Monopoly Pricing With Network Externalities

Luís Cabral
Universidade Nova de Lisboa and CEPR

David Salant
GTE Laboratories Incorporated

Glenn Woroch*
University of California-Berkeley

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Abstract

How should a monopolist price a durable good or a new technology that is subject to network externalities? In particular, should the monopolist set a low "introductory price" to attract a "critical mass" of adopters? In this paper, we provide intuition as to when and why introductory pricing might occur in the presence of network externalities. Incomplete information about demand or asymmetric information about costs are necessary for introductory pricing to occur in equilibrium.

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1 Introduction

How should a monopolist price a durable good or a new technology that is subject to network externalities? Should the monopolist use declining prices to skim off consumer surplus, or, alternatively, should it launch the product with low "introductory" prices to attract a "critical mass" of adopters? These important questions have received surprisingly little attention from the economics literature.

In this paper, we provide intuition as to when and why introductory pricing can occur in the presence of network externalities. Specifically, we consider a monopolist selling a durable good that confers a network externality on a collection of rational buyers. Our principal goal is to establish plausible environments in which equilibrium prices increase over time.

Early development of telephone service supplies a near-perfect example of a monopoly over a service having network externalities. A user derives value from a communications network in proportion to the total number of subscribers. The telephone system in the U.S. was a monopoly based on the 1876 patents over basic telephone technology. Average monthly fees charged by the unregulated telephone companies rose steadily in the early 1880s, nearly doubling over a four-year period. Thereafter the price path flattened, only to plummet when the patents expired in 1893.[1]

On-line information services offer a more up-to-date illustration of introductory pricing. First *CompuServe*, and later *Prodigy*, were introduced with a small sign-up charge and a low monthly fee. As the customer base grew, the services raised prices gradually.

Users need not be connected by a physical network to realize network externalities. For instance, users of computer operating systems and some

[1] Detailed supporting data for this example and others that follow are available from the authors upon request.

general purpose applications packages receive an indirect externality as complementary hardware and software products become available. Computer vendors adopt marketing practices designed to take advantage of this externality. Makers of new hardware platforms are known to offer especially attractive licensing terms to early developers of compatible software. Penetration pricing is also a common strategy when introducing new software operating systems and other general purpose software for which network externalities are important.[2] New versions of existing programs are typically shipped to software developers at negligible cost to encourage them to write compatible applications. Introductory pricing works to enhance the product's quality by attracting these "lead users."

These examples suggest that introductory pricing may be an optimal pricing strategy when network externalities prevail. Our goal is precisely to describe situations in which introductory pricing is an equilibrium outcome. Furthermore, in the models we present, introductory pricing fails to occur *unless network externalities are present*. In this way we have been able to isolate the role played by network externalities in introductory pricing.

Our paper draws on three lines of previous research. The first is the growing literature on the adoption of innovations with network externalities. Farrell and Saloner (1985), Arthur (1989), and others examine equilibrium adoption of "unsponsored" (or nonproprietary) innovations, ignoring the issue of pricing. Katz and Shapiro (1985, 1986), on the other hand, consider the pricing of competing "sponsored" (or proprietary) innovations. They

[2] When first available, Microsoft licensed MS-DOS to Original Equipment Manufacturers for a flat fee, and for a limited time, that fee was reduced by half. See Stephen Manes and Paul Andrews, "Microsoft Monopoly," *Upside*, March 1993, p. 12. Further, new versions of applications packages are often introduced at low prices for a limited time, after which the price jumps dramatically. See *Business Week*, November, 1993, pp.6-8 for some examples. In this case, software suppliers competiting to establish a base of users, and hence some monopoly power, plays an integral role.

also find introductory pricing in equilibrium, but these low first-period prices are caused by a duopolist's desire to establish an installed base ahead of its rival's technology.[3]

The second line of research is the vast literature that endeavors to verify the so-called "Coase conjecture" regarding a monopolist selling a durable good.[4] Coase (1972) claimed that the price set by a monopolist who is unable to commit to future prices will quickly converge to marginal cost as the interval between successive selling periods becomes very short. The Coase conjecture was confirmed and disconfirmed under a variety of conditions.

In all cases the equilibrium solutions obey what Hart and Tirole (1991) call "Coasian dynamics." Coasian dynamics consist of two properties: (i) higher valuation adopters make their purchase no later than lower valuation adopters (the *skimming property*) and (ii) equilibrium price is nonincreasing over time (the *price monotonicity property*). In this paper, we show that the second property need not always hold when network externalities exist.

Finally, the Marketing Science literature has examined pricing with "experience" or "network" effects.[5] These papers typically assume that buyers obey some rule that is not necessarily rational, or alternatively that they are imperfectly informed about the existence or the quality of the good. By contrast, we assume that buyers are perfectly rational agents.

We begin our analysis with the case of perfect information in Section 2. We show that, if each buyer is "small", then discounted prices *must* decrease

[3] Gallini and Karp (1989) also find introductory pricing when there is consumer lock-in, due to firm (or product) specific investments, and repeat purchases.

[4] Stokey (1981); Bulow (1982); Sobel and Takahashi (1983); Fudenberg, Levine and Tirole (1985); Ausubel and Deneckere (1986); Gul, Sonnenschein and Wilson (1986); Kahn (1986); Bagnoli, Salant and Swierzbinski (1989); Von der Fehr and Kühn (1991); Kühn and Padilla (1992).

[5] Bass (1980), Clarke, Darrough and Heineke (1982), Kalish (1983), Besanko and Winston (1990). See also Vettas (1993) who looks at the effects of word of mouth externalities on monopolist prices.

over time. In other words, Coasian dynamics must hold. If, instead, consumers are "large", we can construct examples in which *discounted* prices rise over time by carefully selecting from among multiple equilibria.

In Section 3, we assume imperfect information about consumers' valuations. Again we treat the cases of small and large buyers separately. In both cases, we find equilibria in which prices increase over time. Now, however, the intuition for the result differs between the two cases. When buyers are small, the inducement of a low first-period price is needed to compensate for the uncertainty of an early adoption. When buyers are large, however, delaying a purchase can actually increase the probability that other buyers will eventually adopt. In this case the firm sets a lower first-period price to counteract the tendency to delay.

Finally, in Section 4 we let the firm's cost be unknown to buyers. Again, we find perfect Bayesian equilibria in which prices rise over time. In this case, introductory prices serve as a signal of low cost, thus raising early buyers' expectations about the likelihood of future sales. The lower the seller's cost is, the bigger future sales will be, and thus the higher the expected utility of a purchase today is.

2 The Certainty Case

We begin by assuming that there is perfect information about demand, cost, and the quality of the product. However, we assume that the monopolist is unable to set prices based on buyers' types, either because it cannot observe some individual characteristic, or because it is precluded from price discrimination.

Each buyer's valuation of the good depends on her type as well as on the number of other buyers who have purchased the good—the essence of a network externality. This may be represented as $u_i(n_t)$ the utility derived by consumer i given the cumulative number of purchases through period t, n_t. Each adopter demands at most one unit. And since there is no possibility of resale, a buyer will make a purchase in a period only if she had not done so earlier. Once purchased, the good provides a stream of benefits that each consumer discounts according to the discount factor δ. Finally, we deduct the current price, p_t, to arrive at the net payoff.

We say that a buyer is "small" when her decision to purchase the good has no effect on the payoff to other buyers or on the strategies they choose. This would be true if there were a countably or uncountably infinite number of them. A buyer is "large" if her decision to purchase has a noticeable effect on other buyers' payoffs and decisions. This section demonstrates that the occurrence of introductory pricing depends on the "size" of buyers.

Proposition 1 *If all buyers are small, then in a subgame-perfect Nash equilibrium discounted prices cannot rise between periods in which sales occur.*

Proof: Suppose that sales occur in some period t at a price p_t in equilibrium. For contradiction, suppose that in an earlier period $s < t$ the firm extends an offer of $p_s < \delta^{t-s} p_t$. Any buyer who purchased in period t would be better off by purchasing in period s instead. This would reduce her discounted outlay by $\delta^t p_t - \delta^s p_s$ and also increase her utility by $\sum_{\tau=s}^{t-1} \delta^\tau u_{i\tau}$ (both expressed in present value). As long as the buyer is small, this adjustment in purchasing behavior has no effect either on other buyers' utility or on the firm's profit. Therefore, every buyer who makes a purchase in period t will be induced to advance their purchases, so that in the end no sales will take place at time t. ■

Note that the result does not depend on the strength of network externalities. Discounted prices cannot rise even in complete absence of network effects.[6]

The same result does not hold, however, if buyers are large. An example is presented in the Appendix where discounted prices are increasing along a subgame-perfect Nash equilibrium. The example exploits the multiplicity of equilibria under network externalities: buyers coordinate their purchases so as to "punish" any deviations from equilibrium of the sort considered in the proof of Proposition 1.[7]

Specifically, the equilibrium calls for one buyer to make a purchase in period 1, while three other buyers purchase in period 2 at a higher price. If a period-2 buyer advances her purchase to period 1, then the remaining period-2 buyers "punish" her by not purchasing in period 2. This constitutes a Nash equilibrium of the period-2 subgame, though not the only one.[8]

In the next sections we explore how uncertainty about demand and cost can result in introductory pricing. Once again we consider separately the cases of "large" and "small" buyers, but now increasing prices are possible in both cases.

3 Incomplete information about demand

3.1 Large buyers

Suppose there are two potential buyers and two periods. The i-th buyer's utility is given by v_i if she is the only one who buys, and $v_i + u$ if both

[6] Nominal price can nevertheless rise and the rate of increase will depend on network externalities.

[7] More than introductory pricing is possible when buyers are large. In fact, there may exist a continuum of equilibria. Equilibrium refinements such as coalition proofness or risk dominance, however, may drastically reduce the set of equilibria.

[8] Gul (1987) and Ausubel and Deneckere (1987) show how the multiplicity of equilibria may be used to reverse the Coase conjecture when there is more than one seller, that is, multiple subgame equilibria can be used to sustain equilibria in which prices are (virtually) constant over time.

buyers make a purchase. Here u measures the network externality; it is the same for both buyers and its value is common knowledge to buyers and the seller. v_i measures the "standalone" utility; its value is each buyer's private information. The prior distribution of v_i is uniform on the interval $[0, 1]$. Production cost is assumed to be zero.

This setup is similar to the model of incomplete information presented in Farrell and Saloner (1985). The principal distinction is that we consider a proprietary innovation which is therefore priced. Farrell and Saloner consider an "unsponsored" innovation, thus concentrating on issues of buyer coordination.[9]

We focus on interior equilibria in which, with probability strictly between 0 and 1, a sale is made in each period (provided there is unsatisfied demand). This restriction imposes bounds on the values of δ, the discount factor, and of u, the measure of network externalities.

Basically, in order for the solution to be interior, δ and u cannot be too large. If δ is large, then only a corner solution exists in which no sales occur in the first period. In this extreme case it makes no sense to talk about the evolution of equilibrium prices since no sales are made at the initial price. If, on the other hand, u is very large, then the network externality swamps the uncertainty and the standalone value, and so all consumers who buy will buy early on. Again, the offer of higher second period prices is not exercised. For an open set of values of δ and u, however, we can show that a unique Perfect Bayesian Equilibrium exists that displays introductory pricing with certainty.

Proposition 2 *If* $0 < u < 1/2$ *and* δ *is close to (but lower than)* $\overline{\delta}(u)$,

[9] Also, Farrell and Saloner's payoffs have a more general specification of network externalities.

where

$$\overline{\delta}(u) \equiv \frac{4(1-u)^2}{4(1-u)^2 + 2u^3(1-u) - u^4}, \tag{1}$$

then there exists a unique, interior Perfect Bayesian Equilibrium in which with probability 1 discounted price in the second period exceeds price in the first period.

The proof of the result is in the Appendix. The intuition is as follows. Consider first the case when $\delta \sim 1$ and $u = 0$. It is well known from the literature on bargaining and durable goods pricing that a monopoly seller will not price discriminate over time. Accordingly, buyers will choose to wait to purchase since utility is not discounted and they retain the option of a better outcome. Consequently, discounted prices are nearly constant over time and almost no sales occur in the first period.

Now suppose that $u > 0$. Network externalities introduce a new factor into the buyers' decision besides the time profile of prices. A buyer must weigh the impact of her decision on the likelihood that the other buyer will purchase. As shown in the proof, by foregoing a purchase in the first period, the likelihood of a sale in the second period to the other buyer actually *increases*. The reason is that, if no sales occur in the first period, then both buyers' combined willingness to pay is smaller since neither one is guaranteed the network externality if she buys. For this reason, the seller finds it optimal to set a price much lower if no sale occurs in the first period compared to when a sale does occur.

In contrast to buyers, the seller prefers to make all sales in the first period. Although delaying the purchase increases expected network size, it also decreases expected profits. Therefore, the seller has an incentive to lower first-period prices so as to discourage buyers from delaying first-period purchases in an attempt to force low second-period prices.

When the discount factor is sufficiently small, it can easily be shown that equilibrium price decreases with probability one. The intuition is straightforward. If the future is heavily discounted, the only difference between periods is that in the second period, with positive probability, the monopolist will have lowered its priors with respect to the buyers' valuations, which in turn leads to lower prices. It follows by continuity that for intermediate values of δ and u, equilibrium price increases or decreases with positive probability.

There is an interesting parallel between the equilibrium just described and the one found in Farrell and Saloner's (1985) adoption game with network externalities. In both cases, medium-valuation adopters play "bandwagon" strategies: to adopt (or buy) in the second period if and only if an adoption (purchase) was made in the first period.

Finally, it can also be shown that in equilibrium, whatever the magnitudes of the network externality and the discount factor, the equilibrium is inefficient: welfare-increasing adoptions are delayed from period 1 to period 2, or, in some cases, never made. This is not surprising in light of Farrell and Saloner's "excess inertia" result for unsponsored innovations. When the firm prices a proprietary innovation, the equilibrium can very well be less efficient—and it is.

3.2 Demand Uncertainty and Lead Users

Here we again make the assumption of "small" buyers. Specifically, we assume there is a continuum of buyers who can purchase in one of two periods. Each buyer can be one of two types: H or L. A crucial assumption is that only H-type buyers confer network benefits on other buyers. Accordingly, we can treat these buyers as "lead users," to borrow a common term in the marketing literature. They contribute in a decisive way to the amount of complementary products and services that generate network benefits. For example, if the basic product was a new software operating system, then the lead users would be developers of software applications.

The utility of a H-type buyer is given by $u^H = v + x$, where x is the measure of H-type buyers. L-type buyers receive utility $u^L = x$. For simplicity, we assume there is no discounting or interim utility, so all that matters to buyers is the final number of H-type adopters and the price paid.

The measure of H-type buyers is given by α. The value of α is uncertain to both buyers and seller. It can take on the value $\overline{\alpha}$ or $\underline{\alpha}$. Both buyers and seller hold a common prior probability ρ that $\alpha = \overline{\alpha}$. Finally, the seller has a constant marginal cost c.

Proposition 3 *Suppose that*

$$\underline{\alpha} < c < \overline{\alpha} < \underline{\alpha} + v \tag{2}$$

$$\overline{\alpha} v < (1 - \overline{\alpha})(\overline{\alpha} - c). \tag{3}$$

If ρ is sufficiently low, then there exists a unique Perfect Bayesian Equilibrium in which expected second-period price is higher than the first-period price.

The proof can be found in the Appendix. It is worth noting that the set of parameter values determined by (2)–(3) is non-empty. For instance, they are satisfied by $\underline{\alpha} = 0$, $c = .1$, $\overline{\alpha} = .2$, and $v = .3$.

The intuition for this result can be seen in the following way. The new product can either be a success or a failure (or "good" or "bad"), corresponding to whether the measure of "lead users" is high or low. In equilibrium, this is known at the beginning of period 2. If the product is "good", then the seller prices low in order to attract the largest fraction of buyers. If, on the contrary, the product is "bad", then the seller sets a high pricing, knowing that he will only sell to high-valuation consumers. Now, high-valuation consumers are more optimistic than the seller about the possibility that the product is "good" and a low price will be set in period 2. Therefore, in order for them to buy in the first period, the seller has to set a price which is lower that his (the seller's) expected second-period price.

4 Asymmetric information about cost

Our last explanation for introductory pricing hinges on asymmetric information about production costs. Specifically, we suppose consumers are not perfectly informed about the seller's unit cost. Since we want to concentrate on the effects of asymmetric information, we avoid the issue of consumer's timing of purchases by assuming that the monopolist is selling to two consumers who arrive sequentially.

Each consumer can be one of two types: high valuation (type h) or low valuation (type l) with probability α and $1-\alpha$, respectively. A high valuation consumer has a utility of v if she is the only one to buy the product, and $v+u$ if both consumers buy the product. A low-valuation consumer receives 0 utility if she is the only one to buy the product and u if both consumers buy. Utilities are realized after both adoption decisions have been made.[10]

Finally, the seller can be of two types, high cost (c, type H) and low cost (zero, type L). The seller's cost is unknown to buyers at the time of purchase.[11] In this event we once again find an equilibrium in which the seller sets an increasing price sequence.

[10] Again, this simplification is made with the sole purpose of isolating the effect of asymmetric information about cost.

[11] Since we consider equilibria with separation, the probability that the seller's cost is low is irrelevant so long as it is positive.

Proposition 4 *Suppose that*

$$\frac{u-c}{u+v-c} < \alpha < \frac{u}{u+v} \tag{4}$$

and that

$$\alpha u < \alpha(2\alpha - 1)u + \alpha v + (1-\alpha)c < u. \tag{5}$$

Then, there exists at least one Perfect Bayesian Equilibrium with separation (i.e., different types of firms setting different prices in the first period). Furthermore, every separating equilibrium has the property that the low-cost firm sets first-period price below second-period price.

One such equilibrium has the low-cost firm setting first-period price equal to $p^L = \alpha(2\alpha - 1)u + \alpha v + (1-\alpha)c$. In the second period, it sets price at either u or v according to whether or not a sale was made in the first period. Then $p^L < u$ and $p^L < v$ provided Conditions (4) and (5) are satisfied. They are satisfied, for instance, when $u = c = 2, v = 1$, and $\alpha = 1/2$.

The formal proof of Proposition 4 may be found in the Appendix. The proof is by construction. We first show that Condition (4) implies that, assuming a sale was made in period 1, the monopolist will optimally sell to both types of buyer in period 2 if it has low cost, but only to the high-valuation buyer if it has high cost.

Knowing this, and given that there are network externalities, a buyer's expected utility (and willingness to pay) in the first period is higher the more she believes the seller to be a low-cost firm. This, in turn, creates an incentive for high-cost sellers to masquerade as low-cost sellers in the first period. Finally, to distinguish itself from a high-cost seller, the low-cost seller has to set a very low price in period 1. Conditions (5) guarantees that separation is an equilibrium.

We should note that this equilibrium is very similar to the one found in Bagwell (1989). In his model, consumers must decide whether, after visiting the seller in the first period, to incur a fixed cost to return in the second period. Bagwell finds that a seller will employ introductory pricing to signal low cost. First-period consumers return to the seller when they the seller is low cost, and hence, will charge low second-period price. In our model first-period consumers care about future prices to the extent that the probability of future purchases by *other* consumers depends on future prices.

Finally, notice that while we have only considered separating equilibria, it is fairly straightforward to construct pooling equilibria in which both types of sellers choose the same first-period price. The conditions for a pooling equilibrium to exist depend on the probability that buyers are of type h as well as on the other parameters of the model. There can be two classes of pooling equilibria: One in which only h types buy in the first period, and another in which both h and l types buy in the first period. A straightforward but tedious argument shows that increasing prices can occur in both kinds of pooling equilibria.

5 Conclusion

We have constructed models of pricing a durable good or a new technology that confers network externalities. The models overturn the *price monotonicity property* that is a key element of Coasian dynamics: in each case discounted price rises over time.[12] In addition, discounted prices did not rise whenever network externalities vanished, underscoring the close connection between introductory pricing and network externalities.

These results were derived in settings that were deliberately neutral to-

[12] The *skimming property* remained in tack for those cases in which buyers are unambiguously rank ordered by willingness to pay.

ward introductory pricing. There is no cost escalation or growing demand that would justify increasing prices. Nor do our models allow for intertemporal competition that is often responsible for low initial prices.

There is an interesting question not resolved in this paper: Given that network externalities can work to reverse the direction of Coasian dynamics, could they also refute the Coase conjecture itself? If our work is any indication, plausible demand and cost conditions may call for price to rise initially. Soon thereafter, however, the power of Coasian dynamics will prevail, causing prices to soon after fall toward marginal cost. The extent of such a price cycle remains an open issue.

Table 1: Consumer valuations.

		No. adopters	
Buyer	Type	1 to 3	4
1	A	v_a	v_a
2	B	v_b	$v_b + u$
3	B	v_b	$v_b + u$
4	B	v_b	$v_b + u$

Appendix

■ **Certainty, coordination and increasing prices with large buyers.** With the following example, we show that if buyers are large, then there may exist equilibria with increasing prices.

Suppose that the monopolist sells in two periods to four potential buyers. Table 1 contains the present values of utility for the buyers under the various adoption configurations.[13] There are two types of buyers: The A-type has a positive stand-alone valuation $v_a > 0$ but does not gain from the network externality; the B-types have a smaller (or no) stand-alone valuation $v_b < v_a$ but receive positive network benefits $u \geq 0$. It is crucial for the result that different types derive different utility from the network effect, and that the B-type buyers benefit from the network externality only when *all four* of the consumers purchase the good. Production cost is a constant c per unit.

It can be shown that, if $u = 0$ (i.e., no network externalities), then equilibrium prices must be decreasing, consistent with Coasian dynamics. We will now argue that, if $u > 0$ and if other conditions hold, then there exists a subgame-perfect Nash equilibrium such that discounted prices *increase* over time. The additional conditions are:

[13] Since these are total utilities, they must be multiplied by $(1 - \delta)$ to convert them to (constant) per-period flows.

$$\begin{aligned} v_b &< c & (6) \\ v_a &< \delta(v_b + u) & (7) \\ v_a + 3\delta(v_b + u) &> (1 + 3\delta)c & (8) \end{aligned}$$

We propose strategies for the seller and the buyers that together form an equilibrium:

- The A-type buys as soon as price is lower than v_a.
- The B-types buy in period 1 if and only if $p_1 \leq v_b$. In period 2, they purchase if and only if one of the following is true:
  - the A-type alone purchased in period 1 and $p_2 \leq v_b + u$;
  - the A-type and exactly two B-types purchased in period 1 and $p_2 \leq v_b + u$;
  - some other pattern of purchases occurred during period 1 and $p_2 \leq v_b$;
- The seller sets $p_1 = v_a$ and
  - $p_2 = v_b + u$ if the A-type made a purchase in period 1;
  - $p_2 = v_a$ if the A-type did not make a purchase in period 1.

The equilibrium outcome resulting from the play of these strategies has the A-type buying in period 1 at $p_1 = v_a$, and all B-types buy in period 2 at price $p_2 = v_b + u$. Condition (7) ensures that equilibrium discounted prices rise, that is, $p_1 < \delta p_2$.

To see that the designated strategies form a subgame perfect Nash equilibrium, notice first that the A-type gets a zero equilibrium payoff. Any deviation from these strategies would make her worse off since $p_2 = p_1$ if she does not buy in period 1.

The B-types' strategies at any period-2 subgame clearly constitute a Nash equilibrium: either all adopt and each has willingness to pay $v_b + u$, or not all adopt and willingness to pay is v_b. Notice that, to ensure the B-types will purchase for certain, the seller would have to set $p_2 \leq v_b$, but this would imply a loss, given Condition (6).

Now consider a B-type's strategy in period 1. A unilateral deviation by a single B-type would imply buying at price $p_1 = v_a$. It would also cause the remaining two B-types to forego purchasing in the second period thereby punishing the lone defector according to their candidate strategies. Thus, the B-type can expect a deviation payoff of $v_b - v_a < 0$, compared to an equilibrium payoff of zero.

Finally, let us check that the monopolist's strategy is optimal. In period 2, and assuming that the A-type did not purchase earlier, $p_2 = v_a$ extracts the most surplus the monopolist can get from the A-type; to attract additional buyers, she would have to set $p_2 \leq v_b$, which would result in a negative payoff. (Notice that buyers are pessimistic about their collective ability to coordinate on the Pareto dominating outcome, i.e., all purchase.) The same reasoning applies to the various possibilities that include a purchase by the A-type in period 1.

Similarly, in period 1, $p_1 = v_a$ extracts the most surplus the monopolist can get from the A-type; and to get more buyers, she would have to set $p_1 \leq v_b$, which would result in a negative payoff.

Condition (6) ensures that the firm breaks even.

We thus conclude that the designated strategies constitute a subgame perfect Nash equilibrium; and that, along this equilibrium, price increases from period 1 to period 2. ■

Proof of Proposition 2: We begin by noting that Lemma 10.1 in Fudenberg and Tirole (1991) can be adapted to show that in any subgame in any period there exist critical values v' and $v'' \geq v'$ such that buyers with valuations $v' \leq v \leq v''$ make a purchase in that period. Specifically, denote by v_1 the critical value in the first period and by v_2^k, the two possible values in the second period depending on the number of first-period sales $k = 0, 1$. That is, consumers with valuation greater than v_1 make a purchase in the first period; and consumers with valuation in $[v_0^k, v_1]$ make a purchase in the second period. We establish, by construction, that the equilibrium is unique.

Begin by considering the second-period subgame that follows one sale in the first period. The indifferent buyer's valuation, v_2^1 (period 2, 1 previous sale), is given by

$$u + v_2^1 = p_2^1, \tag{9}$$

where p_t^k is price in period t given that k sales have been made before. This gives the second-period profit function

$$\Pi_2^1 = p_2^1(p_2^1 - u - v_1)/v_1, \tag{10}$$

where $(p_2^1 - u - v_1)/v_1$ is the equilibrium probability that a sale will occur in the second period given that a sale occurred in the first period. Substituting (9) for v_2^1 in (10) and maximizing with respect to p_2^1 yields

$$p_2^1 = (u + v_1)/2, \tag{11}$$

assuming that $u < v_1$. Substituting in (9), we get

$$v_2^1 = (v_1 - u)/2. \tag{12}$$

In order for this to be an interior solution, we require that $v_2^1 > 0$, which implies $v_1 > u$. Notice that, since $v_2^k < v_1$, the requirement that $v_2^k < 1$ is implied by $v_1 < 1$, a condition which we will return to later.

Consider now the second-period subgame that follows no sales in the first period. The indifferent buyer's valuation, v_2^0 (period 2, no previous sales), is now given by

$$v_2^0 + \frac{v_1 - v_2^0}{v_1} u = p_2^0. \tag{13}$$

Here $(v_1 - v_2^0)/v_1$ is the probability that the other buyer will purchase conditional on not having done so earlier. Solving (13) for v_2^0 yields

$$v_2^0 = v_1(p_2^0 - u)/(v_1 - u). \tag{14}$$

Therefore, the profit function is

$$\begin{aligned} \Pi_2^0 &= 2p_2^0(v_1 - v_2^0)/v_1 \\ &\quad + 2p_2^0\Big(v_1 - v_1(p_2^0 - u)/(v_1 - u)\Big)/v_1 \\ &= 2p_2^0(v_1 - p_2^0)/(v_1 - u), \end{aligned} \tag{15}$$

where the second equality comes from substituting for v_2^0 from (14). Maximization results in

$$p_2^0 = v_1/2, \tag{16}$$

which assumes $u < v_1/2$. Substituting (16) for price in (13) we get

$$v_2^0 = v_1 \frac{v_2/2 - u}{v_1 - u}. \tag{17}$$

To ensure that $v_2^0 > 0$, we must have $v_1 > 2u$ which, in turn, requires that $u < 1/2$. For future reference, notice that

$$v_2^1 = v_2^0 + \frac{u^2/2}{v_1 - u}. \tag{18}$$

The second term in the right-hand side is positive, given the condition $v_1 > u$, and so $v_2^1 > v_2^0$. In words, the likelihood that a buyer will purchase in the second period is *higher* when no sales were made in the first than if some had occurred. This apparent contradiction is understandable when one notices that the second-period price is *lower* in absence of any earlier sales. Unsure that they will capture the network externality, buyers' aggregate willingness to pay is lower. Accordingly, the seller is compelled to charge lower prices when no sales are made in the first period.

Having found the solution to both second-period subgames, we now turn to the first period. The indifferent buyer's valuation, v_1, is given by

$$\begin{aligned} v_1 + u\Big((1 - v_1) + \delta(v_1 - v_2^1)\Big) - p_1 = \\ = \delta v_1 + \delta u(1 - v_2^0) - \delta\Big((1 - v_1)p_2^1 + \delta v_1 p_2^0\Big). \end{aligned} \tag{19}$$

On the left-hand side, we have expected net utility from a purchase in the first period. For a price of p_1, the marginal buyer receives a standalone valuation v_1 plus a network benefit when either the other buyer buys today (with probability $1 - v_1$) or the other buyer buys tomorrow (with probability $v_1 - v_2^1$). On the right-hand side, we have expected net utility from a purchase in the second period. For an expected discounted price of $\delta\Big((1 - v_1)p_2^1 + v_1 p_2^0\Big)$, the marginal buyer receives a standalone valuation δv_1 and an expected network externality of $\delta u(1 - v_2^0)$.

¿From (19) we can show that p_1 is increasing in v_1, so that we can form the inverse demand function

$$p_1 = \delta(v_1 p_2^0 + (1 - v_1)p_2^1) + (1 - \delta)(v_1 + u(1 - v_1)) + \delta u(v_2^0 - v_2^1). \tag{20}$$

Expected discounted profit per buyer is given by

$$\Pi_1 = p_1(1 - v_1) + \delta(v_1(v_1 - v_2^0)p_2^0 + (1 - v_1)(v_1 - v_2^1)p_2^1), \qquad (21)$$

where the factors multiplying the prices are the likelihoods of the three sales scenarios. Substituting for p_1 from (20) and simplifying results in

$$\begin{aligned} \Pi_1 &= \delta(v_1(1 - v_2^0)p_2^0 + (1 - v_1)(1 - v_2^1)p_2^1) + \\ &\quad + (1 - v_1)\Big((1 - \delta)(v_1 + u(1 - v_1)) + \delta u(v_2^0 - v_2^1)\Big) \\ &= \delta\Phi + (1 - v_1)\Big((1 - \delta)(v_1 + u(1 - v_1)) + \delta u(v_2^0 - v_2^1)\Big), \end{aligned} \qquad (22)$$

where we have isolated

$$\Phi \equiv v_1(1 - v_2^0)p_2^0 + (1 - v_1)(1 - v_2^1)p_2^1. \qquad (23)$$

This last expression is just *ex ante* expected second-period price.

Next we differentiate (19) with respect to v_1 and evaluate at $v_1 = 1$ to get

$$\left.\frac{\partial \Pi_1}{\partial v_1}\right|_{v_1=1} = \delta \left.\frac{\partial \Phi}{\partial v_1}\right|_{v_1=1} - ((1 - \delta) + \delta u(v_2^0 - v_2^1)) \qquad (24)$$

Setting $\partial\Pi_1/\partial v_1|_{v_1=1} = 0$ yields $\delta = \overline{\delta}(u)$ as the solution. That is, $\delta = \overline{\delta}(u)$ implies that the optimal v_1 equals 1. Note that $\overline{\delta}(u)$ is decreasing in u; when $u < 1/2$, $\overline{\delta}(u)$ ranges from 1 down to 16/17.

When $v_1 = 1$, no sales occur in the first period, and so discounted second-period price is simply δp_2^0 with probability 1. When $v_1 = 1$, (20) reduces to

$$p_1 = \delta p_2^0 + (1 - \delta) + \delta u(v_2^0 - v_2^1). \qquad (25)$$

In that case, the condition for introductory pricing to occur in equilibrium is just

$$(1-\delta)+\delta u(v_2^0 - v_2^1) < 0, \tag{26}$$

where v_2^0 and v_2^1 are evaluated at $v_1 = 1$. Examining (24), this inequality reduces to

$$\left.\frac{\partial \Phi}{\partial v_1}\right|_{v_1=1} < 0. \tag{27}$$

Substituting (11), (12), (16), and (17) for $p_2^1,\ v_2^1,\ p_2^0, v_2^0$ in (23) and simplifying we get

$$\Phi = \frac{v_1^3(1-u) - v_1^2(2-u-u^2) - v_1(3+u)u^2 + (2+u)u^2}{4(u-v_1)} \tag{28}$$

Differentiating with respect to v_1 and evaluating at $v_1 = 1$ gives

$$\left.\frac{\partial \Phi}{\partial v_1}\right|_{v_1=1} = \frac{-u^4}{4(1-u)^2} < 0, \tag{29}$$

ensuring that discounted price strictly increases with probability one.

Since the equilibrium value of v_1 is an increasing function of δ, it follows by continuity that if δ is lower but sufficiently close to $\overline{\delta}(u)$ then the equilibrium is interior (that is, $v_1 < 1$) and discounted price strictly increases with probability one. ■

Proof of Proposition 3: First notice that each buyer's posterior regarding the value of ρ, depending on whether she is an H type or an L type, is given by

$$\rho^H = \frac{\rho\overline{\alpha}}{\rho\overline{\alpha} + (1-\rho)\underline{\alpha}} \tag{30}$$

and

$$\rho^L = \frac{\rho(1-\overline{\alpha})}{\rho(1-\overline{\alpha}) + (1-\rho)(1-\underline{\alpha})}, \tag{31}$$

respectively. In words, being of the H type makes a buyer more optimistic about the measure of H types; conversely, being of the L type makes the buyer more pessimistic about the measure of H types: $\rho^L < \rho < \rho^H$.

The seller has three possibilities. One is to induce both types to buy in the same period (pooling equilibrium). A second one is to induce H types to buy in the first period and L types in the second period. Lastly it could sell to H types only.

Consider first a pooling equilbrium. The highest price the seller can charge is given by the L types' expected utility, namely $\rho^L\overline{\alpha} + (1-\rho^L)\underline{\alpha}$. However, since $\underline{\alpha} < c$ by Condition 2, this price is lower than cost for a sufficiently low ρ.

Consider now a separating equilibrium. In the second period, the measure of adopters in the first period will be known. Based on this value, the seller and the remaining buyers will form a posterior on the value of α. If $\alpha = \overline{\alpha}$ (i.e., the posterior puts weigh 1 on the value $\alpha = \overline{\alpha}$), then the best the seller can do is to set $p_2 = p_2(\overline{\alpha}) = \overline{\alpha}$. It will then sell to the remaining $1-\overline{\alpha}$ consumers (recall that, by assumption, $\overline{\alpha} > c$).

If, on the contrary, $\alpha = \underline{\alpha}$, then no price above cost will induce the L types to buy because, by assumption, $\underline{\alpha} < c$. In equilibrium, any price above $\underline{\alpha}$ is indifferent from the seller's perspective. We assume $p_2 = p_2(\underline{\alpha}) = \underline{\alpha} + v$, as this is the only price that survives the possibility that some H types might have remained to the second period. (Alternatively, we might assume that a fraction ϵ of H types are only born in period 2.) Notice that, given our assumption that $\underline{\alpha} + v > \overline{\alpha}$, the second period price is higher when $\alpha = \underline{\alpha}$, that is, $p_2(\underline{\alpha}) > p_2(\overline{\alpha})$.

Let us now consider the first-period price. The most the seller can charge and still have the high-valuation consumers make a purchase is given by

$$p_1 = \rho^H p_2(\overline{\alpha}) + (1-\rho^H) p_2(\underline{\alpha}) \quad (32)$$

$$= \rho^H \overline{\alpha} + (1-\rho^H)(\underline{\alpha}+v). \quad (33)$$

The justification for this is the following. The high-valuation consumers know that they will always want to make a purchase. Therefore, they only have to compare first-period price with expected second-period price *given their expectations about the value of* α.

Ex-ante expected second-period price, in turn, is given by $\rho p_2(\overline{\alpha}) + (1 - \rho)p_2(\underline{\alpha})$. But then,

$$\begin{aligned} E(p_2) &= \rho p_2(\overline{\alpha}) + (1-\rho)p_2(\underline{\alpha}) \\ &> \rho^H p_2(\overline{\alpha}) + (1-\rho^H)p_2(\underline{\alpha}) \\ &= p_1, \end{aligned} \quad (34)$$

where the inequality follows from the facts $\rho^H > \rho$ and $p_2(\underline{\alpha}) > p_2(\overline{\alpha})$.

To conclude the proof, we have to check that the separating equilibrium is preferable to one in which the seller targets the H-type buyers only. In the latter equilibrium, the maximum price the seller can charge is given by the expected benefit for H-type buyers, that is

$$p_1 = \rho^H(\overline{\alpha}+v) + (1-\rho^H)(\underline{\alpha}+v). \quad (35)$$

By comparison with (33), we can see that the separating equilibrium results in a loss of revenues from sales to H-type buyers given by $\rho^H v$ (the difference in prices) times $\rho\overline{\alpha} + (1-\rho)\underline{\alpha}$ (the expected measure of H types). Given the definition of ρ^H, this product is given by $\rho\overline{\alpha}v$. On the other hand, the separating equilibrium implies an extra profit from sales to L-type buyers, given by $\rho(1-\overline{\alpha})(\overline{\alpha}-c)$. It is a simple exercise to check that Condition (3) implies that the separating equilibrium is preferable. ■

Proof of Proposition 4: Since we are interested in finding Perfect Bayesian Equilibria, we begin by considering the monopolist's problem in the second period. Suppose that the first-period consumer has bought the good. Then, expected profits in the second period for the high-cost monopolist will be $\alpha(u + v - c)$ if she sets a high price ($p_2 = u + v$), and $u - c$ if she sets a low price ($p_2 = u$). (One can easily check that no other price can be optimal for the seller.) By the first part of (4), we conclude that it is optimal for the high-cost monopolist to set a high price in the second period, so that a second adoption occurs with probability α conditional on a first adoption having occurred.

A similar problem arises for the low-cost firm, with the difference that production cost is zero. Given the second part of (4), we conclude that it is optimal for a low-cost firm to set price equal to u in the second period, so that a second adoption occurs with probability 1 conditional on a first adoption having occurred.

Finally, if no purchase has occurred in the first period, then both types of firms set $p_2 = v$.[14]

Condition (4) is crucial for the equilibrium that we will derive. If the first-period consumer believes the firm has low costs, then she will expect a low price to be set in the second period, and a second adoption to occur with probability 1. This implies that her expected valuation equals $u + v$ (or simply u, if she is a low-valuation type). If, on the contrary, the consumer believes the seller has high costs, then her expected valuation is only $\alpha u + v$ (or αu, if she is a low-valuation type).

[14] If $v < c$, then the high-cost firm sets $p_2 = c$. In what follows, we will assume that $v > c$. The result does not depend on this condition, although (4)–(5) would have to be modified.

If there is no uncertainty about the firm's costs, then the optimal price for the first period is easy to derive. A low-cost firm will set a price equal to u, whereas a high-cost firm will set a price equal to $\alpha u + v$. Notice that the first-period price set by a low-cost firm is equal to its second period price, that is, introductory pricing does *not* occur. Also, the *expected* second-period price set by a high-cost firm is equal to the price set in the first period.

We construct equilibria with separation, that is, equilibria in which different types of sellers set different first-period prices. In equilibrium, if each seller sets the designated price, consumers will form posterior beliefs that assign probability 1 to the respective type. If any out-of-equilibrium price is set, then—we assume—consumers believe that the firm has high costs with probability 1.

Given this, the conditions for an equilibrium include: (i) A high-cost firm has no incentive to imitate a low cost firm by setting price at the level a low cost firm would choose, nor would a high-cost firm set any other price which consumers would interpret as indicating the firm has high costs; (ii) A low-cost firm has no incentive to set any price different from the designated price, given that it would then be believed to be a high-cost type.

Suppose that, in the first period, the high-cost firm sets a price $p^H = \alpha u + v$, and let p^L denote the first-period price set by the low-cost firm at the equilibrium we construct. To check condition (i), we have to consider two possible deviations for the high-cost firm. One is for it to set a price equal to the low-cost firm's equilibrium price. The equilibrium condition then becomes

$$\alpha(\alpha u + v - c + \alpha(u + v - c)) + (1 - \alpha)\alpha(v - c) \geq p^L - c + \alpha(u + v - c), \quad (36)$$

assuming that $p^L < u$. The left-hand side of (36) gives the expected profit for a high-cost firm which follows the candidate equilibrium strategy, whereas the right-hand side represents expected profit from imitating the low-cost firm: by setting a price equal to p^L, the high-cost firm is believed to have low cost; and, if $p^L < u$, then both types will buy at this price *if they believe the seller to have low costs*, as they do in equilibrium.

Specifically, let p^L be such that (36) holds as an equality.[15] Simplifying, one gets

$$p^L = \alpha(2\alpha - 1)u + \alpha v + (1 - \alpha)c, \tag{37}$$

so that the second part of (5) implies the equilibrium hypothesis $p^L < u$.

The second possible deviation price for the high-cost firm is αu. At this price, both types of consumers will buy in the first period. However, the first part of (5), together with (36), imply that this is not optimal: compare profit of $1(\alpha u - c) + \alpha(u + v - c)$ with the left hand side of (36).

Intuitively, the high-cost firm is following its first-best action in equilibrium. Therefore, if a buyer is to deviate, it must be that her posterior is affected by the seller's deviation, which only occurs if price is set equal to p^L.

To check Condition (ii), it suffices to consider two deviation values for the first-period price set by the low-cost firm: $p = \alpha u + v$ and $p = \alpha u$. The first possibility leads to the condition

$$p^L + u \geq \alpha(\alpha u + v + u) + (1 - \alpha)v, \tag{38}$$

which is implied by the second part of (5). The second possibility leads to

$$p^L + u \geq \alpha u + u, \tag{39}$$

[15]There exists a continuum of separating equilibria with different low-cost-firm prices p less or equal to p^L. The one we are considering is the unique separating equilibrium that survives Cho and Kreps' (1987) "intuitive" criterion. Note, however, that introductory pricing occurs in *any* separating equilibrium.

which is equivalent to the first part of (5).

Finally, Condition (37) verifies that the low-cost firm engages in introductory pricing. ■

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