Abstract

The Simple Microeconomics of Induced Innovation
A general model analyzes the innovator's decision to perform research and development directed towards process innovation. The innovator chooses expenditures in several research activities. The vector of research expenditures determines the input-output coefficients that describe the innovative technology. The innovator maximizes rents, which with non-drastic innovation equals total savings of variable costs, less research expenditures. If expenditures in different activities exhibit diminishing returns in the savings of all factors, then the optimization problem has a unique solution. Comparative static results are found for changes in factor prices and demand conditions.

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The Simple Microeconomics of Induced Innovation [1]

Jason E. Christian
Department of Agricultural Economics
University of California
Davis, California 95610
Tel. (916) 752-1357
e-mail: jason@primal.ucdavis.edu

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1 Introduction

This paper presents a simple and general microeconomic model of factor-saving process innovation. The model describes the decision to commit costly research and development resources to different lines of research, with the intention of altering the technology by which goods are produced. It describes this decision when there is knowledge about the outcomes of research, which permits the inventor to make research choices in the context of existing or prospective conditions in product and factor markets. It is, then, a microeconomic model of induced innovation, explicitly linking factor prices and the rate and direction of technological change.

Models of induced innovation describe the relationship between the production environment, which can be summarized by factor-market conditions and the demand for the finished product, and the evolution of the production processes actually used. Such a relationship implies that the nature of access to different production environments, such as regimes for the international commercialization and transfer of production technologies, influences the evolution of technology. In turn, the rate and direction of technological change strongly influence the evolution of important social and economic variables. Incorporation of a concise model of endogenous process innovation within a macroeconomic framework permits strong propositions concerning the impact of, for example, technology transfer regimes upon national income (Christian 1990); these propositions gain force with the elaboration and verification of the underlying induced-innovation model.

Analyses of endogenous product innovation within the monopolistic-competition model have supported important applications in the analysis of the international distribution of income, multinational enterprise, and the existence of intra-industry trade (Krugman 1979, Helpman 1984, Helpman & Krugman 1985, Grossman & Helpman 1989). But the product-innovation approach offers no information about the evolution and distribution of production technology, or about the (related) evolution of factor prices. The only possible discussion of income distribution follows from the international distribution of research resources; the attribution of product to factors is ignored. Attention to factor earnings, the principal focus of growth and development economics, is impossible without discussion of factor prices and factor demands, which in turn demands understanding of the effort to modify factor demands, which we call process innovation.

The analysis of the relationship between factor prices and process innovation dates at least to Hicks (1964), who suggested that innovation would more likely demonstrate a labor-saving bias if the price of labor were higher. Hicks' hypothesis was formalized by Salter (1960) and Kennedy (1964, 1973), who posited a given Innovations Possibilities Frontier [IPF] attainable with a fixed (possibly zero) research budget, and considered the choice of points along that frontier. Ahmad (1966) showed that this approach is analogous to the traditional neo-classical cost-minimization problem, and that the IPF corresponds to the isoquant attainable with given research resources. Samuelson (1965) and Nordhaus (1973) critized the IPF theory, both because the replacement of an exogenous production possibilities set by an exogenous innovations possibilities set offers little real advance towards a theory of endogenous technological change, and because the existence of such a frontier cannot be reconciled with theoretical and empirical descriptions of economic growth.

The model proposed by Binswanger (1974, 1978) responded to these criticisms by making endogenous the rate as well as the direction of technological change. Binswanger made research expenditures the choice variable, and analyzed the case where there are diminishing returns to research in different research programs. The induced-innovation program was thus brought firmly within the neoclassical framework, with research explicitly recognized as a costly activity, which if undertaken by profit-maximizing firms should respond to price signals. However, to derive his results Binswanger imposed restrictive assumptions. First, he only analyzed the research choice when when there is independence among research programs: expenditures in one activity have no effect upon the results from any other activity. Second, Binswanger was only able to demonstrate useful comparative-static results within a two-factor model.

The present paper proposes two advances to the theoretical program of endogenous cost-reducing innovation. First, a more general research production function is used, which permits interdependence between different research activities. Second, the research production function is applied to a general J-factor production process. It is established that such a general model is tractable, and the principal comparative-static results of the model are derived.

In Section 2, the assumptions used in this paper are listed. In Section 3, the research choice is laid out as a research-inclusive cost-minimization problem, and a sufficient condition for solution of the problem is identified. Section 4 contains the comparative-static exercises, with seven propositions describing the responses of total research expenditures, unit costs, profits, and factor demands to changes in factor and product prices. Section 5 brings these results together, and suggests several extensions and policy applications. An Appendix contains formal proofs of several intermediate results that are used in the comparative static analysis.

2 Assumptions

1. Production of a commodity takes place under a Leontief-type fixed-proportions production function, where a vector A, whose elements are J input-output coefficients, is endogenous. Prices of the J factors are exogenous, and are describe by $w \in \Re_J^+$.

2. There are M research activities, with the distribution of research resources among activities described by the vector $m \in \Re_M^+$. Total research expenditures are $B \equiv w_R \sum_{k=1}^{M} m_k$, where w_R is the price of research resources.

3. Research expenditures determine the production technology according to the vector function $A : \Re_M^+ \rightarrow \Re_J$, written $A(m)$, which determines the J input-output coefficients as a function of research expenditures m.

4. The component functions $A^j(m)$ are concave, for all $j = 1, \ldots, J$.

5. Production may take place under either the *innovation* $A(m)$ or under a freely-available *public technology* A_0.

6. Access to the innovation is controlled by the *innovator*, who purchases research resources at price w_R and sells licenses to one or several producers. The innovator is rent-extracting, in the sense of Katz & Shapiro (1986) and Kamien & Tauman (1986).

7. The innovation is non-drastic (Arrow 1971): a profit- maximizing monopolist with access to the innovation charges a price equal to the public-technology unit cost $A_0 \cdot w$.

3 The Model

With non-drastic innovation, the rents extracted by the innovator are independent of the licensing structure selected (Arrow 1971, Katz & Shapiro 1986). Furthermore, price and output are predetermined: price P by the public-technology unit cost, output by market demand at price P. If the innovator extracts royalties and fees equal to the difference between receipts and production costs, and pays for the necessary research expenditures, then rent maximization is identical to cost minimization, inclusive of research expenditures. The innovator's problem is then

$$\min_m C(m; X, w, w_R) = \left\{ X[A(m) \cdot w] + w_R \sum_{k=1}^{M} m_k \right\}, \tag{1}$$

with first-order conditions

$$C_k(m) = X[A_k(m) \cdot w] + w_R = 0 \quad k = 1, \ldots, M \tag{2}$$

where $A_k : \Re_M^+ \rightarrow \Re_J$ is the partial derivative $\frac{\partial A(m)}{\partial m_k}$.

Second-order conditions for minimization are given by the positive- definiteness of the Jacobian matrix of C, written $J(C)$, which has typical element $C_{kl} = X[A_{kl} \cdot w]$. Given $X > 0$, this is equivalent to the positive-definiteness of the matrix with typical element $A_{kl} \cdot w$. This $M \times M$ determinant is generally difficult to evaluate for $M \geq 2$ and $J \geq 2$, as each element is the sum of J sub-elements of the form $A^j_{kl} w_j$, where A^j_{kl} can be interpreted as the cross-partial derivative (k, l) of the function $A^j : \Re_M^+ \rightarrow \Re_1^+$ which maps research activities m onto the unit factor requirement for factor j. However, by Lemma 1, proved in the appendix, if the component functions $A^j(m)$ are concave (Assumption 5), then the second-order conditions are satisfied.

Lemma 1 *If the Jacobian of each of the A^j component functions is positive definite, then the Jacobian matrix of $A \cdot w$ is positive definite.*

Concavity of the $A^j(m)$ component functions implies diminishing returns to research. The condition is unnecessarily strong: increasing returns in the saving of a cheap factor could well be overwhelmed by sharply diminishing returns in the savings of a dear factor.

4 Comparative Statics

With prices given, the parameters in the problem are the factor prices w, research remuneration w_R, and output X. To analyze the effects of changes in the parameters on the endogenous variables m, differentiate totally the first-order conditions (2):

$$J(C)\begin{bmatrix} dm_1 \\ dm_2 \\ dm_3 \\ \vdots \\ dm_M \end{bmatrix} = -\begin{bmatrix} (A_1 \cdot w)dX + X\sum_{j=1}^J A_1^j dw_j + dw_R \\ (A_2 \cdot w)dX + X\sum_{j=1}^J A_2^j dw_j + dw_R \\ (A_3 \cdot w)dX + X\sum_{j=1}^J A_3^j dw_j + dw_R \\ \vdots \\ (A_M \cdot w)dX + X\sum_{j=1}^J A_M^j dw_j + dw_R \end{bmatrix}, \qquad (3)$$

where

$$J(C) = \begin{bmatrix} XA_{11}\cdot w & XA_{12}\cdot w & \dots & XA_{1M}\cdot w \\ XA_{21}\cdot w & XA_{22}\cdot w & \dots & XA_{2M}\cdot w \\ \vdots & \vdots & \ddots & \vdots \\ XA_{M1}\cdot w & XA_{M2}\cdot w & \dots & XA_{MM}\cdot w \end{bmatrix}$$

is the Jacobian matrix of $C(m)$.

Proof of Proposition 1, which shows the impact of an increase in demand on research expenditures, is aided by the following Lemma, proved in the appendix.

Lemma 2 *If $J(A\cdot w)$, the Jacobian matrix of $A \cdot w$, is positive definite, then $\mathcal{E}'$ is negative, where*

$$\mathcal{E}' = \begin{vmatrix} 0 & 1 & 1 & \cdots & 1 \\ 1 & A_{11}\cdot w & A_{12}\cdot w & \cdots & A_{1M}\cdot w \\ 1 & A_{21}\cdot w & A_{22}\cdot w & \cdots & A_{2M}\cdot w \\ \vdots & \vdots & \ddots & \vdots & \\ 1 & A_{M1}\cdot w & A_{M2}\cdot w & \cdots & A_{MM}\cdot w \end{vmatrix}.$$

Proposition 1 *An increase in demand, holding price constant, is accompanied by an increase in research expenditures.*

Proof: Proposition 1 Consider the response of research activities to an increase in X.
Applying Cramer's Rule to (3),

$$dm_k = \frac{1}{\mathcal{C}} \begin{vmatrix} XA_{11}\cdot w & XA_{12}\cdot w & \cdots & XA_{1M}\cdot w \\ XA_{21}\cdot w & XA_{22}\cdot w & \cdots & XA_{2M}\cdot w \\ \vdots & \vdots & \ddots & \vdots \\ XA_{k-1,1}\cdot w & XA_{k-1,2}\cdot w & \cdots & XA_{k-1,M}\cdot w \\ -A_1\cdot w & -A_2\cdot w & \cdots & -A_M\cdot w \\ XA_{k+1,1}\cdot w & XA_{k+1,2}\cdot w & \cdots & XA_{k+1,M}\cdot w \\ \vdots & \vdots & \ddots & \vdots \\ XA_{M1}\cdot w & XA_{M2}\cdot w & \cdots & XA_{MM}\cdot w \end{vmatrix} dX \quad (4)$$

where $\mathcal{C} \equiv \det[J(C)]$.

Applying first-order conditions (2), $-A_k \cdot w = 1/X$, hence

$$\frac{\partial m_k}{\partial X} = \frac{X^{M-2}}{\mathcal{C}} \begin{vmatrix} A_{11}\cdot w & A_{12}\cdot w & \cdots & A_{1M}\cdot w \\ A_{21}\cdot w & A_{22}\cdot w & \cdots & A_{2M}\cdot w \\ \vdots & \vdots & \ddots & \vdots \\ A_{k-1,1}\cdot w & A_{k-1,2}\cdot w & \cdots & A_{k-1,M}\cdot w \\ 1 & 1 & \cdots & 1 \\ A_{k+1,1}\cdot w & A_{k+1,2}\cdot w & \cdots & A_{k+1,M}\cdot w \\ \vdots & \vdots & \ddots & \vdots \\ A_{M1}\cdot w & A_{M2}\cdot w & \cdots & A_{MM}\cdot w \end{vmatrix} \quad (5)$$

Moving row k to the top of the determinant,

$$\frac{\partial m_k}{\partial X} = \frac{X^{M-2}(-1)^{k-1}}{\mathcal{C}} \begin{vmatrix} 1 & 1 & \cdots & 1 \\ A_{11}\cdot w & A_{12}\cdot w & \cdots & A_{1M}\cdot w \\ A_{21}\cdot w & A_{22}\cdot w & \cdots & A_{2M}\cdot w \\ \vdots & \vdots & \ddots & \vdots \\ A_{k-1,1}\cdot w & A_{k-1,2}\cdot w & \cdots & A_{k-1,M}\cdot w \\ A_{k+1,1}\cdot w & A_{k+1,2}\cdot w & \cdots & A_{k+1,M}\cdot w \\ \vdots & \vdots & \ddots & \vdots \\ A_{M1}\cdot w & A_{M2}\cdot w & \cdots & A_{MM}\cdot w \end{vmatrix}. \quad (6)$$

Summing over the M research activities yields the change in total research expenditures as

$$\frac{\partial B}{\partial X} = \frac{-X^{M-2}}{\mathcal{C}} \begin{vmatrix} 0 & 1 & 1 & \cdots & 1 \\ 1 & A_{11}\cdot w & A_{12}\cdot w & \cdots & A_{1M}\cdot w \\ 1 & A_{21}\cdot w & A_{22}\cdot w & \cdots & A_{2M}\cdot w \\ \vdots & \vdots & \ddots & \vdots & \\ 1 & A_{M1}\cdot w & A_{M2}\cdot w & \cdots & A_{MM}\cdot w \end{vmatrix}. \quad (7)$$

Applying Lemma 2, the determinant in (7) is negative, establishing the proposition.

□

Corollary There is at least one research activity k such that $\frac{\partial m_k}{\partial X} > 0$.

Proposition 2 *A decrease in unit cost accompanies an increase in demand.*

Proof: Proposition 2 Suppose $\hat{m}$ minimizes $C(m; \hat{X}, w)$. Then

$$\begin{aligned} C(\hat{m}; \hat{X}, w) = \hat{X}[A(\hat{m}) \cdot w] + \hat{B} \quad &\leq \quad \hat{X}[A(M_0) \cdot w] + B_0 \\ &= C(m_0; \hat{X}, w_0), \end{aligned} \quad (8)$$

where $\hat{B} = \sum_k \hat{m}_k$, m_0 minimizes $C(m; X_0, w)$ for $X_0 < \hat{X}$, and $B_0 = \sum_k (m_0)_k$. But using Proposition 1, $B_0 < \hat{B}$, hence $A(\hat{m}) \cdot w < A(m_0) \cdot w$.

□

Proposition 3 *An increase in profit accompanies an increase in demand.*

Proof: Proposition 3 Consider $\hat{X} > x_0$, where $\hat{m}$ minimizes $C(m; \hat{X}, w)$ and m_0 minimizes $C(m; X_0, w)$. Then expressing the maximized profit as $\pi(X)$,

$$\begin{aligned} \pi(\hat{X}) &= (P - A(\hat{m}) \cdot w)\hat{X} - \hat{B} \\ &\geq (P - A(m_0) \cdot w)\hat{X} - B_0 \\ &> (P - A(m_0) \cdot w)X_0 - B_0 = \pi(X_0) \end{aligned}$$

□

Consider next the effects of a factor-price change. A comparative- static expression is found for the difference in expenditures in any research activity following a change in any factor price. In general, the sign of this expression cannot be determined. However, Proposition 4 establishes that in the standard case, which is a research activity whose expenditures increase with output, there is at least one factor for which an increase in price is accompanied by an increase in research expenditures. To derive the comparative-static expression, substitute

$$dX = dw_R = dw_i = 0, \quad i = 1, 2, \ldots, j-1, j+1, \ldots, J$$

into (3), and apply Cramer's Rule, to find

$$\frac{\partial m_k}{\partial w_j} = \frac{-X^M}{\mathcal{C}} \begin{vmatrix} A_{11} \cdot w & A_{12} \cdot w & \cdots & A_{1M} \cdot w \\ A_{21} \cdot w & A_{22} \cdot w & \cdots & A_{2M} \cdot w \\ \vdots & \vdots & \ddots & \vdots \\ A_{k-1,1} \cdot w & A_{k-1,2} \cdot w & \cdots & A_{k-1,M} \cdot w \\ A_1^j & A_2^j & \cdots & A_M^j \\ A_{k+1,1} \cdot w & A_{k+1,2} \cdot w & \cdots & A_{k+1,M} \cdot w \\ \vdots & \vdots & \ddots & \vdots \\ A_{M1} \cdot w & A_{M2} \cdot w & \cdots & A_{MM} \cdot w \end{vmatrix}. \tag{9}$$

Proposition 4 *If an increase in demand x is accompanied by an increase in research expenditures in activity k, then there exists at least one factor j such that an increase in factor price w_j induces an increase in expenditures m_k in activity k.*

Proof: Proposition 4 Multiply the expressions in (9) by w_j and sum over j:

$$\begin{aligned} \sum_{j=1}^{J} w_j \left(\frac{\partial m_k}{\partial w_j} \right) &= -\frac{X^M}{\mathcal{C}} \begin{vmatrix} A_{11} \cdot w & A_{12} \cdot w & \cdots & A_{1M} \cdot w \\ A_{21} \cdot w & A_{22} \cdot w & \cdots & A_{2M} \cdot w \\ \vdots & \vdots & \ddots & \vdots \\ A_{k-1,1} \cdot w & A_{k-1,2} \cdot w & \cdots & A_{k-1,M} \cdot w \\ A_1 \cdot w & A_2 \cdot w & \cdots & A_M \cdot w \\ A_{k+1,1} \cdot w & A_{k+1,2} \cdot w & \cdots & A_{k+1,M} \cdot w \\ \vdots & \vdots & \ddots & \vdots \\ A_{M1} \cdot w & A_{M2} \cdot w & \cdots & A_{MM} \cdot w \end{vmatrix} \\ &= \frac{X^{M-1}}{\mathcal{C}} \begin{vmatrix} A_{11} \cdot w & A_{12} \cdot w & \cdots & A_{1M} \cdot w \\ A_{21} \cdot w & A_{22} \cdot w & \cdots & A_{2M} \cdot w \\ \vdots & \vdots & \ddots & \vdots \\ A_{k-1,1} \cdot w & A_{k-1,2} \cdot w & \cdots & A_{k-1,M} \cdot w \\ 1 & 1 & \cdots & 1 \\ A_{k+1,1} \cdot w & A_{k+1,2} \cdot w & \cdots & A_{k+1,M} \cdot w \\ \vdots & \vdots & \ddots & \vdots \\ A_{M1} \cdot w & A_{M2} \cdot w & \cdots & A_{MM} \cdot w \end{vmatrix} \\ &= X \frac{\partial m_k}{\partial X}. \end{aligned} \tag{10}$$

$$\frac{\partial m_k}{\partial X} > 0 \Longrightarrow \sum_{j=1}^{J} w_j \left(\frac{\partial m_k}{\partial w_j}\right) > 0 \Longrightarrow \exists j \in (1, \ldots, J) : \frac{\partial m_k}{\partial w_j} > 0.$$

□

Corollary There exists some $k \in (1, \ldots, M)$ and $j \in (1, \ldots, J)$ such that $\frac{\partial m_k}{\partial w_j} > 0$.

Proof Apply the corollary to Proposition 1 to Proposition 4.

The three remaining propositions describe the results of a factor price increase, taking into account all induced changes in individual research activities. First, research-inclusive average cost increases with factor prices. Second, increasing a single factor price reduces profits, after taking into account increased research expenditures. Third, increasing a single factor price leads to reduced use of that factor.

Before stating and proving the propositions, is is convenient to reframe the minimization problem in per-unit terms, which involves definition and analysis of research-inclusive minimum unit cost $c(w)$. Minimization (1), for constant X, is equivalent to

$$\min_m \{A(m) \cdot w + b(m)\}, \qquad (1a)$$

where $b(m) = B(m)/X$, with first-order conditions

$$A_k(m) \cdot w + \tfrac{1}{x} w_R = 0, \quad k = 1, \ldots, M, \qquad (2a)$$

which implicitly provides solutions $m(w)$. Then minimum unit cost, including research expenditures, is defined as

$$c(w) \equiv A(m(w)) \cdot w + b(m(w)). \qquad (11)$$

Proposition 5 *Let the price of a factor j increase by one unit. Then research- inclusive unit cost $c(w)$ increases.*

Proof: Proposition 5 Let $m_0 = m(w_0)$ and $\hat{m} = m(\hat{w})$, where

$$\hat{w} = (w_0^1, w_0^2, \ldots, w_0^{j-1}, w_0^j + 1, w_0^{j+1}, \ldots, w_0^J).$$

That is, $\hat{m}$ is the optimal research budget following a unit increase in the price of factor j. Unit cost following this increase is then approximated by

$$A(\hat{m}) \cdot \hat{w} = A(\hat{m}) \cdot w_0 + A^j(\hat{m}) \qquad (12)$$

Since m_0 solves min$\{A(m) \cdot w + b(m)\}$,

$$\begin{aligned} c(w_0) = A(m_0) \cdot w_0 + b(m_0) \quad \leq \quad & A(\hat{m}) \cdot w_0 + b(\hat{(}m) \\ & = A(\hat{m}) \cdot \hat{w} - A^j(\hat{m}) + b(\hat{m}) \\ & < A(\hat{m}) \cdot \hat{w} + b(\hat{m}) = c(\hat{w}) \qquad (13) \end{aligned}$$

□

Proposition 6 *If the price of a single factor increases, then profits fall.*

Proof: Proposition 6 Profits prior to the factor price increase are given by

$$\pi_0 = X[P - A(m_0) \cdot w_0 - b_0], \tag{14}$$

where $b_0 \equiv b(m_0)$. Since m_0 minimizes $c(m; w_0)$,

$$\begin{aligned} X[P - A(m_0) \cdot w_0 - b_0 \quad &\geq \quad X[P - A(\hat{m}) \cdot w_0 - \hat{b}] \\ &= X[P - A(\hat{m}) \cdot \hat{w} - \hat{b}] + XA^j(\hat{m}) \\ &= \hat{\pi} + xA^j(\hat{m}) > \hat{\pi}, \end{aligned}$$

provided that $A^j(\hat{m}) > 0$, where

$$\hat{\pi} = X[P - A(\hat{m}) \cdot \hat{w} - \hat{b}]$$

is profit following the factor-price increase and $\hat{b} \equiv b(\hat{m})$. .

□

Proof of Proposition 7, which follows, requires the following lemmas, which are proved in the Appendix.

Lemma 3 *Let $X(\hat{w}) = X(w_0)$. Then $A^j(m_0) = A^j(\hat{m})$ if and only if $m_0 = \hat{m}$.*

Lemma 4 *The research budget m changes when factor price w_j changes if and only if the use of factor j is influenced by at least one research activity. That is,*

$$\begin{aligned} \exists k \in (1, \ldots, M) : \quad &\tfrac{\partial m_k}{\partial w_j} \neq 0 \\ \Longleftrightarrow \exists \quad &h \in (1, \ldots, M) : \quad A^j_h(m) \neq 0. \end{aligned}$$

Proposition 7 *If the price of factor j increases, then use of factor j falls, for all factors j such that for some k*

$$\frac{\partial A^j}{\partial m_k} \neq 0.$$

Proof: Proposition 7 Since m_0 solves min$\{A(m) \cdot w_0 + b(m)\}$,

$$A(\hat{m}) \cdot w_0 + \hat{b} \geq A(m_0) \cdot w_0 + b_0 = c(w_0) \tag{15}$$

Since $\hat{m}$ solves min$\{A(m) \cdot \hat{w} + b(m)\}$,

$$\begin{aligned} c(\hat{w}) \quad &= \quad A(\hat{m}) \cdot w_0 + A^j(\hat{m}) + b_0 \\ &\leq \quad A(m_0) \cdot \hat{w} + b_0 \\ &\quad = A(m_0) \cdot w_0 + A^j(m_0) + b_0 \end{aligned} \tag{16}$$

Subtracting (15) from (16) gives

$$A^j(\hat{m}) \leq A^j(m_0).$$

From Lemma 4, $\hat{m} \neq m_0$. Applying Lemma 3 then establishes the result.

□

5 Discussion

The principal objective of this paper is to develop an optimizing theory of technological change, which links decisions concerning the amount and allocation of resources for process innovation to variables describing the economic environment. In the discussion which follows, requirements for acceptance of such a theory are laid out, and the contributions of this paper to such a program are identified. Three criteria for a theory are applied: utility for policy, verifiability, and plausibility.

First, a theory must generate propositions which are useful for policy analysis or other applications. Proposition 1, describing Schmooklerian demand-induced research expenditures, and Proposition 7, describing factor-price-induced changes in the distribution of research resources, are products of the optimization hypothesis with direct relevance to a variety of policy questions, such as the choice of regimes for international transfer of technology (Christian 1990).

Theories are rarely accepted simply because they touch upon important subjects. Instead, they gain currency through a more or less formal interaction with empirical work. A theory is strongest that has survived extensive hypothesis-testing over a period of years. One useful avenue for theoretical work is therefore to facilitate empirical verification, by casting the model in terms of observable variables, and by deriving the changes in these variables which should follow shifts in (observable) exogenous variables.

In the context of research choices, the only technology variable that we can really observe is the vector of research expenditures m. Proposition 4 is the key result for empirical work: a change in relative factor prices is predicted to be accompanied by a change in the distribution among activities of research resources.

With only limited opportunities for comparative-static econometric verification of the underlying theory, policy work based on the induced- innovation hypothesis must seek other support. There are two means by which well-framed theory can be seen to be plausible. First, it may produce propositions which are broadly consistent with observed reality and received theory in other areas. Second, it can be built with a minimum of intrusive assumptions: it can be general. The principal contribution of the work reported here can be understood in this light: believable propositions result from a general specification of the research problem.

First, if verified prior conditions are not available, the theorist substitutes hypotheses and postulates. If the resulting conclusions are to be believed, these prior conditions must be reasonable. An important task for theoretical work is thus to lighten the burden of hypotheses. The present paper strengthens the approach taken by Binswanger, by making more general the specification of research activities. Interdependence between activities is not excluded by hypothesis. The load of arbitrary prior assumptions on the form of the mapping between research expenditures and production technologies is otherwise substantially lightened. Indeed, the only remaining prior restriction is the assumed concavity of the component functions $A^j(m)$, a condition which is furthermore shown to be sufficient but not necessary for the conclusions of this paper.

A second approach to increasing the plausibility of a theoretical model is to demonstrate the absence of anomalous results. The bulk of the propositions contained in this paper serve this purpose. There is the Schmookler result of Proposition 1, and the reduction of unit cost which follows, in Proposition 2. If demand for a good increases, post-research profits ought not to fall, a result established in Proposition 3. An increase in a factor price causes reduced use of that factor; Proposition 5 demonstrates that this reduced use is insufficient to actually reduce unit cost, while Proposition 6 shows that post-research profits fall with increasing factor prices.

There are, of course, some remaining assumptions, whose contributions to the results should be considered. First, it was assumed that production takes place under a Leontief fixed-proportions production function. This is less restrictive than it seems: a neoclassical production function is "almost" a special case of the present model. Consider the research budget $B_\epsilon = \epsilon$, for arbitrarily small ϵ. There is an Ahmad isoquant (Ahmad 1966) defining the set of of input-output coefficients acheivable with this (small) level of research expenditures. No restrictions are imposed on the shape of this isoquant. Second, Assumptions 5–7 simplify the problem by making the quantity sold predetermined: the monopolist-innovator is constrained by a limit price. The case of drastic innovation, where the post-innovation profit-maximizing price falls below the public-technology unit cost, is analyzed in Christian (1990); analogous propositions are derived there. Assumptions 2 and 3 are unobtrusive definitions, while Assumption 4 is the critical second-order condition, which cannot be avoided.

There remains considerable work to extend this model to better account for the observed reality of commercial (process) research and development. What is the impact of uncertainty concerning the outcomes of different research activities? Are there conditions where the rational innovator should hold constant the structure of research expenditures in the face of changing conditions? How can the model be specified when research is time-consuming? Can the model be respecified as a model of product innovation, where the demand for a (producer) good is a function of its requirements for complementary inputs,to update McCain (1974)? Resolution of these questions will greatly contribute to the acceptability and utility of the microeconomic model of endogenous process innovation.

Appendix: Proofs to Lemmas

In the proofs which follow, unless otherwise specified it is assumed that the Jacobian matrix of $A(m) \cdot w$, written $J(A \cdot w)$, is positive definite.

Lemma 1 *If the Jacobian of each of the A^j component functions is positive definite, then the Jacobian matrix of $A \cdot w$ is positive definite.*

Proof: Lemma 1 By definition, if $J(A^j)$ is positive definite,

$$\sum_k \sum_l A^j_{kl} h_k h_l > 0, \quad \forall (h_k, h_l) \neq (0, 0).$$

By hypothesis, this holds for all $j = 1, \ldots, J$. Then given non-negative weights w_j, a linear combination of these sums must also be positive, so that for all h_k, h_l,

$$\begin{aligned}\sum_j w_j \left(\sum_k \sum_l A^j_{kl} h_k h_l \right) &= \sum_k \sum_l \left(\sum_j w_j A^j_{kl} \right) h_k h_l \\ &= \sum_k \sum_l (A_{kl} \cdot w)\, h_k h_l > 0.\end{aligned}$$

□

Before proving Lemma 2, I state and prove two intermediate lemmas, which concern the properties of symmetric matrices.

Lemma A.1 *Let $E_N(y)$ be a symmetric $N \times N$ matrix with elements $A_{ij} \cdot w - y$, and let $\mathcal{E}_N(y) = \det[E_N(y)]$. Then $\mathcal{E}' = \mathcal{E}'_N(y)$ is a constant.*

Proof: Lemma A.1

$$\begin{aligned}\mathcal{E}_n(y) &= \begin{vmatrix} A_{11} \cdot w - y & A_{12} \cdot w - y & \ldots & A_{1N} \cdot w - y \\ A_{21} \cdot w - y & A_{22} \cdot w - y & \ldots & A_{2N} \cdot w - y \\ \vdots & \vdots & \ddots & \vdots \\ A_{N1} \cdot w - y & A_{N2} \cdot w - y & \ldots & A_{NN} \cdot w - y \end{vmatrix} \\ &= \begin{vmatrix} 1 & y & y & \ldots & y \\ 0 & A_{11} \cdot w - y & A_{12} \cdot w - y & \ldots & A_{1N} \cdot w - y \\ 0 & A_{21} \cdot w - y & A_{22} \cdot w - y & \ldots & A_{2N} \cdot w - y \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & A_{N1} \cdot w - y & A_{N2} \cdot w - y & \ldots & A_{NN} \cdot w - y \end{vmatrix} \\ &= \begin{vmatrix} 1 & y & y & \ldots & y \\ 1 & A_{11} \cdot w & A_{12} \cdot w & \ldots & A_{1N} \cdot w \\ 1 & A_{21} \cdot w & A_{22} \cdot w & \ldots & A_{2N} \cdot w \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & A_{N1} \cdot w & A_{N2} \cdot w & \ldots & A_{NN} \cdot w \end{vmatrix}\end{aligned}$$

and

$$\mathcal{E}'(y) = \begin{vmatrix} 0 & 1 & 1 & \cdots & 1 \\ 1 & A_{11} \cdot w & A_{12} \cdot w & \cdots & A_{1N} \cdot w \\ 1 & A_{21} \cdot w & A_{22} \cdot w & \cdots & A_{2N} \cdot w \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & A_{N1} \cdot w & A_{N2} \cdot w & \cdots & A_{NN} \cdot w \end{vmatrix} \qquad \text{(A.1)}$$

a constant.

$\square$

Lemma A.2 *There exists some $\gamma > 0$ such that $E(\gamma) = E_M(\gamma)$ is positive definite if and only if $J(A \cdot w)$ is positive definite.*

Proof: Lemma A.2

1. Suppose $J(A \cdot w)$ is positive definite. Then

$$\mathcal{A}_i > 0, \qquad i = 1, \ldots, M,$$

where $\mathcal{A}_i$ is any minor of order i of

$$\mathcal{A}_M = \mathcal{A} = \det[J(A \cdot w)].$$

Let $E_i(y)$ be any matrix formed by deleting $M - i$ rows and the corresponding $M - i$ columns from $E(y)$. By the construction of the E matrices, for each $\mathcal{E}_i(y)$ there is a corresponding $\mathcal{A}_i = \mathcal{E}_i(0)$. Furthermore, by Lemma A.1, for each $\mathcal{E}_i(y)$ there is a corresponding constant $\mathcal{E}_i'(y)$ such that

$$\mathcal{E}_i(y) = \mathcal{A}_i + \mathcal{E}_i'(y) \cdot y.$$

Define

$$\alpha = \min_{i \in M} \mathcal{A}_i$$

and

$$\epsilon = \min_{i \in M} \mathcal{E}_i'(y).$$

Then for $y \geq 0$,

$$\mathcal{E}_i(y) \geq \alpha + \epsilon y, \qquad i \in M.$$

(a) Suppose $\epsilon \geq 0$. Then for any $\gamma > 0$ and for all i,

$$\mathcal{E}_i(\gamma) \geq \alpha + \epsilon\gamma \geq \alpha > 0,$$

hence $E(\gamma)$ is positive definite.

(b) Suppose $\epsilon < 0$. Then

$$0 < \gamma < -\frac{\alpha}{\epsilon} \quad \longrightarrow \quad \mathcal{E}_i(\gamma) \geq \alpha + \epsilon\gamma,$$

hence $E(\gamma)$ is positive definite.

2. Suppose $E(\gamma)$ is positive definite. If G is the $M \times M$ matrix with identical elements $\gamma > 0$, then $E(\gamma) = J(A \cdot w) - G$, where for all $h \neq 0$,

$$\begin{aligned} 0 < h'Eh &= h'[J(A \cdot w) - G]h \\ &= h'[J(A \cdot w)]h - h'Gh \\ &= h'[J(A \cdot w)]h - \gamma h'Uh, \end{aligned} \tag{A.2}$$

where U is the $M \times M$ matix with identical elements $U_{ij} = 1$. Since for $h \neq 0$,

$$h'Uh = \left(\sum_{i=1}^{M} h_i \right)^2 > 0,$$

$$h'[J(A \cdot w)]h > h'[J(A \cdot w)]h - \gamma h'Uh > 0,$$

which establishes that $J(A(m) \cdot w)$ is positive definite.

□

Lemma 2 *If $J(A \cdot w)$, the Jacobian matrix of $A \cdot w$,is positive definite, then $\mathcal{E}'$ is negative, where*

$$\mathcal{E}' = \begin{vmatrix} 0 & 1 & 1 & \cdots & 1 \\ 1 & A_{11} \cdot w & A_{12} \cdot w & \cdots & A_{1M} \cdot w \\ 1 & A_{21} \cdot w & A_{22} \cdot w & \cdots & A_{2M} \cdot w \\ \vdots & \vdots & \ddots & \vdots & \\ 1 & A_{M1} \cdot w & A_{M2} \cdot w & \cdots & A_{MM} \cdot w \end{vmatrix}.$$

Proof: Lemma 2 The proof has two parts. First, I show that assuming $\mathcal{E}' > 0$ generates a contradiction. Second, I show that if $\mathcal{E}' = 0$, then $J(A \cdot w)$ cannot be positive definite. Hence, under the conditions of the Lemma, $\mathcal{E}'$ must be strictly negative.

1. *Show $\mathcal{E}'$ non-positive.* Using Lemma A.2, if $J(A \cdot w)$ is positive definite,

   $$\exists \gamma > 0 : \quad \mathcal{E}(\gamma) = \det[E(\gamma)] > 0.$$

   Furthermore,

   $$E(0) = J(A \cdot w)$$

   is positive definite, so that $\mathcal{E}(0) > 0$. From Lemma A.1, $\mathcal{E}'(y) = \mathcal{E}'$, a constant, hence

   $$\mathcal{E}(y) = \mathcal{E}(0) + \mathcal{E}' \cdot y.$$

   Provided $\mathcal{E}' \neq 0$, the solution ω to $\mathcal{E}(\omega) = 0$ exists, and

   $$\frac{\mathcal{E}(\omega) - \mathcal{E}(0)}{\omega - 0} = -\frac{\mathcal{E}(0)}{\omega} = \mathcal{E}'.$$

   Suppose that $\mathcal{E}' > 0$. Then $\omega < 0$ and

   $$\begin{aligned} h'[E(\omega)]h &= h'[J(A \cdot w) - \omega U]h \\ &= h'[J(A \cdot w)]h - \omega h'Uh \\ &\geq h'[J(A \cdot w)]h > 0. \end{aligned}$$

   That is, $E(\omega)$ is positive definite, which in turn implies that $\mathcal{E}(\omega)$ is strictly positive, a contradiction.

2. *Show $\mathcal{E}'$ non-zero.*

   Suppose $\mathcal{E}' = 0$. That is, one of the columns of the determinant in (A.1) can be formed by a linear combination of the others, which in turn implies there exist a constant C, weights ζ_j and $k \in \{1, 2, \ldots, M\}$ such that

$$A_{ik} \cdot w = \sum_{j=1}^{M} \zeta_j A_{ij} \cdot w + C, \qquad i = 1, \ldots, M,$$
$$\sum_{j=1}^{M} \zeta_j = 1, \quad \text{and} \quad \zeta_k = 0.$$

   Then

$$\det[J(A \cdot w)] =$$
$$\begin{vmatrix} A_{11} \cdot w & \ldots & A_{1,k-1} \cdot w & \sum \zeta_i A_{1i} \cdot w + C & A_{1,k+1} \cdot w & \ldots & A_{1M} \cdot w \\ A_{21} \cdot w & \ldots & A_{2,k-1} \cdot w & \sum \zeta_i A_{2i} \cdot w + C & A_{2,k+1} \cdot w & \ldots & A_{2M} \cdot w \\ \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ A_{M1} \cdot w & \ldots & A_{M,k-1} \cdot w & \sum \zeta_i A_{Mi} \cdot w + C & A_{M,k+1} \cdot w & \ldots & A_{MM} \cdot w \end{vmatrix}$$
$$= \begin{vmatrix} A_{11} \cdot w & \ldots & A_{1,k-1} \cdot w & \sum \zeta_i A_{1i} \cdot w & A_{1,k+1} \cdot w & \ldots & A_{1M} \cdot w \\ A_{21} \cdot w & \ldots & A_{2,k-1} \cdot w & \sum \zeta_i A_{2i} \cdot w & A_{2,k+1} \cdot w & \ldots & A_{2M} \cdot w \\ \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ A_{M1} \cdot w & \ldots & A_{M,k-1} \cdot w & \sum \zeta_i A_{Mi} \cdot w & A_{M,k+1} \cdot w & \ldots & A_{MM} \cdot w \end{vmatrix}$$
$$+ \begin{vmatrix} A_{11} \cdot w & \ldots & A_{1,k-1} \cdot w & C & A_{1,k+1} \cdot w & \ldots & A_{1M} \cdot w \\ A_{21} \cdot w & \ldots & A_{2,k-1} \cdot w & C & A_{2,k+1} \cdot w & \ldots & A_{2M} \cdot w \\ \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ A_{M1} \cdot w & \ldots & A_{M,k-1} \cdot w & C & A_{M,k+1} \cdot w & \ldots & A_{MM} \cdot w \end{vmatrix}$$

   The first term is zero, since the kth column is a linear combination of the other columns. Since $J(A \cdot w)$ is symmetric, the kth row can also be separated into a linear combination of the other rows, plus a row of C. Therefore,

$$\det[J(A \cdot w)] = \begin{vmatrix} A_{11} \cdot w & \ldots & A_{1,k-1} \cdot w & C & A_{1,k+1} \cdot w & \ldots & A_{1M} \cdot w \\ A_{21} \cdot w & \ldots & A_{2,k-1} \cdot w & C & A_{2,k+1} \cdot w & \ldots & A_{2M} \cdot w \\ \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ A_{k-1,1} \cdot w & \ldots & A_{k-1,k-1} \cdot w & C & A_{k-1,k+1} \cdot w & \ldots & A_{k-1,M} \cdot w \\ C & \ldots & C & 0 & C & \ldots & C \\ A_{k+1,1} \cdot w & \ldots & A_{k+1,k-1} \cdot w & C & A_{k+1,k+1} \cdot w & \ldots & A_{k+1,M} \cdot w \\ \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ A_{M1} \cdot w & \ldots & A_{M,k-1} \cdot w & C & A_{M,k+1} \cdot w & \ldots & A_{MM} \cdot w \end{vmatrix}$$

$$
\begin{aligned}
&= C^2 \begin{vmatrix}
0 & 1 & \dots & 1 & 1 & \dots & 1 \\
1 & A_{11}\cdot w & \dots & A_{1,k-1}\cdot w & A_{1,k+1}\cdot w & \dots & A_{1M}\cdot w \\
1 & A_{21}\cdot w & \dots & A_{2,k-1}\cdot w & A_{2,k+1}\cdot w & \dots & A_{2M}\cdot w \\
\vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\
1 & A_{k-1,1}\cdot w & \dots & A_{k-1,k-1}\cdot w & A_{k-1,k+1}\cdot w & \dots & A_{k-1,M}\cdot w \\
1 & A_{k+1,1}\cdot w & \dots & A_{k+1,k-1}\cdot w & A_{k+1,k+1}\cdot w & \dots & A_{k+1,M}\cdot w \\
\vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\
1 & A_{M1}\cdot w & \dots & A_{M,k-1}\cdot w & A_{M,k+1}\cdot w & \dots & A_{MM}\cdot w
\end{vmatrix} \\
&= C^2 \mathcal{E}'_{M-k}(y),
\end{aligned}
$$

where $\mathcal{E}'_{M-k}(y)$ is the minor of order $M-1$ of det$[E(y)]$, formed by striking out the kth row and column from $\mathcal{E}(y)$.

Since $E(\gamma)$ is by assumption positive definite, all of the minors of its determinant are positive, which includes all the minors of the determinant of the matrix formed by striking the kth row and column from $E(\gamma)$. The matrix E_{M-k} is then also positive definite, and satisfies the conditions of Lemma A.1, so that $\mathcal{E}'_{M-k}$ is a constant. It was established earlier in the proof that $\mathcal{E}'_{M-k}$ is non-positive. We have, then

$$\text{det}[J(A\cdot w)] = C^2\mathcal{E}'_{M-k}(y) \leq 0,$$

which contradicts the positive-definiteness of $J(A\cdot w)$.

□

Lemma 3 *Let $X(\hat{w}) = X(w_0)$. Then $A^j(m_0) = A^j(\hat{m})$ if and only if $m_0 = \hat{m}$.*

Proof: Lemma 3 Since $\hat{m}$ minimizes $c(m;\hat{w})$,

$$
\begin{aligned}
c(\hat{m};\hat{w}) &= A(\hat{m})\cdot\hat{w} + \hat{b} \\
&= A(\hat{m})\cdot w_0 + \hat{b} + A^j(\hat{m})\,dw_j \\
&\leq c(m_0;\hat{w}) \\
&\quad = A(m_0)\cdot\hat{w} + b_0 \\
&\quad = A(m_0)\cdot w_0 + b_0 + A^j(m_0)\,dw_j
\end{aligned}
$$

Suppose $A^j(m_0) = A^j(\hat{m})$. Subtracting $A^j(\hat{m})\,dw_j$ from the LHS of the preceding inequality, and $A^j(m_0)\,dw_j$ from the RHS,

$$A(\hat{m})\cdot w_0 + \hat{b} \leq A(m_0)\cdot w_0 + b_0,$$

or $c(\hat{m};w_0) \leq c(m_0;w_0)$. But since m_0 minimizes $c(m_0;w_0)$, and since this minimum is unique, we have

$$c(\hat{m};w_0) = c(m_0;w_0)$$

$$\Longrightarrow \hat{m} = m_0.$$

The proof of the converse is trivial.

□

Lemma 4 *The research budget m changes when factor price w_j changes if and only if the use of factor j is influenced by at least one research activity. That is,*

$$\exists k \in (1, \ldots, M): \quad \frac{\partial m_k}{\partial w_j} \neq 0$$
$$\Longleftrightarrow \exists \quad h \in (1, \ldots, M): \quad A_h^j(m) \neq 0.$$

Proof: Lemma 4 1. Suppose $A_h^j(m) = 0, \quad h = 1, \ldots, M$. Then, using (9),

$$\begin{aligned}\frac{\partial m_k}{\partial w_j} &= -\frac{X^M}{\mathcal{C}} \begin{vmatrix} A_{11} \cdot w & A_{12} \cdot w & \ldots & A_{1M} \cdot w \\ A_{21} \cdot w & A_{22} \cdot w & \ldots & A_{2M} \cdot w \\ \vdots & \vdots & \ddots & \vdots \\ A_{k-1,1} \cdot w & A_{k-1,2} \cdot w & \ldots & A_{k-1,M} \cdot w \\ 0 & 0 & \ldots & 0 \\ A_{k+1,1} \cdot w & A_{k+1,2} \cdot w & \ldots & A_{k+1,M} \cdot w \\ \vdots & \vdots & \ddots & \vdots \\ A_{M1} \cdot w & A_{M2} \cdot w & \ldots & A_{MM} \cdot w \end{vmatrix} \\ &= 0, \quad k = 1, \ldots, M.\end{aligned}$$

2. Suppose the converse is false. That is, the A_h^j are not all zero, yet $\frac{\partial m_k}{\partial w_j} = 0, \ k = 1, \ldots, M$. In particular, suppose that

$$\frac{\partial m_1}{\partial w_j} = -\frac{X^M}{\mathcal{C}} \begin{vmatrix} A_1^j & A_2^j & \ldots & A_M^j \\ A_{21} \cdot w & A_{22} \cdot w & \ldots & A_{2M} \cdot w \\ \vdots & \vdots & \ddots & \vdots \\ A_{M1} \cdot w & A_{M2} \cdot w & \ldots & A_{MM} \cdot w \end{vmatrix} = 0. \tag{A.3}$$

If the determinant in (A.3) equals zero, then there are weights $\theta_i, \quad i = 2, \ldots, M$, such that

$$A_k^j = \sum_{i=2}^M \theta_i \left(A_{ik} \cdot w\right), \quad k = 1, \ldots, M. \tag{A.4}$$

Rearranging (9) and summing over k yields

$$\begin{aligned}\frac{\partial B}{\partial w_j} &= \sum_{k=1}^{M} \left(\frac{\partial m_j}{\partial w_j}\right) \\ &= \frac{x^M}{\mathcal{C}} \begin{vmatrix} 0 & A_1^j & A_2^j & \cdots & A_M^j \\ 1 & A_{11} \cdot w & A_{12} \cdot w & \cdots & A_{1M} \cdot w \\ 1 & A_{21} \cdot w & A_{22} \cdot w & \cdots & A_{2M} \cdot w \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & A_{M1} \cdot w & A_{M2} \cdot w & \cdots & A_{MM} \cdot w \end{vmatrix}\end{aligned} \tag{A.5}$$

If expenditures in none of the programs change in response to a change in w_j, we must have $\frac{\partial B}{\partial w_j} = 0$. Then there are weights $\lambda_i, \quad i = 1, \ldots, M$, such that

$$A_k^j = \sum_{i=1}^{M} \lambda_i \left(A_{ik} \cdot w \right), \quad k = 1, \ldots, M. \qquad \text{(A.6)}$$

Combining (A.4) and (A.6),

$$\sum_{i=2}^{M} \theta_i \left(A_{ik} \cdot w \right) = \sum_{i=1}^{M} \lambda_i \left(A_{ik} \cdot w \right),$$

or

$$A_{lk} \cdot w = \sum_{i=2}^{M} \frac{\theta_i - \lambda_i}{\lambda_i} \left(A_{ik} \cdot w \right), \quad k = 1, \ldots, M. \qquad \text{(A.7)}$$

In other words, the Jacobian matrix $J(A \cdot w)$ is singular, a condition excluded by prior hypothesis.

□

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