1 Introduction

A system is a combination of compatible products which generate value only when consumed together. This paper focuses on the role played by increasing returns to scale in determining the structure of equilibria and the incentives for compatibility in markets for such systems.

It is well known that the complementarity between the component parts of a system can give rise to a form of network effect, even though no *direct* network externality is involved[1]. The network externality is direct when the utility derived from a product by its user depends directly on the number of consumers who have adopted the same product or a compatible one. Communication devices — the telephone or fax machines, for instance — constitute the most often cited illustration of such products: their utility is clearly greatly reduced if no other consumer adopts the same product or some compatible one. In the case of systems, the so-called externality is only *indirect*, or pecuniary, in nature: because of the presence of increasing returns to scale in the production of a component of the system, a greater variety of this component will be available at a lower price, the greater the number of consumers who adopt the same system or a compatible one. The crucial characteristic in the case of systems is that the utility of the basic component of the system is greatly reduced if not combined with some complementary product or service *the production of which reflects increasing returns to scale.*

Examples of systems abound. However, a great number of them would not generally be thought of as representing situations where the number of consumers who adopt it is of crucial significance to the characterization of the market equilibria. This is because, although the necessary complementarity is present, increasing returns to scale in the production of the complementary product may be either totally absent or not very important. Thus pens and their compatible ink cartridges, nuts and bolts, coffee machines and their filters or vacuum

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[1] Amongst the many papers on the subject, see for instance Matutes and Regibeau (1988), Economides (1989, 1991), Economides and Salop (1992), Chou and Shy (1990a, 1990b) and Church and Gandal (1992a, 1992b, 1993). Katz and Shapiro (1994) provide an excellent discussion of the many issues raised by system competition. The interest in network externalities in general has been sparked by the seminal papers of Katz and Shapiro (1985 and 1986) and Farrell and Saloner (1985 and 1986)

cleaners and their vacuum cleaner bags can all be considered as systems for which the effects of increasing returns to scale are negligible. For that reason, they would not naturally spring to mind as examples of markets where the usual issues raised by network externalities are relevant. But things are different for systems such as video cassette players and their video cassettes, computers and their softwares or automobiles and their accompanying service network, which are often cited as examples of network externalities even though there is little or no direct externality present. Thus when studying system competition, it is important to consider the combined presence of complementarity *and* increasing returns to scale in the production of the complementary product or service.

Our purpose is to analyze explicitly the consequences of these increasing returns to scale for the structure of equilibria and for the private and social incentives to render the systems compatible. Increasing returns to scale are assumed to occur in the form of fixed costs in the production of one of the components of each of two systems. These fixed costs have their impact through what we term the "system effect", which captures the positive effect that an increase in the number of consumers who adopt a particular system has on the equilibrium level of utility that each of those consumers derives from adopting it. The size of this system effect is directly related to that of the fixed costs in the production of the complementary product.

We show that the combined presence of fixed costs in the production of the complementary good and of desirability of variety on the part of the consumer can result in an entry coordination problem on the production side of the complementary goods industry[2]. As a result, there always exist market equilibria where no system is produced, whether compatible or not, as well as equilibria characterized by a monopoly of one of the systems under system incompatibility. Symmetric duopolistic equilibria exist if the system effect is not too large. We show that there are two types of such equilibria: some with large fixed costs relative to market size and some with small fixed costs. Each of these types of equilibrium has different

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[2] True network externalities may also give rise to coordination problems, but on the consumers' side. See Farrell and Saloner (1985 and 1986).

comparative statics implications. We also highlight the consequence of the same fixed costs for the private and social incentives to render the systems compatible. In particular, we show that when comparing symmetric equilibria, consumers may easily lose from compatibility — although it increases variety, it also reduces competition between systems — and the private incentive for system compatibility may exceed the social incentive[3].

In section 2 we present the details of the model. Section 3 is devoted to presenting the system effect and to studying the coordination problem of the complementary goods producers. In section 4, we characterize the duopolistic equilibria under both compatibility and incompatibility. We analyze the incentives for compatibility in section 5 and conclude in section 6.

2 The model

We define a system as the combination of two products that provide utility to the consumer only when used jointly. Specifically, we assume that a system is composed of one unit of a basic component, which comes in only one kind, and of a set of differentiated complementary components. An example of such a system is the combination of a VHS (respectively Betamax) VCR, a basic component, and a set of prerecorded VHS (resp. Betamax) videocassettes, a set of differentiated complementary components.

We assume that there are two competing systems, systems 1 and 2. These systems may be compatible or incompatible. Incompatibility means that a complementary component of one system can only be used with the basic component of that system. Compatibility, on the other hand, means that complementary components are not differentiated by system. Every complementary component can be used with any of the two basic components. Under compatibility, there exists only one set of complementary components while under incompatibility there are two such sets (if both systems are available). Each basic component is produced by one firm. We call firm i the firm that produces the basic component of system i.

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[3] With true network externalities, consumers usually gain from compatibility. See Katz and Shapiro (1985).

The two basic components are horizontally differentiated. They are located at the extreme points of the $[-1, 1]$ linear segment, system 1 being located at $t = -1$ and system 2 at $t = 1$. The cost of producing X_i units of the basic component i is $C(X_i)$ if systems are incompatible and $\tilde{C}(X_i)$ if systems are compatible. We assume that the functions C and $\tilde{C}$, which are common to both firms, are increasing and convex. We also assume that $\tilde{C}'(X_i) \geq C'(X_i)$ for all $X_i \geq 0$: the marginal cost of producing a compatible basic component is not lower than the marginal cost of producing an incompatible basic component.

Each set of the complementary components is produced in a monopolistically competitive market à la Chamberlin: each complementary component is produced by one firm with increasing returns to scale; the producers of the complementary components of one system compete against one another; each individual firm neglects its impact on the aggregates of its industry; and there is free entry in the market. This Chamberlinian competition is modelled here as in Dixit-Stiglitz [1977], except that we assume a continuum of producers of the complementary components. Mathematically, we index each producer of a complementary component of system i by the real number l and assume that a segment $[0, L_i]$ of such goods are produced. Thus the number of producers of complementary components of system i, L_i, is a continuous variable. L_i will of course be endogenously determined in equilibrium by the zero-profit condition entailed by the free-entry assumption. All producers of complementary goods share the same cost structure, which is a constant marginal cost of production, c, and a fixed cost, f. The number of producers is also the number of kinds of complementary goods available. Thus L_i is a measure of the available variety of complementary goods that can be used with system i.

Consumers are uniformly distributed along the $[-1, 1]$ segment. Their constant density is equal to a. The closer they are to basic component i, the higher the utility they derive from using system i, everything else being equal. Each consumer has the same revenue, R, which he must allocate between the purchase of the various system components and that of a competitively produced homogeneous good, chosen as the numéraire. If a consumer adopts one system, he will purchase its basic component and a bundle of its complementary components, and spend the rest of his income on the numéraire. Otherwise, he only purchases the numéraire. We assume that preferences are such that a consumer never adopts both systems. Mathematically, let t be the consumer's position on the $[-1,1]$ segment; let $y_i(l)$ be the consumption of complementary good l of system i; and let m be the consumption of the numéraire. The utility function of consumer t is

$$U = \max\{U_1, U_2\} + m \tag{1}$$

where

$$U_i = \left[\int_0^{L_i} u(y_i(l))dl\right]^{\delta} - d_i(t). \tag{2}$$

We make the following assumptions regarding the utility function: $u(y)$ is thrice continuously differentiable, $u'(y) > 0$, $u''(y) < 0$, for all $y \geq 0$; $u(0) = 0$ and the derivatives of $u(y)$ have well-defined limits as y goes to zero; $\lim_{y\to+\infty} u'(y) = 0$; $d_1(t)$ and $d_2(t)$ are twice continuously differentiable; $0 < \delta < 1$; $d_1(t) \geq 0$, $d_1'(t) \geq 0$, $d_2(t) \geq 0$, $d_2'(t) \leq 0$, $d_1(t) = d_2(-t)$, for all $t \in [-1,1]$. For ease of exposition, we will simply assume $d_1(t) = k(1+t)$ and $d_2(t) = k(1-t)$, where k is some positive constant.

The functions $d_1(t)$ and $d_2(t)$ represent the disutility consumer t bears from consuming system 1 or system 2 instead of his ideal system. Obviously, the closer t is to the location of system 1 (resp. system 2), the smaller the disutility, $d_1(t)$ (resp. $d_2(t)$). The assumption that the functions d_1 and d_2 are symmetric around $t = 0$ guarantees that the two systems have identical intrinsic appeal across the whole population. The strict concavity of $u(y)$ means that the consumer will always prefer to allocate a given expenditure on a combination of many different kinds of the complementary product than to allocate the same expenditure on only one kind. In other words, the different complementary goods available for use with a system are not perfect substitutes in the consumers mind. This assumption on the preferences of the consumer embodies the desirability of variety[4]. The fact that $u'(y)$ tends to zero as y becomes infinite indicates that, in the limit, consumers reach satiety in their consumption of one kind of complementary good. The assumption that δ is less than 1 means that U_i is

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[4] This is the same direct approach to modeling the desirablity of variety as in Dixit and Stiglitz (1977).

concave in the total utility derived from consuming the complementary goods of system i. This assumption will ensure that a consumer's indirect utility is concave in the variety of complementary goods offered, L_i.

The producers of the basic components act as leaders of the industry. They move first and simultaneously set their prices, P_1 and P_2, so as to maximize their profits. The producers of the complementary goods of both systems act next. They first make their simultaneous entry decisions and then they simultaneously set their prices, $p_i(l)$ for $0 \leq l \leq L_i$ and $i = 1, 2$. They also aim at maximizing their profits. Finally, the consumers make their consumption decisions based on the prices they face and the availability of complementary goods in order to maximize their utility.

3 The system effect

The effect of an increase in the number of consumers who choose a particular system on that system's level of attractiveness is of particular interest for the characterisation of equilibria in the basic goods market. We will call this effect the system effect. Before turning to the analysis of equilibria in the basic goods market, we will in this section give a precise meaning in terms of the consumers' utility to what we mean by the level of attractiveness of a system and by the system effect.

To do this we need to characterise the subgame perfect equilibria of the model. Since we must solve the game backwards, we start with the consumers' optimal decisions. We then consider the pricing decision of the complementary good producers, followed by the entry decisions in the complementary good industry. The decisions of the basic good producers will be discussed in the next section.

3.1 The consumers' optimal decisions

Consumer t maximizes his utility, U, given by equations (1) and (2), subject to his budget constraint[5]. The decision he has to make is first to choose a system and then to allocate his

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[5] We identify a consumer with his location: "Consumer t" is shorthand for "a consumer located at t."

consumption between that system's components and the numéraire. If consumer t adopts system i, his consumption decision is given by the solution to the following problem

$$V_i(t) = \max\left\{\left[\int_0^{L_i} u(y_i(l))dl\right]^{\delta} - d_i(t) + m\right\} \tag{3}$$

subject to

$$P_i + \int_0^{L_i} y_i(l)p_i(l)dl + m = R \tag{4}$$

The first-order condition of that problem is

$$\delta u'(y_i(l))\left(\int_0^{L_i} u(y_i(l))dl\right)^{\delta-1} - p_i(l) = 0 \tag{5}$$

The second-order condition is automatically satisfied since $u''(y)$ is strictly negative for all $y \geq 0$.

Equations (4) and (5) together give the consumption choice of consumer t provided he adopts system i. Since t does not appear in these two equations, all consumers who adopt system i have the same consumption pattern. Substituting this consumption choice into equation (2) gives consumer t's utility of adopting system i, $V_i(t)$, as a function of prices, revenue and the number of complementary goods of sytem i, L_i. Consumer t's optimal choice of system (system 1, system 2, or neither one) is then found by comparing $V_1(t)$, $V_2(t)$, and R, the utility obtained when he adopts neither system 1 nor system 2. Consumer t's indirect utility, $V(t)$, is equal to the maximum of these three numbers.

3.2 The equilibrium price of the complementary goods

Consider next the pricing decisions of the complementary goods producers once entry decisions have been made. Given the symmetry of the consumers' demand for complementary goods and the fact that all producers of complementary goods have the same cost structure, the equilibrium price, $p_i(l)$, and individual consumption of the complementary goods of system i, $y_i(l)$, will be the same for all such goods. We, therefore, drop the index l from now on.

From the point of view of a producer of a complementary good of system i, the first-order condition of the consumer's problem—equation (5)—represents that consumer's inverse demand function for her product. The price elasticity of this demand, as perceived by the individual producer[6], is equal to $-u'(y_i)/(u''(y_i)\, y_i)$. Since y_i is independent of the consumer's type, t, this elasticity is also that of the total market demand for this complementary good. Then, the profit-maximizing price of its producer must satisfy

$$p_i = \frac{c\, u'(y_i)}{u'(y_i) + y_i u''(y_i)}. \tag{6}$$

The second-order condition for profit maximization is $2u''(y_i)+y_i u'''(y_i) < 0$. We will assume

$$u'(y_i) + y_i u''(y_i) > 0 \quad \text{and} \quad 2u''(y_i) + y_i u'''(y_i) < 0 \quad \text{for all} \quad y_i \geq 0. \tag{7}$$

Therefore, marginal revenue is positive and well defined, as is the price given in equation (6), and this price is indeed the profit-maximizing one for all $y_i \geq 0$.

Before turning to the entry decisions, notice that equations (5) and (6) yield an implicit expression for y_i, the individual consumption of a complementary good of system i, as a function L_i, the variety of such goods, namely

$$\delta(u'(y_i) + u''(y_i)y_i)(u(y_i))^{\delta-1} = L_i^{1-\delta} c\,. \tag{8}$$

By totally differentiating equation (8), it is easy to show that dy_i/dL_i is strictly negative. This means that as a greater variety of complementary goods becomes available, the individual consumption of each kind decreases. We will assume that dp_i/dy_i is nonnegative, or in other words that the elasticity of demand perceived by each complementary goods producer, given by $-u'(y_i)/(u''(y_i)\, y_i)$, does not increase with y_i. Hence the price of each complementary product decreases as the number of complementary goods producers increases.

Equations (3), (4), (6), and (8) together give consumer t's utility of adopting system i, $V_i(t)$, as a function of y_i, which we write

$$V_i(t) = R - P_i - d_i(t) + v(y_i)\ , \tag{9}$$

where

$$v(y_i) = \left(\frac{\delta}{c}\right)^{\frac{\delta}{1-\delta}} \left(1 - \frac{\delta y_i u'(y_i)}{u(y_i)}\right) (u'(y_i) + y_i u''(y_i))^{\frac{\delta}{1-\delta}}\ . \tag{10}$$

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[6] Recall that the monopolistic competition hypothesis means that each individual producer ignores any impact that her decisions have on the aggregates of the complementary goods industry.

Combining equation (9) with the implicit expression of y_i found in equation (8) gives $V_i(t)$ as a function of the variety of complementary goods of system i, L_i, of the consumer's type, t, and the price of system i's basic component, P_i. It is easy to verify that the above assumption that dp_i/dy_i is nonnegative is sufficient for $v'(y)$ to be strictly negative[7].

3.3 Entry in the complementary goods industry

We now are in a position to analyze the determination of the number of producers entering the complementary goods market. We must first note that the combination of a consumer demand for systems of basic and complementary goods characterized by a taste for variety and increasing returns to scale in the production of complementary goods creates a *coordination problem* for complementary good producers. By coordination problem, we mean that it is never profitable for a producer of a complementary good of system i to enter the market unless she knows she will be doing so in the company of other producers of complementary goods of system i. It is easy to understand why. A consumer adopts a system only if the utility he derives from the purchase of its components net of the disutility $d_i(t)$ of not consuming his ideal system exceeds the sum of the component prices. Thus $v(y_i(L_i))$ must exceed $P_i + d_i(t)$, where $y_i(L_i)$ is a shorthand representation of the dependence of y_i on L_i given in equation (8). If this condition is not satisfied, the consumer prefers to purchase only the numéraire. For all consumers, $P_i + d_i(t)$ is at least as large as the basic good's minimum marginal cost of production, $C'(0)$. On the other hand, $v(y_i(L_i))$ goes to zero as L_i goes to zero (see Lemma A-1). Therefore, if the variety of complementary goods of system i, L_i, falls below $\underline{L}$, the strictly positive number defined by $v(y(\underline{L})) = C'(0)$, not a single consumer adopts system i; and the profit of each complementary good producer is strictly negative (it is equal to minus the fixed cost, f). Consequently, whatever pair of prices, P_1 and P_2, the basic good producers announce, the simultaneous decision by all producers of one set of complementary goods (which is to say, the producers of the complementary goods specific to

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[7] The assumption thus rules out the less interesting counterintuitive case where the increase in utility which comes from being better able to diversify consumption as the variety of the complementary good increases is cancelled by an increase in the price of the complementary goods.

one system under system incompatibility, or all the producers of complementary goods under system compatibility) of not entering the market form a Nash equilibrium of the subgame played by these producers. If each of these producers is convinced that no other producer (or less than $\underline{L}$ other producers) will enter the market, it is optimal for her not to enter the market either since she could not recoup her fixed cost of production. This is so even though it might be profitable for a certain number of them to enter the market *together*. The lack of coordination among complementary good producers may result in no supply of the complementary goods of one (or both) system(s).

Suppose now that this coordination problem is overcome and that some producers decide to enter the complementary goods market. In doing so, each complementary good producer of system i neglects any effect that her decision has on L_i. Therefore, in view of equations (8) and (9), she sees her decision as affecting neither y_i nor $V_i(t)$. This implies that each complementary good producer takes as given X_1 and X_2 , the number of consumers who adopt system 1 and system 2.

Under system incompatibility, total demand for each complementary good of system i is equal to $X_i y_i$, its price is given by equation (6), and the profit of its producer is

$$\pi_i = X_i \, c \, \frac{-y_i^2 \, u''(y_i)}{u'(y_i) + y_i \, u''(y_i)} - f \, , \tag{11}$$

where

$$L_i^{1-\delta} c = \delta (u'(y_i) + u''(y_i) y_i)(u(y_i))^{\delta - 1} \, . \tag{12}$$

Under system compatibility, there is only one set of complementary goods: each complementary good is purchased by the consumers who adopt system 1 and system 2. Therefore, total demand for a complementary good is $(X_1 + X_2)y$, and the profit of its producer is

$$\pi = (X_1 + X_2) \, c \, \frac{-y^2 \, u''(y)}{u'(y) + y u''(y)} - f \, , \tag{13}$$

where

$$L^{1-\delta} c = \delta (u'(y) + u''(y) y)(u(y))^{\delta - 1} \, . \tag{14}$$

With free entry of complementary good producers, their equilibrium number is such that their profits are zero. Under incompatibility, the equilibrium number of complementary good producers, L_i^*, is therefore given by $\pi_i = 0$ and the equation giving L_i as a function of y_i. Under system compatibility, the equilibrium number of complementary good producers, L^*, is given by $\pi = 0$ and the equation giving L as a function of y.

3.4 The level of attractiveness of a system and the system effect

Having characterized subgame perfect equilibrium values of y_i, p_i and L_i, we may now provide a precise definition of the level of attractiveness of a system and of the system effect.

Let $w(y) = -(u'(y) + yu''(y))/(y^2u''(y))$. The derivative of w, $w'(y)$, which is given by $((2u''(y) + yu'''(y))u'(y))/((u''(y))^2y^3)$, is everywhere strictly negative. Consequently, the function w is bijective and invertible. Let the function g be the inverse of w. That is to say,

$$g(z) = y \Longleftrightarrow z = -\frac{u'(y) + yu''(y)}{y^2\, u''(y)}\ . \tag{15}$$

The zero-profit conditions under incompatibility and compatibility can then be rewritten, using the function g, as

$$y_i^* = g\left(\frac{X_i c}{f}\right)\ ; \tag{16}$$

$$y^* = g\left(\frac{(X_1 + X_2)c}{f}\right)\ . \tag{17}$$

These two equations together with the equations (12) and (14) determine L_i^* and L^*.[8]

The function g being the inverse of a strictly decreasing function is itself a strictly decreasing function. Therefore, the differentiation of equation (16) (resp. equation (17)) with respect to X_i (resp. $X_1 + X_2$) shows that when the number of consumers of one system increases the individual consumption of each complementary good compatible with that system, y_i or y, decreases. Since dy_i/dL_i (resp. dy/dL) is strictly negative, dL_i/dX_i (resp. $dL/d(X_1 + X_2)$) is strictly positive: As the number of consumers of one system increases, a greater variety of complementary goods of this system becomes available.

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[8] These Nash equilibria of the complementary good producers' entry game can be either locally stable or unstable. For the remainder of this paper, we restrict our attention to the locally stable ones. That is to say, under system compatibility, we only consider the Nash equilibria such that $d\pi/dL < 0$ at $L = L^*$; and under system incompatibility, we only consider equilibria such that the 2x2 matrix whose (i, j) element is $d\pi_i/dL_j$ is negative definite at (L_1^*, L_2^*). We will see in the following section that this restriction insures that the demand for a system varies negatively with its price.

Let us now define the function s as the composite of the functions g and v, that is

$$s(z) = v(g(z)) \text{ for all } z > 0.$$

Combining equations (9) and (16) gives consumer t's utility of adopting system i, V_i, when the systems are incompatible, that is

$$V_i(t) = R - P_i - d_i(t) + s\left(\frac{X_i c}{f}\right) \ . \tag{18}$$

When systems are compatible, the utility of adopting system i is obtained by combining equations (9) and (17) to give

$$V_i(t) = R - P_i - d_i(t) + s\left(\frac{(X_1 + X_2)c}{f}\right) \ . \tag{19}$$

The function $s(z)$ may be referred to as the *level of attractiveness* of the system. As is clear from equation (19), when they are compatible the two systems are equally attractive to the consumer.

The function $g(z)$ and $v(y)$ being both strictly decreasing, the function $s(z)$ is strictly increasing in z. Therefore, equations (18) and (19) show that, under both compatibility and incompatibility, consumer t's utility of adopting one system increases with the number of consumers who adopt that system or a system compatible with it. This dependence of $V_i(t)$ on total consumption of system i and systems compatible with system i is what we call the *system effect*. We measure it by the derivative of $V_i(t)$ with respect to X_i, which is to say $\frac{c}{f}s'(\frac{X_i c}{f})$ under incompatibility and $\frac{c}{f}s'(\frac{(X_1+X_2)c}{f})$ under system compatibility. Thus the system effect corresponds to the effect of a marginal increase in the number of consumers who adopt a particular system on its level of attractiveness.

The system effect plays in the present setting a role similar to the one played by the network effect in the litterature on networks (e.g. Katz and Shapiro [1985], Farrell and Saloner [1985]). The difference between these two effects is in their causes and magnitudes. The network effect is caused by a direct externality in the utility function of consumers, while the system effect is due to a pecuniary externality whose origin is the presence of increasing returns to scale in the production of a good that cannot be consumed alone but only as one part of a system. The magnitude of the network effect depends upon the importance of the direct externality in the consumers' utility function, while *the size of the system effect depends upon both cost and demand parameters*, such as the fixed and marginal costs of production of the complementary good, f and c, and the function u.

4 The nature of duopolistic equilibria

Having analyzed the behavior of consumers and complementary good producers in section 3, we now turn our attention to the actions of the basic good producers. We will restrict our attention to duopolistic equilibria, that is to say equilibria where both basic good producers have strictly positive sales[9]. We will also restrict attention to duopolistic equilibria in which the whole market is covered[10].

4.1 Equilibria under system compatibility

We start with the case of system compatibility. Thus one set of complementary goods is produced and every complementary good can be combined with any of the two basic goods. We saw in section 3 that consumer t's indirect utility of adopting system i, $V_i(t)$, is then given by equation (19). Since $V_1(t)$ decreases with t and $V_2(t)$ increases with t, the consumers to the left of $\tilde{t}$ adopt system 1; the consumers to the right of $\tilde{t}$ adopt system 2; and $\tilde{t}$, the location of the consumers who are indifferent between the two systems is given by the equation $V_1(\tilde{t}) = V_2(\tilde{t})$. Using the definition of V_i, X_1, and X_2, this yields

$$P_1 + \frac{k}{a}X_1 = P_2 + \frac{k}{a}X_2 \ . \tag{20}$$

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[9] For an explicit discussion of monopolistic equilibria, see Desruelle, Gaudet and Richelle (1992). As our above discussion of the coordination problem makes clear, there in fact always exist monopolistic equilibria, where only one of the system is supplied.

[10] Duopolistic equilibria where some of the consumers adopt neither system can be derived exactly in the same manner as monopolistic equilibria. This is true even though when the two systems are compatible the markets are only *quasi-monopolistic*, since the inverse demand faced by each basic goods producer is positively related to the sales of the other producer. Of course if the systems are incompatible, the two markets are then completely independent and the equilibria are replica of truly monopolistic equilibria.

Since $X_1 + X_2 = 2a$, we get, upon substitution, that each firm's inverse demand function is given by:

$$P_i = P_j + \frac{2k}{a}[a - X_i], \quad i,j = 1,2, \quad i \neq j.$$

As is immediately obvious from equation (20), the system effect does not appear here. The reason is that since the two systems are compatible, they have the same level of attractiveness. The problem therefore becomes one of standard price competition between symmetric duopolists on a differentiated products market demand, where each consumer would value good i at $v^* - d_i(t)$, v^* being a high enough strictly positive constant, and each would buy at most one unit of both goods. It is a well known result that the equilibrium will be symmetric and unique. At this equilibrium, the sales of each system will be $X_1 = X_2 = a$ and $P_1 = P_2 = \tilde{C}'(a) + 2k$.

There remains to verify that at this price all consumer wish to purchase one of the systems. The worse-off consumers are the ones furthest away from both systems. Therefore, we simply have to check that $V_i(0) \geq R$. This is so if $s(\frac{2ac}{f}) \geq \tilde{C}'(a) + 3k$. These results are summarized in the following proposition:

Proposition 1 (Symmetric equilibrium under compatibility) *Assume that* $s(\frac{2ac}{f}) \geq \tilde{C}'(a) + 3k$. *Then, under system compatibility, there exists a unique equilibrium with strictly positive sales of systems, such that the whole market is covered. This equilibrium is a symmetric duopoly, in which both basic good producers set their price equal to* $\tilde{C}'(a) + 2k$ *and sell* $X_1 = X_2 = a$ *units.*

4.2 Equilibria under system incompatibility

We now move to the case of system incompatibility. In that case, two sets of complementary goods are produced, each complementary good is specific to one system and it can only be used with the basic good of that system.

When all consumers adopt a system, the location of the consumers who are indifferent between the two systems, $\tilde{t}$, is determined by the equation $V_1(\tilde{t}) = V_2(\tilde{t})$. The sales of firm 1 and firm 2 are then given by $X_1 = a(1+\tilde{t}\,)$ and by $X_2 = a(1-\tilde{t}\,)$. Since the two systems are incompatible, $V_i(t)$ is given by equation (18). The equation therefore yields

$$P_1 + \frac{k}{a}X_1 - s\left(\frac{X_1 c}{f}\right) = P_2 + \frac{k}{a}X_2 - s\left(\frac{X_2 c}{f}\right) \,. \tag{21}$$

This equation differs from the one we obtained when systems are compatible (equation (20)) in one very significant way: the term $s(\frac{X_i c}{f})$, which captures the level of attractiveness of system i, appears in equation (21) while it did not in equation (20). Since the complementary goods are system-specific, the variety of such goods available to consumers of system i depends on the sales of system i and not only, as under compatibility, on the sum of the sales of both systems. Hence the levels of attractiveness of the two systems differ, which is why they appear in equation (21).

Using the fact that $X_1 + X_2 = 2a$, we may again write

$$P_i = P_j + 2k - \frac{2k}{a}X_1 + s\left(\frac{X_1 c}{f}\right) - s\left(\frac{(2a - X_1)c}{f}\right), \quad i,j = 1,2, \quad i \neq j.$$

This is the inverse demand for the basic good i, with the restriction however that it is only defined on a *subinterval* of $[-1,1\,]$. Indeed, because of the coordination problem discussed in the previous section, a system cannot have an arbitrarily small market share whatever the basic good prices. This implies that if one basic good price is sufficiently higher than the other, the system with the low basic good price has a market monopoly[11]. In the range over which it is defined, this inverse demand function is smooth and, as is shown in the Appendix, it is negatively sloped if the equilibria of the entry game played by the complementary goods producers is locally stable (See Lemma A-3 in the Appendix). We will henceforth assume such local stability.

Given this inverse demand function, the first-order conditions for profit maximization by the basic good producers can be written, using the fact that $X_1 + X_2 = 2a$,

$$P_1 = C'(X_1) + \frac{2k}{a}X_1 - \frac{X_1 c}{f}\left[s'\left(\frac{X_1 c}{f}\right) + s'\left(\frac{[2a - X_1]c}{f}\right)\right] \,, \tag{22}$$

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[11] The system with the high basic good price cannot be the one with a market monopoly. If it were, it would have a market monopoly for all pairs of basic good prices given our maintained assumption that market share varies negatively with price. We would then not be studying duopolistic equilibria.

$$P_2 = C'(X_2) - \frac{2k}{a}X_2 - \frac{X_2c}{f}\left[s'\left(\frac{X_2c}{f}\right) + s'\left(\frac{[2a - X_2]c}{f}\right)\right] . \tag{23}$$

Contrary to what was the case under system compatibility, the system effect now appears in the first-order conditions for profit-maximization. In fact it appears twice in each condition, reflecting cross effects between producers. When firm 1 increases its price, it drives consumers away to system 2. This results in more variety and more utility for system 2 consumers, which is captured by the term $\frac{c}{f}s'(\frac{X_2c}{f})$. It also results in less variety and less utility for system 1 consumers, which is captured by the term $\frac{c}{f}s'(\frac{X_1c}{f})$. This reduces firm 1's marginal revenue from raising its price and induces producers to select a lower price than they would in the absence of the system effect. Thus, everything else equal, the presence of the system effect results in a lower price.

It is easy to see that setting both P_1 and P_2 equal to $P^* = C'(a) + 2k - 2\frac{ac}{f}s'(\frac{ac}{f})$, which gives firms 1 and 2 equal sales of a units, satisfies equations (22) and (23) and that this pair of strategies is the only candidate for a symmetric equilibrium. A necessary condition for this price to be positive is $k > zs'(z)$ at $z = ac/f$, which happens to be the condition for the equilibrium of the complementary goods producers' entry game to be locally stable (see Lemma A-3). All consumers will be willing to buy a system at this price provided $V_i(0) \geq R$, which implies $s(z) + 2zs'(z) \geq 3k + C'(a)$ at $z = ac/f$.[12]

To ensure that it actually constitutes an equilibrium strategy profile, one needs to check that the profit of each basic good firm is globally maximized at P^*. This, however, is not always assured, because the system effect may make the profit functions non concave. It is nevertheless possible to find conditions under which setting both P_1 and P_2 equal to P^* does constitute an equilibrium.

Consider first the possibility of existence of such an equilibrium with small fixed costs[13]. When the two systems are compatible, proposition 1 establishes conditions under which the symmetric equilibrium exists, in which case it is unique. It also follows from Lemma A-2

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[12] Notice that since $k > zs'(z)$ at $z = ac/f$, a necessary condition for this condition to be satisfied is $s(z) > zs'(z)$ and hence $s''(z) < 0$ at $z = ac/f$. This means that the level of attractiveness, $s(z)$, must be strictly concave at $z = ac/f$.

[13] That is fixed costs which are small relative to the market size, the important parameter being a/f.

(see the Appendix) that, if $u'(0)$ is finite, the system effect becomes small as the fixed cost of producing one kind of complementary good, f, becomes small[14]. Since the only difference between the first-order conditions for profit maximization under system compatibility and those under system incompatibility is the presence of system effects in the latter, this suggests that if preferences can be represented by a utility function such that $u'(0)$ is finite, if f is low enough, and if the assumptions of proposition 1 are satisfied, then there also exists a symmetric equilibrium under incompatibility and that this equilibrium is the only one with strictly positive sales by both basic good producers. As shown in the Appendix, this is in fact the case. This result is recapitulated in the following proposition.

Proposition 2 (Symmetric equilibrium with small fixed costs) *Assume $u'(0)$ is finite, and let $s^* = \lim_{z \to +\infty} s(z)$ and $s^* > \tilde{C}'(a) + 3k$. Then there is a number N such that if $\frac{ac}{f} > N$, a symmetric market equilibrium exists which is characterized by a common basic good price of $P^* = C'(a) + 2k - 2\frac{ac}{f}s'(\frac{ac}{f})$ and sales of $X_1 = X_2 = a$ units by each basic good producer. Furthermore, this equilibrium is the only one with strictly positive sales of both systems.*

The logic of this proposition is clear. When the fixed cost of production of a complementary good is small (or when the population is large), the variety of complementary goods of one system is large whatever the market share of that system. Therefore, the system effect is small: an increase in market share barely affects the system's level of attractiveness. The equilibrium is very close to what it would be if the system's level of attractiveness were a constant or if systems were compatible.

As already mentionned, the existence of a symmetric equilibrium is not always assured. Obviously, (P^*, P^*) will not be an equilibrium strategy if firms' profits are not locally maximized at that price. Substituting for P^* in the set of second-order conditions for profit maximization, one easily verifies that they are simultaneously satisfied if and only if

$$-\frac{4}{a}k + \frac{4c}{f}s'(\frac{ac}{f}) - C''(a) \leq 0 \ .$$

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[14] This is because if $u'(0)$ is finite, then $s(z)$ is bounded from above.

Since the pair of strategies (P^*, P^*) constitutes the only possible candidate for a symmetric equilibrium, we have therefore shown the following proposition:

Proposition 3 (Non-existence of a symmetric equilibrium) *Assume that the two systems are incompatible and that* $\frac{ac}{f}s'(\frac{ac}{f}) > k + \frac{aC''(a)}{4}$*. Then, there is no symmetric market equilibrium.*

Leaving aside the convexity of the cost function C, the condition of proposition 3 will be satisfied when the product of the system effect and of consumer density, $\frac{ac}{f}s'(\frac{ac}{f})$, is larger than the derivative of the disutility function at $t = 0$, $d_1'(0)(= k)$, which represents the change in preferences for one system over the other as we move around the median consumer location. If all consumers basically have nearly the same intrinsic preferences for the two systems, which is to say that the systems' horizontal differentiation is weak (k is small), the consumers' choices are nearly wholly based on the level of attractiveness of each system, and thus on the variety of its complementary components. There is therefore a strong bandwagon effect: everyone buys the system everyone else buys because it is precisely the system with a large variety of complementary components[15]. On the other hand, if the system effect is weak or if the systems are strongly horizontally differentiated, every consumer ends up buying the system he intrinsically prefers (the one closer to him).

As an immediate corollary of proposition 3, we have

Corollary 1 *If preferences are such that* $zs'(z)$ *is monotone increasing in* z*, a symmetric equilibrium under system incompatibility can exist only for large fixed cost* f*.*

This corollary would be of no interest if there did not exist some admissible representations of preferences for which $zs'(z)$ is monotone increasing. This is however not the case, since one simple utility function which satisfies this property is the constant elasticity utility

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[15] Asymmetric equilibria need not be monopolistic. An example is provided in Desruelle, Gaudet and Lasserre (1992) of an asymmetric duopolistic equilibrium. Such asymmetric equilibria will typically be strongly asymmetric, characterized by one system having a high price, a high level of attractiveness and a high market share, while the other system has a low price, a low level of attractiveness and a low market share.

function $u(y) = y^\rho$, $0 < \rho < 1$. This function is widely used in the literature on network externalities because of its tractibility (see for instance Chou and Shy (1990a and 1990b) and Church and Gandal (1992, 1993a and 1993b)) and has been shown to generate a symmetric duopolistic equilibrium for relatively large fixed costs. We therefore state without proving the following proposition.

Proposition 4 (Symmetric equilibrium with large fixed costs) *Assume* $u(y) = y^\rho$, $0 < \rho < 1$. *Then for some* f *large enough that* $k + \frac{aC''(a)}{4} > \frac{ac}{f}s'(\frac{ac}{f})$ *there exists a unique symmetric equilibrium with the whole market being served. This equilibrium is characterized by a common basic good price of* $P^* = C'(a) + 2k - 2\frac{ac}{f}s'(\frac{ac}{f})$ *and sales of* $X_1 = X_2 = a$ *units by each basic good producer.*

The reason why such a symmetric equilibrium can exist is that if the fixed cost is large, an increase in the number of consumers who adopt a system will have very little effect on the number of producers and on the price of the complementary goods available for that system. As a result, the increase in the level of attractiveness of the system entailed by the increase in the number of consumers will be small. A large fixed cost therefore means a small system effect and hence an equilibrium which is very close to what it would be under system compatibility[16].

That $zs'(z)$ be monotone increasing in z is of course not necessary for a symmetric equilibrium to exist under a large fixed cost. Any utility function which is such that $s'(z)$ is strictly positive and goes to zero as z goes to zero (i.e., f goes to infinity) will exhibit some range of f large such that $k + aC''(a)/4 > zs'(z)$ at $z = ac/f$ and is liable to generate a symmetric equilibrium with large fixed cost. This does not exclude a function for which $u'(0)$ is finite and which therefore also satisfies the conditions of proposition 2. For instance,

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[16] Note that the fixed cost cannot be too large. Indeed, from our discussion of the coordination problem (see also Lemma A.1), we know that the value of a system will become negative for any consumer and any price of the basic good exceeding the marginal cost of the basic good producer whenever the number of complementary goods available for the system falls under a critical level. This will obviously be the case for a sufficiently large fixed cost. Hence, for a sufficiently large fixed cost, there will not exist a symmetric duopoly equilibrium and it will even be possible that no system will be produced.

the simple utility function $u(y) = \log(1 + \alpha y)$, $\alpha \in R^+$, is such that $s'(z)$ is strictly positive and $zs'(z)$ tends to zero both as z goes to infinity (i.e., f goes to zero) and as z goes to zero (i.e., f goes to infinity)[17]. For such a utility function, the condition $k + aC''(a)/4 > zs'(z)$ at $z = ac/f$ may therefore be satisfied for both some large and small fixed costs. The constant elasticity function $u(y) = y^\rho$, $0 < \rho < 1$, on the other hand, can generate a symmetric equilibrium only for large fixed costs, since it has the property that $zs'(z)$ is monotone increasing in z and hence the system effect is monotone decreasing in f: the smaller the fixed cost, the larger system effect.

In fact the crucial condition for a symmetric equilibrium to exist under incompatibility of the systems is that the system effect be sufficiently small, so that an increase in the market share of a system has very little effect on its level attractiveness for the consumer. When this occurs for small fixed costs, it is because the variety of the complementary goods of each system is already large whatever the market share of the system and the consumer does not have an eternal craving for variety (hence the role of the assumption that $u'(0)$ be finite)[18]. When this occurs for large fixed cost, it is because the large fixed cost prevents the number and hence the price of the complementary products from changing very much as the number of consumers who adopt a system increases. The two types of equilibrium have very different comparative statics implications. In an equilibrium with large fixed cost, such as discussed in proposition 4, $zs'(z)$ is an increasing function of z at $z = ac/f$. In an equilibrium with small fixed cost, such as discussed in proposition 2, $zs'(z)$ is a decreasing function of z at $z = ac/f$. This implies that at an equilibrium with large (small) fixed cost, P^* is increasing (decreasing) in f and decreasing (increasing) in a, the consumer density.

One consequence of this difference in the comparative statics is that if one assumes the constant elasticity utility function, then, beginning at a (relatively) high fixed cost symmetric equilibrium, as the number of consumers increases (i.e., as a increases), the equilibrium

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[17] It is easy, though tedious, to show that this property holds for any admissible utility function for which $u'(0)$ is finite.

[18] If $u'(0) = +\infty$, the consumer attaches an infinite value to a small amount of a additional variety no matter how large the variety of goods already offered to him. He thus never reaches satiation in variety, as is the case if $u'(0)$ is finite.

price P^* of the basic good will constantly fall and the equilibrium will eventually become asymmetric and remain asymmetric. On the other hand if one assumes $u'(0)$ finite, then the equilibrium price P^* will at first fall but will eventually rise, and it is conceivable that the equilibrium will go from a large fixed cost symmetric equilibrium to, eventually, a small fixed cost symmetric equilibrium.

5 The incentives for system compatibility

In this section we compare the private and social incentives to make the systems compatible. It is clear that this analysis depends upon what equilibrium we start with under incompatibility and what equilibrium we go to under compatibility. To keep being bogged down in innumerable cases, we focus our attention on two polar facets of the compatibility issue. The first is the link between the compatibility decision and the coordination problem discussed in section 3. The second is the comparison between the private and social incentives to make the systems compatible in duopolistic cases.

As shown in section 3, the presence of fixed costs can result in the sales of one system being zero if complementary good producers cannot coordinate their entry decisions or cannot individually convince themselves that many other complementary good producers are about to enter the market. This problem can be particularly severe if a new product is offered in the form of many incompatible systems. Suppose that different electronic firms had come up with incompatible CD systems[19]. Then, it would have been quite hard for record companies and record stores to know what type of compact disk to produce and display. Furthermore, consumers would have been hesitant to purchase a system whose longevity was not assured. This scenario could very well have ended with the demise of CDs: we would then all still be playing LPs! The complementary good producers' coordination problem is easier to solve when systems are compatible. Then, a complementary good producer only needs to decide whether the new product is likely to be appreciated by consumers. If the answer is yes and

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[19] For the historical record, Philips and Sony each developped their own CD system, but, prior to the launching of the product, they agreed to share their technologies and market only the Philips system.

if one thinks that other people in the trade feel the same way, then it is a fairly safe decision to enter the market. In this case, making the systems compatible shifts the market from an equilibrium with zero sales to one with strictly positive sales. Then, it is both privately and socially beneficial to make the systems compatible.

Consider now the incentives for compatibility in a situation where both before and after compatibility, the market equilibrium is symmetric and the market is covered. Using equations (12) and (14), which must hold in this case under incompatibility and compatibility respectively, we define the function $L(z)$ as

$$L(z) = \left(\frac{\delta}{c}\right)^{\frac{1}{1-\delta}} \frac{\left[u'(g(z)) + u''(g(z))\, g(z)\right]^{\frac{1}{1-\delta}}}{u(g(z))} \;,$$

where g is the function defined in equation (15). Under system compatibility, the common basic good price, P^C, the variety of complementary goods, L^C, and the indirect utility of consumer t, $V^C(t)$, are given by

$$\begin{aligned} P^C &= \tilde{C}'(a) + 2k \;; \\ L^C &= L\left(\frac{2ac}{f}\right) \;; \\ V^C(t) &= s\left(\frac{2ac}{f}\right) - \tilde{C}'(a) - 2k - \min\{d_1(t), d_2(t)\} + R \qquad . \end{aligned}$$

Under system incompability, the common basic good price, P^I, the common variety of complementary goods of each system, L^I, and the indirect utility of consumer t, $V^I(t)$, are

$$\begin{aligned} P^I &= C'(a) + 2k - \frac{2ac}{f} s'\left(\frac{ac}{f}\right) \;; \\ L^I &= L\left(\frac{ac}{f}\right) \;; \\ V^I(t) &= s\left(\frac{ac}{f}\right) + \frac{2ac}{f} s'\left(\frac{ac}{f}\right) - C'(a) - 2k - \min\{d_1(t), d_2(t)\} + R \;. \end{aligned}$$

The function $L(z)$ is strictly increasing since it is the combination of two strictly decreasing functions, $g(y)$ and the mapping between L and y (equation (8)). Therefore, L^C is strictly greater than L^I. As one would expect, each consumer can enjoy a wider variety of complementary goods when systems are compatible than when they are not. In this way, consumers benefit from compatibility. On the other hand, compatibility increases the price they pay for the basic good. The first reason for this is that the marginal cost of producing a compatible good may be higher than the marginal cost of producing a non-compatible good. The second and more fundamental reason is that under incompatibility, the system effect induces basic good producers to lower their prices.

Consider now producers' profits, consumers' surplus, and social welfare under compatibility and incompatibility. We measure social welfare, W, by the sum of the producers' profits and the indirect utility of all consumers. Since the profits of the complementary good producers are always zero, and the profits of the two basic good producers are equal,

$$W = 2\Pi + \int_{-1}^{1} V(t)dt \; .$$

When systems are compatible, profits, Π^C, and social welfare, W^C, are given by

$$\begin{aligned} \Pi^C &= 2ak + a\tilde{C}'(a) - \tilde{C}(a) \\ W^C &= 2as\left(\frac{2ac}{f}\right) - 2\tilde{C}(a) + 2aR - \int_0^1 d_1(t)dt. \end{aligned}$$

When systems are incompatible, profits, Π^I, and social welfare, W^I, are given by

$$\begin{aligned} \Pi^I &= 2ak - \frac{2a^2c}{f}s'\left(\frac{ac}{f}\right) + aC'(a) - C(a) \\ W^I &= 2as\left(\frac{ac}{f}\right) - 2C(a) + 2aR - \int_0^1 d_1(t)dt. \end{aligned}$$

The move to compatibility increases the profits of basic good producers if compatibility causes a price increase higher than the average cost increase, that is to say if

$$\frac{2ac}{f}s'\left(\frac{ac}{f}\right) + \tilde{C}'(a) - C'(a) > \frac{\tilde{C}(a)}{a} - \frac{C(a)}{a} \; . \tag{24}$$

Compatibility increases the indirect utility of each consumer if the increase in the level of attractiveness of the system outweighs the price increase, that is to say if

$$s\left(\frac{2ac}{f}\right) - s\left(\frac{ac}{f}\right) > \tilde{C}'(a) - C'(a) + \frac{2ac}{f}s'\left(\frac{ac}{f}\right) \; . \tag{25}$$

Social welfare increases when systems are made compatible if the gain in the level of attractiveness of the system enjoyed by each consumer is larger than the increase in the average cost of production. This is so if

$$s\left(\frac{2ac}{f}\right) - s\left(\frac{ac}{f}\right) > \frac{\tilde{C}(a)}{a} - \frac{C(a)}{a} \,. \tag{26}$$

Notice immediately that inequality (26) can never be satisfied when $f = 0$. Using Lemma A-2, this is also the case for inequality (25). Thus in the absence of fixed costs in the production of the complementary good, consumers always lose from compatibility and compatibility is never socially desirable.

More generally, when there are fixed costs, it is difficult to compare the private and social incentives for compatibility for all utility functions, since the former involves the derivative of $s(z)$ at one point and the latter a finite difference between two values of $s(z)$. There is however an interesting case where it is possible to do so, which is the case where $s(z)$ is a concave function[20]. Indeed, if $s(\cdot)$ is concave, $s\left(\frac{2ac}{f}\right) - s\left(\frac{ac}{f}\right)$ is smaller than $\frac{ac}{f}s'(\frac{ac}{f})$, which is itself strictly smaller than $\frac{2ac}{f}s'(\frac{ac}{f})$. This means that inequality (25) is not satisfied and consumers lose from compatibility[21]. This also means that inequality (24) is easier to satisfy than inequality (26). The private incentive for compatibility is therefore strictly greater than the social incentive for compatibility. The market may therefore deliver too much compatibility because consumers' surplus decreases when systems are made compatible.

In fact, producers and consumers may be at odds with one another as regards compatibility. As an example, suppose that in addition to $s(z)$ being concave, the move to compatibility changes the basic good's marginal cost of production by a fixed amount and does not affect

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[20] The assumption that $s(z)$ is a concave function is analogous to the assumption of Katz and Shapiro (1985, p. 426) that the "externality function" ($v(y^e)$ in their notation) is concave. We know that $s(z)$ must be concave at $z = ac/f$ (see footnote 12). It does not seem unreasonable to think that $s(z)$ would indeed be globally concave. For instance, this is the case if $u(y) = \log(1 + \alpha y)$, $\alpha \in R^+$. It may also be the case for the constant elasticity function $u(y) = y^\rho$, $0 < \rho < 1$, for some combinations of ρ and δ. Notice that if $s(z)$ were convex, then the system effect would necessarily be everywhere strictly decreasing in f.

[21] Chou and Shy (1990b) also provide an example where this occurs, using the constant elasticity utility function. The result that consumers lose from compatibility is contrary to what was found by Katz and Shapiro (1985). In their case however, compatibility means that the good adopted by a consumer can be used with that adopted by other consumers, whereas here it means that all complementary products can be used with either of the basic components.

the fixed cost of production. Then inequality (24) is always satisfied. The basic good producers always wish to make their products compatible, even though consumers always suffer from compatibility.

6 Conclusion

We have analyzed competition between systems, with emphasis on the role of fixed costs in the complementary goods market on the structure of equilibria and on incentives to compatibility. These fixed costs generate an endogenous network effect, which we have called the system effect. Whereas under compatibility of the systems the only equilibrium is a symmetric duopoly, the equilibria under incompatibility of the systems are monopolistic or duopolistic, symmetric or asymmetric, depending on the size of this system effect.

The size of the system effect in turn depends crucially on that of the fixed costs which generate the increasing returns to scale in the complementary goods industry. Hence the size of these fixed costs is an important determinant of the market structure that may emerge in equilibrium. In particular, the mere presence of fixed costs in the production of the complementary goods alongside the desirability of variety on the part of the consumer may result in an entry coordination problem amongst complementary good producers and as a result, in equilibrium, complementary goods for one or both systems may not be supplied to the market. The existence of a symmetric equilibrium requires that the system effect be not too important. We have shown that this may occur with either large or small fixed costs relative to the total size of the market. The resulting equilibrium are however of different types, with quite different comparative statics implications.

When fixed costs are important, compatibility of the systems may be beneficial to consumers and be welfare improving. In particular, this would be the case if compatibility happens to help solve the complementary good producers' coordination problem. However it may be detrimental to consumers if it is done for systems already available on a symmetric duopolistic market, since it reduces the level of competition among producers, thereby mitigating the gains coming from the increase in variety of the complementary goods.

Appendix

Lemma A–1 $\lim_{L\to 0}\ v(y(L)) = 0$.

Proof. The equations defining $v(y)$ and $y(L)$ are:

$$\begin{aligned} L^{1-\delta}c &= \delta(u'(y)+u''(y)y)(u(y))^{\delta-1}\ , && \text{(A–1)}\\ v(y) &= \left(\frac{\delta}{c}\right)^{\frac{\delta}{1-\delta}}\left(1-\frac{\delta y u'(y)}{u(y)}\right)(u'(y)+yu''(y))^{\frac{\delta}{1-\delta}}\ . && \text{(A–2)}\end{aligned}$$

From equation (A–1), we obtain

$$\lim_{y\to+\infty}\ L = \lim_{y\to+\infty}\ \left(\frac{\delta}{c}\right)^{\frac{1}{1-\delta}}(u'(y)+yu''(y))^{\frac{1}{1-\delta}}(u(y))^{-1} = 0\ ,$$

since, by assumption, $\lim_{y\to+\infty}\ u'(y)=0$, $0 < u'(y)+yu''(y) < u'(y)$ for all y, and $u(y)$ is a positive and increasing function. Therefore $\lim_{L\to 0}\ y(L) = +\infty$. Using equation (A–2) and the assumption that $u'(y)$ goes to zero as y goes to infinity, we get

$$0 \le \lim_{L\to 0}\ v(y(L)) = \lim_{y\to+\infty}\ v(y) \le \lim_{y\to+\infty}\ \left(\frac{\delta}{c}\right)^{\frac{\delta}{1-\delta}}(u'(y))^{\frac{\delta}{1-\delta}} = 0\ .$$

Lemma A–2 *Let* $u'(0)\neq+\infty$. *Then, for all* $x>0$, $\lim_{f\to 0}\frac{c}{f}s'(\frac{xc}{f}) = 0$.

Proof. x and c being both treated as constants here, we must prove that $\lim_{z\to+\infty}\ zs'(z)=0$. We first show that $lim_{z\to+\infty}s(z)$ is finite. The definitions of s, v and g given in section 3 state that $s(z)=v(g(z))$ and $g(z)=y$. Recall that the utility function $u(y)$ is thrice differentiable with $u'(y)>0$ and $u''(y)<0$ for all $y\ge 0$ and its derivatives have well defined limits as y tends to 0. It must also satisfy (see (7)) $u'(y)+yu''(y)>0$ and $2u''(y)+yu'''(y)<0$ for all $y\ge 0$. If in addition $u'(0)$ is finite, then

$$\lim_{y\to 0}\left(1-\frac{\delta y u'(y)}{u(y)}\right)\in[1-\delta,1[\quad\text{and}\quad \lim_{y\to 0}\frac{u'(y)+yu''(y)}{u'(y)}\in]0,1]\ .$$

It follows, from equation (10) or (A–2) above, that we may write

$$\lim_{y\to 0} v(y) = A(0)(u'(0))^{\frac{\delta}{1-\delta}}$$

where $A(0)$ is some finite positive number. Now equation (15) defines the relation between y and z as $z=-[u'(y)+yu''(y)]/y^2u''(y)$, from which we verify that, given the same assumptions on $u(y)$, $\lim_{y\to 0} z=+\infty$ and, consequently, $\lim_{z\to+\infty} y = 0$. Therefore

$$\lim_{z\to+\infty}\ s(z) = \lim_{y\to 0} v(y) = A(0)(u'(0))^{\frac{\delta}{1-\delta}} \equiv s^* \neq +\infty.$$

Finally, one can use the mean value theorem to show that any continuously differentiable function, $f(z)$, defined on the positive real half-line, that tends to some finite number as z goes to $+\infty$ and for which $\lim_{z\to+\infty} zf'(z)$ is well-defined, is such that $\lim_{z\to+\infty} zf'(z) = 0$. But one easily verifies that under the above restrictions on $u(y)$, $\lim_{z\to+\infty} zs'(z) = \lim_{z\to+\infty} v'(g(z))g'(z)z$ is indeed well-defined. Consequently, $\lim_{z\to+\infty} zs'(z) = 0$

Lemma A–3 *Let there be a duopoly of systems 1 and 2. Let the Nash equilibrium of the entry game played by the producers of the complementary goods of both systems be locally stable (i.e. the 2×2 matrix whose (i,j) element is $d\pi_i/dL_j$ is negative definite). Then, $d(P_1 - P_2)/dX_1 < 0$.*

Proof. The profits of the complementary good producers of both systems are

$$\pi_i = \frac{X_i c}{w(y_i)} - f \ , \qquad i = 1, 2 \ ,$$

and, in duopoly, the demand for both systems is given by

$$v(y_1) - k\left[\frac{X_1}{a}\right] - P_1 = v(y_2) - k\left[\frac{X_2}{a}\right] - P_2 \ .$$

The inverse demand function of the basic good producers is

$$s(\frac{X_1 c}{f}) - k\left[\frac{X_1}{a}\right] - P_1 = s(\frac{X_2 c}{f}) - k\left[\frac{X_2}{a}\right] - P_2 \ .$$

At the Nash equilibrium,

$$\begin{aligned} \frac{d\pi_i}{dL_i} &= \left[\frac{as'(\frac{X_i c}{f})}{2k} - \frac{f}{c}\right] \frac{w'(y_i)f}{X_i}\frac{dy_i}{dL_i} \ , \qquad i = 1, 2 \\ \frac{d\pi_i}{dL_j} &= -\frac{as'(\frac{X_j c}{f})}{2k}\frac{w'(y_j)f}{X_i}\frac{dy_j}{dL_j} \ , \qquad i = 1, 2 \text{ and } j \neq i \ . \end{aligned}$$

Since the Nash equilibrium is locally stable,

$$\frac{d\pi_1}{dL_1}\frac{d\pi_2}{dL_2} - \frac{d\pi_1}{dL_2}\frac{d\pi_2}{dL_1} > 0 \ .$$

Substituting for the value of the partial derivatives in the above inequality gives

$$-\frac{2k}{a} + \frac{c}{f}\left(s'\left(\frac{X_1 c}{f}\right) + s'\left(\frac{X_2 c}{f}\right)\right) < 0 \ .$$

The term on the left-hand side of the last inequality is precisely $d(P_1 - P_2)/dX_1$.

Proof of Proposition 2.
We begin by showing that, for a small enough f, $(P_1, P_2) = (P^*, P^*)$ is the only possible Nash equilibrium of the basic good producers' game with strictly positive sales of both systems. In section 4, we saw that, in equilibrium, the consumer who is indifferent between systems 1 and 2, $\tilde{t}$, must be a root of the following equation:

$$\begin{aligned}&2k\tilde{t} + C'(a(1+\tilde{t})) - C'(a(1-\tilde{t})) - \\ &\quad s\left((1+\tilde{t})\frac{ac}{f}\right) - s\left((1-\tilde{t})\frac{ac}{f}\right) - \frac{2ac}{f}\tilde{t}\left[s'\left((1+\tilde{t})\frac{ac}{f}\right) + s'\left((1-\tilde{t})\frac{ac}{f}\right)\right] = 0\ .\end{aligned}$$

Let us call $g_f(\tilde{t})$ the left-hand side of the above equation, and let $g(\tilde{t})$ equal $2k\tilde{t}+C'(a(1+\tilde{t}))-C'(a(1-\tilde{t}))$. Given Lemma A-2, for any $\tilde{t}$ strictly different from +1 and -1, $g_f(\tilde{t})$ converges to $g(\tilde{t})$ as f goes to zero. Therefore, on any compact of $]-1,1[$ (e.g. on any interval $[-1+\epsilon, +1-\epsilon]$ with $\epsilon > 0$), g_f converges uniformly to g as f goes to zero. The function g is strictly increasing and equals zero at $t = 0$. Hence, given the uniform convergence of g_f towards g, the only zeroes of g_f on $[-1+\epsilon, +1-\epsilon]$ (for all $\epsilon > 0$) are in the neighborhood of $t = 0$. At $t = 0$, $g_f(0) = 0$ and $g_f'(0) = 2k + 2C''(a) - \frac{4ac}{f}s'\left(\frac{ac}{f}\right)$. Therefore, if f is small enough, $g_f'(0)$ is strictly positive (see Lemma A-2). This implies that, in the neighborhood of $\tilde{t} = 0$, g_f has only one zero. Putting all these partial results together, one sees that, for a small enough f, the only zero of g_f in $]-1,1[$ is $\tilde{t} = 0$. Plugging this result back in equations (22) and (23) proves that $(P_1, P_2) = (P^*, P^*)$ is indeed the only candidate for a Nash equilibrium with strictly positive sales of both systems, as f goes to zero.

There remains to prove that $(P_1, P_2) = (P^*, P^*)$ is indeed a Nash equilibrium, which is to say that the profit function of firm 1 is maximized at P^*, or equivalently at $X_1 = a$, when firm 2 prices its good at $P_2 = P^*$. We first show that $X_1 = a$ is a local maximum of firm 1's profit function, and then prove that it is a global maximum as well.

When both systems are sold, the profit function of firm 1 is given by $\Pi_1 = P_1X_1 - C(X_1)$, where the price firm 1 receives for its product is

$$P_1 = P^* + k\left[\frac{X_2 - X_1}{a}\right] + s\left(\frac{X_1c}{f}\right) - s\left(\frac{X_2c}{f}\right)\ .$$

The first-order condition for profit maximization, equation (22), is satisfied at $X_1 = a$. At $X_1 = a$, the second-order condition is

$$-\frac{4k}{a} + \frac{4c}{f}s'\left(\frac{ac}{f}\right) - C''(a) < 0\ .$$

Since the system effect, $\frac{c}{f}s'\left(\frac{ac}{f}\right)$, goes to zero as f goes to zero, the second-order condition is satisfied for f close to 0. Consequently, for a small enough f, $X_1 = a$ is a local maximum of the profit function. We now prove that it is a global maximum as well.

With the demand function above, one sees that when both systems are sold, the inverse demand function of firm 1 converges pointwise to its inverse demand function under compatibility, given by

$$P_1 = P_2 + k\left[\frac{X_2 - X_1}{a}\right]\ .$$

The sales of firm 2 are strictly positive if the marginal consumer prefers purchasing system 2 instead of only the numeraire, which is to say if

$$s\left(\frac{(2a-X_1)c}{f}\right) - P^* - k\left[\frac{2a-X_1}{a}\right] > 0 \ .$$

As f goes to zero, $s\left(\frac{(2a-X_1)c}{f}\right)$ goes to s^* for any $X_1 \neq 2a$ and P^* goes to $C'(a)+2k$. Since, by assumption, s^* is strictly greater than $C'(a)+2k$, the above inequality is satisfied in the limit everywhere except at $X_1 = 2a$. In other words, the coordination problem vanishes in the limit and firm 1 is in a duopoly with firm 2 except at $X_1 = 2a$. Hence, we have just shown that the inverse demand function of firm 1 converges pointwise to its inverse demand function under compatibility except at $X_1 = 2a$. The highest price that makes firm 1 a monopoly is the highest price at which there does not exist a stable Nash equilibrium of complementary good producers of both systems. To put it simply, firm 1 becomes a monopoly if it prices its product to make $P_1 - P_2$ negative enough to fall below the inverse demand under duopoly. Since this inverse demand has a negative slope, this implies that P_1 has to be less than

$$P^* + \min\left[k\left[\frac{X_2-X_1}{a}\right] + s\left(\frac{X_1c}{f}\right) - s\left(\frac{X_2c}{f}\right)\right]$$

As f goes to zero, the above expression converges to $P^* - 2k$, which is precisely the price necessary to make firm 1 a monopoly under compatibility. Hence, at $X_1 = 2a$ and as f goes to zero, firm 1's inverse demand function also converges to its inverse demand function under compatibility. This means that this pointwise convergence occurs everywhere on $[0, 2a]$. Since pointwise convergence is equivalent to uniform convergence on a compact, we have shown that the inverse demand function of firm 1 uniformly converges on $[0, 2a]$ to firm 1's inverse demand function under compatibility. Therefore, firm 1's profit function uniformly converges to

$$\Pi_1 = \left[C'(a) + k\left[\frac{X_2-X_1}{a}\right] + 2k\right]X_1 - C(X_1)$$

Under the assumptions of proposition 1, which have been incorporated in proposition 2, this profit function is strictly concave and has a maximum at $X_1 = a$, or $P_1 = P_2 = C'(a) + 2k$. Thus, firm 1's profit function uniformly converges to a strictly concave function with a maximum at $X_1 = a$. This implies that, for a small enough f, $\Pi_1(X_1)$ has a global maximum on $[0, 2a]$ in the neighbourhood of $X_1 = a$. And, since $X_1 = a$ is a local maximum of the profit function, it is that global maximum.

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Complementarity, Coordination and Compatibility:
The Role of Fixed Costs in the Economics of Systems [†]

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[†] We wish to thank Jiang Bian for his help in clarifying one of our proofs and especially Nicholas Economides, Neil Gandal and Claude Henry for their helpful comments on a previous version of the paper. Desruelle and Gaudet acknowledge the financial support of the Social Sciences and Humanities Research Council of Canada while working on this project. The views expressed here are those of the authors and do not necessarily represent those of the IMF.

Abstract

We analyze industry equilibrium and incentive to compatibility when goods produced by different producers generate utility only when consumed as component parts of a system. We assume the presence of two systems, each composed of some basic component and a set of differentiated complementary products. The combination of complementarity between the two components of the system and of fixed costs in the production of the complementary product results in a form of network effect. We focus on the role played by the size of the fixed costs in the production of the complementary products in determining the size of this system effect and, by this means, the structure and types of equilibria that may be observed: monopolistic or duopolistic, symmetric or asymmetric. We also highlight the consequence of the same fixed costs for the private and social incentives to render the systems compatible.