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# Rational Nonprofit Entrepreneurship*


**Abstract**

This paper models nonprofit entrepreneurship as the equilibrium outcome of a multistage game among individuals who would like a public good to be provided. The model predicts that if individuals will voluntarily contribute towards provision of the public good, then it is in the private interest of the entrepreneur to impose a non-distribution constraint on himself by founding a nonprofit firm. This decision also improves the allocation of resources, in the sense that it results in greater voluntary contributions than if the firm that provides the public good is proprietary.



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# 1 Introduction

Why would anyone invest time and money to found a *nonprofit* enterprise[1] ? More specifically, how could it be rational for the individual who founds an enterprise and might still control it to deny herself the opportunity to appropriate the fruits of her skills and efforts? Yet nonprofit firms constitute a significant sector of most modern economies, and appear in a wide variety of industries, providing everything from health and educational services to charitable activities and entertainment. In the years 1929-85, the percentage of U.S. national income originating in the nonprofit sector has varied between 2.0 and 4.5.%[2] The number of nonprofit organizations operating in the U.S. grew from 309,000 to 845,000 between 1967 and 1983, an increase of 170%, over a period in which the number of proprietary firms less than doubled.[3]

Of course, rational nonprofit entrepreneurs must have a reason for choosing to incorporate their enterprises under nonprofit regulation, but what is it? The motivations of nonprofit entrepreneurs are important. If those who control nonprofit enterprises cannot appropriate any profits, it is unlikely that profit-maximization is a useful behavioral assumption to employ in studying them. And, as Steinberg(1993) has noted, it is difficult to offer advice to governments regarding how to regulate, tax and subsidize nonprofit enterprises if the objectives of

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[1] There is by now a fairly large literature devoted to developing models of the behavior of nonprofit organizations. See James and Rose-Ackerman(1986), Powell(1987), Gassler(1987) and Steinberg(1993) for surveys of this literature.

[2] Source: Weisbrod (1988) Table A.5, p.172.

[3] Source: Weisbrod(1988) Table A.1, p. 169

those controlling these organizations are not well understood. According to Steinberg(1993), the question 'If not for profit, for what?' posed by Young(1983) still remains unanswered.

One approach to understanding the behavior of nonprofit managers and entrepreneurs has been to assume that the non-distribution constraint on the net cash flows of nonprofit enterprises is not perfectly enforceable in any case. This implies that a nonprofit manager can at least partially circumvent it by using some of the firm's revenues for 'perks' or an inflated salary. So a manager who is more skilled at cheating or who prefers perks to cash might maximize her full income by seeking employment in-or founding- a nonprofit rather than a profit-taking enterprise. This is the approach taken by Eckel and Steinberg(1993).

While there is probably some truth to the hypothesis that the non-distribution constraint is not always perfectly enforced, this approach begs the question as to why anyone would voluntarily impose such a constraint in the first place. After all, the founder of a profit-taking firm can also pay inflated salaries and provide perks to herself and her managers, and can do so legally. To understand nonprofit firm formation, we need models in which it is in the entrepreneur's own interest to impose a non-distribution constraint on her firm's net cash flows. Of course, once profits have been realized ex-post, it may never be in the entrepreneur's interest to obey the non-distribution constraint, and there may be some shirking if it is not rigidly enforced, but at least it should be in her interest ex-ante to impose it. Would anyone found a nonprofit enterprise even if they knew that the non-distribution constraint would be strictly enforced? The model we present below answers this question in the affirmative.

A second approach related to the first is to assume that there are returns to being a nonprofit entrepreneur which do not accrue to the founder of a proprietary firm. For example, the entrepreneur's motivations could include the desire for status or recognition, or feelings of 'warm glow', which she would not receive if she founded a profit-taking enterprise. Without denying that these factors could influence the choice of whether to found a profit-taking or nonprofit enterprise, we wish to ask whether a rational entrepreneur would found a nonprofit firm even in their absence. In the model we present below, an entrepreneur whose only motivation is to maximize the utility derived from the consumption of a private and a public good nonetheless chooses rationally to found a nonprofit enterprise.

The starting point for the model presented here is the observation that many nonprofit institutions receive a portion of their revenues from private voluntary contributions. Most are also in the business of producing a commodity or service which has some of the attributes of a public good.[4] Universities, charitable organizations and medical research institutions are obvious examples, but hospitals, museums, and theatrical or musical companies can also be viewed in this way, since the existence of any of these agencies makes available to an entire community a flow of services which (may) generate consumer surplus. The presence of a new hospital, museum or theatre in a community has an option-value which in turn can be viewed formally as a 'commodity' which is nonrival in consumption.

This paper presents a model of the behavior of institutions which provide commodities, services or facilities which have such a public good component - institutions that often

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[4] There is a large literature on voluntary contributions to public goods, but it has so far developed with little reference to other work on nonprofit firms, in spite of the fact that virtually all of the institutions to which individuals voluntarily contribute are nonprofit. Recent work in this vein includes that of Bernheim(1986), Bergstrom, et.al.(1986), Andreoni (1988),(1989), Sugden (1982), and Varian(1994).

solicit voluntary contributions from the public to fund their activities. Just as in the theory of proprietary, profit-taking firms, we concentrate on the motivations of the individuals who found and operate them.

The model allows the entrepreneur to choose to operate either a nonprofit or profit-taking firm to produce the public good. It predicts that in situations in which a firm would receive voluntary contributions, it will be in the entrepreneur's own interest to found a nonprofit rather than a proprietary enterprise. We thus provide a simple explanation for the commonplace observation that virtually all firms receiving private donations are nonprofit[5] . Further, it is shown that a nonprofit firm will elicit a higher level of total private donations than would an otherwise identical profit-taking firm, thus providing an efficiency rationale for the existence of such enterprises.[6]

The next section lays out the structure of the model, and Section 3 then derives the unique subgame-perfect equilibrium for the case of a two-person community. The structure of equilibria for a community of $n$ identical individuals is derived in the Appendix.

[5] Fama and Jensen(1983) advance the hypothesis that the non-distribution constraint provides assurance to donors that their contributions will not be expropriated by residual claimants, without providing an explicit model of the nature of that assurance, or its consequences. The idea that 'contract failure' is a primary reason for the existence of nonprofit firms is due to Hansmann(1980).

[6] Easley and O'Hara (1983) show that a non-profit firm can be the solution to an optimal social contracting problem with asymmetric information. However, they do not consider whether it is incentive compatible for the entrepreneur to found a nonprofit firm, and the possibility of private donations is not modelled.

# 2 Model

## 2.1 The Institutional Context

Consider a group of individuals who have preferences over commodity bundles consisting of their own private consumption expenditures, and the level of provision of a non-excludable public good.[7] Suppose, however, that initially there is no institution engaged in producing the public good. Then even if individuals wanted to voluntarily contribute toward its provision, there is no one to collect such contributions and use them to produce the public good. If any of the public good is to be provided, then someone must *organize* its production.

We assume that doing this requires someone - an entrepreneur - to set up a firm. A decision that must be made by the entrepreneur at the outset is whether to incorporate this firm as 'nonprofit' or as 'profit-taking'. Since part of our purpose is to determine whether it can be in a utility-maximizing entrepreneur's self-interest to impose a non-distribution constraint on herself, we assume that this non-distribution constraint is perfectly enforced. By assumption, the entrepreneur's only motivation is to maximize the utility she derives from her consumption of private and public goods, just like everyone else. It is only this role as owner/operator of the firm that provides the public good which distinguishes her from other individuals in the population.

Once a firm has been set up, the next phase in the provision process is to gather funds

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If the good in question is, for example, the building of a new hospital wing or the founding of a theatre company, then some or all of the goods and services such a facility ultimately provides may be excludable. However, so long as individuals anticipate future receipt of surplus from the *existence* of such a facility, (even if its services will be charged for), the facility itself has a non-excludable public good aspect and individuals might want to contribute voluntarily to insure it is made available. Those who see no value in the existence of the facility will of course not voluntarily contribute funds for its establishment.

to cover the costs of producing the public good. As founder, the entrepreneur always has the option to contribute some of her own private funds to the firm before appealing to the public for donations[8] . She also has the option of contributing again after the fund-raising campaign is over, if she feels that a higher level of provision is preferable. Indeed, it will become apparent that the fact that the entrepreneur *cannot commit to not contribute more* after the public has contributed has important consequences on the equilibrium outcome.

Finally, in the last stage, the entrepreneur chooses the production technology and the quantity of the public good to be produced by her firm. To abstract from issues of output quality, X-efficiency and agency problems within the firm, we assume that there is only one production process for the public good and that output quality and costs of production are certain and immediately observable by everyone. This means that in the last stage, the only decision left to the entrepreneur is how much of the public good to provide with the money collected by the firm.[9] In particular, if her firm is profit motivated, she could choose to produce less of the public good than would be possible with the funds collected, and appropriate the residual to increase her private consumption.

This model is intended to capture the essence of what entrepreneurs actually do. Whether profit motivated or nonprofit, entrepreneurs are individuals who 'found' new enterprises and have the last word as to how they operate. It is shown below that in this extremely stark

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For simplicity, we assume that the entrepreneur does not need to raise funds in the capital market, and does not receive government grants.

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That is, how many meals to provide to the indigent, how large and well-equipped should be the new hospital burn unit, or how many plays the theatre company should produce in a season.

setting (the non-distribution constraint is perfectly enforced, there are no non-pecuniary rewards to nonprofit entrepreneurship, no issues of asymmetric information between the firm and others, and no scope for influencing the characteristics of the output) it is in a rational entrepreneur's self-interest to found a nonprofit firm to produce the public good.

## 2.2 Formalization

Let $N = \{0, 1, ..., n\}$ be the set of $n+1$ individuals in the community. Let $x_i$ be individual $i$'s private consumption and let $Z$ be the quantity of the public good. Let $w_i$ denote $i$'s endowment of private consumption (his wealth), and suppose there is no endowment of the public good.

Assume that preferences can be represented by continuous, strictly quasi-concave utility functions $U_i(x_i, Z)$ for all $i$. Let $F(Z)$ be the level of donations required to produce $Z$ units of the public good and assume that the cost function $F(\cdot)$ is continuous, strictly increasing and convex. Without loss of generality, we will from now on write individual utility functions as $u_i(x_i, D) = U_i(x_i, F^{-1}(D))$.[10] Let $d_i$ be $i$'s contribution to the public good producing firm, $D = \sum_{i \in N} d_i$ be the sum of all contributions, and $D_{-i} = D - d_i$ be the total contributions by everyone except $i$.

We postulate the following sequence of actions:

1. Individuals choose simultaneously whether or not to volunteer as the agent who will organize the production of the public good. The individual who so volunteers is called

10 The convexity of $F(\cdot)$ guarantees that the functions $u_i(\cdot)$ are quasi-concave in $x_i$ and $D$ .

the *entrepreneur.* If no one volunteers, no public good is provided.

2. The entrepreneur elects to found either a nonprofit or a profit-taking enterprise to produce the public good.

3. The entrepreneur may invest some of her private wealth in the firm she has founded before soliciting voluntary contributions from the general public.

4. After observing the entrepreneur's actions, other members of the community contribute simultaneously to the public good producing enterprise.

5. The entrepreneur may decide to make a final (possibly negative) contribution to the enterprise, and then produces the public good using all the funds received by her firm.

This sequence of decisions defines a multistage game with observed actions in which the set of players is $N$ and the payoff functions are $u_i(.)$. The strategy sets include the choices $\{$*volunteer, do not volunteer*$\}$ and $\{$*for-profit, nonprofit*$\}$ in stages 1 and 2, respectively. In stage 3, the entrepreneur, who will be designated from now on as individual 0, chooses an initial contribution $d_0 \in [0, w_0]$. In stage 4, everyone else simultaneously chooses a contribution level $d_i$, knowing $d_0$ and knowing whether the firm is for profit or nonprofit. Finally, in stage 5, the entrepreneur chooses whether to add (or subtract) an additional amount $e$, knowing what has been contributed already and whether she is subject to a non-distribution constraint.

# 3 Equilibrium strategies

Before proceeding, it will be useful to restate the standard results for the situation in which a simultaneous-contributions game determines the level of public goods provision. The Nash equilibrium contribution levels in such a game will be $d^s = (d_0^s, d_1^s, ..., d_n^s)$, where

$$d_i^s = \max\{0, h_i(w_i + D_{-i}^s) - D_{-i}^s\} \quad i = 0, 1, ..., n$$

and where

$$h_i(y) = \arg \max_{D \in [0,y]} \{u_i(y - D, D)\}$$

is $i$'s 'private demand' for $D$ when his full income is $y$. Assuming that both $x_i$ and $Z$ are normal for all $i$, Bergstrom et. al.(1986) show that $d^s$ is unique.[11]

## 3.1 The entrepreneur has the final word

To determine the subgame perfect equilibria of this game, we begin by analyzing the behavior of the entrepreneur in the subgames beginning in the last stage. Upon reaching this stage, the entrepreneur has already contributed $d_0 \geq 0$, and the public has contributed $D_{-0} = \sum_{j=1}^{n} d_j$.

If at stage 2 the entrepreneur had set up a profit-taking enterprise, her only subgame perfect equilibrium strategy is to contribute

$$e^p(d_0, D_{-0}) = \max\{-D, h_0(w_0 + D_{-0}) - (D_{-0} + d_0)\}. \tag{1}$$

That is, she contributes the difference between the level of $D$ she considers optimal, $h_0(w_0 + D_{-0})$, and what she and the public have already contributed, $D_{-0} + d_0$. Note that $e^p(.)$ may

[11] While it is not essential to our analysis, we will simplify matters by assuming that $d_i^s > 0$ for all $i = 0, 1, 2, ..., n$.

be negative if $h_0(w_0 + D_{-0}) < D$, in which case the entrepreneur would choose to repossess some of her initial contribution or even to appropriate for her personal consumption some or all of the donations made by others to the enterprise.[12]

If instead she set up a nonprofit firm, her only subgame perfect equilibrium strategy is to contribute

$$e^n(d_0, D_{-0}) = \max\{0, h_0(w_0 + D_{-0}) - (D_{-0} + d_0)\}. \tag{2}$$

The non-distribution constraint prevents the entrepreneur from appropriating the donations of others and from reneging on her own initial contribution.

Equation (1) illustrates the agency problem posed by general donations to a profit-taking firm. Individuals may want to give it money to finance the production of a public good but if the firm is profit-taking they have no assurance that their contributions will be used for this purpose. By comparison when the firm is nonprofit, the non-distribution constraint assures individuals that each dollar they contribute will increase the expenditures earmarked for the public good's production by a dollar, provided of course that the non-distribution constraint is binding on the entrepreneur.

## 3.2 The public knows what she's up to

The subgames beginning in Stage 4 involve simultaneous contributions by the $n$ other community members, given the entrepreneur's choices of institutional form and initial contribution $d_0$. In a subgame-perfect equilibrium, the public contributors also anticipate that the

[12] Note also that in the limiting case in which the entrepreneur gets no utility from $Z$, $h_0(\cdot) \equiv 0$, and hence $e^p(\cdot) \equiv -D$.

entrepreneur will play optimally in the last stage.

Suppose first that the entrepreneur has chosen to found a profit-taking enterprise, and made an initial contribution of $d_0$. Contributors then realize that total contributions of $D_{-0}$ at this stage will result in total expenditures of

$$d_0 + D_{-0} + e^p(d_0, D_{-0}) = \max\{0, h_0(w_0 + D_{-0})\}$$

towards production of the public good. Since normality of all goods implies that $h_0(.)$ increases at a rate less than one, then a dollar contributed by the public will result in a less than one dollar increase in expenditures to produce the public good.

Letting $D_{-0i} = D_{-0} - d_i$, then individual contributions will be characterized by

$$d_i^p = \arg \max_{d_i \in [0.w_i]} \{u_i\left(w_i - d_i, h_0(w_0 + D^p_{-0i} + d_i)\right)\}, i = 1, 2, ..., n. \tag{3}$$

in a subgame perfect equilibrium. In determining their optimal contribution to the profit-taking firm, individuals anticipate that the entrepreneur will implicitly 'tax' their contributions by diverting a fraction of each dollar they contribute to the firm toward her own private consumption. Note that $d_0$ does not appear in (3). Since the entrepreneur remains free to take back any amount she previously contributed to her enterprise, this money is not 'committed' to be spent on the production of the public good. The equilibrium actions of the other individuals at this stage are therefore independent of the entrepreneur's initial contribution.

If instead the entrepreneur had founded a nonprofit firm, public contributors can calculate that total donations of $D_{-0}$ will result in equilibrium contributions of

$$d_0 + D_{-0} + e^n(d_0, D_{-0}) = \max\{d_0 + D_{-0}, h_0(w_0 + D_{-0})\}$$

toward production of the public good. Individual contributions at this stage, contingent on a given initial contribution of $d_0$ by the entrepreneur, will then satisfy

$$\delta_i(d_0) = \arg \max_{d_i \in [0, w_i]} \left\{ u_i \left( w_i - d_i, d_0 + d_i + \sum_{j \neq i} \delta_j(d_0) + e^n(d_0, d_i + \sum_{j \neq i} \delta_j(d_0)) \right) \right\} \qquad (4)$$

in a subgame perfect equilibrium.

The level of individual contributions will depend on whether individual contributors anticipate that the non-distribution constraint will be binding on the entrepreneur or not. If $d_0 + D_{-0} < h_0(w_0 + D_{-0})$, the non-distribution constraint is not binding, so that the public expects the entrepreneur to contribute again. In this case, they anticipate that each additional dollar they contribute to the firm induces the entrepreneur to reduce her final contribution by a fraction of a dollar. Individual equilibrium contributions are then identical to (3). Clearly, the imposition of a non-distribution constraint cannot have any impact on equilibrium behavior if it is not binding. When the non-distribution constraint is binding on the entrepreneur, then $e^n(d_0, d_i + \sum_{j \neq i} \delta_j(d_0)) = 0$ and an additional dollar contributed to the firm increases the expenditures dedicated to the public good's production by a dollar.

Note that when the firm is nonprofit, individual equilibrium contributions generally depend on $d_0$ because a larger $d_0$ increases the funds earmarked for the public good's production and decreases the entrepreneur's available wealth. This, in turn, reduces the level of public donations that is necessary to cause the non-distribution constraint to bind.

## 3.3 Contributing once is enough

In this section, we want to find the subgame perfect equilibrium continuations of the subgames starting with each of the entrepreneur's possible choices of an initial contribution level. However, the logic of these equilibria is considerably more transparent in the case in which there is only one other contributor besides the entrepreneur. So in what follows we will assume that $n = 1$. The extension to the case of $n$ identical contributors is presented in the Appendix.

The decision problem facing both individuals is illustrated in **Figure 1**. The two loci $d_0 = h_0(w_0 + d_1) - d_1$ and $d_1 = h_1(w_1 + d_0) - d_0$ correspond to the reaction functions of the two individuals in a simultaneous contributions game.[13] The unique Nash equilibrium would then be at point $S$ where each would contribute $d_0^s$ and $d_1^s$ respectively and total provision would be $D^s$.

In the multistage game which interests us here, if the entrepreneur has chosen to operate a profit-taking firm, then her final equilibrium contribution is given by (1), and the equilibrium contribution of the other individual is given by (3), with $i = 1$ and $D_{-0i} = 0$. This corresponds to point $P$ in **Figure 1**. Individual 1 anticipates that the entrepreneur will adjust her final contribution such that, for any initial $d_0$, and any contribution by him of $d_1$, the ultimate total donations satisfy $d_0 + e_0 = h_0(w_0 + d_1) - d_1$, and so chooses $d_1^p$ such that his indifference curve is tangent to the entrepreneur's reaction curve, just as in a 'Stackelberg'

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These are drawn as being linear in Figure 1, but all that is important is that they have slopes between 0 and $-1$.

game in which two individuals contribute sequentially. So the 'profit-taking' subgame has many equilibria in which the entrepreneur contributes any $d_0$ initially, the other individual contributes $d_1^p$, and the entrepreneur makes up the difference (positively or negatively) in the last stage to attain a level $D^p$ of total donations. As noted, $d_1^p$ is independent of $d_0$, and if $h_0(.)$ is increasing and weakly concave, then $d_1^p$ is unique. The entrepreneur always contributes $d_0^p$ in total.

Note that if the entrepreneur derives no utility from the public good, $h_0(y)$ is zero for all values of $y$, and individual contributors will expect her to contribute nothing and put every dollar they contributed into her purse, so they will then also contribute nothing. However, individuals may be willing to contribute voluntarily even to a proprietary firm ($d_1^p > 0$), provided its founder/manager cares enough about the public good aspect of what the firm provides.

Now suppose that the entrepreneur has founded a nonprofit enterprise, so that her final contribution is given by (2), and the other individual's equilibrium contribution function is given by (4), (again, with $D_{-0i} = 0$). The key to the analysis of this case is to note that for any value of $d_0$ there is a level of contributions, $\overline{D}(d_0)$, which makes the non-distribution constraint just binding on the entrepreneur.[14] Note that $\overline{D}(d_0)$ is decreasing in $d_0$ because the more money the entrepreneur commits to the enterprise initially, the less wealth she has left at the last stage, and hence the smaller the level of total contributions required to induce her to contribute no further.

[14] Formally, $\overline{D}(d_0) = \max\{D | h_0(w_0 + D - d_0) \geq D\}$,so that $h_0(w_0 - d_0 + \overline{D}(d_0)) \equiv \overline{D}(d_0)$.

The typical decision problem facing individual #1 is illustrated in **Figure 2**. Contributing any $d_1$ such that $d_1 + d_0 < \overline{D}(d_0)$ leaves the entrepreneur wanting to contribute again, so she reduces her contribution at the rate implicit in $h_0(.)$ for each dollar #1 contributes. On the other hand, a contribution greater than this makes the non-distribution constraint binding on the entrepreneur, so that each dollar contributed increases $D$ by one dollar. Therefore, #1's choice set is bounded by the outermost of curves $D = h_0(w_0 + w_1 - x_1)$ and $D = d_0 + w_1 - x_1$.

For example, if the initial entrepreneurial contribution is $d_0^a$, individual 1 chooses $d_1^p$ such that $d_0^a + d_1^p < \overline{D}(d_0^a)$, and the entrepreneur contributes again in the last stage. On the other hand, an initial contribution of $d_0^b > d_0^a$ induces him to contribute $d_1^b > d_1^p$. Interestingly, *by increasing her initial contribution sufficiently, the entrepreneur can induce the other to contribute more.* This happens because when the enterprise is nonprofit, a bigger initial contribution decreases the threshold at which the non-distribution constraint becomes binding. This makes it easier for the entrepreneur to convince the public that she does not intend to contribute again and forces the public to take responsibility for the final level of provision.

The essential point here is that the only effect a variation in $d_0$ has on the continuation of the game is that it changes the location of the 'kink' in the other individual's choice set. Note also that the segment of #1's choice set given by $D = h_0(w_0 + w_1 - x_1)$ is a subset of the one he faces in the profit-taking subgame, so a small enough initial contribution by the entrepreneur would be followed by $d_1^p$. It follows that there is a unique $d_0^*$, which, if contributed by the entrepreneur initially, leaves the other individual indifferent between contributing $d_1^p$ or a larger amount $d_1^*$ which is just sufficient to make the non-distribution constraint exactly binding on the entrepreneur.

In **Figure 1**, if the non-distribution constraint is not binding, then the equilibrium continuation again yields an outcome at $P$, with individual 1 on indifference curve $u_1$. But if it is binding, then the equilibrium continuation is on the locus $d_1 = h_1(w_1 + d_0) - d_0$. So if the entrepreneur contributes $d_0^*$ initially, the other individual will be indifferent between contributing $d_1^p$ (and letting the entrepreneur contribute again to obtain the outcome at $P$) or contributing $d_1^*$ (and obtaining the outcome at $N$).

It follows that when the firm is nonprofit, the unique subgame perfect equilibrium has the entrepreneur contributing $d_0^*$ initially and the other individual responding by contributing $d_1^*$. The entrepreneur gives no less than $d_0^*$ because this would result in a discontinuous decrease in the contribution of the other individual. In **Figure 1**, any $d_0$ smaller than $d_0^*$ would induce the other individual to contribute only $d_1^p$ which would force her to contribute again, up to $d_0^p$ in total. The outcome would be at $P$, and she would be on indifference curve $u_0$. Thus any $d_0 < d_0^*$ would result in a larger contribution in total by the entrepreneur, but less total contributions than if she contributes $d_0^*$. On the other hand, any larger $d_0$ causes $d_1$ to fall in response at a less than one for one rate and yields an outcome on #1's reaction function $d_1 = h_1(w_1 + d_0) - d_0$ to the north-west of point $N$. In equilibrium then, the entrepreneur makes an initial donation which is just large enough to 'commit' her to not contribute again after the other individual has contributed optimally in response. As for the other individual, even though he is indifferent between contributing $d_1^p$ and $d_1^*$, only $d_1^*$ is an equilibrium response to $d_0^*$ because if he contributed $d_1^p$ the entrepreneur would be better off contributing $d_0^* + \varepsilon$ initially.

### 3.4 To profit or not to profit?

So far we have found that in the 'nonprofit' equilibrium continuation (point $N$ in **Figure 1**), the entrepreneur contributes $d_0^*$ and total contributions are $D^*$, while in the 'profit-taking' equilibrium continuation (point $P$), she contributes $d_0^p$ and total contributions are $D^p$. Which will she prefer? The answer is immediate from **Figure 1**: $D^* > D^p$ while $d_0^* < d_0^p$. The entrepreneur gets to consume *more* public good while contributing *less* from her own pocket to its provision when the firm is nonprofit.

So an entrepreneur who wants to produce a nonexcludable public good will always find it in her interests to found a nonprofit firm. The non-distribution constraint provides the entrepreneur with a means of committing to not appropriate funds which others wish to assign to the provision of the public good, and so it induces higher donations by the public.[15] Imposing a perfectly enforced non-distribution constraint on herself is in the entrepreneur's interest because it allows her to contribute less and consume more of the public good than if she were not subject to this constraint.

Note that an essential ingredient in this model is the assumption that the entrepreneur herself gets utility from the firm's output. If she does not, she will contribute nothing whether

[15] This is exactly the argument of Fama and Jensen(1983). Hansmann(1980) also refers to nonprofit firms as being viewed as more 'trustworthy' in situations of asymmetric information.

the firm is profit-taking or not, and so has no reason to prefer the nonprofit form.

Another essential aspect of this model is that the firm provides a *non-excludable public good.* If no one but the entrepreneur derived utility from jointly consuming the firm's output, then no one else would be willing to contribute to its provision, whether the firm was profit-taking or not. In that case, there would be no advantage to the entrepreneur from choosing the nonprofit form either. And if the good in question is easily excludable, customers would demand (and the entrepreneur would have to provide them with) an immediate *quid pro quo* for their money - access to consuming the good. So for example, a firm would sell admission tickets for an indoor concert, which is easily excludable, instead of relying on voluntary donations. On the other hand, the organizers of an outdoor street concert may be unable to sell admission tickets if exclusion is impossible or too costly, and have to rely on voluntary contributions.[16]

When there are just two individuals, the public donor is indifferent as to whether the public good producing firm is profit-taking or nonprofit. However, since the entrepreneur is strictly better off with a nonprofit firm, nonprofit institutions Pareto-dominate profit-taking firms when it comes to producing public goods. This rare opportunity to make possible actual Pareto improvements could explain why governments have enacted legislation allow-

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Note that this does not imply that theaters and orchestras performing indoors could not also solicit and receive voluntary donations. Some nonprofit firms, e.g., hospitals, theater groups or philarmonic orchestra, receive revenues from the sale of excludable goods and user fees, while at the same time receiving donations to ensure their continued existence. Such firms could be viewed formally as producing two goods: a private good sold to consumers(e.g., medical care, a concert, a play) and a public good (the option value of their existence) to which individuals may contribute voluntarily. The model in this paper assumes that the firm produces a single public good. Extension of this model to potentially profitable multiproduct firms producing both a non-excludable public good and a private good could possibly shed some light on the coexistence of both nonprofit and proprietary firms in some industries.

ing the incorporation of nonprofit organizations and provided the institutional mechanisms for the enforcement of the non-distribution constraint on the net cash flows of nonprofit enterprises.[17] However, possible welfare gains need not be relevant to the *entrepreneur*'s decision. A rational self-interested entrepreneur will always choose to incorporate his public good producing organization under nonprofit regulations, not because this is socially optimal in some sense, but because it is in her best interest to do so.

### 3.5 Why me?

So far, we have found that in any subgame perfect equilibrium, the entrepreneur would found a nonprofit firm. She will also invest sufficiently in it that the non-distribution constraint will be binding on her. The last question that remains to be answered is whether it can be in anyone's interest to be a nonprofit entrepreneur.

An important factor affecting individuals' decisions to volunteer or not to perform this task is the fact that not only will the entrepreneur not profit from this venture, but there is a *cost* to being the entrepreneur. To see this, consider **Figure 1** again. In equilibrium, the entrepreneur contributes $d_0^* > d_0^s$ while the other individual contributes $d_1^* < d_1^s$. But if the roles were reversed, it is individual 0 who would contribute less than $d_0^s$ while individual 1 as entrepreneur would contribute more than $d_1^s$. The individual who volunteers must therefore always contribute more than he would if someone else organized the public good's production. Everyone would therefore prefer to let someone else be the provider of nonprofit

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This is not a rationale for tax exemption of nonprofit organizations, nor for the tax-deductibility of contributions made to them, however. See Hall(1987) for an historical overview of the evolution of legislation governing the nonprofit sector in the U.S.A.

entrepreneurial services.

While it is true that both would prefer to let the other volunteer nonprofit entrepreneurial services, this does not mean that no one will do so. On the contrary, anyone can reason that if no one volunteers, the public good will not be provided at all. Any individual who does 'volunteer' to be the entrepreneur will be better off with her resulting consumption bundle than with $(w_i, 0)$, because as the entrepreneur she has available the strategy of choosing $d_i = e_i = 0$ . So if no one else volunteers, $i$ would rather incur the cost of entrepreneurship than consume no public good at all. If we also assume that if two individuals volunteer simultaneously, they both have to incur at least a part of the entrepreneurship cost, then the game at the first stage is simply a *game of chicken*. In any pure strategy equilibrium, either individual will volunteer nonprofit entrepreneurial services. Therefore, even though there is a cost to doing so, it is consistent with an individual's self-interest to volunteer nonprofit entrepreneurial services[18] .

# 4 Conclusion

This paper has provided a simple rationale for the existence of nonprofit firms: to provide nonexcludable public goods. In any instance in which the production of such a good is financed by voluntary contributions, it is in both the private interest of the entrepreneur and in the wider public interest that the firm which produces the good be nonprofit.

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An interesting question is whether it is possible to determine, with a heterogeneous population, the traits of that individual who becomes the entrepreneur. On this, see Bliss and Nalebuff (1984) and Bilodeau and Slivinski(1994a,b).

# A Appendix: Many contributors

When there are more than one potential contributors, analysis of the model becomes considerably more complicated. In particular, the equilibrium continuation in the 'nonprofit' subgame is not unique. We simplify matters in this paper by assuming that all individuals have the same preferences.[19]

Consider first the subgame which follows a choice by the entrepreneur to found a profit-taking enterprise. It is still true that her final contribution is determined by (1). Then it follows that if $h_0()$ is concave, and $x_j$ and $D$ are normal goods for each $j$ with respect to the derived utility function $u_j(x_j, h_0(w_0 + D))$, then there is again a unique equilibrium set of contributions $d^p = (d_0^p, d_1^p, ..., d_n^p)$. Further, it is also the case that $D^p < D^s$, that $D_{-0}^p < D_{-0}^s$, and $d_0^p > d_0^s$. Thus, the choice of a profit-taking institutional form will again result in the entrepreneur contributing more than she would in a simultaneous contributions game, with others contributing less in aggregate.

As to the nonprofit sub-game, it is characterized by the following facts:

**Fact 1** *If the entrepreneur initially contributes the amount $d_0^s$, then it is not an equilibrium response for the public to contribute the amounts $[d_j^s]$. Similarly, they cannot respond in equilibrium to an initial contribution of $d_0^p$ by contributing $[d_j^p]$.*

The intuition behind Fact 1 is identical to that which arose with a single contributor. Because $d_0^s$ would be the entrepreneur's best-response to $D_{-0}^s$ in a simultaneous contributions

[19] All we need for the results in this section is that all individuals other than the entrepreneur have identical preferences, and even that is not necessary for all of our results.

game, just as $d_0^p$ would be to $D_{-0}^p$, these cannot occur in a game in which the entrepreneur has an opportunity to make a final contribution. In the first case, if the $n$ general contributors were to contribute $d_j^s$ in response to $d_0^s$, every public contributor for whom $d_j^s > 0$ would have an incentive to deviate to a lower contribution level, so as to cause the entrepreneur to contribute more later. Similarly, a general response of $d_j^p$ to $d_0^p$ would leave every general contributor for whom $d_j^p > 0$ with an incentive to deviate to a contribution which is large enough to induce the entrepreneur to give no more than the initial $d_0^p$ at the last stage.

**Fact 2** *There exist critical values of initial contributions in the non-profit sub-game, denoted as $d_0^l$, and $d_0^u$, such that:*

*(i) A contribution of $d_j^p$ for each contributor $j$ is an equilibrium response to $d_0$ if and only if $d_0 \leq d_0^u$, and*

*(ii) A contribution of $\delta_j^n(d_0)$ for each $j$ is an equilibrium response to $d_0$ if and only if $d_0 \geq d_0^l$.*

Here, $\delta_j^n(.)$ refers to the unique subgame perfect equilibrium response of individual $j$ in the last stage of a game in which the entrepreneur makes an initial contribution $d_0$, followed by simultaneous contributions by all others, at which point the game ends, and the entire sum of the $n+1$ contributions is turned into the public good.[20]

Fact 2 expresses the idea that if the entrepreneur initially gives a sufficiently small amount, the unique equilibrium response of all public contributors is to take advantage of the fact that he cannot commit to not contribute again after they have, so they respond

[20] Varian (1994) has analyzed this two-stage game in some detail.

with contributions of $d_j^p$-just as if the entrepreneur had founded a profit-taking firm. Then, $d_0^u$ is the upper bound on initial contributions which necessarily elicit this equilibrium response. If the entrepreneur contributes any amount greater than $d_0^u$ initially, some other contributor $j$ will have an incentive to deviate from contributing $d_j^p$ -by contributing more-when all $k \neq j$ contribute $d_k^p$. On the other hand, the entrepreneur can commit to give no more by making a sufficiently large initial contribution, so that other contributors must respond in equilibrium as though he will not-that is, they respond as if the game had no final stage in which the entrepreneur can contribute further. On the other hand, $d_0^l$ is the lower bound on $d_0$ values which necessarily elicit this equilibrium response from contributors. Any initial entrepreneurial contribution less than $d_0^l$ will leave some $j$ with a positive incentive to contribute less than $\delta_j^n(d_0)$ when all $k \neq j$ contribute $\delta_k^n(d_0)$.

To define the critical values of $d_0$, we need some notation and definitions. First, consider a two-stage game in which the entrepreneur contributes some $d_0$, after which the others contribute simultaneously, and the game then ends. For any $d_0$ then, there is a unique vector of subgame perfect equilibrium responses, $[\delta_j^n(d_0)]$, using Theorem 3 of Bergstrom, Blume and Varian (1986). Let $\Delta^n(d_0) = \sum_{j=1}^n \delta_j^n(d_0) + d_0$ and $\Delta_{-j}^n(.) = \sum_{i \neq j} \delta_i^n(d_0) + d_0$, and let $\Delta_{-0}^n(d_0) = \Delta^n(d_0) - d_0$, and $x_j^n(d_0) = w_j - \delta_j^n(d_0)$.

Now, for any $d_0, j \neq 0$ and $D_{-0j}$, let

$$k_j(d_0, D_{-0j}) = \max\{0, \overline{D}(d_0) - D_{-0j} - d_0\}$$

and

$$K_j(d_0, D_{-0j}) = \min\{w_j, k_j(d_0, D_{-0j})\}.$$

When $k_j(.) \in ]0, w_j[$, this is the level of $d_j$ at which the 'kink' occurs in the boundary of $j$'s choice set.

Now let $U_j^n(d_0) = u_j(w_j - \delta_j^n(d_0), d_0 + \Delta^n(d_0))$ and $U_j^p = u_j(w_j - d_j^p, D^p)$ be $j$'s payoffs in the "nonprofit' and 'profit-taking' equilibrium respectively. Note that since $x_j^n(.)$ is increasing when it is less than $w_j$, and $\Delta^n(.)$ is increasing, then $U_j^n(.)$ is increasing with $d_0$. Let

$$A_j^n(d_0) = \max_{d_j}\{u_j(w_j - d_j, h_0(w_0 + \Delta_{-0j}^n(d_0) + d_j) | 0 \leq d_j \leq K_j(d_0, \Delta_{-0j}^n(d_0))\}$$

be $j$'s payoff if he 'deviates down' from a contribution of $\delta_j^n(d_0)$ when all $k \neq j$ are contributing $\delta_k^n(d_0)$, after the entrepreneur has contributed $d_0$. Similarly, let

$$A_j^p(d_0) = \max_{d_j}\{u_j(w_j - d_j, D_{-0j}^p + d_0 + d_j) | K_j(d_0, D_{-0j}^p) \leq d_j \leq w_j\}$$

be $j$'s payoff if he 'deviates up' from a contribution of $d_j^p$ when all others are contributing $d_j^p$ after the entrepreneur has contributed $d_0$.

Note that if $K_j(d_0, \Delta_{-0j}^n(d_0)) = 0$, then it must be that $h_0(w_0 + \Delta_{-0j}^n(d_0)) < \Delta_{-j}^n(d_0)$, so that $A_j^n(d_0) = u_j\left(w_j, h_0(w_0 + \Delta_{-0j}^n(d_0))\right) < u_j(w_j, \Delta_{-j}^n(d_0)) \leq U_j^n(d_0)$. Also, if $K_j(d_0, \Delta_{-0j}^n(d_0)) = w_j$, then $h_0(w_0 + \Delta_{-0j}^n(d_0) + d_j) > \Delta_{-j}^n(d_0) + d_j$, for any $d_j \in [0, w_j]$, so $A_j^n(d_0) > U_j^n(d_0)$.

Similarly, if $K_j(d_0, D_{-0j}^p) = 0$, then $D_{-0j}^p + d_0 + d_j^p > h_0(w_0 + D_{-0}^p) = D^p$, and so $A_j^p(d_0) > U_j^p$. Also, if $K_j(d_0, D_{-0j}^p) = w_j$, then for any $d_j \in [0, w_j]$ we have that $h_0(w_0 + D_{-0j}^p + d_j) > D_{-0j}^p + d_0 + d_j$, so that $U_j^p \geq u_j(w_j - d_j, h_0(w_0 + D_{-0j}^p + d_j)) > u_j(w_j - d_j, D_{-0j}^p + d_j)$, and therefore, $U_j^p > A_j^p(d_0)$.

Since $\overline{D}(.)$ is decreasing in $d_0$, $K_j(.)$ is non-increasing in both it's arguments. Since $\Delta_{-j}^n(d_0)$ is increasing in $d_0$, then $\overline{D}(d_0) - \Delta_{-j}^n(d_0)$ is also decreasing in $d_0$, and thus $K_j(d_0, \Delta_{-0j}^n(d_0))$

is non-increasing in $d_0$. This then implies that $A_j^p(d_0)$ is continuous and non-decreasing in $d_0$, while $U_j^n(d_0)$ is increasing and continuous in $d_0$, and $A_j^n(d_0)$ is continuous and non-increasing in $d_0$.

Now let

$$\Omega^n(d_0) = \max_j \{A_j^n(d_0) - U_j^n(d_0)\},$$

be the gain of the individual who can gain the most from 'deviating down'. This must be non-increasing in $d_0$. We have already that if $D_{-0}^s > 0$, then $\Omega^n(d_0^s) > 0$. Further, since $K_j(w_0, \Delta_{-0j}^n(w_0)) = 0$, for all $j$, then $A_j^n(w_0) < U_j^n(d_0)$ for all $j$, so that $\Omega^n(w_0) < 0$. Therefore we can uniquely define

$$d_0^l \equiv \min\{d_0 | \Omega^n(d_0) \leq 0\}$$

This is the unique smallest $d_0$ for which no contributor wants to unilaterally 'deviate down', and is greater than $d_0^s$, and less than $w_0$. Now define

$$\Omega^p(d_0) \equiv \max_j \{A_j^p(d_0) - U_j^p\}$$

which is non-decreasing in $d_0$ by similar arguments. We then define

$$d_0^u \equiv \max\{d_0 | \Omega^p(d_0) \leq 0\}.$$

The two Facts then imply that $d_0^s < d_0^l$, and $d_0^p > d_0^u$, so that there are two broad cases to consider.

**Case I**: $d_0^l \leq d_0^u$.

In this case, the two critical initial contribution levels are bracketed by $d_0^s$ and $d_0^p$, and the only possible equilibrium outcomes are analogous to that of the single contributor case.

For any $d_0 < d_0^l$, the only equilibrium response is $[d_j^p]$, while for any $d_0 > d_0^u$, only $[\delta_j^n(d_0)]$ can be an equilibrium response by the other contributors. For $d_0$ values between these two critical values, both $[d_j^p]$ and $[\delta_j^n(d_0)]$ are possible joint equilibrium responses by the general public.

To see how an equilibrium is constructed, note that if the entrepreneur chooses any $d_0$ which is responded to with $d_j^p$ by all contributors, he must end up contributing $d_0^p$ in total, with $D^p$ being the resulting total contributions. Any $d_0$ which elicits $\delta_j^n(d_0)$ as a response for all $j$, yields total contributions of $\Delta^n(d_0)$, with the entrepreneur contributing no more than the initial $d_0$. Further, if $d_0 > d_0^s$, the entrepreneur's utility, $u_0(w_0 - d_0, \Delta^n(d_0))$, is decreasing in $d_0$.

Now, with the responses of the contributors to $d_0$ values between $d_0^l$ and $d_0^u$ being either of these two possibilities, the entrepreneur's unique equilibrium strategy must be to choose the smallest value in that interval which elicits a response of $\delta_j^n(d_0)$. The only requirement on the contributors' strategies then is that there must exist such a minimum value. By choosing appropriately the public contributors' strategies for $d_0$ values in the interval $[d_0^l, d_0^u]$, any $d_0$ in this interval can be the entrepreneur's equilibrium choice, which we henceforth denote as $d_0^*$. These findings are summarized in the following proposition.

**Proposition 1** *If $d_0^l < d_0^u$, $d_i^s > 0 \quad \forall i$, and both the public good and private consumption are normal, then for any $n$ every equilibrium outcome has the following properties:*

*(i) The entrepreneur chooses to found a non-profit firm, and chooses an initial contribution $d_0^*$ from the interval $[d_0^l, d_0^u]$, which is therefore greater than $d_0^s$, but less than $d_0^p$.*

*Formally,* $d_0^* = \min\{d_0 | \forall j, \delta_j(d_0) = \delta_j^n(d_0)\}$*.*

*(ii) Other contributors all respond by contributing the amounts* $\delta_j^n(d_0^*)$*, resulting in total contributions of* $\Delta^n(d_0^*)$*.*

*(iii) The entrepreneur contributes nothing further, so that the level of total contributions remains* $\Delta^n(d_0^*)$*, which is more than both* $D^s$ *and* $D^p$*. Further, any level of* $d_0 \in [d_0^l, d_0^u]$ *can be an equilibrium choice by the entrepreneur, for an appropriate choice of the n general contributors' strategies.*

This implies that the entrepreneur's equilibrium initial (and therefore total) contribution is more than $d_0^s$, and less than he would contribute were he to found a profit-taking firm ($d_0^p$). Further, since the functions $\delta_j^n(.)$ are non-increasing, the general public in aggregate give less than $D_{-0}^s$ but more than $D_{-0}^p$, with total donations (including that of the entrepreneur) nonetheless being greater than $D^s$, and therefore also greater than $D^p$. This is why the entrepreneur will never choose to found a profit-taking firm when the non-profit subgame is characterized as in Case I: founding a nonprofit results in a smaller personal contribution, but a larger level of total donations.

**Case II**: $d_0^u < d_0^l$

The definitions of these critical values imply that in this case, there can be no subgame perfect equilibrium in which only *pure* strategies are employed. The only possible pure strategy responses to any $d_0$ are $[d_j^p]$ or $[\delta_j^n(d_0)]$, and neither of these can be an equilibrium response for initial contributions by the entrepreneur which fall between $d_0^u$ and $d_0^l$. This is because someone would have an incentive to deviate from either of these strategy profiles when the initial contribution by the entrepreneur is in this range. It may be that there are equilibrium continuation strategies for these $d_0$ values in which the contributors randomize over their contributions, but determining the structure of an equilibrium with mixed strategies is problematic in a model with non-linear payoff functions. So we have not characterized the equilibria for this case.

Note however that as long as $d_0^l < d_0^p$, choosing a nonprofit firm and then contributing $d_0^l$ will always be a strategy that is available to the entrepreneur, and which results in the outcome $d_0^l, \Delta^n(d_0^l)$, which dominates the profit-taking outcome $d_0^p, D^p$ for the entrepreneur, since $\Delta^n(d_0^l) > \Delta^n(d_0^s) = D^s > D^p$. The following result is therefore of interest:

**Proposition 2** *If preferences are identical, then* $d_0^l < d_0^p$*, and* $d_0^u > d_0^s$.

Proposition 2 implies that *if* an equilibrium exists under case II, and preferences are identical, then the entrepreneur will still never choose the profit-taking institutional form, as the option of choosing a nonprofit firm, and contributing $d_0^l$ is preferable. The significance of the result that $d_0^u > d_0^s$ is only that there is a range of positive levels of contribution which will result in the 'profit-taking outcome' even if $d_0^u < d_0^l$.

Since we have only characterized the equilibria for Case I, the following proposition is presented as assurance that we are not investigating the properties of a non-existent equilibrium.

**Proposition 3** *If* $x_j$ *and* $Z$ *are both normal,* $d_0^s$ *and* $D_{-0}^s$ *are positive,* $w_j = w$ *for all* $j$*, and individuals have identical, affinely homothetic preferences*[21] *then* $d_0^l \leq d_0^u$.

[21]

Before proving propositions 1-3, we start with some preliminary lemmas. Throughout these proofs, the assumption that $d_0^s > 0$, $D_{-0}^s > 0$, and that $x_j$ and $Z$ are normal is maintained.

**Lemma 1** *$d^p$ is unique.*

**Proof.** In the profit-taking subgame, the $n$ contributors payoff functions are:

$$u_j(w_j - d_j, h_0(w_0 + D_{-0})) \equiv v_j(w_j - d_j, D_{-0}),$$

so we can define $g_j(y) \equiv \arg\max_{D_{-0}}\{v_j(y - D_{-0}, D_{-0})|y \in [0, y]\}$, and we now have a simultaneous contributions game with utility functions $v_j(.)$. If $h_0(.)$ is concave, then the $v_j(.)$ are continuous, and strictly quasi-concave in $d_j$, and a Nash equilibrium will exist. If $x_j$ and $D_{-0}$ are both normal in $v_j(.)$, then Theorem 3 of Bergstrom, et. al.(1986) applies to guarantee uniqueness of $d_1^p, ..., d_n^p$, and the uniqueness of $d_0^p$ is immediate. □

**Lemma 2** *$d_j^p \geq g_j(w_j + D_{-0j}) - D_{-0j}$, and if $d_j^p > 0$, then $d_j^p < h_j(w_j + D_{-j}^p) - D_{-j}^p$.*

**Proof.** The first claim follows immediately from standard arguments regarding simultaneous contributions games. The second claim is immediate because $h_j(w_j + D_{-j}^p) - D_{-j}^p$ is the amount $j$ would contribute for a given $D_{-j}^p$, if the entrepreneur gave the full amount $d_0^p$ initially and could not change this later. Thus, it is the amount he would give if his contributions went 'untaxed' by $h_0(.)$, whereas $d_j^p$ is the amount given when taxed; in either

That is, individual preferences are an affine transformation of some homothetic preference ordering over $(x_j, D)$ pairs. See Blackorby, Boyce and Russell (1978).

case, a contribution of $d_j^p$ by $j$ results in $D^p$, so the result follows from a Slutsky substitution effect argument. □

**Lemma 3** *$D^p < D^s$, $d_0^p > d_0^s$, and $D_{-0}^p < D_{-0}^s$.*

**Proof.** Suppose that $D^p > D^s$, b.w.o.c. Then $h_0(w_0 + D_{-0}^p) = D^p > D^s = h_0(w_0 + D_{-0}^s)$ so that $h_0(.)$ increasing implies $D_{-0}^p > D_{-0}^s$, which in turn implies that some $j \neq 0$ has $d_j^p > d_j^s$. However, $d_j^p > 0$ then, so that $h_j(w_j + D_{-j}^p) - D_{-j}^p > d_j^p > d_j^s \geq h_j(w_j + D_{-j}^s) - D_{-j}^s$. So $D_{-j}^p < D_{-j}^s$, and therefore $D^p < h_j(w_j + D_{-j}^p) < h_j(w_j + D_{-j}^s) \leq D^s$, a contradiction.

Suppose now that $D^p = D^s$. Then again, $h_0(w_0 + D_{-0}^p) = D^p = D^s \geq h_0(w_0 + D_{-0}^s)$, so that $D_{-0}^p \geq D_{-0}^s$, and $d_0^p \leq d_0^s$. If $D_{-0}^s < D_{-0}^p$, we get a contradiction as above, so this leaves only the possibility that $D_{-0}^s = D_{-0}^p$, which implies $d_0^s = d_0^p$. If all $j \neq 0$ have $d_j^p = 0$, then $D_{-0}^p = D_{-0}^s = 0$, contrary to our assumption, and so some $d_j^p > 0$, implying that for that j, $h_j(w_j + D_{-j}^s) \leq D^s = D^p < h_j(w_j + D_{-j}^p)$, and therefore, $D_{-j}^p > D_{-j}^s$. Then for such a contributor, $j$, it follows that $d_j^s \geq h_j(w_j + D_{-j}^s) - D_{-j}^s > h_j(w_j + D_{-j}^p) - D_{-j}^p > d_j^p > 0$.

Thus we have that if any $d_j^p > 0$, then $d_j^s > d_j^p$, so that $D_{-0}^s > D_{-0}^p$, a contradiction. Hence $D^p < D^s$, so that $h_0(w_0 + D_{-0}^p) = D^p < D^s = h_0(w_0 + D_{-0}^s)$, so $D_{-0}^p < D_{-0}^s$, and thus $d_0^p = h_0(w_0 + D_{-0}^p) - D_{-0}^p > h_0(w_0 + D_{-0}^s) - D_{-0}^s = d_0^s$. □

Establishing the properties of $d_0^u$ requires the identicality assumption, which is used explicitly in the following results. In particular, we simply note without proof, that with identical preferences, there exists $x^p$, and an increasing function $x^n(d_0)$, such that for every $j$, $d_j^p = \max\{0, w_j - x^p\}$, and $\delta_j^n(d_0) = \max\{0, w_j - x^n(d_0)\}$. Proofs are identical to those in Bergstrom, Blume and Varian (1986).

**Lemma 4** *With identical preferences:*

*(i)* $\forall j, d_j^p \leq d_j^s$.

*(ii)* $\Omega^p(d_0^s) \leq 0$, *so that* $d_0^s < d_0^u$.

*(iii)* $\Omega^n(d_0^p) \leq 0$, *so that* $d_0^l < d_0^p$.

**Proof.** **(i)** Suppose, by way of contradiction, that for some $j, d_j^p > d_j^s$, which then implies $d_j^p > 0$. Then by Lemma 1, $h(w_j + D_{-j}^p) - D_{-j}^p > d_j^p > d_j^s \geq h(w_j + D_{-j}^s) - D_{-j}^s$. Then $D_{-j}^p < D_{-j}^s$, and since $d_0^p > d_0^s$, there must be some $k \neq 0, j$ for which $0 \leq d_j^p < d_j^s$. Then we have $x^p = w_j - d_j^p < w_j - d_j^s \leq x^n(d_0^s) \equiv x^s$, and $x^p \geq w_k - d_k^p > w_k - d_k^s = x^s$, a contradiction.

**(ii)** Suppose not, so that for some $j$, $A_j^p(d_0^s) > U_j^p$. Then it must be that $d_j' \equiv \arg\max_{d_j}\{u(w_j - d_j, d_0^s + D_{-0j}^p + d_j) | K_j(d_0^s, D_{-0j}^p) \leq d_j \leq w_j\}$ satisfies $D' \equiv d_0^s + D_{0j}^p + d_j' > \overline{D}(d_0^s) = D^s > D^p$, and $u(w_j - d_j', D') > u(w_j - d_j^p, D^p)$. Letting $D_{-j}' \equiv d_0^s + D_{-0j}^p$, we have $D' = h(w_j + D_{-j}') > D^s \geq h(w_j + D_{-j}^s)$, so $D_{-j}' > D_{-j}^s$, and therefore there must be some $k \neq 0, j$ for whom $d_k^p > d_k^s$, contradicting (**i**).

**(iii)** We first prove two useful facts:

**Fact 3** $\delta_j^n(d_0^p) > 0$ *implies* $\delta_j^n(d_0^p) > d_j^p$.

Suppose this is not so. Then $x^n(d_0^p) = w_j - \delta_j^n(d_0^p) \geq w_j - d_j^p$, and if $x^p > w_j - d_j^p$, then $d_j^p = 0$, and $\delta_j^n(d_0^p) > d_j^p$, a contradiction. So it must be that $x^p = w_j - d_j^p$, hence $x^n(d_0^p) \geq x^p$, which means that $\delta_k^n(d_0^p) \leq w_k - x^n(d_0^p) \leq w_k - x^p, \forall k$. In particular, then, $\delta_k^n(d_0^p) > 0$ implies that $d_k^p = w_k - x^p \geq \delta_k^n(d_0^p) > 0$, while $\delta_k^n(d_0^p) = 0$ implies $d_k^p \geq \delta_k^n(d_0^p)$, so we get that $\Delta^n_{-0}(d^s_0) = D^s_{-0} \geq D^p_{-0} \geq \Delta^n_{-0}(d^p_0)$, so that $\Delta^n_{-0}(.)$ being decreasing implies $d^s_0 \leq d^p_0$, contradicting Lemma 2.

**Fact 4** $\delta^n_j(d^p_0) = 0$ *implies* $d^p_j = 0$.

Suppose not, so that $w_j - x^p = d^p_j > 0 = \delta_j{}^n(d^p_0) \geq w_j - x^n(d^p_0)$, so $x^p < x^n(d^p_0)$. So if any $\delta^n_k(d^p_0) > 0$, then $\delta^n_k(d^p_0) = w_k - x^n(d^p_0) < w_k - x^p$, so if any $d^p_k > 0$, then $d^p_k = w_k - x^p > \delta^n_k(d^p_0)$, contradicting Fact 1. So $\delta_k{}^n(d^p_0) > 0$ must imply that $d^p_k = 0$, and so $x^n(d^p_0) < w_k - x^p$, which is a contradiction, also.

Therefore it must be that all $\delta^n_k(d^p_0) = 0$, so that $\Delta^n_{-0}(d^p_0) = 0$, which would then imply that $\Omega^n(d^p_0) \leq 0$, as we are trying to prove. So suppose that some $\delta^n_k(d^p_0) > 0$, so that Facts 3 and 4 must hold.

We can now prove the Lemma.

Suppose that for some $j$, $\delta^n_j(d^p_0) > 0$, and it pays $j$ to 'deviate down' to some $d'_j < \delta_j{}^n(d^p_0)$. Then $d'_j + \Delta^n_{-0j}(d^p_0) < \overline{D}_{-0}(d^p_0) = D^p_{-0}$, and $D' = h_0(w_0 + d'_j + \delta^n_{0j}(d^p_0)) < h_0(w_0 + D^p_{-0}) = D^p$. Then, since Facts 3 and 4 imply that $\Delta^n_{-0j}(d^p_0) \geq D^p_{-0j}$, it must be that $d'_j < d^p_j$, and thus $d^p_j > 0$. Then we have that $h_j(w_j + \Delta^n_{-0j}(d^p_0)) + d^p_0 \leq D' < D^p = h_j(w_j + D^p_{-0j}) + d^p_0$ and this implies that $\Delta^n_{-0j}(d^p_0) < D^p_{0j}$, a contradiction. □

**Proof.** The results above establish that under identical preferences, it must be that $d^u_0$ and $d^l_0 \in [d^s_0, d^p_0]$. All that remains to prove proposition 3 is to show that under these assumptions $d^l_0 < d^u_0$.

Note that affinely homothetic preferences imply that $h_j(y) = a_j(y - b_j)$ for some $a_j < 1$, and any $y > b_j$. Thus, $h_j(y) = \arg\max_D u_j(y - p_0 D, D)$, where $p_0 \equiv 1/a_0$. Further, the indirect utility function for any individual with such preferences can be written as $v_j(p, m_j) = \frac{m_j - \Lambda_j(p)}{\pi_j(p)}$, where $p$ is the relative price of $Z$. Thus we have that

$$A_j^p = \frac{w_j + D_{-0j}^p + \omega - \Lambda_j(p_0)}{\pi_j(p_0)}$$

where $\omega = w_0 - b_0$.

Identical $u_j(.)$ and $w_j$ imply that (dropping the $j$ subscript)

$$\forall j, d_j^p = d^p = h(w + (n-1)d^p + \omega) - [d_0^p + (n-1)d^p]$$

and that

$$\forall j, U_j^p = U^p = \frac{w + (n-1)d^p + \omega - \Lambda(p_0)}{\pi(p_0)} \equiv \frac{I - \Lambda_0}{\pi_0}$$

Further,

$$A^p(d_0^u) \equiv \frac{w + (n-1)d^p + d_0^u - \Lambda(1)}{\pi(1)} \equiv \frac{I^u - \Lambda_1}{\pi_1} = U^p.$$

To prove the result we need to show that $\Omega^n(d_0^u) \leq 0$. Now note that

$$U_j^n(d_0^u) \equiv U^u \equiv \frac{w + (n-1)d^u + d_0^u - \Lambda_1}{\pi_1} \equiv \frac{J^u - \Lambda_1}{\pi_1}$$

where $d^u = \delta^n(d_0^u)$, and also

$$A_j^n(d_0^u) \equiv A^u \leq \frac{w + (n-1)d^u + \omega - \Lambda_0}{\pi_0} \equiv \frac{J - \Lambda_0}{\pi_0}$$

Also note that $I^u - I = d_0^u - \omega = J^u - J$. It remains to show that $A^u < U^u$, but substituting the expression for $U^p$ into the one for $U^u$, and manipulating yields:

$$\frac{J^u - \Lambda_1}{\pi_1} = \frac{J - \Lambda_0}{\pi_0} + \frac{(\pi_0 - \pi_1)(J - I)}{\pi_0 \pi_1}.$$

Since $\pi(.)$ is increasing, then $\pi_0 < \pi_1$, and the result follows if $(n-1)d^u = \Delta^n_{-0j}(d^u_0) > D^p_{-0j} = (n-1)d^p$. However, we already demonstrated that with identical preferences alone, $\Delta^n_{-0j}(d^p_0) > D^p_{-0j}$, and since $d^p_0 > d^u_0$ and $\Delta^n_{-0j}()$ is decreasing, this must hold. □

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