An evolutionary model for the dynamics of vertical integration and network-based production

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Abstract

The concept of modularity has gained recently a growing attention in the management literature as a key to explain the contemporary trends of industrial dynamics. A stronger exploitation of external «network-based» economies with respect to internal «bureaucracy-based» economies is one of the major consequences of the diffusion of flexible production systems and of modular architectures for products. To explain this connection, in this paper a model is presented which tries to explain the co-evolution of technology and organization as the outcome of a complex evolutionary process. In particular, through a set of exploratory agent-based simulations we try to show the existence of qualitatively different dynamic processes in coincidence of different phases and specific conditions of product and technology development, as well as the existence of a relationship between this dynamics and other competitive factors, determining a wide set of structurally different dynamical patterns in coincidence of different combinations of factors.

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1. Modularity and modular networks

Recently the concept of modularity has gained a growing attention in the management literature as a key to explain a set of emerging trends that are progressively and profoundly modifying the industrial landscape of developed countries [Baldwin and Clark 1997, 2000; Schilling 2000; Fleming and Sorenson 2001a, 2001b and 2003; Galunic and Eisenhardt 2001; Schilling and Steesma, 2001; Langlois 2002a; Sturgeon 2002]. In the last two decades firms have redefined their boundaries and relations with markets (through an extensive resort to outsourcing), internal functioning processes (with a growing flexibilisation of the labor force), as well as the geographical scale of their operations (through delocalisation and foreign direct investments). This passage took place under the double pressure of technological change and of final demand: the former has given to firms the opportunity to reduce the costs of information exchange and elaboration thanks to the cluster of innovations linked to the «Information Revolution» [Freeman e Louçã 2001; Mokir 2002; Perez 2002]; the latter has progressively differentiated and become more difficult to predict, asking for more flexibility in production processes and for more speed in product innovation [Langlois e Robertson 1992], thus favoring the birth and development of new systems oriented towards «flexible production» [Milgrom and Roberts 1990; 1995].

Business-oriented studies generally agree that the major benefits of flexible production systems are realized when the latter combine with a modular design of products, that fosters a higher speed of innovation and an easier differentiation of product themselves. Following Ulrich [1995], we can define an architecture of a product as the combination of three design specifications: i) the pattern of functional components; ii) the mapping between functional and physical components; iii) the definition of the interfaces between linked physical components[1]. An architecture can be defined as modular when there is a one-to-one mapping between functional and physical components, and interfaces are de-coupled. The latter condition means that an interface is designed in such a way as to minimize the need to change a physical component as a consequence of a change in another component linked to the first one through that interface. When one of the two conditions is not fulfilled, the architecture is said to be «integral». The main advantage of modularity consists in a greater easiness to change product functionality, because each functional element can be varied simply changing the corresponding physical component without any further change on other components. An architecture of this kind makes easier for firms to increase product variety, because each change in function needs only one physical component to be updated, and variety can be

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[1] Ulrich's definition draws on previous studies, which had focused on the relationships between innovation dynamics and product architecture, as for example that of Abernathy e Utterback [1975] and more recently those of Henderson and Clark [1990], and Henderson [1992]. Subsequently the notion of modularity in design has been reconsidered and further developed by Baldwin e Clark [2000].

achieved in a combinatorial way without the risk of a complexity explosion due to uncontrolled interdependencies.

As long as product variety is the main goal of flexible production systems, a modular architecture is thus particularly fit for them. At the same time a modular architecture helps components' standardization, because if each component performs only one function there is a higher probability that this function is shared by different products or different versions of the same product. Further, there is a likely relationship between product architectures and organizational models which, when a modular architecture with standardized components is available, should favor the possibility to outsource the production of those components. This relationship seems to be confirmed by the fact that a stronger exploitation of external «network-based» economies with respect to internal «bureaucracy-based» economies [Chandler 1977; 1990] is one of the major consequences of the diffusion of flexible production systems and modular architectures [Sturgeon 2002; Langlois 2002b]. The growing diffusion of the so-called «modular networks» has occurred as a consequence of many interdependent factors, converging in a coherent set of technological, organizational and institutional changes. Two were the major success factors of this new model: i) the reduction of fixed-capital sunk costs, thanks to the possibility to quickly and economically re-allocate the existing productive capacity according to market dynamics [Sturgeon 2002]; ii) a greater speed of innovation, thanks to the increasing returns generated by the growing specialization of firms on a limited number of core competences [Prahalad e Hamel 1990]. As far as the first point is concerned, nowadays it is recognized to be essential for many firms in many industries to posses a strong and diversified network of suppliers, because in the prevailing market conditions it is usually not possible to plan production flows beyond a limited time horizon. By resorting to suppliers, client firms can increase production without the need to incur in fixed-capital investments, or decrease it without destroying existing capacity, because suppliers, having themselves a strong and diversified network of clients, can compensate the declining demand of one client firm with that of many others. Thanks to these different and partially overlapping networks, economies of scale are not achieved anymore internally as in the vertically-integrated firm [Chandler 1990], but externally through the interaction of clients and suppliers. As far as the second point is concerned, innovation is fostered also by the diffusion of decentralized (cooperative and competitive) search processes which are typical of these networks. These processes are sustained also by a reduction of entry barriers, as newcomers can make use of the available capacity of suppliers to enter the market.

In order to realize the advantages of modular networks, two main preconditions must be fulfilled. The first one is the reduction of coordination costs among different firms made possible through information technologies and internationalization of markets. The convergence process of national economies, favoring a growing extension of markets, has created wider opportunities for deeper specialization and horizontal integration over different markets, i. e. for stronger economies of scope; while information technologies have reduced the push towards vertical integration, by separating, through networks, the achievement of scale economies from internal growth of firms. In the new context the internal resources of firms focus on developing and exploiting, over different market and products, core competences and specializations, while vertical integration is not anymore decisive, and the production of intermediate inputs is more and more often outsourced. The second precondition concerns the existence of standards, and it is fulfilled when products are available that posses a stable and open architecture, which is also codified and shared by the main competitors of one industry. The need for standards depends from the fact that in modular networks cooperation is assured by market mechanisms, that require, to work properly, a flux of codified information (specifying the technical, commercial and operative details of each transaction), which is more easily achieved if standards are available. The fulfillment of these two preconditions (through information technologies and internationalization on one side, and modular architectures paving the way for a wider use of standards on the other) has freed modular networks from geographical constrains, making them more efficient (with respect to more geographically-clustered networks, such as industrial districts) in a context of growing international trade, and helping a huge re-allocation of productive capacity, with the emergence of new global leaders in many industries[2].

Even if that of modular networks is indeed a success story, nonetheless any organizational model is the outcome of a partial and temporary equilibrium, whose boundaries are defined by the particular conditions that have produced it and reveal themselves in the trade-offs to which this model is subject. As we have underlined above, in the case of modular production we need the product architecture to be relatively well known and stable for the discovery process of innovation to be partitioned among independent (and partially uncoordinated) agents, yielding what Henderson and Clark [1990] call «modular innovation». This stability is essential to avoid the disruptive consequences of the parallel discovery processes, going on in each module, on the global performance of the product, but such a level of reliability is more a consequence than a starting point of the discovery process [Baldwin e Clark 1997: 86]. When we are confronted with a new product or technology an early and definitive adoption of a specific architecture implies a higher probability of performance losses. In fact, if interdependencies between components are not well known, that architecture can partition subsets of components without taking properly into account

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[2] For example, since the half of the Nineties the electronic industry assisted to the spectacular growth of a set of 5 suppliers (Solectron, Flextronics, Sanmina/SCI, Celestica, Jabil Circuit), which have been able to gain increasing market's shares offering to the (former) final assemblers the complete («turn-key») production of their products, while the latter renounced in many cases completely to their internal productive capacity, selling all their plants to the former [Sturgeon 2002].

those interdependencies, thus causing unexpected and undesired feedbacks over the global functioning of the product. In order to overcome these deficiencies we need to recombine components without the constrain of a fixed architecture, and rather focusing exactly on the test of alternative architectures.

The general problem we face is that of the connection between the way in which a search process is carried out, and the possible outcomes of the research itself. From an evolutionary perspective this connection is not surprising, because it widely recognized that innovation is a path dependent process, i. e. one in which the point where we are determines where we can get to in a given amount of time. Path dependency implies that not all the possible points of the «innovation space» are immediately and equally accessible to all agents, and consequently that the latter have generally different «visions» of the same problem, which on its turn means that they search using different strategies and from different starting points. Following this perspective, the trade-offs of alternative search strategies and their consequences on search performance have been studied and discussed by a recent strand of economic literature. These contributions have showed that search processes in complex environments are exposed to a trade-off between speed of exploration (which must be higher if selective pressure is higher) and quality of results (which is lower if the search process is faster) [Marengo *et al.* 1999: 15; Frenken, Marengo and Valente 1999: 12; Marengo and Dosi 2003]. In particular Marengo and Dosi [2003: 17] have demonstrated that the size of the smallest subproblem of a given problem (which represents a measure of the time requested to solve that problem [Holland 1975; Page 1996]) is weakly decreasing in the number of «satisfying» solutions, i. e. those accepted «as good» by the search agent [Marengo and Dosi 2003: 17]. This trade-off is stronger if the problem to be solved is highly complex (i. e. if the interdependencies between the different parts of the problem make it very difficult to separate it in a set of simpler sub-problems [Simon 2002]), while less complex problems can be solved quickly with a smaller loss of performance. As a consequence of this trade-off, in highly selective environments those strategies are more productive that (all other things being equal) pursue a finer decomposition of the original problem, even if they incur in a lower probability to reach the best solution for that problem. Frenken, Marengo and Valente [1999] have showed through micro-simulations the effects of this trade-off on a population of agents exposed to a selection process that acts on the basis of the different performance of solutions achieved through alternative search strategies. Agents that use search strategies based on finer decompositions of the problem generally dominate agents using a global strategy, thanks to the higher speed of the former in reaching «satisfying» levels of fitness, while the latter reach the highest fitness over a longer period. As a consequence, the advantage of a finer decomposition will emerge only if the selective pressure on agents is strong enough to «kill» agents pursuing a coarser decomposition before their reach the best solution. In this sense, selection pushes against the long-period performance of the search process, while it enhances its short-period performance.

Other works have extended these general results to the analysis of product development and modularity [Dosi, Levinthal and Marengo 2002; Ethiraj and Levinthal 2003; Frenken 2001; 2004], highlighting two fundamental facts:

1. The complexity of innovation (conceived as a collective discovery process going on through variation and selection) changes during the life-cycle of one product or technology; in general, complexity will be higher during the first stages of product/technology development, when agents do not dominate fully the task and the discovery process is exposed to frequent failures because of the existence of unforeseen connections among different parts of the task; it will be lower when the product's or technology's architecture is stable enough to be efficiently modularized, opening the opportunity to exploit the increasing returns yielded by the division of «innovative labor».
2. There are different conceivable search strategies to explore the «landscape» of possible innovations: at one side of the spectrum we have a completely decentralized search strategy, in which the discovery process is subdivided among many agents which compete in a (at least partially) uncoordinated fashion to solve the different parts of the «innovative puzzle»; at the other side we have a fully centralized strategy, in which the discovery process is planned and centrally controlled, and the different parts of the puzzle are solved together, in order to minimize the probability of unexpected negative feedbacks.

A combination of the two alternatives exposed above defines four possible situations which can be summarized as in Table 1. In general, a decentralized search process offers a higher speed of adaptation (i. e. a faster performance increase), all other things being equal, and especially when it can exploit the potential of parallel search, through which many competing agents can explore a bigger portion of the innovative possibilities (even if at a higher cost because of redundancy). The comparative performance of the two strategies depends on the development stage of the technology or product which is undergoing innovation: centralized search helps finding solutions with a higher fitness at the cost of a slower discovery process, while the fitness of the solutions yielded by the decentralized strategy will be lower, when compared to the first ones, when the complexity of innovation is higher. Thus a lack of coordination among agents can represent a big gap in the initial phases of development of a new product or technology.

Table 1. Relationship between the complexity of innovation and the performance of alternative search strategies

Search strategy	*Complexity of innovation*	
	Higher	*Lower*
Decentralized (market)	Higher adaptation speed when compared to centralized strategy (fitness increase very fast at the beginning of the search process)	
	Higher fitness' loss (stronger damages caused by uncontrolled interdependencies)	Low fitness' loss (weaker damages caused by uncontrolled interdependencies)
Centralized (bureaucracy)	Lower adaptation speed (fitness increase very slow at the beginning of the search process) Highest *fitness* (stronger control over interdependencies)	

The main drawback of decentralized search in situations of higher complexity is to accept too many local innovations from different agents, without being able to control for their reciprocal compatibility[3]. This trade-off is not linear, i. e. not necessarily in situations of higher complexity more centralized strategies will prevail: their advantages could not be able to reveal themselves if the selective pressure is too strong, simply because the slower centralized agents are removed from competition before they can take the lead. In other terms, the combination of a highly selective environment and a decentralized search process can produce negative effects on innovation in the development of a complex technology: if «decentralized» agents are systematically selected because the selective pressure is high, the average quality of the product or technology will be lower in the long run.

Our schema is of course extremely simplifying. In particular we have assumed a fixed structure (and complexity) of the task, while in real innovation processes complexity can be defined only in connection with the amount of knowledge possessed by the agents. The complexity of a technology contains an inherently subjective component, and should be conceived more properly as the outcome of a dynamic interconnection between the «objective» dimension of technological problems and the «subjective» exploration strategies of those problems. When a high coherence between the objective and subjective dimension is achieved, the problem is not anymore locally (i. e. for that agent and that environment) complex. Given these premises, the trade-offs of our schema give rise, from a dynamic point of view, to an open evolutionary process, which is based on two main cyclical phases:

1. *architectural analysis and development of the product or technology*, whose goal is to identify a satisfying decomposition of that product or technology in functional and physical components; in this phase, if local search processes are indeed necessary and useful, they must be strictly coordinated and thus, following the taxonomy proposed by Henderson and Clark [1990], in this

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[3] On this point see Milgrom and Roberts [1995].

phase architectural innovation is prevailing;

2. *architecture exploration*: once that a stable decomposition is achieved, the necessity to coordinate local discovery processes becomes lower and these can be entrusted to independent agents; following Henderson and Clark [1990], in this phase modular innovation prevails.

The adoption of a modular design is thus coincident with a process of complexity reduction, rather than with one of complexity management as Ethiraj and Levinthal [2003: 1] seem to suggest, because the proper complexity management lies in the dynamical cycle between architectural and modular innovation, which alternate in different moments of a product's life-cycle. Because of the inherent and inescapable non-linearities of evolution at every level of reality, unexpected variations in internal and external factors can indeed force agents to re-frame even the most stable and affordable design. This necessity can emerge, for example, when we need to extend the duration, resistance or performance of one product. In these cases it is likely that, in order to reach our goal, we have to introduce innovations in one module which can act negatively on the performance of other modules, showing the existence of (now relevant but previously neglected) interactions which must be adequately deal with if we want to obtain the expected result. Since we are moving in an unexplored territory, it is not possible to exclude that we need to re-discuss and re-structure the whole product architecture. This possibility highlights that complexity can never be completely deleted in a complex world as ours, and consequently, even if modular architectures are very useful, the innovation process can never be perfectly and uniquely modular[4].

The limits of modularity are thus the consequence of a more general principle: when we confront with complex problems every modularization is necessarily limited and imperfect. A perfect modularization could be accomplished only if there where no interdependencies between different modules, but this eventuality coincides with a situation of «non complexity»[5]. In order to see that we confront with complex phenomena we can focus on the non linear connections linking the different domains in which the concept of modularity can be defined when it is applied to firms, namely: i)

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[4] The limits of modularity are seldom discussed, especially in the management literature. An exception is represented by Fleming and Sorenson [2001a; 2001b; 2003; 2004], who have suggested to extend the idea of the life-cycle of technologies [Abernathy and Utterback 1978; Klepper 1997], in order to include the possibility of multiple dynamical development cycles, carried on through successive «modularizations» and «integrations», where the former help in controlling complexity by making technologies more stable, and the latter become necessary when the returns of the discovery process in the frame of a given modular architecture decrease.

[5] A more detailed explanation of this assertion goes beyond the scope of this paper. In order to give a hint on this point we can refer to Simon's [1962] original concept of «nearly decomposable» systems, which is based on the idea that the distinction between the faster micro-dynamics occurring inside highly interconnected subsystems (i. e. what we call modules) and the slower macro-dynamics occurring between those subsystems allows a rigorous separation of time scales in describing the whole system. Here we can remark that the violation of time (and space) scale separation is exactly a necessary condition for the emergence of complex systems [Bar-Yam 1997: 89-95], and thus complexity occurs because of the approximative validity of the distinction between inside-modules and between-modules dynamics.

technology, in which modularity is a property of product and process design; ii) organization, in which modularity deals both with internal configuration of organization and with organizational boundaries; iii) knowledge processes, in which this notion regards individual and collective problem solving strategies in complex environments. The underlying hypothesis of the majority of existing studies on this topic is that there is a strong linear connection between these three domains[6]. On the other hand recent contributions [Brusoni and Prencipe 2001] have suggested that modularization in one of these dimensions doesn't imply a parallel trend in the others. For example, in modular production systems it is very often necessary, for the information flows and problem solving processes to work properly, that some firms act as «system integrators», developing the «architectural» knowledge which is necessary to coordinate the activities of the single components' producers[7]. Generally this role is played by the final assemblers, which consequently tend to focus progressively on design, R&D and marketing activities, while they outsource production. Modularization in technology doesn't exclude, in other terms, the need for cognitive integration, and the lack of correspondence between the two dimensions helps to explain why some activities (as design) cannot be modularized and outsourced as easily as others [Brusoni and Prencipe 2001: 195]. From the point of view of external cooperation, the «persistence» of cognitive integration implies the need of a tight coordination between cooperating firms, which goes far beyond standard market transactions as conceived by mainstream theory, and involves trust and other relational factors, which cannot be completely dispensed with even if transactions are based on an extensive use of codified information and standards.

2. Rationale of the model

Our model is intended to represent the trade-offs of alternative search strategies in a dynamic competitive context, i. e. one where firms have to cope continuously with external solicitations coming from both market competition and technological change. Since there is not an unique optimal strategy, our goal is to identify a set of factors whose variation can explain the prevalence of alternative strategies in different situations. In order to achieve this goal, as the problem is indeed very general and involves potentially many interconnected domains, we need to narrow our scope. We did this firstly by focusing on the comparison between decentralized and centralized search strategies, i. e. leaving aside further potential differences; and secondly by evaluating this

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[6] See for example Nightingale [2000], Schilling e Steensma [2001], Galunic and Eisenhardt [2001], but also Baldwin and Clark [1997], while a more mixed judgment is given by Langlois [2002b].

[7] The necessity of cognitive and relational «hubs» has been detected in other kind of production networks, as industrial districts [Lombardi 2003], and it has been recently recognized as a more general feature of social networks [Barabási e Bonabeau 2003].

comparison in the context of a concrete organizational alternative, namely that of internal versus outsourced production. In particular, we compare firms that produce internally the components of which their final product is made up, with firms that buy the same components from the market. Our goal is thus to compare the performance of the two alternative production patterns just described (which could be labeled respectively as «verticalized» and «deverticalized») competing in the same environmental conditions. By doing this, our model focuses only on one of the possible ways in which the alternative between centralization and decentralization can manifest in the domain of organization. There are indeed many other contexts in which this comparison could be usefully performed[8].

The dynamics of the model is generated by technological innovation, which is represented according to an idea originally proposed by Sah and Stiglitz [1986]. Their model is an attempt to deal with the problem of decisional architecture through a very simple representation. The notion of architecture means here the pattern of connections between the units that contribute to a decisional process, with a particular reference to the distribution of authority, information flows and elaboration capacity. The model compares two alternative architectures: i) *poliarchy*, i. e. a system where many competing units can pursue projects independently one from the other; ii) *hierarchy*, i. e. a system where only some agents can decide whether to pursue one project, while others are charged to supply useful information to those who decide. If agents are subjects to informational and cognitive limits, when they have to evaluate innovation projects they can make two kind of mistakes: i) to reject projects that should be accepted because they are found (ex post) to yield positive results; ii) to accept projects that should have been rejected because they subsequently fail and eventually cause damage. The two typologies of errors can be assimilated to Type I and Type II errors in the standard theory of statistical inference. The two architecture have a different bias towards the two typologies of errors. Given the same set of agents, those organized in polyarchies will collectively tend to make more Type II errors, because this architecture has lower resources for evaluating one project, while it admits that one rejected project can be accepted by another agent. On the other hand agents organized in hierarchies have a stronger bias towards Type I errors, because they accept a lower number of projects, and loose consequently some favorable opportunities, while they accept projects which have on the average better results thanks to their better ability to monitor and evaluate the projects themselves.

The two architectures are confronted with the problem to evaluate and eventually adopt one project picked up from a portfolio of N alternative projects. Each project has a net return x, which can

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[8] For example, decentralized architectures are feasible inside single organizations too, if these adopt non-bureaucratic configurations, open to frequent recombinations of internal resources, as well as competition and parallel search processes [Simon 1991].

be positive or negative. The density function of projects, with respect to their return, is $g(x)$. The probability that one project is selected is denoted by p, and its value depends from the return of that project. The function $p(x)$ is called screening function for the projects. In the simplest case this function can find a linear formulation, for a project i, as $p(x)=p(\mu)+p_x(x_i-\mu)$, where $\mu=E[x]$ is the expected value of each project given the density function of the initial portfolio, and p_x denotes the partial derivative with respect to x, evaluated for $x=x_i-\mu$. In a condition of imperfect information and bounded rationality of the agents, we have that $p(x)<1$.

The selection process occurs differently in each of the two architectures. In polyarchies projects are posted randomly to the decisional units: if one project is rejected by one unit, another one can examine it and eventually adopt it; if it is accepted, it cannot be anymore evaluated and adopted by other units; no project can be examined twice. The selection is process is thus limitedly parallel, and the total probability of one project to be adopted is equal to the sum of the probabilities that it is adopted by each unit, conditioned by the probability that the same project is rejected by the other units. In the case of two units this probability is $f^P=p(x)+(1-p(x))p(x)=p(x)(2-p(x))$ (where P stands for polyarchy), under the condition that the probability of adoption is the same for every unit. In hierarchies each project goes through a double screening, i. e. it is firstly examined by a lower-level unit and, if accepted by this one, it is submitted to a higher-level unit which re-examines the project and takes the final decision. The selection process is serial, and the probability of adoption is given by the joint probability that the project is accepted by both of the two levels. In the case of two units with equal probabilities of adoption the joint probability is $f^H=p^2(x)$, where H stands for hierarchy.

Limiting themselves to the two-units case and assuming identical screening functions for the two architectures, Sah and Stiglitz [1986] derive the main implications of this model. In particular they demonstrate that: i) given the same initial portfolio, polyarchy selects a higher number of projects when compared to hierarchy; ii) a worse initial portfolio implies a worsening in the performance of polyarchy with respect to hierarchy. If the first point is more evident, the second can be explained as follows. Let's define $z_1>0$ as the return of a successful project and $-z_2<0$ as the return of an unsuccessful project; $p_1=p(z_1)$ is the probability that the successful project is adopted, and $p_2=p(-z_2)$ is the probability that the unsuccessful project is adopted, with $p_1>p_2$; finally α is the proportion of successful projects in the initial portfolio. If $Y^s=E[xf^s]$ is the total return yielded by the projects adopted by one of the two architectures (where $s=P$ stands for polyarchy and $s=H$ stands for hierarchy), and $\Delta Y=Y^P-Y^H$, we obtain that

$\Delta Y = 2[z_1 \alpha p_1(1-p_1) - z_2(1-\alpha) p_2(1-p_2)]$. From this formula we can derive that: i) polyarchy performs better when the value of the initial portfolio, which is represented by the ratio $a = z_1 \alpha / z_2(1-\alpha)$, is higher; ii) for «neutral» portfolios (i. e. those in which $z_1 \alpha = z_2(1-\alpha)$ and thus $a=1$) polyarchy performs better only if $1-p_1 > p_2$ (remembering that $p_1 > p_2$). This result can be explained if we think that $1-p_1$ e p_2 are respectively the probability of making Type I errors (reject a good project) and that of making Type II errors (adopt a bad project). If the former probability is higher than the latter, the architecture which yields a greater reduction of Type I errors (i. e. polyarchy) will perform better because it rejects a higher share of bad projects; if the latter is higher than the former, hierarchy will outperform polyarchy, because of its opposite bias towards a greater reduction of Type II errors[9].

At this point it is quite easy to see that this model captures the trade-off between alternative strategies that we have mentioned in the previous paragraph. From the fact that $f_P > f_H$ for $p(x) < 1$ we can draw two main implications: i) polyarchies accept a higher share of projects at the cost of a lower expected return for each project[10], exactly as we supposed that decentralized architectures should behave; ii) symmetrically hierarchies reach on the average a higher return at the cost of a lower number of adopted projects, which can lead to a lower total return, representing fairly well the fact that centralized architectures are slower than decentralized ones, as we have advocated above. The total return for each architecture is thus the result of two independent factors, which combine differently in the two architectures: i) the average value of the adopted projects; ii) the number of adopted projects. Consequently hierarchy prevails only if the combination between the share of bad projects and the ratio between the damages caused by bad projects and the benefits of good projects is such as to cause for polyarchies a loss which is higher than the additional return stemming from the higher number of good projects which the latter adopt. This particular conditions can be interpreted as those prevailing in the initial phases of development of a new product or technology, as well as, more in general, in those phases in which architectural recombination is occurring. In those conditions agents face a higher difficulty in finding good solutions and can undergo strong damages because of wrong projects.

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[9] Drawing conclusions on the comparative performance of the two architectures becomes more difficult if we admit that units, foreseeing their own limits, can rationalize their decisional criteria [Sah and Stiglitz 1986: 271-275]. Yet we must note that, if we admit this possibility, the informational and cognitive limits assumed in the model loose most of their explanatory force, especially from a dynamical point of view. In our model we refer to the basic version of the model, and assume consequently that informational and cognitive limits cannot be completely endogenized and optimized in the decision process.

[10] Even if Sah and Stiglitz don't treat the last point explicitly, it follows from the observation that, if $f_P > f_H$ for the same project, when we have two projects i and j with respective return x_i and x_j , $f_P(x_i) = f_H(x_j)$ only if $x_j > x_i$.

3. Model description

Our model represents the competitive dynamics of one industry, in which there are two subpopulations of firms with different characteristics: i) «verticalized», i. e. firms that produce internally the components of their final product; ii) «deverticalized», i. e. firms that buy one or more of those components from the market. More in detail, the model contains the following main entities:

1. one *industry*, which produces one final product P, made of n components, so that $P=\{c_1, \ldots, c_n\}$;
2. a set of $n+1$ *sectors*, i. e. n intermediate sectors which produce components plus one final sector;
3. *firms* that populate the $n+1$ sectors and compete on the respective markets;
4. *final demand* which is specified as an aggregate entity whose variation depends on the average quality and price of the final products supplied by firms;
5. finally, a set of T periods in which firms compete, for each period t, on the basis of the existing conditions at $t-1$, determining in this way the conditions of the industry at $t+1$.

In the following paragraphs we describe in detail the characteristics of our model, in order to introduce the results of a set of «agent-based» micro-simulations performed on a first and partial computational implementation of the same model, which are subsequently discussed in par. 4-6[11].

3.1. Market dynamics

The products supplied by the final firms (i. e. those operating in the final sector) are homogeneous with respect to their final use, but heterogeneous for quality characteristics and price [Lancaster 1966 and 1971]. To represent this kind of heterogeneity we can describe each product as a set of three elements: i) a vector of technical characteristics; ii) a vector of service characteristics for the final user; iii) a map linking the two vectors [Saviotti and Metcalfe 1984]. Gallouj and Weinstein [1997] have extended this frame to include a vector of capabilities which is combined with the technical characteristics of inputs to obtain the service characteristics of the output. This solution has been adopted by Ciarli and Valente [2003] to represent productive processes in a model with

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[11] The model has been implemented in LSD (*Laboratory for Simulation Development*), a simulation environment specifically designed to develop and analyze «agent-based» economic models. This tool has been developed by an Italian researcher, Marco Valente, and is particularly suited for building economic models. For additional information see http://www.business.aau.dk/~mv/. For an introduction to the theoretic foundations of «agent-based» modeling we can refer to Lane [1993], Axtell [2000], Valente [2002] and Tesfatsion [2003].

similar characteristics to the one presented here. In our case, to keep as simple as possible our model, the productive process has not been fully specified as above, but with a simplified schema which is nevertheless perfectly compatible with the models just mentioned, and could be easily extended in that direction. In particular, to each component c_i of the final product it is associated a value $x_i \geqslant 1$, which measures the contribution of that component to the global performance of the final product in terms of service characteristics for the final user (i. e. what for sake of brevity we will call from now on its «quality»). The quality Y of the final product for each firm j of the final sector is then equal to[12]:

$$[1] \quad Y_j = \prod_{i=1}^{n} x_i^{a_i} \quad ,$$

where the exponents $a_i > 0$ are parameters that represent the weight of the quality of each component on the quality of the final product, and the values of the x_i are determined by the dynamics of technological innovation (see par. 3.4). We should note that the functional form of the expression [1] is supermodular [Milgrom and Roberts 1995], i. e. it takes into account the interdependency between the qualities of the different components: according to the definition of supermodularity, an increase in the quality of one component yields a higher contribution to global quality when the quality of the other components is on a similar level too.

Final demand is determined by an aggregate index which increases in direct proportion with technological progress, since innovation is embodied in the service characteristics of the final products sold by firms operating in the final sector [Ciarli and Valente 2003]. More specifically demand is computed at each period with the following formula:

$$[2] \quad D = \left\{ H \left[1 + \left(1 + \frac{1}{\bar{P}} \right)^{a_p} \prod_{i=1}^{n} \bar{x}_i^{a_i} \right] \right\} * \sum_{j=1}^{m} Prd_j[t-1] \quad ,$$

where $H \geqslant 1$ is a constant; $Prd_j \geqslant 1$ is a variable that represents the productive capacity achieved by each of the m final firms; a_p and a_i, positives, represent respectively the elasticities with respect to variations of price and quality of components; $\bar{P}$ is the average price and $\bar{x}_i$ is the average quality of the components, weighted on the market shares of the m firms operating for the final market: $\bar{P} = \sum_{j=1}^{m} P_j * MS_j$ and $\bar{x}_i = \sum_{j=1}^{m} x_{ij} * MS_j$, where P_j and x_{ij} are determined respectively by firms' costs (see below, par. 3.3) and by the dynamics of innovation (see below, par. 3.4).

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[12] For the sake of simplicity temporal indexes are always omitted if in the same equation there are not variables referred to different time periods.

Competition in the final sector is based on the global «performance» of products, measured by the quality of their components and the price of the product itself. For each firm a «potential» market share, based on the comparative performance of its product with respect to the others, is computed as follows:

[3] $MS_j^{pot}=\frac{I_j}{\sum I}$,

where $I_j=\left(1+\frac{1}{P_j}\right)^{a_P} Y_j$ is a competitiveness index for firm j and $\sum I$ is the sum of the values of I_j for all the m firms of the final sector.

Taking into account the size of final demand and the potential market share of the firm, the output produced by each final firm j for each period t is given by:

[4] $Q_j=\left[(1-\zeta) MS_j^{pot}+\zeta \frac{Prd_j}{\sum Prd}\right]*D$,

The meaning of the expression [4] can be explained as follows. The quantity produced by the firm j is a fraction of total demand, determined by two distinct factors, whose weight is regulated by the parameter $0\leqslant\zeta\leqslant1$: i) the comparative performance of the product supplied by the firm, represented by MS_j^{pot} ; ii) the market position of the firm, represented by the ratio $Prd_j/\sum Prd$. We suppose, in other terms, that consumers can be characterized by a higher or lower level of inertia, i. e. a stronger or weaker ability to recognize the improvements occurring in the performance of the products supplied by each firm, and a corresponding weaker or stronger bias towards products supplied by firms with a stronger market position, irrespective of their relative quality and price. Once Q_j is computed, it is possible to determine the final market share for each firm in the period t simply as:

[5] $MS_j=\frac{Q_j}{\sum Q}$.

3.2. Suppliers

Firms in the final sector are divided in two subpopulations which differ for the productive pattern they adopt. For each component c_i , each firm has two options: i) to produce it directly; ii) to buy it on the market from one (and only one) of the firms operating in the n intermediate sectors. In the first case the quality of that component is determined by the innovation process that occurs inside the final firm. In the second case the final firm includes in its final product the quality of the component bought on the market, i. e.:

[6] $x_{ij}=y_{ik}$

for each final firm j which buys the component c_i from the supplier k operating in the *i*-th intermediate sector.

The selection of suppliers by final firms occurs on the basis of the «perceived quality» of their components and of the price at which the former wish to sell. With regard to the first point, final firms have an imperfect ability to read the real quality of the components produced by the potential suppliers [Valente 1999]. Perceived quality $\overset{\circ}{y}_i$ is given by:

[7] $\overset{\circ}{y}_i \sim N(y_i, \sigma_{\overset{\circ}{y}_i})$,

where $\sigma_{\overset{\circ}{y}_i}=(1-MaxLearn)*DevLearn*y_i$ [13]. If the selection is made on the basis of perceived quality, the final firm must nevertheless compete on the basis of the real quality of the components it has bought. Consequently firms can make wrong choices with a probability which is higher when the ability to evaluate ex ante the quality of components is lower, i. e. when $\sigma_{\overset{\circ}{y}_i}$ is higher.

We can now consider how the price of the component c_i enters in the selection procedure of the supplier. On the one side final firms compare the price offered by the potential supplier (p_{c_i}) with the average price of suppliers in the previous period ($\bar{p}_{c_i}[t-1]$), while on the other side they compare the perceived quality of the potential supplier ($\overset{\circ}{y}_i$) with the average perceived quality in the previous period ($\bar{\overset{\circ}{y}}_i[t-1]$). Then they choose that supplier only if the expected benefits on quality exceed or at least equate the eventual additional costs:

[8] $$\frac{(\overset{\circ}{y}_i[t]-\bar{\overset{\circ}{y}}_i[t-1])}{\bar{\overset{\circ}{y}}_i[t-1]} \geqslant \frac{(p_{c_i}[t]-\bar{p}_{c_i}[t-1])}{\bar{p}_{c_i}[t-1]}$$

A second condition to be satisfied is that the potential supplier must have enough productive capacity available to satisfy the needs of the client firm, i. e. $Prd_k^{av} \geqslant Prd_j$. This condition limits the number of clients a supplier can serve, and can eventually force final firms to a second best choice with respect to quality. Once the first choice of one supplier is made at t_0 , final firms can change supplier in each of the subsequent periods comparing price and quality of the other potential suppliers with those of the actual one. Also in this case the new supplier is selected only if it yields a quality increase which is at least equal to the eventual cost increase and if it has enough productive capacity to satisfy the demand, but this choice entails additional uncertainty, because final firms can read with no distortions only the quality of their actual suppliers, while that of the potential

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[13] On the basis of this expression, the average distance of perceived and real quality increases in direct proportion to the value of the parameter $DevLearn$, equal for all firms, with $0<DevLearn<1$; and in inverse proportion to the value of $MaxLearn$.

alternative is known only imperfectly, according to the expression [7]. For this reason we introduce three additional conditions which aim to capture the fact that generally final firms would be unwilling to change supplier, even if there is an expected benefit, if the latter is not large enough to justify the risk of lowering the performance of the firms as a consequence of a wrong choice. More in detail the conditions to be satisfied are the following:

1. the candidate h must ensure at least an apparent quality improvement over the actual supplier k : $\mathring{y}_{ih}[t] > y_{ik}[t]$;
2. this improvement must be relevant: to obtain a non arbitrary threshold we assume that this condition is satisfied if $\frac{(\mathring{y}_{ih}[t] - y_{ik}[t])}{y_{ik}[t]} > RND$, where RND is a random number extracted from a uniform distribution in the interval $\{0,1\}$ ($RND \sim U(0,1)$);
3. the competitive performance of the firm is not positive: looking at the difference between the market share of the leader firm l and that of a final firm j ($\Delta MS = MS_l - MS_j$), the firm j will search a new supplier if $\Delta MS > RND$, with RND defined as above.

We must note that, since the three conditions are stochastic, final firms will generally make different decisions in similar conditions. If the three conditions are jointly satisfied, the selection process of a new supplier goes through the following steps:

1. candidates are ordered for decreasing values of $\mathring{y}_i$;
2. following this order, the client firm search the first supplier with available productive capacity;
3. the client firm reads the price offered by the candidate supplier;
4. if the price offered by the candidate supplier is lower than that of the actual supplier, the candidate is selected and the search process is concluded;
5. if the price is higher but the expected improvement in quality is higher or equal to the price increase, the candidate is selected and the search process is concluded;
6. if neither of the two conditions at step 4) or 5) are satisfied, the client firms asks the candidate supplier to specify a new price, lower than the first one;
7. if with the new price the candidate doesn't satisfy at least one of the conditions specified at steps 4) or 5), the candidate is rejected and the next one is examined;
8. if none of the candidates satisfies at least one of the conditions at step 4) or 5) the client firm retains the old supplier.

3.3. Costs, investments and profit

We have explained above how the price enters in the market competition of the final firms. On its part the price offered by the final firm j is determined by the unitary variable costs:

[9] $P_j = c_{u_j}(1 + mkp_j)$,

where $mkp_j > 0$ is the mark-up of firm j and c_{u_j} is the unitary cost of the same firm. The value of the mark-up can be controlled by firms, above a minimum threshold equal for all of them. In particular we suppose that firms can increase their mark-up when cost reductions due to technical progress occur, in order to appropriate of (at least) some of the potential benefits for consumers. If $c_{u_j}[t+1] < c_{u_j}[t]$ the new mark-up is computed as follows:

[10] $$mkp_j[t+1] = \left[mkp_j[t] + \frac{mkp_j[t] * c_{u_j}[t] - \Delta c_{u_j}}{c_{u_j}[t+1]} \right] / 2$$

where $\Delta c_{u_j} = c_{u_j}[t+1] - c_{u_j}[t]$, and the second term of the sum on the right hand side of the expression [10] is the mark-up value which should be necessary to keep unchanged the price after a cost reduction. The increase of the mark-up is constrained by an additional condition, which is fulfilled with the following probability:

[11] $Pr[mkp_j[t+1] > mkp_j[t]] = Pr[RND > \Delta MS]$

with RND and ΔMS defined as above. In other terms the tendency to increase mark-ups will be stronger the smaller is the distance of one firm from the leader firm in terms of market shares, and this tendency will be thus highest when the firm is the market leader itself. In a dynamical competitive environment this kind of opportunistic behavior can be hidden to consumers by quality improvements and price reductions which can be higher than those of the competitors, because they are financed by higher volumes of resources (as we shall see more in detail below). It is consequently not irrational for firms to pursue such a behavior, if they can hide to market and competitors their true production costs when they have a dominant position, and if they can revert their strategy, lowering their own prices, when they perceive relevant negative signals from their sales. The only constrain imposed by the model on this regard is that leader firms are not perfectly reactive to these signals. On the other hand, the behavior of the followers is opposite to that of the leader: they will tend to reduce their mark-up down to the minimum threshold to re-gain market shares through price reduction. The probability that the reduction of mark-up occurs depends from a complementary condition with respect to the expression [11]:

[12] $Pr[mkp_j[t+1] < mkp_j[t]] = Pr[RND < \Delta MS]$.

In other terms this condition is fulfilled with a higher frequency for those firms which have a wider market share gap from the leader.

The second factor used to compute price is unitary cost, which is calculated differently depending on the productive pattern of the firm:

$$[13] \quad c_{u_j} = \sum_{i=1}^{n} \alpha_i p_{c_i} + \sum_{i=1}^{n} (1-\alpha_i)\left(1+\frac{1}{t_j}\right) \quad ,$$

where $\alpha_i = 1$ if the *i*-th component is bought from the market, and $\alpha_i = 0$ if it is produced internally; p_{c_i} is the price offered by the supplier for the component i ; t_j is an index of the technological level achieved by the firm in the production process, whose computation is specified below. Regarding the costs of external supplies, we must precise that the prices offered by suppliers is computed in the same way as in the expression [9], i. e. it depends from the unitary variable costs and mark-up of the suppliers. The comparative (static) convenience of «internalized» versus «externalized» production depends from the relative value of the terms appearing in the expression [13]. In particular this convenience is determined by the value of the price offered by the supplier k , $p_{c_i k} = (1+\frac{1}{t_k})(1+mkp_k)$, compared with the efficiency achieved by the process technology of the final firm j , $1+\frac{1}{t_j}$. In order to be $p_{c_i k} \leqslant 1+\frac{1}{t_j}$, we must have that t_k is higher than t_j by a difference which must be sufficient to compensate the presence of the mark-up of the supplier, i. e. $t_j \leqslant \frac{1}{(1+1/t_k)*(1+mkp_k)-1}$ or alternatively $t_k \geqslant \frac{1}{(1+1/t_j)/(1+mkp_k)-1}$.

The unitary variable cost enters of course also in the computation of the total costs of the firm, which are obtained as follows:

$$[14] \quad C_j = c_{u_j} Q_j + c_{fix_j}$$

with Q_j defined by the expression [4] and c_{fix_j} which stands for the fixed costs of the firm:

$$[15] \quad c_{fix_j} = Prd_j * \left(\sum_{i=1}^{n} \alpha_i c_{trans} + \sum_{i=1}^{n} (1-\alpha_i) c_{tec} + c_{gen} \right)$$

with α_i and Prd_j defined as above, and c_{trans} , c_{tec} and c_{gen} equal parameters for all the firms. These parameters represent respectively the fixed costs deriving from transactions with suppliers, those deriving from the technological characteristics of production, determining the minimum scale of plants in the industry, and the general administrative costs. All these costs are fixed in the sense that they don't depend from the output produced in each period, but they change in proportion with the productive capacity of the firm (Prd_j). Further we have that $c_{trans} < c_{tec}$, so that firms relying on external suppliers have lower fixed costs with respect to firms which produce components internally.

Once total costs are computed, profits of the firm j are given by:

[16] $\pi_j = R_j - Inv_j$,

where $R_j = Q_j * P_j - C_j$. Investments Inv_j are on their part calculated as follows:

$$[17] \quad Inv_j = \begin{cases} \dfrac{S_{invest_j} * R_j}{n} * \sum_{i=1}^{n} (1-\alpha_i) & \text{, if } R_j > 0 \\ \dfrac{S_{invest_j} * \pi_{cum_j} / s}{n} * \sum_{i=1}^{n} (1-\alpha_i) & \text{, if } R_j \leq 0 \end{cases}$$

where $0{,}1 < S_{invest_j} < 0{,}9$ and $\pi_{cum_j} = \sum_{t=0}^{t=s} \pi_{jt}$ are cumulated profits until the period s by the firm j . Since we don't introduce credit in the model, we suppose that firms, when they are in difficulty ($R_j \leqslant 0$), can draw on a part of the funds cumulated in the past[14] in order to finance their own investments. In analogy with the expression [13], the weight of the single components on investments is equal, and investments are lower for firms which rely on external suppliers, for which $\alpha_i = 1$. The share of revenues to be used for investments S_{invest} is increased if a stochastic condition is satisfied according to a probability which is similar to that of the expression [11]:

[18] $Pr[S_{invest_j}[t+1] > invest_j[t]] = Pr[RND < \Delta MS]$

As a consequence of this expression, the follower firm will tend to increase its share of investments in order to get more competitive, up to the maximum threshold. The behavior of the leader firm is also in this case partially myopic, as it holds, in analogy with the expression [12], that:

[19] $Pr[S_{invest_j}[t+1] < invest_j[t]] = Pr[RND > \Delta MS]$

The leader firms will tend to reduce their investments share down to the minimum threshold, as they can rely generally on a higher amount of investments, because of their higher amount of sales. Once their overall amount is computed, funds for investments are divided in two separate chapters: i) technological innovation; ii) increase of productive capacity. The investments in innovation are computed simply as follows:

[20] $Inv_{inn_j} = \gamma_j Inv_j$

with $0 < \gamma_j < 1$, and investments in productive capacity are determined residually:

[21] $Inv_{cap_j} = Inv_j - Inv_{inn_j}$

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[14] The rule we adopt here is really oversimplified, i. e. we suppose that the investments represent a share of the average profits of the previous s periods. The funds employed for investments are consequently subtracted from the cumulated profits: if $R_{jt} \leqslant 0$, $\pi_{cum_j}[s+1] = \pi_{cum_j}[s] - \pi_{cum_j}[s] / s$, and if eventually $\pi_{cum_j} < 0$ the firm is removed from the market.

Each firm is thus characterized by a different tendency to invest in innovation and consequently in productive capacity, but this tendency is also conditioned by the prevailing strategies on the market:

[22] $\gamma_j = [\phi_j + \sum(\phi * MS)]/2$

where ϕ_j represents the tendency of the single firm to invest in innovation and $\sum(\phi * MS)$ is the weighted average tendency for final firms. In other terms, firms will tend to adapt to the winning strategy (i. e. that associated with the larger market shares), without being able to eliminate completely some irreducible differences which can be explained as a consequence of different organizational capabilities and cultures.

As we shall see below, the higher or lower investments in innovation condition, although not linearly, the speed of technological innovation inside firms. For the moment we focus on the effects of investments in productive capacity. In general $Prd_j[t+1] > Prd_j[t]$ when $\sum_{t=0}^{|t|} Inv_{cap_{jt}} \geq Res_{Cap_j}$, where $|t|$ represents the number of periods passed since the last increase of productive capacity, and Res_{Cap_j} are the resources needed to increase it, i. e. $Res_{Cap_j} = \left(\sum_{i=1}^{n} \alpha_i c_{trans} + \sum_{i=1}^{n} (1-\alpha_i) c_{tec} + c_{gen} \right) * \Gamma$, with $\Gamma > 1$. The latter parameter governs the ratio between the resources needed to increase productive capacity (i. e. those necessary to build a new plant) and the fixed costs determined by this increase (i. e. those necessary to keep efficient the plant), represented by the first term of the product.

3.4. The dynamics of innovation

In our model technological innovation regards both the improvement of products and of production processes. The mechanism through which innovations are adopted and developed in the two cases is the same, even if they are independent processes, which condition in different ways the competitive dynamics of the model. Regarding the former, product innovation means an increase in the quality of the components, which on its part is reflected in the quality of the final product and in the competitiveness of the firms, as specified in expressions [1] and [4]. Regarding the latter, process innovation is represented by a simple index of the efficiency level gained by the single firm, which determines production costs, as specified in the expression [13].

Following the representation of Sah and Stiglitz [1986] described above, the dynamics of innovation is specified through the adoption process of ideas or inventions, which is structured in two steps: i) the first one involves the selection of ideas, performed through a stochastic process; ii) the second one involves the application of ideas, or the real innovation, which gives to the firm a (negative or positive) payoff. During the first step ideas (for example, in the case of product innovation) are selected according to the probability given by the following formula:

[23] $$Pr_{j[x_i[t+1]=x_i[t]+\Delta x_i]} = Pr_j\left[RND < \left\{(1-F_{rate}) * \left[e^{[(x_i[t]+\Delta x_i)-E(x_i[t+1])]}\right]^{\lambda}\right\}^2\right]$$

where Δx_i can be higher or lower (even negative) in the case respectively of a «good» or «bad» idea; the parameter $0 < F_{rate} < 1$ represents the failure rate of innovative ideas, giving an indirect measure of the difficulty the firm faces in developing and improving the quality x_i of the component c_i ; and the parameter $\lambda > 0$ can be used to enlarge or reduce the strength of the negative or positive signal that we suppose firms receive from a «good» or «bad» idea respectively. The overall meaning of the formula is the following: the probability to accept an idea is equal to the expected proportion of good ideas (represented by the first term of the multiplication on the right hand side of the expression [23]), increased (or lowered) by the positive (negative) signal that the firm receives from the particular idea it is examining (given by the second term of the multiplication). We can reproduce in this way the rationally more conservative behavior of firms confronted with a higher uncertainty, due to unforeseeable developments regarding the components undergoing innovation. This tendency is partly contrasted by the fact that a high level of uncertainty lowers the expected value of ideas, namely:

[24] $$E(x_i[t+1]) = F_{rate} * (x_i[t] + \Delta x_i^{fall}) + (1-F_{rate}) * (x_i[t] + \Delta x_i^{succ})$$

When the failure rate is high, all other things being equal, the difference between the value of a good idea and the expected value of one idea, and thus the second term of the multiplication in the expression [23], is increased. We reproduce in this way the fact that, in a situation of higher difficulty to develop a product, the value of the few good ideas is sensibly higher.

The parameter F_{rate} is a proxy that tries to represent, although in a very simplified way, the presence of high levels of complexity in the innovation process. With regard to the development of the component x_i , a value of F_{rate} near the unity stands for a high level of uncontrolled interdependencies between the development of that component and that of the other ones that compose the final product. On the contrary a level of F_{rate} near to zero stands for the opposite situation, in which the single component can be modified without fearing for unforeseeable negative feedbacks, stemming from uncontrolled interactions with the others.

Before we can examine the second step of the innovation process, we must make an additional comment. As we shall see in the next paragraph, the probability to select an idea is lower for final firms with respect to intermediate firms (i. e. firms operating in the intermediate sectors), and thus final firms accept on the average a lower number of innovative ideas with respect to suppliers. This different tendency implies comparative advantages or disadvantages depending on the parameters that regulate the innovation process, with a particular regard to the failure rate and to the ratio between positive and negative payoffs. When the probability to fail is very high and/or bad ideas yield a relevant damage compared to the benefits of good ideas, a more conservative strategy can be more effective.

During the second step of the innovation process, if the idea has been selected according to the probability specified in the expression [23], it yields to the firm its positive or negative payoff. The evolution of the positive payoff is non linear in the model, and is represented by a logistic equation:

[25] $$x_i[t+1] = \frac{MaxTec}{1+(MaxTec/x_i[t]-1)*e^{-TecR_j}}$$

while the adoption of a bad idea yields a temporary damage, which can be interpreted as a «false step» in the development cycle of one product, represented in a simpler way as:

[26] $$x_i[t+1] = \psi\, x_i[t]$$

with $\psi \leqslant 1$, fixed in such a way that the value of the expression [26] is strictly lower than that of the expression [25][15]. The logistic shape of the positive payoff has been adopted for two reasons: i) it reproduces the existence of limits in technological progress; ii) it helps to identify different phases in the development cycle of one product (namely, an initial phase, with limited returns and slow progress; an «explosion» phase, with increasing exponential returns; a maturity phase, with progressively decreasing returns), whose existence has found confirmation in many cases; iii) it allows to introduce in a simple way a variable ($TecR_j$), which regulates differently for each firm the speed of development of its product depending on the value of its investments. In addition, the adoption of a bounded increase curve for the variable referring to technological progress is a guarantee that the simulations of the model don't produce explosive dynamics. In fact, considering how the model is built, in the absence of improvements in the quality of the components (variable x_i) or in the efficiency of the production process (variable t_j), no competitive dynamics is possible, and the market share of firms cannot change from the initial values.

A limit of the model can be identified in the supposition that the difficulty to improve components within a given architecture (measured by F_{rate}) is independent from their development cycle, where the latter (as from the expression [25]) concerns the non linear dynamics of the returns from the innovative efforts of firms. This supposition is clearly unrealistic, as in the models that explore the links between the diffusion of new products and the emergence of «dominant designs» [Dosi 1982; Tushman and Rosenkopf 1992] it has been often highlighted that the latter, defining a

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[15] This condition is necessary in order to guarantee the existence of technological progress, even the slower, in situations of highest uncertainty, in which failed innovations are the majority.

common background for different firms, not only stimulates a higher innovation rate (i. e., in our model, a lower value of F_{rate}), but also increases the returns of innovation. At any rate, analysing the interdependence of these two variables is not necessary for our present goals, while their separation helps to obtain more general results, because we can examine the combined effects of the two variables over the entire range of their possible values, without adding specific hypotheses on the way they connect each other.

The last point to be examined is the role of investments in the innovation process. Given two firms i and j , $TecR_i > TecR_j$ if and only if $Inv_{inn_i} > \overline{Inv}_{inn}$ and $Inv_{inn_j} < \overline{Inv}_{inn}$, where $\overline{Inv}_{inn}$ is the average investment in innovation. If n and m are respectively the number of final firms and of suppliers, $\overline{Inv}_{inn}$ is given by

$$[27] \quad \overline{Inv}_{inn} = \frac{\sum_{i=1}^{n} Inv_{inn_i} + \sum_{j=1}^{m} Inv_{inn_j}}{n+m}$$

The average investment in innovation is computed summing together final firms and suppliers, because both compete to innovate the same components. Given the value of the investments of one firm compared to the average, the rate of technological progress is represented by a random variable extracted by two different uniform distributions: $TecR_1 \sim U(a,b)$ (if $Inv_{inn} > \overline{Inv}_{inn}$) and $TecR_2 \sim U(b,c)$ (if $Inv_{inn} < \overline{Inv}_{inn}$), with $1 > a > b > c$. As specified by the expression [25], the real rate is equal to $TecR_1$ or $TecR_2$ only if the firm selects a good idea, while if the idea is bad the firm faces a regression of its technological level. In addition there is also a third possibility: when the firm doesn't select any idea[16], quality increases nonetheless according to the expression [25], through a small incremental improvement that takes place at a rate $TecR_3 \sim U(c,0)$, lower than the one of the firms that introduce a successful innovation. This possibility is introduced in order to avoid that firms go through a continuous decrease of their technological levels when they are confronted with high levels of difficulty in improving products and processes.

3.5. The behavior of suppliers

Firms operating in intermediate sectors follow in general the same rules as those operating in the final sector, so that we need only to specify the equations that work differently in the former from the latter. As we have already mentioned, the first difference between final and intermediate firms lies in the higher probability for the latter to select an innovative idea regarding product or process

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[16] This possibility occurs when $RND \geqslant \left\{(1 - F_{rate}) * \left[e^{[(x_i[t] + \Delta x_i) - E(x_i[t+1])]}\right]^{\lambda}\right\}^2$ as specified by the expression [23].

innovation. This probability is given for the intermediate firm k by:

$$[28] \quad Pr_{k}[x_i[t+1]=x_i[t]+\Delta x_i]=Pr_{k}\left[RND<\left\{(1-F_{rate})*\left[e^{[(x_i[t]+\Delta x_i)-E(x_i[t+1])]}\right]^{\lambda}\right\}\right]$$

with $Pr_k > Pr_j$, as it is clear confronting the expressions [28] and [23]. The basic intuition behind this difference is that «verticalized» production (represented in our case from the final firm that produce internally its components) is linked to a more integrated decisional and research architecture, realized through the coordination of the decisional units of the firm. An architecture of this kind is characterized by a more «conservative» innovation strategy, because many connections are considered together before a decision is taken. On the opposite side intermediate firms (and consequently final firms that buy their products) tend to accept a comparatively higher number of innovative ideas. Their strategy can be explained through the notion of modular innovation: this firms innovate on the components they produce independently both from their competitor and from the firms that operate in the other intermediate sectors, and thus collectively tend to explore directly a wider portion of the innovative landscape. The effectiveness of this strategy depends on the level of interdependence among different components. The measure for complexity adopted in the expressions [23] and [28] (F_{rate}) should be interpreted as a simplified representation exactly of the level of interdependence of the functional structure of the different components that build the final product. As we shall see better below, if this interdependence exceeds a critical threshold, the gains derived from an extended research among the alternative innovative combinations, made in parallel by a plurality of competing agents, can be outperformed by those of a more conservative strategy.

The second difference between intermediate and final firms is that the former face a «real» demand, i. e. the behavior of their clients is micro-specified in the model. We thus have a different, and simpler, specification of the output produced by each supplier:

$$[29] \quad Q_k=\sum_{j=1}^{p} Q_j$$

The quantity of output produced by the intermediate firm k is equal to the sum of the quantities produced for its p clients. Here we adopt for simplicity a fixed proportion of 1:1 between component and final product: to produce a unit of final product we need a unit of that component. The price at which intermediate firms sell their product is in the first place equal for all their clients, and it is computed in the same way as in the expression [9], including also the possibility to change the mark-up according to the rule specified in the expression [10]. The only difference becomes apparent when the client firms ask a lower price (see above). In that case intermediate firms compute the following new price:

[30] $P_{res_k}=\begin{cases} P_k & \text{, if } RND>\Delta MS \\ \bar{P}[t-1] & \text{, if } RND\leq\Delta MS \end{cases}$

with RND and ΔMS defined as above. In other terms, suppliers tend to refuse to lower their price if they are in a position of leadership in their market, i. e. if ΔMS is small. We leave thus open the opportunity of a replacement of the leader firms by new entrants and followers, both readier to catch market opportunities reducing their prices in order to gain (or not to loose) a client.

Profits of the intermediate firms are computed as in the expression [16], but have to take into account the eventual price differences among different clients:

[31] $\pi_k=\sum_{j=1}^{p}(P_j*Q_j)-C_{tot_k}-Inv_k$

where P_j and Q_j are respectively the price offered to and the quantities produced for the single client j . Regarding the costs, the only difference lies in the fixed ones, which are determined in a slightly different way from the expression [15]:

[32] $c_{fix_k}=Prd_k*\left(p_k*c_{trans}+c_{tec}+c_{gen}\right)$,

i. e. with transaction costs proportional to the number p of clients, and Prd_k which represents the productive capacity of the firm, computed in the same way as for final firms.

The last point to consider is the entry mechanism for new firms (while the exit dynamics is the same of final firms, i. e. the intermediate firm k is removed from the market if $\pi_{cum_k}<0$). The entrance of new firms occurs according to a Poisson process[17], allowing to test the differences produced by different birth rates in the intermediate sectors on the competitive performance of the final firms. New firms are initialized with values of the fundamental variables (product quality, technological level, productive capacity, mark-up, investments share, cumulated profits) equal to the weighted average of the same variables referred to the firms which existed on the market in the previous period. In this way new firms don't locate themselves on the «frontier» of their sector, which we assume is hardly reachable by a new entrant, but rather in line with the «state of the art» of that sector.

4. Setting the stage: competition of «verticalized» firms

In order to expose the fundamental competitive dynamics of the model we consider firstly the case of firms with the same production pattern. Consequently, even if we are now going to analyze the

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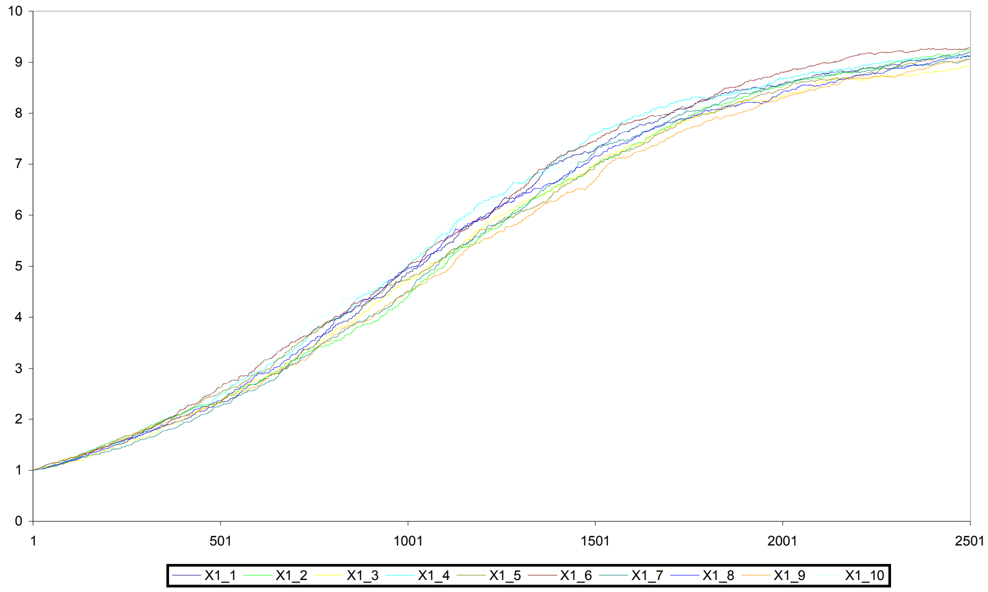
[17] Namely, the probability of n new entries obeys to the following limiting distribution: $Pr_{[Entry=n]}=(\lambda t)^n*(e^{-\lambda t}/n!)$ where t is the number of periods under consideration, n represents the number of new entries of which we wish to compute the probability and λ represents the intensity of the process, i. e. the expected number of entries in each period. The latter variable is used in our model to allow a higher or lower number of entries.

behavior of «verticalized» firms, our attention will be focused on those variables that characterize the behavior of firms independently from the advantages or disadvantages of a «verticalized» versus «deverticalized» production pattern, which will be examined subsequently. In the first place we examine a competitive context in which there is not the «inertial» effect given by the accumulation of productive capacity (see the expression [4]). In graph 1 we draw an illustrative path of quality improvement for one component, determined by the innovation dynamics in a simulation computed starting from a set of given initial conditions[18] on a sample of 10 «verticalized» firms with identical initial values for all the (lagged) endogenous variables and parameters. The logistic shape of the improvements is the one expected from the expression [25], with the value of the quality of the component c_1 that approaches to the maximum ($MaxTec=10$) at the end of the simulated periods.

Graph 1. An example of the dynamics of quality improvement for the component c₁



The differences among single firms depend both from the stochastic nature of the innovative process (see the expression [23]) and from the influence of investments (which are different because of the different amount of profits depending on the respective market shares) on the speed of the innovation process (see the expression [25]). The quality improvement of the component c_1 and the progressive price reduction allowed by the improvement of the production process (see the expression [13]) determine the expected growth of demand and of total output, which coincide by

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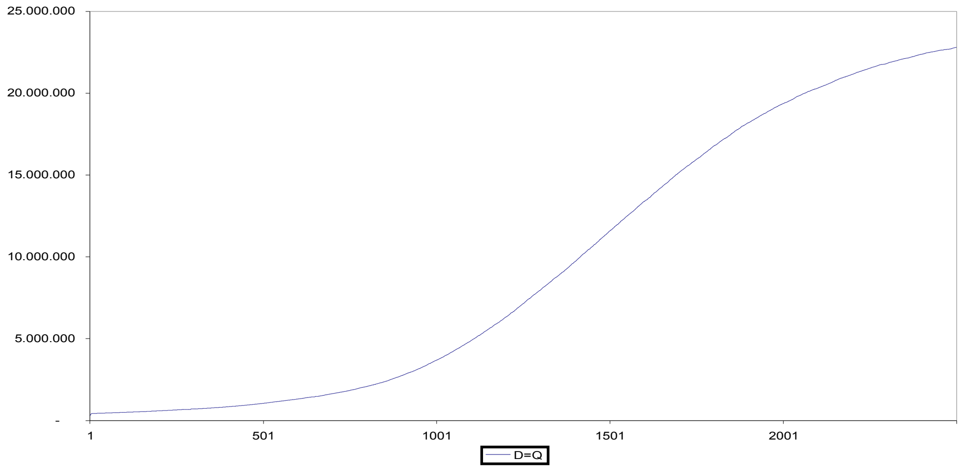
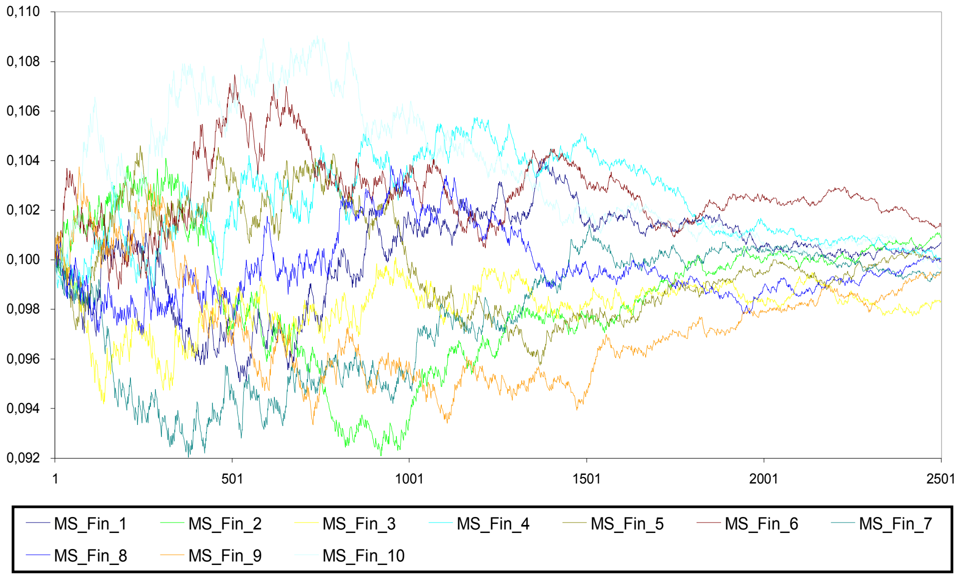
[18] The values for which the simulations have been computed, except where differently indicated, are the following: $t_j=1$, $x_i=1$, $mkp_j=0{,}3$, $c_{trans}=1$, $c_{tec}=100$, $c_{gen}=10$, $F_{rate}=0{,}3$, $MaxTec=10$. With regard to demand, we have assumed equal elasticities with value 1 (see expression [2]). In the case of t_j , x_i and mkp_j , which are endogenous lagged variables of the model, the values just indicated are used to initialize the simulations. With regard to x_i , to simplify the analysis the possibility of improvement is introduced only for c_1, while c_2 and c_3 will be always considered as fixed parameters with a value equal to 1.

definition (see expressions [2] and [4]). The increase of demand and output, as we can see from graph 2, follows a logistic curve which parallels that of technological progress.

Graph 2. An example of the evolution of final demand (D) and of total output of the industry (Q) as a consequence of innovation (D=Q by definition)



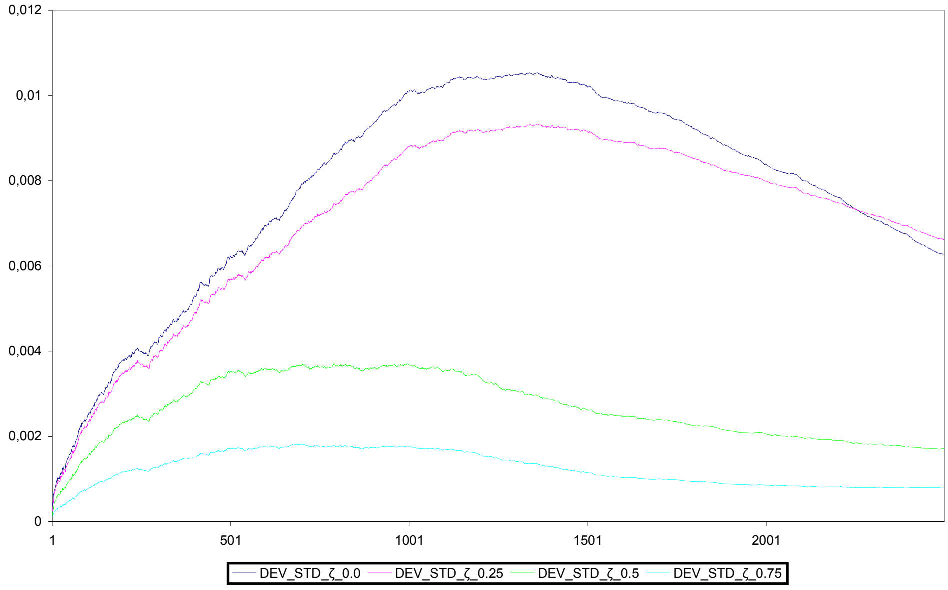
Graph 3. An example of market shares' dynamics (without effects of cumulated productive capacity)



The dynamics of technological progress conditions that of the market shares, which are in the first place determined by the quality of the final product alone, i. e. for $\zeta = 0$ (see expression [4]). A simple comparison of graph 1 with graph 3, where it is represented the market shares dynamics of the same sample of firms, highlights a strict correspondence between quality and competitive performance. The leader firms (as, for example, the firms n° 6 in the final periods of the simulation) are also those with a better performance for the component c_1[19], while those with the widest gap in quality (as the firm n° 9) are also those with the lowest market shares. Further, the distribution of the market shares increases when the qualitative performance diverge (as it is the case until the 1.000th period of the simulation), while it tend to an even distribution when quality differences lower (as in the periods between the 1.000th and 2.500th time step of the simulation).

The effect of the introduction of an «inertial» effect due to the accumulation of productive capacity can be easily verified by graph 4, which represents the average values, for ten simulations repeated for different values of ζ , of the standard deviation of the market share of the sample of ten firms already considered. The higher the value of ζ , the more the differences between market shares of firms lower, i. e. we get a «smoothing» of the competitive dynamics, even if market shares are still proportional to respective qualities. Further, when quality differences between products decrease, as it is the case in the final periods of the simulations, a stronger weight of cumulated productive capacity determines the persistence of stronger differences between market shares than in the case where competition is determined solely by quality, as we can see from the fact that the standard deviation for $\zeta = 0$ tends to decrease more quickly in the final periods when compared to the other cases.

*Graph 4. Connection between the weight of cumulated productive capacity in competition and the distribution of market shares**



* Average values of the standard deviation of market shares computed for 10 firms and for 10 simulations repeated for each value of ζ .

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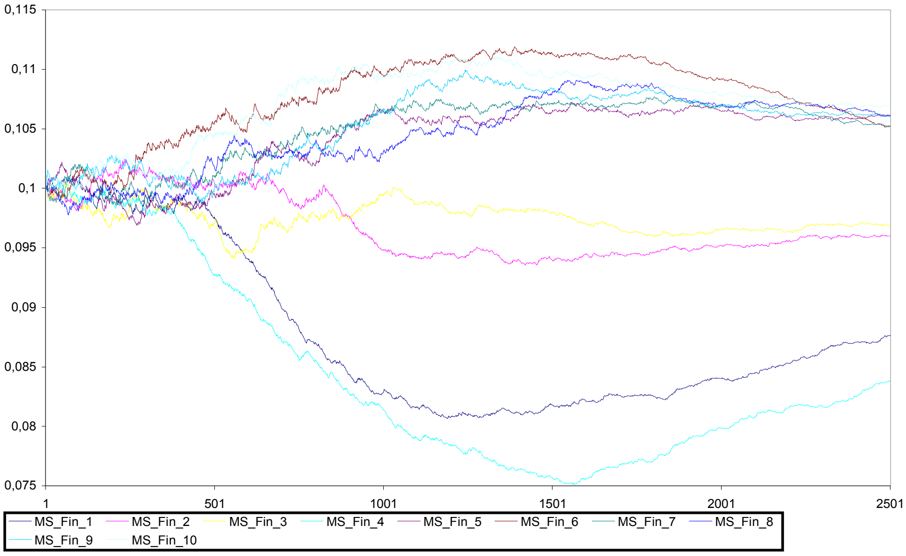
[19] We must remember that for sake of simplicity the performance of the other components is kept fixed at the value of 1.

Now we can verify what happens when we introduce the possibility for firms to change mark-ups, as it is described by expressions [10] – [12]. This opportunity allows firms that posses relatively efficient production technologies (see the expression [9]) to widen their profit margins as an alternative to a price reduction. If a decision of this kind has obvious negative effects on the price competitiveness of the firm, on the other hand firms can compensate this disadvantage assigning to investments in innovation a greater part of the additional profit they gain from this decision. In the latter case they can defend and even strengthen their market leadership simply by producing better products than their competitors. An example of a dynamics of this kind is depicted in graph 5, that draws the evolution of market shares when mark-ups are variable, for a simulation computed on 10 firms with the same characteristics as those observed up to this point[20].

Graph 5. An example of market shares' dynamics in presence of variable mark-ups



The dispersion of market shares is much higher than the one recorded above, with some firms cumulating an irrecoverable competitive lag during the simulation. Further we add the apparent paradox for which (as depicted in graph 6) the leading firms are the ones that offer the highest prices. This paradox is explained by graph 7, in which we draw the global product performance for each firm, i. e. the value of the variable I, which takes into account both quality and price, entering directly into the computation of the market shares (see the expression [3]). A visive comparison of graphs 5 and 7 shows that market shares are still proportional to product quality: since price is nothing more than a component of global performance, leading firms occupy their position exactly because they are more able to strengthen the quality of their components, with a positive effect on global performance which is higher

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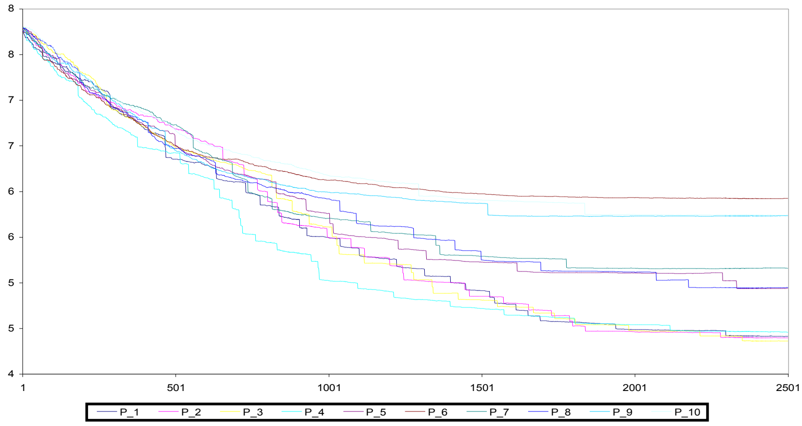
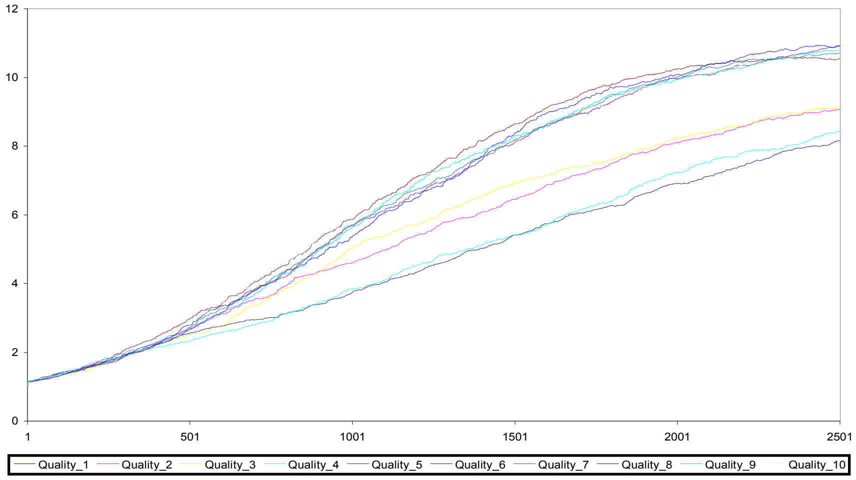
[20] To simplify we assume equal investments strategy, with $\phi = 0{,}2$, and $\zeta = 0{,}5$.

than the negative effect due to the higher prices. In these conditions the initial gap cannot be anymore covered by the followers, because a virtuose cycle of positive effects occurs for those firms that are able to get an initial advantage in process technology or product quality: a higher initial efficiency or quality makes available a higher amount of funds to invest, and these on their part increase both the product quality and productive efficiency, thus yielding an increased amount of resources to invest. Further, the followers cannot fill the gap, even if they increase their tendency to invest, because the bigger returns of leader firms guarantee a bigger amount of investments even when the latter have a lower tendency to invest than the former[21].

Graph 6. An example of price dynamics in presence of variable mark-ups



Graph 7. An example of products' global performance dynamics in presence of variable mark-ups



The effects just described of course depend from the elasticity of demand with respect to the variations of price and quality of the final product (see the expressions [1] – [4]). If we change the

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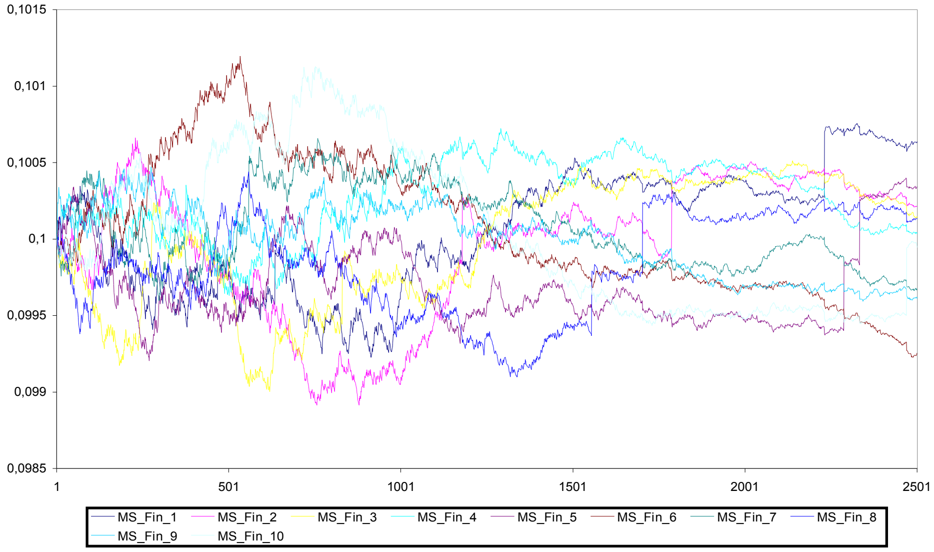
[21] All these effects are even amplified if $\zeta = 0$, as we can easily imagine from our previous arguments.

values of these elasticities we have used up to now ($a_i = a_p = 1$), introducing a higher elasticity for price variations ($a_p = 1{,}5$) and a lower one for variations in quality of c_1 ($a_1 = 0{,}1$), the differences between market shares becomes much smaller, and firms which were lagging behind in the previous case are not anymore so distant from the market leaders (Graph 8). In particular demand conditions, then, a «monopolistic» strategy of the kind depicted above can be no more advantageous[22].

*Graph 8. An example of market shares' dynamics in presence of variable mark-ups and particular conditions of final demand**



* High elasticity to price variations ($a_p = 1{,}5$) and low elasticities to variations in the quality of c_1 ($a_1 = 0{,}1$).

5. Competition of «deverticalized» firms

In the model we can identify three main differences between «verticalized» and «deverticalized» firms: i) the cost structure (see above, par. 3.3); ii) the dynamics of innovation processes, on which we will come back in the next paragraph; iii) the computation of the quality of components, which occurs for the latter through the dynamical relationship with suppliers. In this paragraph we will focus on the latter, with a particular regard to the following topics: i) the different possibility to switch from one supplier to the other in different competitive conditions; ii) the different effects of these changes in connection with a different ability of «deverticalized» firms to «read» in advance the quality of the components supplied by intermediate firms (see the expression [7]).

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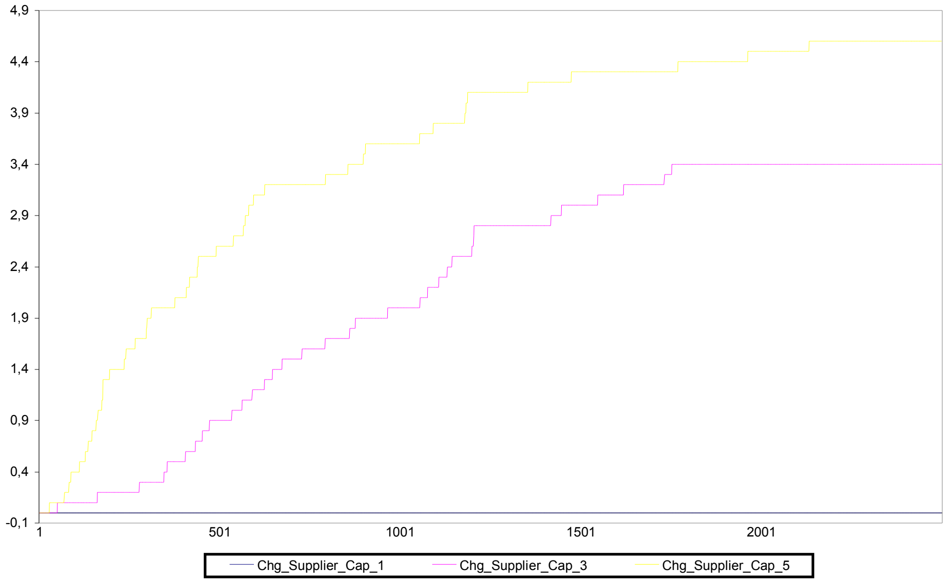
[22] A systematic analysis of the effects produced by different values of the parameters of final demand goes beyond the scope of out actual interests. In the following pages we will refer always to the benchmark values which we have already defined ($a_i = a_p = 1$).

The opportunity for «deverticalized» firm to change their actual supplier is constrained by the existence of other suppliers with available productive capacity in the quantity required to satisfy their own demand (see above, par. 3.2). Graph 9 offers a clear confirmation of the relationship between these two aspects, drawing the cumulated number of switches occurring in a simulation for different initial productive capacities of each supplier of c_1. The graph reproduces average data over 10 simulations repeated on two samples of 10 «deverticalized» firms and 10 suppliers. The meaning of these results is clear: «deverticalized» firms can change supplier only if the initially available productive capacity exceeds demand in a sufficient measure to ensure the permanence of a relevant amount of unused productive capacity throughout the simulated periods. The subsequent increases of productive capacity of suppliers occur in fact as a consequence of an adaptation to the output growth of clients, so that the new productive capacity is immediately absorbed by the demand of the actual clients.

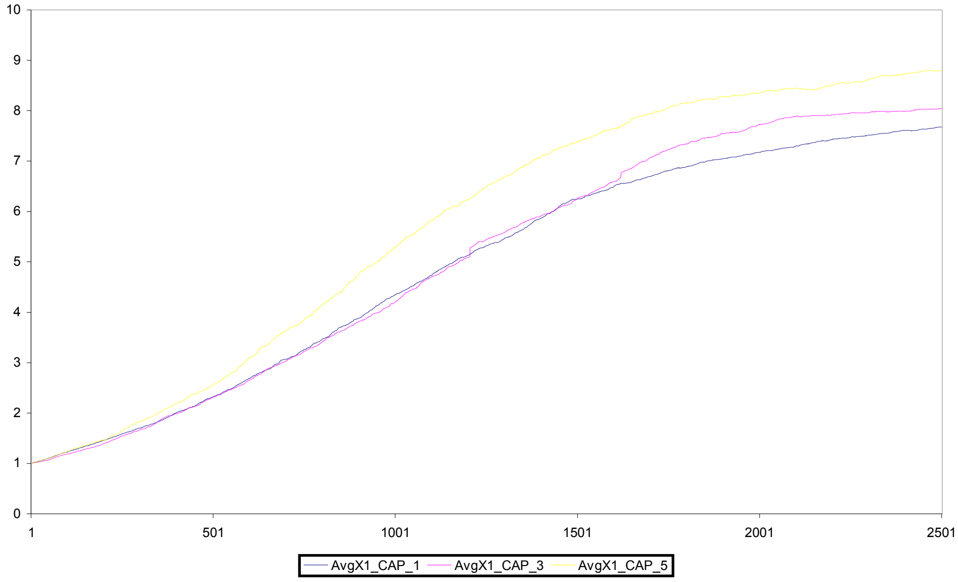
Graph 9. Relationship between the initial productive capacity of suppliers of the component c_1 *and the total number of changes of suppliers performed by «deverticalized» firms**



* Average number of cumulated switches, computed for 10 simulations repeated for each value of the initial productive capacity on a sample of 10 «deverticalized» firms and 10 suppliers .

The positive effects induced by the possibility to change frequently suppliers are depicted in graph 10: the higher the initially available productive capacity (and consequently the fewer the limits in the possibility to change suppliers, choosing those with the better performances), the better the average performance of the components bought by «deverticalized» firms (see the expression [6]). As a consequence, the competitiveness of «deverticalized» firms is strictly dependent from the stronger or weaker development of intermediate sectors. In other terms the model is able to reproduce dynamically the positive relationship between market extension and specialization, which has been very often stressed by economic theory [Young 1928; Stigler 1951], and has gained new attention as a consequence of the most recent trends of industrial dynamics [Langlois 2002b]. From an evolutionary perspective the most interesting fact is that, if we want the «selective virtues» of market to work effectively, we must make a «wasteful» use of available resources. The selection of best suppliers can occur only if these take the risk to anticipate potential demand, and this possibility involves necessarily a congruence in the expectations of many actors, i. e. a condition which is not so simple to achieve. The benefits of competition are less evident for medium values of productive capacity (in our case equal to 3), because the probability to change is in that case lower and the occurrence of a switch becomes more exposed to chance, as we see from the existence of two clear «steps» in the performance curve, that coincide exactly with a change of suppliers.

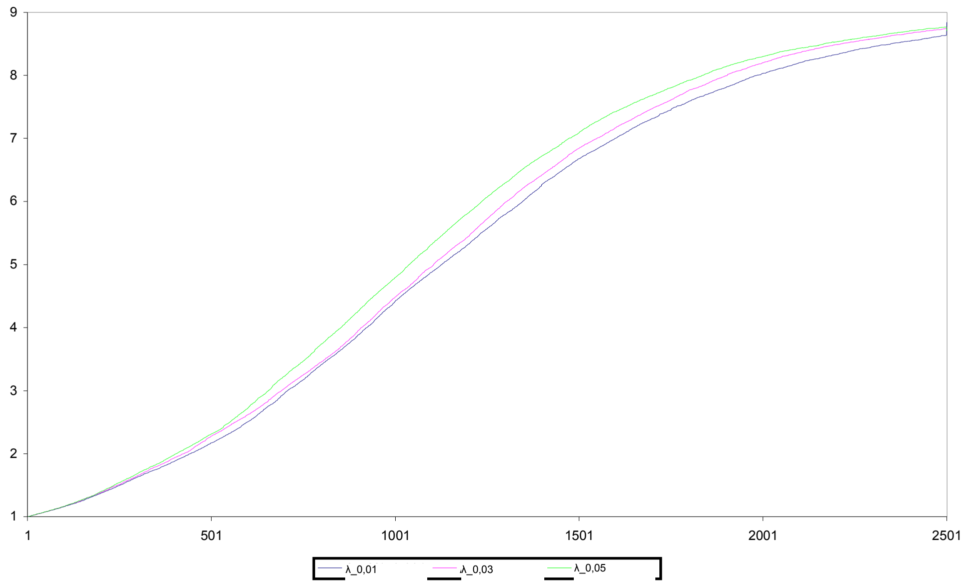
*Graph 10. Relationship between the initial productive capacity of suppliers and the average quality of the components**



* Average values computed for 10 simulations repeated for each value of the initial productive capacity on a sample of 10 «deverticalized» firms and 10 suppliers.

From graph 11 we can see the effect on the average quality of component c_1 of the introduction of an entry mechanism for new suppliers (see above, par. 3.5), evaluated at the same value of initial productive capacity (set equal to 3). When the average number of new entrants per period (regulated by the intensity λ of the Poisson process) increases, the average performance of the components improves as a consequence of the higher number of switches which are possible. This connection highlights the fact that the efficiency of the selection process is not linked to the productive capacity of each single supplier, but rather to the total capacity available on the market[23]. From a comparison of graphs 10 and 11 we note, on the other hand, a more limited effect of a variation of λ with regard to the one produced by an increase of the initially available productive capacity. This difference is caused by the fact that new entrants are initialized with quality values equal to the average, and they are consequently appealing only for a part of «deverticalized» firms (namely those resorting to suppliers with a lower quality than the average). The number of switches is thus significantly lower than the one we could have in case the new entrants would locate themselves on the efficiency and quality «frontier» of their sector.

*Graph 11. Connection between the average performance of components and the number of new entrants in the intermediate sectors**



* Average values computed for 10 simulations repeated for each value of λ (parameter for the intensity of a Poisson process) on a sample of 10 «deverticalized» firms and 10 suppliers.

Finally we can examine the consequences for «deverticalized» firms of a stronger difficulty to gather in advance a good information on the quality yielded by the suppliers. The existence of informational distortions is formalized by the expression [7], and regulated by the parameters $DevLearn$ and $MaxLearn$. In the simulations we have presented above we have assumed $MaxLearn = 0{,}9$ and $DevLearn = 0{,}1$. The effect of an increase in informational distortion is

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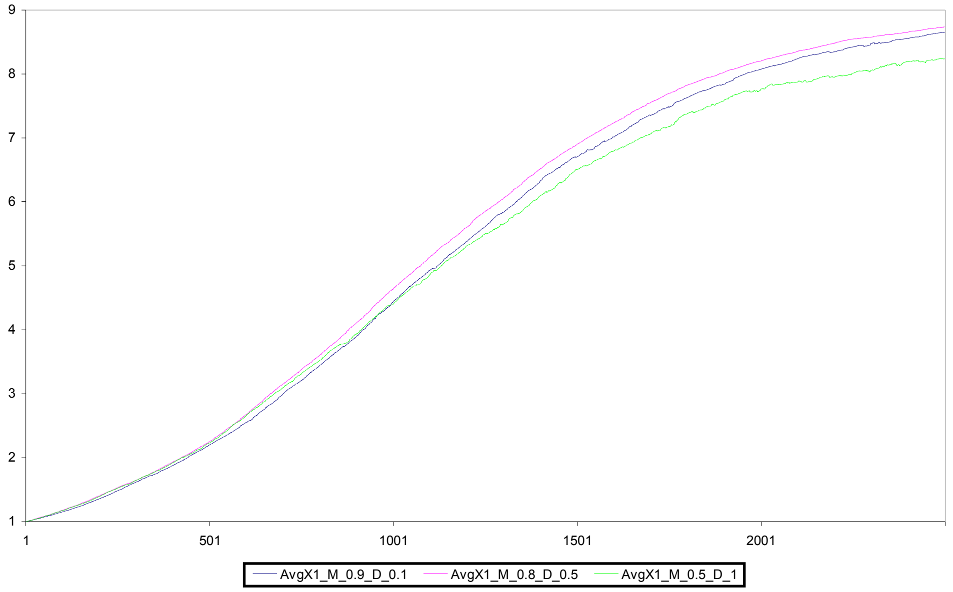
[23] In this regard a dynamic entry process going on throughout the simulated periods is much more effective than an initially higher number of suppliers. In the latter case discarded suppliers soon exit from the market, reducing quickly the supplied productive capacity at the demand level. On the contrary, a permanent flow of new firms can help supply to exceed demand on a continuous basis, leaving more room for eventual switches in case that one of the new suppliers yields a satisfying performance for one or more of the «deverticalized» firms.

depicted in graph 12, in which the average quality of c_1 for the above indicated values (with initial productive capacity equal to 3 and $\lambda=0{,}01$) is compared to the one we get respectively for $MaxLearn=0{,}8$ and $DevLearn=0{,}5$, and for $MaxLearn=0{,}5$ and $DevLearn=1$, i. e. in progressively worse conditions.

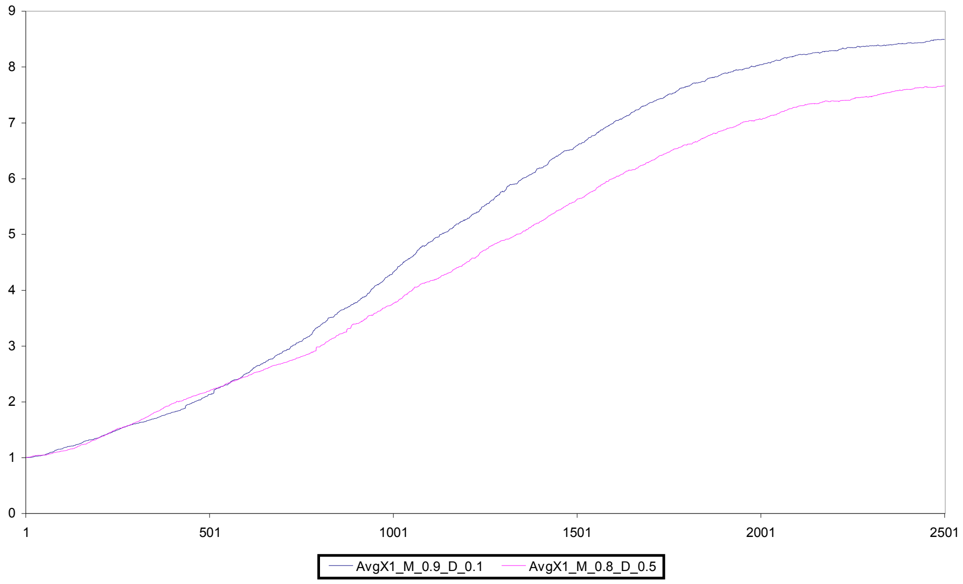
*Graph 12. Connection between the average quality of the components and the level of informational distortion, with entry of new suppliers**



* Average values computed for 10 simulations repeated for each value of $MaxLearn$ and $DevLearn$ on a sample of 10 «deverticalized» firms and 10 suppliers, with $\lambda=0{,}01$.

It is rather surprising that the average quality, when informational distortion increases, is firstly higher, and then lower than the one we get from our benchmark values. This dynamics is explained by a positive effect of a worse information on the tendency of «deverticalized» firms to change supplier, because the former produces on the average a greater difference between the real quality of the actual supplier and the perceived quality of the candidate supplier. This effect is stronger, given what we have said above, when there is more productive capacity available, i. e. with a higher initial productive capacity and/or with a higher number of new entrants. In particular, it is the entry rate of new firms to produce the stronger effect because, while the new firms have a performance in line with the average, the worse suppliers are continuously removed from the market. As a consequence, the average quality of suppliers tends continuously to converge upwards, and the eventual losses caused by a wrong decision to switch are on the average lower than the benefits of a higher number of switches. In these conditions changing more suppliers is thus more efficient, explaining the positive effect of a stronger informational distortion. This supposition can be verified directly looking at the effects of a stronger informational distortion when the entry of new firms is not allowed. From graph 13 it is evident that, all other things being equal, the positive effect disappears for $\lambda = 0$.

*Graph 13. Relationship between the average quality of the components and the informational distortion, without entry of new suppliers**



* Average values computed for 10 simulations repeated for each value of *MaxLearn* and *DevLearn* on a sample of 10 «deverticalized» firms and 10 suppliers, with $\lambda = 0$.

In conclusion, our analysis has highlighted that the competitiveness of «deverticalized» firms increases with a higher availability of productive capacity, be it achieved from a higher capacity of the single suppliers or from a higher number of suppliers on the market, where the latter is best obtained dynamically through a continuous entry of new firms. On the other hand competitiveness decreases, even if not linearly, when client firms have a worse ability to gain correct information on the quality of suppliers. A lower level of informational distortion can be explained by a higher availability of communication infrastructures, thus capturing, although in a very stylized way, the effects of a development of these infrastructures on the competitive fitness of a «deverticalized» production pattern.

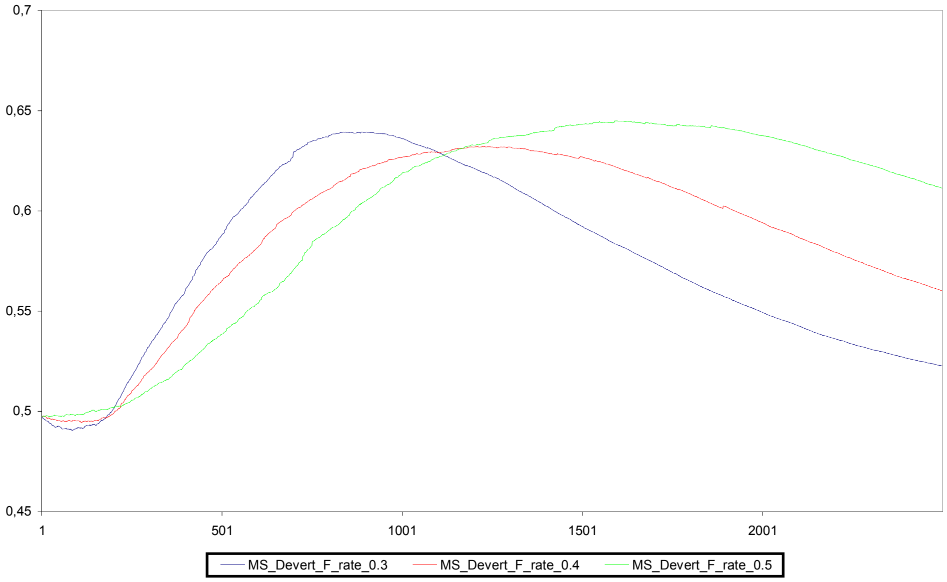
6. Innovation dynamics and competition between «verticalized» and «deverticalized» firms

Now that we have evaluated the behavior of the model with respect to a limited set of «interesting» parametric values, we can focus on our main goal, namely to compare the competitive performance of «verticalized» and «deverticalized» firms for different levels of complexity of the innovation process. As we have specified in par. 3.4 and 3.5, the innovation process of the two typologies is different, with a supposed inverse relationship between complexity and innovation performance of «deverticalized» firms and their suppliers, considered as a global decentralized search architecture. In the model the level of complexity is regulated by the parameter F_{rate} (see expressions [23] and [28]). Consequently our first goal is to verify the distribution of market shares between the two typologies at different values of F_{rate}. To complete the analysis we will then evaluate, for the same value of F_{rate}, the effects of the variation of some of the parameter already considered above.

As we had supposed presenting the model, when the levels of complexity are relatively low ($0{,}3 \leqslant F_{rate} \leqslant 0{,}5$) «deverticalized» firms have a competitive advantage over «verticalized» firms, although this advantage doesn't appear linearly. From graph 13 we see that in the initial periods of the simulations the global market share of «deverticalized» firms is stationary if not declining, and that the advantage achieved in intermediate periods tends subsequently to decrease in the final ones. Even in conditions of limited complexity, the initial and final phases of the development cycle of a product are characterized by a relative advantage for «verticalized» firms, which depends from the logistic shape of the payoffs deriving from the innovation process (see the expression [25]). In the first periods the positive payoffs grow slowly and thus the difference between positive and negative payoffs is still limited, favoring the strategy of «verticalized» firms that get on the average a lower number of negative payoffs. The advantage of «deverticalized» firms increases when the positive payoffs grow exponentially, because at the same time the negative one increases linearly (see expression [26]), and thus the difference between the two becomes bigger and bigger. In this situation we observe a relative advantage of «deverticalized» firms, that accept a higher number of innovations, because the benefits of successful innovations (i. e. those with positive payoffs) exceed largely the damages caused by negative payoffs.

In the final periods the advantage of «deverticalized» firms is decreasing because these, being more efficient in their innovation process, approximate more quickly to the upper bound of the logistic curve, leaving thus room for the «verticalized» firms to catch up. This supposition is confirmed by the fact that the advantage of «deverticalized» firms increases for $F_{rate}=0{,}5$. When F_{rate} gets higher the share of unsuccessful innovations increases, and thus it is increased the number of periods that are necessary for both typologies of firms to reach the upper bound of the logistic curve. This lag enables «deverticalized» firms to exploit their advantage in the intermediate periods for a higher number of periods, reaching in this way a total market share which is higher than that achieved for $F_{rate}=0{,}3$, and leaving for the same time period less opportunities for l«verticalized» firms to catch up.

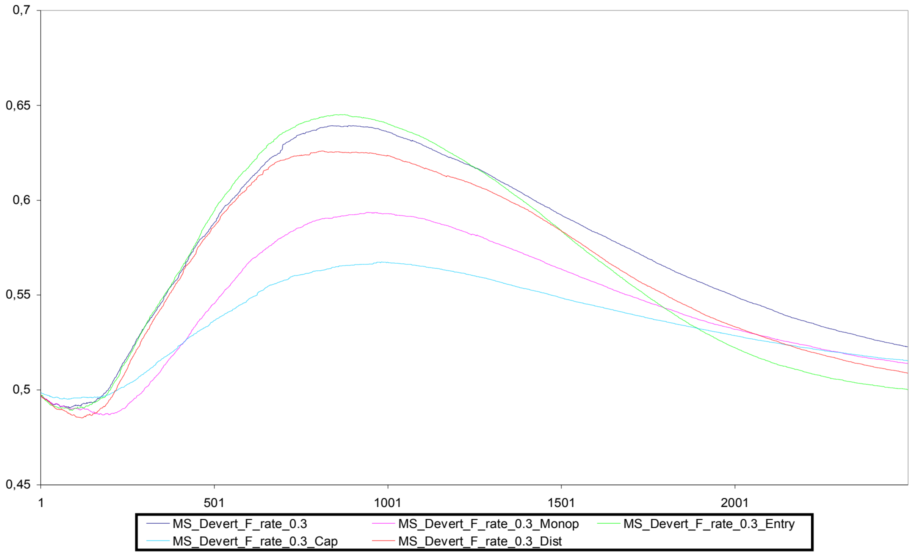
*Graph 13. Total market share of «deverticalized» firms for relatively low complexity levels of the innovation process**



* Average values computed for 10 simulations repeated for each value of F_{rate} on a sample of 10 «deverticalized» firms, 10 «verticalized» firms and 10 suppliers. Competition occurs with constant mark-ups and without entry of new suppliers. Further, we have that $\zeta=0$, $MaxLearn=0{,}9$ and $DevLearn=0{,}1$.

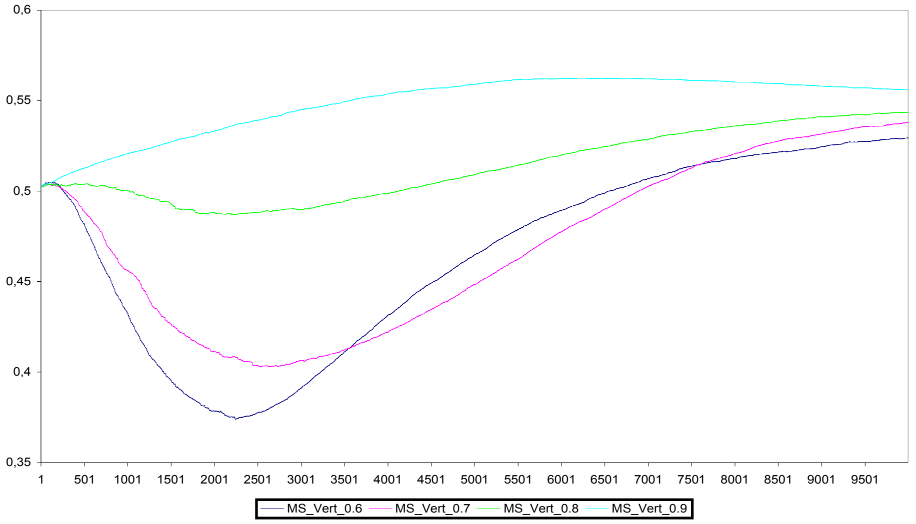
Now we can focus on the effects, for equal values of complexity ($F_{rate}=0{,}3$), of the different competitive conditions described in the previous paragraphs. Graph 14 highlights that, with respect to the benchmark values already depicted in graph 13, only the possibility of new entrants among suppliers produces effects on the competitiveness of «deverticalized» firms (the faster decline with respect to benchmark values is caused by the fact that «deverticalized» firms reach even more fastly the upper bound of the logistic curve, allowing an anticipate catch up of «verticalized» firms). If the negative effect of a high informational distortion for «deverticalized» firms is not surprising, given what we have already said above, the effects of the introduction of an inertial effect in competition (due to the role of existing productive capacity) and of the possibility for final firms to change mark-ups need to be briefly explained. As we have seen above, the first factor tends to «smooth» the competitive dynamics, causing in this case a disadvantage for «deverticalized» firms, that have on the average a better competitiveness. The second factor amplifies the initial differences, producing in our case an advantage for «verticalized» firms, that have a better performance in the initial periods. The temporary decline of market shares for «deverticalized» firms is in fact much more prolonged in this condition than for the benchmark values.

*Graph 14. Total market share of «deverticalized» firms, for the same complexity level of the innovation process, in different competitive conditions**



* Average values computed for 10 simulations repeated with $F_{rate}=0{,}3$ on a sample of 10 «verticalized» firms, 10 «deverticalized» firms and 10 suppliers, for the following variations of the benchmark condition (graph 13): Monop= competition with variable mark-ups; Entry= possibility for new suppliers to enter the market, with $\lambda=0{,}03$; Cap= competition with «inertial» effect of existing productive capacity, with $\zeta=0{,}5$; Dist=presence of a high information distortion for «deverticalized» firms, with $MaxLearn=0{,}8$ and $DevLearn=0{,}5$.

*Graph 15. Total market share of «verticalized» firms for high complexity levels of the innovation process**



* Average values computed for 10 simulations repeated for each value of F_{rate} on a sample of 10 «verticalized» firms, 10 «deverticalized» firms and 10 suppliers. Competition occurs with constant mark-ups and no entry of new suppliers. Further $\zeta=0$, $MaxLearn=0{,}9$ and $DevLearn=0{,}1$.

Finally, in coincidence with relatively high values of complexity ($0{,}6\leq F_{rate}\leq 0{,}9$) we have a progressive loss of performance for «deverticalized» firms. This dynamics is indirectly depicted in graph 15, which draws the total market share of «verticalized» firms in coincidence of different values of F_{rate} . In particular, the disadvantage of «verticalized» firms is reduced progressively when F_{rate} steps from a value of 0,6 to one of 0,7, and then even more steadily for $F_{rate}=0{,}8$. Lastly, for $F_{rate}=0{,}9$ we find an advantage for «verticalized» firms over the entire simulated time period[24], showing for this value of F_{rate} the existence of a *trade-off* between decentralization and complexity, as we had supposed above.

7. Concluding remarks

The basic idea underlying this paper is that only a complex evolutionary process can generate the non-linear phenomena which characterize the reality of industrial dynamics, where alternative organizational patterns exist often one beside to the other, and the metamorphosis of «dominant forms» determines, through epidemic imitation and/or a repositioning inside «competitive niches» which are continuously changing, dynamic paths that are difficult to predict and always open to the emergence and disappearance of organizational and technological equilibria which are necessarily partial. More in detail, the exploratory simulations we have presented and discussed above have shown that our model reproduces the trade-off between decentralization and complexity, suggested by the literature briefly exposed in par. 1, and reputed to offer a first hint for a broader explanation of the co-evolution of organizational patterns and technology. In this direction, for example, the model highlights a comparative advantage of «centralization», which occurs, for all levels of complexity, in the initial and final phases of the development cycle of one product. The latter effect is produced in the model simply by the existence of differences in the returns of innovation in different phases of the same development cycle, which is indeed a well established stylized fact in the studies on technological change. When the return of a successful innovation is much higher that the damage caused by a failure, a «decentralized» search strategy yields better results even if it implies a higher number of failures, because at the same time it guarantees also a higher number of successes. This advantage disappears only when the probability to fail is much higher than the probability to succeed. In our simulations the trade-off reveals itself only for very high values of F_{rate} , which on the other part (we must not forget) are entirely arbitrary, because they depend on the respective values of positive and negative payoffs, which have been fixed for our simulations in a discretionary way. Any hypothesis on the plausibility or on the relevance in the real world of the different parametric values examined in the paper is nonetheless beyond the scope of our actual analysis. For the moment it is enough for us to have shown the existence of a qualitatively different

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[24] The simulations presented in graph 15 are computed over 10.000 periods, because the time required for firms to cover the logistic curve of the innovation process is longer for higher levels of complexity.

dynamics in coincidence of different levels of complexity (i. e. in different phases of product and/or technology development), as well as the fact that this dynamics is influenced by many other competitive factors (as for example the stronger or weaker development of intermediate sectors), determining a wide set of structurally different dynamic patterns in coincidence of different combinations of factors.

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