

# A Note on Bank Behavior and Monetary Policies in an Oligopolistic Market

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## Abstract

We extend Freixas and Rochet (1997) to two-stage game: In the first stage each bank simultaneously decides on amounts of deposits. Then each bank decides on loan amounts in the second stage. Each stage is a Cournot game in quantity. Scope economies between loans and deposits also arise explicitly in our model. We show that in the case where large economies of scope an increase in the interbank interest rate leads to an decrease in the interest rate on deposits and to an increase in the interest rate on loans.

*Keywords:* Bank behavior, Strategic effects, Monetary policies, Scope economies.

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# 1 Introduction

Freixas and Rochet (1997), preceded by the seminal works on monopolistic bank model by Klein (1971) and Monti (1972), investigates the effects of a monetary policy on bank behavior in a Cournot oligopoly. Assuming that each bank determines the volumes of deposits and loans simultaneously in order to maximize their profits (i.e., *simultaneous portfolio choice*), and that scope economies (cost complementarities) between loans and deposits does not exist<sup>1</sup>, Freixas and Rochet (1997) shows that an increase in the interbank interest rate leads to an increase in the interest rates on deposits and loans<sup>2</sup>. However, it is often pointed out that each bank determines their volumes of loans given the volumes of deposits, and some empirical studies show that the scope economies between deposits and loans exists<sup>3</sup>. Nevertheless, There are few theoretical studies in consideration of these points.

The purpose of this paper is to construct a model in which each bank determines its volumes of deposits and loans sequentially (i.e., *sequential portfolio choice*), and investigates the effects of a monetary policy on the volumes of deposits, loans and corresponding interest rates in an oligopoly. For this purpose, we extend Freixas and Rochet (1997) in the following two directions. First, the deposit and loan decisions are made sequentially, rather than simultaneously. In the first stage each bank simultaneously decides on amounts of deposits. Then, once that information is revealed, each bank simultaneously decides on loan amounts in the second stage. This sequential portfolio choice formulation of deposits and loans competition seems appropriate when the choice of deposits are less flexible and less frequently changed than that of loans. The second extension is that scope economies

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<sup>1</sup>This also implicitly assume that the bank's decision problem is separable. That is, if there is no scope economies (i.e., the cost function is separable), the optimal volumes of loans (and the corresponding interest rate) is independent of the properties of the deposit market, and the optimal volumes of deposits (and the corresponding interest rate) is independent of the properties of the loan market. As a result, there is no link between credit and deposit activities.

<sup>2</sup>Assuming constant elasticities of demand of loans and supply of deposits, Freixas and Rochet (1997) also shows that loans rate (resp. deposits rate) becomes less (resp. more) sensitive to changes in the interbank rate as the intensity of competition increases.

<sup>3</sup>See Gilligan et al. (1984), Hirota and Tsutsui (1992) among others.

arises explicitly in our model. Especially, we focus on the role economies of scope play in producing a counter-intuitive relationship between the interbank interest rate and the endogenous interest rates.

Our model suggests that the result of Freixas and Rochet (1997) may not continue to hold when we introduce sequential portfolio choice and scope economies. In particular, we can identify the situations where an increase in the interbank interest rate leads to an decrease in the interest rate on deposits and to an increase in the interest rate on loans.

Most closely related to this paper is the above mentioned literature on the industrial organization approach to banking<sup>4</sup>. Besides them, Toolsema and Schoonbeek (1999) introduces asymmetries in the cost functions of banks<sup>5</sup>, or in their way of conduct (Cournot or Stackelberg) and demonstrates that for the Cournot version with asymmetric costs as well as for the Stackelberg version of the model, the same results as Freixas and Rochet (1997) hold for the *total* volumes of loans and deposits and the corresponding interest rates, and that, on the other hand, in the asymmetric cost Cournot version the results of Freixas and Rochet (1997) do not necessarily hold for the *individual* volumes of loans and deposits of the the bank with the smallest costs. Compared with the work by Toolsema and Schoonbeek, our analysis lays particular emphasis on the strategic aspects of deposits on loans which depend on scope economies.

The rest of the paper is organized as follows. Section 2 introduces the model and characterizes the equilibrium outcomes. In section 3 we investigate the effects of changing the interbank interest rate on the equilibrium quantities and correponding interest rates. Section 4 concludes. In Appendix we also investigate the effects of a change in the reserve rate on the equilibrium quantities and interest rates.

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<sup>4</sup>This paper is also related to a standard strategic commitment games literature. For example, Brander and Spencer (1983), Fudenberg and Tirole (1984) consider the case where in the first stage firms take some action (e.g. R&D, advertising) which alter the conditions of future competition, then competition is performed based on the action made in the previous stage. ( For a survey, see Tirole (1988).)

<sup>5</sup>As in the Freixas and Rochet (1997), Toolsema and Schoonbeek (1999) also considers the case in which cost complementarities (scope economies) between loans and deposits does not exist.

## 2 The Model

There are two banks, bank A and bank B.<sup>6</sup> They operate on the market for loans as well as on the market for deposits. The difference between the volume of loans  $L_i$  and the volume of deposits  $D_i$  of the bank  $i$  can be borrowed (or lent, if negative) on an interbank market. Denote the interest rates on the loan market and deposit market by  $r_L$  and  $r_D$ , respectively. The inverse demand function for loans is given by  $r_L(L)$ ,  $L := L_i + L_j$ , with derivative  $r'_L(L) < 0$ , and the inverse supply function of deposits is  $r_D(D)$ ,  $D := D_i + D_j$ , with derivative  $r'_D(D) > 0$ . The cost of managing an amount  $L$  of loans and an amount  $D$  of deposits is given by  $C(D_i, L_i)$ . Let  $r$  denote the exogenous interest rate on the interbank market, and  $\alpha$  ( $0 \leq \alpha < 1$ ) be the exogenous fraction of deposits that is required as a non-interest bearing reserve by the government or the central bank.

The profit function of bank  $i$  is given by

$$\pi_i(L_i, D_i) = r_L(L)L_i + rM_i - r_D(D)D_i - C(D_i, L_i), \quad i = A, B \quad (1)$$

where  $M_i$ , the net position of the bank  $i$  on the interbank market, is given by

$$M_i = (1 - \alpha)D_i - L_i, \quad i = A, B. \quad (2)$$

In this paper we specify a nonlinear cost function, which allows for scope economies between deposits and loans, as follows.

$$C(L_i, D_i) = \theta(D_i)L_i + \phi D_i, \quad i = A, B \quad (3)$$

where  $\phi > 0$  is the constant unit cost of deposits and  $\theta(D_i) > 0$  is marginal cost of loans. We assume that the same technology is available to all banks and that  $C(L_i, D_i)$  is continuously differentiable up to any order. Note that the sign of the cross derivative of this cost function  $\partial C / \partial L_i \partial D_i$  depends on the sign of  $\theta'(D_i)$ . The economic interpretation of the condition of  $\partial C / \partial L_i \partial D_i$  is related to the notion of scope economies. When  $\partial C / \partial L_i \partial D_i < 0$  ( $\theta'(\cdot) < 0$ ), there exists economies of scope. This implies that a universal bank that jointly offers

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<sup>6</sup>In what follows, unless otherwise noted, subscripts  $i, j = A, B$  and  $i \neq j$ .

loans and deposits is more efficient than two separate entities, specialized respectively on loans and deposits. On the contrary, when  $\partial C/\partial L_i \partial D_i > 0$  ( $\theta'(\cdot) > 0$ ), there exists diseconomies of scope. Finally, there exists no scope economies when  $\partial C/\partial L_i \partial D_i = 0$  ( $\theta'(\cdot) = 0$ )<sup>7</sup>.

In what follows, we focus on the case where  $\theta''(\cdot) = 0$ , and  $r_L(L)$ ,  $r_D(D)$  are linear function such as  $r_L = a - bL$ ,  $r_D = \beta + \gamma D$ , where  $a, b, \beta, \gamma > 0$ .

Now, under these specification, the equation (1) can be rewritten as

$$\pi_i(L_i, D_i) = [a - b(L_i + L_j) - r - \theta(D_i)]L_i + [r(1 - \alpha) - \beta - \gamma(D_i + D_j) - \phi]D_i. \quad (4)$$

The timing of the game is as follows: In the first stage, each bank simultaneously chooses the volume of deposits. Then they simultaneously choose the volume of loans in the second stage. Therefore, each bank engages in sequential portfolio choice problem. We assume that there is a well-defined Nash equilibrium in the second stage competition. In what follows, we adopt the subgame-perfect equilibrium as equilibrium concept. Thus, we can solve the game by backward induction.

We start to analyze the second stage competition among banks. Each bank chooses  $L_i$  to maximize  $\pi_i$  in (4). The first-order conditions are

$$\frac{\partial \pi_A}{\partial L_A} = a - 2bL_A - bL_B - r - \theta(D_A) = 0 \quad (5)$$

and

$$\frac{\partial \pi_B}{\partial L_B} = a - 2bL_B - bL_A - r - \theta(D_B) = 0. \quad (6)$$

The second-order conditions are also satisfied. Combining (5) and (6) yields the equilibrium amounts of loans in the second-stage subgame as

$$L_A = \frac{a - 2\theta(D_A) + \theta(D_B) - r}{3b} \quad (7)$$

and

$$L_B = \frac{a - 2\theta(D_B) + \theta(D_A) - r}{3b}. \quad (8)$$

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<sup>7</sup>Note that  $\theta'(\cdot) > 0$  (resp.  $\theta'(\cdot) < 0$ ) is not the definition but the sufficient condition for economies of scope (resp. diseconomies of scope). See Baumol et al. (1982).

From (7) and (8), One can easily confirm the strategic effects of the deposits,  $D_i$  and  $D_j$ , and of the parameter  $\alpha, r$  on  $L_A, L_B$ .

$$\frac{\partial L_i}{\partial D_i} = \frac{-2\theta'(D_i)}{3b}, \quad \frac{\partial L_i}{\partial D_j} = \frac{\theta'(D_j)}{3b} \quad (9)$$

$$\frac{\partial L_i}{\partial \alpha} = 0, \quad \frac{\partial L_i}{\partial r} = -\frac{1}{3b} \quad (10)$$

Note that strategic effects of  $D_i, D_j$  depend on the sign of  $\theta'(\cdot)$ . In the case of economies of scope ( $\theta'(\cdot) < 0$ ),  $\partial L_i/\partial D_i > 0$ ,  $\partial L_i/\partial D_j < 0$ . On the other hand,  $\partial L_i/\partial D_i < 0$ ,  $\partial L_i/\partial D_j > 0$  in the case of diseconomies of scope ( $\theta'(\cdot) > 0$ ).

Now turning to consider the first stage. By substituting (7) and (8) into (4), we obtain the profit function of Bank  $i$  in term of  $D_i$  and  $D_j$ :

$$\pi_i(D_i, D_j) = \frac{1}{b} \left[ \frac{a + \theta(D_j) - 2\theta(D_i) - r}{3} \right]^2 + [r(1 - \alpha) - \beta - \gamma(D_i + D_j) - \phi] D_i. \quad (11)$$

Where  $i, j = A, B, i \neq j$ . By differentiating (11) with respect to  $D_i$  and setting this first order derivative to zero, we have following first-order conditions.

$$\frac{\partial \pi_A}{\partial D_A} = -\frac{4}{3b} \left( \frac{a + \theta(D_B) - 2\theta(D_A) - r}{3} \right) \theta'(D_A) - 2\gamma D_A - \gamma D_B + r(1 - \alpha) - \beta - \phi = 0. \quad (12)$$

$$\frac{\partial \pi_B}{\partial D_B} = -\frac{4}{3b} \left( \frac{a + \theta(D_A) - 2\theta(D_B) - r}{3} \right) \theta'(D_B) - 2\gamma D_B - \gamma D_A + r(1 - \alpha) - \beta - \phi = 0. \quad (13)$$

The second-order conditions require that  $\frac{8(\theta'(D_i))^2}{9b} - 2\gamma < 0$  for  $\forall i, i = A, B$ .

The equation (12) (resp.13) gives the best-response function of bank A (resp. bank B) in the deposits:  $BR_A(D_B)$  (resp.  $BR_B(D_A)$ ). One can confirm that these reaction functions are downward sloping as usual in the model of Cournot competition (i.e. *strategic substitutes* in Bulow-Geanakoplos-Klemperer (1985) terms). In what follows, We impose following assumptions for the stability of equilibrium<sup>8</sup>.

**Assumption 1**  $|BR'_i(D_j)| = |BR'_j(D_i)| < 1$  for  $i = A, B, i \neq j$ .

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<sup>8</sup>See Martin (1993).

Under the *Assumption1*, Combining (12) and (13) yields the following (first-stage) equilibrium amounts of deposits for each bank,  $D_i^*$  ( $i = A, B$ ), which is a function of  $r, \alpha$ ;

$$D_i^* = D_i^*(r, \alpha), \quad i = A, B. \quad (14)$$

By substituting (14) into (7), (8), we can obtain subgame-perfect equilibrium amounts of loans for each bank. Let denote them by considering the functional relationship as follows:

$$L_i^* = L_i^*(r, \alpha, D_i^*(r, \alpha), D_j^*(r, \alpha)), \quad i, j = A, B, i \neq j. \quad (15)$$

### 3 Comparative Statics

In this section, we investigate the effects of changing the interbank interest rate on the equilibrium quantities and the corresponding interest rates. We focus on a symmetric subgame-perfect equilibrium.

Totally differentiating (12) and (13) with respect to endogenous variables and exogenous variables  $\alpha$  and  $r$ , we obtain

$$\begin{pmatrix} \frac{8\theta'(D_A)^2}{9b} - 2\gamma & -\frac{4\theta'(D_A)\theta'(D_B)}{9b} - \gamma \\ -\frac{4\theta'(D_A)\theta'(D_B)}{9b} - \gamma & \frac{8\theta'(D_B)^2}{9b} - 2\gamma \end{pmatrix} \begin{pmatrix} dD_A^* \\ dD_B^* \end{pmatrix} = \begin{pmatrix} rd\alpha - \frac{[9b(1-\alpha)+4\theta'(D_A)]}{9b}dr \\ rd\alpha - \frac{[9b(1-\alpha)+4\theta'(D_B)]}{9b}dr \end{pmatrix} \quad (16)$$

where the determinant for the matrix on the left hand side is given by

$$\Delta \equiv \left( \frac{8\theta'(D_A)^2}{9b} - 2\gamma \right) \left( \frac{8\theta'(D_B)^2}{9b} - 2\gamma \right) - \left( \frac{4\theta'(D_A)\theta'(D_B)}{9b} + \gamma \right) \left( \frac{4\theta'(D_A)\theta'(D_B)}{9b} + \gamma \right).$$

The sign of  $\Delta$  is positive under the *assumption1*.

We can solve (16) for  $dD_i^*/dr$  as,

$$\frac{\partial D_i^*}{\partial r} = -\frac{1}{9b\Delta} \left\{ \frac{[9b(1-\alpha) + 4\theta'(D_i)][8\theta'(D_j)^2 - 18b\gamma]}{9b} + \frac{[9b(1-\alpha) + 4\theta'(D_j)][4\theta'(D_i)\theta'(D_j) - 9b\gamma]}{9b} \right\} \quad (17)$$

for  $i, j = A, B, i \neq j$ . In a symmetric case ( $D_s = D_i = D_j$ )<sup>9</sup>, the equation (17) implies

$$\frac{\partial D_s^*}{\partial r} = \frac{\partial D_i^*}{\partial r} = \frac{\partial D_j^*}{\partial r} = -\frac{1}{9b\Delta} \left\{ \frac{[9b(1-\alpha) + 4\theta'(D_s)][12\theta'(D_s)^2 - 27b\gamma]}{9b} \right\}. \quad (18)$$

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<sup>9</sup>Where subscript  $s$  means a symmetric equilibrium.

Under the *Assumption 1*, we can confirm that  $\{12[\theta'(D_s)]^2 - 9b\gamma\}/9b < 0^{10}$ .

Thus, the sign of  $\partial D_s^*/\partial r$  accords with the sign of  $[9b(1 - \alpha) + 4\theta'(D_s)]$ . The first term in this bracket captures the benefit from the increased marginal revenue from a different increase in the interbank rate, which is always positive. The second term reflects the effect through the scope economies, which is positive (resp. negative) if  $\theta'(\cdot) > 0$  (resp.  $\theta'(\cdot) < 0$ ). If  $\theta'(\cdot) \geq 0$ , the sign of  $[9b(1 - \alpha) + 4\theta'(D_s)]$  is always positive. Thus each bank increases the volumes of deposits. Freixas and Rochet (1997) considers the case where  $\theta'(\cdot) = 0$ . In this simple case, the sign of  $\partial D_s^*/\partial r$  is always positive. Thus, a rise in  $r$  always increases the volumes of deposits and corresponding interest rate. In the case of  $\theta'(\cdot) < 0$ , however, the sign of  $[9b(1 - \alpha) + 4\theta'(D_s)]$  is complicated and it depends on the range of underlying parameters. Denote the deposit rate in a subgame-perfect equilibrium by  $r_D^* = r_D^*(D^*)$  with derivative  $r_D'(\cdot) > 0$ . The results are summarized as follows.

**Proposition 1** *The effect of  $r$  on  $r_D^*$  depends on the scope economies. (i) If  $\theta'(\cdot) \geq 0$ , then  $\partial r_D^*/\partial r > 0$ . (ii) If  $\theta'(\cdot) < 0$  and  $|\theta'(\cdot)| < [9b(1 - \alpha)]/4$ , then  $\partial r_D^*/\partial r > 0$ . (iii) If  $\theta'(\cdot) < 0$  and  $|\theta'(\cdot)| > [9b(1 - \alpha)]/4$ , then  $\partial r_D^*/\partial r < 0$ .*

The economics interpretation behind *proposition 1* is as follows. In the case where (i) and (ii) (resp. (iii)), a rise in  $r$  gives rise to an outward (resp. inward) shift of each bank's reaction curve in deposits respectively. It then follows that each bank increases (resp. decreases) the volumes of deposits and corresponding interest rate.

From the equation (15), the effect of  $r$  on  $L_i^*$  ( $i = A, B$ ) is as,

$$\begin{aligned} \frac{\partial L_i^*}{\partial r} &= \frac{\partial L_i^*}{\partial r} + \frac{\partial L_i^*}{\partial D_i} \frac{\partial D_i^*}{\partial r} + \frac{\partial L_i^*}{\partial D_j} \frac{\partial D_j^*}{\partial r} \\ &= -\frac{1}{3b} - \frac{2\theta'(D_i)}{3b} \frac{\partial D_i^*}{\partial r} + \frac{\theta'(D_j)}{3b} \frac{\partial D_j^*}{\partial r} \end{aligned} \quad (19)$$

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<sup>10</sup>By differentiating the equation (12)(or (13)),  $BR_i'(D_j) = \frac{9b\gamma + 4\theta'(D_i)\theta'(D_j)}{8[\theta'(D_i)]^2 - 18b\gamma}$  for  $i = A, B, i \neq j$ . The numerator is positive due to same technologies among banks. On the other hand, the denominator is negative from the second order condition. Therefore,  $|BR_i'(D_j)| < 1$  implies that  $\frac{8[\theta'(D_i)]^2 + 4\theta'(D_i)\theta'(D_j) - 9b\gamma}{9b} < 0$  for  $i = A, B, i \neq j$ . In a symmetric equilibrium ( $D_s = D_i = D_j$ ), this implies that  $\{12[\theta'(D_s)]^2 - 9b\gamma\} < 0$ .

The last equality follows from (9) and (10). For a symmetric case,  $D_s = D_i = D_j$ , by using (18), the equation (19) can be rewritten as

$$\frac{\partial L_s^*}{\partial r} = \frac{\partial L_i^*}{\partial r} = \frac{\partial L_j^*}{\partial r} = -\frac{1}{3b} - \frac{\theta'(D_s)}{3b} \frac{\partial D_s^*}{\partial r}. \quad (20)$$

The first term in the right-hand sides of (20) reflects the direct effect of an increase in  $r$ . This direct effect is always negative because a rise of interbank rate means a rise of the opportunity cost of loans. On the other hand, there also exists the indirect effect (strategic effect) from a change in deposits. Recall that this strategic effect depends on the sign of  $\theta'(\cdot)$ . Freixas and Rochet (1997) considers the case where each bank determines the volumes of deposits and loans simultaneously and scope economies does not exist ( $\theta'(\cdot) = 0$ ). In this simple case, strategic effect is not present in (20) and the sign of  $\partial L_s^*/\partial r$  depends on only direct effect. Thus, a rise in  $r$  always decreases the volumes of loans and increases the corresponding interest rate. As in the present paper, however, if each bank determines the volumes of deposits and loans sequentially and scope economies do exist, the sign of  $\partial L_s^*/\partial r$  depends on both direct effect and strategic effect. Denote the loan rate in a subgame-perfect equilibrium by  $r_L^* = r_L^*(L^*)$  with derivative  $r_L'(\cdot) < 0$ . The results are summarized as follows.

**Proposition 2** *The effect of  $r$  on  $r_L^*$  depends on the scope economies. (i) If  $\theta'(\cdot) \geq 0$ , then  $\partial r_L^*/\partial r > 0$ . (ii) If  $\theta'(\cdot) < 0$  and  $|\theta'(\cdot)| > [9b(1 - \alpha)]/4$ , then  $\partial r_L^*/\partial r > 0$ . (iii) If  $\theta'(\cdot) < 0$  and  $|\theta'(\cdot)| < [9b(1 - \alpha)]/4$ , then the sign of the  $\partial r_L^*/\partial r$  is indeterminate.*

The economics interpretation behind *proposition 2* is as follows. In the case of (i) and (ii), both the direct effect and the indirect effect in (20) work in the same direction. Thus, a rise in  $r$  decreases the volumes of loans and increases the corresponding interest rate. In the case (iii), however, these two effects work in the opposite direction. We cannot determine whether the direct effect or the indirect effect dominates the other.

Table 1 summarizes the effects of a change in  $r$  on  $r_D^*$  and  $r_L^*$ .

|                  |                                      | $\partial r_D^*/\partial r$ | $\partial r_L^*/\partial r$ |
|------------------|--------------------------------------|-----------------------------|-----------------------------|
| $\theta' \geq 0$ |                                      | +                           | +                           |
| $\theta' < 0$    | $ \theta'  < \frac{9b(1-\alpha)}{4}$ | +                           | $\pm$                       |
|                  | $ \theta'  > \frac{9b(1-\alpha)}{4}$ | -                           | +                           |

Table 1: The effects of a change in  $r$  on  $r_D$  and  $r_L$

## 4 Concluding Remarks

Freixas and Rochet (1997) considers simultaneous portfolio choice and the linear cost function between deposits and loans (i.e.  $\theta'(\cdot)$ ). They show that an increase in the interbank interest rate leads to an increase in the interest rates on deposits and loans<sup>11</sup>.

The present paper describes a two-player, two-stage game. Our model follows the previous literature with two exceptions. First, the deposit and loan decisions are made sequentially, rather than simultaneously. Second, we specify a nonlinear cost function, which allows for economies of scope between deposits and loans.

We show that economies of scope can overturn the findings of previous studies. Especially, we show that in the case where large economies of scope an increase in the interbank interest rate leads to a decrease in the interest rate on deposits and to an increase in the interest rate on loans.

We note that our results have been obtained in a specific functional forms with linear demand function and the supply function. Notwithstanding those specifications, however, we believe that our results obtained in this paper are fundamentally robust for a more general functional forms. The question of how robust they really are will be left for further research.

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<sup>11</sup>Early work by Klein (1971) and Monti (1972) show that similar results hold in a monopolistic bank model.

## Appendix

In this appendix, we investigate the effects of changing the reserve rate on the equilibrium quantities and interest rates.

We can solve (16) for  $dD_A^*/d\alpha$  and  $dD_B^*/d\alpha$  respectively as,

$$\frac{dD_A^*}{d\alpha} = \frac{r}{\Delta} \left\{ \frac{8[\theta'(D_B)]^2 + 4\theta'(D_A)\theta'(D_B) - 9b\gamma}{9b} \right\} \quad (21)$$

and

$$\frac{dD_B^*}{d\alpha} = \frac{r}{\Delta} \left\{ \frac{8[\theta'(D_A)]^2 + 4\theta'(D_B)\theta'(D_A) - 9b\gamma}{9b} \right\}. \quad (22)$$

In a symmetric equilibrium ( $D_s = D_i = D_j$ ), (21) and (22) imply that

$$\frac{dD_s^*}{d\alpha} = \frac{dD_i^*}{d\alpha} = \frac{dD_j^*}{d\alpha} = \frac{r}{\Delta} \left\{ \frac{12[\theta'(D_s)]^2 - 9b\gamma}{9b} \right\}. \quad (23)$$

Therefore, in a (symmetric) subgame-perfect equilibrium, an increase of reserve ratio decreases the amounts of deposits for each bank:  $dD_s^*/d\alpha = dD_i^*/d\alpha = dD_j^*/d\alpha < 0$ ,  $i = A, B, i \neq j$ . Denote the deposit rate in a subgame-perfect equilibrium by  $r_D^* = r_D^*(D^*)$  with derivative  $r_D'(\cdot) > 0$ . The results are summarized as follows;

**Proposition 3** *In a symmetric subgame-perfect equilibrium, an increase of reserve ratio always decreases interest rate on the deposit. That is,  $\partial r_D^*/\partial\alpha < 0$ .*

Let us turn to see the effect of  $\alpha$  on  $L$ . From (15) we have

$$\begin{aligned} \frac{\partial L_i^*}{\partial\alpha} &= \frac{\partial L_i^*}{\partial\alpha} + \frac{\partial L_i^*}{\partial D_i} \frac{\partial D_i^*}{\partial\alpha} + \frac{\partial L_i^*}{\partial D_j} \frac{\partial D_j^*}{\partial\alpha} \\ &= -\frac{2\theta'(D_i)}{3b} \frac{\partial D_i^*}{\partial\alpha} + \frac{\theta'(D_j)}{3b} \frac{\partial D_j^*}{\partial\alpha}. \end{aligned} \quad (24)$$

The last equality comes from (9) and (10). For a symmetric case ( $D_s = D_i = D_j$ ), by using (23), the equation (24) can be rewritten as

$$\frac{\partial L_s^*}{\partial\alpha} = \frac{\partial L_i^*}{\partial\alpha} = \frac{\partial L_j^*}{\partial\alpha} = -\frac{\theta'(D_s)}{3b} \frac{\partial D_s^*}{\partial\alpha}. \quad (25)$$

From the *Proposition 1*, the sign of  $\partial D_s^*/\partial\alpha$  is negative, so the sign of  $\partial L_s^*/\partial\alpha$  depends on the sign of  $\theta'(\cdot)$ . Therefore, if there exists economies of scope (resp. diseconomies of scope), then  $\partial L_s^*/\partial\alpha < 0$  (resp.  $\partial L_s^*/\partial\alpha > 0$ ). Denote the loan rate in a subgame-perfect equilibrium by  $r_L^* = r_L^*(L^*)$  with derivative  $r_L'(\cdot) < 0$ . The results are summarized as follows.

**Proposition 4** *The effect of  $\alpha$  on  $r_L^*$  depends on the scope economies. (i) If there are economies of scope ( $\theta'(\cdot) < 0$ ), then  $\partial r_L^*/\partial\alpha > 0$ . (ii) If there are no economies of scope ( $\theta'(\cdot) = 0$ ), then  $\partial r_L^*/\partial\alpha = 0$ . (iii) If there are diseconomies of scope ( $\theta'(\cdot) > 0$ ), then  $\partial r_L^*/\partial\alpha < 0$ .*

Table 2 summarizes the effects of a change in  $\alpha$  on  $r_D$  and  $r_L$ .

|                      | $\partial r_D^*/\partial\alpha$ | $\partial r_L^*/\partial\alpha$ |
|----------------------|---------------------------------|---------------------------------|
| $\theta'(\cdot) < 0$ | –                               | +                               |
| $\theta'(\cdot) = 0$ | –                               | 0                               |
| $\theta'(\cdot) > 0$ | –                               | –                               |

Table 2: The effects of a change in  $\alpha$  on  $r_D$  and  $r_L$

## References

- [1] Baumol, W.J., J.C.Panzar and R.D.Willing (1982) *Contestable Markets and the Theory of Industrial Structure*, Harcourt Brace Jovanovich.
- [2] Brander, J. and B. Spencer (1983) “Strategic Commitment with R&D: the symmetric case”, *Bell Journal of Economics*. 14: 225-235.
- [3] Bulow, J., J. Geanakoplos and P. Klemperer (1985) “Multimarket Oligopoly: Strategic Substitutes and Complements”, *Journal of Political Economy*. 93: 488-511.
- [4] Freixas, X. and Rochet, J-C (1997) *Microeconomics of banking*, The MIT Press, Cambridge.
- [5] Fudenberg, D. and J. Tirole (1984) “The Fat-Cat Effect, the Puppy Dog Ploy and the Lean and Hungry Look”, *American Economic Review*. 74: 361-366.
- [6] Gilligan, T., Smillock, M. and W. Marshall (1984) “Scale and Scope Economies in the Multiproduct Banking Firm”, *Journal of Monetary Economics*. 13: 393-405.
- [7] Kasuya, M. (1986) “Economies of Scope no Riron to Ginkogyo eno Tekiyo”, *Kinyukenkyu*. 8: 79-118. (in Japanese)
- [8] Klein, M. (1971) “A theory of the banking firm”, *Journal of Money, Credit, and Banking*. 3: 205-218.
- [9] Martin, S. (1993) *Advanced Industrial Economics*, Blackwell Publishers Inc, Oxford and Cambridge, Massachusetts.
- [10] Monti, M. (1972) “Deposit, Credit, and interest rate determination under alternative bank objectives”, In *Mathematical methods in investment and finance*, edited by G.P.Szego and K.Shell. Amsterdam: North-Holland.
- [11] Suzumura, K. (1991) “Ginkougyouniokeru KyouyouEkiseiEKeizaikouseih”, *Kinyukenkyu*. 9: 17-39. (in Japanese)

- [12] Tirole, J. (1988) *The Theory of Industrial Organization*, The MIT Press, Cambridge.
- [13] Toolsema, L.A. and Lambert Schoonbeek (1999) “Bank Behavior and the Interbank Rate in an Oligopolistic Market”, *mimeo*.