# Equilibrium Vertical Foreclosure in the Repeated Game

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**Abstract**

This paper analyzes if vertical foreclosure can emerge as an equilibrium outcome of an infinitely repeated game. Foreclosure is profitable due to a "raising rival's costs" effect but it is not a Nash equilibrium of the static game. The results are that foreclosure is in fact a subgame perfect Nash equilibrium of the repeated game, and it may facilitate collusion compared to the nonintegrated industry. The possibility of a counter merger of the nonintegrated firms negatively affects the likelihood and profitability of collusive foreclosure.

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# 1 Introduction

One of the main arguments in the foreclosure literature is that vertical integration may enable to "raise rival's costs". The argument was first put forward by Ordover, Saloner and Salop (1990) (henceforth OSS) and is as follows. A vertically integrated firm has an incentive to foreclose nonintegrated downstream firms because this reduction of upstream competition leads to higher input cost for the nonintegrated downstream rivals. Since the downstream segment of the integrated firm benefits when rivals' costs are raised, the integrated firm is better off pursuing such a foreclosure strategy.

Hart and Tirole (1990) and Reiffen (1992) criticized this argument. They pointed out that the result depends on the assumption that integrated firms can commit not to deliver nonintegrated downstream rivals. Without commitment, vertically integrated firms will compete just like the other upstream firms. In particular, it is a best reply not to foreclose the nonintegrated downstream firms and so Nash equilibrium prices are the same with and without vertical merger.

This paper investigates whether foreclosure can be an equilibrium in the infinitely repeated game. This idea has been proposed by Riordan and Salop (1995), and the intuition is straightforward. Repeated interaction (Macauley, 1963) can help the integrated firm establishing a reputation for staying out of the nonintegrated market and this will induce nonintegrated upstream firms to charge higher prices. Since all upstream firms benefit from vertical foreclosure in the long run, it could be an equilibrium in the repeated game if firms are sufficiently patient.

The results in this paper show that this intuition is correct. In an oligopoly model similar to the one in OSS, foreclosure is indeed a subgame perfect Nash equilibrium of the infinitely repeated game, provided firms' discount factor is sufficiently high. In a general model, we show that in fact any individual rational price charged to the nonintegrated downstream firm can be part of a collusive equilibrium. Equilibrium refinements help reducing the set of plausible prices as they suggest that the non-integrated firm will be charged a price at least as high as the monopoly price. Using a parametrized model, we further show that collusive foreclosure with vertical integration often requires a lower minimum discount factor than collusion under vertical separation. The possibility of a counter merger of the nonintegrated firms negatively affects the likelihood and profitability of such collusive foreclosure. Finally, we discuss what happens if collusion at the downstream level is a possibility.

## 2 The Model and static Nash equilibrium

The market has two upstream firms and two downstream firms, as in OSS. Call the two upstream firms $U1$ and $U2$, and the two downstream firms $D1$ and $D2$. Upstream firms have constant marginal cost which are normalized to zero. For simplicity, the (linear) input prices the upstream firms post constitute the only cost of the downstream firms.[1]

At the downstream level, there is differentiated price competition. Denote by $Q_i(p_i, p_j)$, $i, j = 1, 2, i \neq j$, the demand function of $Di$. When $Di$ pays a price of $c_i$ per unit of the input, its profits are

$$\pi_{Di} = (p_i - c_i)Q_i(p_i, p_j), \quad i, j = 1, 2, i \neq j. \tag{1}$$

We impose the following assumptions on demand. Demand functions $Q_i(p_i, p_j)$ are twice continuously differentiable with $\partial Q_i/\partial p_i < 0$, $\partial Q_i/\partial p_j > 0$, and $\partial Q_i/\partial p_i - \partial Q_i/\partial p_j < 0$, $i, j = 1, 2$, $i \neq j$. These assumptions ensure downward sloping demand with substitutes goods. Further, we assume that goods are strategic complements, that is, $\partial^2 Q_i/\partial p_i \partial p_j > 0$. A final assumption is that $\partial^2 Q_i/\partial p_i^2 + \partial^2 Q_i/\partial p_i \partial p_j < 0$. These assumptions imply that the upward sloping best-reply functions have a slope of less than one and, hence, they are sufficient to ensure the existence of a unique Nash equilibrium of the stage game.[2]

Let $p_i^*(c_i, c_j)$, $i, j = 1, 2$, $i \neq j$, denote the static Nash equilibrium prices at the $D$ level. In the static Nash equilibrium, the input price vector $(c_i, c_j)$ sufficiently describes downstream competition, and we will often use $Q_i^*(c_i, c_j)$ as a shotcut for $Q_i(p_i^*(c_i, c_j), p_j^*(c_j, c_i))$, and $\pi_{Di}^*(c_i, c_j)$ for $\pi_{Di} = (p_i^*(c_i, c_j) - c_i)Q_i^*(c_i, c_j)$.

Given the above assumptions, it is easy to verify that raising the cost of a downstream rival is profitable, that is,

$$\frac{\partial \pi_{Di}^*(c_i, c_j)}{\partial c_j} = \frac{\partial \pi_{Di}}{\partial p_j}\frac{\partial p_j^*}{\partial c_j} > 0, \; i, j = 1, 2, i \neq j, . \tag{2}$$

Note that $\partial \pi_{Di}/\partial p_j > 0$ follows from $\partial Q_i/\partial p_j > 0$, and $\partial p_j^*/\partial c_j > 0$ follows from comparative statics of the first order condition $\partial \pi_{Dj}/\partial p_j = 0$.

It is useful to derive the static Nash equilibrium of the game at the upstream level. Without vertical integration, the two upstream firms compete à la Bertrand for both $Di$. The upstream firm posting the lower price for the input in market $Di$ obtains a profit of $c_i Q_i^*(c_i, c_j)$, $i, j = 1, 2$, $i \neq j$, and, in the case of

[1] This can be easily be generalized to more complex downstream cost functions. See OSS.

[2] The assumptions to ensure a unique static Nash equilibrium are merely made to simplify the analysis. Under weaker assumptions, the stage game has multiple equilibria and one would need to distinguish between stable and unstable equilibria (see OSS). Note also that somewhat weaker conditions might be sufficient to guarantee existence and uniqueness (see Vives, 1999).

a tie, this profit is split equally between the two upstream firms. In the unique static Nash equilibrium, both upstream firms charge a price equal to marginal cost, i.e., equal to zero.

Now consider vertical integration and call the integrated firm of $U1$-$D1$. When $U1$ and $D1$ are integrated, the downstream segment of $U1$-$D1$ is delivered internally at marginal cost and the two upstream firms compete for $D2$ only. As emphasized by Hart and Tirole (1990) and Reiffen (1992), the unique static Nash equilibrium is has $U1$-$D1$ and $U2$ charging a price equal to marginal cost, just as in the case without integration. $U2$ earns zero profits and $U1$-$D1$ earns $\pi^*_{D1}(0,0)$ in this static Nash equilibrium.[3]

# 3 Collusive Foreclosure in the Repeated Game

In this section, we will analyze under which conditions foreclosure emerges as an equilibrium outcome in the general model. Collusion at the upstream level with vertical integration will be analyzed in this section—this is the case where foreclosure may occur. Collusion under vertical separation will be discussed in Section 5, and collusion at the downstream level will be considered in Section 7.

We suppose that firms try to implement the following *collusive foreclosure strategy.* $U2$ charges $D2$ an input price $c \geq 0$ and $U1$-$D1$ does not deliver market $D2$ as part of a collusive strategy in the infinitely repeated game. Alternatively, $U1$-$D1$ could also post a price larger than $c$ as part of the collusion. This alternative implies minor modifications of the results which we discuss at the end of this section.

The nature of this collusive strategy is different from that of normal oligopoly collusion. When adhering to the collusive agreement, $U1$-$D1$ stays out of the market while, when defecting, it enters (at a price smaller than $c$). $U2$ is simply a monopolist when collusion is successful but $c$ will generally not be its preferred monopoly price. Many values of $c$ can potentially be part of an equilibrium in the infinitely repeated game, and only careful application of equilibrium selection criteria can help identifying more plausible values of $c$. When defecting $U2$, is still a monopolist and then it will surely charge its monopoly price.

Before solving the repeated game, it is useful to define three particular levels of $c$. The first is the one just mentioned where $U2$ charges the price $c$ which maximizes it's own profits. Denote this price by $c_2^{mon}$. Formally,

$$c_2^{mon} \equiv \arg\max_c \; cQ_2^*(c,0). \tag{3}$$

The second benchmark is the one that maximizes $U1$-$D1$ profits if it serves the $D2$ market in addition

[3] Also OSS acknowledge that this is the Nash equilibrium without commitment.

to the $D1$ market as a monopolist. Define this level formally by

$$c_1^{mon} \equiv \arg\max_c \pi_{D1}^*(0,c) + cQ_2^*(c,0). \quad (4)$$

Note that $c_1^{mon} > c_2^{mon}$ due to $\partial\pi_{D1}^*/\partial c > 0$, and that both $c_1^{mon}$ and $c_2^{mon}$ are unique. This second benchmark will be important when we analyze defection by $U1$-$D1$. Finally, it is useful to define the level of $c$ where $U2$'s output (and profit) becomes zero, denoted by $\overline{c}$. Formally

$$\overline{c} \equiv \{Q_2(\overline{c},0) = 0\}. \quad (5)$$

Note that $\overline{c} > c_2^{mon}$ but $c_1^{mon}$ may be smaller or larger than $\overline{c}$.

We now analyze collusion in the infinitely repeated game. Time is indexed from $t = 0, ..., \infty$. Firms discount future profits with a factor $\delta$, where $0 < \delta < 1$. When analyzing the repeated game, denote by $\pi_1^c$ and $\pi_2^c$ the profit $U1$-$D1$ and $U2$ earn when both firms adhere to collusion, respectively. Let $\pi_i^d$ denote the profit when a firm defects, and $\pi_i^p$ is the average profit per period when a punishment path is triggered.

Collusive equilibria should be subgame perfect Nash equilibria. Subgame perfect Nash equilibria satisfy

$$\sum_{t=0}^{\infty} \delta^t \pi_i^c \geq \pi_i^d + \sum_{t=1}^{\infty} \delta^t \pi_i^p \quad (6)$$

or

$$\delta \geq \frac{\pi_i^d - \pi_i^c}{\pi_i^d - \pi_i^p} \equiv \delta_i, \quad (7)$$

where $i = 1, 2$, and $\delta_i$ denotes the minimum discount factor required for firm $i$ to adhere to collusion.

Consider the collusive profits, $\pi_i^c$, first. $U_1$-$D_1$'s collusive profit is $\pi_1^c = \pi_{D1}(c,0)$ since $U_1$-$D_1$does not make any profit at the upstream level in equilibrium. $U_2$ makes a collusive profit of $\pi_2^c = cQ_2^*(c,0)$.

Defection from this collusive foreclosure strategy is as follows. If $U2$ defects, it charges $c_2^{mon}$ no matter which $c$ is part of the collusive strategy (aside, this implies $\pi_2^d = \pi_2^c$ if $c = c_2^{mon}$). If $U1$-$D1$ defects it must undercut $U2$'s price, $c$, in order to gain the profit in market $D2$. If $c \leq c_1^{mon}$, $U1$-$D1$ will simply charge a price infinitesimally smaller than $c$. For $c > c_1^{mon}$, $U1$-$D1$ will defect charging $c_1^{mon}$ by definition of $c_1^{mon}$.

Consider now the punishment following a defection. Since there is perfect Bertrand competition at the upstream level, simple trigger strategies with Nash reversion imply $c = 0$ and so upstream profits are zero in the unique static Nash equilibrium. These are also the maximin profits and more severe punishment strategies do not exist. We obtain $\pi_1^p = \pi_{D1}^*(0,0)$ and $\pi_2^p = 0$ for $U1$-$D1$ and $U2$, respectively.

Collusive equilibria must be individually rational, that is, firms must get at least their maximin profits in a collusive equilibrium of the repeated game. Maximin profits $\pi_1 = \pi^*_{D1}(0,0)$ and $\pi_2 = 0$ result when $c = 0$. $U2$ also gets zero profits with $c = \bar{c}$. Hence, any positive $c < \bar{c}$ implies $\pi^c_1 = \pi_{D1}(c,0) > 0$ (from (2)) and $\pi^c_2 = cQ_2(c,0) > 0$ therefore and fulfills the individual rationality requirement.

We are now ready to prove

**Proposition 1** *Vertical foreclosure is a subgame perfect Nash equilibrium in the repeated game, provided firms' discount factor $\delta$ is sufficiently high. Moreover, any individually rational collusive input price $0 < c < \bar{c}$ can be sustained with a high enough discount factor.*

**Proof.** We prove the more general second part by showing that $\delta_i < 1$ for $i$=1,2, and for all $0 < c < \bar{c}$. Now, $\delta_i < 1$ if $\pi^d_i \geq \pi^c_i > \pi^p_i$. First, recall $\pi^c_1 = \pi^*_{D1}(0,c) > \pi^*_{D1}(0,0) = \pi^p_1$ and $\pi^c_2 = cQ^*(c,0) > 0 = \pi^p_2$. This establishes the strict inequalities. Second, to prove $\pi^d_1 \geq \pi^c_1$ suppose that first $c \leq c^{mon}_1$ such that, when defecting, $U1$-$D1$ gets $\pi^d_1 = \pi^c_1 + \pi^c_2 > \pi^c_1$. If $c > c^{mon}_1$, $U1$-$D1$'s optimal defection is to charge $c^{mon}_1$ and we have $\pi^d_1 = \pi^c_1 + c^{mon}_1 Q^*(c^{mon}_1, 0) > \pi^c_1$. To prove $\pi^d_2 \geq \pi^c_2$, note that $\pi^d_2 = c^{mon}_2 Q^*(c^{mon}_2, 0) \geq \pi^c_2$ by definition of $c^{mon}_2$. ■

Proposition 1 shows that vertical foreclosure can indeed by sustained as a subgame-perfect Nash equilibrium in the infinitely repeated game. The result counters the criticism the OSS result received (not being a Nash equilibrium) whenever there is repeated interaction and the discount factor is sufficiently high.

Note that for all $c < \bar{c}$, a strictly positive minimum discount factor is required.[4] The reason why foreclosure may not be a subgame-perfect Nash equilibrium for small $\delta$ is that the firms cannot reduce $\max\{\delta_1, \delta_2\}$ arbitrarily close to zero. This is in contrast to some Cournot oligopoly models and models with differentiated price competition where a collusive equilibrium always exist since firms can simply select actions sufficiently close to the static Nash equilibrium actions.

Proposition 1 contains a *Folk Theorem*-like message on the action domain. All individually rational collusive input prices can be part of an equilibrium in the repeated game, suggesting a coordination problem. This raises the question if equilibrium selection criteria help reducing the set of plausible collusive equilibria.

**Proposition 2** *Only collusive equilibria with $c \geq c^{mon}_2$ are Pareto efficient.*

[4] This follows from $\pi^d_1 > \pi^c_1 > \pi^p_1 > 0$ and therefore $\delta_1 > 0$ unless $c = \bar{c}$. Similarly $\delta_2 > 0$, unless $c = c^{mon}_2$. Hence $\max\{\delta_1, \delta_2\} > 0$ for all $c$.

**Proof.** By definition of $c_2^{mon}$, $\partial\pi_2^c/\partial c > 0$ when $c < c_2^{mon}$ and $\partial\pi_2^c/\partial c \leq 0$ when $c \geq c_2^{mon}$. From (2), $\partial\pi_1^c/\partial c > 0$ for any $c$. Hence, choices of $c < c^{mon}$ are not Pareto efficient but those in the interval $[c^{mon}, \bar{c})$ are. ■

Pareto efficiency (from the firms' point of view) suggests that the collusive prices we may expect to occur will be at least as high as $c_2^{mon}$. The intuition is that, when $c$ is increased beyond $c_2^{mon}$, $U1$-$D1$ gains unambiguously while firm $U2$ loses for either $c < c_2^{mon}$ and $c > c_2^{mon}$. Hence, the bargaining situation implicit in the repeated game can plausibly lead to such an outcome. By contrast, in the one-shot game analyzed in OSS (assuming $U1$-$D1$ can commit not to deliver $D2$), $U2$ is a monopolist in the $D2$ market and would therefore never charge a price other than $c_2^{mon}$.

The characterization of Pareto efficient actions does say not anything whether or not these actions are likely to meet the incentive constraint (7). To answer this question, the following lemma is helpful. It states how minimum discount factors $\delta_1$ and $\delta_2$ as in (7) respond to changes of $c$.

**Lemma 3** *Let $\delta_1(c)$ and $\delta_2(c)$ denote the minimum discount factors required by $U1$-$D1$ and $U2$ respectively as functions of the collusive price $c$.*

*(i) $\partial\delta_1(c)/\partial c \gtreqless 0$ if and only if $(\partial\pi_2^c/\partial c)(\pi_1^c - \pi_1^p) - (\partial\pi_1^c/\partial c)(\pi_2^c) \gtreqless 0$.*

*(ii) $\partial\delta_2(c)/\partial c \gtreqless 0$ if and only if $c \gtreqless c_2^{mon}$.*

**Proof.** Consider part (i). If $c \leq c_1^{mon}$, $\pi_1^d = \pi_1^c + \pi_2^c$ and $\delta_1(c) = \pi_2^c/(\pi_2^c + \pi_1^c - \pi_1^p)$. Hence

$$\frac{\partial\delta_1}{\partial c} = \frac{(\partial\pi_2^c/\partial c)(\pi_1^c - \pi_1^p) - (\partial\pi_1^c/\partial c)(\pi_2^c)}{(\pi_1^c + \pi_2^c - \pi_1^p)^2} \tag{8}$$

which yields the condition. If $c > c_1^{mon}$, $\pi_1^d = \pi_1^c + c_1^{mon} Q_2^*(c_1^{mon}, 0)$. So, $\delta_1 = c_1^{mon} Q_2^*/(c_1^{mon} Q_2^* + \pi_1^c - \pi_1^p)$ and

$$\frac{\partial\delta_1}{\partial c} = \frac{-(\partial\pi_1^c/\partial c)c_1^{mon} Q_2^*}{(c_1^{mon} Q_2^* + \pi_1^c - \pi_1^p)^2} < 0. \tag{9}$$

But in this case $c > c_2^{mon}$ so that $(\partial\pi_2^c/\partial c)(\pi_1^c - \pi_1^p) - (\partial\pi_1^c/\partial c)(\pi_2^c) < 0$. Therefore, the condition in the lemma gives the correct sign of $\partial\delta_1/\partial c$. Then consider part (ii). Note that $\pi_2^d = \pi_2^c(c_2^{mon}, 0)$ is a constant, hence, we get $\partial\delta_2/\partial c = -(\partial\pi_2^c/\partial c)/(\pi_2^d)^2$. Hence, $\partial\delta_2/\partial c \gtreqless 0$ if and only if $c \gtreqless c_2^{mon}$. ■

The next proposition puts the lemma and Proposition 2 together. It characterizes extremal equilibria, defined as those subgame perfect Nash equilibria which give the highest profit to the firms subject to the incentive constraint (7).

**Proposition 4** *Extremal subgame perfect Nash equilibria involve $c \geq c_2^{mon}$, provided $(\partial\pi_2^c/\partial c)(\pi_1^c - \pi_1^p) - (\partial\pi_1^c/\partial c)(\pi_2^c) < 0$.*

**Proof.** Assume $c < c_2^{mon}$. If the condition in the proposition is met, one gets $\partial\delta_1/\partial c < 0$ and $\partial\delta_2/\partial c < 0$ from the lemma. From Proposition 2, $\partial\pi_2^c/\partial c > 0$ and $\partial\pi_1^c/\partial c > 0$. Hence, extremal subgame perfect Nash equilibrium must involve $c \geq c_2^{mon}$, provided $(\partial\pi_2^c/\partial c)(\pi_1^c - \pi_1^p) - (\partial\pi_1^c/\partial c)(\pi_2^c) < 0$. ■

Observe that the condition $(\partial\pi_2^c/\partial c)(\pi_1^c - \pi_1^p) - (\partial\pi_1^c/\partial c)(\pi_2^c) < 0$ does not appear to be particularly restrictive. It holds, for example, in the model with linear demand (see below). Further $\partial\pi_1^c/\partial c > 0$, so, the condition will be met when $\partial\pi_2^c/\partial c$ or $\pi_1^c - \pi_1^p$ are small.[5] The proposition implies that firms are likely to collude on a price $c \geq c_2^{mon}$ because any $c$ below $c_2^{mon}$ reduces both firms' profits and makes collusion more difficult to sustain for both firms.

Figure 1 summarizes the discussion of the general model. It shows minimum discount factors $\delta_1$ and $\delta_2$ for values of $c$ between zero and $\overline{c}$. Figure 1 is drawn with the help of the lemma and the following properties of $\delta_i$. It is straightforward to verify $\delta_2(0) = 1$. Further, $\delta_2(c_2^{mon}) = 0$ and $\delta_2(\overline{c}) = 1$ follow from the definitions of $c_2^{mon}$ and $\overline{c}$. Regarding $\delta_1$, note $0 < \delta_1(c) < 1$ for all $c < \overline{c}$ due to $\pi_1^d > \pi_1^c > \pi_1^p > 0$.[6] Further $\delta_1(\overline{c}) = 0$ if, as assumed the figure, $c_1^{mon} > \overline{c}$.[7] Finally, the figure is based on the assumption that $(\partial\pi_2^c/\partial c)(\pi_1^c - \pi_1^p) - (\partial\pi_1^c/\partial c)(\pi_2^c) < 0$ holds.

[Figure 1 about here.]

The figure illustrates $\underline{\delta} = \min_c \max\{\delta_1(c), \delta_2(c)\} > 0$, and $\partial\delta_i(c)/\partial c < 0$, $i$=1, 2, if $c < c_2^{mon}$. Foreclosure as a collusive strategy will fail when $\delta < \underline{\delta}$ and will typically involve $c \geq c_2^{mon}$ otherwise.

Finally, we need to discuss what happens if $U1$-$D1$ does not completely withdraw from the $D2$ market but posts a price $c+\varepsilon$, $\varepsilon$ being small, as part of the collusion instead. The only thing that would change in this case is that $U2$ could not defect profitably any more when $c < c_2^{mon}$. To see this, note that $U2$ wants to defect by charging the price $c_2^{mon}$, but since $U1$-$D1$ charges $c+\varepsilon < c_2^{mon}$, this is not possible. This implies $\pi_2^d = \pi_2^c$ and so $\delta_2 = 0$ if $c < c_2^{mon}$. Everything else and in paricular the results in this section remain unchanged.

[5]Note that $\partial\delta_1/\partial c < 0$ if $c \geq c_2^{mon}$. If $c < c_2^{mon}$, $sign[\partial\delta_1/\partial c]$ is ambiguous.

[6]We cannot directly determine $\delta_1(0)$ since $\delta_1(c) = \pi_2^c/(\pi_2^c + \pi_1^c - \pi_1^p)$ but $\pi_2^c(0) = 0$ and $\pi_1^c(0,0) - \pi_1^p = 0$. L'Hôpital's rule yields

$$\lim_{c\to 0} \delta_1(c) = \lim_{c\to 0} \pi_2^c(c)'/(\pi_2^c(c)' + \pi_1^c(c)') < 1$$

from $\pi_2^c(c)' = Q_2$ and $\pi_1^c(c)' > 0$.

[7]This follows from $\pi_2^c(\overline{c}) = 0$ and so $\pi_1^d = \pi_1^c > \pi_1^p$. If $c_1^{mon} < \overline{c}$, $\delta_1(\overline{c}) > 0$ follows from $\pi_1^d = \pi_1^c + c_1^{mon}Q_2^*(c_1^{mon},0)$ so that $\delta_1 = c_1^{mon}Q_2^*/(c_1^{mon}Q_2^* + \pi_1^c - \pi_1^p) > 0$.

## 4 A Parametrized Model

In this section, we will develop a parametrized version of the model which is useful to derive further results. The market model is similar to the one in OSS (Appendix) and has linear demand. Demand is symmetric and the demand intercept is, without loss of generality, normalized to one

$$Q_i(p_i, p_j) = 1 - kp_i + dp_j, \ i, j, = 1, 2; i \neq j. \tag{10}$$

where $k > d > 0$. Products are entirely heterogenous if $d = 0$ while $d = k$ would imply perfectly homogenous goods. $Di$'s profit is

$$\pi_{Di} = (1 - kp_i + dp_j)(p_i - c_i), \ i, j, = 1, 2; i \neq j. \tag{11}$$

Myopic maximization at the downstream level yields Nash equilibrium prices

$$p_i^*(c_i, c_j) = \frac{2k + d + k^2 c_i + kdc_j}{4k^2 - d^2} \tag{12}$$

and equilibrium outputs

$$Q_i^*(c_i, c_j) = k\frac{2k + d - (2k^2 - d^2)c_i + kdc_j}{4k^2 - d^2}. \tag{13}$$

Downstream profits are $\pi_{Di}^*(c_i, c_j) = (Q_i^*)^2/k$.

Consider the infinitely repeated game with integration. As above, $U_1$-$D_1$ forecloses $D_2$, and $U_2$ delivers $D_2$ at a collusive price $c$. The integrated firm delivers its downstream unit at zero cost. From (13), $U_1$-$D_1$ and $D_2$ will sell the following quantities

$$Q_1^*(0, c) = k\frac{2k + d + kdc}{4k^2 - d^2}, \tag{14}$$

$$Q_2^*(c, 0) = k\frac{2k + d - (2k^2 - d^2)c}{4k^2 - d^2}. \tag{15}$$

$U_1$-$D_1$'s downstream profit is

$$\pi_{D1}^*(0, c) = k\left(\frac{2k + d + kdc}{4k^2 - d^2}\right)^2 \tag{16}$$

while $D2$'s profit is

$$\pi_{D2}^*(c, 0) = k\left(\frac{2k + d - (2k^2 - d^2)c}{4k^2 - d^2}\right)^2. \tag{17}$$

$U_1$-$D_1$ does not make any profit at the upstream level. $U_2$ makes a profit of $cQ_2$ or

$$\pi_{U2}^* = ck\frac{2k + d - (2k^2 - d^2)c}{4k^2 - d^2}. \tag{18}$$

For $U_1$-$D_1$ punishment profits are

$$\pi_1^p = \pi_{D1}^*(0,0) = \frac{k}{(2k-d)^2} \tag{19}$$

while $\pi_2^p = 0$.

The benchmark prices can easily be obtained as

$$c_2^{mon} = \frac{2k+d}{2(2k^2-d^2)}. \tag{20}$$

and

$$c_1^{mon} = \frac{(2k+d)(4k^2+2kd-d^2)}{2(8k^4-7k^2d^2+d^4)}. \tag{21}$$

The third benchmark is

$$\bar{c} = \frac{2k+d}{2k^2-d^2}. \tag{22}$$

Note $\bar{c} = 2\, c_2^{mon}$ but $\bar{c} \gtreqless c_1^{mon}$ is ambiguous.

First, consider $U_1$-$D_1$'s incentives to collude. If $c < c_1^{mon}$, $U_1$-$D_1$'s defection profit is the sum of $\pi_{U2}$ as in (18) and $\pi_{D1}^*$, its own equilibrium profit as in (16). If $c \geq c_1^{mon}$, $U_1$-$D_1$'s defection profit is the sum of $c_1^{mon} Q_2^*(c_1^{mon}, 0)$ and $\pi_{D1}^*(c_1^{mon}, 0)$. The trigger-strategy implies a punishment profit of $\pi_{D1}^*(0,0) = k/(2k-d)^2$. The minimum discount factor required for $U1$-$D1$ to adhere to collusion is

$$\delta_1 = \begin{cases} \dfrac{(4k^2-d^2)(2k+d-c(2k^2-d^2))}{8k^2(k+d)-d^3-c(8k^4-7k^2d^2+d^4)} \text{ if } c < c_1^{mon} \\[2ex] \dfrac{(8k^2(k+d)-d^3)c_1^{mon} - (8k^4-7k^2d^2+d^4)(c_1^{mon})^2 - 2ckd(2k+d) - c^2k^2d^2}{8k^2(k+d)-d^3)c_1^{mon} - (8k^4-7k^2d^2+d^4)(c_1^{mon})^2}, \text{ if } c \geq c_1^{mon} \end{cases} \tag{23}$$

where $c_1^{mon}$ is defined as in (21). In the general model, when $c < c_1^{mon}$, $\partial\delta_1/\partial c < 0$ if and only if a regularity condition holds. For the parametrized model, the condition is met since

$$sign\left[\frac{\partial \delta_1}{\partial c}\right] = -\frac{(4k^2+kd-2d^2)c^2k^3d}{(4k^2-d^2)^2(2k-d)} < 0. \tag{24}$$

When $c \geq c_1^{mon}$, we know $\partial\delta_1/\partial c < 0$ from the general model.

Now turn to $U_2$'s incentives to collude. $U_2$'s collusive profit is $\pi_{U2}(c)$, and $U_2$'s defection profit is $\pi_{U2}(c^{mon})$. The punishment profit is $\pi_{U2}(0) = 0$. Plugging these expressions into (7), one obtains

$$\delta_2 = \left(\frac{d+2cd^2+2k-4ck^2}{2k+d}\right)^2. \tag{25}$$

We know $\partial\delta_2/\partial c$ from the general model.

# 5 Does Vertical Integration Facilitate Collusion?

The result that vertical foreclosure can emerge as an equilibrium outcome of the repeated game is an important one. However, it does not necessarily suggest a policy against vertical integration. The make a point against vertical integration, one needs to show that the industry is more prone to collusion with integration than without, that is, one needs to show that integration facilitates collusion. In an infinitely repeated game, an industrial policy can be said to facilitate collusion if the industry requires a lower minimum discount factor than without the policy. To investigate this, we now compare the required minimum discount factors with and without vertical integration.

Without integration, it is straightforward to solve for the minimum discount factor. There are now two independent $U$ firms competing in both downstream markets, $D1$ and $D2$. Suppose the firms collude on arbitrary collusive prices $c_1$ and $c_2$ in markets $D1$ and $D2$ respectively. Let $\pi^c$ denote the sum of profits made in the two markets by a firm and denote the defection profit by $\pi^d$. If a firm defects, it will do so in both markets (Bernheim and Whinston, 1990). Hence, $\pi^d = 2\pi^c$. Finally, already simple Nash reversions yield $\pi^p = 0$. It follows that the collusive prices $c_1$ and $c_2$ can be supported as a subgame perfect Nash equilibrium if and only if $\delta \geq 1/2$. This minimum discount factor does not depend on $c_1$ and $c_2$ or any functional forms. Further, the fact that firms collude in two markets here does not affect to propensity to collude (Bernheim and Whinston, 1990).

Now we compare this to the minimum discount factor required for collusion under vertical integration.

**Proposition 5** *In the parametrized model, vertical integration facilitates collusion if and only if $d/k >$ 0.380.*

**Proof.** To prove that vertical integration facilitates collusion, we need to show that $\max\{\delta_1(c), \delta_2(c)\} <$ $1/2$ for some $c$. To do this, we solve for $\delta_i(c) = 1/2$, $i$=1,2, and then search for $c$ values such that both $\delta_i(c)$ are below the threshold.

First, we look for solutions to $\delta_2(c) = 1/2$. It is straightforward to verify that $\delta_2(c) \leq 1/2$ if and only if $c_2^{mon}(1 - \sqrt{1/2}) \leq c \leq c_2^{mon}(1 + \sqrt{1/2})$.

Next, we look for solutions to $\delta_1(c) = 1/2$. Consider $c < c_1^{mon}$ and define $\widehat{c} \equiv \{\delta_1(c) = 1/2 | c < c_1^{mon}\}$. We obtain a unique solution,

$$\widehat{c} = \frac{8k^3 - 4kd^2 - d^3}{8k^4 - 5k^2d^2 + d^4}. \tag{26}$$

Then consider $c \geq c_1^{mon}$ and define $\widetilde{c} \equiv \{\delta_1(c) = 1/2 | c \geq c_1^{mon}\}$ here. Solving $\delta_1(c) = 1/2$ for $c$ yields two solutions. The negative root can be dismissed as it implies $\widetilde{c} < 0$. The positive root is

$$\widetilde{c} = \frac{-2(2k+d)+\sqrt{m}}{2kd} \tag{27}$$

where $m = 2(8k^2+8dk+2d^2+c_1^{mon}((8k^2(k+d)-d^3)-c_1^{mon}(8k^4-7k^2d^2+d^4)))$. From $\partial\delta_1/\partial c < 0$, it follows that all $c > \widetilde{c}, \widehat{c}$ yield $\delta_1(c) < 1/2$.

We need to ensure $\widehat{c} \leq c_1^{mon}$ and $\widetilde{c} \geq c_1^{mon}$. There are two unknown variables, $d$ and $k$, but we can express the solution in terms of $d/k$, the ratio of the slope parameters indicating the degree of product differentiation. Note that $0 < d/k < 1$. Now, solving for $\widehat{c} = \widetilde{c} = c_1^{mon}$, it turns out $\widehat{c} \leq c_1^{mon}$ if and only if $d/k \geq 0.541$, and $\widetilde{c} \geq c_1^{mon}$ if and only if $d/k \leq 0.541$.

We can now compare $\widehat{c}$ and $\widetilde{c}$ to the $c$ values for which $\delta_2(c) < 1/2$. To do this, it is useful to express $\widehat{c}$ and $\widetilde{c}$ in terms of $c_2^{mon}$. First, we analyze $\widehat{c}$ for $0.541 \leq d/k \leq 1$. If $d/k = 0.541$, we get $\widehat{c} = 1.354c_2^{mon}$. If $d = k$, $\widehat{c} = c_2^{mon}/2$. Further $\widehat{c}/c_2^{mon}$ monotonically decreases in $d$. This implies $1.354c_2^{mon} \geq \widehat{c} \geq c_2^{mon}/2$ and so $c_2^{mon}(1-\sqrt{1/2}) < \widehat{c} < c_2^{mon}(1+\sqrt{1/2})$. Second, we check $\widetilde{c}$ for $0.541 \geq d/k \geq 0$. If $d/k = 0.541$, then $\widetilde{c} = 1.354c_2^{mon}$ as above, but $\lim_{d\to 0}\widetilde{c} = +\infty$. Because $\widetilde{c}/c_2^{mon}$ decreases monotonically in $d$, we need to find the minimum $d/k$ such that $\widetilde{c} \leq c_2^{mon}(1+\sqrt{1/2})$. This threshold is $d/k \geq 0.380$ as stated in the proposition. If this condition is met, $c_2^{mon}(1-\sqrt{1/2}) < \widetilde{c} < c_2^{mon}(1+\sqrt{1/2})$.

To conclude, if $0 < d/k \leq 0.380$, $\max\{\delta_1(c), \delta_2(c)\} \geq 1/2$. If $0.380 < d/k \leq 0.541$, there are $c$ satisfying $\widehat{c} < c < c_2^{mon}(1+\sqrt{1/2})$ such that $\max\{\delta_1(c), \delta_2(c)\} < 1/2$. If $0.541 \leq d/k < 1$, there are $c$ satisfying $\widetilde{c} < c < c_2^{mon}(1+\sqrt{1/2})$ such that $\max\{\delta_1(c), \delta_2(c)\} < 1/2$. ■

The proposition shows that the collusive foreclosure strategy can be sustained with a discount factor lower than $1/2$ if there is not too much product differentiation. The proof of the proposition gives a complete characterization of the range of $c$ values for which the result holds. The interval includes $c > c_2^{mon}$, that is, some of the Pareto efficient extremal equilibria prices.

Drawing policy conclusions from this result, one has to be cautious. Note that the proposition does not imply that vertical separation is always preferable from a policy point of view even if the condition on product differentiation is met. For $\delta < \underline{\delta} = \min_c \max\{\delta_1(c), \delta_2(c)\}$, neither the integrated nor the separated industry are collusive and market outcomes are the same. For discount factors between $\underline{\delta}$ and 0.5, only the integrated industry is collusive (provided $d/k \geq 0.380$), and this is the case an active policy would want to prevent. Finally, for $\delta > 0.5$, both industries are collusive but note that both downstream markets are supplied at $c > 0$ under separation, while this is the case only for the $D2$ market with integration ($D1$ is delivered at marginal cost). This suggests the possibility that vertical integration is preferable from a policy point of view if $\delta > 0.5$, although this depends on the specific collusive prices charged by firms.

## 6 The Possibility of a $U2$-$D2$ Merger

One of the key results in OSS is to show that $U1$-$D1$ has an incentive to commit to a price *lower* than $c_2^{mon}$ to prevent $U2$ and $D2$ from merging. The logic is that, for some values of $c$, the joint profits of $U2$ and $D2$ can be increased by vertically integrating and thus supplying $D2$ internally at marginal cost. This level of $c$ is called $c^{mer}$ and is obtained by solving $\pi_{D2}(c) + \pi_{U2}(c) = \pi_{D2}(0)$ for $c$. For the parametrized model, the solution is

$$c^{mer} = \frac{(2k + d)d^2}{2(2k^2 - d^2)k^2} \tag{28}$$

(see also OSS, Appendix). This upper bound on $c$ is also relevant in the repeated game. We obtain

**Proposition 6** *If $U2$ and $D2$ can merge, firms will collude on a price $c \leq c^{mer} < c_2^{mon}$ in the parametrized model. In the extremal subgame perfect Nash equilibrium, $c = c^{mer} < c_2^{mon}$.*

**Proof.** If $U2$ and $D2$ can merge, $c \leq c^{mer}$ is binding. In the parametrized model, $c^{mer} = c_2^{mon} d^2/k^2 < c_2^{mon}$. From the lemma and the proof of Proposition 2, $c = c^{mer}$ is the extremal equilibrium as $c < c^{mer}$ lowers both firms' profits and requires a higher discount factor. ■

The possibility of a $U2$-$D2$ merger forces firms to collude on a price $c < c_2^{mon}$, that is, a Pareto inferior outcome. Therefore, collusion is generally less likely as a higher $\delta$ is required, and less damaging for welfare as a lower $c$ will occur. This possibility will also reduce the scope for which vertical integration facilitates collusion. Generally, the threat of a $U2$-$D2$ merger is beneficial from a policy perspective.

## 7 Collusion at the downstream level

We finally discuss what happens if collusion at the downstream level is possible. Generally, downstream collusion might affect the desirability of the collusive foreclosure strategy, so we need to discuss this possibility. We do not provide a complete analysis here. Instead, only the most important points which relate to foreclosure will be discussed.

Consider the vertically integrated industry and suppose that the discount factor is sufficiently high to support collusion at both levels of the industry. Where will collusion most likely occur? At the $U$ level or at the $D$ level? Or even at both levels?

We discard the possibility of collusion at both levels in two steps. The first step is to note that $U1$-$D1$ can choose at which level to collude since it operates at both levels of the industry. It can simply decide to price competitively at one level of the industry even if the discount factor would allow collusion at both levels. The second step is the observation that collusion at both levels is Pareto inferior. Assume there is collusion at both levels. $U$-level (foreclosure-type) collusion benefits $U1$-$D1$ and $U2$ but hurts $D2$, while $D$-level collusion benefits $U1$-$D1$ and $D2$ but hurts $U2$. In other words, collusion at both levels imposes a negative externality on both $U2$ and $D2$. By contrast, collusion at only one level deletes one of these externalities and is therefore Pareto superior (for the two remaining colluding firms). It follows that collusion at both levels will not occur as $U1$-$D1$ can increase it profits by pricing competetively at one level of the industry.

The question remains whether $U1$-$D1$ prefers to collude upstream or downstream.[8] An answer to this question is difficult as the outcomes of the collusive foreclosure strategy and downstream collusion are usually rather different in nature and not easy to compare. We can, however, compare discount factors when the outcomes of upstream and downstream collusion coincide. We will do this here for the parametrized model.

For a given $c$, upstream (foreclosure-type) collusion yields a unique outcome. The collusive foreclosure strategy implies downstream prices of $p_1^*(0,c)$ and $p_2^*(c,0)$ for $U1$-$D1$ and $D2$ respectively. Now assume that the same outcome occurs as a result of downstream collusion. There is upstream competition in this case, so $c = 0$. Downstream firms implement collusive prices of $p_i^c(0,0)$, $i$=1,2, identical to those which occur with foreclosure, that is, $p_1^c(0,0) = p_1^*(0,c)$ and $p_2^c(0,0) = p_2^*(c,0)$. Clearly, $U1$-$D1$ benefits from this (exactly as much as with upstream collusion) but $D2$ also benefits since $c = 0$ now and its profits $p_2^c Q_2^*$ accordingly.

The Appendix contains the derivation of the relevant minimum discount factor required for downstream collusion. Here, we discuss the most important results qualitatively. A first observation is that, by choosing a sufficiently low $c$, the two $D$ firms can lower the minimum discount factor required for collusion arbitrarily close to zero. Intuitively, colluding on a low $c$ yields prices close to the static Nash equilibrium where incentives to deviate are small. This is an advantage of collusion at the $D$ level. Above, we saw that foreclosue-type collusion requires $\underline{\delta} = \min_c \max\{\delta_1(c), \delta_2(c)\} > 0$. Here, for $\delta < \underline{\delta}$, collusion at the $D$ level is still feasible.

[8]Note that the integrated firm does not have a preference for upstream or downstream collusion per se. Its profit is ultimately affected only by the prices at the downstream level and whether downstream prices are higher because of foreclosure or because of downstream collusion is immaterial.

However, collusion at the $D$ level as specified also has disadvantages. The minimum discount factor required increases monotonically in $c$, and it is smaller than 1 if only if $c < c_2^{mon} d^2/k^2 < c_2^{mon}$. In words, the Pareto efficient collusive prices $c \geq c_2^{mon}$ cannot be sustained with downstream collusion at all. By contrast, we know that these prices were an equilibirum of the repeated game with the collusive foreclosure strategy, sometimes even when the discount factor was less than 1/2. Since higher $c$ raise $U1$-$D1$'s profit, it will go for upstream rather than downstream collusion whenver the constraint on $\delta$ allows to do so. The conclusion is that the collusive foreclosure strategy is not always less attractive and certainly not redundant. By contrast, it is a very profitable strategy whenever the incentive constraint is met.

There are, of course, plausible forms of collusion at the $D$ level other than $p_1^c(0,0) = p_1^*(0,c)$ and $p_2^c(0,0) = p_2^*(c,0)$. But note that in the implicit bargaining situation with $D2$, $U1$-$D1$ can credibly threat to switch to upstream (foreclosure-type) collusion. That is, collusion at the $D$ level should give $U1$-$D1$ no less than it would get in the foreclosure outcome and so collusion on $p_1^c(0,0) = p_1^*(0,c)$ and $p_2^c(0,0) = p_2^*(c,0)$ is focal.

# 8 Conclusions

This paper shows that vertical foreclosure can by sustained as the outcome of a subgame-perfect Nash equilibrium of an infinitely repeated game where the market model has a raising rival's cost effect. The result counters the criticism that foreclosure is not a Nash equilibrium of the static game analyzed by OSS. The results also indicate that this collusive foreclosure strategy, if successful, can be very damaging from a policy perspective as equilibrium selection criteria suggest that non-integrated firms will be charged at least the monopoly price for the input good. Comparing the industry with and without vertical integration, the paper shows that the minimum discount factor required for collusion is lower with vertical integration unless products are very differentiated, so, vertical integration often facilitates collusion. As already observed by OSS for the static game, the possibility of a counter merger weakens or even reverses these results. Collusive foreclosure is generally less likely as a higher discount factor is required, and less worrisome as a lower input prices will occur. Finally, the paper shows that vertical foreclosure can be preferrable to downstream collusion.

Following OSS and the critique by Hart and Tirole (1990) and Reiffen (1992), various papers have shown that a raising rivals' costs effect of vertical integration can be rigorously derived from game-theoretic models. Ordover, Saloner and Salop (1992) re-establish their result in a descending-price auction. Riordan (1998) analyses backward integration by a dominant firm with a cost advantage. Choi and Yi (2000) and Church and Gandal (2000) show the result if upstream firms can commit to a technology which makes the input incompatible to nonintegrated downstream firms. In Chen (2001) downstream firms strategically choose upstream suppliers, and Riordan and Chen (2003) investigate the connection between vertical integration and exclusive dealing contracts. This paper contributes to this literature by showing the foreclosure result in the repeated game of the original OSS setting.

# Appendix

Here, we want to analyze collusion at the $D$ level. Collusive prices $p_i^c$, $i$=1,2, are *as if* $D2$ pays $c$ with noncooperative pricing. $D1$ is gets the input at marginal cost, as above. The collusive downstream prices are

$$p_1^c = p_1^*(0,c) = \frac{2k+d+kdc}{4k^2-d^2}, \tag{29}$$

$$p_2^c = p_2^*(c,0) = \frac{2k+d+2k^2c}{4k^2-d^2}. \tag{30}$$

Collusive quantities and profits are immediate

$$\pi_{D1}^c = k\frac{(2k+d+kdc)^2}{(4k^2-d^2)^2}, \tag{31}$$

$$\pi_{D2}^c = k\left(2k+d-2k^2c+d^2c\right)\frac{2k+d+2k^2c}{(4k^2-d^2)^2}. \tag{32}$$

Note that $\pi_{D1}^c = \pi_{D1}^*$ as in (16). Further, recall that $c=0$ so that $\pi_{D2}^c = p_2^c Q_2^*$.

When analyzing defection, we need to solve for best-reply prices. Now, $p_1^c = p_1^*(0,c)$ is a already best reply to $p_1^c = p_2^*(c,0)$. This implies $\pi_{D1}^d = \pi_{D1}^c$ and hence $\delta_{D1} = 0$. Therefore, we can henceforth focus on $D2$. $D2$'s best reply is

$$p_2^d = \frac{1+dp_1}{2k} = \frac{4k+2d+cd^2}{2(4k^2-d^2)}, \tag{33}$$

which imply a defection profit of

$$\pi_2^d = \frac{k}{4}\left(\frac{4k+2d+cd^2}{4k^2-d^2}\right)^2. \tag{34}$$

We know the static Nash equilibrium profit is $\pi_{D2}(0,0) = \pi_{D2}^p = k/\left(2k-d\right)^2$ as above. Plugging $\pi_{D2}^c$, $\pi_{D2}^d$ and $\pi_{D2}^p$ into (7), we obtain

$$\delta_{D2}(c) = \frac{c\left(4k^2-d^2\right)^2}{\left(8k+cd^2+4d\right)d^2}. \tag{35}$$

$\delta_{D2}(0)$ and $\partial\delta_{D2}(c)/\partial c > 0$ are immediate. Further $\delta_{D2}(c) \leq 1$ if and only if $c \leq c_2^{mon}d^2/k^2 = c^{mer}$.[9]

[9] The condition $\delta_{D2}(c) \leq 1$ if and only if $c \leq c^{mer}$ is intuitve since $\pi_{D2}^c$ exactly equals the profit $U2$ and $D2$ could make if they were integrated.

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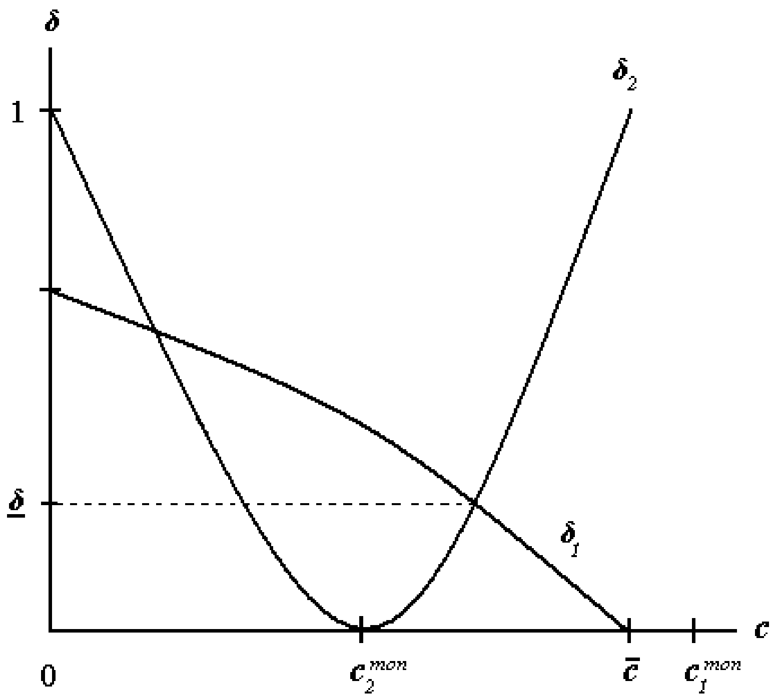


Figure 1. Critical discount factors as functions of $c$.