Equilibrium Foreclosure and Complementary Products

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November 8, 1993

Abstract

In this paper we address the possibility of horizontal foreclosure in markets for complementary services (software) where the consumption value of durables (hardware) depends on the availability of software. Horizontal foreclosure occurs when a hardware firm merges with a software firm and the integrated firm ceases to supply compatible software for a rival technology. We find that horizontal foreclosure can be an equilibrium outcome where both the merger and compatibility decisions are part of a multistage game which permits the foreclosed firm to play a number of counter- strategies. Moreover, foreclosure may result in monopolization of the hardware market. We find that the foreclosure equilibrium is inefficient: total surplus would be higher without foreclosure.

We would like to thank C. Fershtman, A. Fishman, P. Kennedy, O. Shy, D. Sibley and seminar participants at the Summer in Tel-Aviv Mini-Conference on Networks, Simon Fraser University, and the University of Victoria. Church gratefully acknowledges the financial support of the Research Grants Committee at the University of Calgary.

Keywords: Integration, Complementary Products, Foreclosure

1. Introduction

In recent years, economists have explored a wide range of exclusionary strategies that can be used by firms to increase their market share and profits.[1] The hallmark of recent work has been the emphasis on examining the effectiveness of these strategies in a fully specified game-theoretic model. In particular the thorny questions of the foreclosure effects of vertical integration and tying have been recently addressed in equilibrium models by Ordover, Saloner, and Salop (OSS, 1990) and Whinston (1990), respectively.[2] In this paper we ask whether horizontal foreclosure can emerge in equilibrium and if it does what are the efficiency effects in oligopolistic markets where complementary products are important. In particular we are interested in industries where the consumption benefit of a durable or hardware good is a function of the variety of complementary products or software available. We define horizontal foreclosure to occur when a software firm merges with a hardware firm and the integrated firm ceases to supply compatible software for a rival hardware technology. This action is demand impairing or revenue reducing since the reduction in the number of software varieties compatible with the competing hardware technology decreases its value to consumers.

The recent trend toward integration in the entertainment industry highlights the possibilities for this type of behaviour. In the music industry, three of the largest electronics firms control or own three of the six big record companies.[3] In 1993, the next great format war will start over the technology that will replace the audio cassette.[4] The two combatants are the Digital Compact Cassette (DCC) developed by Philips and Matsushita and the Mini Disc (MD), a Sony product. The outcome of the "war" will largely depend on the availability of software. At this juncture, it appears that all of the major record companies, including Sony, will release titles (software) in the DCC format, but Phillips, Matsushita, Time-Warner, and

[1] In general these strategies work by disadvantaging rivals either by raising the costs of rivals or reducing demand for a rival's product. This work is surveyed in Ordover and Saloner (1989). Seminal contributions are Salop and Scheffman (1983) and two papers by Krattenmaker and Salop (1986a, 1986b).

[2] See also Hart and Tirole (1990) for the foreclosure effects of vertical integration.

[3] In the late 1980s, Sony purchased Columbia Records and Columbia Pictures (Business Week, 10/09/89). Matsushita acquired MCA, an entertainment conglomerate which included a record company and a major film studio (Economist, 12/01/90). Philips owns Polygram (Economist, 8/11/90). Moreover, Toshiba has a minority stake in Time-Warner (Economist, 11/2/91). The other two major record companies are Bertelsmann and Thorn-EMI.

[4] See Economist, 11/07/92; Business Week, 06/15/92; and Music-Week, 06/20/92; for details.

Bertelsmann are hedging on whether or not they will release titles in the MD format.[5]

Video delivery in the home provides a second example. At present this service is almost exclusively provided by the cable television industry in Canada and the United States. Integration between film studios, cable channels, and local cable franchise operators is quite prevalent. Moreover, there are numerous instances where a film studio and a network have entered an exclusive licensing agreement and where local cable operators have exclusive broadcast rights within their franchise areas for cable channels.[6] The prevalence of integration and its substitute, exclusive contracting, may make entry by new hardware technologies very difficult. The ability of both Direct Broadcast Satellite and the telephone companies to provide a competitive home video delivery alternative will be impaired if they offer insufficient programming.

We address the question of horizontal foreclosure by modelling both the merger and foreclosure decisions as part of a multistage game. As Rasmussen et al (1991) and OSS (1990) emphasize, only in a fully specified equilibrium model can the rationality and the welfare implications of exclusionary strategies be assessed. Developing an equilibrium model permits explicit consideration of the objections of the 'Chicago School' to the theoretic possibility, welfare implications, and jurisprudence of exclusionary practices.[7]

In the context of the possibility of horizontal foreclosure of compatible software there are three prima facie reasons for why it is unlikely to be an equilibrium phenomena. The first is the availability of a *counter-strategy*. If a foreclosed hardware firm has an opportunity to respond in a similar manner, the initial foreclosure will be ineffective. The second is that there is likely to be a *commitment problem*. The hardware firm that forecloses is forgoing a potentially very profitable software market. It may well be more profitable for the integrated firm to supply software in both formats, and if so, it will be difficult to credibly commit to foreclose. The last objection is that there is likely to be a *hold-up* problem. Suppose initially there are two software firms and one merges with a hardware firm and the integrated firm forecloses on the second hardware technology. The remaining software

[5]Music Week, 06/20/92.

[6]For more on the prevalence of exclusive contracts in the cable television industry and on the incentives to acquire programming on an exclusive basis, see Gandal and Salant (1991). An exclusive licensing agreement provides a network with exclusive broadcast rights and is very similar in effect to integration and foreclosure. See Mathewson and Winter (1984).

[7]For a summary of the objections of the 'Chicago School' to foreclosure theory, see the discussions in OSS (1990) and Whinston (1990) and the references therein.

firm will be a monopoly provider of software for the second hardware technology as well as a duopolist in the software market for the integrated firm's hardware. This may be very profitable, suggesting the possibility that it is better not to be the software firm that merges and forecloses, but to remain independent.

In the model we develop there are two hardware firms who produce differentiated products. The willingness of a consumer to pay for the hardware goods depends on her preferences about hardware, software and the number of compatible software varieties available. Software is provided by two multiproduct software firms. Each software firm, in the absence of foreclosure, can supply software in formats compatible with both hardware technologies.

In the initial stage of the game, the hardware firms have the opportunity to acquire one of the two independent software firms. If a bid for a software firm is accepted, then the integrated or merged firm must determine the formats in which it will offer software. The integrated firm can elect either to make the software available only in a format compatible with its own hardware or it can make its software available in formats compatible with both hardware technologies.

Having observed both the integration and foreclosure decisions of the first hardware firm, the remaining hardware firm then has the opportunity in the second stage to acquire the other independent software firm and if it does so, it also makes a compatibility decision. Thus, there are two strategic decisions hardware firms make. The first is whether or not to integrate or acquire a software firm. The second is, having merged with a software firm, whether or not to make software compatible with the competing hardware technology. Foreclosure occurs when an integrated firm makes its software incompatible.

We find that there are two possible equilibria. In the first, the industry structure which emerges endogenously is unintegrated. In this equilibrium neither hardware firm merges or acquires a software firm. In the second, we get a foreclosure outcome. One of the hardware firms merges with one of the software firms and forecloses on the remaining hardware firm. The remaining hardware firm does not respond by integrating with the remaining independent software firm and foreclosing. Horizontal foreclosure emerges as an equilibrium even though we have explicitly allowed for the remaining hardware firm to counter by integrating and foreclosing, addressed the potential for a hold-up problem, and explicitly considered the format choice by the foreclosing firm.

There are three effects associated with unilateral foreclosure which underlie our results: a demand effect, a software profit effect and a hardware price effect. Unilateral foreclosure provides a hardware firm with a relative advantage since there will be twice as many software varieties available for it. This demand effect increases the hardware market share of the foreclosing firm.

Unilateral foreclosure also has an effect on the software markets for the two technologies. The foreclosing technology remains a duopoly, but the software market for the foreclosed technology is now a monopoly. The integrating firm which forecloses forgoes duopoly software profits in the other technology.

There is also a price effect associated with integration and foreclosure. Integration and foreclosure results in an increase in price competition in the hardware market. Equilibrium hardware prices are lower when hardware firms integrate and foreclose than in the unintegrated market structure.

If the degree of hardware differentiation is not too large relative to the preferences of consumers for variety, then a hardware firm will find it profitable to integrate and foreclose if it anticipates no response from the other hardware firm. In these circumstances, the increase in hardware market share and profits from the demand effect more than compensates for the lost software profits.

We show that it is never optimal for a foreclosed hardware firm to respond by integrating and foreclosing. While this nullifies the demand effect, the resulting low hardware pricing (the price effect) and the loss in software duopoly profits result in lower profits than the hardware firm and the software firm would earn if they remained independent.

An alternative counter-strategy is for the foreclosed hardware firm to integrate but not foreclose. This allows it to credibly commit to low hardware prices, since by charging a low hardware price it extends its hardware market share and thus the software market in which it is a monopolist. This will be a profitable counter-strategy if demand for hardware is relatively price elastic, ie the degree of hardware differentiation is not too large.

The foreclosure equilibrium arises if the degree of hardware differentiation is small enough relative to the preferences of consumers for software variety so that the increase in hardware market share from the demand effect is relatively significant. However, the degree of hardware differentiation must also be large enough so that the increase in the hardware market share from the price effect of the counter-strategy is small. Essentially what is

required is that the demand for hardware be elastic with respect to changes in the number of software varieties (thus making foreclosure profitable), but relatively inelastic with respect to changes in the price of hardware (thus making the counter-strategy ineffective).

Moreover if the demand effect is sufficiently large, the foreclosure equilibrium will involve standardization on the hardware technology of the foreclosing hardware firm. Foreclosure can be an effective means to monopolize the hardware market.

We show that the foreclosure equilibrium is inefficient. Total surplus is greater in the unintegrated industry structure than in the foreclosure equilibrium.

The structure of our model is closely related to that of OSS (1990). They consider whether vertical or supply side foreclosure can emerge in an industry initially characterized by two upstream suppliers, who provide an input used in production by two downstream firms. In their paper, firms have an opportunity to acquire one of the upstream suppliers in stage one. Then in stage two, input prices are determined. If a merger took place in stage one, the other downstream firm has the opportunity to acquire the remaining upstream firm in stage three. Finally, downstream prices are set in stage four. They find that foreclosure can emerge in equilibrium.

Hart and Tirole (1990) and Reiffen (1992) argue that the OSS result (that foreclosure can emerge in equilibrium) depends on the ability of a merged firm to commit to a price in stage two. That is, the merged firm must be able to commit not to compete aggressively with the remaining supplier to supply the other downstream firm for foreclosure to be an equilibrium. If the input prices were determined by Bertrand competition (Nash equilibrium in prices), foreclosure would not arise in equilibrium.

In our model, the long-run strategic decisions (integration and compatibility) are undertaken before any prices are determined. The desirability of either integration or foreclosure is based on the effect that it has on price competition in the hardware market. An integrated hardware firm that has foreclosed has no incentive to supply software to the other hardware firm, even after the second hardware firm has decided not to integrate: to do so would lower its profits and thus the decision not to supply compatible software is credible.

The plan of the paper is as follows. In section 2, we specify technology and the preferences of consumers. In section 3, we derive the equilibria to the hardware pricing game for all possible industry structures. In section 4 we determine the equilibrium industry structure when foreclosure does not result in standardization. In section 5 we consider

whether foreclosure can result in monopolization and standardization in the hardware market. We address the welfare implications of the foreclosure equilibrium in section 6 and section 7 provides brief concluding remarks.

2. The Model

In this section we develop the model. We begin by describing technology.

2.1 Technology

We consider a situation where there are four firms: two manufacturers of hardware and two software firms. We denote the two hardware firms by A and B. The hardware products are differentiated along the unit interval. The locations of the hardware firms are fixed: technology A is at the left-end point, technology B at the right-end point.

The software firms are multiproduct and each has a stock or inventory of N software products, where $N > 1$. The development costs for this inventory of software products are sunk and we assume that the number cannot be augmented. The software of both software firms is initially available in formats compatible with both of the hardware technologies. Finally, with no loss of generality, we assume that the marginal cost of producing a unit of hardware or a unit of software is equal to zero.

2.2 Preferences of Consumers

We now specify the preferences of consumers over hardware, software, and an outside good. In modelling preferences over hardware and software we explicitly recognize the following:

(i) The value of hardware depends primarily on the availability of software. Without the provision of compatible software, hardware provides minimal consumption benefit.

(ii) The greater the variety of software, the greater the benefit or value of a hardware technology. However, the marginal value of additional software is decreasing.

(iii) The demand for both hardware and a variety of software is perfectly inelastic.

From (iii) consumers purchase only one unit of hardware and one unit of each variety of software. However, from (ii) they will, in general, purchase more than one variety of software. We assume that the benefit consumers receive from consuming N varieties of

software is given by N^{β}, where, β, the value of variety, is less than one but greater than zero by (*ii*).

The preferences of consumers for hardware are represented using an address model. The tastes of consumers are distributed uniformly along a line of unit length, the population is normalized to one, and all consumers have income *y*. The consumption of a hardware technology different from the most preferred type imposes a utility cost on the consumer that is proportional to the distance separating the two types.

The utility function of a consumer located distance t_i from hardware *i* is therefore[8]

$$U_i = N_i^{\beta} + x + \alpha - kt_i, \tag{1}$$

where α is the stand-alone benefit of hardware, N_i is the number of software varieties consumed, *x* is consumption of a competitively supplied outside good, and *k* measures the extent of hardware differentiation. The budget constraint for a consumer is:

$$\sum_{j=1}^{N} \rho_{ji} + x = y - p_i, \tag{2}$$

where ρ_{ji} is the price of a unit of software variety *j* available for hardware technology *i*, *y* is the income of the consumer, p_i is the price of hardware technology *i*, and *N* is the number of software varieties purchased. Maximization of (1) is a two-stage procedure. In the second stage, (1) is maximized subject to (2) for each hardware technology *i*. In this stage, the consumer selects which varieties of software and the total number of varieties to consume. Substituting this into (1) gives indirect utility for technology *i*. In the first stage the consumer selects the hardware variety for which indirect utility is greatest.[9]

To solve the second-stage for technology *i*, rank the software varieties in ascending order by price. The marginal benefit of another software variety is $\beta N^{\beta-1}$.[10] Ignoring the integer problem, the number of varieties consumed is then implicitly defined by $\rho_N = \beta N^{\beta-1}$, where ρ_N is the price of the N^{th} most expensive software variety.

In other words, the consumer purchases one unit of the *N* lowest-priced varieties, where *N* is such that the marginal benefit of the N^{th} software product equals ρ_N. If the marginal

[8]The network benefit, N_i^{β}, is only obtained if hardware *i* is purchased.

[9]We assume that α is large enough so that the purchase of one of the two hardware technologies is optimal.

[10]To reduce notational clutter, we temporarily drop the *i* subscript.

benefit of the N^{th} software product exceeds ρ_N, one unit of all software varieties is consumed.

We assume that consumers purchase hardware first, then software.[11] Before considering the indirect utility or consumption benefit that a consumer receives from consuming technology *i*, we state the following lemma regarding the price of software.[12]

Lemma 1 *If N software varieties are supplied for technology i, then the symmetric Nash equilibrium software price is*

$$\rho_i = \beta N_i^{\beta-1}. \tag{3}$$

In equilibrium, software is priced such that each consumer who purchased technology *i* is just willing to purchase one unit of each software variety provided. From Lemma 1, the price of software is determined solely by the number of software varieties supplied and it is a decreasing function of the number of software varieties.

A symmetric software price equal to $\beta N_i^{\beta-1}$ implies that the budget constraint can be written as $\beta N_i^{\beta} + x = y - p_i$. Solving for x and substituting into (1) gives the indirect utility function of a consumer located distance t_i from technology *i*:

$$V_i = (1-\beta)N_i^{\beta} + y + \alpha - p_i - kt_i. \tag{4}$$

A consumer purchases hardware A if the benefit from adopting system A (V_A) exceeds the benefit from adopting system B (V_B). If we measure t from the left-end point,[13] then the marginal consumer is defined implicitly by $V_A = V_B$. Using equation (4) and rearranging terms, the value for the equilibrium market share for technology A is

$$t = \frac{((1-\beta)(N_A^{\beta} - N_B^{\beta}) - (p_A - p_B) + k)}{(2k)}. \tag{5}$$

The equilibrium market share for technology B is simply $(1-t)$. The market share for either hardware firm cannot exceed 1. If $t = 1$ then technology A is exclusively adopted.

[11] The import of this assumption is that software vendors cannot affect the market share of a hardware technology by their pricing decision.

[12] The proofs of all lemmas and propositions are in the appendix.

[13] In other words, $t_A = t$ and $t_B = 1 - t$.

3. Timing of the game

We model a three-stage game. In the initial stage, the hardware firms have the opportunity to bid for one of two independent software firms. If the software firm is acquired, we assume without loss of generality that it is acquired by hardware firm A. The integrated or merged firm then must determine the formats in which it will offer software. The integrated firm can elect either to make the software available only in a format compatible with its own hardware or it can market software available in formats compatible with both hardware technologies. If one of the hardware firms acquires one of the software firms in the first stage, the remaining hardware firm then has the opportunity in the second stage to acquire the other independent software firm, and if it does so, it also makes a compatibility decision. It does this after observing the compatibility decision of the first integrated firm. In the final stage hardware prices are determined and consumers make their adoption decisions. The subgame perfect equilibrium is found by backwards induction.

3.1 Hardware Price Competition

There are four possible industry structures.[14] These four subcases are:

(1) Unintegrated: Neither hardware firm acquires a software firm.
(2) Bilateral Foreclosure: Both hardware firms merge with an independent software firm and foreclose.
(3) Integrated: Both hardware firms have acquired independent software firms, but only one has foreclosed.
(4) Foreclosure: Only one hardware firm has acquired a software firm and it has foreclosed.

Without loss of generality we assume that whenever only one of the hardware firms integrates and forecloses, it is firm A. In cases (3) and (4), firm A forecloses. The difference between the two is that in (3) firm B integrates, but continues to make its software compatible, whereas in (4), firm B does not integrate. If the industry structure is (1), then $N_A = N_B = 2N$,[15]

[14]There might appear to be two additional industry structures: (1) both hardware firms integrate, but neither foreclose and (2) one hardware firm integrates, but does not foreclose, and the other hardware firm does not integrate. These cases are formally equivalent to the unintegrated case since they do not change the number of software varieties available for either technology or the pricing behaviour of the hardware firms. There is neither a demand nor a pricing effect.

[15]Recall that in the absence of integration, all software products are compatible with both hardware

in (2), $N_A = N_B = N$, while for (3) and (4) $N_A = 2N$, but $N_B = N$. The marginal consumer and the market shares of the two hardware firms as a function of the prices of hardware are found by substituting in the relevant number of software varieties available for each hardware technology into (5). In the first two industry structures the number of software products supplied for each hardware platform is the same and thus,

$$t = \frac{p_B - p_A + k}{2k}. \tag{6}$$

In cases (3) and (4), the foreclosure cases,

$$t = \frac{(1-\beta)\left(2^{\beta}-1\right)N^{\beta} - (p_A - p_B) + k}{2k}. \tag{7}$$

Foreclosure provides firm A with a competitive advantage. Comparing (6) and (7) illustrates that a greater supply of software leads to a market share advantage since $(1-\beta)\left(2^{\beta}-1\right) > 0$ for all $0 < \beta < 1$. This is the demand effect.

3.2 Industry Structure Subgames

We now consider each of the four possible industry structures (subgames) in turn.

3.2.1 Unintegrated Industry

In this case each hardware firm only derives profits from sales of hardware. The respective profits of hardware firms A and B are

$$\pi_A = tp_A = \frac{[(p_B - p_A) + k]}{2k}p_A, \tag{8}$$

and

$$\pi_B = (1-t)p_A = \frac{[(p_A - p_B) + k]}{2k}p_B. \tag{9}$$

The following Lemma summarizes the equilibrium hardware prices and profits in the unintegrated case.

Lemma 2 *The unique equilibrium hardware prices in the unintegrated industry structure are $p_U = p_A = p_B = k$, equilibrium market share of each firm is $t = 1/2$, and each hardware firm earns profits of $\pi_U = k/2$.*

technologies.

The profits earned by each of the independent software firms in this case are

$$\begin{aligned}\pi_s^U &= \rho_A tN + \rho_B (1-t) N \\ &= \beta(2N)^{\beta-1} tN + \beta(2N)^{\beta-1} (1-t) N \\ &= \beta(2N)^{(\beta-1)} N, \end{aligned} \tag{10}$$

where we have made use of $N_A = N_B = 2N$ and Lemma 1.

3.2.2 Bilateral Foreclosure

In this setting, each hardware firm earns profits from the sales of both hardware and software. The software provided by each hardware firm is only compatible with its hardware and consequently, each hardware firm is the monopoly supplier of software for its technology. From Lemma 1, hardware firm A will charge $\rho_A = \beta N^{(\beta-1)}$ for each of its software products and software revenues will be $t\beta N^{\beta}$. The profits of hardware firm A, after substituting (6) for t, are

$$\pi_A = t(p_A + \beta N^{\beta}) = \left(\frac{(p_B - p_A) + k}{2k} \right) (p_A + \beta N^{\beta}). \tag{11}$$

The following Lemma provides the equilibrium prices and profits in the bilateral foreclosure case.

Lemma 3 *The equilibrium hardware price in the bilateral foreclosure (BF) case is* $p_{BF} = p_A = p_B = k - \beta N^{\beta}$ *, equilibrium market share is* $t = 1/2$*, and* $\pi_{BF} = \pi_A = \pi_B = k/2$.

3.2.3 Integrated

In this case both hardware firms have acquired control of a software firm. However, hardware firm A makes its software available in a format compatible only with its hardware. Hardware firm B on the other hand elects to make its software available in formats compatible with both hardware technologies. The two hardware firms compete as duopolists in the software market for technology A and from Lemma 1, the price of software will be $\rho_A = \beta(2N)^{\beta-1}$, while in the software market for technology B, firm B is a monopolist and the price of software will be $\rho_B = \beta(N)^{\beta-1}$. We can state the following lemma, which summarizes the equilibrium prices, market shares and profits in the integrated case. In advance of the lemma, define

$$k_i = \frac{\left(2^{\beta} - 1\right) N^{\beta}}{3}. \tag{12}$$

Lemma 4 *When $k > (\leq) k_i$, an interior (standardization) equilibrium obtains. The equilibrium hardware prices, market shares, and profits when both hardware firms integrate, but only A forecloses in an interior equilibrium are:*

$$p_A^I = \frac{\left(2^{\beta-1}(2-3\beta)-1\right)N^\beta + 3k}{3},$$

$$p_B^I = \frac{\left(\left(2^{\beta-1}+2^\beta-3\right)\beta - 2^\beta + 1\right)N^\beta + 3k}{3},$$

$$t^I = \frac{\left(2^\beta - 1\right)N^\beta + 3k}{6k},$$

$$\pi_A^I = \frac{\left(\left(2^\beta - 1\right)N^\beta + 3k\right)^2}{18k},$$

$$\pi_B^I = \frac{\left(3k - (2^\beta - 1)N^\beta\right)\left(3k + \left(2^{\beta-1}(3\beta-2)+1\right)N^\beta\right)}{18k} + \frac{3\beta 2^{\beta-1}N^\beta\left(3k + (2^\beta-1)N^\beta\right)}{18k}.$$

In a standardization equilibrium, the equilibrium prices, market shares and profits are:[16]

$$p_A^{IS} = ((1-\beta)(2^\beta - 1) + (2^{\beta-1}-1)\beta)N^\beta - k,$$

$$p_B^{IS} = (2^{\beta-1}-1)\beta N^\beta,$$

$$t^{IS} = 1,$$

$$\pi_A^{SI} = \left(2^\beta - 1\right)N^\beta - k,$$

$$\pi_B^{IS} = \beta 2^{\beta-1}N^\beta.$$

Foreclosure provides firm A with a market share advantage in equilibrium since $\left(2^\beta - 1\right) > 0$. If the parameters are such that $k_i \geq k$, then the variety advantage accorded to technology A is large enough vis-a-vis the degree of hardware differentiation that all consumers purchase technology A, ie, the standardization equilibrium obtains.

[16] In a standardization equilibrium, an integrated firm B continues to earn profits from its "technology A" compatible software.

3.2.4 Foreclosure

This scenario is very similar to the integrated case. In both cases hardware firm A acquires one of the software firms and forecloses on firm B. However, in this scenario firm B does not respond by acquiring the second software firm. The remaining independent software vendor provides software for both hardware technologies. Thus, in both of these industry structures there are $2N$ software products provided for technology A and only N software varieties provided for technology B. In advance of the following lemma, define

$$k_f = \frac{fN^\beta}{3}, \tag{13}$$

where $f = 2^{\beta-1}(2-\beta)+\beta-1$.

Lemma 5 *When $k > (\leq) k_f$, an interior (standardization) equilibrium obtains. The equilibrium hardware prices, market shares, and profits in an interior equilibrium when only firm A integrates and forecloses are:*

$$p_A^f = \frac{\left((1-2\beta)\,2^\beta - 1 + \beta\right)N^\beta + 3k}{3},$$

$$p_B^f = \frac{3k - fN^\beta}{3},$$

$$t^f = \frac{fN^\beta + 3k}{6k},$$

$$\pi_A^f = \frac{\left(3k + fN^\beta\right)^2}{18k},$$

$$\pi_B^f = \frac{\left(3k - fN^\beta\right)^2}{18k}.$$

In a standardization equilibrium, the equilibrium prices, market shares and profits are:

$$p_A^{fs} = (1-\beta)\left(2^\beta - 1\right)N^\beta - k,$$

$$p_B^{fs} = 0,$$

$$t^{fs} = 1,$$

$$\pi_A^{fs} = fN^\beta - k,$$

$$\pi_B^{fs} = 0.$$

Foreclosure increases the market share of firm A since $f > 0$. If the variety advantage is large relative to the degree of hardware differentiation, then a standardization equilibria results and all consumers purchase technology A.

In the interior equilibrium, the profit of the independent software firm equals the sum of its monopoly profits in the software market for technology B and its duopolist profits in the software market for technology A:

$$\begin{aligned} \pi_s^f &= \rho_A N t + \rho_B N (1-t) \\ &= \beta (2N)^{\beta-1} N t + \beta (N)^{\beta-1} N (1-t) \\ &= \frac{\left(\left(2^{\beta-1}-1\right) N^\beta f + 3\left(2^{\beta-1}+1\right) k\right) \beta N^\beta}{6k,} \end{aligned} \tag{14}$$

where the last equality is obtained by substituting the reduced form expression for the equilibrium market share from Lemma 5.

In the standardization case, the profits of the independent software firm are simply duopoly profits from the entire market:

$$\pi_s^{fs} = \beta 2^{\beta-1} N^\beta. \tag{15}$$

3.3 Implications of the Alternative Structures

In both the unintegrated and bilateral foreclosure cases the two firms split the market in half. The demand effect ensures that in the integrated and foreclosure cases, the market share of firm A is greater than one half and in both of these cases, it may be large enough that standardization may arise.

We can also rank equilibrium hardware prices. The highest hardware prices occur in the unintegrated market structure, the lowest in the bilateral foreclosure case. In the bilateral foreclosure case, integration and foreclosure increase the incentives for a hardware firm to price its hardware more competitively relative to a non-integrated hardware firm. In this case a hardware firm which has integrated and foreclosed has an additional incentive to lower the price of hardware since this increases the market for its software sales. An increase in the number of software products makes a hardware firm 'tougher' in the terminology of Fudenberg and Tirole (1984) .

In the foreclosure and integrated case, firm A (the foreclosing firm) has an incentive to price its hardware more competitively only when the demand effect is large. If the demand

effect is small, increases in the number of software products make firm A 'softer:' it is more profitable to charge higher prices to captive customers than fight for a larger market share.[17]

In both of these cases, however, firm B (the foreclosed firm) has an incentive to price its hardware more competitively relative to the unintegrated market structure. In the foreclosure case, it has an incentive to compete more aggressively because its market share is smaller and hence the loss on inframarginal units less. This effect is such that even if the demand effect is small, prices in the foreclosure equilibrium are lower than in the unintegrated case.

In the integrated case, firm B is a monopoly provider of software for its own technology and a duopolist in the software market for technology A. This differential in software market structure provides it with an incentive to increase its hardware market share by lowering its hardware price in order to extend its monopoly in software. The market share of technology A in the foreclosure case is greater than in the integrated case, because the price effect when B is integrated is greater than when it is not.

4. Non-Standardization Equilibrium

In the preceding section, we determined the equilibrium prices and profits for each of the four possible industry structures. In this section we use that analysis to determine the equilibrium to the full three-stage game under the assumption that the resulting equilibrium in any of the subgames does not does not entail standardization. Since $k_f > k_i$, this means we restrict the parameter space to $k > k_f$. We begin by considering the effect of the two counter-strategies available to firm B if firm A integrates and forecloses.

4.1 Optimal Response to Foreclosure

Firm B has three options when faced with integration and foreclosure by Firm A. It can integrate and foreclose, integrate but not foreclose, or remain unintegrated. We determine the optimal response of firm B in the following two propositions. In Proposition 1, we compare the stand-alone profits of firm B and the remaining independent software firm to the profits that firm B would earn if it integrated and foreclosed. In Proposition 2, we

[17]The hardware price reaction function of firm A in both of these cases is given by (26) in the appendix. When $\beta < .22179$ increases in the number of software varieties makes firm A softer, when $\beta > .22179$, increases in the number of software varieties makes firm A tougher.

compare these stand-alone profits to the profits that firm B would earn if integrated but did not foreclose.

Proposition 1 *If firm A forecloses, firm B will never find it optimal to acquire the remaining independent software firm and foreclose on firm A.*

The intuition for the non-optimality of an integration and foreclosure response by firm B is straight forward. Integration and foreclosure significantly increase the degree of price competition in the hardware market, since increasing hardware market share increases the size of the software market in which each integrated firm has a monopoly. However, the increased hardware price competition dissipates any software profits. Moreover, the profitability of the independent software firm has increased since if it remains independent it will earn monopoly profits in the software market for technology B as well as duopoly profits in the market for technology A.

Firm B has at its disposable a second counter-strategy, which is to integrate, but not foreclose. In advance of the following proposition, define

$$k_{min} = \frac{1}{3}\frac{\left(\left(2^{\beta+1} - 2^{2\beta-1} - 2\right)\beta + 2^{2\beta-1} - 32^{\beta-1} + 1\right)}{2^{\beta-1} - 1}N^{\beta}. \tag{16}$$

Proposition 2 *If a merged firm forecloses, the unintegrated hardware firm will find it profitable not to integrate if $k > k_{min}$. If $k < k_{min}$, the unintegrated hardware firm will integrate with the remaining independent software firm and continue to supply compatible software for both hardware technologies.*

Proposition 2 indicates that if the degree of hardware differentiation is relatively small then firm B will find it optimal to respond to foreclosure by firm A by integrating but not foreclosing. The intuition for this is that if the degree of differentiation is relatively insignificant, the market share increase for firm A from the demand effect associated with foreclosure is relatively large. Moreover, B can effectively restore its market share by lowering its price of hardware since hardware will be relatively price elastic. Integrating into software provides it with a means to credibly lower its hardware price, increasing both hardware market share and extending the market of its software monopoly.

4.2 Optimality of Foreclosure

In this section we consider the incentives of a merged firm A to foreclose. The advantages of foreclosure arise from the demand effect: the favorable differential in software variety increases hardware market share. The cost of foreclosure is the forgone software profits for the other technology. The circumstances when a merged firm will find it profitable to foreclose are detailed in the next proposition. Define

$$k_{max} = \frac{f^2 N^{\beta}}{18\beta 2^{\beta-1} - 6f}. \tag{17}$$

Proposition 3 *Suppose that hardware firm A and one of the independent software firms have merged and that firm B will not integrated with the remaining independent software firm. The merged firm will foreclose (not foreclose) if* $k < k_{max}$ *(*$k > k_{max}$*).*

Proposition 3 indicates that if the hardware products are highly differentiated firm A will prefer not to foreclose. The reason is that in these circumstances, the market share gain from the demand effect of foreclosure is small and is more than offset by the decrease in hardware prices and lost software profits. If the hardware products are not highly differentiated then the demand effect will be large and a hardware firm will find it profitable to integrate and foreclose, since the increase in the hardware market from the demand effect more than makes up for the decrease in hardware prices and lost software profits.

The premise of Proposition 3 is that firm B will not respond by integrating. Proposition 2 indicated that for certain parameter values, firm B will in fact respond to integration and foreclosure by integrating but not foreclosing. The next proposition shows that if Firm A anticipates this response, it will not integrate and foreclose.

Proposition 4 *If firm B finds it profitable to respond to integration and foreclosure by firm A by integrating but not foreclosing, then firm A will not integrate and foreclose.*

Proposition 4 indicates that if the foreclosed hardware firm responds with the counter-strategy of integrating but not foreclosing, then the foreclosure strategy of the first hardware firm to foreclose is in fact ineffective. The incentive that the second firm has to lower its hardware price in order to expand the software market in which it has a monopoly sufficiently mitigates the demand effect associated with foreclosure.

Combining Propositions 2, 3, and 4, a necessary condition for foreclosure to be an equilibrium is $k_{max} > k > k_{min}$. In other words, the degree of hardware differentiation must be low enough ($k < k_{max}$) that the demand effect of foreclosure makes foreclosure profitable, but high enough ($k > k_{min}$) that the price effect of the counter-strategy is relatively small.[18]

The condition, $k_{max} > k > k_{min}$, is not sufficient for equilibrium, however. For a foreclosure equilibrium to exist a software firm must voluntarily agree to merge with one of the hardware firms. A software firm will not do so if it anticipates that its profits would be greater if it was the independent software firm in a foreclosure equilibrium. Since it will be a monopoly provider of software for technology B and remain a duopolist in the software market for technology A, its profits will increase relative to the unintegrated market structure. The maximum a hardware firm will bid to acquire a software firm in the first stage is the difference between what it earns if it forecloses and its profits if it is foreclosed upon. The next proposition provides a sufficient condition for when this difference is greater than the profits of the independent software firm in the foreclosure equilibrium.

Proposition 5 *For* $0 < \beta < .17287$*, there is not a hold-up problem.*

Proposition 5 indicates that when β is small, there is no hold-up problem and software firms prefer to merge and foreclose, rather than remain independent. For greater values of β, the software firm prefers to remain independent and become a monopolist in the software market for technology B. Large values of β imply a high willingness to pay for software and hence software prices and profits are high. Another implication of Proposition 5 is that if there is in fact a bidding process for the initial software firm, we would expect that the winning bid would be the difference between the profits a hardware firm obtains if it forecloses and the profits it earns if it is foreclosed upon. Thus, the payoffs of the two hardware firms will be identical regardless of whether they foreclose or are foreclosed upon if foreclosure is in fact an equilibrium outcome. In the next proposition we show that there are parameter values such that foreclosure is an equilibrium outcome.

Proposition 6 *Integration and foreclosure by firm A and no response by firm B will be an equilibrium outcome when foreclosure leads to an interior equilibrium when* $0 < \beta \leq .098733$*, and* $k_{max} > k > k_{min}$*. If either of these parameter restrictions do not hold, the equilibrium industry structure will be unintegrated for all* $k > k_f$*.*

[18] It is easy to show that $k_{max} \geq k_{min}$ if $\beta \leq .098733$.

Proposition 6 indicates that there are two possible equilibria to the full game when the parameters are restricted such that foreclosure leads to an interior equilibrium. If the parameter restrictions hold, then in equilibria foreclosure occurs: firm A integrates and forecloses on firm B, firm B remains unintegrated. If the parameter restrictions are not satisfied, then neither firm integrates and the equilibrium industry structure is unintegrated.

5. Standardization Equilibrium

In this section we briefly consider the case in which foreclosure results in standardization. If the degree of hardware differentiation is sufficiently small, in particular if, $k < k_f$, then the relative variety advantage provided to a foreclosing hardware firm is significant enough that the market share of the rival technology is reduced to zero if it does not retaliate. We can state the following proposition.

Proposition 7 *For values of $k_f > k > k_i$, the equilibrium industry structure is unintegrated.*

The intuition for this result is that for $k > k_i$, by responding to foreclosure by integrating, firm B obtains a positive market share in the hardware market. Similar to Proposition 4, this makes foreclosure unattractive to the merged firm.

We now consider the equilibrium for values of $k_i > k$. In advance of the following proposition, define

$$k_s = \frac{2(1-\beta)\left(2^{\beta}-1\right)N^{\beta}}{3}. \tag{18}$$

Proposition 8 *For $k_s > k$, firm A will always find it profitable to foreclose if B does not retaliate. For $k < k_i$, it is not profitable for firm B to retaliate by acquiring the remaining independent software firm.*

The intuition for the optimality of a non-response by firm B is that even if it responds by merging and not foreclosing, it does not make any hardware sales and hence the combined profits of the merged firm are equivalent to the profits earned by an independent software firm in a standardization equilibrium. If k is relatively small vis-a-vis β and N, then de facto standardization by foreclosing is profitable for firm A. Whether firm A forecloses or not its software profits are the same since in both cases it is a duopoly supplier of software to the entire market. Hence whether or not it forecloses depends on the impact foreclosure

has on hardware profits. If the degree of hardware differentiation is large ($k > k_s$), then the price effect makes foreclosure unprofitable. If the degree of hardware differentiation is small ($k_s > k$), then the price effect is small and foreclosure profitable. There is no hold-up problem, since the profits that the independent software firm will earn in a foreclosure equilibrium are simply duopoly profits in the software market for technology A. This is also the profit that firm A earns from software sales, whether it forecloses or not. The increase in hardware profits when firm A forecloses provides the requisite surplus to make a merger between firm A and a software firm voluntary. Finally, we can state the following proposition, which follows immediately from Propositions (7) and (8).

Proposition 9 *For values of $k < min[k_i, k_s]$ there is a foreclosure equilibrium in which technology A is exclusively adopted. Otherwise, the equilibrium industry structure is unintegrated.*

6. Social Welfare

In this section we evaluate the social desirability of the foreclosure equilibrium. We do this by comparing the total surplus associated with the unintegrated equilibrium to the total surplus associated with the foreclosure equilibrium.

6.1 Unintegrated Total Surplus

Lemma 2 provides details on the unintegrated equilibrium. Total surplus is the sum of hardware profits, software profits, and consumers' surplus. Using (4), the surplus of consumers is

$$\begin{aligned} CS^U &= 2\int_0^{1/2} \left((1-\beta)(2N)^\beta + y + \alpha - k - kt\right) dt \\ &= (1-\beta)(2N)^\beta + y + \alpha - \frac{5k}{4}. \end{aligned} \tag{19}$$

The combined profits of the four firms are, using Lemma 5 and (10):

$$\Pi^U = k + \beta(2N)^\beta. \tag{20}$$

Consequently, total surplus in the unintegrated industry structure is, summing (19) and (20):

$$TS^U = (2N)^\beta + y + \alpha - \frac{k}{4}. \tag{21}$$

6.2 Total Surplus, Foreclosure Equilibrium

The surplus of consumers in the foreclosure equilibrium is, using (4), $N_A = N$, and $N_B = 2N$

$$\begin{aligned} CS^F &= \int_0^t \left((1-\beta)(2N)^{\beta} + y + \alpha - p_A - kt\right) dt + \\ &\quad \int_t^1 \left((1-\beta)(N)^{\beta} + y + \alpha - p_B - k(1-t)\right) dt \\ &= \left((1-\beta)(2N)^{\beta} + y + \alpha - p_A\right) t + \left((1-\beta)(N)^{\beta} + y + \alpha - p_B\right)(1-t) - \\ &\quad \frac{k}{2}\left(t^2 + (1-t)^2\right) \end{aligned} \tag{22}$$

Hardware and software profits equal

$$\Pi^f = p_A t + \beta(2N)^{\beta-1} Nt + (1-t)\, p_B + \beta(2N)^{\beta-1} Nt + \beta(N)^{\beta-1} N(1-t) \tag{23}$$

Total surplus in the foreclosure equilibrium is the sum of (22) and (23):

$$TS^f = y + \alpha + N^{\beta}\left(\left(2^{\beta} - 1\right) t + 1\right) - \frac{k}{2}\left(t^2 + (1-t)^2\right). \tag{24}$$

Setting $t = 1$ in (24) yields total surplus if foreclosure involves standardization.

Proposition 10 *The foreclosure equilibrium is inefficient.*

It is perhaps not surprising that the foreclosure equilibrium is inefficient. While the hardware firm and software firm which merge clearly gain, as does the remaining independent software firm, these gains are more than offset by the reduction in the surplus of consumers and the profits of the unintegrated hardware firm. In the foreclosure equilibrium, customers of the unintegrated hardware firm are supplied with only half the number of software products and at significantly higher prices, compared to the unintegrated industry equilibrium. The reduction in the hardware price for both technologies engendered by the integration of firm A is not enough to compensate. The decrease in the price of hardware also impacts negatively on the profits of the unintegrated hardware firm.

7. Conclusion

In this paper we developed a model to address whether or not horizontal foreclose in the markets for complementary services for consumer durables is an equilibrium outcome and,

if it is, what the welfare implications are. We find that horizontal equilibrium foreclosure can occur as the outcome of a fully specified equilibrium model, and that when it does occur, it is inefficient. We find that for certain parameter values, in equilibrium, one hardware will merge with a software firm and discontinue software support for a rival hardware technology. Moreover, the remaining hardware firm will not respond in kind. Thus, foreclosure occurs even though a successful counter-strategy is available. Furthermore, the integrated firm has no incentive to supply software compatible with the hardware of its rival and neither of the software firms has an incentive to hold-out when approached to merge. In addition, we found that when the degree of differentiation of the hardware products is small the foreclosure equilibrium can result in de facto standardization. The technology of the foreclosing firm is exclusively adopted and foreclosure is an effect monopolization strategy.

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Appendix

Proof of Lemma 1. If the price of a software variety exceeds $\beta N_i^{\beta-1}$, consumers will not purchase it. A price less than $\beta N_i^{\beta-1}$ reduces profits: sales are unchanged as the demand by a consumer for a variety of software is perfectly inelastic and the market size is determined by hardware sales. *Q.E.D.*

Proof of Lemma 2. Maximizing (8) and (9) with respect to the relevant hardware price yields the following best response functions:

$$p_i = \frac{p_j + k}{2}, i, j = A, B, i \neq j.$$

The expressions in Lemma 2 follow immediately from the best-response functions. *Q.E.D.*

Proof of Lemma 3. Maximizing (11) with respect to p_A and the analogous profit function for firm B with respect to p_B yields price best response functions for firms A and B:

$$p_i = \frac{p_j + k - \beta N^{\beta}}{2}, i, j = A, B, i \neq j.$$

The equilibrium expressions in Lemma 3 follow immediately from the best-response functions. *Q.E.D.*

Proof of Lemma 4. The profits of firm A from its hardware and sales of N software products are

$$\pi_A = \left(p_A + \beta(2N)^{\beta-1}N\right) t. \tag{25}$$

The best-response function for firm A is found by substituting (7) for t into (25) and maximizing with respect to p_A:

$$p_A = \frac{\left((1-\beta)(2^{\beta}-1) - \beta 2^{(\beta-1)}\right) N^{\beta} + p_B + k}{2} \tag{26}$$

Firm B derives profits from hardware, software for technology A, and software compatible with technology B. The profits of firm B are

$$\pi_B = \left(p_B + \beta N^{\beta}\right)(1-t) + \beta(2N)^{\beta-1}Nt \tag{27}$$

Substituting in (7) for t into (27) and maximizing with respect to p_B gives the reaction function for firm B:

$$p_B = \frac{\left(p_A + k + \left(\left(2^{\beta-1} - 1\right)\beta - (1-\beta)\left(2^{\beta} - 1\right)\right)N^{\beta}\right)}{2} \tag{28}$$

Then, the equilibrium prices and market shares in an interior equilibrium follow immediately from the best-response functions, (26) and (28). Substitution of the equilibrium values for p_A^I, p_B^I, and t^I into the profit functions for the two hardware firms, (25) and (27), gives profits in the interior equilibrium.

For parameter values where $k_i \geq k$, there will be a standardization equilibrium since $t^I \geq 1$. The equilibrium price in the standardization equilibria for firm A follows from substituting B's best response function, (28) into the expression for market share, (5), and setting the resultant expression equal to one. Substituting the standardization equilibrium price of firm A into (28) yields the standardization equilibrium price for firm B. The standardization profits follow immediately. *Q.E.D.*

Proof of Lemma 5. The profits of firm B arise only from the sale of hardware and are simply

$$\pi_B = (1-t)p_B \tag{29}$$

Substituting in (7) for t and maximizing (29) with respect to p_B yields the best-response function for firm B:

$$p_B = \frac{p_A + k - (1-\beta)(2^{\beta} - 1)N^{\beta}}{2}. \tag{30}$$

The best response function for firm A is again given by (26). The interior equilibrium expressions follow from the two best response functions, and equations (29) and (25).

A standardization equilibrium will result if $k_f \geq k$ since $t^f \geq 1$. The standardization price for technology A is found by substituting B's reaction function, (30), for p_B into the expression for market share, setting the result equal to one, and solving for p_A. Substituting into B's reaction function yields $p_B = 0$. The standardization profits follow immediately.

Q.E.D.

Proof of Proposition 1. Denote the sum of the profits of firm B and the independent software firm in the foreclosure equilibrium as stand-alone (SA) profits. Using Lemma 5,

$$\pi_{SA}^f = \pi_B^f + \pi_S^f = \frac{\left(\left(2^{\beta-1} - 1\right)\beta + f/3\right)N^{2\beta}f + \left(3\left(2^{\beta-1} + 1\right)\beta - 2f\right)N^{\beta}k}{6k} + \frac{k}{2}. \tag{31}$$

From Lemma 3, the profits of firm B if it responds by integrating and foreclosing are $k/2$. The difference between stand alone profits and integrate and foreclose profits is

$$\pi_{SA}^{f} - \pi_{B}^{BF} = \frac{\left(\left(2^{\beta-1}-1\right)\beta + f/3\right)N^{2\beta}f + \left(3\left(2^{\beta-1}+1\right)\beta - 2f\right)N^{\beta}k}{6k}. \tag{32}$$

Substituting the value for the lower bound on k,

$$k_f = \frac{fN^{\beta}}{3}$$

into (32) gives

$$\pi^{SA} - \pi_{B}^{f} = \frac{\left(2^{\beta}\beta - f/3\right)N^{2\beta}f}{6k},$$

which is greater than zero for $0 < \beta < 1$. Since the numerator of (32) is increasing in k, it will still be positive for values of k greater than the lower bound. *Q.E.D.*

Proof of Proposition 2. From Lemma 4, the difference between the profits of following this strategy and stand alone profits is:

$$\pi_{SA}^{f} - \pi_{B}^{i} = -\frac{\left(\left(\left(2^{2\beta-1} - 22^{\beta+1} + 2\right)\beta + 32^{\beta-1} - 2^{2\beta-1} - 1\right)\beta\right)N^{2\beta}}{18k} - \frac{3\left(\left(2^{\beta-1}-1\right)\beta\right)N^{\beta}k}{18k}, \tag{33}$$

which is greater than zero if $k > k_{min}$. *Q.E.D.*

Proof of Proposition 3. If the merged firm did not foreclose on firm B it would derive profits from hardware sales and software sales for both technologies:

$$\pi_A = p_A t + \rho_A t + \rho_B (1-t)$$

or recognizing that the number of software varieties available for each technology will be the same and substituting in from Lemma 1 for the price of software,

$$\pi_A = \left(p_A + \beta 2^{\beta-1}N^{\beta}\right)t + \beta 2^{\beta-1}N^{\beta}(1-t) \tag{34}$$

or

$$\pi_A = p_A t + \beta 2^{\beta-1}N^{\beta}. \tag{35}$$

Merger with non-foreclosure yields an expression for the profit of firm A identical to the unintegrated case except for an additive constant equal to the profits of an independent software firm in the unintegrated case.[19] Consequently, substituting into (35) from the results of Lemma 2 for the hardware profits of firm A, the profits of the merged firm when it does not foreclose and B does not integrate, are simply the sum of an independent software firm and an unintegrated hardware firm in the unintegrated industry structure:

$$\pi_A^{nf} = \frac{k}{2} + \beta 2^{\beta-1} N^{\beta}. \tag{36}$$

The incentive for the merged firm to foreclose is the difference between foreclosure profits for firm A from Lemma 5 and non-foreclosure profits, from (36):

$$\pi_A^{f} - \pi_A^{nf} = \frac{\left(fN^{\beta} + 3k\right)^2}{18k} - \frac{k}{2} - \beta 2^{\beta-1} N^{\beta}. \tag{37}$$

This is positive if $k_{max} > k$. *Q.E.D.*

Proof of Proposition 4. If firm A does not integrate and foreclose, then the profits earned by it and the independent software firm equals (36) when B does not integrate. If A forecloses and B retaliates by merging but not foreclosing, the profits of firm A are given by Lemma 4. The difference between these indicates whether A will foreclose even if B retaliates. This difference is equal to

$$\pi_A^{f} - \pi_A^{nf} = \frac{\left(2^{2\beta} - 2^{\beta+1} + 1\right) N^{2\beta} + 6\left(2^{\beta} - 1 - 32^{\beta-1}\beta\right) N^{\beta} k}{18k}. \tag{38}$$

For an interior equilibrium, the market share of firm A cannot exceed 1. Thus a lower bound on k is given by (13), k_f , which is greater than k_i. From (12),

$$k_i = \frac{(2^{\beta} - 1)N^{\beta}}{3}. \tag{39}$$

Substituting this lower bound for k into (38) gives

$$\pi_A^{f} - \pi_A^{nf} = \frac{3\left(2^{\beta} - 1\right)\left(2^{\beta}(1-\beta) - 1\right) N^{2\beta}}{18k},$$

[19] It is for this reason that integration without foreclosure by both firms and integration without foreclosure by one firm when the other firm is unintegrated are formally identical to the unintegrated industry structure, ie there is no price effect.

which is less than zero for $0 < \beta < 1$. Since the numerator of (38) is decreasing in k, as k increases, the numerator remains negative. *Q.E.D.*

Proof of Proposition 5. The maximum a hardware firm will bid for a software firm is the difference between its profits if it forecloses and if it is foreclosed upon. In order for a bid to be accepted, it must be greater than the profits a software firm would earn if it was the independent software firm in the foreclosure equilibrium. Thus in order for a successful bid to be made,

$$\pi_A^f - \pi_B^f > \pi_S^f \tag{40}$$

or

$$\pi_A^f > \pi_S^f + \pi_B^f$$

or

$$\pi_A^f > \pi_f^{SA}.$$

From Lemma 5,

$$\pi_A^f = \frac{\left(3k + fN^\beta\right)^2}{18k}. \tag{41}$$

Stand alone (hardware and software) profits in the foreclosure industry structure are given by (31):

$$\pi_{SA}^f = \frac{\left(\left(2^{\beta-1} - 1\right)\beta + f/3\right)N^{2\beta}f + \left(3\left(2^{\beta-1} + 1\right)\beta - 2f\right)N^\beta k}{6k} + \frac{k}{2}.$$

The difference between these two is

$$\pi_A^f - \pi_{SA}^f = \frac{\left(1 - 2^{\beta-1}\right)N^{2\beta}f\beta + \left(4f - 3\left(2^{\beta-1} + 1\right)\beta\right)N^\beta k}{6k}, \tag{42}$$

which simplifies to

$$\pi_A^f - \pi_{SA}^f = \frac{\left(4f - 62^{\beta-1}\beta\right)N^{2\beta}f}{18k}. \tag{43}$$

when we substitute in the lower bound for k, k_f, where, $k_f = f\frac{N^\beta}{3}$. For $0 < \beta < .56421$ (43) is positive and the numerator of (42) is increasing in k for $< \beta < .17287$. *Q.E.D.*

Proof of Proposition 6. From Proposition 3, Firm A finds it profitable to foreclose, given no retaliation when $k_{max} > k$. From Proposition 2, Firm B will, however, retaliate if $k < k_{min}$. For a foreclosure equilibrium to exist, $k_{max} \geq k \geq k_{min}$. Such a range for k will exist if

$k_{max} \geq k_{min}$. This is true for $0 < \beta \leq .098733$. There is not a holdup problem since from Proposition 5, a software firm will prefer to merge and foreclose if $\beta < .17287$. We also check that $k_{min} > k_f$ for $0 < \beta < 1$, so that for $k > k_{min}$, the foreclosure subgame equilibrium is in fact an interior equilibrium. Furthermore, we need to check that $k_{max} > k_f$ so that when A forecloses the equilibrium is in fact an interior equilibrium and not a standardization equilibrium. For $0 < \beta < .22179$, it is straightforward to show that $k_{max} > k_f$. If the two parameter restrictions do not hold, then either firm A does not find it profitable to foreclose, even if firm B does not merge or even if A finds it profitable to foreclose if B does not respond, it is profitable for firm B to respond by integrating and not foreclosing which makes foreclosure by A non-optimal. *Q.E.D.*

Proof of Proposition 7. We first show that for $k_f > k > k_i$, firm B always responds to foreclosure by firm A by integrating but not foreclosing. If $k = k_i$, then the variety advantage afforded firm A if it forecloses and B retaliates is such that the retaliation is ineffective and $t = 1$. In this case, the profits of B if it integrates arise only from the sale of software to technology A. However, this is also the value of stand alone profits. For $k > k_i$, $1 > t^I$, p_B^I is increasing in k, t^I decreasing in k, and the profits of firm B,

$$\pi_B^I = \left(p_B^I + \beta N^\beta\right)\left(1 - t^I\right) + \beta 2^{\beta-1} N^\beta t^I,$$

are increasing and therefore greater than stand alone profits.

From Proposition (4), it will not be profitable for firm A to foreclose if firm B retaliates when $k > k_i$. Q.E.D.

Proof of Proposition 8. Foreclosure profits are, from Lemma 5

$$\pi_A^{fs} = \left(2^{\beta-1}(2-\beta) + \beta - 1\right) N^\beta - k. \tag{44}$$

or, breaking out hardware and software profits,

$$\pi_A^{fs} = (1-\beta)\left(2^\beta - 1\right) N^\beta - k + \beta 2^{\beta-1} N^\beta.$$

From (36) the non-foreclosure profits of firm A are

$$\pi_A^{nf} = \frac{k}{2} + \beta 2^{\beta-1} N^\beta.$$

The difference between these two is positive when $k_s > k$. We now show that no integration by firm B is the best response to foreclosure by firm A. We first show that firm B will never find it optimal to acquire the remaining software firm and foreclose on A. If foreclosure results in standardization, then in the foreclosure equilibrium the profits of firm B are zero and hence stand-alone profits are simply duopoly software profits. The difference between stand-alone profits and bilateral foreclosure profits is:

$$\pi^f_{SA} - \pi^{BF} = \beta 2^{\beta-1} N^\beta - k/2.$$

This is greater than zero if $2^\beta \beta - f/3 > 0$, which is identical to the condition for an interior equilibrium and it holds for all $0 < \beta < 1$.

We now argue that firm B will never find it optimal to acquire the independent software firm and not foreclose. If firm A has foreclosed, the stand alone profits of firm B and the independent software firm are simply duopoly profits from software sales to technology A. If B integrates, the equilibrium is still a standardization equilibrium and the integrated firm earns profits equal to duopoly profits from software sales to technology A. *Q.E.D.*

Proof of Proposition 10. The total surplus associated with the foreclosure equilibrium is given by (24). The middle term, $N^\beta \left(\left(2^\beta - 1 \right) t + 1 \right)$, is increasing in t since $2^\beta - 1 > 0$. The last term,

$$\frac{k}{2} \left(t^2 + (1-t)^2 \right),$$

is minimized when $t = 1/2$ and is increasing in t for $t > 1/2$. Substituting in $t = 1$ for t in the middle term and $t = 1/2$ for the last, a strict upper bound on TS^f is,

$$TS^f < y + \alpha + (2N)^\beta - \frac{k}{4}. \tag{45}$$

The right-hand side is identical to (21), the total surplus in the unintegrated equilibrium.

Q.E.D.