Profit maximisation and alternatives in oligopolies*

Alexander Klemm†

Institute for Fiscal Studies

August 6, 2004

Abstract

This paper analyses oligopolies using the Cournot/Stackelberg framework, but allowing some firms to be pursueing aims other than profit maximisation. The existence of even a single output maximising firm can have dramatic effects on outputs, prices and welfare, even if such a firms faces additional costs.

*The author wishes to thank Steffen Huck, Marco Schonborn and seminar participants at UCL for helpful comments.

†IFS, 7 Ridgmount Street, London WC1E 7AE; Tel: 0044 20 7291 4800; Fax: 0044 20 7323 4780; e-mail: a.klemm@ifs.org.uk.

JEL classifications: L13, L21, L31, D43.

Key words: profit maximisation, oligopoly, non-profit organisations

1. Introduction

There is now a large literature on why firms may not always be profit maximising.[1] The reasons include the following possibilities, but are not restricted to them. 1. Firms may wish to promote a certain output out of ideological convictions(fan-clubs, churches, certain cultural organisations). 2. Owner managers may not be willing to put effort into increasing profits above some minimum level. 3. Owners may value the quality of their output.

The aim of this paper is to analyse the effects that the existence of firms that maximise something other than profits will have on markets. It is not an aim of this paper to discuss the reasons for such behaviour, as the focus is purely on the effects. While examples will be given, these are only meant to provide some intuition rather than to imply that the results only apply in these cases.

Throughout the paper an oligopolistic market will be assumed. This is because in such a market firms earn economic rents that they can choose to forgo without risking to go out of business. In a perfectly competitive market however, firms

[1] See *inter alia* Rose-Ackerman (1996) for a survey.

make zero economic profits, even when maximising profits and there is thus less scope for alternatives to profits maximisation.

If firms do not maximise profits, there are infinite possibilities of what they may do instead. As early as in De Scitovszky (1943) the idea was expressed that managers may in fact maximise something else. One possibility is that they might try to maximise output, subject to making a minimum profit of M.[2] An alternative interpretation of the same formulation is that the maximise output subject to the constraint that they wish to break even, but that they face fixed costs of M. An example for such a behaviour would be a cultural organisation that wishes to promote a certain cultural activity. Other examples include churches and fan-clubs. Note that if such an organisation received a subsidy, then M would be negative, i.e. they would be allowed to be loss-making to some extent.

Without wishing to be exhaustive about the possible maximisation functions, one alternative is considered. This is that managers might minimise output subject to a minimum profit of M. An example of such an incentive would be an owner-manager who values nothing more than his spare time and the avoidance of effort, but who needs a certain income.

[2] More common in the literature is the assumption that managers maximise utility, which includes both profits and sales (*inter alia* Vickers (1985)). While the set-up proposed here seems very similar (with a zero weight on profits), the results are quite different due to the non-linearity caused by the existence of a minimum profit.

2. The theoretical set-up

The model used is the well-known oligopoly model of quantity competition developed by Cournot.[3] The inverse demand function is assumed linear and normalised to $p = 1 - Q$, where p is price and $Q \equiv \sum q_i$ is total quantity produced, where q_i is output of firm i. Marginal cost are assumed away for now.

The reaction function of profit-maximising firms in this setting are obtained by maximising profits taking the other firms' output decisions as given. They take the form:

$$q_i = \frac{1}{2}\left(1 - \sum_{j \neq i} q_j\right) \tag{2.1}$$

For firms that maximise quantity instead of profits, the reaction function is obtained by maximising output subject to profits being greater than M. Given that profits π_i are decreasing in output for any output greater than the profit-maximising one, the solution can be obtained by solving:

[3] This is covered by most microeconomic textbook, *inter alia* Varian (1992), chapter16.

$$\pi_i = \left(1 - \sum_{j=1}^{N} q_j\right) q_i - M = 0 \tag{2.2}$$

$$\Longleftrightarrow q_i = \frac{1}{2}\left(1 - \sum_{j \neq i} q_j \pm \sqrt{\left(1 - \sum_{j \neq i} q_j\right)^2 - 4M}\right) \tag{2.3}$$

The relevant solution clearly is the one where both terms are added, as the aim is maximise quantity. However in the case of the effort-avoiding quantity-minimising owner-manager, the relevant solution is the one where both terms are subtracted.

3. Quantity maximisation

3.1. Assuming no fixed costs

Assuming fixed costs away simplifies the algebra and reveals an interesting special case. The reaction function of the quantity-maximising firm in that case reduces to:

$$q_i = 1 - \sum_{j \neq i} q_j \tag{3.1}$$

Solving for a Nash-equilibrium (NE) leads the following result:

Proposition 1. *In the absence of fixed costs, quantity competition will lead to marginal cost pricing and zero profits if there is at least one firm that maximises output subject to non-negative profits. This result does not depend on whether players move simultaneously or sequentially.*

In the case of simultaneous moves, the best reply of the quantity-maximising firm is always to increase output until the total quantity in the market reaches 1, at which point prices and profits become 0. The profit-maximising firms then however always face an incentive to cut their output to earn at least some scarcity rents. Therefore the only NE is the one in which only quantity-maximising firms will produce. If the quantity maximiser is a Stackelberg leader, the same result is obtained, because such a firm will choose to serve the whole market, knowing that profit-maximising firms will then cut their outputs to zero. The situation is slightly more complicated if the profit maximiser is leader. Then the profit maximiser knows that his profits will be nil, irrespective of the output choice. If he has even an infinitesimal preference for larger output, he may well choose to serve the whole market himself, given that no profits are lost, but output is gained. If he has no such preference, he can randomly pick any output including zero.

At first sight this result is attractive, as it shows that efficient outcomes can be obtained by ensuring that one firm does not maximise profits. Regulation of uncompetitive industries could thus easily be achieved by setting up a public firm. Furthermore this is costless, as this firm is breaking even. The result however also has a number of drawbacks. First, it is not very attractive that profit-maximising firms cease producing (except possibly when they are Stackelberg leaders), because that leaves one quantity maximiser as a monopolist. Second, it is not clear how the efficiency of quantity-maximising managers can be ensured. After all, they make zero profits, whatever their actions. The game may become more realistic by re-introducing fixed costs or minimum profits. These could either be because the quantity-maximising firm has higher cost because of some inefficiency, or because the minimum profit is needed to pay managers a salary if they successfully maximise output.

3.2. Allowing fixed costs or minimum profits

In order to facilitate the exposition, it will from now on be assumed that there are only two firms, firm 1 which maximises output and firm 2 which maximises profits.[4] The reaction functions are then:

[4] To allow comparisons with the Cournot case, the results obtained with two firms facing identical reaction functions and no fixed or marginal cost are repeated here: $q_i = \frac{1}{3}, Q = \frac{2}{3}, p = \frac{2}{3}$ and $W = \frac{4}{9}$, where W is welfare, i.e. the sum of consumer surplus and profits.

$$
\begin{aligned}
q_1 &= \frac{1}{2}\left(1 - q_2 + \sqrt{(1-q_2)^2 - 4M}\right) \\
q_2 &= \frac{1}{2}(1 - q_1)
\end{aligned}
\tag{3.2}
$$

A problem with this set of reaction functions is that the reaction function of the quantity-maximising firm is not defined over the whole relevant interval $[0,1]$ for any $M > 0$. This is because for a positive M, there will always be an output of firm 2 that is too high so that no reaction of firm 1 can ensure the minimum profit of M. In that case it will be assumed that firm 1 will simply try to come as close to its required profit as possible, i.e. in that case firm 1 will maximise profits and its reaction function will equal the Cournot one. The complete reaction function can thus be described as a combination of an iso-profit curve for profits of M and a linear Cournot reaction function. Formally, the reaction function of firm 1 thus is:

$$
\begin{aligned}
q_1 &= \frac{1}{2}\left(1 - q_2 + \sqrt{(1-q_2)^2 - 4M}\right) && \forall 0 \leq q_2 \leq 1 - 2\sqrt{M} \\
&= \frac{1}{2}(1 - q_2) && \forall 1 - 2\sqrt{M} \leq q_2 \leq 1
\end{aligned}
\tag{3.3}
$$

3.2.1. Simultaneous moves

There can be one, none or even multiple equilibria, depending on the level of M. The following levels of M are interesting cases for which reaction functions are depicted in figure 1:

- $M = 0$, i.e. no profits required and the solution discussed above is obtained.
- $M = 1/9$, i.e. the Cournot profit.
- $M = 1/8$, i.e. the Stackelberg profit.

The reaction functions of firm 2 is depicted as a dashed line. The ones of firm 1 are the solid lines, plotted for different values of M. As M increases, firm 1's reaction function shifts downwards. The lower bound is the Cournot reaction function, which applies when minimum profits cannot be reached at any point, so that the firm just maximises profits. The upper bound is the straight line obtained in the case of zero-profit requirement. In all intermediate cases the reaction function will be a combination of an iso-profit curve and the Cournot reaction function.

In the case of a zero-profit requirement, the reaction functions cross in the corner at point A as discussed above. When the required minimum profit is the

Cournot one, the reaction functions cross twice, including at point C, the Cournot outcome. The other equilibrium is at point B, where the quantity maximiser produces more than the other firm. Total profits must be lower, as firm 1 is still making Cournot profits and firm 2 clearly makes less than that. While the equilibrium at point B is stable, the Cournot equilibrium at point C is stable from the right only. An infinitesimal cut in firm 2's output will thus lead to the other equilibrium. For any minimum profit between 0 and the Cournot profit, there will be a unique equilibrium between points A and B, where total output will be greater than in the Cournot case and profits lower.

If profits are required to be at the Stackelberg level, there are again two possible equilibria, one at point D, at which the Stackelberg outcome is reached, and one at point C, where the Cournot outcome is reached. The Stackelberg outcome is stable from the left only. Any random increase by the profit-maximiser will lead to the Cournot outcome. At the Cournot outcome, the quantity maximiser is not making the required profit though. He may give up so that a monopoly would result. For profits between the Cournot and the Stackelberg ones, there will be three equilibria, one between B and D (stable), one between D and C (unstable) and one at C (stable). In that case the efficiency enhancing equilibrium is stable, although the Cournot outcome, which may lead to monopoly, is so as well.

The stable equilibria on the line A to D can be easily calculated from equation (3.2). They are:

$$\begin{aligned} q_1 &= \frac{1}{2} + \frac{1}{2}\sqrt{1-8M} \\ q_2 &= \frac{1}{4} - \frac{1}{4}\sqrt{1-8M} \\ Q &= \frac{3}{4} + \frac{1}{4}\sqrt{1-8M} \qquad \forall 0 < M \leq \frac{1}{8} \end{aligned} \tag{3.4}$$

Finally, if required profits are more than Stackelberg ones, the only equilibrium will be the Cournot one, where however the required profits are not reached.

3.2.2. Sequential moves

While the sequence of moves did not matter much in the case of no fixed costs, the outcomes are now affected by who moves first.

Assume first that the quantity-maximising firm is the Stackelberg leader. It will pick the point with the highest output for its required level of minimum profits that lies on the other firms reaction function. In other words, the leader will choose the point with the highest output of those points where its iso-profit curve cuts the other firm's reaction function. But remember that the upper part of the quantity-maximiser's reaction function is just the iso-profit curve. Hence nothing changes in those case where there is already a unique equilibrium on the interval (A, B) in Figure 1, i.e. when the required profits are less than Cournot-profits. For minimum profits at or above the Cournot level, the only change is that now the equilibrium with the higher output, i.e. the on on the interval $[B, D)$, will now be the unique equilibrium. For minimum profits at the Stackelberg level, unsurprisingly exactly the Stackelberg solution is obtained.

The situation is different if the profit maximiser is leader. Suppose first that minimum profits are relatively high. In that case, the quantity maximiser's reaction function will be identical to the Cournot reaction function at the range that is relevant. The outcome will thus be the standard Stackelberg solution with the leader producing $q_2 = \frac{1}{2}$ at a profit of $\pi_2 = \frac{1}{8}$ and the follower producing $q_1 = \frac{1}{4}$ at a profit of $\pi_1 = \frac{1}{16}$. This solution is obtained for any minimum profit of at least the Stackelberg follower profits. Note though that any minimum profit in excess of Stackelberg follower profits cannot be reached by the follower. For lower profits, the solution will be different, because the follower's reaction function will not be identical to the Cournot one anymore on the relevant range. The optimal solution for the leader then is locate at the corner of the followers reaction function, i.e. at the point where the reaction function changes from an iso-profit line to the

Cournot reaction function. The solution to the game where the profit-maximising firm is leading therefore are:

$$
\begin{aligned}
q_1 &= 1 - 2\sqrt{M} \\
q_2 &= \sqrt{M} \\
Q &= 1 - \sqrt{M} \qquad\qquad \forall 0 < M \leq \frac{1}{16}
\end{aligned} \tag{3.5}
$$

Note that in that case the quantity-maximising firm reaches its minimum profit level. Total output is higher than in the pure original Stackelberg case ($1-\sqrt{M} \geq \frac{3}{4} \forall 0 \leq M \leq \frac{1}{16}$), but lower than under simultaneous moves ($1 - \sqrt{M} \leq \frac{3}{4} + \frac{1}{4}\sqrt{1-8M} \forall 0 \leq M \leq \frac{1}{8}$).

3.3. Welfare

If M represents a minimum profit or a payment to managers, then the welfare analysis is straightforward. Any outcome with outputs higher than in the Cournot case will be welfare enhancing. If M however represents a cost, then its detrimental effect on output should be accounted for. In the simultaneous move case, total output is always greater than or equal to the Stackelberg output (of 3/4). Welfare must be higher than in the standard duopoly case if M represents minimum profits or a payment to managers. If M however represents an efficiency cost, then welfare is given by:

$$\begin{aligned} W &= Q - \frac{1}{2}Q^2 - M \\ &= \frac{7}{16} + \frac{1}{16}\sqrt{1-8M} - \frac{3}{4}M \end{aligned} \tag{3.6}$$

Note that welfare is decreasing in M. By comparing it to the Cournot welfare (of 4/9), we can calculate the maximum M, up to which having a quantity-maximising firm with fixed costs enhances welfare:

$$\begin{aligned} \frac{7}{16} + \frac{1}{16}\sqrt{1-8M} - \frac{3}{4}M &= \frac{4}{9} \\ \Longrightarrow M = \frac{\sqrt{6}-1}{27} &\approx 0.054 \end{aligned} \tag{3.7}$$

Hence even if there are fixed costs to being a non-profit maximising firm, welfare will be enhanced as long as these costs are less than about half the Cournot profits.

A similar result is obtained for marginal costs. Suppose the quantity-maximising firm faces fixed marginal costs of c. The reaction functions then become:

$$\begin{aligned} q_1 &= 1 - c - q_2 \\ q_2 &= \frac{1}{2}(1 - q_1) \end{aligned} \tag{3.8}$$

The equilibrium is then given by:

$$\begin{aligned} q_1 &= 1 - 2c \\ q_2 &= c \\ Q &= 1 - c \end{aligned} \tag{3.9}$$

Output in the Cournot case would be 2/3. Therefore a market with a quantity maximising firm which faxes a marginal cost, will lead to higher total output as long as marginal costs are not greater than 1/3. If costs are any higher, the outcome will be worse than Cournot (but better for the profit maximiser). As costs reach 0.5, the quantity-maximising firm will not produce any output and the maximiser becomes a monopolist. Welfare must now be adjusted for the cost incurred by the quantity-maximising firm:

$$\begin{aligned} W &= Q - \frac{1}{2}Q^2 - q_1 c \\ &= \frac{3}{2}c^2 - c + \frac{1}{2} \end{aligned} \tag{3.10}$$

This is higher than the Cournot welfare up to a marginal cost of $1/3\left(1-\sqrt{2/3}\right) \approx 0.061$.

4. Quantity minimisation

We now turn to the "lazy" owner-manager. The set-up of the problem is exactly the same, just that know the two terms in equation 2.3 are subtracted rather than added. In the case of zero required profits, this unsurprisingly leads to the result that such a firm will not produce. However, for a positive amount of minimum profits, both firms will produce. As shown in figure 2, equilibrium can be reached for minimum profits up to and including the Cournot profit. The single equilibria are always between A and B. If profits are required to be larger than Cournot ones, the reaction functions still only cross in the Cournot point. Higher profits are thus not feasible in equilibrium. The equilibria as a function of M are given by:

$$\begin{aligned} q_1 &= \frac{1}{2} - \frac{1}{2}\sqrt{1-8M} \\ q_2 &= \frac{1}{4} + \frac{1}{4}\sqrt{1-8M} \\ Q &= \frac{3}{4} - \frac{1}{4}\sqrt{1-8M} \end{aligned} \tag{4.1}$$

The welfare analysis is straightforward. Even without any extra costs, be they fixed or marginal, welfare is generally lower than in the Cournot case, except when required profits are the Cournot ones, in which case welfare is the same. Intuitively this is because the quantity maximising firm, will always reduce output if its profits turn out to be higher than expected. The profit-maximising firm will react to that by increasing output, although by less than the reduction, because the profit-maximiser takes the marginal revenue effect of higher output into account.

5. Conclusion

This paper has shown that the welfare in an oligopoly can be enhanced if there are firms that maximise output instead of profits. If they require zero profits, they will in most circumstances drive out any profit-maximising firms. If required profits are positive, but below Cournot profits, market output will be higher and prices lower. If profits are required to be even higher, but below Stackelberg ones, there will be multiple equilibria, more than one of which may be stable. Welfare is generally found to be increased by the presence of such firms, unless they face substantial additional efficiency costs.

It is also shown that there can be equilibria with output-minimising firms, which can be thought of as lazy owner managers. The presence of such firms can only diminish welfare.

At the risk of overstretching this simple model, the following conclusions for policy can be drawn. It may well be in the public interest to have a firm which is not forced to maximises profits in an oligopolistic industry. This is even beneficial if there are efficiency costs, provided they are not too large. Hence, simply showing that a nationalised firm is less efficient than the private sector is not a sufficient argument for privatisation, as the net effect on welfare could still be positive.

There is a lot of anecdotal evidence of non-profit maximising behaviour in small owner-managed establishments. Take as an example a small shop offering bad service despite being in competition with a supermarket. The model shows that such a shop is not necessarily doomed to failure, but may survive in equilibrium.

This short paper can at best shed light on some of the issues of non-profit maximising firms in oligopolistic markets. Dynamic models could change the results, particularly if firms have the opportunity of pretending to be of one type in some periods and then to change tactics.

6. References

De Scitovszky, T. (1943) "A Note on Profit Maximisation and its Implications" Review of Economic Studies, Vol.11-1, pp57-60

Rose-Ackerman, S. (1996) “Altruism, Nonprofits and Economic Theory” Journal of Economic Literature Vol. 34-2, pp 701-128

Varian, H.R. (1992) "Microeconomic Analysis" New York: Norton.

Vickers, J. (1985) “Delegation and the Theory of the Firm” Economic Journal Vol.95 Issue Supplement: Conference Papers (1985)

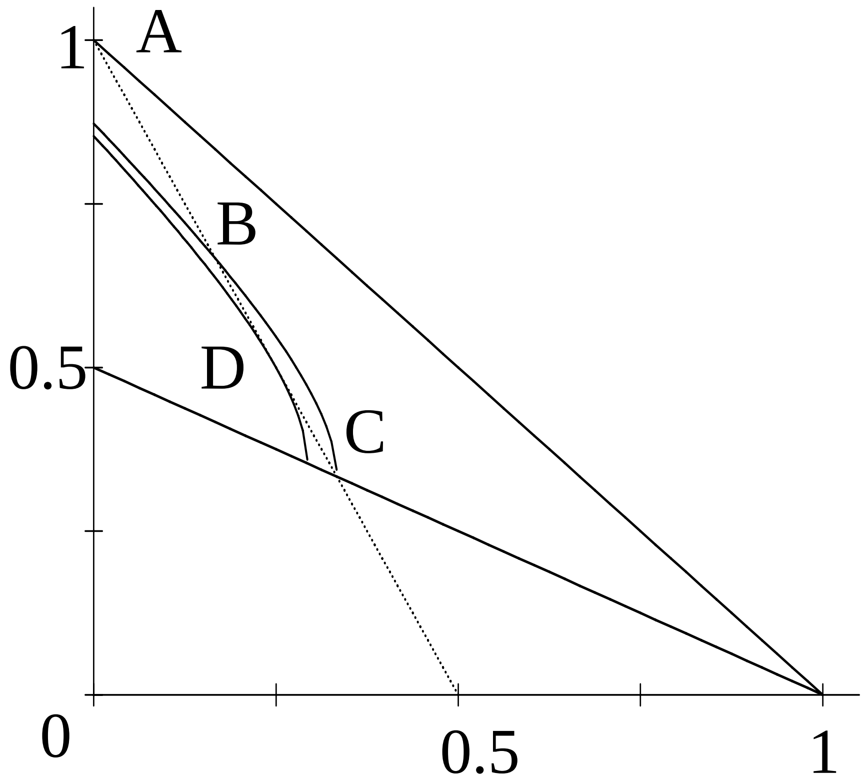

Figure 1: Reaction functions for different minimum profits of a quantity-maximising firm.

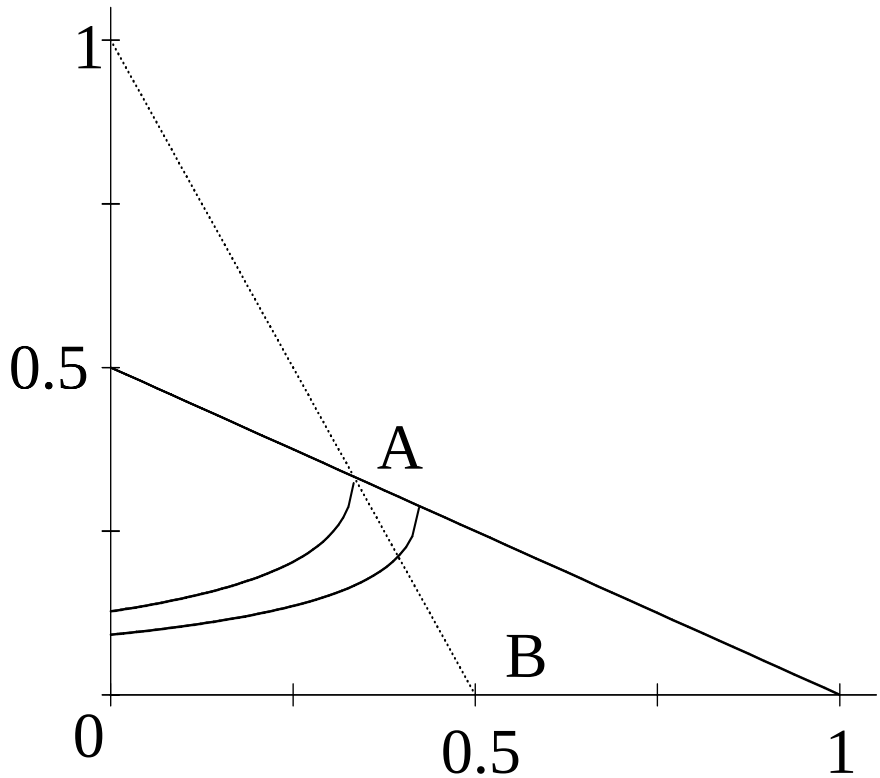

Figure 2: Reaction functions for different minimum profits of a quantity-minimising firm