract: There are many products which have little or no value in isolation, but if combined with other products or services, they generate more value. The utility that a given user derives from these products depends upon the number of other users who consume the same products. This kind of positive consumption externality is defined as a network externality. Network externalities are pervasive in the computer in the computer software market. While software vendors consider pricing strategies, they must also take into account the impact of network externalities on their sales. Our main interest in this paper is to describe the firm's strategies and behavior in the presence of network externalities. Based on the two-stage model in this paper, we find that the software firm will charge a lower price to attract more users in the first stage, then charge a higher price as it does in monopoly as traditional economic theory indicates. And our dynamic model shows that the optimal upgrade time occurs when gross profit of the first edition equals the gross profit of the second edition of the software. This implies that either too earlier or late promotion of the new software version will causes profit loss.

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I. Introduction

There are many products for which the utility derived by a consumer depends on the number of the agents consuming the good. In other words, positive consumption externalities arise. Katz and Shapiro (1985) defined and illustrated three types of network externality effects. (1) Direct physical effects; generated through the direct physical effect of the number of agents purchasing the product. Examples include fax machines, telephone and data networks, etc. (2) Indirect effects; or what might be called the hardware-software paradigm. For example, an agent buying a PC will not merely take the number of PCs sold into account, he will also consider software availability. Similar examples are video games and video players. (3) Postpurchase service network effects; for which vivid case is the automobile market. An agent will take into consideration the postpurchase network service size in addition to the car's quality.

As Katz and Shapiro (1994) pointed out these kinds of network externality effects will trouble economic theory both in market equilibrium and in the market performance. That is, if the market presents such network externalities, an equilibrium might not exist, or multiple equilibria might exist. And the fundamental theorems of welfare economics may not apply in this situation. Lots of interesting problems arise in markets with such network externalities.

The personal computer industry is perhaps the best case of network externalities, both for hardware and software. The hardware competition between Apple and IBM compatible PCs has prevailed for years. The operating system war between Microsoft and IBM has become a hot issue recently. The new upgrade information about some important software such as Lotus 1-2-3 spreadsheet is usually paid attention by the people as well as stock market. The issues in which we are interested in the software industry are what the pricing and promotion strategies are given the network externalities in the market.

In this paper, a two-stage static model and a simple dynamic model of software monopoly will be introduced. I examine the monopolistic firm's pricing strategies both for software introduction and upgrading. The dynamic model indicates the optimal upgrade time of the software. The paper is organized as follows: section II discusses consumers' decision behavior, based on consumers' behavior, the two-stage static model in section III demonstrates the firm's pricing policy in section III. I then apply optimal control theory to analyze the firm's dynamic behavior in the presence of network externalities in section IV. This model also proposes the optimal upgrade time for the software monopolist. Some conclusions are drawn in section V.

II. Choices of Consumers and Market Equilibrium

Assume the software, x, exhibits network externalities, i.e., utility comes not only from consuming software x itself, but also from the number of other consumers who adopt the same products. There are k potential consumers and one firm in this market.

Without taking the network externalities into account initially, the stand-alone willingness to pay for the software x is $R_i(h)$, i = 1, 2,..., k. R_i's are ranked by the willingness to pay, i denotes consumer's rank order, and h denotes an individual's preference parameter, which can be viewed as a technology index. Superior technology enables consumers to enjoy more with the new technology, so consumers are willing to pay more for it, i.e., $\partial R_i(h) / \partial h > 0$. It is assumed to be constant here for present discussion in this paper and we will suppress $R(h)$ in following analysis. Assume every potential consumer demands either one unit of product or none, each makes a discrete choice.

In the absence of network externalities, the consumers' demand schedule D for this product could be drawn in the order of the willingness to pay. (See, Figure 1)

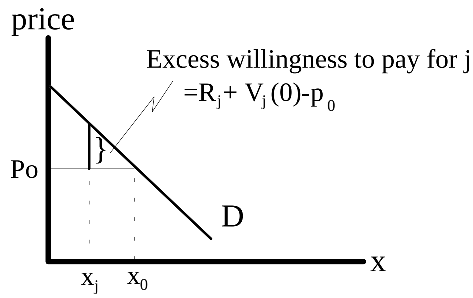

Figure 1.

Following Katz and Shapiro's (1985) assumptions, suppose the network externalities function for ith individual is $v_i(x)$, where $\frac{\partial v_i}{\partial x} > 0$, $\frac{\partial^2 v_i}{\partial x^2} < 0$, $v_i(0) = 0$, and $\lim_{x \to \infty} v'(x) = 0$. So, in the presence of network externalities, the willingness to pay for x is $R_i(h) + v_i(x)$.

In stage 0, first the firm chooses p_0, and x_o consumers whose willingness to pay is greater than or equal to the price , i.e., $R_i(h) - p_0 \geq 0$, will purchase in the first period. Notice that network externalities do not yet influence consumer choice since the product is not yet established. However, stage 0 consumption knocks the market out of equilibrium, for network externalities now emerge. Marginal consumers who originally didn't enter the market now want to join the network because they perceive positive network externalities. So Δx consumers, where $\Delta x = x_1 - x_0 > 0$, enter the network in stage 1. Those whose willingness to pay are greater than or equal to p_0 will enter the network, i.e., $R_i(h) + v_i(x_0) - p_0 \geq 0$, $i = x_0 + 1, x_0 + 2, \ldots, k$. D_1 denotes demand in the second stage. By dynamic adjustment the network will grow at a declining rate until the effect of network externalities vanishes. Finally, in stage t, the network reaches equilibrium with demand denoted by D* as shown in Figure 2. Suppose the firm set an initial price P_0. Then a corresponding network growth function, $G(v(x), p_0)$, associated with this pricing strategy P_0. emerges (Figure 2). At time t = 0, the network size is X_0. Because prior period purchase increases the present willingness to pay for potential users, some of those marginal users who didn't enter the network previously will now join the network. Therefore, at time t=1, the network grows to X_1. For the same reasons, users join the network continuously until the excess willingness to pay vanishes. The network will follow along a concave growth function and reach the equilibrium $X_{t^*} = X_0^*$. The equilibrium set is (X_0^*, p_0). If the firm set a different price, P_1, instead of P_0 initially, a corresponding network growth function, $G(v(x), p_1)$ would lead to a different equilibrium, $X_2 = X_1^*$, the equilibrium set is (X_1^*, p_1). The firm's different initial pricing strategies, P_j , collectively map out the different long run equilibrium network sets $\{(X_j^*, p_j)\}$, based on the network growth function, $G(v(x), p_j)$, yielding a *the network demand function*, D*, as indicated in figure 2. Thus, the long-run market equilibrium condition can be expressed as,

$$R_{x^*}(h) + v_{x^*}(x^*) - p_j = 0$$

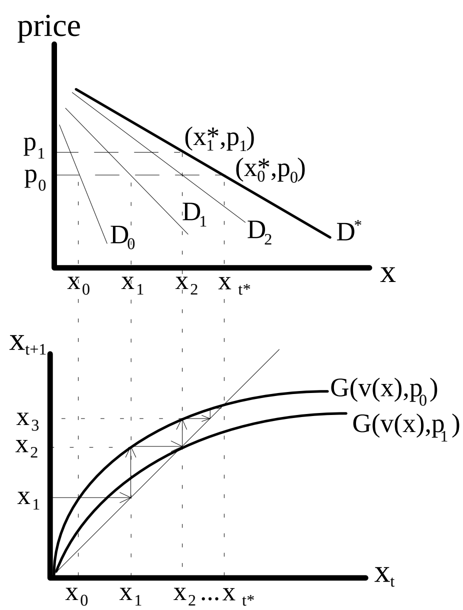

Figure 2. Dynamic adjustment of network growth.

III. Two-Stage Static Model

Following the consumer behavior and long-run market equilibrium condition presented in section II, now consider a two-stage static model of the software monopolist's behavior. Assume the monopolist provides two successive versions of a software package. In the first stage he offers an introductory version, x^I; in the second stage he offers an upgrade version, x^{II}. We assume that the constant marginal costs of these two versions are c_1 and c_2 respectively. The monopolist looks for prices p_1 and p_2 (the prices for x^I and x^{II}) that maximize the present value of the profit stream from the two successive versions of the software, given the existence of network externalities. The effect of network externalities is assumed to be lagged. That is, consumers' decisions to purchase the introduction version does not expect the network externalities for the present version. When the upgrade version becomes available, consumers consider the existing network size for the introductory version in determining whether to upgrade or not. Hence, the monopolist's profit function is[1],

$$\Pi = \pi_1 + \pi_2 = (p_1 - c_1)x^I(p_1) + \delta\Big((p_2 - c_2)x^{II}\big(x^I(p_1), p_2\big)\Big), \qquad (1)$$

where $\delta = 1/(1+r)$, a discounted factor and r is the discount rate.

The first order conditions require,

$$\frac{\partial \Pi}{\partial p_1} = (p_1 - c_1)\frac{\partial x^I}{\partial p_1} + x^I(p_1) + \delta(p_2 - c_2)\frac{\partial x^{II}}{\partial x^I}\frac{\partial x^I}{\partial p_1} = 0 \qquad (2)$$

$$\frac{\partial \Pi}{\partial p_2} = \delta\left((p_2 - c_2)\frac{\partial x^{II}}{\partial p_2} + x^{II}(p_2)\right) = 0, \qquad (3)$$

[1]Software vendors usually provide two prices for upgrades; one for owners of the first version, and a higher price for new users. This is conventional price discrimination. It is easy to show there are similar results under this situation if imposed by price discrimination. However, we don't examine the price discrimination policy here.

Assume the second order conditions hold. Define price elasticity as $\varepsilon_i = \frac{\partial x_i}{\partial p_i}\frac{p_i}{x_i}$, , $i = 1, 2$.

Equation (3) can be rewritten as

$$p_2^*(1+\frac{1}{\varepsilon_2}) = c_2, \tag{4}$$

where p_2^* is optimal upgrade price. This is the standard monopolistic pricing policy; it means marginal revenue equals marginal cost. If there is only one upgrade, the monopoly follows the pricing principle of MR = MC. Furthermore, of particular interests is to look further at equation (2). After multiplying both sides with $\partial p_1 / \partial x_1$ and rearranging it, we can obtain

$$p_1^*(1+\frac{1}{\varepsilon_1}) = c_1 - \delta(p_2^* - c_2)\frac{\partial x^{II}}{\partial x^{I}}. \tag{5}$$

where p_1^* is optimal upgrade price. Since $(p_2^* - c_2)$ is nonnegative[2], $\partial x_2 / \partial x_1$, be the network externalities, should be positive. Therefore, we find that in the presence of network externalities, a profit-maximizing monopolist charges a lower price for the introductory version of the software than without the network externalities (where $\partial x^{II} / \partial x^{I}$ = 0). It is intuitive that a monopolist sets a lower introductory price to attract more users in order to generate greater demand in next round since the firm knows there exist network externalities. Equation (5) indicates that the monopolist equalizes present marginal revenue and the present marginal cost plus a (negative) present opportunity cost caused by future network externalities effects.

IV. The Dynamic Model and The Decision of Optimal Upgrade Time

[2]Equaiton (3) can be rearranged as $x^{II}(p_2^*) = -(p_2^* - c_2)\partial x^{II} / \partial p_2$. Since $x^{II}(.) \geq 0$, and $\partial x^{II} / \partial p_2 < 0$, $(p_2^* - c_2) \geq 0$.

The firm is assumed to be able to provide enough amount of the network good to satisfy fully demand over the product life cycle. Based on the assumptions given in earlier sections, we form the dynamic adjustment of the network size which can be expressed by the differential equation,

$$\frac{dx}{dt} = G(v(x), p), \tag{6}$$

G(.) is the network growth function which is a function of network externalities and price. $\frac{\partial G(.)}{\partial v}$ is defined as *marginal growth of network effect*. As v(.) increases, the positive growth of the network, G(.), gradually vanishes. i.e., $\frac{\partial^2 G(v)}{\partial v^2} < 0$.

The monopoly firm chooses an optimal pricing strategy to maximize discounted present value of the profit stream, taking into account the impacts of the different network growth paths of the network associated with different prices.

Sales in time t are represented as y_t, which is the change of the network size in the infinitesimal time interval t, i.e., $y_t = \frac{dx}{dt} = G(\cdot)$. Hence, the total sales from time 0 to time T is given by $x(T) = \int_0^T y_t dt$. For simplicity, assume the firm sets a fixed price over the this product's lifetime. The firm's problem becomes

$$\text{Max } \int_0^T e^{-rt}(p-c)y_t dt \tag{7}$$

$$\text{s.t. } \frac{dx}{dt} = G(v(x), p),$$

$$x(o)=0.$$

where r is the discount rate.

The Hamiltonian is

$$\text{H} = e^{-rt}(p-c)y_t + \phi_t\left[G(v(x_t), p)\right] \tag{8}$$

Define $\lambda_t = e^{-rt}\phi_t$ by the current value ofϕ_t, then we have current value Hamiltonian

$$\begin{aligned} H &= (p-c)G(.) + \lambda_t G(.) \\ &= (p-c+\lambda_t)G(.) \end{aligned} \tag{9}$$

Assume that there exists interior solution. The necessary conditions for this maximization problem are,

$$\frac{\partial H}{\partial p} = (\hat{p} - c + \hat{\lambda}_t)\frac{\partial G(v(\hat{x}_t), \hat{p})}{\partial p} + G(v(\hat{x}_t), \hat{p}) = 0 \qquad (10)$$

$$\frac{d\lambda}{dt} = r\lambda_t - \frac{\partial H}{\partial x_t}$$

$$= r\hat{\lambda} - (\hat{p} - c + \hat{\lambda}_t)\frac{\partial \hat{G}}{\partial v}\frac{\partial \hat{v}}{\partial x_t}$$

(11)

$$\frac{dx_t}{dt} = \hat{y}_t = G(v(\hat{x}_t), \hat{p}) \text{, and } x(0)=0. \qquad (12)$$

where "^" means the optimal solutions for each variable.

These necessary conditions yield the optimal time paths for $\hat{\lambda}_t$ *and* $\hat{x}_t$, and then $\hat{y}_t = G(v(\hat{x}_t), \hat{p})$ and optimal $\hat{p}$. $\hat{\lambda}_t$ is conventionally interpreted as the shadow value or shadow price of network growth, *G(.)*. The transversality condition tells us that $\hat{\lambda}_t \geq 0,\ \hat{\lambda}_T G(T) = 0$, where T is the terminal time.

Rearrange equation (10) and substitute $\hat{y}_t$ for *G(.)* to get

$$\hat{p}\left[1 + \frac{\partial p}{\partial \hat{y}_t}\frac{\hat{y}_t}{p}\right] = (c - \hat{\lambda}_t) \qquad (13)$$

Define the elasticity of demand by $\varepsilon_t = \frac{\partial p}{\partial \hat{y}_t}\frac{\hat{y}_t}{p}$, and rewrite equation (13),

$$\hat{p}\left[1 + \frac{1}{\hat{\varepsilon}_t}\right] = (c - \hat{\lambda}_t) \qquad (14)$$

This is just traditional theory of monopoly pricing; the monopolist produces where marginal revenue equals marginal cost, i.e., $MR = \hat{p}\left[1 + \frac{1}{\hat{\varepsilon}_t}\right] = c = MC$. In the presence of network externalities, the monopolist chooses a price lower than c because the value of marginal network growth, $\hat{\lambda}_t$, also affects marginal cost. As Figure 3 indicates graphically, the monopolist lowers his price (P_n) below that which prevails in the absence of network externalities (P_m) to induce more present demand because this ultimately leads to larger sales volume and profits.

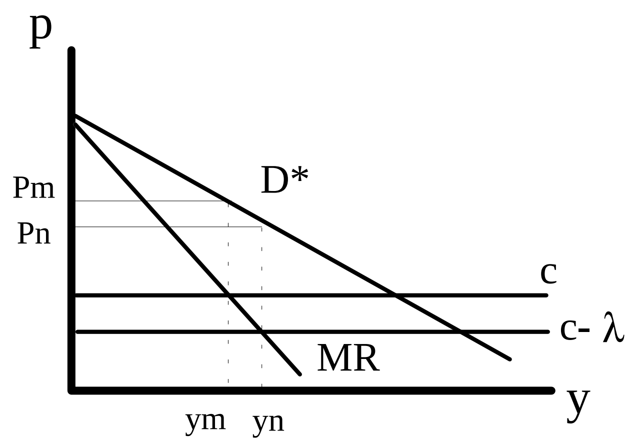

Figure 3. Monopolistic pricing in the presence of network externalities.

Optimal Upgrade Time

Now I extend the above one-generation model to a two-generation setting problem and try to find the optimal software upgrade time.

Assume the firm will offer only one upgrade version of the software. Given the network externalities, first we assume that the second edition, x^{II} is perfectly backward compatible with the first edition, x^{I}. It means $x^{II}(t_1) = x^{I}(t_1)$ where t_1 denotes the terminal time for the first edition. There won't be any first edition of the software available after the introduction of the second edition. Under such assumptions, the optimal control problem becomes

Maximizing

$$\int_0^{t_1} e^{-rt}(p_1 - c_1)y_{1t}dt + \int_{t_1}^{T} e^{-rt}(p_2 - c_2)y_{2t}dt \qquad (15)$$

s.t. $y_{1t} = G(v_1(x^I), p_1)$, and $x^I(0) = 0$,

$y_{2t} = G(v_2(x^{II}), p_2)$, and $x^{II}(t_1) = x^I(t_1)$

Substitute $y_{1t} = dx^I / dt$ and $y_{2t} = dx^{II} / dt$ into equation (15), it can be rewritten as

$$\int_0^{t_1} e^{-rt}(p_1 - c_1)\frac{dx^I}{dt}dt + \int_{t_1}^{T} e^{-rt}(p_2 - c_2)\frac{dx^{II}}{dt}dt$$

$$= \int_0^{t_1} e^{-rt}(p_1 - c_1)dx^I + \int_{t_1}^{T} e^{-rt}(p_2 - c_2)dx^{II}$$

$$= (p_1 - c_1)\int_0^{t_1} e^{-rt} dx^I + (p_2 - c_2)\int_{t_1}^{T} e^{-rt} dx^{II}$$

$$= (p_1 - c_1)\left(e^{-rt} x^I(t)\right)\Big|_0^{t_1} + (p_2 - c_2)\left(e^{-rt} x^{II}(t)\right)\Big|_0^{t_1}$$

$$= (p_1 - c_1)\left(e^{-rt_1} x^I(t_1) - 0\right) + (p_2 - c_2)\left(e^{-rT} x^{II}(T) - e^{-rt_1} x^{II}(t_1)\right) \quad (16)$$

In order to maximize the sum of the two generations' profits over time. The first order condition is to take the derivative of equation (16) with respect to t_1 and set it to be zero. Then we obtain

$$(p_1 - c_1)\left(e^{-rt_1} \frac{dx^I(t_1)}{dt}\right) + (p_2 - c_2)\left(-e^{-rt_1} \frac{dx^{II}(t_1)}{dt}\right) = 0 \quad (17)$$

Multiplying both sides with e^{rt} and rearranging it, Equation (17) can be rewritten as

$$\frac{dx^I / dt}{dx^{II} / dt} = \frac{y_1(t_1)}{y_2(t_2)} = \frac{G_1(.)}{G_2(.)} = \frac{p_2 - c_2}{p_1 - c_1}, \text{ or} \quad (18)$$

$$(p_1 - c_1)G_1(.) = (p_2 - c_2)G_2(.) \quad (19)$$

According to equation (19), the optimal upgrade time occurs when gross profit of the first edition, $(p_1 - c_1)G_1(.)$, equals the gross profit, $(p_2 - c_2)G_2(.)$, of the second edition of the software, or at the time when the ratio of the network growth of two generations is equal to the reciprocal of the gross profit ratio. The optimal upgrade time and the firm's profit flow are shown in figure 4.

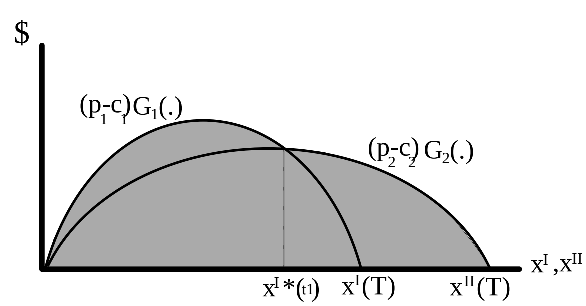

Figure 4. The optimal upgrade time and the firm's revenue.

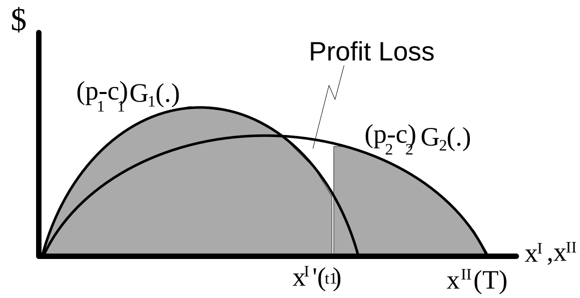

Figure 5a. A case of later promotion.

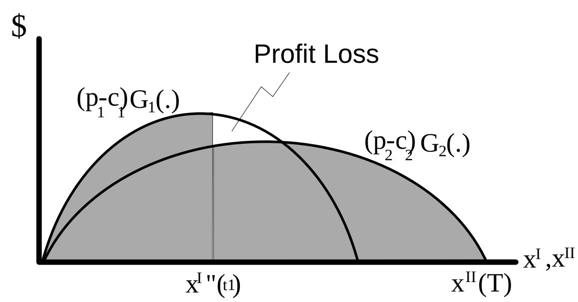

Figure 5b. A case of earlier promotion.

Figure 5a and 5b show the two cases in which software promotion occurs too early or too late shown in figure 5a and 5b. We imagine that sometimes when software vendors announce a delay in introducing next version of software, the reasons to delay usually is that the new version has not been well developed yet. However, according to our model, another interesting explanation might be that the present version is still relatively profitable in the mean time.

V. Conclusions

According to our two-stage static model, we found that the monopolist will charge a lower price for the introduction version of software than a monopolist usually does. It makes sense that a monopolistic firm sets a lower price to induce more users to get into the network in the first stage in order to extract more profit from the network effects in the second stage. The firm's strategy is nothing but setting the price at the point where marginal revenue equals marginal cost which is the sum of marginal cost at present period and discounted opportunity cost in the future.

In our dynamic model, the similar pricing strategies of the monopolistic firm are also observed at which marginal revenue is equal to present marginal cost minus shadow value of the network growth. And the firm's optimal upgrade time is shown in our two-generation dynamic model. We can see that the optimal upgrade time is at the time when gross profit of the first version is equal to the gross profit of the second version.

There are a lot of ways in which the models could be generalized and extended. For example, it would be more general to assume there is monopolistic competition in the software market. If we extend the models to more than one firm in competing the network market, it will be much interesting to see what the firms' behaviors and pricing strategies including the cooperation and predation. The models can also be extended to link the hardware and software since there must be some network-effect relationship between them. We already saw that Novell, a big network software provider, is going to incorporate hardware production. Intel, a biggest 80x86 CPU supplier, has entered the software market to try to get more business based one the network effect of it CPU product. As we see, the linkage between hardware-software paradigm will become an interesting issue and it deserves more researches.

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