Minimum Quality Standards and Equilibrium Selection with Asymmetric Firms.

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Abstract

In a vertically differentiated market with cost asymmetries, the risk dominance criterion selects the equilibrium where the high quality is produced by the efficient firm. We show that a sufficiently high Minimum Quality Standard reverses equilibrium selection. Hence, MQS may be used in order to increase a domestic firm's profit at the expense of a more efficient foreign rival. This produces higher domestic and lower world welfare. Since the protectionist impact of MQS comes through equilibrium targeting rather than directly affecting equilibrium outcomes, it cannot be easily detected.

JEL classification: L13, L5, F13; *Keywords*: Vertical product differentiation, Minimum quality standards, Equilibrium selection, Protectionism.

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1 Introduction

In this paper we examine the impact of a Minimum Quality Standard (MQS) on firms' profits in a vertically differentiated duopoly with cost asymmetric firms. Our analysis applies to the case where a domestic firm faces competition from a more efficient foreign rival. We suggest that a seemingly inoffensive MQS may have a strong protectionist impact by allowing an inefficient domestic firm to undertake the production of the high quality product.

In vertically differentiated markets the asymmetry in equilibrium qualities guarantees better profit for the firm that produces the high quality, even when the firms are symmetric in terms of cost. Provided that cost asymmetries are not too large, the high quality producer obtains higher profits even if it is less cost-efficient than its rival. Thus, instead of offering an artificially higher market share to its domestic firm through tariffs and import quotas, a regulator can favour that firm by simply helping it to take up the role of the high quality producer.

For cost asymmetries that are not too large, either the less or the more efficient firm producing the high quality can be an equilibrium. The situation resembles to an asymmetric *Battle of the Sexes* game, in which the only available equilibrium selection criterion is that of Risk-Dominance (RDSC) introduced by Harsanyi and Selten (1988). In the past few years an increasing number of studies have relied on the RDSC. Van Huyck et al. (1990) shows empirically the RDSC to be a good predictor of a game's outcome. Motta et al. (1997) studies the opening of trade between two countries and shows that in the integrated market the RDSC selects the equilibrium where the high quality is produced by the firm that was also producing a higher quality in autarky (persistence of quality leadership). Cabrales et al. (2000) shows that in games of vertical differentiation with asymmetric cost, the RDSC selects the equilibrium where the efficient firm produces the high quality (and, of course, makes higher profits), and supports this prediction experimentally. In Moraga-Gonzàlez and Viaene (2005) (hereafter, M-V), the use of tariffs and subsidies increases the relative efficiency of an otherwise inefficient domestic firm. As a result, the RDSC selects the equilibrium where this firm is the high quality producer.

In this paper we consider a situation where a domestic firm competes against a more efficient foreign rival and show that the use of MQS can tip the RDSC towards selecting the equilibrium that favors the domestic firm. This produces higher domestic and lower world welfare.

Unlike the use of tariffs and subsidies considered in M-V, the MQS constitutes a non-discriminatory policy since its impact on either the high or the low quality does not depend on the identity (domestic-foreign) of the firm firm producing that quality. Moreover, instead of improving domestic firm's equilibrium payoffs–like most of the well-known protectionist policies do–it operates by *lowering* these payoffs in any given equilibrium. The equilibrium reversing MQS is such that, had the domestic firm remained in the production of the low quality, it would have been eliminated from the market. On the other hand, recall that under Bertrand competition in the last stage, MQS have been shown to *always hurt the profit of the firm producing the high quality.*[1] Since in the selected equilibrium the high quality is produced by the domestic firm, considered *ex post*, that firm's profit would have been higher in the absence of MQS. In other words, the implementation of MQS seems to be working against the domestic firm's profit, it can hardly, therefore, be accused as protectionist measure! This conclusion neglects, of course, the role that the MQS may have played in *selecting* the domestic firm's preferred equilibrium.

[1]See Ronnen (1991) and Jinji and Toshimitsu (2004). As it turns out, in order to reverse equilibrium selection the MQS must be so high that it hurts the domestic firm's profit even in the equilibrium where the latter produces the low quality and could have been favoured by a lower MQS.

Several papers have investigated the impact of MQS on trade relying on a given quality hierarchy between the domestic and the foreign firm's product.[2] The possibility of quality reversals is considered in Herguera *et al.* (2002) and Boccard and Wauthy (2005). However, in both these papers the reversal is due to the elimination of one equilibrium through discriminatory treatment of the foreign firm. In the first, the foreign firm lowers its quality in order to reduce its anticipated tariff payment while in the second, the fact that the foreign firm faces a quota makes the domestic firm aggressive in terms of quality choice. Equilibrium selection is exogenous in Herguera and Lutz (1998) where an extremely high MQS forces an *inefficient* foreign firm out of the market. Such MQS though cannot help the domestic firm when it has a cost disadvantage *vis-à-vis* its foreign rival and may also create strong reactions.[3]

By fully endogenizing equilibrium selection, the present work is, to our knowledge, the first to show that a non-discriminatory measure, like a MQS that does not deter foreign entry, can induce quality leapfrogging in favour of an otherwise inefficient domestic firm.

2 The model

Consider a vertically differentiated product market where a domestic and a foreign firm, denoted respectively as D and F, compete in prices. Consumers buy either a single unit of a given quality or nothing at all. A consumer k purchasing quality $q_i \geq 0$ at price p_i, $i = D, F$, has utility

[2]E.g. Das and Donnenfeld (1989), Boom (1995), and Fisher and Serra (2000).

[3]In the late 1990's the Canadian government tried to set sufficiently high health standards that would eliminate imports of cheese from non-pasteurized milk, in which French producers had cost advantage and a quality leading role. Local consumer and foreign producer reaction cancelled this effort.

$U^k = \theta^k q_i - p_i,$

while the utility in the case of non-purchase is zero. Each consumer is characterized by θ^i, his willingness to pay for quality increments; the θ's of the entire consumer population are distributed uniformly on the $[0, 1]$ interval.

Variable production costs are constant, independent of quality, equal for both firms and for simplicity assumed also equal to zero. The choice of a given quality entails a sunk fixed cost

$F_i = \lambda_i \frac{q^2}{2}$

where λ_i represents firm i's technology, $i = D, F$. We assume that the foreign firm is more efficient and set, without loss of generality, $1 = \lambda_F \leq \lambda_D = \lambda$. Firms play a two-stage game. In the first stage they choose simultaneously their respective quality level In the second stage they choose simultaneously their price. Quality decisions are observable by both rivals before price decisions are made. Firm i is allowed to choose $q_i = 0$ in the first stage, which corresponds to the no-entry option. When $q_i = 0$ and $q_j \neq 0$, at the second stage firm j acts as a monopolist. The solution concept adopted is that of subgame perfect Nash equilibrium.

Taking qualities as given we have two possible kinds of subgames according to whether it is the foreign or the domestic firm that produces the high quality.[4] The superscript $i = F, D$, characterizes the subgame by indicating the firm that produces the high quality in the subgame considered. As before, subscripts characterize firms, so similar (different) superscript and subscript indicate the high (low) quality producer in the subgame. Starting from the price stage, we compute optimal prices as

[4]Bertrand competition in homogeneous products is ignored as it can never be an equilibrium in this game.

functions of qualities:[5]

$$p_i^i = \frac{2\left(q_i^i - q_j^i\right)q_i^i}{4q_i^i - q_j^i}, \qquad p_j^i = \frac{\left(q_i^i - q_j^i\right)q_j^i}{4q_i^i - q_j^i} \tag{1}$$

Substituting optimal prices back into the profit functions we obtain profits as functions of qualities. Hence, the high and low quality producer maximize respectively[6]

$$\pi_i^i = 4q_i^{i^2}\frac{q_i^i - q_j^i}{\left(4q_i^i - q_j^i\right)^2} - \lambda_i^i\frac{q_i^{i^2}}{2},\ \pi_j^i = \frac{q_i^i q_j^i\left(q_i^i - q_j^i\right)}{\left(4q_i^i - q_j^i\right)^2} - \lambda_j^i\frac{q_j^{i^2}}{2} \tag{2}$$

$i, j = F, D$, $i \neq j$, and $\lambda_i^i, \lambda_j^i = 1, \lambda$. It is *important* to note that $\lambda_F = 1$, and $\lambda_D = \lambda$ whatever the subgame considered, i.e. whatever the superscript.

The first order condition for the choice of the high quality in both subgames is:

$$q_i^i = \frac{1}{\lambda_i^i}\frac{-8q_j^{i^2}q_i^i + 12q_j^i q_i^{i^2} - 16q_i^{i^3}}{(q_j^i - 4q_i^i)^3} \tag{3}$$

while the corresponding condition for the low quality is:

$$q_j^i = \frac{1}{\lambda_j^i}\frac{7q_j^i q_i^{i^2} - 4q_i^{i^3}}{(q_j^i - 4q_i^i)^3} \tag{4}$$

Expressions (3) and (4) represent *two* systems of two equations in two unknowns. Within each subgame one should now obtain reaction functions in the quality space and from them the equilibrium qualities of the subgame. However, this task proves to be extremely complex at an analytical level. For this reason we re-write (3) and (4) as functions of the *degree of differentiation*, defined as $r^i \equiv \frac{q_i^i}{q_j^i} > 1$, $i, j = F, D$, $i \neq j$:

$$q_i^i = \frac{1}{\lambda_i^i}\frac{-8r^i + 12r^{i^2} - 16r^{i^3}}{(1 - 4r^i)^3}, \tag{5}$$

[5]The procedure is very standard and therefore, omitted. Second order conditions are respected (full proofs available by the authors).

[6]We do not assign qualities to firms since this will be endogenously determined. According to our notation, similar superscript and subscript indicate high quality.

$$q^i_j = \frac{1}{\lambda^i_j}\frac{7r^{i^2} - 4r^{i^3}}{(1-4r^i)^3} \tag{6}$$

Dividing (5) by (6) and re-arranging terms, we obtain the following equilibrium condition on r^i:

$$\lambda^i_i(4r^{i^3} - 7r^{i^2}) - \lambda^i_j(16r^{i^2} - 12r^i + 8) = 0 \tag{7}$$

Solving Eq.(7) and substituting the equilibrium value of r^i back to Eq.(5) and Eq.(6) we obtain equilibrium qualities and profits for both firms, denoted with a *. Let $E^F = (q^{F*}_F, q^{F*}_D)$ and $E^D = (q^{D*}_D, q^{D*}_F)$ represent the equilibrium of each reduced form subgame where the role of the high quality producer has been *exogenously assigned* to the foreign and domestic firm, respectively. Obviously, in a pure strategy equilibrium each firm will choose among its equilibrium strategies in the subgames, therefore, we only need to consider the set $\omega_F = \{q^{F*}_F, q^{D*}_F\}$ for firm F and the set $\omega_D = \{q^{F*}_D, q^{D*}_D\}$ for firm D in order to find equilibria for the entire game. Let $a_{HL} \equiv \pi^F_F(q^{F*}_F, q^{F*}_D)$, $a_{HH} \equiv \pi_F(q^{F*}_F, q^{D*}_D)$, $a_{LL} \equiv \pi_F(q^{D*}_F, q^{F*}_D)$, and $a_{LH} \equiv \pi_F(q^{D*}_F, q^{D*}_D)$ represent firm F's payoffs from all possible combinations of the four strategies that can be part of a subgame equilibrium. In an analogous manner, let $b_{HL} \equiv \pi^F_D(q^{F*}_F, q^{F*}_D)$, $b_{HH} \equiv \pi_D(q^{F*}_F, q^{D*}_D)$, $b_{LL} \equiv \pi_D(q^{D*}_F, q^{F*}_D)$, and $b_{LH} \equiv \pi_D(q^{D*}_F, q^{D*}_D)$ represent the corresponding payoffs of firm D.[7] Hence, the payoff matrix of the entire game can be summarized as:

[7] Firm F's (D's) payoffs are indicated by a (b). The first letter in the payoff subscripts indicates the strategy of firm F while the second, that of firm D. In order to avoid heavy notation in subscripts we denote each firm's strategies by H, L according to whether the firm acts as the high or the low quality producer. For instance, b_{HL} indicates firm D's payoff when firm F acts as high quality producer and firm D as low quality producer. It needs to be recalled that firm F's H-strategy is $q^{F*}_F \neq q^{D*}_D$ which is firm D's H-strategy.

		Firm D	
		q_D^F	q_D^D
Firm F	q_F^F	a_{HL}, b_{HL}	a_{HH}, b_{HH}
	q_F^D	a_{LL}, b_{LL}	a_{LH}, b_{LH}

Hereafter, we assume that $\lambda \leq \overline{\lambda} \simeq 1.6$, which, according to Zhou *et al.* (2002), implies that *both* E^F *and* E^D *are equilibria of the entire game.* Thus, at the begining of the game firm i needs to anticipate the subgame that will be played in order to choose between its strategies in ω_i, $i = F, D$. According to Harsanyi and Selten (1988) the focal point will be the equilibrium that, if wrongly neglected, reduces payoffs more sharply. This is the main idea behind the *Risk Dominance Selection Criterion* (RDSC) which selects equilibrium E^F (E^D) when[8]

$$RD(\lambda) = (a_{HL} - a_{LL})(b_{HL} - b_{HH}) - (a_{LH} - a_{HH})(b_{LH} - b_{LL}) > (<)0$$

The term $(a_{HL} - a_{LL})$ can be interpreted as the net gain for firm F from correctly anticipating the q_D^{F*} move from its rival rather than wrongly anticipating q_D^{D*} and playing accordingly. The term $(b_{HL} - b_{HH})$ represents the corresponding net gain for firm D. Hence, the first product in the above expression shows how important is for the two firms not to play wrong if their rival is going to play the strategy corresponding to E^F. Similarly, the second product shows the importance of correctly anticipating E^D when the rival is going to play his strategy corresponding to E^D.[9]

Cabrales et al. (2000) show that in games of vertical differentiation with asymmetric cost the RDSC selects the equilibrium where the quality leader is the low

[8]Notice also that none of the subgame equilibria is Pareto optimal, therefore the only available equilibrium-selection criterion is the RDSC. An alternative would have been an equilibrium in mixed strategies.

[9]Since both E^F and E^D are Nash equilibria the terms in parenthesis are all positive. For a more detailed discussion of the criterion the reader can see Motta *et al.* (1997).

cost firm. In our context, this implies that E^F risk-dominates E^D, *i.e.,* that it is the foreign competitor who will produce the high quality and enjoy higher profits. The domestic government can reverse this outcome by using discriminatory measures against the foreign firm (tariffs, quotas or subsidies to the domestic firm, as in M-V). Such measures, however, invite retaliation and may be constrained by international agreements.

Consider now the imposition of a MQS at a quality level $\widetilde{q}$. The following results are known to hold under Bertrand competition in the last stage:[10] a) as long as both qualities remain in the market, the profit of the high quality decreases monotonically with $\widetilde{q}$; b) the profit of the low quality, while initially increasing in $\widetilde{q}$, past a certain level it becomes monotonically decreasing. Moreover, c) unless there is an abrupt change in market structure, the profit of the low quality is continuous in $\widetilde{q}$; d) a MQS equal to the high quality generates no revenues, due to Bertrand competition in homogeneous products. Combining b), c) and d) implies the existence in every subgame $i = F, D$, of a $\widetilde{q} \equiv \overline{q}^i > q_j^{i*}$, such that $\pi_j^i(\overline{q}^i, q_i^{i*}(\overline{q}_j^i)) = 0$ and $\forall \widetilde{q} > \overline{q}^i$, $\pi_j^i(\overline{q}^i, q_i^{i*}(\overline{q}^i)) = 0$. We term as *radical* MQS any $\widetilde{q} > \overline{q}^i$; obviously, a radical MQS induces exit of the lower quality in the subgame.[11] Let a tilde $\widetilde{}$ on a variable indicate its equilibrium value in the presence of MQS. The value of $\overline{q}^i$ as function of the high quality is obtained from Eq.(2) by setting $\widetilde{\pi}_j^i \equiv \pi_j^i(\widetilde{q}_j^i = \overline{q}^i, \widetilde{q}_i^i) = 0$:

$$\overline{q}^i = \frac{2(\widetilde{q}_i^{i2} - \overline{q}^i \widetilde{q}_i^i)}{\lambda_j^i(\overline{q}^i - 4\widetilde{q}_i^i)^2} = \frac{2(\overline{r}^{i2} - \overline{r}^i)}{\lambda_j^i(1 - 4\overline{r}^i)^2} \tag{8}$$

$i = F, D$, with $\overline{r}^i \equiv \frac{q_i^i}{\overline{q}_j^i}$. If the low quality decides to remain in the market, the implicit reaction function of the high quality is described by Eq.(5) with q_i^i replaced by $\widetilde{q}_i^i$ and q_j^i by $\widetilde{q}$. Dividing Eq.(5) by Eq.(8) and re-arranging terms, we obtain the

[10] The first two are due to Ronnen, 1991.

[11] Leapfrogging is ruled out within a subgame as it leads to the other subgame.

following equilibrium condition on $\overline{r}^i$:

$$\lambda_i^i(\overline{r}^i - 5\overline{r}^{i^2} + 4\overline{r}^{i^3}) - \lambda_j^i(4 - 6\overline{r}^i + 8\overline{r}^{i^2}) = 0 \tag{9}$$

which is equivalent to expression (7) in the unconstrained case. We solve Eq.(9) for the equilibrium value of $\overline{r}^i$, which substituted back into Eq.(8) yields $\overline{q}^i = \overline{q}^i\left(\lambda_i^i, \lambda_j^i\right)$. Since for $\widetilde{q} \geq \overline{q}^i$ the low quality makes nonpositive profits and exits the market, the high quality producer chooses the same quality that a monopolist would choose, *i.e.,* $\widetilde{q}_i^i = q_M^i$, provided of course that the latter is higher than $\widetilde{q}$. Lemma 1 below provides a technical result necessary for what follows.

Lemma 1: *For all* $\lambda \leq \overline{\lambda}$, $\overline{q}^F < \overline{q}^D$.

Proof. Note that in the E^F equilibrium the MQS applies to the domestic firm and vice-versa for E^D. Since the domestic firm is less cost efficient, it needs more differentiation in order to survive, therefore, *for any given high quality* $\overline{q}^F < \overline{q}^D$. However, the high quality is not the same in the two subgames: $\forall\widetilde{q}$, $\widetilde{q}_F^F > \widetilde{q}_D^D$, due again to the higher efficiency of the foreign firm. Thus, we have two effects working in opposite directions and we need to proceed computationally. The proof is obtained numerically by computing $\overline{q}^D$, $\overline{q}^F$, for all relevant values of λ.[12] Figure 1 reports these results. ■

Now we are able to show that:

Proposition 1 *:* $\forall\lambda \in (1, 1.09]$, $RD\left(\widetilde{q}\right) < 0$*,* $\forall\widetilde{q}$ *in the neighborhood to the right of* $\overline{q}^F$*.*

[12]Figure 1 constitutes full proof since it computes for all values of λ and not just a sample of such values. Unfortunately, from the moment qualities and/or the RDSC need to be determined one can only hope to find numerical solutions, as this has always been common practice in the literature (see for instance, Motta, 1993, Motta *et al.*, 1997, or Moraga-Gonzàlez and Viaene, 2005).

Proof. We compute $RD(\overline{q}^F)$ for all relevant values of λ (see Appendix 1). From Figure 2 which reports these results it is clear that $RD(\overline{q}^F) < 0$.[13] It is easy to see that within a given market structure all the payoffs are continuous in $\widetilde{q}$ and so is $RD(\widetilde{q})$. Therefore, there exists an interval in the right neighborhood of $\overline{q}^F$ in which $RD(\widetilde{q}) < 0$, Q.E.D. ■

The solid line in Figure 3a shows the typical evolution of RD as function of the MQS.[14] Part b of the figure computes RD for all $\widetilde{q}$ in the $(\overline{q}^F, \overline{q}^D)$ interval, for $\lambda = 1.05$. When firms are symmetric, $\overline{q}^F = \overline{q}^D = \overline{q}$ and any $\widetilde{q} \geq \overline{q}$ induces monopoly of one or the other firm. With asymmetric firms, all levels of MQS between $\overline{q}^F$ and $\overline{q}^D$ instead of resulting in a monopoly of the efficient firm, they induce a role switch and result in a duopoly where the inefficient firm produces the high quality. This is a point that has been neglected in the literature.[15]

That RD becomes abruptly negative when $\widetilde{q}$ reaches $\overline{q}^F$ and the domestic firm is forced out of the market in the E^F equilibrium, is not a coincidence. The dashed line in Figure 3a shows the value of RD, let it be $\widehat{RD}(\widetilde{q})$, in the hypothetical case where

[13] Since at $\widetilde{q} = \overline{q}^F$ the domestic firm is indifferent between staying or exiting the market, $RD(\overline{q}^F)$ may take two values, one positive, the other negative. Following convention and the fact that $RD(\overline{q}^F + \varepsilon) < 0$, for any ε arbitrarily small, we consider that at $\widetilde{q} = \overline{q}^F$ the low quality exits the market in E^F.

[14] Due to scale problems, figure 1a is "drawn" instead of being the result of computations based on specific parameter values. What is important is that in all the computations we performed, RD had the shape presented in figure 1a.

[15] As a matter of fact, one can show numerically that for $\lambda \leq 1.09$ RD remains negative in the entire interval $(\overline{q}^F, \overline{q}^D)$ (for $\lambda \leq 1.085$) or in the neighborhood to the right of $\overline{q}^F$, while for $1.09 \leq \lambda \leq 1.6$ the RDSC becomes again positive for all values of $\widetilde{q}$ (computations available by the authors). These results are of little importance as long as the regulator cares about the domestic firm's profit, since the latter decreases monotonically with $\widetilde{q}$ in the E^D equilibrium. They have some importance when total domestic welfare is considered (see below).

the domestic firm is not allowed to exit the market, despite its negative profits. Note that $\forall \widetilde{q} \in \left[\overline{q}^F, \overline{q}^D\right]$, $\widehat{RD}\left(\widetilde{q}\right) > 0$, which implies that, had the domestic firm been forced to stay in the market in the E^F equilibrium, it would most likely have ended up as the low quality producer in the equilibrium of the entire game. As revealed by Table 1, which compares the elements of $\widehat{RD}$ and RD for $\lambda = 1.05$ and $\widetilde{q} = \overline{q}^F$, exit reduces RD by substantially reducing the $(a_{HL} - a_{LL})$ term.[16] The latter represents the profit difference for the foreign firm between choosing the optimal *versus* a suboptimal quality, given q_D^D. As long as the low quality stays in the market, such a mistake has both a direct effect and, most importantly, a strategic effect on the foreign firm's profit. When $\widetilde{q} \geq \overline{q}^F$, the domestic firm's exit in the E^F equilibrium eliminates the strategic effect and therefore makes this difference much less significant: wrongly ignoring the E^F equilibrium is no longer that damaging for the foreign firm's profit as wrongly ignoring E^D.[17]

What we have shown, so far, is that a carefully selected MQS allows the domestic firm to make more profit than its foreign rival. This, of course, is not per se a policy objective. Bearing in mind the results of Ronnen (1991)[18] the regulator needs to examine whether the domestic firm is better-off with the MQS that maximizes its

[16] The results of the table 1 are qualitatively robust in varying λ.

[17] More rigorously, the term $(a_{HL} - a_{LL})$ can be approximated as $d\pi_F = \frac{d\pi_F}{dq_F} dq_F$, where $dq_F = q_F^{F*} - q_F^{D*}$. Let a hat$\widehat{}$ characterize all relevant variables in the $E^F\left(\widetilde{q}\right)$ equilibrium when the domestic firm is constrained not to exit the market. Note first that $|d\widehat{q}_F| > |d\widetilde{q}_F|$: due to the strategic effect, $\widehat{q}_F^{F*} > \widetilde{q}_F^{F*}$, while $\widehat{q}_F^{D*} = \widetilde{q}_F^{D*} = \widetilde{q}$. Also, $\frac{d\widetilde{\pi}_F}{dq_F} = \frac{\partial\widetilde{\pi}_F}{\partial q_F}$ while $\frac{d\widehat{\pi}_F}{dq_F} = \frac{\partial\widehat{\pi}_F}{\partial q_F} + \frac{\partial\widehat{\pi}_F}{\partial p_D}\frac{\partial p_D}{\partial q_F}$, with $\frac{\partial\widehat{\pi}_F}{\partial p_D}\frac{\partial p_D}{\partial q_F} > 0$ (the strategic effect of quality choice). On the other hand, exit reduces the domestic firm's damage from wrongly anticipating $E^F\left(\widetilde{q}\right)$,*i.e.,* $(\widehat{b}_{LH} - \widehat{b}_{LL}) > (\widetilde{b}_{LH} - \widetilde{b}_{LL})$,it tends, therefore, to make the $E^F\left(\widetilde{q}\right)$ equilibrium more likely. This second effect of exit on RD is always less significant than the first..

[18] *I.e.,* that a MQS a) always reduces the profit of the high quality, and b) if set at a moderate level increases the profit of the low quality.

profit as low quality producer or with the MQS that reverses equilibrium selection. Let $\max \widetilde{\pi}_D^F$ represent the domestic firm's profit in the former case. The following proposition clears the issue:

Proposition 2 *:* $\forall \widetilde{q} \in (\overline{q}^F, \overline{q}^D)$, $\pi_D^D(\widetilde{q}) > \max \widetilde{\pi}_D^F$.

Proof. We compute $\pi_D^D(\widetilde{q} = \overline{q}^F)$ and $\max \widetilde{\pi}_D^F$ for all relevant values of λ. The results are reported in Figure 4 where one can see that $\forall \lambda \in (1, 1.09]$, $\pi_D^D(\widetilde{q} = \overline{q}^F) > \max \widetilde{\pi}_D^F$. Since the profit of the high quality firm is a decreasing function of MQS, we also compute $\pi_D^D(\widetilde{q} = \overline{q}^D)$ which again is higher than $\max \widetilde{\pi}_D^F$. Thus, any MQS that reverses the equilibrium increases the domestic firm's profit, QED. ■

Notice that, while a mild MQS can also have a positive impact on π_D^F, the local firm benefits a lot more from any MQS that reverses equilibrium selection. Furthermore, recall that the profit of the high quality is always lower in the presence of MQS (Ronnen, 1991). Hence, had the domestic firm been able to secure its role as the high quality producer by some other means, it would have preferred the MQS to be abolished. In other words, *given that the production of the high quality has been attributed to the domestic firm,* the presence of MQS hurts the domestic firm's profit, and even more so, since it has been imposed at a very high level. Therefore, MQS can hardly be accused as being a protectionist measure. What this argument obscures is, of course, the role of MQS in affecting equilibrium selection.

To this point we have implicitly assumed that the regulator cares only about the domestic firm's profit, perhaps for being subject to intense lobbying from the producer. In such scenario, the optimal choice of MQS is obviously just above $\overline{q}^F$. In what follows, we show that the regulator that maximizes domestic welfare will also choose a MQS that reverses equilibrium selection, albeit higher than the

MQS that maximizes domestic profit. Any such choice, however, does not maximize world welfare. In particular, we show that MQS choices that reverse equilibrium selection are inferior to a range of high MQS that respect the selection of E^F. Let W_W^j, W_D^j, W_C^j, $j = F, D$, represent word welfare, domestic welfare, and (domestic) consumer surplus, respectively, in each equilibrium. Let also $\widetilde{q}_1, \widetilde{q}_2$ be any two values of $\widetilde{q}$ such that $\widetilde{q}_1 < \overline{q}^F < \widetilde{q}_2 < \overline{q}^D$.[19] Then,

Proposition 3 *When the equilibrium is selected according the RDSC, i)* $\forall \{\widetilde{q}_1, \widetilde{q}_2\}$ $W_D(\widetilde{q}_2) > W_D(\widetilde{q}_1)$*; ii)* $\exists \xi > 0$ *such that* $\forall \{\widetilde{q}_1, \widetilde{q}_2\}$ *with* $\widetilde{q}_1 \in \left(\overline{q}^F - \xi, \overline{q}^F\right)$, $W_W(\widetilde{q}_2) < W_W(\widetilde{q}_1)$*; iii)* $\exists \zeta > 0$ *such that* $\forall \{\widetilde{q}_1, \widetilde{q}_2\}$ *with* $\widetilde{q}_1 \in \left(\overline{q}^F - \zeta, \overline{q}^F\right)$, $W_C(\widetilde{q}_2) < W_C(\widetilde{q}_1)$.

Proof. *The proof of the proposition consists of showing that, according to the RDSC, the evolution of* $W_D(\widetilde{q})$, $W_W(\widetilde{q})$, *and* $W_C(\widetilde{q})$ *is as shown in Figures* 5*a* , 5*b, and* 5*c, respectively. In the appendix it is shown that within each equilibrium,* E^F, E^D, *as long as both firms remain active (i.e.,* $\forall \widetilde{q} < \overline{q}^j$*) a)* $dW_i^j/d\widetilde{q} > 0$, $i = F, D$, $j = W, D, C$, *(lemma 2), and b)*$W_W^F(\widetilde{q}) > W_W^D(\widetilde{q})$, $W_C^F(\widetilde{q}) > W_C^D(\widetilde{q})$ *while* $W_D^F(\widetilde{q}) < W_D^D(\widetilde{q})$ *(lemma 3). These results imply that at* $\widetilde{q} = \overline{q}^F$, $W_D(\widetilde{q})$ *jumps upwards, which suffices for the proof of part i). They also imply that* $W_W(\widetilde{q})$ *and* $W_C(\widetilde{q})$ *have a downward jump. For ii) we also need that* $W_W^F(\overline{q}^F) > W_W^D(\overline{q}^D)$, *and, similarly, for iii) that* $W_C^F(\overline{q}^F) > W_C^D(\overline{q}^D)$, *both shown to hold in the appendix, QED.* ■

Thus, if the domestic regulator is interested in either domestic firm's profit or total domestic welfare, he must choose a MQS in the $\left(\overline{q}^F, \overline{q}^D\right)$ region. If the target is to maximize former, the MQS must be just to the right of $\overline{q}^F$, while for maximizing the latter an MQS just to the left of $\overline{q}^D$ must be chosen. Any value between these two extremes may be optimal when domestic profit wheighs more than consumer surplus

[19]Only effective levels of MQS are considered.

in the regulator's welfare function.[20] In all cases the MQS is too high relative to what maximizes domestic consumer surplus and world welfare. Any MQS beyond $\overline{q}^D$ establishes a monopoly of the foreign firm and, therefore, lowers domestic welfare.[21]

3 Conclusion

In a vertically differentiated duopoly with asymmetric firms, unless cost asymmetries are too large, there exist two equilibria. Without MQS, the RDSC selects the equilibrium where the more efficient firm produces the high quality (see Cabrales *et al.*, 2000) . Within this equilibrium, there exists a level of MQS, $\overline{q}^F$, sufficiently high as to induce exit of the less efficient firm (low quality producer) and market monopolization. However, this conclusion is based either on cost symmetric firms (Ronnen, 1991) or on an exogenous choice of equilibrium (Jinji and Toshimitsu, 2004). We have shown that with asymmetric firms, a MQS at the $\overline{q}^F$ level, or slightly above, tips the RDSC towards selecting the equilibrium where the less efficient firm produces the high quality: both firms remain in the market and the less efficient firm makes higher profits.[22]

Our analysis suggests that an appropriately chosen MQS can be used in order

[20]Because at $\overline{q}^F$ the jump in the value of domestic profits is substantial, $\widetilde{q} = \overline{q}^F$ may be optimal even when consumer surplus weighs more (but not that much more) than profit. When the regulator cares little about profits he will choose a MQS either above or below the $\left(\overline{q}^F, \overline{q}^D\right)$ region.

[21]Now, both consumer surplus and domestic profit are reduced, not only compared to situations with $\widetilde{q} \in \left(\overline{q}^F, \overline{q}^D\right)$, but also compared to MQS lying in the left neighborhood of $\overline{q}^F$.

[22]If one keeps increasing the level of MQS, a second equilibrium reversal will occur at $\overline{q}^D > \overline{q}^F$ and the less efficient firm will be eventually eliminated. Traditional intuition remains, therefore, ultimately valid. What our analysis suggests is that a) market monopolization will occur at levels of MQS higher than expected, and b) at some MQS levels expected to eliminate the less efficient firm, the latter will instead make higher profits by producing the high quality.

to target the equilibrium that confers a domestic firm higher profit at the expense of a more efficient foreign rival. Since a MQS always hurts high quality profits, once the equilibrium is selected the presence of MQS appears to reduce the profit of the domestic producer (who is now producing the high quality). What the above argument hides is, of course, the role of the MQS in selecting the equilibrium that is more favorable to the domestic firm. Hence, compared to other measures aiming to protect domestic producers, the protectionist action of the MQS is less obvious and creates less friction with international agreements.

Admittedly, like in almost all the papers in the related literature,[23] some of our crucial results rely on numerical analysis, for it is practically impossible to analytically compute the value of RDSC. This should not obscure the value of this paper's main intuition, namely that in asymmetric vertical differentiation models, a) analyzing policy measures whithin an exogenously selected equilibrium may be seriously misleading, and b) equilibrium targeting may be a policy objective per se.

Let us finally note that equilibrium targeting may apply in a more general context than that of the MQS, presented in this paper. First, our main results are qualitatively unaltered if we consider that in the second stage firms compete in quantities (as in Valletti, 2000). Second, it is conceivable that the players themselves engage in equilibrium targeting by selecting specific actions prior to the game. For example, one can imagine a three stage game where in the first stage some firm (or both firms) make strategic choices in order to tip the RDSC towards their favored outcome in subsequent stages. Investigating such possibility features in our research agenta.

[23]E.g., Motta (1993), Echia and Lambertini (1997), Scarpa (1998), Motta et al., (1997), or Moraga-Gonzàlez and Viaene (2005), among others.

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Figures:

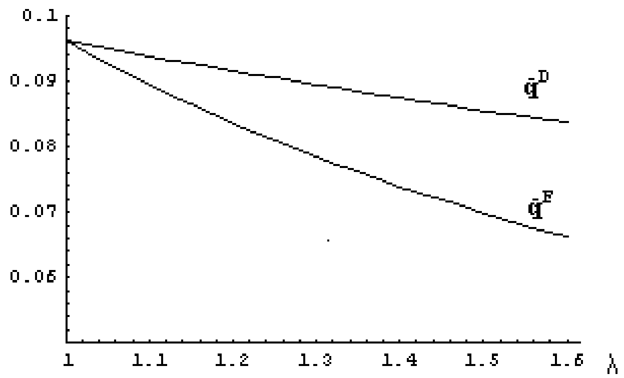


Figure 1: The evolution of $\overline{q}^D, \overline{q}^F$with λ, $\forall\lambda \in (1, 1.6)$.

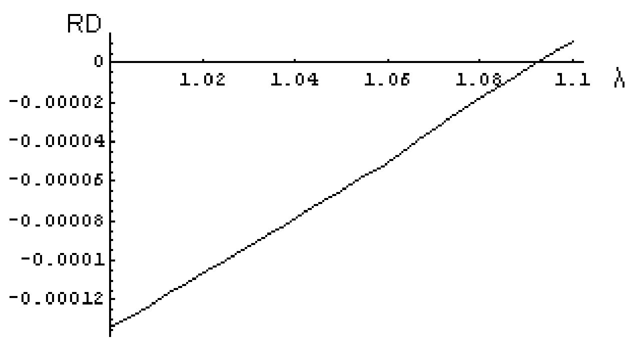
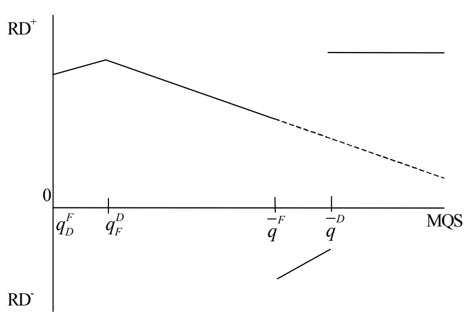


Figure 2: $RD\left(\widetilde{q} = \overline{q}^F\right)$ for $\lambda \in (1, 1.1)$.



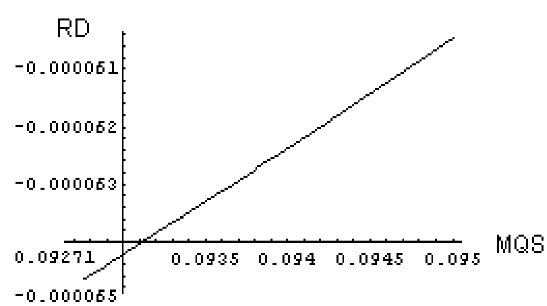


Figure 3a: $RD\left(\widetilde{q}\right)$ for all values of the MQS. for $\lambda \in (1, 1.085]$

Figure 3b: $RD\left(\widetilde{q}\right)$ for all relevant values of the MQS $\widetilde{q} \in \left[\overline{q}^F, \overline{q}^D\right]$, for $\lambda = 1.05$.

	RD	a_{HL}-a_{LL}	b_{HL}-b_{HH}	a_{LH}-a_{HH}	b_{LH}-b_{LL}
Exit is allowed	-6.5×10^{-5}	0.03125 - 0.01888 =0.01237	0 - (-0.03249) =0.03249	0.00014 - (-0.03105) =0.03119	0.01496 - 0 =0.01496
Exit is not allowed	9×10^{-5}	0.01661- (-0.00430) =0.02091	0 - (-0.03213) =0.03213	0.00014 - (-0.02975) =0.02989	0.01496 - (-0.00451) =0.01947

Table 1: The components of RD according to wether is or is not allowed, for λ=1.05 and $\widetilde{q} = \overline{q}^F(\lambda = 1.05)$.

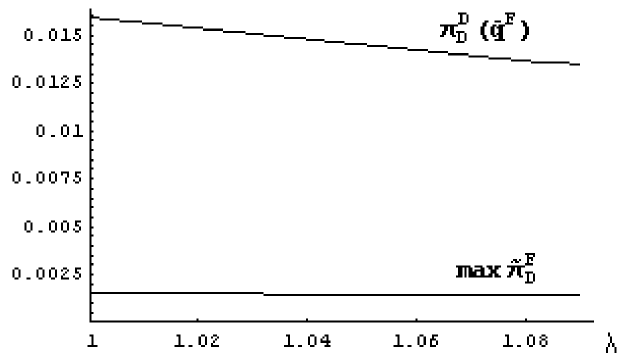
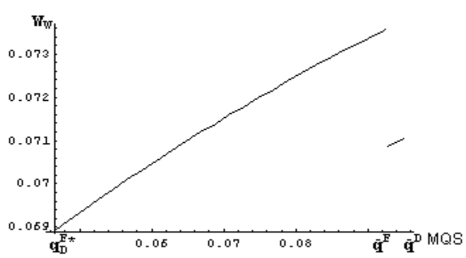


Figure 4: $\pi_D^D(\overline{q}^F)$ and $\max \widetilde{\pi}_D^F$ for $\lambda \in (1, 1.09)$.



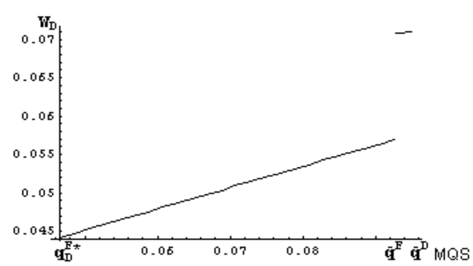


Figure 5a: $W_W(\widetilde{q})$ for $\lambda = 1.05$ for all effective values of the MQS

Figure 5b: $W_D(\widetilde{q})$ for $\lambda = 1.05$ for all effective values of the MQS

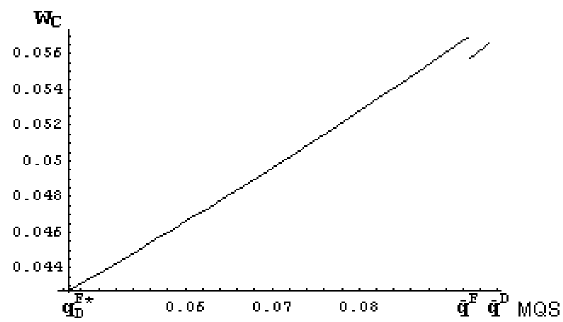


Figure 6: $W_C(\widetilde{q})$ for $\lambda = 1.05$ for all effective values of the MQS

Appendix 1

We impose a MQS at a level $\widetilde{q} = \overline{q}^F(\lambda)$, so that it will be restrictive in the E^F and E^D equilibrium, and the firm D will go out of business in the E^F equilibrium. Hence, the foreign firm is a monopolist in the E^F equilibrium and its optimal quality, denoted as $\widetilde{q}^F_F = q^F_M = \frac{1}{4}$, is derived accordingly. We have $\widetilde{q}^D_D = q^D_D(\overline{q}^F, \lambda)$, $\widetilde{q}^D_F = \overline{q}^F(\lambda)$, and $\widetilde{q}^F_D = 0$. Combining the Eq.(8) and (9) we obtain $\overline{q}^F(\lambda)$, $\widetilde{q}^D_D$ is given by Eq.(3).

The pay-offs under the presence of $\overline{q}^F$ are:

$\widetilde{b_{HL}} = 0$

$\widetilde{a_{HL}} = \pi^{FM}_F(q^F_M) = \frac{1}{4}(q^F_M - 2q^{F^2}_M)$

$\widetilde{b_{LH}} = \pi^D_D(\overline{q}^F, \widetilde{q}^D_D) = 4\widetilde{q}^{D^2}_D \frac{\widetilde{q}^D_D - \overline{q}^F}{\left(4\widetilde{q}^D_D - \overline{q}^F\right)^2} - \lambda\frac{\widetilde{q}^{D^2}_D}{2}$

$\widetilde{a_{LH}} = \pi^D_F(\overline{q}^F, \widetilde{q}^D_D) = \frac{\widetilde{q}^D_D \overline{q}^F\left(\widetilde{q}^D_D - \overline{q}^F\right)}{\left(4\widetilde{q}^D_D - \overline{q}^F\right)^2} - \frac{\overline{q}^{F^2}}{2}$

$\widetilde{b_{HH}} = \pi_D(q^F_M, \widetilde{q}^D_D) = 4\widetilde{q}^{D^2}_D \frac{\widetilde{q}^D_D - q^F_M}{\left(4\widetilde{q}^D_D - q^F_M\right)^2} - \lambda\frac{\widetilde{q}^{D^2}_D}{2}$

$\widetilde{a_{HH}} = \pi_F(q^F_M, \widetilde{q}^D_D) = \frac{\widetilde{q}^D_D q^F_M\left(\widetilde{q}^D_D - q^F_M\right)}{\left(4\widetilde{q}^D_D - q^F_M\right)^2} - \frac{q^{F^2}_M}{2}$

$\widetilde{b_{LL}} = 0$

$\widetilde{a_{LL}} = \pi_F(\overline{q}^F) = \frac{1}{4}(\overline{q}^F - 2\overline{q}^{F^2})$

Based on the corresponding expressions for optimal qualities and the previous expressions we compute $RD\left(\overline{q}^F(\lambda), \lambda\right)$ for the all values of λ. This yields that $\forall\lambda \in [1, 1.09]$, $RD(\overline{q}^F(\lambda), \lambda) < 0$.

Appendix 2: Proof of Proposition 3

In order to prove the proposition we need to show two auxiliary results. In what follows we only consider levels of $\widetilde{q}$ for which the MQS is effective, thus, the lower quality is equal to $\widetilde{q}$. Let $\theta^i_1 = \theta^i_1\left(q^i_i\left(\widetilde{q}, \lambda\right), \widetilde{q}, \lambda\right) = \left(2q^i_i - \widetilde{q}\right) \big/ \left(4q^i_i - \widetilde{q}\right)$, $i = F, D$, represent the taste parameter of the consumer just indifferent between buying the high or the low quality in the i equilibrium; similarly, let $\theta^i_2 = \theta^i_2\left(q^i_j\left(\widetilde{q}, \lambda\right), \widetilde{q}, \lambda\right) =$

$\left(q_i^i - \widetilde{q}\right) / \left(4q_i^i - \widetilde{q}\right)$ the taste parameter of the consumer just indifferent between buying the low quality or nothing at all. Hence, $W_W^F(\widetilde{q}) = \int_{\theta_2^F}^{\theta_1^F}(\theta\widetilde{q})d\theta + \int_{\theta_1^F}^{1}(\theta q_F^F)d\theta - \frac{q_F^{F^2}}{2} - \frac{\lambda\widetilde{q}^2}{2}$, $W_W^D(\widetilde{q}) = \int_{\theta_2^D}^{\theta_1^D}(\theta\widetilde{q})d\theta + \int_{\theta_1^D}^{1}(\theta q_D^D)d\theta - \frac{\lambda q_D^{D^2}}{2} - \frac{\widetilde{q}^2}{2}$, $W_D^F(\widetilde{q}) = \int_{\theta_2^F}^{\theta_1^F}(\theta\widetilde{q})d\theta + \int_{\theta_1^F}^{1}(\theta q_F^F - p_F^F)d\theta - \frac{\lambda\widetilde{q}^2}{2}$, $W_D^D(\widetilde{q}) = \int_{\theta_2^D}^{\theta_1^D}(\theta\widetilde{q} - p_F^D)d\theta + \int_{\theta_1^D}^{1}(\theta q_D^D)d\theta - \frac{\lambda q_D^{D^2}}{2}$, $W_C^F(\widetilde{q}) = \int_{\theta_2^F}^{\theta_1^F}(\theta\widetilde{q} - p_D^F)d\theta + \int_{\theta_1^F}^{1}(\theta q_F^F - p_F^F)d\theta$ and $W_C^D(\widetilde{q}) = \int_{\theta_2^D}^{\theta_1^D}(\theta\widetilde{q} - p_F^D)d\theta + \int_{\theta_1^D}^{1}(\theta q_D^D - p_D^D)d\theta$.

Lemma 2: In any subgame equilibrium, as long as both firms are active we have $dW_k^i/d\widetilde{q} > 0$, $k = W, D, C$, $i = F, D$

Proof. i) In order to prove that $\frac{dW_W^F}{d\widetilde{q}} = \frac{\partial W_W^F}{\partial\widetilde{q}} + \frac{\partial W_W^F}{\partial q_F^F}\frac{dq_F^F}{d\widetilde{q}} > 0$ we examine separately the sign of each term in the middle part. From Ronnen, 1991, we know that $\frac{dq_F^F}{d\widetilde{q}} > 0$. Further:

- $\frac{\partial W_W^F}{\partial\widetilde{q}} = \frac{1}{2}q_F^{F^2}\frac{20q_F^F - 17\widetilde{q}}{\left(4q_F^F - \widetilde{q}\right)^3} - \lambda\widetilde{q} > \frac{1}{2}q_F^{F^2}\frac{20q_F^F - 17\widetilde{q}}{\left(4q_F^F - \widetilde{q}\right)^3} - 1.09 * 0.1$

  since $\lambda = 1.09$ is the highest value of λ considered (see Proposition 1) and $\widetilde{q} = 0.1$ is an upper bound for $\widetilde{q}^D$ (see Figure 1).[24] The RHS of the above is equal to

  $0.001\frac{3024q_F^{F^3} - 3268q_F^{F^2}\widetilde{q} - 1308q_F^F\widetilde{q}^2 + 109\widetilde{q}^3}{\left(4q_F^F - \widetilde{q}\right)^3} > 0$ for $\widetilde{q} < 0.72q_F^F$.

  The latter is always true since $\widetilde{q} < 0.1 < 0.72 \times q_F^{*F} < 0.72 \times q_F^F$ for all level of λ considered, indeed a) as $\frac{dq_F^{*F}}{d\lambda} = \frac{\partial q_F^{*F}}{\partial q_D^{*F}}\frac{dq_D^{*F}}{d\lambda} < 0$ (see Zhou and al., 2000 and 2002) and $q_F^{*F}(\lambda = 1.09) = 0.2528$ then $\widetilde{q} < 0.1 < 0.72 \times q_F^{*F}$ for all level of λ considered, and b) as $\frac{dq_F^F}{d\widetilde{q}} > 0$ then $q_F^F \geq q_F^{*F}$.[25]

- $\frac{\partial W_W^F}{\partial q_F^F} = \frac{-18q_F^{F^2}\widetilde{q} + 5\widetilde{q}^2 q_F^F + \widetilde{q}^3 + 24q_F^{F^3}}{\left(4q_F^F - \widetilde{q}\right)^3} - q_F^F$

  Note that q_F^F is the marginal cost (with respect to quality) of the foreign firm. Since we are only interested in equilibrium situations, marginal cost can be

[24] It can be easily shown that $\overline{q}^D(\lambda)$ is decreasing in λ and that $\overline{q}^D(1) = \overline{q}^F(1) < 0.1$.

[25] q_F^{*F} is the high quality in E^F equilibrium without SQM.

replaced by marginal revenue which equals $-4q_F^F \frac{4q_F^{F^2} - 3q_F^F \widetilde{q} + 2\widetilde{q}^2}{\left(-4q_F^F + \widetilde{q}\right)^3}$. Substituting the latter for q_F^F we may write

$$\frac{\partial W_W^F}{\partial q_F^F} = \frac{-18q_F^{F^2}\widetilde{q} + 5\widetilde{q}^2 q_F^F + \widetilde{q}^3 + 24q_F^{F^3}}{\left(4q_F^F - \widetilde{q}\right)^3} - \left(-4q_F^F \frac{4q_F^{F^2} - 3q_F^F \widetilde{q} + 2\widetilde{q}^2}{\left(-4q_F^F + \widetilde{q}\right)^3}\right) = \frac{2q_F^{F^2} - q_F^F\,\widetilde{q} - \widetilde{q}^2}{\left(4q_F^F - \widetilde{q}\right)^2} > 0,$$

Hence, $\frac{dW_w^F}{d\widetilde{q}} > 0$. In a similar fashion we can show that $\frac{dW_W^D}{d\widetilde{q}} = \frac{\partial W_W^D}{\partial \widetilde{q}} + \frac{\partial W_W^D}{\partial q_F^F}\frac{dq_F^F}{d\widetilde{q}} > 0$.

ii)In order to prove that $\frac{dW_D^F}{d\widetilde{q}} = \frac{\partial W_D^F}{\partial \widetilde{q}} + \frac{\partial W_D^F}{\partial q_F^F}\frac{dq_F^F}{d\widetilde{q}} > 0$ we examine separately the sign of each term in the middle part. Further:

- $\frac{\partial W_D^F}{\partial \widetilde{q}} = \frac{9}{2}\frac{q_F^{F^2}}{\left(-4q_F^F + \widetilde{q}\right)^2} - \lambda\widetilde{q} > \frac{9}{2}\frac{q_F^{F^2}}{\left(-4q_F^F + \widetilde{q}\right)^2} - (1.09 * 0.1) > 0.001\frac{2756q_F^{F^2} + 872q_F^F\widetilde{q} - 109\widetilde{q}^2}{\left(4q_F^F - \widetilde{q}\right)^2} > 0$

  see i) for the substitution of $\lambda\widetilde{q}$ by $(1.09 * 0.1)$.

- $\frac{\partial W_D^F}{\partial q_F^F} = \frac{2q_F^{F^2} - q_F^F\widetilde{q} - \widetilde{q}^2}{\left(4q_F^F - \widetilde{q}\right)^2} > 0$

  Hence, $\frac{dW_w^F}{d\widetilde{q}} > 0$.

In order to prove that $\frac{dW_D^D}{d\widetilde{q}} = \frac{\partial W_D^D}{\partial \widetilde{q}} + \frac{\partial W_D^D}{\partial q_D^D}\frac{dq_F^D}{d\widetilde{q}} > 0$ we examine separately the sign of each term in the middle part. Further:

- $\frac{\partial W_D^D}{\partial \widetilde{q}} = \frac{3}{2}\frac{q_D^{D^2}}{\left(4q_D^D - \widetilde{q}\right)^2} > 0$

- $\frac{\partial W_D^D}{\partial q_D^D} = 3q_D^D \frac{2q_D^D - \widetilde{q}}{\left(4q_D^D - \widetilde{q}\right)^2} - \lambda q_D^D$

  Note that λq_D^D is the marginal cost (with respect to quality) of the domestic firm. Since we are only interested in equilibrium situations, marginal cost can be replaced by marginal revenue which equals $-4q_D^D \frac{4q_D^{D^2} - 3q_D^D\widetilde{q} + 2\widetilde{q}^2}{\left(-4q_D^D + \widetilde{q}\right)^3}$. Substituting the latter for λq_D^D we may write

  $\frac{\partial W_D^D}{\partial q_D^D} = 3q_D^D \frac{2q_D^D - \widetilde{q}}{\left(4q_D^D - \widetilde{q}\right)^2} - \left(-4q_D^D \frac{4q_D^{D^2} - 3q_D^D\widetilde{q} + 2\widetilde{q}^2}{\left(-4q_D^D + \widetilde{q}\right)^3}\right) = q_D^D \frac{8q_D^{D^2} - 6q_D^D\,\widetilde{q} - 5\widetilde{q}^2}{\left(4q_D^D - \widetilde{q}\right)^3} > 0$ for $\widetilde{q} < \frac{4}{5}q_F^F$ (this is always true, see above), Hence, $\frac{dW_D^D}{d\widetilde{q}} > 0$.

iii) In order to prove that $\frac{dW_C^F}{d\widetilde{q}} = \frac{\partial W_C^F}{\partial \widetilde{q}} + \frac{\partial W_C^F}{\partial q_F^F}\frac{dq_F^F}{d\widetilde{q}} > 0$ we examine separately the sign of each term in the middle part. Further:

- $\frac{\partial W_C^F}{\partial \widetilde{q}} = \frac{1}{2} q_F^{F2} \frac{28 q_F^F + 5\widetilde{q}}{\left(4q_F^F - \widetilde{q}\right)^3} > 0$
- $\frac{\partial W_C^F}{\partial q_F^F} = q_F^F \frac{8{q_F^F}^2 - 6q_F^F\,\widetilde{q} - 5\widetilde{q}^2}{\left(4q_F^F - \widetilde{q}\right)^3} > 0$ for $\widetilde{q} < \frac{4}{5} q_F^F$ (this is always true)

Hence, $\frac{dW_C^F}{d\widetilde{q}} > 0$ and $\frac{dW_C^D}{d\widetilde{q}} > 0$.

■

Lemma 3: $\forall \widetilde{q} \leq \overline{q}^F$, i) $W_W^F(\widetilde{q}) > W_W^D(\widetilde{q})$, ii)$W_C^F(\widetilde{q}) > W_C^D(\widetilde{q})$, while iii)$W_D^F(\widetilde{q}) < W_D^D(\widetilde{q})$

Proof. i) We prove that $\forall \widetilde{q} \leq \overline{q}^F$, a) $W_P^F(\widetilde{q}) > W_P^D(\widetilde{q})$, with W_P^j the sum of both firms profit in both E^F, E^D equilibria, and b) $W_C^F(\widetilde{q}) > W_C^D(\widetilde{q})$.

a) The profit of the producer of high quality (noted πh in this proof) corresponds to π_F^F in the equilibrium E^F, and π_D^D in the equilibrium E^D. $\forall \widetilde{q} \leq \overline{q}^F$, we have $\pi h = 4qh^2 \frac{qh - \widetilde{q}}{(4qh - \widetilde{q})^2} - \lambda h \frac{qh^2}{2}$. In E^F, the producer of high quality is firm F, we have, therefore, $qh = q_F^F$ and $\lambda h = \lambda_F = 1$. In E^D the producer of high quality is firm D, thus we have $qh = q_D^D$ and $\lambda h = \lambda_D > 1$.

The profit of the low quality producer (denoted as πl in this proof) corresponds to π_D^F in equilibrium E^F and π_F^D in equilibrium E^D. $\forall \widetilde{q} \leq \overline{q}^F$ we have $\pi l = \frac{qh\ \widetilde{q}(qh - \widetilde{q})}{(4qh - \widetilde{q})^2} - \lambda l \frac{\widetilde{q}^2}{2}$. In E^F the producer of low quality is firm D, we have therefore, $\lambda l = \lambda_D > 1$, in E^D the producer of low quality is the firm F, we have $\lambda l = \lambda_F = 1$.

The variations of πh and πl between both equilibria E^F and E^D are the consequence of a *positive variation* of λh (i.e. $\lambda_D - 1$), and a *negative variation* of λl (i.e. $1 - \lambda_D$).

$\forall \widetilde{q} \leq \overline{q}^F$, we have :

$\frac{d\pi h}{d\lambda h} + \frac{d\pi l}{d\lambda h} - (\frac{d\pi h}{d\lambda l} + \frac{d\pi l}{d\lambda l}) = \frac{\partial \pi h}{\partial qh}\frac{dqh}{d\lambda h} + \frac{\partial \pi h}{\partial \lambda h} + \frac{\partial \pi l}{\partial qh}\frac{dqh}{d\lambda h} - (\frac{\partial \pi h}{\partial qh}\frac{dqh}{d\lambda l} + \frac{\partial \pi l}{\partial qh}\frac{dqh}{d\lambda l} + \frac{\partial \pi l}{\partial \lambda l})$

As $\frac{\partial \pi h}{\partial qh} = 0$, $\frac{\partial \pi l}{\partial qh} = \widetilde{q}^2 \frac{2qh+\widetilde{q}}{(4qh-\widetilde{q})^3} > 0$, $\frac{dqh}{d\lambda h} < 0$ (see Zhou and al., 2000 and 2002) and $\frac{dqh}{d\lambda l} = \frac{\partial qh}{\partial \widetilde{q}}\frac{d\widetilde{q}}{d\lambda l} = 0$, then $\frac{d\pi h}{d\lambda h} + \frac{d\pi l}{d\lambda h} - (\frac{d\pi h}{d\lambda l} + \frac{d\pi l}{d\lambda l}) = -\frac{1}{2}(qh^2 - ql^2) + \frac{\partial \pi l}{\partial qh}\frac{dqh}{d\lambda h} < 0$, $W_P^F(\widetilde{q}) > W_P^D(\widetilde{q})$.

b) The variation of consumer surplus between both equilibriums E^F and E^D is the consequence of the *positive variation* of λh, and the *negative variation* of λl.

$\forall \widetilde{q} \leq \overline{q}^F$, we have :

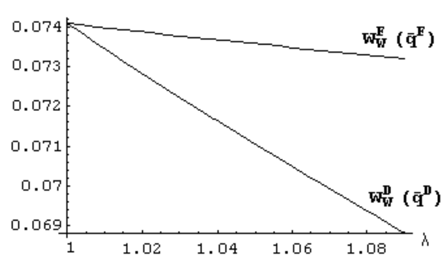
$\frac{dW_C}{d\lambda h} - \frac{dW_C}{d\lambda l} = \frac{\partial W_C}{\partial qh}\frac{dqh}{d\lambda h} - \frac{\partial W_C}{\partial qh}\frac{dqh}{d\lambda l}$

As $\frac{\partial W_C}{\partial qh} = \frac{(2qh^2+qh\,\widetilde{q})(4qh-5\widetilde{q})}{(4qh-\widetilde{q})^3} > 0$ ($\forall \lambda \in (1, 1.09]$, we have $\widetilde{q} < \frac{4}{5}qh$), $\frac{dqh}{d\lambda h} < 0$ and $\frac{dqh}{d\lambda l} = 0$, then $\frac{dW_C}{d\lambda h} - \frac{dW_C}{d\lambda l} = \frac{\partial W_C}{\partial qh}\frac{dqh}{d\lambda h} < 0$, $W_C^F(\widetilde{q}) > W_C^D(\widetilde{q})$.

ii) It is proved by i) b).

iii) For $\lambda = 1$, then $\forall \widetilde{q} \leq \overline{q}^F$ we have $W_C^F(\widetilde{q}) = W_C^D(\widetilde{q})$, $\pi_D^F(\widetilde{q}) < \pi_D^D(\widetilde{q})$ and $W_D^F(\widetilde{q}) < W_D^D(\widetilde{q})$. When λ increases we have $W_C^F(\widetilde{q}) > W_C^D(\widetilde{q})$ (see ii) and $\pi_D^F(\widetilde{q}) < \pi_D^D(\widetilde{q})$. As $W_C^F(\widetilde{q})$, $W_C^D(\widetilde{q})$, $\pi_D^F(\widetilde{q})$, $\pi_D^D(\widetilde{q})$ are continous in λ, we may say that for λ marginally superior to 1, $W_D^F(\widetilde{q}) < W_D^D(\widetilde{q})$. ■

For prove the proposition 3, we also need that $W_W^F(\overline{q}^F) > W_W^D(\overline{q}^D)$, and similarly that $W_C^F(\overline{q}^F) > W_C^D(\overline{q}^D)$. The following figures demonstrate it.



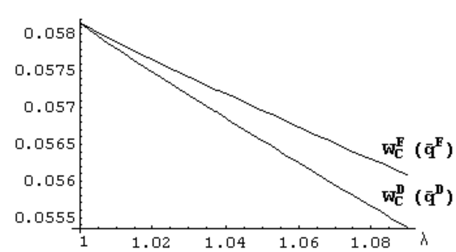


Figure A1: $W_W^F(\overline{q}^F)$ and $W_W^D(\overline{q}^D)$ for $\lambda = 1.05$ Figure A2: $W_C^F(\overline{q}^F)$ and $W_C^D(\overline{q}^D)$ for $\lambda = 1.05$