A Simple Model of Informative Advertising

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ABSTRACT

This paper presents an oligopolistic model of informative advertising, where firms simultaneously choose prices and advertising intensities. For this game there is a dispersed price equilibrium in which the amount of advertising by each firm is socially optimal. The advertising technology considered is more general than in Butters', however his results can be obtained as the number of firms tends to infinity. For some advertising technologies entry leads to a market contraction; we can also observe situations where increases in advertising costs lead to higher profits. This model can be used as a "benchmark" against which other models of informative advertising can be compared.

Keywords: informative advertising, price dispersion, oligopoly

JEL Classification: D43,D83

I. Introduction

The literature on markets with imperfectly informed buyers has in most cases modeled buyers as active agents seeking price information in a market where firms set prices. In this setting it is up to the individual buyer to decide how much price information to acquire. Buyers' search costs are the ultimate determinant of the level of information in the marketplace, and that information, in turn, is used by firms in selecting their pricing strategies. Two extreme examples are a world with perfectly informed buyers in which firms behave as Bertrand players and a world with uninformed buyers that has all firms behaving as monopolists. The intermediate cases have been shown to involve equilibria with price dispersion.

In the real world buyers acquire price information through several means. Of these, advertising is one of the most common, at least for some product categories. An interesting aspect of advertising is that it is the firm who controls the flow of information. Under this information structure the firm can manipulate its sales not only through the choice of price but also by selecting an advertising intensity which then affects the market equilibrium. More specifically, the role of advertising introduces several questions of interest: Will the equilibrium be different from those of other models with imperfect information? What factors determine the level of prices and the extent of buyer information? Will firms behave in a socially desirable way? What welfare consequences will result?

Butters (1977) was the first to study the role of firms as suppliers of price information. In his model firms provide buyers with price information through purely informative advertising. Because firms must bear the cost of providing the price information, the Bertrand result does not attain. Butters showed that a monopolistically competitive market has an equilibrium with firms advertising different prices with different intensities. His result confirms other findings [e.g., Salop

and Stiglitz (1977); Wilde and Schwartz, (1979); MacMinn, (1982); Burdett and Judd (1983)] that equilibrium in markets with imperfectly informed buyers is characterized by price dispersion. More unusual was his finding that the equilibrium amount of advertising was socially optimal.

Grossman and Shapiro (1984) extended the Butters model to an oligopolistic environment in which firms sell a spatially differentiated product. In this setting they were able to show that there was a single price equilibrium and that firms would not behave in a socially optimal manner; they would most likely engage in excessive advertising. Finally, Stegeman (1990) modified Butters' model to allow buyers to have different reservation prices. He showed that the resulting equilibrium still involved price dispersion but found that firms would choose a socially sub-optimal level of advertising.

In this paper we construct an oligopolistic variant of the Butters (1977) model, and show that his most important results hold when different information technologies are considered. In particular, price dispersion prevails and the amount of advertising is socially optimal in equilibrium. Price dispersion occurs because firms play mixed strategies in prices. Butters' result is obtained if we let the number of firms tend to infinity and assume that costs per unit of advertising are constant.

This model differs from that of Grossman and Shapiro (1984) in a single way: it assumes that firms are selling a homogeneous product while in Grossman and Shapiro (1984) the product is spatially differentiated. However, as will be shown, their implicit generalization could be misleading; in their model the homogenous goods case does not obtain as the degree of product differentiation tends towards zero. Rather, when product differentiation tends toward zero, firms cease to advertise and select a price equal to the marginal cost. This suggests that the case for the homogeneous good is a special one that needs to be addressed separately.

II. Model Assumptions

It is assumed that there are many prospective buyers, all with the same reservation price p_m and each demanding one unit of a homogeneous product. These buyers receive ads free of charge from existing firms. The ads contain information about prices (and firm existence) and are truthful; that is, firms honor the advertised prices[1]. To simplify notation the number of buyers is normalized to unity. These buyers act in a rather mechanical fashion and their strategies are quite simple. Having received ads from the different firms, they acquire one unit of the good from the firm advertising the lowest price, so long as that price is below p_m. If a buyer is not contacted he will not enter the market. In the event that a buyer is indifferent between two or more prices, he selects one firm through a random process, giving an equal chance to any of the firms that contacted him. Ads are the only source of information available to prospective buyers.

There are N firms selling a homogeneous product, who simultaneously and noncooperatively choose advertising intensity, Φ_j, and price, p_j, j=1,...,N. Since the number of buyers is normalized to one the advertising intensity, Φ_j, can be interpreted as the number of buyers who receive ads from firm j. It is assumed that firms cannot target specific buyers; firms reach buyers in a random fashion. It is also assumed that the probability that any buyer receives an ad from any firm is independent of the probability that he receives an ad from another firm. All firms have the same marginal production cost, which is assumed to be zero. By construction, Φ_j belongs to the unit interval, and p_j to the $[0,p_m]$ interval.

Following Grossman and Shapiro (1984), it is assumed that the cost of reaching buyers increases at an increasing rate. Several justifications support this assumption. Grossman and

[1] Although no explicit mechanism is considered to guarantee that firms are honest, standard arguments include the loss of reputation, legal sanctions, etc.

Shapiro (1984) point to the possibility of media saturation or the existence of different predispositions to view ads on the part of the target population. Another justification has to do with the advertising technology itself. If random messages are sent at a fixed cost per message, then the probability of reaching a buyer not yet informed decreases with the amount already advertised.

The cost of reaching a proportion of the buyers is given by the function $A(\Phi;\alpha)$, with $A_\Phi>0$ and $A_{\Phi\Phi}>0$[2]. The argument α is a shift parameter in the cost function. Increases in the value of α reflect increases in the cost of reaching buyers, and consequently $A_\alpha>0$ and $A_{\Phi\alpha}>0$. There are no fixed advertising costs, so $A(0)=0$. These assumptions exclude the possibility of increasing returns in the advertising technology. Finally, advertising will only be feasible if $A_\Phi(0)<p_m$, that is, if the cost of advertising to the first buyer is lower than the potential revenue that a sale to that buyer would generate.

III. Advertising Technologies

The above assumptions are natural ones. They hold for all the specific advertising cost functions that have been used in this literature, including the Quadratic Advertising function [Tirole; 1988], the Butters technology, and the Constant Reach Independent Readership (CRIR) technology of Grossman and Shapiro (1984).

The Quadratic Advertising function is not based upon an underlying technology of message production. It has the advantage of being extremely simple to manipulate algebraically and is given by $A(\Phi;\alpha) = \frac{\alpha}{2}\Phi^2$.

[2] In general subscripts will denote partial derivatives. Whenever it is clear from the context we will abstain from using function arguments.

Butters (1977) derives his cost function explicitly from the random allocation mechanism employed by the firms. He notes that if one firm sends L ads at random to M buyers and if both L and M are large, then the fraction of buyers who do not receive any ad is $1-\Phi=(1-\frac{1}{M})^{L}\simeq\exp^{-\frac{L}{m}}$.

If each ad has a fixed cost of δ, then the total advertising cost is δL, or, in terms of Φ, $-\delta M\log(1-\Phi)$. If we let α equal δM we obtain $A(\Phi;\alpha)=-\alpha\log(1-\Phi)$. The parameter α can be identified with either the cost per ad or with the number of buyers. In this paper the number of buyers is normalized to one, so the former interpretation is appropriate.

The CRIR starts from the assumption that an advertiser can place ads in any of a set of magazines or newspapers. These ads are the only means of conveying price information to buyers. Furthermore, the newspapers have independent readerships of equal size. If each magazine charges a fixed advertising fee per reader, then the cost of achieving a given reach ,Φ, can be computed[3]. The resulting function is the same as Butters' but multiplied by $-\frac{r}{\log(1-r)}$, where r is the readership of each magazine. In practical terms this means that the α parameter can also be identified with changes in r. The similarity between the two cost functions is not accidental. Butters' technology can be obtained as a limiting case of the CRIR technology[4].

IV. Equilibrium

The strategy space for any firm i consists of the set $S_i=[0,1]\times[0,p_m]$, and its payoff of

[3] The logic used to derive the advertising cost function is analogous to that used for the Butters technology. For details see Grossman and Shapiro (1984).

[4] See Grossman and Shapiro (1984).

Π_i:S→R where S is the Cartesian product of all strategy spaces. Firms simultaneously select advertising intensity and price. The ranking of firms' price is important in the model, since buyers will buy at the lowest price they have learned. It is assumed that the firm advertising the lowest price can sell to all buyers that it has reached. More generally, a firm can sell to a buyer it reaches only if that buyer does not receive an ad for a lower price from any other firm.

Following the usual practice, we restrict the analysis to symmetric Nash equilibria. In the first proposition we show that there is no single price equilibrium.

Proposition 1 : There is no symmetric Nash equilibrium in pure strategies.

Proof - See the Appendix.

Proposition 1 shows that, in contrast to the Grossman and Shapiro model, there is no equilibrium in pure strategies in both advertising intensity and price. Since the two models are similar in other ways, it is apparent that the product homogeneity assumption prevents the single price equilibrium. In this model buyers respond solely to prices and this leads to discontinuities in the payoffs of the firms. This, in turn, results in the existence of a Bertrand-Edgeworth effect. When a firm undercuts prices it always experiences a discrete jump in profits. This incentive to undercut is reversed once it becomes more profitable to advertise the highest price to the captive clientele. These Bertrand-Edgeworth effects are not present in the Grossman and Shapiro model because product differentiation "softens" price competition[5]. As in the Hotelling model, when a Grossman-Shapiro firm undercuts prices it attracts only the marginal buyer. There is, however, a symmetric Nash equilibrium involving mixed strategies in price as the next proposition demonstrates.

[5] The reason why these Bertrand-Edgeworth effects do not come into play in the Grossman and Shapiro model has to do with the assumption that the degree of product differentiation is continuous and not with product differentiation *per se*. With a finite number of product types we would probably have the same kinds of discontinuities as are present in this model.

Proposition 2 : There exists a symmetric mixed strategy equilibrium in which all N firms play the strategy (Φ^*,$f^*(p)$), where Φ^* is the implicit solution to $p_m(1-\Phi)^{N-1} = A_\Phi$ and $f^*(p)$ is the density function associated with the following cumulative density function,

$$F^*(p) = \begin{cases} 1 & \text{for} \quad p \geq p_m \\ \dfrac{1}{\Phi^*} - \dfrac{(1-\Phi^*)}{\Phi^*}\left(\dfrac{p_m}{p}\right)^{\frac{1}{N-1}} & \text{for} \quad p_o \leq p < p_m \\ 0 & \text{for} \quad p < p_o \end{cases} \qquad \textbf{(1)}$$

with $p_o = A_\Phi(\Phi^*)$.

Proof - See the Appendix.

This equilibrium is qualitatively different from those of both Butters and Grossman and Shapiro. In this model's equilibrium all firms advertise with equal intensity but play mixed strategies in price. Thus price dispersion results, even if we are dealing with a small number of firms.

V. Comparative Statics

The optimal level of advertising, Φ^* depends only on α and N. Solving for the implicit derivatives that define Φ^* (see Proposition 2), we obtain,

$$\Phi^*_\alpha = \frac{-A_{\Phi\alpha}}{(N-1)(1-\Phi^*)^{-1}A_\Phi + A_{\Phi\Phi}} < 0 \ , \qquad \textbf{(2)}$$

and treating N as a continuous variable,

$$\Phi^*_N = \frac{\log(1-\Phi^*)A_\Phi}{(N-1)(1-\Phi^*)^{-1}A_\Phi + A_{\Phi\Phi}} < 0 \ . \qquad \textbf{(3)}$$

Both results are intuitively sensible and they are also true in the other models of informative advertising. Firms adjust to increases in the cost of advertising by lowering their reach. On the other hand, increased competition tends to reduce the expected yield per ad and consequently firms respond by lowering advertising intensity. As Grossman and Shapiro note, this is consistent with the observation that advertising is especially important in industries that are relatively concentrated.

We can also infer comparative static effects for the size of the market. In equilibrium there will be $1-(1-\Phi^*)^N$ informed (active) buyers. An increase in α will lower Φ^* and consequently market size. However, for N the effect is ambiguous. As the number of firms increases, reach per firm decreases, but the advertising of the additional firm may not be sufficient to offset the decrease in the advertising of all other firms. For example, whenever advertising technologies satisfy the condition $A_{\Phi\Phi}(1-\Phi^*) < A_{\Phi}$, entry will lead to a market contraction[6].

For the Butters technology market size is $1-\alpha/p_m$, which is invariant with the total number of firms. This comes from its specific equilibrium condition, $p_m(1-\Phi^*)^N = \alpha$ that stipulates that in equilibrium firms should advertise up to the point where the cost of the last ad sent equals the expected revenue of a sale at the highest price to an uninformed buyer. The impact of the exogenous variables on the level of prices is also a question of interest. Advertising costs affect the price distribution through Φ^*. We know that increases in α lower Φ^* and consequently the price distribution shifts to the right leading to higher prices. This can be confirmed by differentiating (1) with respect to α,

$$\frac{\partial F^*(p_i)}{\partial \alpha} = -\frac{\Phi^*_\alpha}{\Phi^{*2}}\left[1-\left(\frac{p_m}{p_i}\right)^{\frac{1}{N-1}}\right] < 0 \ . \qquad \textbf{(4)}$$

[6] An example of such technology is $A(\Phi;\alpha) = \alpha(k^{\Phi}-1)$ for $1<k<e$.

The impact of changes in N is ambiguous and depends on two effects. An increase in the number of firms reduces reach per firm Φ^*, which tends to raise prices. This negative effect may be offset by a positive effect of competition. The increase in the number of firms makes it more likely that any buyer receives more than one ad, consequently lowering the probability of sales at higher prices.

The Butters equilibrium price distribution can be found as a limiting case of F^* when N approaches towards infinity. For the Butters technology, $\lim_{N\to\infty} p_o = \alpha$ and

$\lim_{N\to\infty} F^*(p) = \frac{\log(p) - \log(\alpha)}{\log(p_m) - \log(\alpha)}$, which is also the cumulative price distribution in Butters[7].

VI. Welfare Effects

The welfare properties of advertising have stimulated a lot of discussion among economists. This debate can be traced back to Kaldor (1950) who initially argued that advertising was produced in excess. This controversy extends to models of informative advertising, since existing models differ in their welfare properties.

In the simplest model, the advertiser has two conflicting incentives to provide price information. The first effect concerns uninformed buyers and the size of the market. An ad that reaches any of uninformed buyer generates a sale regardless of price. This action is socially desirable because it increases market size and consequently welfare. However, because of the competitive pressure, the firm can not charge the highest price and fully appropriate the social

[7] See Butters (1977), Proposition 2.4.

surplus. Consequently, it will provide a sub-optimal level of price information. The second "business-stealing" effect occurs because an individual firm does not account for reduced profits at other firms as it increases its own advertising intensity and captures customers from rivals. The mere shuffle of customers among firms is wasteful and consequently socially undesirable. From this point of view firms tend to provide an excessive amount of advertising.

Butters (1977) showed that in a monopolistically competitive market the equilibrium amount of advertising was socially optimal. This result is of particular interest because there are no *a priori* reasons to expect that the opposing externalities would always "cancel out". In contrast, Grossman and Shapiro (1984) found that in an oligopolistic market for a differentiated product, firms would not provide the optimal level of advertising. Moreover, they showed that with a large number of firms, advertising is excessive, and they conjectured that the result would hold in homogeneous goods markets as well[8]. The next proposition shows that in this model firms behave in a socially desirable way.

Proposition 3 : The equilibrium amount of advertising is socially optimal.

Proof. Total Social Welfare is the value of the good for all buyers that enter the market minus total advertising costs, that is,

$$W = \left[1 - (1 - \Phi)^N\right] p_m - NA(\Phi) \ . \qquad \textbf{(5)}$$

A social planner would choose Φ_W that would maximize Total Welfare. Solving the first order condition for the level of advertising we obtain

$$(1 - \Phi_W)^{N-1} p_m = A_\Phi(\Phi_W) \ , \qquad \textbf{(6)}$$

[8] In the above mentioned models, as well as in the present model, potential welfare distortions may only result from advertising intensity, given that all buyers have a unit demand function with equal reservation prices. The only paper built around the assumption that buyers have heterogeneous reservation values is that of Stegeman (1990). In his model Stegeman shows that firms will always advertise less than the social optimum.

the same condition that defines the optimal advertising level.Δ

This intriguing result shows that the Butters finding not only holds in an oligopolistic environment, but also for a wider class of advertising technologies. Apparently welfare distortions are not a necessary consequence of imperfect information alone as previous results suggest.

Once one establishes that in the simpler model analyzed here the two contradictory effects of advertising on welfare cancel out, we can interpret the other results in terms of deviations from this simpler model. Because of its simple structure and striking welfare consequences, the model developed here provides a useful benchmark for assessing other models of informative advertising. For example, Grossman and Shapiro's result of excessive advertising seems to be the exclusive result of the assumption of differentiated products. In such a market when a buyer receives an ad he is informed of both price and "location". This additional effect that advertising has of matching buyers with brands provides an incentive to supply more information than in the simpler case considered here, and therefore results in excessive advertising. Likewise, the Stegeman (1990) finding that information is always under provided can also be understood. In his model the problem of the non-appropriability of the social surplus is a consequence of his assumption that buyers have heterogeneous reservation values. Under this assumption, firms cannot fully appropriate the social surplus of the uninformed buyers and consequently lack the incentive to provide an optimal amount of price information.

The impact of changes in advertising costs on welfare can be assessed by differentiating (5) with respect to α,

$$W_\alpha = \Phi^*_\alpha N(1-\Phi^*)^{N-1} p_m - NA_\alpha - N\Phi^*_\alpha A_\Phi = -NA_\alpha \ . \qquad \textbf{(7)}$$

As expected, increases in advertising cost lower Total Welfare. Advertising cost affects welfare directly through a change in the cost function and indirectly through Φ^*. However, the

direct effect completely accounts for the reduction in welfare, since the decrease in welfare caused by less advertising is exactly compensated by the savings in advertising costs that result from a decrease in Φ^*.

How does entry affect Total Welfare? Differentiating (5) with respect to N gives

$$W_N = -\log(1-\Phi^*)A_\Phi(1-\Phi^*) - A(\Phi^*) \ . \qquad \textbf{(8)}$$

The sign of this expression is indeterminate unless we specify a particular advertising technology. For the Butters and CRIR technologies (8) equals zero, so Total Welfare is unchanged with the number of firms. This result is at first surprising because one would expect to have Total Welfare increase with entry. But as was shown earlier, for these technologies the market size is unchanged with the number of firms. This means that only advertising costs could affect Total Welfare, but it turns out that, in equilibrium, the total number of ads sent is also independent of N (recall that for the Butters technology, the total number of ads sent by all N firms is NL which equals $\log(p_m/\alpha)$, and is therefore independent of N).

One of Grossman and Shapiro's key findings is that profits may increase with advertising costs. As we will see, this result also prevails in this paper's homogeneous product model. Expected profits are given by

$$E\Pi = p_m\Phi^*(1-\Phi^*)^{N-1} - A(\Phi^*) \ . \qquad \textbf{(9)}$$

Differentiating with respect to α, we get

$$E\Pi_\alpha = A_{\Phi\alpha}\Phi^* + A_{\Phi\Phi}\Phi^*_\alpha\Phi^* - A_\alpha \ . \qquad \textbf{(10)}$$

This derivative can be either positive or negative. For the two advertising technologies considered in this paper, profits will (for high values of Φ^*) increase with α. Consider for the

moment the quadratic advertising technology. For this technology, profits will increase with α as long as $\Phi^* > 1/N$. The same general pattern prevails for the Butters technology, but there is no explicit solution for the turning point value of Φ^*[9].

Additional advertising when firms have low levels of advertising is more likely to increase market size than to shuffle customers among firms. The probability of reaching an uninformed buyer is high so firms will set high prices and not attempt to steal others' customers. However, as advertising increases, there will be more buyers aware of several prices. Firms select lower prices because the danger of losing buyers to other firms is greater and the number of uninformed buyers is smaller. An increase in α induces less advertising, relieves competitive pressures and allows firms to increase prices by enough to offset the direct effect on costs.

Finally, we can look at the impact that the number of firms has on profits. As the number of firms increases profits per firm decrease and tend towards zero. This is an expected result which ensures the existence of a free-entry equilibrium to this game.

VII. Conclusion

This paper presents a contribution to the ongoing debate concerning the welfare properties of informative advertising. By exploring a simple oligopolistic model where identical firms simultaneously choose prices and advertising intensities, we find the existence of a symmetric equilibrium which involves some price dispersion. The advertising technology herein employed is quite general, and includes that of Butters (1977) as a particular case. Nevertheless, we find that Butters' result that firms provide the socially optimal amount of price information holds for a

[9] Although no general proof is given, all numerical simulations carried out by the author for this technology produced the same pattern.

larger set of advertising technologies and in an oligopolistic environment.

For some advertising technologies this model has other interesting properties. One of them, as yet unobserved in other models of this type, is the possibility that entry may lead to a market contraction. The other property, also present in the Grossman and Shapiro model, permits increases in advertising costs to lead to higher expected profits.

The insights provided here may also help settle some of the controversy surrounding the welfare properties of informative advertising. By interpreting other models as deviations from the simple benchmark model provided here one can also better understand the conflicting results. This model is suitable for this task because of the simplicity of its assumptions, its use of a general advertising technology, and its finding of a socially optimal level of advertising.

APPENDIX

Proof of Proposition 1:

Suppose that (0,p') $\in S_i$ is a symmetric Nash equilibrium. The expected profit of any firm would be zero since it does not realize any sales. Let our deviant firm use the following strategy

$$\begin{cases} (\Phi_i^o, p_m) & \text{if} \quad p_m \le A_\Phi(1) \\ (1, p_m) & \text{if} \quad p_m > A_\Phi(1) \ , \end{cases} \qquad \textbf{(A.1)}$$

where Φ_i^o is given by $p_m=A_\Phi(\Phi_i^o)$. This strategy would always yield a positive profit of

$$\begin{cases} p_m \Phi_i^o - A(\Phi_i^o) & \text{if} \quad p_m \le A_\Phi(1) \\ p_m - A(1) & \text{if} \quad p_m > A_\Phi(1) \ . \end{cases} \qquad \textbf{(A.2)}$$

This means that a symmetric Nash equilibrium in pure strategies, if it exists, entails some positive level of advertising. Advertising a price of zero is not a symmetric Nash equilibrium because in this situation firms would earn negative profits (less than they would by not advertising).

Consider now the possibility of a symmetric Nash equilibrium where both advertising intensity and price are positive. Let such proposed symmetric Nash equilibrium be (Φ^*,p^*). In this situation their expected profits would be

$$E\Pi = p' \frac{[1-(1-\Phi^*)^N]}{N} - A(\Phi^*) \ . \qquad \textbf{(A.3)}$$

By undercutting price by an infinitesimal amount ϵ, a deviant would get,

$$E\Pi = (p' - \epsilon)\Phi^* - A(\Phi^*) \ , \qquad \textbf{(A.4)}$$

an amount clearly superior to that given in (A.3), because the discrete jump in sales outweighs the marginal decrease in price.Δ

Proof of Proposition 2:

$F^*(p_o)=0$, $F^*(p_m)=1$ and $F^*(p)$ is continuous and monotone nondecreasing hence it is a proper cumulative distribution function.

Given other players' optimal proposed strategies, the expected profit of a deviant firm i, (Φ_i,p_i) is

$$E(\Pi_i / \Phi_i, p_i) = \Phi_i p_i [1 - \Phi^* F^*(p_i)]^{N-1} - A(\Phi_i) \ . \qquad \textbf{(A.5)}$$

If $p_o \le p_i \le p_m$ the expected profit of firm i would be

$$E(\Pi_i / \Phi_i, p_i) = p_m \Phi_i (1-\Phi^*)^{N-1} - A(\Phi_i) \ , \qquad \textbf{(A.6)}$$

which, when maximized with respect to Φ_i, would show $\Phi_i=\Phi^*$ and yields the same expected profit as for all other firms.

Advertising a price above p_m would be unprofitable and a price below p_o would, for all values of Φ_i, yield a lower expected profit than the optimal strategy.Δ

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