CAPACITY-CONSTRAINED PRICE COMPETITION WHEN UNIT COSTS DIFFER

by

Raymond J. Deneckere[*]

and

Dan Kovenock**

December 1988
Revised May 1994

---

[*] Department of Economics, University of Wisconsin-Madison, Madison, Wisconsin 53706-1393.

**Department of Economics, Krannert School of Management, Purdue University, West Lafayette, IN 47907.

We are grateful to Joseph Harrington, Marie Thursby, Casper de Vries and, especially, William Novshek for helpful discussions and comments. Thomas Faith and Ioannis Tournas provided valuable research assistance. This paper was presented at the Winter Meetings of the Econometric Society in December 1988, the Midwest Mathematical Economics Conference in April 1989, the Sixteenth Annual Congress of the European Association for Research in Industrial Economics in August 1989, the European Meetings of the Econometric Society in September 1989, and in seminars at the Ecole Nationale des Ponts et Chaussées, Erasmus University Rotterdam, Indiana University, INSEAD, Texas A&M University, Tilburg University, the University of Bonn and the University of Florida. Deneckere acknowledges financial support through National Science Foundation Grant SES-8619012 and the Kellogg Graduate School of Management's Beatrice/Esmark Research Chair. Kovenock acknowledges financial support through Erasmus University Rotterdam, the Purdue Research Foundation, the Ford Motor Company Fund, and an Ameritech Foundation Summer Faculty Research Grant. The authors bear full responsibility for any errors or omissions.

"Capacity-Constrained Price Competition when Unit Costs Differ"

by

Raymond Deneckere and Dan Kovenock

Abstract

This paper characterizes the set of Nash equilibria in a price setting duopoly in which firms have limited capacity, and in which unit costs of production up to capacity may differ. Assuming concave revenue and efficient rationing, we show that the case of different unit costs involves a tractable generalization of the methods used to analyze the case of identical costs. However, the supports of the two firms' equilibrium price distributions need no longer be connected and need not coincide. In addition, the supports of the equilibrium price distributions need no longer be continuous in the underlying parameters of the model.

Two applications of our characterization are pursued. In the Kreps-Scheinkman model of capacity choice followed by Bertrand-Edgeworth price competition we show that, unlike in the case of identical costs, Cournot equilibrium capacity levels need not arise as subgame-perfect equilibria. The low-cost firm has greater incentive to price its rival out of the market than exists under Cournot behavior. Our second application is to the analysis of the effects of tariffs and quotas in a model in which a domestic market is supplied by a price setting duopoly consisting of a domestic and a foreign firm. We obtain a strong nonequivalence result.

I. Introduction

In recent years there has been a resurgence of interest in the Bertrand-Edgeworth model of capacity constrained price competition (see Kreps and Scheinkman (1983), Osborne and Pitchik (1986), and Allen and Hellwig (1986)). The impetus for this resurgence can be traced back to three separate sources. First, economists have realized that the Bertrand-Edgeworth model provides an attractive model of short-run competition. It models price formation directly, without making use of the (already overworked) auctioneer. And unlike in the Cournot model, capacity bottlenecks here do not directly constrain the strategic variables (Krishna, 1989b). Second, the Bertrand-Edgeworth model displays some features that make it attractive for industrial organization purposes. Its (typically) mixed strategy equilibria capture the observation that many markets display price fluctuations (Sweeney and Comanor (1989)) and/or persistent price dispersion (Varian (1980)). In addition, as emphasized by Ghemawat (1988), unlike models of quantity-based competition, the Bertrand-Edgeworth model does not force equality of competitors' prices, and hence admits interesting short-run share-profitability and concentration-profits relationships. Finally, the Bertrand-Edgeworth model provides an ideal framework for investigating international trade questions: capacity constraints are then interpreted as quotas or VER's (Krishna (1989a,b)), whereas differences in production costs may result from the imposition of tariffs.

As the last example illustrates, many applications of the Bertrand-Edgeworth model are impossible, or do not become interesting, unless differences in productive efficiency are allowed. Unfortunately, almost without exception, existing treatments have focused on the case where short run production costs are identical for all participants.[1] This paper demonstrates that the analysis of Bertrand-

---

[1] The one exception which we are aware of is the paper by Gelman and Salop (1983), which deals with the incentive of a cost-disadvantaged entrant to keep capacity small. Gelman and Salop assume that the low cost firm has enough capacity to serve the whole market, and that the high cost firm acts as a price

Edgeworth competition with different unit costs is indeed tractable, allowing interesting applications to be pursued. We characterize equilibrium profits and strategies for a class of aggregate demand functions yielding concave revenue, assuming demand is rationed efficiently. Our assumptions on demand are weaker than in Kreps and Scheinkman (1983), but avoid the complications that may arise when using the general demand of Osborne and Pitchik (1986).

Incorporating differences in productive efficiency involves a tractable generalization of the methods used to analyze the model with identical costs of production up to capacity. However, unlike in the case of identical production costs, under our demand assumptions the supports of the two firms' equilibrium price distributions need not be connected and need not coincide; they may differ by a single point. Somewhat surprisingly, we show that these phenomena appear over some portion of capacity space whenever unit costs are not identical and the low cost firm does not have a drastic cost advantage. We also show that the supports of the equilibrium price distributions need not be continuous in the underlying unit costs and capacities.

Two applications of our characterization are pursued, indicating how it provides a richer set of models with which to analyze economic phenomena. In the Kreps-Scheinkman model of simultaneous capacity choice followed by simultaneous price setting, we show that, unlike for the case of identical costs, Cournot equilibrium capacity levels (for a marginal cost equal to the sum of the marginal capacity cost and the unit cost of production up to capacity) need not arise as subgame perfect equilibria. Under Cournot, the low cost firm must assume that price will always adjust to clear all quantities supplied to the market. In contrast, in the two-stage game, when deciding on its optimal response to a given capacity of its rival, the low cost firm need not assume that price will adjust to clear all capacity available for supply to the

---

leader. Deneckere and Kovenock (1992) endogenize the sequencing of moves in the price setting subgame, and argue that over this region of capacity space the low cost firm should act as a price leader.

market. In fact, the low cost firm often finds it profitable to choose a high capacity level and price its less efficient opponent out of the market. As a result, subgame perfection may require nondegenerate mixed strategies at the capacity setting stage. However, for the case where the cost of capacity is negligible, we provide a necessary and sufficient condition for the Cournot outcome to arise.

Our second application is to the analysis of the effect of tariffs and quotas in an international trade model in which a domestic market is supplied by a price-setting duopoly consisting of a domestic and a foreign firm. A tariff acts to raise the foreign firm's unit costs up to capacity, and a quota acts to reduce the foreign firm's capacity. We derive a strong nonequivalence result. If a positive tariff (binding quota) is levied and the resulting equilibrium is one in which neither firm is driven entirely from the market, then there exists no binding quota (positive tariff) that generates the same equilibrium price distribution.

In Section II we present the basic model. Section III derives the rule for determining equilibrium profits. Section IV characterizes the regions of capacity space in which pure strategy equilibria exist. Under somewhat more stringent conditions on demand, it also characterizes equilibrium profits over the remainder of the capacity space. Section V derives the corresponding nondegenerate mixed strategy distributions. Section VI concludes with the applications.

II. The Model

Consider a market in which two firms produce a homogeneous good. Aggregate demand for the firms' output as a function of price is $d(p): R_+ \to R_+$. We assume that d(p) satisfies the following assumptions:

<u>A.1</u>: $\exists\ 0 < p_0 < \infty$ such that $d(p) > 0$ if $p < p_0$, and $d(p) = 0$ if $p \geq p_0$. The function $d(p)$ is continuous and strictly decreasing on $[0,p_0]$, and twice continuously differentiable on $(0,p_0)$. Furthermore, $pd(p)$ is strictly concave on $[0,p_0]$, with maximizer p^m.

Each firm i (i = 1,2) produces the good at a constant unit cost $0 \leq c_i < p_0$ up to a capacity level of $k_i > 0$. Note that, unlike previous treatments, we do not assume that production costs are the same for both firms. Since players compete in prices and may not be able to serve the entire market, we need to specify a rule that allocates demand in terms of the prices. Following the example of Levitan and Shubik (1972), Kreps and Scheinkman (1983), and Osborne and Pitchik (1986), we assume that demand is allocated efficiently.[2] Thus, if $p_i < p_j$ and $d(p_i) > k_i$, the demand facing firm j is $\max(0, d(p_j) - k_i)$. We also assume that in the case of a tie in prices, the low cost firm sells its capacity first.[3,4] Thus, if $p_i = p_j = p$ and $c_i < c_j$, the demand faced by firm j is $\max(0, d(p) - k_i)$.

Under these assumptions the profit to firm i when it sets price p_i and firm j sets price p_j is

$$(2.1)\quad \pi_i(p_i,p_j) = \begin{cases} L_i(p_i) \equiv (p_i-c_i)\min(k_i,d(p_i)), & \text{if } p_i < p_j \\ T_i(p_i) \equiv (p_i-c_i)\min(k_i, \max(0,d(p_i)-Ik_j)), & \text{if } p_i = p_j \\ H_i(p_i) \equiv (p_i-c_i)\min(k_i, \max(0, d(p_i)-k_j)), & \text{if } p_i > p_j \end{cases}$$

---

[2] Davidson and Deneckere (1986) discuss the merits of alternative rationing schemes.

[3] We make this assumption to simplify some of our proofs. In Section V we show that equilibrium profits are unaffected by the tie breaking rule, and that equilibrium distributions are altered only in the classical Bertrand region when the low cost firm does not have a drastic cost advantage.

[4] To break ties when $c_1 = c_2$, we arbitrarily let firm 1 sell its capacity first.

$\lfloor$

where I is an indicator that takes on the value 1 if $c_i > c_j$, or $c_1 = c_2$ and i = 2, and takes on the value 0 if $c_i < c_j$, or $c_1 = c_2$ and i = 1. Here $L_i(p_i)$ refers to the profit from being the low priced seller at p_i, $H_i(p_i)$ the profit from being the high priced seller at p_i, and $T_i(p_i)$ the profit from tying at p_i. Note that depending on the value of I, the function $T_i(p_i)$ will coincide with either L_i or H_i. The functions L_i and H_i are illustrated in Figure 1.

Let $S_i = [c_i, p_0]$ denote the pure strategy set of firm i, and Σ_i the corresponding set of mixed strategies (cumulative distribution functions on S_i). Note that we have ruled out strategies for firm i which involve pricing below its unit cost.[5] The domain of firm i's profit function can be extended in a natural way to Σ_i x Σ_j. For $F_j \in \Sigma_j$ define

$$(2.2) \qquad \pi_i(p,F_j) = H_i(p)(F_j(p)-\alpha_j(p)) + T_i(p)\alpha_j(p) + L_i(p)(1 -F_j(p))$$

where $\alpha_j(p)$ is the size of the masspoint in F_j at p (if one is present). Let $J(F_j) = \{p : \alpha_j(p) > 0\}$ be the set of jumppoints in F_j, for j=1,2. Finally let $\Pi_i(F_i, F_j) =$ Install Equation Editor ar
click here to view equatio

For any quadruple (k_1,k_2,c_1,c_2) with $k_1 > 0$ and $k_2 > 0$ we will now analyze the normal form game $G(k_1,k_2,c_1,c_2)$ with strategy sets Σ_i and payoff functions $\Pi_i(F_i, F_j)$.

III. Uniqueness of the Equilibrium Profits in $G(k_1, k_2, c_1, c_2)$

---

[5] Although any such price is weakly dominated by setting c_i this assumption is not completely innocuous. See footnote 7 for details.

Theorem 5 of Dasgupta and Maskin (1986) guarantees the existence of a mixed strategy Nash equilibrium to the game $G(k_1,k_2,c_1,c_2)$. The only potentially problematic condition of this theorem, the upper semi-continuity of $\pi_1(p_1,p_2) + \pi_2(p_1,p_2)$ in (p_1,p_2), holds because the sum is continuous at all off-diagonal points and because along the diagonal our tie breaking rule minimizes the total cost of providing the good. Since revenue is continuous, this means that in approaching a point on the diagonal total profit cannot jump down.[6] In the remainder of this section we characterize the Nash equilibrium profits of the game $G(k_1,k_2,c_1,c_2)$ for all vectors (k_1,k_2,c_1,c_2) and show that they are uniquely determined.

We first need to establish some notation. Let P be the set of prices that maximize $L_i(p_i)$ and P be the set of prices that maximize $H_i(p_i)$. Given our assumptions on demand P consists of the singleton p , and $L_i(p_i)$ is continuous and strictly increasing in p_i for $p_i < p$. Let $H= H_i(P)$. If H is nonzero P consists of the singleton p, and $H_i(p_i)$ is continuous and strictly increasing in p_i for $p_i < p$. If $H = 0$ we define $p \equiv c_i$.

Note that p and H depend on the capacity pair (k_1,k_2). However, for k_i sufficiently large, firm i is no longer capacity constrained when it is highest priced at p. More specifically, let P(q) be the inverse

---

[6] It is interesting to note that without the rule which allows the low-cost firm to sell its capacity first in the event that $p_1 = p_2$, neither the Dasgupta and Maskin (1986) existence theorems nor the existence results of Simon (1987) apply. In particular, with other tie breaking rules the sum of the two firms' profits will generally not satisfy the D-M requirement of upper semi-continuity in (p_1, p_2). The expected profit functions of the two firms will also generally not satisfy Simon's (1987) "complementary discontinuity" property on $\Sigma_1 \times \Sigma_2$ (for a counterexample see footnote 4 of Deneckere and Kovenock (1989)). D-M's Theorem 5b proves existence using a related complementary discontinuity assumption on $S_1 \times S_2$. However, the theorem requires the strategy spaces to coincide making it inapplicable in our context. Enlarging the high cost firm's strategy set to equate it with the low cost's causes the complementary discontinuity property to be violated. Despite the fact that these conditions are violated, we show in Section V that an equilibrium of $G(k_1,k_2,c_1,c_2)$ exists regardless of the details of the tie breaking rule.

demand function corresponding to $d(p)$, and let $r_i(k_j|c_i) \equiv \text{argmax}_x\, (P(x + k_j) - c_i)x$ be firm i's Cournot best response function. For $k_i \geq r_i(k_j|c_i)$ the functions p and H become independent of k_i. We will denote the unconstrained maximizer and maximum of H_i as $e_i(k_j) \equiv p(\infty,k_j)$ and $E_i(k_j) \equiv H(\infty,k_j)$.

Let $(F_1(p),F_2(p))$ be a pair of equilibrium price distributions. Let $s_i = \inf\{p: F_i(p) = 1\}$ and $\underline{s}_i = \sup\{p: F_i(p) = 0\}$ be the bounds of the support of F_i, i = 1,2. From our restriction on the strategy spaces, $c_i \leq \underline{s}_i \leq s_i \leq p_0$, i = 1,2.

We first demonstrate the important result that in Bertrand-Edgeworth equilibrium at least one firm must be held down to its minmax profit level.

<u>Lemma 1</u>: There exists a firm i such that $\Pi_i(F_1,F_2) = H$.

<u>Proof</u>: Since $0 \leq H_i(p) \leq \Pi_i(p,F_j) \leq \Pi_i(F_1,F_2)$ for all $p \in S_i$ we know $\Pi_i(F_1,F_2) \geq H$, i = 1,2. Suppose $s_i > s_j$, or $s_1 = s_2 = s$ and firm j has no mass point at s. Since the probability of a tie at s_i is zero, $\Pi_i(s_i, F_j) = H_i(s_i) \leq H$. Since $\Pi_i(s_i, F_j) = \Pi_i(F_1,F_2)$, the two inequalities imply $\Pi_i(F_1,F_2) = H$. Next, suppose $s_1 = s_2 = s$, both firms have mass points at s, but $\Pi_i(F_1,F_2) > H$ for each i. Since $H \geq 0, s > \max\{c_1,c_2\}$. For neither firm to want to place its masspoint slightly below s we must have $L_i(s)=T_i(s)$ for each i, or $s \leq P(k_1+k_2)$. This implies that $\Pi_i(F_1,F_2) = (s - c_i)k_i \leq H$, contradicting $\Pi_i(F_1,F_2) > H$. #

In order to determine the equilibrium payoffs, define

$$\underline{p}_i = \min\{p_i: L_i(p_i) = H\}, \quad i = 1,2.$$

Note that $\underline{p}_i \leq p < p$, $i = 1,2$. Since $0 \leq H_i(p) \leq \Pi_i(p,F_j) \leq \Pi_i(F_1,F_2)$ for all $p \in S_i$ we know $\Pi_i(F_1,F_2) \geq H$, $i = 1,2$. In equilibrium firm i will therefore never set a price below $\underline{p}_i$.

<u>Theorem 1</u>: Let $\underline{p}_j \geq \underline{p}_i$. Then $\Pi_j(F_1,F_2) = H$. If $H > 0$ then $\Pi_i(F_1,F_2) = L_i(\underline{p}_j)$. If $H = 0$ then $\Pi_i(F_1,F_2) = \max_{p\in[c_i,c_j]} L_i(p)$.

<u>Proof</u>: If $\Pi_j(F_1,F_2) > H$ then $\underline{s}_j > \underline{p}_j$, implying $\Pi_i(F_1,F_2) > L_i(\underline{p}_j) \geq H$, a contradiction to Lemma 1. If $H > 0$ and $\Pi_i(F_1,F_2) > L_i(\underline{p}_j)$ then $\underline{s}_i > \underline{p}_j$, implying $\Pi_j(F_1,F_2) > L_j(\underline{p}_j) = H$, contradicting $\Pi_j(F_1,F_2) = H$. If $H = 0$ then $\Pi_j(F_1,F_2) = 0$ so that $F_i(c_j) = 1$ and $c_i \leq c_j$. But then firm i must place all mass on $\operatorname{argmax}_{p\in[c_i,c_j]} L_i(p)$[7]. #

Theorem 1 establishes that in order to determine equilibrium profits we only need to calculate $\underline{p}_1$ and $\underline{p}_2$, and rank them. While this is also true in the special case of identical costs, to our knowledge the first explicit definition and comparison of these constructs appear in Deneckere and Kovenock (1992). This is perhaps because, in the case of identical costs, for each pair (k_1,k_2) with $k_2 > k_1$, $\underline{p}_2 \geq \underline{p}_1$ with

---

[7] When $\underline{p}_j > \underline{p}_i$ and $H = 0$, our restriction that Σ_j include only prices at or above c_j is essential in pinning down $\Pi_i(F_1,F_2)$. If firm j could set prices below c_j, and the conditions $P(k_i) < c_j$ and $P(k_i) < p$ hold, there would exist a continuum of equilibria in which firm i receives an equilibrium profit less than $\operatorname{Max}_{p\leq c_j} L_i(p)$. Any price p_i of firm i between $\max(P(k_i),\underline{p}_i)$ and $\min(p,c_j)$ could be supported as a pure strategy equilibrium price for firm i if j put enough mass at, or in every neighborhood above, $\underline{p}$. We rule out these equilibria because in discrete approximations to the game they would require that firm j play a weakly dominated strategy.

equality if and only if there is a pure strategy equilbrium in the simultaneous move game. This property does not carry over to the case of different unit costs (as is demonstrated in Section IV).

IV. The Ranking of the $\underline{p}_i$

The critical prices $\underline{p}_i$ defined in Section III are functions of (k_1,k_2,c_1,c_2). Without loss of generality we assume that $c_1 \leq c_2$ and analyze the dependence of these prices on k_1 and k_2. Figure 2 illustrates the ranking of the $\underline{p}_i$ for the case where $r_1(0|c_1) < d(c_2)$, i.e. firm 1's unconstrained monopoly price is greater than c_2. Figure 3 illustrates the case where firm 1 has a drastic cost advantage, i.e. $r_1(0|c_1) \geq d(c_2)$.

Let $A = \{(k_1,k_2) \in R^+ : k_1 \leq r_1(k_2|c_1) \text{ and } k_2 \leq r_2(k_1|c_2)\}$ be the region bounded by the lower envelope of the Cournot reaction functions. Let $C = \{(k_1,k_2) \in R^+ : k_1 \geq d(c_2) \text{ and } k_2 \geq \phi_2(c_1,c_2)\}$ be the region where firm 1 has enough capacity to drive firm 2 out of the market and prefers doing so to selling to residual demand after firm 2 has sold its capacity k_2. Thus $\phi_2(c_1,c_2) = \min \{k_2 \geq 0 : E_1(k_2) \leq \max_{p\leq c_2} (p-c_1)d(p)\}$. Note that (A.1) implies that regions A and C are nonempty and do not intersect.

Theorem 2 shows that a pure strategy equilibrium exists only in regions A and C. In region A, capacities are so low that firms prefer to sell everything they can produce, so that price adjusts to clear the market. In this region $\underline{p}_1 = \underline{p}_2 = p = p = P(k_1+k_2)$. Region C is the classical Bertrand region, in which $\underline{p}_1 \leq c_2 = \underline{p}_2 = p$ and firm 2 is priced out of the market. The equilibrium over this region is unique only if $c_1 = c_2$:

<u>Theorem 2</u>: Suppose (A.1) holds and $c_1 \leq c_2$. Then a pure strategy equilibrium exists if and only if capacities lie in regions A or C. In region A equilibrium occurs necessarily in pure strategies, with both firms charging $P(k_1+k_2)$. In region C, if $p \leq c_2$ then in any equilibrium firm 1 sets p with probability one and firm 2 uses any strategy placing all mass at or above c_2. If $p > c_2$ then in any equilibrium firm 1 sets

c_2 with probability one and firm 2 uses any strategy which deters firm 1 from raising price; one such strategy is charging c_2 with probability one.

Proof: Region A is characterized by $p = P(k_1+k_2)$ for $i = 1,2$ and so both firms have $H = k_i(P(k_1+k_2)-c_i)$. Consequently, $\underline{p}_1 = \underline{p}_2 = P(k_1+k_2)$ and by Theorem 1, $\Pi_i(F_1,F_2) = H = k_i(P(k_1+k_2)-c_i)$. Suppose $s = \max \{s_1,s_2\} > P(k_1+k_2)$. As argued in the proof of Lemma 1 the probability of a tie at s equals zero so there exists a firm i such that $\Pi_i(F_1,F_2) = H_i(s)$. Now $s > P(k_1+k_2)$ implies $H_i(s) < H$, a contradiction. Since $\underline{s}_i \geq \underline{p}_i$, we conclude that in region A both firms have $\underline{s}_i = s_i = P(k_1+k_2)$.

In region C, we claim that $\underline{p}_2 = c_2 \geq \underline{p}_1$. Theorem 1 then yields $\Pi_2(F_1,F_2) = H = 0$ and $\Pi_1(F_1,F_2) = \max_{p\in[c_1,c_2]} L_1(p)$, immediately implying the desired result. The equality $\underline{p}_2 = c_2$ holds because in region C $k_1 \geq d(c_2)$. If $p \leq c_2$ then $\underline{p}_1 \leq p \leq c_2$. If $p > c_2$, then either $\underline{p}_1 \leq P(k_1) \leq c_2$, or $\underline{p}_1 > P(k_1)$. In the latter case, $L_1(\underline{p}_1) = (\underline{p}_1 - c_1)\, d(\underline{p}_1) = H(k_1,k_2) = E_1(k_2) \leq (c_2-c_1)d(c_2)$. This implies $\underline{p}_1 \leq c_2$ since $L_1(p)$ is increasing below p.

Suppose a pure strategy equilibrium with $p_i < p_j$ exists for (k_1,k_2) C. Since $L_i(p)$ must be maximal at p_i and $p \geq P(k_i)$, $\Pi_j(F_1,F_2) = 0$. For firm j to be maximizing it must be that $p \leq c_j$. Together these inequalities imply that $i = 1$ and $\underline{p}_1 \leq c_2 = \underline{p}_2$, contradicting (k_1,k_2) C.

Finally, suppose there exists a pure strategy equilibrium with $p_1 = p_2 = p$ for (k_1,k_2) $A\cup C$. Outside of region $A\cup C$ there necessarily exists i such that $p > \underline{p}_i > P(k_1 + k_2)$. Outside region C either $k_1 < d(c_2)$ implying $\underline{p}_2 > c_2$ or $k_1 \geq d(c_2)$ and $\underline{p}_1 > c_2$. We conclude that $p \geq \max \{P(k_1 + k_2),c_2\}$. But then some firm can gain by slightly undercutting p. #

In region B = $R^+ \setminus A \cup C$ equilibria necessarily occur in nondegenerate mixed strategies. Note that (A.1) implies that B is nonempty. In this region, the ranking of the $\underline{p}_i$'s is greatly simplified when reaction functions in the quantity setting game with constant marginal costs of production c_i are downward sloping and the game has a unique Cournot equilibrium $k^c = (k,k)$. Since A.1 is insufficient to guarantee this, we will henceforth slightly strengthen the concavity of revenue pd(p):

<u>A.2</u>: The function $d'(p) + pd''(p)$ is strictly negative on $(0,p_0)$.

Under A.2, Theorem 3 below shows that there exists a curve $(k_1,\theta(k_1))$ passing through k^c and the southwest boundary of region C. Capacity pairs in region B for which $k_2 < \theta(k_1)$ have $\underline{p}_1 > \underline{p}_2$, so that in equilibrium the low cost firm is held to its minmax profit level. Capacity pairs in region B for which $k_2 > \theta(k_1)$ have $\underline{p}_2 > \underline{p}_1$.

<u>Theorem 3</u>: Suppose (A.1) and (A.2) hold, and suppose $c_1 < c_2$. Then:

(i) If $r_1(0|c_1) < d(c_2)$, there exists a continuous function $\theta: [0,\infty) \rightarrow [0,d(c_1)]$ such that $\underline{p}_2 \ \underline{p}_1$ whenever $k_2 \ \theta(k_1)$ and $(k_1,k_2) \ A$. Furthermore, the function θ satisfies $\theta(k_1) = r_2(k_1|c_2)$ for $k_1 \in [0,k]$, $r_2(k_1|c_2) < \theta(k_1) < k_1$ for $k_1 \in (k,d(c_2)]$, and $\theta(k_1) = \phi_2(c_1,c_2)$ for $k_1 \in [d(c_2),\infty)$.

(ii) If $r_1(0|c_1) \geq d(c_2)$, then $\underline{p}_2 > \underline{p}_1$ whenever $(k_1,k_2) \ A$.

<u>Proof</u>: See Appendix A.

Figure 2 illustrates the function $\theta(k_1)$ for the case of linear demand. The functions $\psi_i(k_j,c_i)$, i = 1,2, indicated in the figure are defined by $\psi_i(k_j,c_i) = \max_q \{[P(q) - c_i]q = E_i(k_j)\}$. Alternatively, $\psi_i(k_j,c_i)$ is defined implicitly by $d(\underline{p}_i) = k_i$, so that for $k_i \leq \psi_i(k_j,c_i)$ firm i is capacity constrained at $\underline{p}_i$. In the case of identical unit costs up to capacity, analyzed by Kreps and Scheinkman (1983), Osborne and Pitchik (1986), and Deneckere and Kovenock (1992), the curves $\psi_1(k_2,c)$ and $\psi_2(k_1,c)$ intersect along the diagonal, as do the curves $r_1(k_2|c)$ and $r_2(k_1|c)$. The curve $\theta(k_1)$ then coincides with the diagonal for $k_1 \in [k,d(c)]$, and $\phi_2(c,c)$ coincides with the vertical line $k_2 = d(c)$.

When firm 1 has a drastic cost advantage, i.e. $r_1(0|c_1) \geq d(c_2)$, and A.2 holds, the Cournot best reply functions do not intersect in the positive quadrant, as is illustrated in Figure 3. This greatly facilitates the analysis since then $\underline{p}_2 > \underline{p}_1$ for all (k_1,k_2) A.

V. Equilibrium Distributions

In order to characterize the nondegenerate mixed strategy equilibria in region B, we first bound the supports of the equilibrium price distributions. To understand Lemma 2 below, note that in region B it is necessarily the case that $\underline{p}_j \geq \underline{p}_i$ implies $p > \underline{p}_j$.[8]

Lemma 2: Suppose (A.1) holds and $(k_1,k_2) \in$ B. Then $\underline{p}_j \geq \underline{p}_i$ implies $\underline{s}_1 = \underline{s}_2 = \underline{p}_j$. Furthermore $\underline{p}_j > \underline{p}_i$ or $\underline{p}_1 = \underline{p}_2$ and $p \leq p$ imply $s_i \leq p = s_j$.

Proof: In region B, $H > 0$ so that $p \geq P(k_i) > p > \underline{p}_j$. Hence $L_j(p)$ is increasing on $[\underline{p}_j,p]$ and $L_i(p)$ is increasing on $[c_i,p]$. By Theorem 1 $\Pi_i(F_1,F_2)= L_i(\underline{p}_j)$; since L_i is increasing on $[c_i,\underline{p}_j]$, $\underline{s}_i \geq \underline{p}_j$. Now $\underline{s}_i > \underline{p}_j$ would imply the contradiction $\Pi_j(F_1,F_2) \geq L_j(p) > H$ for some $p \in (\underline{p}_j,\underline{s}_i)$, so that $\underline{s}_i = \underline{p}_j$. Finally, $\underline{s}_j < \underline{p}_j$

---

[8] The inequality $p = \underline{p}_j \geq \underline{p}_i$ implies (k_1,k_2) B. Indeed, if $p = \underline{p}_j$ then $L_j(p) = H_j(p)$; by (2.1) either $p = \underline{p}_j = P(k_1+k_2) \geq \underline{p}_i$ or $p = \underline{p}_j = c_j \geq \underline{p}_i$. The first inequality implies $p = \underline{p}_i = P(k_1+k_2)$, so that $(k_1,k_2) \in$ A, while the second inequality is equivalent to $(k_1,k_2) \in$ C.

yields the contradiction $\Pi_j(F_1,F_2) = L_j(\underline{s}_j) < H$, and $\underline{s}_j > \underline{p}_j$ yields the contradiction $L_i(\underline{p}_j) = \Pi_i(F_1,F_2) \geq \Pi_i(\underline{p}_j + \varepsilon,F_j) = L_i(\underline{p}_j+\varepsilon)$, for sufficiently small $\varepsilon > 0$.

If $\underline{p}_j > \underline{p}_i$ and $s_j < s_i$ then $\Pi_i(F_1,F_2) = H_i(s_i) \leq H$, contradicting Theorem 1. Since $\Pi_i(F_1,F_2) > H$ and $s_j \geq s_i$, it must be that $\Pi_j(F_1,F_2) = H_j(s_j)$ so $s_j = p$ (If $s_j = s_i$ this follows from our tie breaking rule).

Finally, suppose $\underline{p}_1 = \underline{p}_2$ and $p \leq p$. First observe that $s_i > p$ yields the contradiction $\Pi_j(F_1,F_2) \geq \Pi_j(p,F_i) > H$. Now $s_i \leq p$ implies $s_j \leq p$, for $s_j > p$ implies the contradiction $\Pi_j(F_1,F_2) = H_j(s_j) < H$. Suppose $s_j < p$. Since L_i and H_i are increasing on $[s_j,p)$, F_i places no weight on $[s_j,p)$ (with our tie breaking rule there cannot be common mass points at s_j). This yields the contradiction $\Pi_j((s_j + p)/2,F_i) > \Pi_j(s_j,F_i) = \Pi_j(F_1,F_2)$. #

Let j be such that $\underline{p}_j > \underline{p}_i$ or $\underline{p}_1 = \underline{p}_2$ and $p \leq p$. We have just shown that the supports of the equilibrium price distributions are contained in $[\underline{p}_j,p]$. To construct F_j, let $\Pi \equiv \Pi_i(F_1,F_2) = L_i(\underline{p}_j)$, and for $p \in [\underline{p}_j,p]$ define

$$Q_j(p;\Pi) = (L_i(p) - \Pi)/(L_i(p) - H_i(p)).$$

The inequalities $c_i \leq \underline{p}_i \leq \underline{p}_j < p < P(k_i) \leq p$ imply that on the above interval $L_i(p) > H_i(p)$ and $L_i(p) \geq L_i(\underline{p}_j) = \Pi \geq H_i(p)$, so that $0 \leq Q_j(p;\Pi) \leq 1$. Now for all p in the support of F_i except possibly a set of F_i - measure zero, N_i, we have $\Pi_i(p,F_j) = \Pi$. Thus, if p is not a mass point of F_j, then $p \in (\text{supp } F_i)\backslash N_i$ and (2.2) imply that $F_j(p) = Q_j(p;\Pi)$. Moreover, for p $(\text{supp } F_i)\backslash N_i$, we must have $F_j(p) \geq Q_j(p;\Pi)$. Setting

$$F_j(p) = \begin{cases} 0 & p < \underline{p}_j \\ Q_j(p; \Pi) & \underline{p}_j \le p < p \\ 1 & p \ge p \end{cases}$$

would yield an equilibrium strategy for firm j if, given firm i's equilibrium strategy, it received an expected profit of Π at all points in the interval $[\underline{p}_j, p]$, and if $Q_j(p; \Pi)$ were nondecreasing in p. Similarly, $F_i(p)$ defined in an analogous fashion would yield an equilibrium strategy for firm i if it received an expected profit of Π at all points in $[\underline{p}_j, p)$ given firm j's equilibrium strategy, and if $Q_i(p; \Pi)$ were nondecreasing in p. Unfortunately, while it is easily shown that $Q_i(p; \Pi)$ must be increasing on $[\underline{p}_j, p]$, with $Q_i(\underline{p}_j; \Pi) = 0$ and $Q_i(p; \Pi) = 1$, when $c_1 < c_2$ it is not always the case that $Q_j(p; \Pi)$ is nondecreasing :

Lemma 3: Suppose (A.1) holds and $(k_1,k_2) \in B$. Let j be such that $\underline{p}_j > \underline{p}_i$ or $\underline{p}_1 = \underline{p}_2$ and $p \le p$. Then $Q_i(p)$ satisfies the following properties:

(a) $Q_i(\underline{p}_j) = 0$, $Q_i(p) = 1$.

(b) Q_i is differentiable at every point in $(\underline{p}_j, p)$, except at $P(k_j)$, when $P(k_j) \in (\underline{p}_j, p)$.

(c) Q_i is strictly increasing on $[\underline{p}_j, p)$.

(d) Q_i is concave on the interval $[\underline{p}_j, p]$ and is twice continuously differentiable except at $P(k_j)$, when $P(k_j) \in (\underline{p}_j, p)$. In that case, $\lim_{p\uparrow P(k_j)} dQ_i/dp > \lim_{p\downarrow P(k_j)} dQ_i/dp$.

The function $Q_j(p)$ satisfies the following properties:

(e) $Q_j(\underline{p}_j) = 0$, and $Q_j(p) \le 1$, with equality if and only if $\underline{p}_1 = \underline{p}_2$ and $p = p$.

(f) Q_j is differentiable at every point in $(\underline{p}_j,p)$, except at $P(k_j)$ when $P(k_j) \in (\underline{p}_j,p)$. In that case, $\lim_{p\downarrow P(k_j)} dQ_j/dp > 0$ and $\lim_{p\uparrow P(k_j)} dQ_j/dp < \lim_{p\downarrow P(k_j)} dQ_j/dp$.

(g) Q_j is strictly increasing on the interval $[\underline{p}_j,p]$ except possibly on a single subinterval of the form $[p\ ,b]$ where $p \geq \max\{\underline{p}_j,p\}$ with equality only if $p = p > \underline{p}_1 = \underline{p}_2$, and $b = \min[P(k_j),p]$. On (p,b) Q_j is decreasing. If $P(k_j) \leq \underline{p}_j$, Q_j is increasing everywhere on $[\underline{p}_j,p]$.

(h) Q_j is locally concave where it is nondecreasing and differentiable.

(i) A necessary but not sufficient condition for Q_j to decrease in the interval is that $c_j < c_i$.

Proof: See Appendix B. #

Intuitively, Lemma 3 can be understood as follows. Q_i is the probability with which j must be undercut in order to give him equilibrium profit over $[\underline{p}_j, p]$. Since H_j and L_j are increasing over this range, and since $H_j < L_j$, Q_i must increase in order to make profit constant. Now over $(\underline{p}_j, b)$, the range where Q_j could turn down, $Q_j = [(k_i/k_j)(p-c_j)/(p-c_i)]Q_i$. When $c_1 = c_2$, this implies that Q_j must increase whenever Q_i is increasing. When $c_1 < c_2$ and $j = 1$, however, the same equation shows that Q_1 must be decreasing whenever Q_2 is flat, a condition that holds at $p = p$.

Lemma 3 facilitates the construction of a pair of equilibrium strategies. Following Osborne and Pitchik (1986), let

$$IQ_j(p;\Pi) = \max_{\underline{p}_j \leq x \leq p} Q_j(x;\Pi)$$

be the nondecreasing cover of Q_j on $[\underline{p}_j,p]$. Lemma 3(g) implies that IQ_j equals Q_j except possibly on an interval of the form (p,p) or $(p,p]$, wherep $= \sup \{p \leq p \leq p : Q_j(p) \leq Q_j(p)\}$. Then the strategy

$$(5.1) \qquad F_j(p) = \begin{cases} 0 & p < \underline{p}_j \\ IQ_j(p;\Pi) & \underline{p}_j \leq p < p \\ 1 & p \geq p \end{cases}$$

is an equilibrium strategy for firm j. To understand why $F_j(p)$ is an equilibrium strategy note first that $F_j(p)$ is nondecreasing, nonnegative, right-continuous, and is less than or equal to one for all p. It is therefore a strategy. When firm i sets a price $p \in [\underline{p}_j, p]$ for which $Q_j(p;\Pi) = IQ_j(p;\Pi)$ it earns its equilibrium profit. If firm i sets a price $p \in [\underline{p}_j, p]$ for which $Q_j(p;\Pi) < IQ_j(p;\Pi)$ it earns strictly less than its equilibrium profit. Thus, no such price will be set by firm i. From Lemma 3 we already know that $Q_i(p;\Pi)$ is increasing over $[\underline{p}_j,p]$. Given F_j, if firm i were indifferent between all prices in the interval, Q_i would be an equilibrium strategy, since it makes j indifferent between all prices in the interval, and earns a strictly lower profit elsewhere. However, since $F_j(p)$ may be strictly greater than $Q_j(p;\Pi)$ for some $p \in [\underline{p}_j, p]$ firm i may not be indifferent; it will attach zero measure to the set of prices for which a strict inequality holds. Since firm j also attaches zero probability to intervals where the strict inequality holds (except at p, which may be a mass point) we know that $F_i(p) \geq Q_i(p;\Pi)$ over these intervals. Since firm i must set $F_i(p) = Q_i(p;\Pi)$ at points in the support of $F_j(p)$, in order to remain an admissible strategy $F_i(p)$ must place a mass point atp , the size of which equals the increase in $Q_i(p;\Pi)$ over the interval $[p,p]$. Formally, $F_i(p)$ is an equilibrium strategy for firm i if :

$$(5.2)\qquad F_i(p) = \begin{cases} 0 & p < \underline{p}_j \\ Q_i(p;\Pi) & p \in [\underline{p}_j, p]\backslash[p,p) \\ Q_i(p;\Pi) & p \in [p,p) \\ 1 & p > p \end{cases}$$

When $c_1 = c_2$, Lemma 3(e) and (i) imply that the supports of the equilibrium distributions coincide, and are equal to the interval $[\underline{p}_j, p]$. In addition, only firm j can have a masspoint, at p. As the above discussion suggests, with $c_1 < c_2$ the equilibrium supports need no longer be connected and need not coincide. Indeed, Lemma 3(g) and (i) show that two types of gaps may arise over the range of capacities where $\underline{p}_1 > \underline{p}_2$. If $p < p$ the supports of the equilibrium distributions coincide, but have a common gap (p,p) inside the interval $(\underline{p}_1, p)$. When $p < p = p$, firm 2 has a connected support, but firm 1's support has a gap caused by an isolated masspoint at p. In this case the two supports differ by the point p.[9] The construction of the equilibrium distributions also shows that in either of these cases firm 2 has a

---

[9] In Osborne and Pitchik's (1986) analysis (which has more general assumptions on demand but identical unit costs) the supports of the equilibrium distributions also need not be connected and need not coincide. However, Osborne and Pitchik themselves describe these possibilities as "degeneracies." Indeed, without concave revenue the degeneracies can be very extreme, such as equilibrium supports consisting of a countably infinite union of disjoint intervals! With concave revenue and identical unit costs, all degeneracies disappear (see Osborne and Pitchik (1986, p. 247)); the supports of the equilibrium distributions are connected. Thus, our analysis shows that the introduction of different unit costs in and of itself can make the structure of the equilibria more complicated (yet still tractable); quite unlike in the case of identical unit costs there is no reasonable assumption on demand which would make the complications described in our model disappear.

masspoint at p and firm 1 may have a masspoint at p. When unit costs differ both firms thus may have a masspoint in their distributions (though each firm has at most one).

In our model, the occurrence of disconnected and nonidentical supports is not a degeneracy. As Theorem 4 below shows, whenever firm 1 does not have a drastic cost advantage and $c_1 < c_2$, there exist open sets in capacity space where each phenomenon appears, even when using strong restrictions on demand (such as linearity). To state the theorem, we first partition region B into four subregions, depending upon the nature of the equilibrium supports. Let $\eta(k_2) = \{k_1 : P(k_1) = e_1(k_2)\}$, and define

$$B_1 = \{(k_1,k_2) \in B : k_1 < d(c_2),\ k_2 \geq \theta(k_1)\}$$

$$B_2 = \{(k_1,k_2) \in B : \eta(k_2) \geq k_1 > r_1(k_2|c_1),\ k_2 < \theta(k_1)\}$$

$$B_3 = \{(k_1,k_2) \in B : \psi_1(k_2) > k_1 > \eta(k_2),\ k_2 < \theta(k_1)\}$$

$$B_4 = \{(k_1,k_2) \in B : k_1 \geq \psi_1(k_2),\ \phi_2(c_1,c_2) > k_2 > 0\}$$

Figure 4 illustrates the different regions for the case of linear demand. We can now state the main result of this section :[10]

<u>Theorem 4</u> : Suppose (A.1) and (A.2) hold and $c_1 < c_2$. Then for $(k_1,k_2) \in B$ the equilibrium distributions are unique and given by (5.1) and (5.2). If $r_1(0|c_1) < d(c_2)$ the supports of the equilibrium distributions are as follows :

---

[10] Lemmas 2 and 3, the construction of the equilibrium distributions in (5.1) and (5.2), and the uniqueness of the equilibrium distributions in region B (proven in Appendix C) all rely on assumption (A.1) only. In Theorem 4, we make assumption (A.2) only to simplify the mapping from capacity pairs to properties of the equilibrium supports.

(i) In B_1, supp F_1 = supp $F_2 = [\underline{p}_2, p]$.

(ii) In B_4, supp F_1 = supp $F_2 = [\underline{p}_1, p]$.

(iii) In B_2, supp $F_2 = [\underline{p}_1, p]$ and supp $F_1 = [\underline{p}_1, p] \cup \{p\}$.

(iv) Region B_3 can be further partitioned into three subsets with nonempty interiors.

(a) In B_{31}, supp $F_2 = [\underline{p}_1, p]$ and supp $F_1 = [\underline{p}_1, p] \cup \{p\}$.

(b) In B_{32}, supp F_1 = supp $F_2 = [\underline{p}_1, p] \cup [\hat{p}, p]$, with $\hat{p} < p$.

(c) In B_{33}, supp F_1 = supp $F_2 = [\underline{p}_1, p]$.

If $r_1(0|c_1) \geq d(c_2)$ then supp F_1 = supp $F_2 = [\underline{p}_2, p]$ for all $(k_1,k_2) \in B$.

Proof: We have already shown that (5.1) and (5.2) constitute an equilibrium; uniqueness is argued in Appendix C.

Define $\Theta = \{(k_1,k_2) : k_2 = \theta(k_1) \text{ and } d(c_2) > k_1 > k\}$. Suppose there existed $(k_1,k_2) \in \Theta$, such that $p < p$. Note that (k_1,k_2) C and Theorem 3(i) imply $p < P(k_1) < P(k_2)$. By Lemma 3(a), $Q_2(p) = 1$, and by Theorem 1 and equation (B.1) of Appendix B, $Q'(p) = 0$. Equation (B.6) of Appendix B then implies $Q'(p) = [(k_2/k_1)(c_1-c_2)/(p-c_2)^2] < 0$, a contradiction to Lemma 3(g). We conclude that $p \leq p$ for all $(k_1,k_2) \in \Theta$. Now for $(k_1,k_2) \in B_1 \setminus \Theta$, Theorem 3(i) implies $\underline{p}_2 > \underline{p}_1$. Part(i) now follows from Lemma 3(c) and 3(i).

Part (ii) follows from the fact that in region B_4 the inequality $P(k_1) \leq \underline{p}_1$ holds, Theorem 3(i), and Lemma 3(c) and (g).

To prove part (iii) observe that in region B_2 the inequality $\underline{p}_1 > \underline{p}_2$ holds, and so $\Pi = H_1(p)$. In conjunction with $H'(p) = 0$ formula (B.1) of Appendix B then yields $\lim_{p \uparrow p} dQ_2(p)/dp = 0$. Now in region B_2 $P(k_1) \geq p$, so that equation (B.6) of Appendix B yields $\lim_{p \uparrow p} dQ_1(p)/dp = [(k_2/k_1)(c_1-c_2)/(p -$

$c_2)^2]Q_2(p) < 0$. Part (g) of Lemma 3 then implies that there exists $p \in (\underline{p}_1,p)$ such that Q_2 is increasing on $[\underline{p}_1,p)$ and decreasing on $(p,p]$. By construction of the set I, $p = \inf I$.

For part (iv), let $k = \min \{\theta(k_1), k_1 \in [k,d(c_2)]\}$. Observe that $k > 0$ and that for $k_2 \in (0,k)$, $(k_1,k_2) \in B_3$ if and only if $k_1 \in (\eta(k_2),\psi_1(k_2))$. For fixed $k_2 \in (0,k)$ define $h(k_1) = \lim_{p\uparrow P(k_1)} Q'(p)$. Note that by Lemma 3(g), a necessary and sufficient condition for Q_1 to turn down in the interval $[\underline{p}_1,p]$ is that $h(k_1) < 0$, since in region B_3 we have $\underline{p}_1 < P(k_1) < p$. We will now examine the behavior of $h(k_1)$ on $(\eta(k_2),\psi_1(k_2))$.

First, observe that $k_1 \rightarrow \psi_1(k_2)$ implies $P(k_1) \rightarrow \underline{p}_1$. By equation (B.4) of Appendix B, $h(k_1) \rightarrow (k_2 \Pi)/L_2(\underline{p}_1)^2 > 0$. We conclude that there exists $\varepsilon_1(k_2) > 0$ so that for $k_1 \in (\psi_1(k_2)-\varepsilon_1(k_2),\psi_1(k_2))$ Q_1 is increasing on $[\underline{p}_1,p)$. Lemma 3(c), (5.1) and (5.2) then imply iv(c).

Next, note that $k_1 \downarrow \eta(k_2)$ implies $P(k_1) \rightarrow p$. Hence $h(k_1) \rightarrow \lim_{k_1\uparrow\eta(k_2)} Q'(p) < 0$, as shown in part (iii). For k_1 sufficiently close to $\eta(k_2)$ there therefore exists $p \in (\underline{p}_1,p)$ such that $Q'(p) = 0$. Since p, p and Q_1 are continuous in k_1, and since (as shown in part (iii)) the inequality $Q_1(p) > Q_1(p)$ holds at $k_1 = \eta(k_2)$, there exists $\varepsilon_2(k_2) > 0$ such that $Q_1(p) > Q_1(p)$ for $k_1 \in (\eta(k_2),\eta(k_2)+\varepsilon_2(k_2))$. Lemma 3(c), (5.1) and (5.2) then imply iv(a).

Finally, since $h(k_1)$ is continuous on $(\eta(k_2),\psi_1(k_2))$ there exists $k \in (\eta(k_2),\psi_1(k_2))$ such that $h(k) = 0$ and $h(k_1) < 0$ on $(k - \varepsilon,k)$, for some $\varepsilon > 0$. Define $\rho(k_1) = \operatorname{argmax}_{\underline{p}_1 \leq p \leq P(k_1)} Q_1(p)$. By Lemma 3(g), $\rho(k_1)$ is single-valued and hence continuous. Furthermore, we have just argued that for $k_1 \in (k - \varepsilon,k)$ $\rho(k_1) < P(k_1)$. Let $V(k_1) = \max_{\underline{p}_1 \leq p \leq P(k_1)} Q_1(p)$. For $k_1 \in (k - \varepsilon,k)$ $V(k_1) > Q_1(P(k_1))$. Consider $m(k_1) = Q_1(p) - V(k_1)$. We will show that $\lim_{k_1\uparrow k} m(k_1) > 0$, implying that $Q_1(\hat{p}) = V(k_1)$ for some $\hat{p} \in (P(k_1),p)$. Observe that $\lim_{k_1\uparrow k} m(k_1) = Q_1(p) - Q_1(P(k))$. Now $Q_1(p) - Q_1(P(k)) = (\Pi/L_2(P(k)) - (\Pi$

$/L_2(p)) > 0$ since $P(k) < p$. Hence there exists $\varepsilon_3(k_2) > 0$ such that on $(k - \varepsilon_3(k_2),k)$ $Q_1(p) > Q_1(p)$. Lemma 3(c), (5.1) and (5.2) then imply iv(b). #

Theorem 4 shows that the equilibrium distributions can be one of four different types. In region B_1, F_1 is atomless and F_2 has a masspoint at p. In region $B_4 \cup B_{33}$, F_2 is atomless and F_1 has an atom at p . In region $B_2 \cup B_{31}$, F_1 and F_2 are atomless on the interior of their respective supports, but F_2 has a masspoint at p and F_1 has a masspoint at p. Since p < p, firm 1 has a larger support than firm 2. Finally, in region B_{32}, the supports are identical and disconnected, firm 2 places a masspoint at p and firm 1 places a masspoint at p.

These equilibrium distributions have an interesting economic interpretation. Whenever $k_2 \geq \theta(k_1)$, i.e. capacities lie in region B_1, the high cost firm is sufficiently large that it finds it optimal to price as a residual demand monopolist with positive probability. Consequently, Firm 2 has a "high" normal price, and both firms hold random sales that undercut this price. When $k_2 < \theta(k_1)$, i.e. capacities belong to $B_2 \cup B_3 \cup B_4$, the high cost firm is sufficiently small that the low cost firm now finds it optimal to price as a residual demand monopolist with positive probability. In region $B_4 \cup B_{33}$, both firms randomly undercut this high normal price. In region $B_2 \cup B_{31}$, the high cost firm sets a "low" normal price that discretely undercuts p, and both firms hold random sales that undercut the "low" normal price. Finally, in region B_{32}, both normal prices are randomly undercut, but there is an intermediate range of prices which are never observed. These predictions are potentially empirically testable.[11]

---

[11] See Ghemawat (1988) for an interpretation of the empirical implications of equilibrium for the case of identical unit costs.

Theorem 4 shows that the supports of the equilibrium distributions differ for capacity pairs belonging to $B_{31} \cup B_2$, are disconnected for capacity pairs belonging to B_{32}, and that these regions are nonempty as long as $c_2 > c_1$ and firm 1 does not have a drastic cost advantage. It also follows from Lemma 3(i) that when $c_2 = c_1$ and capacities belong to $B_2 \cup B_3$ the supports of the equilibrium distributions coincide with the interval $[\underline{p}_1,p]$. This raises the interesting question of how the gaps in the equilibrium supports come to disappear as c_2 approaches c_1. We address this issue in our next result.

Theorem 5 : Suppose (A.1) and (A.2) hold. Let $c \downarrow c_1$, and denote the region of nondegenerate mixed strategies by B^n. Also let B^∞ be the corrresponding region when $c_2 = c_1$.

(i) If $(k_1,k_2) \in B$, then for n sufficiently large $(k_1,k_2) \in B$. Furthermore, $\lim_{n\to\infty p} {}^n = p$.

(ii) If $(k_1,k_2) \in B$, then for n sufficiently large $(k_1,k_2) \in B_3$.

Proof : The boundaries η and ψ_1 are invariant under c_2, and equation (A.1) of Appendix A shows that $\theta(.)$ converges to a segment of the diagonal. Hence $(k_1,k_2) \in B$ implies $(k_1,k_2) \in B$ for sufficiently large n, and similarly $(k_1,k_2) \in B$ implies $(k_1,k_2) \in B$ for sufficiently large n.

To prove the remainder of (i), suppose that $(k_1,k_2) \in B$ for $n \geq n$. For such n, Q_2 is independent of c_2. Sincep_n is defined as the solution to $Q'(p_n) = 0$, it follows from equation (B.6) of Appendix B, Lemma 3(g), and the implicit function theorem thatp_n is increasing in n. Suppose now that $\lim_{n\to\infty p} {}^n$ $\equiv p_\infty < p$. Then (B.6) yields the contradiction $0 = \lim_{n\to\infty} Q'(p_n) = (k_2/k_1)\, Q'(p_\infty) > 0$.

To prove the remainder of (ii), observe that $(k_1,k_2) \in B$ implies $P(k_1) < p$, so that $Q'(p) > \varepsilon_4 > 0$ for all $p < P(k_1)$. But then (B.6) yields $\lim_{p\uparrow P(k_1)} Q'(p) > 0$ for n sufficiently large, implying the desired result. #

The proof of Theorem 5 actually shows that the lower boundary of region B_{33} converges to η as c_2 decreases to c_1, so that regions B_{31} and B_{32} vanish in the limit. Furthermore, any point (k_1,k_2) belonging to B_{31} for some $c_1 > c_2$ must go through region B_{32} before ending up in region B_{33}.[12] Thus, as c_2 decreases, the equilibrium supports continue to grow, and coincide with the interval $[\underline{p}_1,p]$ when c_2 is sufficiently close to c_1. Region B_2, on the other hand, does not vanish in the limit when c_2 approaches c_1. Capacity pairs in this region see their equilibrium supports grow to $[\underline{p}_1,p]$, but there remains a gap in the support of F_1 no matter how close c_2 is to c_1.

A final consequence of the above analysis is that the supports of the equilibrium price distributions need not be continuous in the underlying parameters (k_1,k_2,c_1,c_2). Indeed, suppose $k_2 = \theta(k_1)$ so that $\underline{p}_1 = \underline{p}_2$ and $s_1 = s_2 = p \leq p$. When $p < p$, a small change in any parameters making $\underline{p}_1 > \underline{p}_2$ (for example, a slight increase in c_1) would lead firm 1 to not only place mass in some neighborhood above p but, also, to place a mass point (albeit a small one) at p. Thus, there is a discrete jump in the upper bound of the support of firm 1's distribution.

In concluding our characterization of the Nash equilibria of the game $G(k_1,k_2,c_1,c_2)$ it is important to note that equilibrium payoffs are invariant with respect to the sharing rule in the event of a tie in prices.[13] Equilibrium strategies are also invariant except for capacity values $(k_1,k_2) \in C$ when $r_1(0|c_1) \leq d(c_2)$.

To see this, note that in region C, when $c_1 < c_2$, it is necessarily the case that $L_1(c_2) > T_1(c_2)$ under any alternative tie breaking rule. If $r_1(0|c_1) \leq d(c_2)$ this destroys the pure strategy equilibrium in which

---

[12] This follows from the reasoning in the proof of iv(b) of Theorem 4.

[13] A prominent alternative to our tie breaking rule is $T_i(p_i) = (p_i - c_i) \min(k_i, k_i d(p)/(k_1 + k_2))$.

both firms charge the price c_2. The mixed strategy equilibria in which firm 2 distributes all of its mass in the half-open interval above c_2 in such a way as to yield $p_1 = c_2$ a best response obviously remain equilibria under alternative specifications of the tie breaking rule.

Next, let us consider capacity pairs in region $A \cup B$. Let (F_1,F_2) be the equilibrium under our tie breaking rule (TB1), and let TB2 be any other tie breaking rule. Observe that $\Pi^{B2}(p,F_j) = \Pi^{B1}(p,F_j)$ for $p \ \ J(F_j)$. Now suppose that $\Pi^{B2}(p,F_j) > \Pi^{B1}(F_1,F_2)$ for $p \in J(F_j)$. Since outside region C it is necessarily the case that $p > c_2$, we can then define a sequence $p_n \uparrow p$, such that $\lim_{p_n \uparrow p} \Pi^{B1}(p_n,F_j) \geq \Pi^{B2}(p,F_j) > \Pi^{B1}(F_1,F_2)$. This contradicts the fact that F_i is a best response to F_j under TB1, proving that (F_1,F_2) remains an equilibrium under TB2. A parallel argument establishes that over region $A \cup B$ any equilibrium under TB2 must remain an equilibrium under TB1.

VI. Applications

a. Capacity Choice

Kreps and Scheinkman (1983) study the game in which firms first simultaneously choose capacities and then simultaneously select prices. Assuming that firms have identical unit cost of production (up to capacity), that inverse demand is concave,[14] and that the cost of capacity is increasing and convex, they show that the unique subgame perfect equilibrium outcome of the game[15] coincides with the Cournot outcome (where both production and capacity costs are taken into account). In this

---

[14] Osborne and Pitchik (1986) analyze this game under more general assumptions on demand, but still with identical production and capacity costs (implying existence, but not uniqueness of Cournot).

[15] Kreps and Scheinkman actually show that the game has a unique Nash equilibrium. We will not consider imperfect equilibria in this paper.

section, we analyze the two-stage game, but relax the assumption of identical unit production costs. We also consider a somewhat larger class of demand functions than Kreps and Scheinkman. For simplicity, we assume that the capacity cost is constant per unit; we denote this constant by $r > 0$.

The requirement of subgame perfection allows us to reduce our study of equilibria of the two-stage game to those of a single-stage game, where the payoffs accruing to firms after simultaneous capacity choices correspond to the ones in G, minus capacity costs. We will refer to this game and its associated payoff functions as $\Gamma(c_1,c_2,r)$ and $\Pi_i(k_1,k_2,c_1,c_2,r)$, respectively. The existence of a Nash equilibrium (in mixed strategies) to Γ is immediate from the continuity of $\Pi_i = L_i$ (min (p, max $(\underline{p}_1,\underline{p}_2)$)) and Glicksberg's (1952) theorem. Under (A.1) and (A.2), it is easy to show that any pure strategy equilibrium of Γ must be the Cournot equilibrium.[16] However, as the following example demonstrates, when $c_1 \neq c_2$ there may be no equilibrium in which firms choose determinate capacities.[17]

---

[16] If $(k_1,k_2) \in A$, then the outcome necessarily coincides with the Cournot outcome. If $(k_1,k_2) \in C$, then firm 2's equilibrium profits in the pricing subgame must be zero. Since $r > 0$, this requires $k_2 = 0$, so that firm 1 has a drastic cost advantage. The equilibrium where firm 1 has a monopoly then coincides with the Cournot outcome. Finally, (k_1,k_2) cannot belong to B, for then there exists a firm that has $k_i > r_i(k_j)$ and whose payoff in the pricing subgame equals $H(k_j)$. Since $r > 0$, this firm could profitably deviate by choosing $k_i = r_i(k_j)$.

[17] Osborne and Pitchik (1986) also obtain the result that there are Cournot equilibria that are not subgame perfect equilibria of Kreps and Scheinkman's two-stage game. They show that the set of pure strategy equilibria of $\Gamma(c,c,r)$ is a subset of the set of Cournot equilibria, and argue by example that this subset may be proper. The essential feature of the Osborne-Pitchik example is that Cournot reaction functions do not slope downward. Also, in their example there exists a Cournot equilibrium which corresponds to a subgame perfect equilibrium of the two-stage game (though the authors warn that they do not know whether there always exists such an equilibrium).

Suppose that $d(p)=\max(0,1 - p)$ and let $c_1=0$, $c_2 =.3$, and $r = 0^+$.[18] Figure 5 illustrates the best response correspondences for this case. As can be seen in the figure, when the high cost firm's capacity is not too large, the low cost firm's best response coincides with the Cournot best response function for unit cost $c_1 + r = 0^+$. When the high cost firm's capacity reaches a critical level, k (which depends on c_1,c_2 and r), the low cost firm's best response jumps to $k_1 = 1 - c_2$.[19] Firm 1 finds it most profitable to respond to a capacity greater than k by choosing a capacity level that would allow it to accommodate all demand when setting $p_1 = c_2$. This enables it to price its rival out of the market in the price setting subgame. In the example here, k is strictly less than k, the Cournot output level for firm 2. When this occurs, no equilibrium of $\Gamma(c_1,c_2,r)$ will coincide with the Cournot equilibrium. Moreover, since the best response correspondences do not intersect, no pure strategy equilibrium exists.

Our analysis shows that when unit costs differ the Cournot equilibrium need not be a subgame perfect equilibrium of the two-stage game, even under assumptions which guarantee downward sloping Cournot best response functions and a unique Cournot equilibrium. Unlike Osborne and Pitchik, we also provide an economically important reason for why (as well as when) such a result obtains.

[18] By 0^+ we mean an infinitesimally small positive number.

[19] The discontinuity in $r_2(k_1)$ illustrated in Figure 5 can be explained as follows. As is evident from the kink in $L_1(p)$ at $p = P(k_1)$ in Figure 1, $\lim_{k_2 \uparrow \psi^1(k_1)} |d\underline{p}_1/dk_2| > \lim_{k_2 \downarrow \psi^1(k_1)} |d\underline{p}_1/dk_2|$. Now in region B_4, firm 2's profits are independent of k_1, making the optimal capacity response within this region constant in k_1. Let w denote this optimal response. At $k_1 = \psi_1(w,c_1)$, the above inequality implies that firm two can increase its profits by increasing its capacity into region B_3, causing the discontinuity in $r_2(k_1)$. However, as Figure 2 indicates, this discontinuity must occur for $k_1 > k$, and hence is irrelevant for the question of whether Cournot is an outcome of Γ.

A complete characterization of the equilibria of the two-stage game when Cournot does not result is beyond the scope of this paper.[20] In the remainder of this section we deal with the problem of determining when the Cournot outcome obtains. The next two theorems analyze the case where the cost of capacity is negligible, $r = 0^+$. Theorem 6 provides a necessary and sufficient condition for Cournot to arise as an equilibrium in the game $\Gamma(c_1,c_2,0^+)$. With some additional restrictions on demand, Theorem 7 provides a partition of the cost space (c_1,c_2) into regions where Cournot does and does not hold. We conclude our analysis with a discussion of the case where r is not negligible. To this end, let (k,k) and (Π,Π) denote the Cournot outputs and profits when $r = 0$.

<u>Theorem 6</u>: Suppose (A.1) and (A.2) hold, and $c_1 \leq c_2$. A necessary and sufficient condition for (k,k) to be an equilibrium of the game $\Gamma(c_1,c_2,0^+)$ is that:

(a) For every $k_1 > k$, $\underline{p}_1(k_1,k) \geq \underline{p}_2(k_1,k)$

(b) For every $k_2 > k$, $\underline{p}_2(k,k_2) \geq \underline{p}_1(k,k_2)$.

<u>Proof</u>: We will prove that firm 1 has no incentive to deviate from (k,k) if and only if (a) holds. The proof for firm 2 is analogous.

(Sufficiency) Suppose that (a) holds. Then, by Theorem 1, for every $k_1 > k$, $\Pi_1(k_1,k) = H$. But $H = \Pi$, since H is invariant with respect to k_1 above k.

---

[20] In our opinion, when Γ has no pure strategy equilibrium, a single capacity setting stage does not adequately model competitive interaction. Indeed, ex-post the capacity choices in a mixed strategy equilibrium will typically not be best responses to each other. The outcome of a game with multiple capacity setting stages will depend on the timing of the available moves and the extent to which capacity costs are sunk.

(Necessity) Suppose $\underline{p}_1(k_1,k) < \underline{p}_2(k_1,k)$ for some $k_1 > k$. Then by Theorem 1, $\Pi_1(k_1,k) > H = \Pi$. #

In the analysis that follows, we restrict our attention to cost pairs for which $r_1(0|c_1) < d(c_2)$. If the reverse inequality holds, firm 1 may price at its monopoly level and undercut firm 2's unit cost. Therefore, in the game $\Gamma(c_1,c_2,0^+)$, firm 1 sets its capacity equal to its monopoly output and firm 2 chooses a capacity of zero. This outcome coincides with the Cournot equilibrium when unit costs are (c_1,c_2).

Suppose now that $r_1(0|c_1) < d(c_2)$. Part (i) of Theorem 3 then implies that $\underline{p}_2(k,k_2) \geq \underline{p}_1(k,k_2)$ for all $k_2 > k$, so that firm 2 <u>never</u> has an incentive to deviate from k. It can also be seen from Figure 2 that when $k > \phi_2(c_1,c_2)$, condition (a) of Theorem 6 will necessarily be violated at $k_1 = d(c_2)$, and hence Cournot will not arise as a subgame perfect equilibrium of the two-stage game. Referring back to the definition of ϕ_2, this will happen whenever $(c_2 - c_1)d(c_2) > E_1(k)$: by choosing a capacity of $d(c_2)$ and charging a price of c_2 firm 1 can credibly threaten to drive firm 2 out of the market, and thereby increase its profits above the Cournot level.

As should be clear from Figure 2, it is possible that Cournot does not arise even when $(c_2 - c_1)d(c_2) \leq E_1(k)$. Nevertheless, as is shown in Theorem 7, there exists a restricted (but still interesting) class of demand functions for which Cournot does obtain when $(c_2 - c_1)d(c_2) \leq E_1(k)$:

<u>Theorem 7</u>: Suppose that (A.1) and (A.2) hold, $(1/d')$ is nondecreasing and concave, and $r_1(0|c_1) < d(c_2)$. A necessary and sufficient condition for Cournot to be an equilibrium of the game $\Gamma(c_1,c_2,0^+)$ is then that $E_1(k) \geq (c_2 - c_1)d(c_2)$.

<u>Proof</u>: See Appendix D.

Figure 6 shows the cost pairs (c_1,c_2) for which Cournot is not an equilibrium outcome of $\Gamma(c_1,c_2,0^+)$ when $d(p) = \max(0,1 - p)$. While Theorem 7 analyzes only the case in which $r = 0^+$, it is easily shown that for a range of capacity costs above zero there will still be unit cost pairs (c_1,c_2) for which the Cournot result does not hold. For these cost pairs, there will be no subgame perfect equilibrium in which the two firms use pure strategies in setting capacities. As the cost of capacity becomes larger, the range of unit costs up to capacity for which Cournot does not hold gets smaller. Computations carried out for the linear demand example show that when $r \geq .075$ all equilibria involve Cournot capacities.

b. Tariffs vs. Quotas

Our model provides a natural framework for examining the effects of tariffs and quotas in a duopolistic setting.[21] Suppose firm 1 is a domestic firm and firm 2 a foreign firm, each producing for the domestic market only. We assume that the firms are capacity-constrained price setters with given capacities and unit costs of production up to capacity. In the absence of intervention in the market through a tariff or quota, the firms play the game $G(k_1,k_2,c_1,c_2)$. In contrast to the analysis of Sections IV-VI we no longer impose the assumption that $c_1 \leq c_2$.

The imposition of a tariff at a fixed level t is assumed to raise the unit cost of the foreign firm in providing the good to the domestic market to $c \equiv c_2 + t$. Thus, when a tariff is levied the firms play the game $G(k_1,k_2,c_1,c)$. The imposition of a quota at a level strictly less than the foreign firm's capacity

---

[21] For a treatment of some of these issues in the context of a capacity constrained price game with differentiated products see Krishna (1989a,b). Hwang and Mai (1988) examined the equivalence of tariffs and quotas in a conjectural variations model.

restricts that capacity to the level of the quota. We shall refer to such a quota as a "binding quota."[22] The quota-constrained capacity level of the foreign firm will be denoted k. Thus, if a binding quota is levied on the foreign firm, firms play the game $G(k_1,k,c_1,c_2)$.

While there are many intriguing questions which arise in the context of this model, one topic which has received widespread attention in the literature is whether tariffs and quotas are in any sense equivalent. For instance, in an influential paper, Kala Krishna (1989a) considers a model with imperfect substitutes in which an unconstrained domestic firm competes with a foreign firm capacity constrained (by a voluntary export restraint) at or near its Bertrand quantity. For this example, Krishna demonstrates that a tariff and an "import equivalent" quota (VER) are not equivalent, in the sense of generating the same equilibrium prices. Since our model is a homogeneous goods analogue to that described in Krishna, and since her characterization of equilibrium does not hold over the complete capacity space (for parts of this space, the characterization is an open question, but clearly involves nondegenerate mixed strategies) and does not hold if the goods are imperfect but very close substitutes (this is inconsistent with her concavity assumption), the following theorem gives a very strong nonequivalence result:

Theorem 8: Suppose (A.1) holds. Starting from an initial position (k_1,k_2,c_1,c_2), suppose a positive tariff (binding quota) is levied such that in the resulting equilibrium neither firm is driven entirely from the market. Then there exists no binding quota (positive tariff) which generates the same equilibrium price distributions.

---

[22] This terminology has been used elsewhere to refer to a quota which strictly reduces the quantity sold in the market by the foreign firm at given prices. In our model, since firms may not sell all of their capacity in equilibrium, this need not be the case.

<u>Proof</u>: We prove the statement for a tariff levied. The converse will then follow immediately. A tariff transforms the game to $G(k_1,k_2,c_1,c)$ where $c > c_2$. Let $\underline{p}_i(t)$, $p(t)$, $i = 1,2$, be the critical prices of the two firms in the transformed game. In equilibrium, one of the following must hold: (i) $\underline{p}_1(t) > \underline{p}_2(t)$, (ii) $\underline{p}_2(t) > \underline{p}_1(t)$, or (iii) $\underline{p}_2(t) = \underline{p}_1(t)$.

We look first at case (i). Since by assumption neither firm is driven from the market, we know $H > 0$. With $\underline{p}_1(t) > \underline{p}_2(t)$, $\underline{p}_1(t) < p(t)$, and by Lemma 2, $\underline{s}_1 = \underline{s}_2 = \underline{p}_1(t)$ and $s_2 \leq s_1 = p(t) = p(k_2,c_1) = e_1(k_2)$. By Lemma 3, firm 1 has a masspoint at $p(t)$, $\alpha_1(p(t)) > 0$.

Given any binding quota k, the resulting price distribution will be identical to that under the tariff only if these same conditions hold. Let $\underline{p}_i(q)$ and $p(q)$, $i = 1,2$, be the critical prices under the quota. Then we can obtain an identical distribution only if $\underline{p}_1(q) = \max(\underline{p}_1(q),\underline{p}_2(q))$ and $\alpha_1(p(q)) > 0$ for, otherwise, from the uniqueness of the equilibrium distribution and the characterization of equilibrium in sections IV and V, we could not obtain a nondegenerate mixed strategy equilibrium in which, for some price p, $s_2 \leq s_1 = p$ and $\alpha_1(p) > 0$. However, since $p(q) > p(t)$ the resulting equilibrium distributions are not identical.[23]

Case (ii) follows by a similar argument applied to $\underline{p}_2(t)$ and $p(t)$. With $\underline{p}_2(t) > \underline{p}_1(t)$ and $H > 0$, $\underline{p}_2(t) < p(t)$, and by Lemma 2, $\underline{s}_1 = \underline{s}_2 = \underline{p}_2(t)$ and $s_1 \leq s_2 = p(t) = p(k_1,c) = e_2(k_1,c)$. By Lemma 3, firm 2 has a masspoint $\alpha_2(p(t)) > 0$. If a quota is to duplicate this distribution it too must yield a nondegenerate mixed strategy distribution in which $p(t)$ is the upper bound of the union of the firms' supports and firm

---

[23] In this case a nonbinding quota will also not duplicate the equilibrium distributions under a tariff since it can be shown that, with $p(t) > \underline{p}_1(t) > \underline{p}_2(t)$, $Q(p;L_2(\underline{p}_1(t))) < Q(p;L_2(\underline{p}_1(t)))$, $\forall\ p \in (\underline{p}_1(t),p(t))$, where the superscripts 0 and t denote the function Q_1 calculated for the pair (k_2,c_2) and (k_2,c), respectively.

2 has a masspoint at this price. This can only happen if $\underline{p}_2(q) = \max(\underline{p}_1(q),\underline{p}_2(q))$ and $p(q) > \underline{p}_2(q)$. But if this holds then $p(q) < p(t)$, since the former price equals $p(k_1,c_2) = e_2(k_1,c_2)$.

Suppose now that case (iii) holds, $\underline{p}_1(t) = \underline{p}_2(t) = \underline{p}(t)$. Then from Lemma 2, $\underline{s}_1 = \underline{s}_2 = \underline{p}(t)$. We consider two subcases.

(a) Suppose $\underline{p}(t) = P(k_1 + k_2)$. Then we are in a pure strategy region and a binding quota must have $p = P(k_1 + k) > P(k_1 + k_2)$. (Note that this is the only part of the theorem where a nonbinding quota will duplicate a tariff. See footnote 23.)

(b) Suppose $\underline{p}(t) > P(k_1 + k_2)$. If $p(t) < p(t)$, from Lemma 3(c),(e) and (g), and by (5.1) and (5.2), $s_i = s_j = p(t)$ and firm i has a masspoint at $p(t)$. If $i = 1$ then the result follows from an argument similar to that of case (i) (the case of a nonbinding quota is covered by an argument similar to that in footnote 23). If $i = 2$, then the result follows from an argument similar to that in case (ii). Finally, suppose $p(t) = p(t) = p^H(t)$. Then neither firm has a masspoint at the upper bound of the equilibrium supports, $s_1 = s_2 = p^H(t)$. In order to duplicate the equilibrium distributions the critical prices under a quota must satisfy $\underline{p}_1(q) = \underline{p}_2(q) = \underline{p}(t)$ and $p(q) = p(q) = p^H(t)$ (otherwise, there would exist a masspoint). But $p(t) < p(q)$ implies that this cannot hold. #

It should be noted that in Theorem 8 the one case where the restriction of levying only binding quotas is of importance in establishing nonequivalence is the case where $G(k_1,k_2,c_1,c)$ is such that $k_1 \leq r_1(k_2|c_1)$ and $k_2 \leq r_2(k_1|c)$. When capacities are below the Cournot best response functions given the tariff levied, the equilibrium price distributions under the tariff are pure strategies, with each firm charging $p = P(k_1 + k_2)$. In this case a nonbinding quota will yield the same price, but a binding quota will not. However, although the price distributions are the same under the tariff and nonbinding quota, the government obtains revenue under the tariff but does not under the quota.

It should also be noted that there are cases of nonequivalence in other ranges of the parameter space (k_1,k_2,c_1,c_2). Suppose, for instance, that we start from an initial position in which $c_1 > c_2 > 0$ and $k_1,k_2 > d(c_2)$. Since the capacity constraints will never be binding in equilibrium, this is very much like the classical Bertrand model. Now suppose a tariff is levied at a positive level t, so that $c_1 < c_2 + t < p$. In the equilibrium of the game $G(k_1,k_2,c_1,c_2 + t)$, firm 1 drives firm 2 out of the market and charges the price $p_1 = c_2 + t < p$. This result cannot be obtained with a quota. Since $c_2 < c_1$, for firm 2 to be driven out under a quota the quota must be set at zero $(k = 0)$. But in this case firm 1 charges p.

The question of the nonequivalence of tariffs and quotas is just one of many applications of our model in the trade context outlined. One interesting application, which appears immediate from our treatment of the game of capacity choice, is to compare the effects of tariffs and quotas on investment in capacity by foreign and domestic firms. Another application would involve embedding the one shot model in a supergame model of collusion allowing one to examine the effect of levying tariffs and quotas on the sustainability of collusion.[24] Finally the price leadership model of Deneckere and Kovenock (1992) can be used to analyze the effects of tariffs and quotas on the endogenous determination of a price leader (see Deneckere, Kovenock and Sohn (1991)).

---

[24] See Davidson (1984) and Rotemberg and Saloner (1989).

Appendix A

Proof of Theorem 3: First, observe that $k_i \leq r_i(k_j)$ implies $\underline{p}_i = p = P(k_1 + k_2)$, and that $k_i > r_i(k_j)$ implies $P(k_1 + k_2) < \underline{p}_i \leq p$ (where equality is possible only in region C). Consequently, $\underline{p}_i = P(k_1 + k_2) < \underline{p}_j$ for (k_1,k_2) A with $k_i \leq r_i(k_j)$. We are left with the ranking of the $\underline{p}_i$ in the region where $k_1 > r_1(k_2|c_1)$ and $k_2 > r_2(k_1|c_2)$.

It is important to first establish the locus of points where firm i is exactly capacity constrained at $\underline{p}_i$, i.e., $d(\underline{p}_i) = k_i$. Observe that since $k_i \geq r_i(k_j)$, $H(k_i,k_j) = E_i(k_j)$ and hence that k_i must satisfy $[P(k_i) - c_i]k_i = E_i(k_j)$. Our assumptions on $d(\bullet)$ imply that the function $q \rightarrow [P(q) - c_i]q$ is strictly quasiconcave, so that for each $k_j > 0$, there are exactly two solutions in $[0,d(c_i)]$ to this equation. The smallest of these solutions necessarily satisfies $k_i < r_i(k_j)$ and hence is inadmissible. Hence, for each $k_j \geq 0$ there is a $k_i = \psi_i(k_j) \equiv \max\{q: [P(q) - c_i]q = E_i(k_j)\}$ such that $d(\underline{p}_i) = k_i$. Furthermore, since $E_i(k_j)$ is decreasing in k_j, ψ_i is increasing in k_j.

Next, we claim that $k_2 \geq \psi_2(k_1)$ and $k_1 > 0$ implies $\underline{p}_2 > \underline{p}_1$. Indeed, if $\psi_2(k_1) \leq k_2 < d(c_1)$ and $\underline{p}_1 \geq \underline{p}_2$, then $d(p) \leq d(\underline{p}_1) \leq d(\underline{p}_2) \leq k_2$ so that $H(k_1,k_2) = 0$, contradicting $k_2 < d(c_1)$. If $k_2 \geq d(c_1)$, then $\underline{p}_1 = c_1 < c_2 \leq \underline{p}_2$. A similar argument establishes that $\psi_1(k_2) \leq k_1 < d(c_2)$ and $k_2 > 0$ implies $\underline{p}_1 > \underline{p}_2$.

Let us now investigate the region where $k_1 \geq d(c_2)$, so that $\underline{p}_2 = c_2$. First, consider the case where $r_1(0|c_1) \leq d(c_2)$. Then $\underline{p}_1$ satisfies the equation $L_1(\underline{p}_1) = E_1(k_2)$. For $k_2 \geq d(c_1)$, $\underline{p}_1 = c_1 < c_2 = \underline{p}_2$, and for $k_2 = 0$, $\underline{p}_1 = e_1(0) \geq c_2 = \underline{p}_2$. Since $\underline{p}_1$ is strictly decreasing in k_2 on $[0,d(c_1)]$ there exists, for each k_1, a unique value of k_2 such that $\underline{p}_1 = c_2 = \underline{p}_2$. Since $k_1 \geq d(c_2)$, $k_2 = \theta(k_1)$ must satisfy:

$$L_1(c_2) = (c_2 - c_1)d(c_2) = E_1(k_2)$$

for all $k_1 \geq d(c_2)$. Now consider the case where $r_1(0|c_1) > d(c_2)$. For $d(c_2) \leq k_1 \leq r_1(k_2|c_1)$, we already showed that $\underline{p}_2 > \underline{p}_1$ (except at $k_1 = d(c_2)$ and $k_2 = 0$, where equality holds). A similar argument to the case $r_1(0|c_1) \leq d(c_2)$ then establishes that $\underline{p}_2 > \underline{p}_1$ for all $k_1 \geq d(c_2)$.

We are left with the region where $r_1(k_2|c_1) \leq k_1 \leq \min\{d(c_2),\psi_1(k_2)\}$ and $r_2(k_1|c_2) \leq k_2 \leq \psi_2(k_1)$. We will refer to this region as Ω (observe that Ω may be empty when $r_1(0|c_1) > d(c_2)$). Observe that on Ω each firm i is capacity constrained at $\underline{p}_i$, so that $\underline{p}_i = c_i + E_i(k_j)/k_i$. Let $\Phi = (\underline{p}_2 - \underline{p}_1)k_1k_2$ be viewed as a function of k_2. Then:

(A.1)
$$\Phi = (c_2 - c_1)k_1k_2 + k_1E_2(k_1) - k_2E_1(k_2)$$
$$\Phi' = (c_2 - c_1)k_1 + [e_1(k_2) - c_1][k_2 - r_1(k_2|c_1)]$$
$$\Phi'' = e'(k_2)[k_2 - r_1(k_2|c_1)] + [e_1(k_2) - c_1][1 - r'(k_2|c_1)].$$

Now, the f.o.c. for $e_1(k_2)$ implies that $e_1(k_2) - c_1 = -r_1(k_2|c_1)/d'(e_1(k_2))$. Also, since $d(e_1(k_2)) = k_2 + r_1(k_2|c_1)$, we have $d'(e_1(k_2))e'(k_2) = 1 + r'(k_2|c_1)$. Thus, we obtain:

$$\Phi'' = -\{2r_1(k_2|c_1) - k_2 - k_2r'(k_2|c_1)\}/d'(e_1(k_2)) .$$

For each k_1 for which there exists k_2 such that $(k_1,k_2) \in \Omega$ let $(k_1) = \min\{k_2\colon (k_1,k_2) \in \Omega\}$ and $(k_1) = \max\{k_2\colon (k_1,k_2) \in \Omega\}$. We wish to study the behavior of Φ on [,]. Observe that for $k_2 \geq {}_2 \equiv r_1({}_2|c_1)$, Φ' is positive. Observe also that for $k_2 \leq {}_2$, $\Phi'' > 0$. We conclude that either $\Phi'(k_2) > 0$ at $k_2 = (k_1)$ so that $\Phi' > 0$ on $[(k_1),(k_1)]$, or else there exists a uniquely defined $k_2 = \mu(k_1) \in ((k_1),{}_2)$ such that $\Phi' < 0$ on $[(k_1),\mu(k_1))$ and $\Phi' > 0$ on $(\mu(k_1),(k_1)]$.

First, suppose $k_1 > k$. Then $\Phi((k_1)) < 0$, so that $\Phi(k_2) < 0$ on $[(k_1),\mu(k_1)]$, and $\Phi' > 0$ on $(\mu(k_1), (k_1)]$. Hence there exists at most one value of $k_2 \in (\mu(k_1),(k_1)]$ such that $\Phi(k_2) = 0$. In fact, since $\Phi((k_1)) > 0$, the existence of such a solution is guaranteed. Denote this solution by $k_2 = \theta(k_1)$.

Finally, let $k_1 \leq k$. Observe that $\Phi(k_2) \geq 0$ at $k_2 = (k_1) = r^1(k_1|c_1)$, with strict inequality for $k_1 < k$. We now claim that $\Phi'(k_2) \geq 0$ at $k_2 = (k_1)$, so that $\Phi > 0$ on $((k_1),(k_1)]$. Define $\omega(k_2) = \Phi'(k_2)|_{k_1=r_1(k_2|c_1)}$. Then:

$$\begin{aligned}\omega(k_2) &= (c_2 - c_1)r_1(k_2|c_1) + [e_1(k_2) - c_1][k_2 - r_1(k_2|c_1)] \\ &= [e_1(k_2) - c_1]k_2 - [e_1(k_2) - c_2]r_1(k_2|c_1).\end{aligned}$$

Now if $r_1(0|c_1) > d(c_2)$, then $c_1 \leq e_1(k_2) < e_1(0) \leq c_2$, so that $\omega \geq 0$ at $k_2 = r^1(k_1|c_1)$. If $r_1(0|c_1) \leq d(c_2)$, then $\omega(k) = 0$. To see this, note that $\omega(k) = [e_1(k) - c_1]k - [e_1(k) - c_2]k$. From the f.o.c. for $e_2(k_1)$ and $e_1(k_2)$ we see that $e_2(k) - c_2 = -r_2(k|c_2)/d'(e_2(k))$ and $e_1(k) - c_1 = -r_1(k|c_1)/d'(e_1(k))$. Substituting this into the expression for $\omega(k)$, and noting that $e_2(k) = e_1(k)$ then yields $\omega(k) = 0$. Observe also that $\omega'(k_2) = e'(k_2)[k_2 - r_1(k_2|c_1)] + [e_1(k_2) - c_1] - [e_1(k_2) - c_2]r'(k_2|c_1)$. Now if $k_2 \leq {}_2 = r_1({}_2|c_1)$, then $k_2 \leq r_1(k_2|c_1)$ and so $\omega'(k_2) > 0$ on $[k,{}_2]$. For $k_2 \geq {}_2$, direct inspection of the expression for $\omega(k_2)$ reveals that $\omega(k_2) > 0$. Consequently, $\omega(k_2) > 0$ on $(k,r^1(0|c_1)]$, establishing the claim. We conclude that $\underline{p}_2 > \underline{p}_1$ for $k_1 \leq k$ and $k_2 \geq (k_1) = r^1(k_1|c_1)$ (except at (k,k), where equality holds). #

Appendix B

Proof of Lemma 3: By definition, $Q_i(p) = (L_j(p) - \Pi)/(L_j(p) - H_j(p))$. (a) is immediate from the definitions of $\underline{p}_j$ and p and from Theorem 1. (b) follows immediately from the fact that, throughout the interval, $H_j(p)$ is differentiable, $L_j(p)$ is differentiable at every point except $P(k_j)$, and $L_j(p) > H_j(p)$. (c) follows from the fact that, from (b),

$$dQ_i/dp = \frac{[L'(p)(\Pi - H_j(p)) + H'(p)(L_j(p) - \Pi)]}{(L_j(p) - H_j(p))^2} \tag{B.1}$$

With $\Pi > H_j(p)$ on $[\underline{p}_j,p)$ from Theorem 1, $L_j(p) > \Pi$ from the fact that $L_j(p)$ is strictly increasing on $[c_j,p)$ and $\Pi = L_j(\underline{p}_j)$, and both L' and H' positive on $[\underline{p}_j,p)$ where they exist, we know that (B.1) is positive where these derivatives exist. At $P(k_j)$ the function Q_i is continuous and, for $P(k_j) \in (\underline{p}_j,p)$, $\lim_{p\uparrow P(k_j)} dQ_i/dp > \lim_{p\downarrow P(k_j)} dQ_i/dp$.

The last inequality follows from the fact that for $p > P(k_j)$, $L'(p) = (p - c_j)d'(p) + d(p)$, while for $p < P(k_j)$, $L'(p) = k_j$ and the fact that L'(p) appears in (B.1) with a positive coefficient. To prove the rest of (d), note that L_j and H_j are twice continuously differentiable except for L_j at the point $P(k_j)$. Furthermore,

$$d^2Q_i/dp^2 = \left\{\frac{(L''(\Pi-H_j)+H''(L_j-\Pi))(L_j-H_j) - 2(L'-H')(L'(\Pi-H_j)+H'(L_j-\Pi))}{(L_j - H_j)^3}\right\} \tag{B.2}$$

It is easily verified that for $p \neq P(k_j)$, $L' > 0$, $H' > 0$, $L' - H' > 0$, $L'' \leq 0$ and $H'' < 0$. With $\Pi - H_j > 0$ and $L_j - H_j > 0$ in the open interval, the numerator in (B.2) is negative and the denominator is positive. Thus, except for $p = P(k_j)$, d^2Q_i/dp^2 is negative and continuous. This, together with the limit result at $P(k_j)$ tells us that Q_i is concave.

We now check the properties of Q_j. By definition, $Q_j(p) = (L_i(p) - \Pi)/(L_i(p) - H_i(p))$. The claim in (e) is immediate from Theorem 1. (f) follows from the fact that, throughout the interval, $L_i(p) = (p_i - c_i)k_i$ is differentiable, $H_i(p)$ is differentiable at every point except $P(k_j)$, and $L_i(p) > H_i(p)$. With

$$\text{(B.3)} \qquad dQ_j/dp = [L'(\Pi - H_i) + H'(L_i - \Pi)]/(L_i - H_i)^2,$$

all the terms in this expression are continuous on the interval except $H'(p)$ at $P(k_j)$, which satisfies $\lim_{p\uparrow P(k_j)} H'(p) = (P(k_j) - c_i)d'(P(k_j)) < 0$, and $\lim_{p\downarrow P(k_j)} H'(p) = 0$ as long as $P(k_j) \in (\underline{p}_j, p)$. Substituting this into (B.3) and noting that the coefficient of $H'(p)$ in (B.3) is positive, we obtain $\lim_{p\downarrow P(k_j)} dQ_j/dp > \lim_{p\uparrow P(k_j)} dQ_j/dp$. Since H' is the only expression that can be negative in (B.3), we see that $\lim_{p\downarrow P(k_j)} dQ_j/dp > 0$, proving (f).

To prove (g), note that $L' = k_i > 0$ and $L'' = 0$. Also note that for $p < P(k_j)$, $H' = d(p) - k_j + (p - c_i)d'(p) \;\; 0$ and $H'' = 2d'(p) + (p - c_i)d''(p) < 0$; for $p > P(k_j)$, $H' = H'' = 0$. When $P(k_j) > \underline{p}_j$, for $p < P(k_j)$, (B.3) cannot be signed since H' may be negative. However, at $p = \underline{p}_j$, $L_i(\underline{p}_j) = \Pi$ from Theorem 1, so

$$\text{(B.4)} \quad (dQ_j/dp)(\underline{p}_j) = [L'(\underline{p}_j)(\Pi - H_i(\underline{p}_j))]/[L_i(\underline{p}_j) - H_i(\underline{p}_j)]^2 > 0.$$

Since dQ_j/dp is continuous except at $P(k_j)$, (B.4) implies that there is some neighborhood above $\underline{p}_j$ in which Q_j is increasing. For $\underline{p}_j < p < p$, (B.3) implies $dQ_j/dp > 0$; consequently $p \geq \max\{\underline{p}_j, p\}$, with equality only when $p = p$ and the numerator in (B.3) is zero at $p = p$. This requires $\Pi = H_i(p)$, so that $\underline{p}_1 = \underline{p}_2 < p = p$. Now $p > P(k_j)$ implies $dQ_j/dp > 0$ since $H' = 0$. Therefore, to prove the claim we need only to show that if Q_j turns down in the interval then it does not turn up again unless it hits $P(k_j)$. To show this note that

$$\text{(B.5)}\quad d^2Q_j/dp^2 = \left\{\frac{(L''(\Pi-H_i)+H''(L_i-\Pi))(L_i-H_i)-2(L'-H')(L'(\Pi-H_i)+H'(L_i-\Pi))}{(L_i-H_i)^3}\right\}$$

The sign of this expression is equal to the sign of the numerator. Suppose $dQ_j/dp = 0$. Then $L'(\Pi - H_i) + H'(L_i - \Pi) = 0$, which implies that $\mathrm{sgn}(d^2Q_j/dp^2) = \mathrm{sgn}(H''(L_i - \Pi)(L_i - H_i)) < 0$. (Here we make use of the fact that $L'' = 0$). Thus, at a critical point of Q_j (other than $P(k_j)$), the function is locally concave: once Q_j turns down it cannot turn up again until it hits $P(k_j)$. This proves (g). To prove (h) note, more generally, that since $L' - H' = k_i + k_j - d(p) - (p - c_i)d'(p) > 0$, from (B.5) we see that $\mathrm{sgn}(d^2Q_j/dp^2) < 0$ whenever $\mathrm{sgn}(dQ_j/dp) > 0$. Thus, whenever Q_j is locally nondecreasing (except at $P(k_j)$) it is locally concave. Finally, to show (i), note that from (g), Q_j can be decreasing only on a subinterval of $(\underline{p}_j, \min(P(k_j),p))$. For this range of prices it is easily verified that $Q_j = [(k_i/k_j)(p - c_j)/(p - c_i)]Q_i$, which implies that

$$\text{(B.6)}\quad \frac{dQ_j}{dp} = \frac{k_i}{k_j}\,\frac{(c_j - c_i)}{(p - c_i)^2}\,Q_i + \frac{k_i}{k_j}\,\frac{(p - c_j)}{(p - c_i)}\,\frac{dQ_i}{dp}.$$

Since the second summand is positive, if $c_j \geq c_i$, Q_j is increasing. Thus, a necessary condition for Q_j to decrease is that $c_j < c_i$. #

Appendix C

In this Appendix, we will demonstrate the uniqueness of the mixed strategy equilibrium in Region B. Lemma 2 then implies that there exists j such that $\underline{p}_j \geq \underline{p}_i, \underline{s}_i = \underline{s}_j = \underline{p}_j$ ands$_i \leq s_j = p$. The first lemma, which is due to Osborne and Pitchik (1983, p.18), demonstrates that--except possibly for the single point p--the supports of the equilibrium strategies coincide.

Lemma C.1: Supp F_j = supp $F_i \cup \{p\}$.

Proof: Suppose $p \in (\underline{p}_j,p)$ and p supp F_i. Then, since L_j and H_j are increasing at p, so is $\Pi_j(p,F_i)$. Hence p supp F_j.

Next, suppose $p \in (\underline{p}_j,p)$ and p supp F_j. Let $x = \max\{[\underline{p}_j,p) \cap$ supp $F_j\}$ and $y = \min\{(p,p] \cap$ supp $F_j\}$. Observe that $F_j(x) < 1$ and that for $s \in (x,y)$: $\Pi_i(s,F_j) = (1 - F_j(x))L_i(s) + F_j(x)H_i(s)$. Observe also that since L_i is linear on $[\underline{p}_j,p]$ and since H_i is strictly concave on $[\underline{p}_j,P(k_j)]$ and identically zero on $[P(k_j),p_0]$, $\Pi_i(s,F_j)$ is increasing and/or strictly concave on $[\underline{p}_j,P(k_j)]$ and increasing on $[P(k_j),p]$. We conclude that if $p \in$ supp F_i, then p maximizes $\Pi_i(p,F_j)$ on (x,y) and $p \in J(F_i)$. Now clearly x $J(F_i)$ (this is obvious if $x \in J(F_j)$; if x $J(F_j)$ then $\Pi_i(x,F_j) < \Pi_i(p,F_j)$ by the maximization property of p). But then $\Pi_j(F_j,F_i) = \Pi_j(x,F_i) = F_i(x)H_j(x) + (1 - F_i(x))L_j(x)$. We then obtain an immediate contradiction to equilibrium, since on [x,p) both L_j and H_j are increasing and since i puts no mass on [x,p). #

Our next lemma shows that gaps in i's support can occur only on the set $\{p: Q_j(p; \Pi) < IQ_j(p; \Pi)\}$.

Lemma C.2: Let $p \in (\underline{p}_j,p)$ with p supp F_i. Then $Q_j(p; \Pi) < IQ_j(p; \Pi)$.

Proof: First, we deal with the case where $F_i(p) = 1$. Then $s_i < p < p$, and so $F_j(s_i) = \lim_{s\uparrow p} F_j(s)$. Also s_i $J(F_j)$, since L_j and H_j are increasing on $[s_i,p]$ and since $\Pi_j(p,F_i) = \Pi$. We conclude that $H_i(s_i)F_j(s_i) + L_i(s_i)(1 - F_j(s_i)) = \Pi_i(s_i,F_j) = \Pi = H_i(s_i)Q_j(s_i) + L_i(s_i)(1 - Q_j(s_i))$, and hence that $F_j(s_i) = Q_j(s_i) < 1$. Observe now that $Q'(s_i) = 0$. Indeed, if $Q'(s_i) < 0$, then there would exist $s < s_i$ such that $Q_j(s) > F_j(s)$ and if $Q'(s_i) > 0$, then there would exist $s > s_i$ such that $Q_j(s) > F_j(s) = F_j(s_i)$. In both cases, by playing s, firm i would net $F_j(s)H_i(s) + (1 - F_j(s))L_i(s) > Q_j(s)H_i(s) + (1 - Q_j(s))L_i(s) = \Pi$, a contradiction to equilibrium. Observe also that $Q_j(s) \leq F_j(s_i)$ for all $s \in (s_i,p)$, for otherwise there would exist $s \in (s_i,p)$ such that $Q_j(s) > F_j(s)$, yielding the same contradiction. From Lemma 3, we conclude that $Q_j(s) < IQ_j(s)$ for all $s \in (s_i,p)$.

Next, let us suppose that $F_i(p) < 1$. Let $x = \max\{[\underline{p}_j,p) \cap \text{supp } F_i\}$ and $y = \min\{(p,p] \cap \text{supp } F_i\}$. Since $\Pi_j(\bullet,F_i)$ is increasing on $[x,y)$, it must be that $\lim_{s\uparrow y} F_j(s) = F_j(x)$. Next we claim that y $J(F_j)$. Otherwise y $J(F_i)$ and since $y \in$ supp F_i we would have $\Pi = \Pi_i(y,F_j) = H_i(y)F_j(y) + L_i(y)(1 - F_j(y))$. This contradicts the fact that for all $s \in [x,y)$: $\Pi \geq \Pi_i(s,F_j) = H_i(s)F_j(x) + L_i(s)(1 - F_j(x))$. Suppose then that y $J(F_j)$. Then $\Pi_i(y,F_j) = \Pi$ and so $F_j(x) = F_j(y) = Q_j(y; \Pi)$.

Next, observe that $x \in J(F_i)$. For if not, then since by Lemma C.1, $x \in$ supp F_j, $\Pi = \Pi_j(x,F_i)$. This would contradict equilibrium, as $\Pi_j(s,F_i)$ is increasing on (x,y). Now $x \in J(F_i)$ implies $F_j(x) = Q_j(x; \Pi)$ and so $Q_j(x; \Pi) = F_j(x) = F_j(y) = Q_j(y; \Pi)$. As in the case of $F_i(p) = 1$, we conclude that $Q'(x) = 0$ and that $Q_j(s) < F_j(x)$ for all $s \in (x,y)$. Hence, $Q_j(s) < IQ_j(s)$ for all $s \in (x,y)$, proving the desired result. #

The next two lemmas allow us to completely pin down the supports of the distributions.

Lemma C.3: Suppose $Q_j(p) < IQ_j(p; \Pi)$. Then $p \quad \text{supp } F_i$.

Proof: Suppose to the contrary that $p \in \text{supp } F_i$. By Lemma C.1, $p \in \text{supp } F_j$. Furthermore, $F_j(p) \geq IQ_j(p; \Pi) > Q_j(p)$. Now $p \quad J(F_i)$, since otherwise $p \quad J(F_j)$ and so $\Pi = H_i(p)F_j(p) + L_i(p)(1 - F_j(p))$, implying $F_j(p) = Q_j(p; \Pi)$. For the same reason, F_i is not right increasing at p. We conclude that F_i is left increasing at p, and so there exists $\{p_n\} \subset \text{supp } F_i$, $p_n \uparrow p$ such that $\Pi_i(p_n,F_j) = \Pi$. Hence, $F_j(p_n) = Q_j(p_n; \Pi)$. However, for large n, $Q_j(p_n; \Pi) < IQ_j(p_n; \Pi)$, yielding the contradiction $F_j(p_n) = Q_j(p_n; \Pi) < IQ_j(p_n; \Pi) \leq F_j(p_n)$. #

Lemma C.4: Suppose $p \in \text{supp } F_i$. Then $\exists\, \varepsilon > 0$ such that $\forall\, p \in (p - \varepsilon,p)$, $Q_j(p; \Pi) = IQ_j(p; \Pi)$.

Proof: Suppose not. Then from Lemma 3 $\exists\, \varepsilon > 0$ such that $\forall\, p \in (p - \varepsilon,p)$, $Q_j(p) < IQ_j(p)$. From Lemma C.3, for every p in this interval $p \quad \text{supp } F_i$. Furthermore, p cannot be a masspoint of firm i since then $\lim_{p \uparrow p} F_i(p)H_j(p) + (1 - F_i(p))L_j(p) > H_j(p) = H$. Thus, $p \quad \text{supp } F_i$. #

Combining Lemmas C.1-C.4 we have $\text{supp } F_j = [\underline{p}_j,p]\backslash(p,p)$ and $\text{supp } F_i = \text{closure}(\text{supp } F_j \backslash p)$. Finally, it is easy to argue that masspoints in the distributions can occur for i only atp and for j only at p (otherwise, there would have to be additional gaps in the support). This completely pins down the distribution functions, since if $p \in \text{supp } F_k$ and $p \quad J(F_l)$ (for $l \neq k$), $\Pi_k(p,F_l) = \Pi$ and so $F_l(p) = Q_l(p; \Pi)$.

Appendix D

Proof of Theorem 7: We need only show that if $E_1(k) \geq (c_2 - c_1)d(c_2)$ then firm 1 has no incentive to deviate from $k_1 = k$. First, observe that $\Pi_1(k_1,k) \leq E(k)$ for all $k_1 \geq d(c_2)$. Indeed, if $k_1 \geq d(c_2)$ then $H(k_1,k) = 0$ and so $\underline{p}_2 = c_2$. If $\underline{p}_1 \leq c_2$, then $\Pi_1(k_1,k) = (c_2 - c_1)d(c_2) \leq E_1(k)$. If $\underline{p}_1 > c_2 = \underline{p}_2$, then $\Pi_1(k_1,k) = E_1(k)$, and so firm 1 has no incentive to deviate to $k_1 \geq d(c_2)$.

Suppose now that $k < k_1 < d(c_2)$. Then $c_2 < P(k_1) < P(k)$. The remainder of the proof is broken up in two cases, depending upon the relationship between $\underline{p}_1$ and $P(k_1)$. First, if $\underline{p}_1 \geq P(k_1)$, then $H_2(\underline{p}_1) = 0$ and so $p \leq \underline{p}_1$. This implies $\underline{p}_2 \leq \underline{p}_1$, and so $\Pi_1(k_1,k) = H(k_1,k) = E_1(k)$.

Suppose now that $\underline{p}_1 < P(k_1)$. We now claim that $\underline{p}_2 < P(k)$. Indeed, if $\underline{p}_2 \geq P(k)$, then $\underline{p}_2 \geq P(k) > P(k_1) > c_2$ and so $H_2(p) = 0$ implying $\underline{p}_2 = c_2$, a contradiction. Hence:

$$\underline{p}_1 = c_1 + E_1(k)/k_1$$

$$\underline{p}_2 = c_2 + E_2(k_1)/k.$$

Let $\zeta(k_1) = k_1k(\underline{p}_1 - \underline{p}_2) = k_1k(c_1 - c_2) + kE_1(k) - k_1E_2(k_1)$. Then $\zeta(k) = 0$ and $\zeta(d(c_2)) = k\{E(k) - (c_2 - c_1)d(c_2)\} \geq 0$. Analogous to Appendix A, we may now calculate :

$$\zeta'(k_1) = -(c_2 - c_1)k + [e_2(k_1) - c_2][k_1 - r_2(k_1|c_2)]$$

$$\zeta''(k_1) = -\{2r_2(k_1|c_2) - k_1(1 + r'(k_1|c_2))\}/d'(e_2(k_1)).$$

Also observe that the sign of ζ'' is equal to the sign of $-d'(e_2(k_1))\zeta''(k_1)$, and that the latter expression has derivative $r'(k_1|c_2) - 1 - k_1r''(k_1|c_2)$. Our assumptions on demand can be shown to imply that $r'' \geq 0$, so that the above derivative is negative. Analogous to the argument in Appendix A, we may prove that $\zeta'(k) = 0$.

There are now two cases to consider. First, assume that $\zeta''(k) \leq 0$. Then $\zeta''(k_1) < 0$ on $(k,d(c_2)]$, and hence $\zeta'(k_1) < 0$ on $(k,d(c_2)]$. The latter statement contradicts $\zeta(k) = 0$ and $\zeta(d(c_2)) \geq 0$. Thus it must be that $\zeta''(k) > 0$. Since $\zeta''(d(c_2)) < 0$, we know that ζ' is increasing on $(k,{}_1)$, where ${}_1$ is the unique solution to $\zeta''(k_1) = 0$ in $(k,d(c_2))$, and decreasing thereafter. Since $\zeta(k) = 0$ and $\zeta(d(c_2)) \geq 0$, this proves $\zeta(k_1) \geq 0$ on $[k,d(c_2)]$ so that $\Pi_1(k_1,k) = H(k_1,k) = E_1(k)$. We conclude that firm 1 has no incentive to deviate to $k_1 \in (k,d(c_2)]$ either.

Finally, for $k_1 \in [0,k)$, $\Pi_1(k_1,k) = k_1[P(k_1 + k) - c_1]$, an expression that is maximized as $k_1 = k$. #

References

Allen, B. and M. Hellwig (1986), "Bertrand-Edgeworth Oligopoly in Large Markets," Review of Economics Studies 53, 175-204.

Dasgupta, P. and E. Maskin (1986), "The Existence of Equilibrium in Discontinuous Economic Games, I: Theory," Review of Economic Studies 53, 1-26.

Davidson, C. (1984), "Cartel Stability and Tariff Policy," Journal of International Economics 17, 219-237.

Davidson, C. and R. Deneckere (1986), "Long-Run Competition in Capacity, Short-Run Competition in Price, and the Cournot Model," The Rand Journal of Economics 17, 404-415.

Deneckere, R. and D. Kovenock (1989), "Capacity-Constrained Price Competition When Unit Costs Differ," CMSEMS Discussion Paper No. 861, Northwestern University.

Deneckere, R., D. Kovenock and Y. Sohn (1991), "Quotas and Tariffs with Endogenous Conduct," Institute for Research in the Behavioral, Economic and Management Sciences, Discussion Paper No. 1006, Purdue University.

Deneckere, R. and D. Kovenock (1992), "Price Leadership," Review of Economic Studies 59, 143-162.

Gelman, J. and S. Salop (1983), "Judo Economics: Capacity Limitation and Coupon Competition," Bell Journal of Economics 14, 315-325.

Ghemawat, P. (1988), "Short Run Competition: IO Models for Cross-Sectional Analysis," mimeo, Harvard University.

Glicksberg, I. (1952), "A Further Generalization of the Kakutani Fixed Point Theorem with Application to Nash Equilibrium Points," Proceedings of the AMS 3, 170-174.

Hwang, H. and C. Mai (1988), "On the Equivalence of Tariffs and Quotas Under Duopoly: A Conjectural Variation Approach," Journal of International Economics 24, 373-380.

Kreps, D. and J. Scheinkman (1983), "Quantity Precommitment and Bertrand Competition Yield Cournot Outcomes," Bell Journal of Economics 14, 326-337.

Krishna, K. (1989a), "Trade Restrictions as Facilitating Practices," Journal of International Economics 26, 251-270.

Krishna, K. (1989b), "What Do VER's Do?," in : Beyond Trade Friction : Japan-U.S. Economic Relations, R. Sato and J. Nelson (Eds.), Cambridge University Press, 76-92.

Levitan, R. and M. Shubik (1972), "Price Duopoly and Capacity Constraints," International Economic Review 13, 111-123.

Osborne, M. and C. Pitchik (1983), "Price Competition in a Capacity Constrained Duopoly," Columbia University, Department of Economics, Working Paper No. 185.

Osborne, M. and C. Pitchik (1986), "Price Competition in a Capacity-Constrained Duopoly," Journal of Economic Theory 38, 238-260.

Rotemberg, J. and G. Saloner (1989), "Tariffs versus Quotas with Implicit Collusion," Canadian Journal of Economics 22(2), 237-244.

Simon, L. (1987), "Games with Discontinuous Payoffs," Review of Economic Studies 54, 569-597.

Sweeney, J. and W. Comanor (1989), "Customer Loyalty and Price Fluctuations," mimeo, Stanford University.

Varian, H. (1980), "A Model of Sales," American Economic Review 70, 651-659.