The Other Side of Limited Liability: Predatory Behavior and Investment Timing

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Christian Bayer[*†]

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Abstract

This paper investigates the interplay of investment irreversibility, predatory behavior, and limited liability in a duopoly with aggregate demand uncertainty. We find that limited liability and investment irreversibility is likely to produce predatory behavior in very competitive industries in which prices react strongly to changes in quantity and capacity increases are not too costly.
The rationale for this may be summarized as follows: Under limited liability, the owners of a firm have to decide whether they are willing to finance losses from private funds, or whether they rather default on the firms obligations in adverse states. However, market conditions themselves become endogenous in a duopoly since the quantity decisions of all competitors determine the market price. If now investment is irreversible, it is a strong commitment. It hence becomes a device to force others to leave early and allows oneself to commit to leave late. If the ability to promote the exit of a competitor is strong, it may then even result in firms investing only to prey, i.e. firms invest only to consequently monopolize the market.
Therefore, the model of this paper explains predatory behavior in a duopoly without invoking reputational, network- or learning-effects. Moreover, this paper's model also does not define predatory behavior as deviations from tacit collusion.

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[*]Universität Dortmund, Graduiertenkolleg "Allokationstheorie, Wirtschaftspolitik und kollektive Entscheidungen", Dept. of Economics, D-44221 Dortmund,
phone: +49-231-755-5403, e-mail: *christian.bayer@wiso.uni-dortmund.de*

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1 Introduction

Predatory behavior as some special form of anti-competitive practices has a long tradition of debate both among academic economists and practitioners. It has been studied widely at least since Selten (1978) introduced his "chain store paradox". Selten claimed that predatory behavior could not emerge among rational competitors even though predation—especially in a dynamic context—has some intuitive appeal.

The intuitive predatory outcome could be reestablished by later contributions. However, most of these contributions either modelled predatory behavior as a static and once-and-for-all increase in quantity, modelled predation as a result of learning or network effects, or defined predation as temporary deviations from tacit collusion. Yet, the present paper shows that it is not necessary to restrict the analysis of predatory behavior to these phenomena. In our approach predatory behavior emerges dynamically from time to time, but is not modelled as a break-down of collusion. Instead, predation results from the interaction of investment-irreversibility and exit decisions. If investment is irreversible, it has a strong strategic influence on exit decisions. It is a commitment to leave the market late and at the same time it promotes the exit of the competitor. Therefore, firms may wish to invest upon a decrease in demand. This temporarily depresses prices further and forces the competitor to exit. Consequently, a market decline triggers a predatory race for market shares.

Predation was firstly re-established after Selten's "paradox" by Kreps and Wilson (1982) and by Milgrom and Roberts (1982). They modified Selten's model by introducing asymmetry of information among competitors. Yet, predatory behavior in their models is a static phenomenon as a consequence of the inherently static nature of information. Firms act more aggressively all the time to deter entry respectively to promote exit of competitors. Analogously, when strategic commitment triggers predation, in most models (e.g. Brander and Lewis 1986, Bolton and Scharfstein, 1990, or Glazer, 1994) there is no change of behavior over time. Again firms act more aggressively in any state of the world. Empirically however, the aggressiveness of competitors seems to switch from periods of less competition to periods of aggressive competition and vica versa.[1]

Existing dynamic theories of predation often take tacit collusion as starting point. Examples for this are models which study price wars.[2] In these models, market conditions are uncertain (in the future), and so firms have symmetric but imperfect information. Changes in demand may then trigger deviations from collusion when deviations become profitable. As a result, firms start price wars from time to time, and this approach gives predation as a dynamic phenomenon a theoretical underpinning. Yet, the focus on tacit collusion remains a caveat. To avoid this, alternative approaches have been suggested

[1] See for example Busse's (2002) analysis of price wars in the airline industry.

[2] See Ordover and Saloner (1989) for a summary or for more recent contributions Fershtman and Pakes (2000) or Busse (2002).

for network industries[3] and industries with prominent learning-curve effects (Cabral and Riordan, 1994 and 1997). Nevertheless, these models remain only valid for to certain types of industries.

However, the strategic commitment approach integrates naturally into a dynamic framework of predatory behavior, once one accepts the assumption of (partial) irreversibility of investment. Then, strategic real options theory[4] becomes a valid tool to analyze competition, exit, and irreversible investment. Although irreversible investment is the possibly strongest form of commitment, and may thus have a strong strategic influence on other decisions of a firm, most of the real options literature has ignored the strategic interaction of (potential) exit and investment.

If a firm can exit a market at will and has only limited liability, its owners have to decide whether they are willing to finance possibly negative cash flows from private funds, or whether they rather default on the firms obligations in adverse states. Consequently, firms need to determine at which market conditions they will optimally default and exit. However, market conditions themselves become endogenous in a duopoly since the investment and thus quantity decisions of all competitors determine the market price.

If investment alters the market price of produced goods, investment decisions of one firm alter the likelihood of exit for the other firm: A firm that invests and expands production, receives higher earnings at the expense of other firms. Upon exit, this firm looses more income while a non-investing firm looses less income when it leaves. Thus, the investing firm wishes to delay exit after investment, while the other firm wishes to exit more early. In consequence, investment is not only a commitment to leave the market late, but also a device to force others to leave early, so that the interdependence of both firms' earnings transforms investment into a device with a twofold strategic value. This strategic value gives firms a strong incentive to commit themselves and invest early and in the extreme, the ability to promote the exit of a competitor may even result in firms investing only to prey. They invest only to consequently monopolize the market.

This incentive to monopolize can be substantial and hence, the interplay of limited liability and irreversibility of investment influences investment decisions becomes strategically important. Against the strategic incentive firms need to trade off the gain from waiting and obtaining more information, the value of waiting. One of the main points of our paper is the analysis of these two countervailing forces, strategic commitment and value of waiting.

As both forces counteract, predatory behavior does not take the form of stronger competition in all states of the world. In our approach, predatory behavior emerges as a policy triggered by adverse market conditions. Moreover, we do not model predation as a

[3]See Athey and Schmutzler (2001) for a general model of investment and increasing dominance that includes network-industries as a special case.

[4]See for example Huisman and Kort (1999), Sparla (2001), Weeds (2003), or Murto (2004).

break down of some collusive situation. The break down of collusion has been studied by Grenadier (1996) in a real options duopoly-model of investment. In his model investment cascades are triggered by a market decline that breaks up collusion. In our model, we also find investment in declining markets, but we explain this as a result of an interaction of investment and exit decisions.

This interaction and predatory behavior are our two points of focus, and that focus clearly distinguishes our work from existing ones which study exit decisions in a duopoly using a real options framework: Sparla (2001) discusses partial but irreversible capacity reductions, Lambrecht (2001) and Murto (2004) both analyze complete exit. In all three papers (Sparla, Murto and Lambrecht), firms are assumed to be unable to increase capacity, so that predatory behavior cannot emerge. Yet, Lambrecht also studies sequential market entry and exit decisions. There, he finds that entrants sometimes crowd out an existing monopolist upon market entry. Nevertheless, there is no market entry in a declining market in his model and the roles of the firms are preassigned with respect to who enters first.

Both, irreversible investment and exit decisions, have been studied before by Joaquin and Khanna (2001), but predatory investment cannot occur in their model of potential competition, because they assume that (rational) exit of the competitor imposes a *loss* on the remaining firm. For a monopoly, Jou (2001) models both, entry and exit, and relates them to the issue of optimal financing. Obviously predation is no issue in monopoly.

Wether predatory behavior occurs in equilibrium in our model depends on idiosyncratic and on aggregate factors. In the aggregate the "competitiveness" of the market is important. If adjustment costs are high or prices hardly react to changes in quantity, predatory investment never occurs.

In the idiosyncratic domain, the difference between firms in fixed running (overhead) costs determines their propensity for predation. In this respect, the most closely related papers are Fershtman and Pakes' (2000) theoretical work and the empirical paper of Busse (2002). Busse finds for the airline industry that financial leverage is one of the main determinants for starting a price war. This result is important for our findings, as interest payments on debt are naturally one important source of fixed cost. Moreover, Busse finds that the probability of starting a price war reacts in a non-linear fashion to changes in the financial situation of a firm. We find a similar result in our theoretical model.

The theoretical analysis is complemented by numerical examples. The mathematical structure only allows to generate analytical conditions for optimal strategies. Closed-form solutions for optimal investment and exit strategies cannot be derived when both decisions are to be considered simultaneously, see Dixit and Pindyck (1994). In consequence, only numerical simulations can provide some insight on the magnitude and economic significance of the strategic effects studied. Therefore, one section of the paper provides some numerical examples. In these examples, strategic motives are indeed very important. Firms invest predatorily, i.e. not because investment has a positive present value on its own, but because it allows to drive the competitor out of the market. If strong price reactions enable predatory behavior upon a market decline, firms also invest early when the market grows and investment does not immediately force the competitor to exit. In consequence, the gain from waiting for more information is sometimes even fully offset by strategic incentives and in this extreme, the first investor behaves in equilibrium as if it follows a naive net-present-value rule.

The rest of the paper proceeds as follows. Section 2 outlines the model and presents the basic assumptions. Section 3 discusses non-strategic investment and exit decisions of monopolists. Section 4 derives firm value in duopoly and the corresponding price triggers for investment and exit. Section 5 presents our numerical results. Section 6 concludes. Detailed proofs are available in the appendix.

2 Model setup

2.1 General assumptions

We model a market with stochastic demand fluctuations in continuous time t, $t \in [0, \infty[$. In this market, up to two risk-neutral firms can produce and sell. Total production given, the price process $(P_t)_{t \geq 0}$ is assumed to be a geometric Brownian motion and shall be given by

$$P_t = D(Q_t) Y_t \quad , \tag{1}$$

$$dY_t = Y_t (\mu dt + \sigma dB_t) \quad . \tag{2}$$

Y_t measures the aggregate state of demand and B_t denotes a standard Brownian motion. Q_t denotes aggregate industry production and the inverse demand D maps these quantities to prices which are then shifted by Y. Production of each individual firm i at time t shall be denoted by $q_{i,t}$. For the sake of simplicity, we assume that output is solely produced by a capital good which does not depreciate.

For $t = 0$ we assume that both firms already operate in the market, each with some initial production $\underline{q}_i$. Moreover, both firms may exit irreversibly at no cost at any time, i.e. irreversibly chose $q = 0$. Additionally, we assume that a firm is unable to temporarily suspend production. As long as a firm operates it has to pay some fixed costs of operation b_i, e.g. coupon payments for debt, overhead costs etc. Therefore, instantaneous profits of firm i are given by

$$q_{i,t} P_t(Q_t, Y_t) - b_i \quad . \tag{3}$$

Both firms may also invest and irreversibly (except for potential permanent exit) increase production to $\bar{q}_i$ at cost C_i. Hence, there are 9 possible states of production (q_1, q_2). The set of possible states is given by $\{0, \underline{q}_1, \bar{q}_1\} \times \{0, \underline{q}_2, \bar{q}_2\}$.

Whenever one of the firms invests or exits, the price changes instantaneously. When for example one firm quits the price jumps up immediately. For notational convenience, we define a function Δ for the relative price change induced by a change in aggregate supply from Q_- to Q_+.

$$\Delta_{Q_-,Q_+} := \frac{P_t(Q_+)}{P_t(Q_-)} = \frac{D(Q_+)}{D(Q_-)} \tag{4}$$

2.2 Firm Value

The firms are assumed to have unlimited access to external resources, i.e. they can finance any losses and investment costs if they wish. Both firms discount future profits at the risk-adjusted discount rate ρ, and seek to maximize firm value. The discount rate ρ shall be larger than the drift of the price μ to obtain a finite expected firm value. Moreover, we assume $|\mu| < \frac{\sigma^2}{2}$. This assures that P_t reaches any finite value in finite time.[5]

Under these assumptions, the roots of the so called "fundamental quadratic equation" (see e.g. Dixit and Pindyck, 1994) are given by

$$\beta_{1,2} = \frac{1}{2} - \frac{\mu}{\sigma^2} \pm \sqrt{\left[\frac{1}{2} - \frac{\mu}{\sigma^2}\right]^2 + \frac{2\rho}{\sigma^2}} \ , \tag{5}$$

which implies $\beta_1 > 1$ and $\beta_2 < 0$ and $\beta_1 + \beta_2 = 1 - 2\frac{\mu}{\sigma^2} > 0$.

Therefore, as in Jou (2001, p. 72), the general solutions for the firm value $V_i(P, q_i, q_{-i})$ of firm i is given by the following equation.[6]

$$V_i(P, q_i, q_{-i}) = q_i \tfrac{P}{\rho-\mu} - \tfrac{b_i}{\rho} + a_{i1}(q_i, q_{-i})P^{\beta_1} + a_{i2}(q_i, q_{-i})P^{\beta_2}. \tag{6}$$

The constants a_{i1} and a_{i2} reflect not only the (q_i, q_{-i})-state dependent option value of the investment and exit option, but also the expected change in profits due to potential actions of the competitor. Consequently, strategic considerations have an important influence on both parameters, and a_{i1} and a_{i2} will vary with the state of production (q_i, q_{-i}).

2.3 Game sequence and equilibrium concept

Therefore, a_{i1} and a_{i2}, have to be solved for by deriving further conditions that reflect the dynamic optimality of investment and exit plans. Hence, it is useful to recall the timing structure of the game. At each point in time an *active* firm may chose to

(1) invest and increase capacity to $\bar{q}_i$ if it has not invested yet,

(2) exit and become inactive from then on,

(3) or keep production constant and wait.

[5] As Sparla (2001) argues, if the drift μ is strong compared to the variance σ^2, the probability that firms will not exit in finite time is strictly positive. However, this causes notational inconvenience as one root of the "fundamental quadratic equation" (see below) has to be "adjusted" to derive the correct value functions, see Sparla (2001) for details.

[6] See appendix for details. As usual, we denote by firm $-i$ the competitor of firm i.

Figure 1: Examples of possible sequences of events

Consequently, each state (q_1, q_2) can be seen as subgame (stage) and the following sections will derive and specify firm value and strategies at the various states (q_1, q_2).

Figure 1 displays some of the possible sequences of action that may occur depending on the realization of the Brownian motion.

We do not assume which firm invests or exits first, but let this be determined in equilibrium. Therefore, whenever the first firm acts (invests or exits) the other firm must be at least indifferent between acting somewhat earlier to be the first mover or taking the action later as the second mover. Consequently, it is necessary to discuss first the behavior of both firms as second mover (for investment just as for exit) to obtain the valuation of both competitors for the second mover's position. This valuation is crucial for solving the competition for the position of the first mover using the just outlined indifference principle for equilibrium investment and exit. For the exit game the position of the second mover coincides with the position of a monopolist.

When positions are valued differently, the analysis greatly simplifies. To obtain this simplification, the fixed costs of both firms shall differ. There may also (but not necessarily) be a difference between both firms with respect to the quantities the two firms produce both, at high and at low capacity. All differences, both in costs and quantities, are summarized in the function $l_i(q_i) := \frac{b_i}{q_i D(q_i)}$, the ratio of fixed costs to monopoly earnings at $Y_t = 1$. Without loss of generality, we assume firm 2 is the firm with the the larger ratio of fixed costs to earnings once both firms have invested. In other words firm 1 is more efficient with respect to overhead costs.

Assumption 1: $l_1(\bar{q}_1) < l_2(\bar{q}_2)$.

The cost difference now allows us to concentrate on simple trigger strategies to characterize firm behavior. These strategies define a price trigger for investment and exit of a firm for any state of the game in which that firm is active. Firms can chose a **non-predatory strategy** that is characterized by exit at low prices and by a single price trigger for investment for each state $\left(\underline{q}_i, q_{-i}\right)$. If a firm chooses a **predatory strategy** it will not exit at low prices, but will instead use its investment option and invest, since this promotes the exit of the competitor.

Definition 1 *A vector $P_i^{\#} \in \mathbb{R}_+^{11}$ of (q_i, q_{-i})-state-contingent price-triggers for investment $P_{\underline{q}_i,q_{-i}}^{inv,i}$, predation $P_{\underline{q}_i,q_{-i}}^{pred,i}$ and exit $P_{q_i,q_{-i}}^{exit,i}$*

$$P_i^{\#} = \left(\begin{array}{c} \underbrace{P_{\underline{q}_i,\bar{q}_{-i}}^{inv,i} \quad P_{\underline{q}_i,\underline{q}_{-i}}^{inv,i} \quad P_{\underline{q}_i,0}^{inv,i}}_{investment}, \underbrace{P_{\underline{q}_i,\bar{q}_{-i}}^{pred,i} \quad P_{\underline{q}_i,\underline{q}_{-i}}^{pred,i}}_{predation}, \\ \underbrace{P_{\bar{q}_i,\bar{q}_{-i}}^{exit,i} \quad P_{\bar{q}_i,\underline{q}_{-i}}^{exit,i} \quad P_{\bar{q}_i,0}^{exit,i} \quad P_{\underline{q}_i,\bar{q}_{-i}}^{exit,i} \quad P_{\underline{q}_i,\underline{q}_{-i}}^{exit,i} \quad P_{\underline{q}_i,0}^{exit,i}}_{exit} \end{array} \right) \tag{7}$$

*characterizes a stationary **Markov-strategy** of firm i.*

This notation of a strategy allows for both predatory and non predatory behavior. Given strategy $P_i^{\#}$ firm i invests when the price is equal to or is larger than the investment price trigger for the first time in the current state (q_1, q_2). Firm i exits when the price *equals* the respective exit-price trigger for the first time.

If the strategy is *non-predatory,* we have $P_{\underline{q}_i,\bar{q}_{-i}}^{pred,i} < P_{\underline{q}_i,\bar{q}_{-i}}^{exit,i}$ and $P_{\underline{q}_i,\underline{q}_{-i}}^{pred,i} < P_{\underline{q}_i,\underline{q}_{-i}}^{exit,i}$. These values make sure that firm i never invests predatorily, since it will have left before the triggers are reached.

In a *predatory strategy* at least one of the triggers $P_{\underline{q}_i,\bar{q}_{-i}}^{pred,i}$ or $P_{\underline{q}_i,\underline{q}_{-i}}^{pred,i}$ falls between the respective exit and investment price-triggers. In this case firm i invests also when the price falls to the predatory investment price-trigger. It does so, since investment promotes the exit of the competitor.

Predatory price triggers are defined for any state in which the firm can still increase capacity. Therefore, a strategy defines for each state a price trigger for investment, for predation and for exit, although prices may actually never reach that price triggers of firm i in equilibrium. If for example in state $\left(\bar{q}_i, \underline{q}_{-i}\right)$ the competitor of firm i exits before the price-trigger $P_{\bar{q}_i,\underline{q}_{-i}}^{exit,i}$ is actually reached, the state changes, and the trigger now effective is $P_{\bar{q}_i,0}^{exit,i}$. Therefore, the strategy definition defines also (most) behavior out of equilibrium. Out of equilibrium situations are only ignored in our definition of a strategy for firm i if they result form a mistake of firm i itself. This simplification allows to describe strategies in the convenient price trigger form.

So defined strategies are Markovian and stationary as they only condition on the current price and state but neither on the history of the game (Markovian) nor on time itself (stationarity). Our focus on pure strategies is motivated by notational convenience, but underlying is the continuous time equilibrium concept of Fudenberg and Tirole (1985).

This concept uses mixed strategies and an extended strategy space. However, equilibrium outcomes are equivalent to those in which agents employ pure strategies, see section 3 in Fudenberg and Tirole (1985) for details. With the definition of a strategy, we can now describe what is a best response and an equilibrium in the game studied.

Definition 2 *A vector of* (q_i, q_{-i})*-state-contingent price-triggers* $\bar{P}_i^{\#}$ *is a* ***Markov-perfect best-response in pure strategies*** *of firm i to the vector of price triggers* $P_{-i}^{\#}$ *of firm* $-i$*, if:*

(a) *for all* (q_i, q_{-i}) *it is credible not to exit before the declared exit price-trigger*

$$\forall P \quad > \quad \bar{P}^{exit,i}_{q_i,q_{-i}} : V_i \left(P, q_i, q_{-i} | \bar{P}_i^{\#}, P_{-i}^{\#} \right) > 0. \qquad \textit{(limited liability)}$$

(b) *Moreover, firm i has no incentive to preempt on its own price triggers for investment:*

$$\forall P \in \left[\bar{P}^{pred,i}_{\underline{q}_i,q_{-i}}, \bar{P}^{inv,i}_{\underline{q}_i,q_{-i}} \right] : V_i \left(P, \underline{q}_i, q_{-i} | \bar{P}_i^{\#}, P_{-i}^{\#} \right) \geq V_i \left(P, \overline{q}_i, q_{-i} | \bar{P}_i^{\#}, P_{-i}^{\#} \right) - C_i \qquad \text{(no preemption)}$$

(c) *And using other price triggers* $P_i^{\#}$ *that fulfill the above credibility constraints does not increase value at the proposed price triggers:*

$$\begin{aligned} V_i \left(\bar{P}^{exit,i}_{q_i,q_{-i}}, q_i, q_{-i} | \bar{P}_i^{\#}, P_{-i}^{\#} \right) &\geq V_i \left(\bar{P}^{exit,i}_{q_i,q_{-i}}, q_i, q_{-i} | P_i^{\#}, P_{-i}^{\#} \right) \\ V_i \left(\bar{P}^{pred,i}_{q_i,q_{-i}}, q_i, q_{-i} | \bar{P}_i^{\#}, P_{-i}^{\#} \right) &\geq V_i \left(\bar{P}^{pred,i}_{q_i,q_{-i}}, q_i, q_{-i} | P_i^{\#}, P_{-i}^{\#} \right) \qquad \text{(optimality)} \\ V_i \left(\bar{P}^{inv,i}_{q_i,q_{-i}}, q_i, q_{-i} | \bar{P}_i^{\#}, P_{-i}^{\#} \right) &\geq V_i \left(\bar{P}^{inv,i}_{q_i,q_{-i}}, q_i, q_{-i} | P_i^{\#}, P_{-i}^{\#} \right). \end{aligned}$$

Definition 3 *A* ***Markov-perfect equilibrium in pure strategies*** *is a pair of vectors* $\left(P_i^{\#}, P_{-i}^{\#} \right)$*, so that each vector is a best response to the other.*

The first two constraints reflect the credibility of a strategy. If a firm likes to act before a proposed price trigger has been reached, a threat to use this trigger cannot be credible. In fact, the "limited liability" is a "no-preemption" constraint for the exit decision. When the limited liability constraint is binding, firm i chooses to exit before the exit price trigger is actually reached, i.e. it preempts on its own exit. Note, however, the constraint *does not* imply that the firm does never have negative profits. Only the expected value of future (and current) profits must be positive. Therefore, the constraint is conditional on the strategy of the competitor–especially conditional on whether firm i expects to leave first or expects to leave second.

The optimality constraint implies that the equilibrium price triggers need to be best response price triggers also for any (q_1, q_2) subgame. They cannot be just "good threats" at the very beginning of the game. Therefore, we can and need to solve for the equilibrium by backward induction over the states (not over time). Because of this, we start with the analysis of a monopoly, as this is the "subgame" reached, once one firm has left.

Yet, the definitions above are somewhat imprecise from a very formal perspective. Strictly speaking, the strategies firms use should be sets of prices and firms exit or invest once the price hits *any* boundary of these sets for the first time. However, we have only given upper bounds for exit and lower bounds for investment. Murto (2004) shows that firms may optimally use disconnected sets as optimal strategies for exit. This means a firm may exit when prices reach some high price interval, it will not exit on an interval of smaller prices, but exit again when prices decrease further (gap-equilibrium). The intermediate interval of inaction can of course only be reached by intermediate initial prices or by mistake. Our equilibrium- and best-response definitions understand the firms' strategies in Murto's way, but we restrict our analysis only on the largest interval of prices for exit. This is reflected in our definitions: They give no restriction for firm i for any prices below its own exit price-trigger. However, this procedure is only justified in case strategies are in fact disconnected sets if the initial price is not intermediate.

The following assumption ensures, that (a) price levels always exist, so that investment is profitable and (b) allows us to concentrate on simple price triggers for the exit decisions:

Assumption 2: (a) Investment increases revenues of the investing firm, regardless of whether the other firm has invested or not, i.e.

$$D(\underline{q}_i + q_{-i})\underline{q}_i < D(\bar{q}_i + q_{-i})\bar{q}_i, \; q_{-i} \in \{0, \underline{q}_{-i}, \bar{q}_{-i}\} \tag{8}$$

(b) Moreover, the initial price-level P_0 shall be such that none of the firms optimally exits at $t = 0$ or exits when prices increase.

Part (b) of the assumption avoids the kind of difficulties of non-unique exit equilibria studied by Murto (2004).

3 Firm Value, and the Timing of Investment and Exit for a Monopolist

3.1 Monopolist with large capacity

We begin our analysis with a monopolist that has already carried out its investment option. This subgame is reached when a monopolist with low capacity invests or when the competitor of a duopolist with high capacity exits. In other words, the monopolist's position is the position of the second mover with respect to exit.

The case of a monopolist with an exit option is well studied which allows us to primarily build on established results–e.g. from Jou (2001). For a monopolist who already operates at high capacity the following rationale determines the value of a_{i1} and a_{i2} and leads to

Proposition 1. When price P tends to infinity the option to exit becomes worthless and so $a_{i1}(\bar{q}_i, 0) = 0$. When exit is timed optimally a value matching and a smooth pasting condition must hold, so that we can infer $P^{exit,i}_{\bar{q}_i,0}$ and $a_{i2}(\bar{q}_i, 0)$ from these two conditions which are given by the following equations

$$V_i(P^{exit,i}_{\bar{q}_i,0}, \bar{q}_i, 0) = 0 \tag{9}$$

$$\frac{\partial V_i(P^{exit,i}_{\bar{q}_i,0}, \bar{q}_i, 0)}{\partial P} = 0. \tag{10}$$

Proposition 1 *Having invested, the monopolist´s firm value is*

$$V_i(P, \bar{q}_i, 0) = \bar{q}_i \frac{P}{\rho - \mu} - \frac{b_i}{\rho} + \frac{b_i}{\rho(1-\beta_2)} \left(\frac{P}{P^{exit,i}_{\bar{q}_i,0}} \right)^{\beta_2}. \tag{11}$$

The price trigger for exit is given by $P^{exit,i}_{\bar{q}_i,0} = \frac{\beta_2(\rho-\mu)}{(\beta_2-1)\rho} \frac{b_i}{\bar{q}_i}$.

Proof. *Denote the revenues process by $R := \bar{q}_i P$. This process has exactly the same properties as the price process in Jou (2001). The proposition then follows straightforward from Jou´s Proposition 1.* ∎

From this proposition we can see that holding the option to exit adds $\frac{b_i}{\rho(1-\beta_2)} \left(\frac{P}{P^{exit,i}_{\bar{q}_i,0}} \right)^{\beta_2}$ to the expected value of profits $\bar{q}_i \frac{P}{\rho-\mu} - \frac{b_i}{\rho}$ the firm would obtain when continuing operation infinitely.

3.2 Monopolist with an investment option

If the monopolist is able to increase capacity, a_{i1} is no longer zero. Instead, the ability to increase capacity adds another pair of value-matching and smooth-pasting conditions, which are optimality conditions for investment. Now a_{i1} and a_{i2} both have to be simultaneously solved for from the system of equations generated by smooth-pasting and value-matching conditions for both, exit and investment. The conditions for investment are given by

$$V_i(P^{inv,i}_{\underline{q}_i,0}, \underline{q}_i, 0) = V_i(\Delta_{\underline{q}_i,\bar{q}_i} \cdot P^{inv,i}_{\underline{q}_i,0}, \bar{q}_i, 0) - C_i, \tag{12}$$

$$\left.\frac{\partial V_i(P, \underline{q}_i, 0)}{\partial P}\right|_{P=P^{inv,i}_{\underline{q}_i,0}} = \left.\frac{\partial V_i(\Delta_{\underline{q}_i,\bar{q}_i} \cdot P, \bar{q}_i, 0)}{\partial P}\right|_{P=P^{inv,i}_{\underline{q}_i,0}}. \tag{13}$$

The former condition equalizes the value before investment and value after investment, taking into account the cost of investment C_i. The latter condition ensures that value changes smoothly in Y. Due to the change in production, prices react differently to changes in Y before and after investment. The $\Delta_{\underline{q}_i,\bar{q}_i}$ term corrects for this.

For the exit price decision, the conditions remain similar to the case without investment.

$$V_i(P^{exit,i}_{\underline{q}_i,0}, \underline{q}_i, 0) = 0 \tag{14}$$

$$\frac{\partial V_i(P^{exit,i}_{\underline{q}_i,0}, \underline{q}_i, 0)}{\partial P} = 0 \tag{15}$$

Yet, the value function is more complicated and involves terms to the power of β_1 and β_2. This no longer allows to solve for the optimal price triggers analytically. However, the trigger prices and a_{i1} and a_{i2} can be determined numerically from equations $(12) - (15)$.

4 Firm Value, and the timing of investment and exit in duopoly

In contrast to monopoly, behavior in duopoly is determined by strategic considerations. The complexity of the strategic situation increases with the number of firms that can increase capacity. Therefore, we begin with the analysis of a duopoly in which both firms have already exercised their investment option and operate at high capacity $\bar{q}_i$.[7] Thereafter, we analyze the situation which naturally precedes this one: One firm is already at high capacity, whereas the other firm still operates at low capacity and may invest. At last the situation is studied where both firms still have the option to invest.

4.1 Both firms operate at high capacity

When both firms have invested, so that we are in state $(\bar{q}_1, \bar{q}_2)$, both firm nevertheless have to decide whether and when to exit. This decision in particular determines who leaves first and who monopolizes the market. However, a priori it is not obvious which firm will leave first. But since we assumed the two firms to differ in their fixed costs of operation, the only Markov-perfect equilibrium of the resulting exit game is the one in which the firm with the larger overhead exits at its monopoly exit price. This is shown by the proposition below, which is similar to Murto's (2004, p. 13) result when gap equilibria do not exist or Lambrecht's (2001) result for the sub-game perfect equilibrium of the exit game.[8]

Proposition 2 *In all Markov-perfect equilibria in pure strategies of the $(\bar{q}_1, \bar{q}_2)$-subgame (exit after investment), firm 2, the firm with the larger fixed costs, chooses its monopoly exit price as the price trigger for bankruptcy $P^{exit,2}_{\bar{q}_2,\bar{q}_1} = P^{exit,2}_{\bar{q}_2,0}$, whereas firm 1 chooses as exit-price trigger some $P^{exit,1}_{\bar{q}_1,\bar{q}_2} \in \left]\Delta_{\bar{q}_1+\bar{q}_2,\bar{q}_1}{}^{-1} P^{exit,1}_{\bar{q}_1,0}, \Delta_{\bar{q}_1+\bar{q}_2,\bar{q}_2}{}^{-1} P^{exit,2}_{\bar{q}_2,0}\right[$.*

Proof. *See appendix.* ■

[7] This is similar to the pure exit games studied by Lambrecht (2001) and Murto (2004).

[8] As there are equal costs of exit (they are zero for both firms) in our model no gap equilibria can arise when firms only decide on exit.

Firm 2 exits at its monopoly exit price trigger, but this does not mean firm 2 quits at the same state of demand Y as in monopoly. Total production is larger in duopoly and this lowers the price keeping the state of aggregate demand Y constant. Thus, as exit occurs at the same price, in duopoly the exit state of demand Y must be larger than in monopoly.

Since firm 1 leaves second in equilibrium, its exit price-trigger, $P^{exit,1}_{\bar{q}_1,\bar{q}_2}$, is in a sense just "virtual". Along the equilibrium path, this price trigger will never be reached: Firm 2 quits before the price drops to $P^{exit,1}_{\bar{q}_1,\bar{q}_2}$, the new state is $(\bar{q}_1, 0)$, and the effective exit price-trigger becomes $P^{exit,1}_{\bar{q}_1,0}$. Thus, $P^{exit,1}_{\bar{q}_1,\bar{q}_2}$ only determines what firm 1 would do if firm 2 by mistake (with zero probability) had not left when the price drops to $P^{exit,1}_{\bar{q}_1,\bar{q}_2}$.

Due to this virtuality, $P^{exit,1}_{\bar{q}_1,\bar{q}_2}$ is partly undetermined. Firm 1 can choose any exit price trigger that firstly would never result in its immediate exit once firm 2 has left $\left(P^{exit,1}_{\bar{q}_1,\bar{q}_2} > \Delta_{\bar{q}_1+\bar{q}_2,\bar{q}_1}{}^{-1} P^{exit,1}_{\bar{q}_1,0}\right)$ and secondly does not allow firm 2 to chose a lower exit price-trigger $\left(P^{exit,1}_{\bar{q}_1,\bar{q}_2} < \Delta_{\bar{q}_1+\bar{q}_2,\bar{q}_2}{}^{-1} P^{exit,2}_{\bar{q}_2,0}\right)$.

With exit price-triggers determined on the basis of Proposition 2, we can now compute the value functions when both firms are at high capacity. According to Proposition 2, firm 2 behaves myopically and so its value function is the same as when it is a monopolist.

Firm 2's potential exit, however, changes firm 1´s value, and so, its value function needs to be determined anew; especially the "option values" a_{11}, a_{12} in (6) have to be re-calculated.

Again to determine a_{11} we suppose price tends to infinity. Then the exit option becomes worthless and hence $a_{11} = 0$.[9] The other constant, a_{12}, can be solved from the following value-matching condition at the exit price of firm 2

$$V_1(P^{exit,2}_{\bar{q}_2,0}, \bar{q}_1, \bar{q}_2) = V_1(\Delta_{\bar{q}_1+\bar{q}_2,\bar{q}_1} P^{exit,2}_{\bar{q}_2,0}, \bar{q}_1, 0). \tag{16}$$

This condition equalizes firm 1's value at the logical second before and after the exit of firm 2. The condition yields for a_{12} after some algebraic calculations

$$a_{12}(\bar{q}_1, \bar{q}_2) = g \cdot \frac{1}{1-\beta_2} \frac{b_1}{\rho} \left(\frac{1}{P^{exit,1}_{\bar{q}_1,0}} \right)^{\beta_2}, \tag{17}$$

with g defined as

$$g := \underbrace{(\Delta_{\bar{q}_1+\bar{q}_2,\bar{q}_1})^{\beta_2}}_{<1} \underbrace{-\beta_2 \left(\Delta_{\bar{q}_1+\bar{q}_2,\bar{q}_1} - 1\right)}_{>0} \left(\frac{\bar{q}_1 b_2}{\bar{q}_2 b_1} \right)^{1-\beta_2} > 1.$$

The ratio $\frac{\bar{q}_1 b_2}{\bar{q}_2 b_1} = \frac{P^{exit,2}_{\bar{q}_2,0}}{P^{exit,1}_{\bar{q}_1,0}}$ is the ratio of monopoly exit prices; the stated inequalities are shown to hold in the appendix. Substituting (17) in (6) we obtain for the value of firm 1

[9] See e.g. Jou (2001) for details.

Figure 2: Equity-value of firm 1

(a) g is large, V non-monotonous (b) g is small, V monotonous

the following expressions

$$V_1(P, \bar{q}_1, \bar{q}_2) = \bar{q}_1 \frac{P}{\rho - \mu} - \frac{b_1}{\rho} + g \cdot \frac{b_1}{(1-\beta_2)\,\rho} \left(\frac{P}{P^{exit}_{\bar{q}_1,0}} \right)^{\beta_2}, \; P \geq P^{exit,2}_{\bar{q}_2,0}. \tag{18}$$

As one can now easily see, a factor $g > 1$ is added in (11) to the price of the exit option in the presence of a competitor who leaves the market when prices decline. Factor g is composed of the costs of postponed exit $(\Delta_{\bar{q}_1+\bar{q}_2,\bar{q}_1})^{\beta_2}$ and the gain when firm 2 exits, which is a "hedge" against bad states. This hedge outweighs the cost of waiting, thus increases value, and—importantly for what follows—kinks value of firm 1 at the price at which firm 2 exits. Once firm 2 has left, only the limited liability (partly) shields firm 1 against losses induced by a decrease in price. In contrast, before firm 2 has left, when prices drop firm 1 can expect also to gain from firm 2 leaving .

How much value firm 1 gains upon exit of firm 2 especially depends on the demand function. When prices react very strongly to changes in quantity, the value gain and hence g is large. In such a case firm 1's value may even decrease in price when the price is near the exit price of firm 2–see figures 2(a),(b).

This point is of importance, as it is crucial for enabling predatory investment, as we will see later. Algebraically one obtains for the right-hand first derivative of V_1 w.r.t. P, $\frac{\partial V_1}{\partial P}^+$, at $P^{exit,2}_{\bar{q}_2,0}$, i.e. when firm 2 exits:

$$\frac{\partial V_1(P^{exit}_{\bar{q}_2,0}, \bar{q}_1, \bar{q}_2)}{\partial P}^+ = \bar{q}_1 \frac{1}{\rho - \mu} + g \cdot \frac{\beta_2 b_1}{(1-\beta_2)\,\rho} \left(\frac{P^{exit}_{\bar{q}_2,0}}{P^{exit}_{\bar{q}_1,0}} \right)^{\beta_2 - 1} \frac{1}{P^{exit}_{\bar{q}_1,0}}. \tag{19}$$

This derivative is negative if

$$-g \cdot \frac{\beta_2 b_1}{(1-\beta_2)\,\rho} \left(\frac{P^{exit}_{\bar{q}_2,0}}{P^{exit}_{\bar{q}_1,0}} \right)^{\beta_2 - 1} \frac{1}{P^{exit}_{\bar{q}_1,0}} > \bar{q}_1 \frac{1}{\rho - \mu} \Leftrightarrow g \cdot \left(\frac{P^{exit}_{\bar{q}_2,0}}{P^{exit}_{\bar{q}_1,0}} \right)^{\beta_2 - 1} > 1, \tag{20}$$

and factor g can be arbitrarily large when $\Delta_{\bar{q}_1+\bar{q}_2,\bar{q}_1}$ is large. For the last inequality to hold, it is sufficient that $-\beta_2\,(\Delta_{\bar{q}_1+\bar{q}_2,\bar{q}_1} - 1) > 1$.

4.2 One firm operates at high capacity while the other firm has an investment option

Before both firms reach high capacity, firms invest sequentially. As Huisman and Kort (1999) show, non (tacit)-collusive simultaneous investment does not occur in equilibrium. Although both firms may become leader in equilibrium in the homogeneous setting they study, almost surely the firms never invest simultaneously if not colluding. So without collusion we can concentrate the analysis on sequential investment.

Concentrating on sequential investment will help to keep our analysis focused. Therefore, we do not explicitly study collusive simultaneous investment nor do we study collusive inaction. Collusive equilibria actually may arise, but the analysis for this carries over straightforward from Huisman and Kort (1999). Will will turn again to this point when we discuss the competition for the leader's position.

To study this competition, we analyze the investment decisions inducing backwardly over the (q_i, q_{-i})-states. So we begin with the situation where firm $-i$ already increased capacity and firm i may now follow. We have seen that the firms differ substantially once both are at high capacity. Due to this asymmetry, we need to study the behavior of both firms as followers separately. We begin with firm 2. It turns out that this is the easier case to analyze.

4.2.1 Firm 2 as follower

We have seen that firm 2 will leave the market first when both firms are at high capacity. At low capacity, firm 2's overhead costs are even larger relative to its earnings than they are when firm 2 operates at high capacity. Thus, before investment firm 2 is in a weaker position than after investment, see figure 3. Therefore, firm 2 will also exit first if it operates at low capacity and firm 2's investment will delay firm 2's own exit. Moreover, investment does not alter this outcome of the exit game, once firm 1 already operates at high capacity. Thus, in state $\left(\bar{q}_1, \underline{q}_2\right)$ firm 2 leaves first just as in state $(\bar{q}_1, \bar{q}_2)$. This allows firm 2 to completely ignore the strategic character of the situation:

Proposition 3 *As a follower, firm 2 behaves myopically. It chooses the same exit and investment price-triggers that a monopolist on the residual demand function $D\left(q_2 + \bar{q}_1\right)$ would choose. Firm 1 uses an exit price trigger analogously to proposition 2 and exits second.*

Proof. *See appendix.* ■

That firm 2 as follower behaves myopically is a result similar to other games with preemption (e.g. Weeds, 2003).

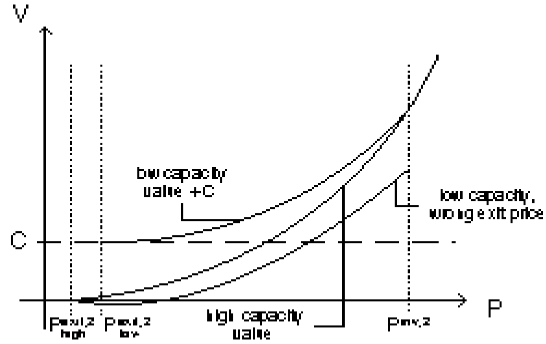

Figure 3: Exit-prices and value of firm 2

4.2.2 Firm 1 as follower

If firm 1 is the follower the strategic situation changes dramatically: In contrast to firm 2, firm 1's actions affect the likelihood of its competitor (firm 2) leaving the market. This increases the value of investment for firm 1 and induces firm 1 to exercise its investment option more early.

Yet, more interesting is what happens at low prices. Consider the following situation: Suppose at low capacity firm 1 has a much smaller production than firm 2, but both firm have similar overhead costs. Without any investment option firm 1 would clearly leave first. Suppose that firm 1 still exits first if it only invests when the price rises. This may now lead to the interesting situation shown in figure 4. In this figure, at high capacity V_1 is falling in P, which is possible as we have seen. For intermediate prices the gain in expected earnings does not cover the costs of investment. However, for both high and low prices there is a gain from investing. At high prices investment pays and is carried out at a high price-trigger. For low prices firm 1 has a high probability to monopolize the market soon if it invests and so investment becomes profitable at low prices, too. In consequence, firm 1 would not exit, but rather *predatorily* invest when prices decline. Firm 2 does not have this opportunity to prey.

Whether or not firm 1 preys, we need to determine in order to calculate the equilibrium of the exit game. The decision to prey depends for firm 1 on firm 2's exit price $P^{exit,2}_{\bar{q}_2,\underline{q}_1}$. For the moment, this price-trigger can be taken as given, but will be determined endogenously in equilibrium in a later section. Yet, individual optimality already puts a restriction to the exit price-trigger, as the following Lemma shows. This Lemma proves useful in discussing the existence of predatory investment in our model.

Lemma 1 ***(a)*** *If firm 2 leaves the market first,* $P^{exit,2}_{\bar{q}_2,\bar{q}_1} < P^{exit,2}_{\bar{q}_2,\underline{q}_1}$ *holds and* $P^{exit,2}_{\bar{q}_2,\bar{q}_1} = P^{exit,2}_{\bar{q}_2,0}$ *as derived in Proposition 3.*
(b) *Moreover, in all cases* $P^{exit,2}_{\bar{q}_2,\underline{q}_1} < \Delta^{-1}_{\underline{q}_1+\bar{q}_2,\bar{q}_1+\bar{q}_2} P^{exit,2}_{\bar{q}_2,\bar{q}_1}$.

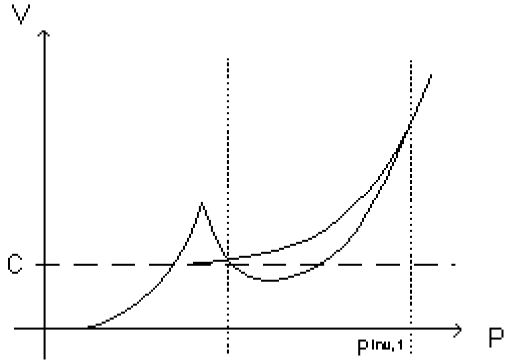

Figure 4: Possibility of predatory investment

Proof. *See appendix.* ■

Now, what determines whether firm 1 invests predatorily? For predatory investment to occur, firm 1's gain in value from increasing capacity must exceed investment costs also at low prices. This value gain, is the difference of firm 1's value at high capacity, $V_1(\Delta_{\underline{q}_1+\overline{q}_2,\overline{q}_1+\overline{q}_2}P,\overline{q}_1,\overline{q}_2)$ and firm 1 value at low capacity, $\hat{V}_1(P,\underline{q}_1,\overline{q}_2)$, assuming firm 1 can only invest at a single high price trigger. If the value gain from investment exceeds investment costs at prices below the single price trigger, firm 1 enjoys a capital gain by investing at low prices, too. On the extreme, this can mean that firm 1 invests at all prices above $\Delta^{-1}_{\underline{q}_1+\overline{q}_2,\overline{q}_1+\overline{q}_2}P^{exit,2}_{\bar{q}_2,\bar{q}_1}$, which happens when the value gain from investing is not smaller than investment costs, C_1, for intermediate prices. When investment costs exceed the returns from investment for intermediate prices, like they do in figure 4, then firm 1 will predatorily invest at a low price additionally to its investment at a high price trigger.

For this high price trigger, it is necessary to know at least a lower bound that separates the regular investment from predatory investment. Otherwise, the price triggers cannot be distinguished numerically and the hypothetical value function $\hat{V}_1$ cannot be determined.

To obtain this lower bound to the investment price trigger, we construct another hypothetical value function for firm 1 under the assumption that firm 1 has no investment option at all. For this hypothetical situation, the exit equilibrium can be easily found by applying proposition 2, and so the hypothetical value function, $\tilde{V}_1$, is well defined. The function itself is a lower bound to the true value of firm 1, just as it is a lower bound to $\hat{V}_1$.

Having constructed $\tilde{V}_1$, we then compare this theoretical value with the value of firm 1 at high capacity. For very large prices $\tilde{V}_1(P,\underline{q}_1,\overline{q}_2) < V_1(\Delta_{\underline{q}_1+\overline{q}_2,\overline{q}_1+\overline{q}_2}P,\overline{q}_1,\overline{q}_2) - C_1$ must hold, since investment pays due to assumption 2(a). Therefore, the (largest) solution to

$$\tilde{V}_1(P,\underline{q}_1,\overline{q}_2) + C_1 = V_1(\Delta_{\underline{q}_1+\overline{q}_2,\overline{q}_1+\overline{q}_2}P,\overline{q}_1,\overline{q}_2)$$

is our lower bound to regular investment.

If on the one hand investment *does not pay* for all prices, but on the other hand predatory investment is profitable, then there must be a price region Z where

$$\hat{V}_1(P, \underline{q}_1, \overline{q}_2) > \widetilde{V}_1(P, \underline{q}_1, \overline{q}_2) > V_1(\Delta_{\underline{q}_1+\overline{q}_2, \overline{q}_1+\overline{q}_2} P, \overline{q}_1, \overline{q}_2) - C_1.$$

holds for $P \in Z$, but for some $P' < \min Z$

$$\hat{V}_1(P', \underline{q}_1, \overline{q}_2) < V_1(\Delta_{\underline{q}_1+\overline{q}_2, \overline{q}_1+\overline{q}_2} P', \overline{q}_1, \overline{q}_2) - C_1$$

holds, so that investment is profitable at P'. Since $\widetilde{V}_1$ is a lower bound for $\hat{V}_1$, the last inequality implies

$$\widetilde{V}_1(P', \underline{q}_1, \overline{q}_2) < V_1(\Delta_{\underline{q}_1+\overline{q}_2, \overline{q}_1+\overline{q}_2} P', \overline{q}_1, \overline{q}_2) - C_1.$$

Consequently, since both functions are continuous, it is *necessary* for predation to occur that

$$\widetilde{V}_1(P, \underline{q}_1, \overline{q}_2) + C_1 = V_1(\Delta_{\underline{q}_1+\overline{q}_2, \overline{q}_1+\overline{q}_2} P, \overline{q}_1, \overline{q}_2) \tag{21}$$

has more than one solution in P, recall figure 4 and see figure 5. The largest solution to this equation defines the lower bound for the non-predatory investment price-trigger. At most we may have 4 different price regions that differ in the profitability of investment:

Lemma 2 *Other things being equal* (21) *has at most three solutions in* $P > \max\{P^{exit,2}_{\overline{q}_2, \underline{q}_1}, P^{exit,1}_{\underline{q}_1, \overline{q}_2}\}$. *At most two of these solutions can be larger than* $\Delta^{-1}_{\underline{q}_1+\overline{q}_2, \overline{q}_1+\overline{q}_2} P^{exit,2}_{\overline{q}_2, \overline{q}_1}$.
We denote the solutions with $P^*(< P^{**})(< P^{***})$ *respectively and the set of solutions by* S.

Proof. *See appendix.* ■

Hence, formally $\max(S)$ is our lower bound for non-predatory investment. Figure 5 displays possible solutions to (21) that are non-unique.

If there is no solution, firm 1 will invest at any price. This may occur when the costs of investing are low, the competitor's exit is very likely, and prices react strongly to changes in quantity. In this case monopoly is a very valuable position and firm 1's investment speeds up the competitor's exit substantially. Then investment allows firm 1 to monopolize the market much sooner, so that firm 1 likes to invest at any price level. In such a situation however, firm 2 has no incentive at all to be the first firm to invest. We will not further focus on this case.

If there is only one solution, we know that there will be no predatory investment. Given the exit equilibrium, in the single solution case investment is determined by the

Figure 5: Multiple solutions to (21)

(a) two solutions (b) three solutions

usual smooth-pasting and value-matching conditions for firm 1:

$$V_1(P^{inv,1}_{\underline{q}_1,\bar{q}_2},\underline{q}_1,\bar{q}_2) = V_1(\Delta_{\underline{q}_1+\bar{q}_2,\bar{q}_1+\bar{q}_2}P^{inv,1}_{\underline{q}_1,\bar{q}_2},\bar{q}_1,\bar{q}_2) - C_1, \tag{22}$$

$$\left.\frac{\partial V_1(P,\underline{q}_1,\bar{q}_2)}{\partial P}\right|_{P=P^{inv,1}_{\underline{q}_1,\bar{q}_2}} = \left.\frac{\partial V_1(\Delta_{\underline{q}_1+\bar{q}_2,\bar{q}_1+\bar{q}_2}P,\bar{q}_1,\bar{q}_2)}{\partial P}\right|_{P=P^{inv,1}_{\underline{q}_1,\bar{q}_2}}. \tag{23}$$

If (21) has only one solution, we can solve for the investment price-trigger and value function, for any given constellation of exit price-triggers $\left(P^{exit,2}_{\bar{q}_2,\underline{q}_1},P^{exit,1}_{\underline{q}_1,\bar{q}_2}\right)$.

If there is more than one solution to (21) we need to calculate the value of firm 1 if it was restricted to invest at a price above $\max(S)$. This is the value function $\hat{V}$ and the corresponding hypothetical investment price-trigger $\hat{P}^{inv,1}_{\underline{q}_1,\bar{q}_2}$ can again be calculated for any constellation $\left(P^{exit,2}_{\bar{q}_2,\underline{q}_1},P^{exit,1}_{\underline{q}_1,\bar{q}_2}\right)$. For this calculation, we simply use the above two conditions (22) and (23), but constrain $\hat{P}^{inv,1}_{\underline{q}_1,\bar{q}_2}$ to be larger than $\max(S)$.

We then compare the value after investment $V_1(\Delta_{\underline{q}_1+\bar{q}_2,\bar{q}_1+\bar{q}_2}P,\bar{q}_1,\bar{q}_2)$ with the hypothetical value $\hat{V}(P,\underline{q}_1,\bar{q}_2)$ taking investment cost C_1 into account. If there is a price $P\in\left[\max\{P^{exit,2}_{\bar{q}_2,\underline{q}_1},P^{exit,1}_{\underline{q}_1,\bar{q}_2}\},\hat{P}^{inv,1}_{\underline{q}_1,\bar{q}_2}\right[$ such that the gain in value from investment, $\hat{V}(P,\underline{q}_1,\bar{q}_2)-V_1(\Delta_{\underline{q}_1+\bar{q}_2,\bar{q}_1+\bar{q}_2}P,\bar{q}_1,\bar{q}_2)$, outweighs investment costs C_1, then firm 1 has an incentive to invest predatorily. This means, we search for solutions to

$$\hat{V}(P,\underline{q}_1,\bar{q}_2) = V_1(\Delta_{\underline{q}_1+\bar{q}_2,\bar{q}_1+\bar{q}_2}P,\bar{q}_1,\bar{q}_2) - C_1 \tag{24}$$

on the interval $\left[\max\{P^{exit,2}_{\bar{q}_2,\underline{q}_1},P^{exit,1}_{\underline{q}_1,\bar{q}_2}\},\hat{P}^{inv,1}_{\underline{q}_1,\bar{q}_2}\right]$, where $\hat{P}^{inv,1}_{\underline{q}_1,\bar{q}_2}$ is a solution by definition. Two possible structures may emerge as displayed in figure 6:

1. If (24) has one further solution, there is an Investment/ No-Investment/ Investment scheme, i.e. a low price-trigger for which investment occurs and a high price trigger for investment and a region of inactivity in between. See figure 6(a).

2. If (24) has three solutions, the situation gets more complex. If occasionally P is

Figure 6: Solutions to (24)

(a) two solutions (b) three solutions

very low, firm 1 has no incentive to invest because firm 2's exit is very likely anyway. When the price rises, firm 1 finds it profitable to invest predatorily. Yet, as the price rises further firm 1 again becomes inactive, but invests again when prices get very large. See figure 6(b). Starting between the two largest solutions, we obtain the same Investment/ No-Investment/ Investment scheme as above.

Now, when does predatory investment occur? The following proposition gives sufficient conditions

Proposition 4 *Suppose V_1 is* ***not monotonically increasing*** *at high capacity, i.e.* $g > \left(\frac{P^{exit,}_{\bar{q}_2,0}}{P^{exit,1}_{\bar{q}_1,0}}\right)^{1-\beta_2}$.

(a) *Then $V_1\left(P, \bar{q}_1, \bar{q}_2\right)$ obtains its minimum on $P > P^{exit,2}_{\bar{q}_2,0}$ at $P^{\min} = g^{\frac{1}{1-\beta_2}} P^{exit,1}_{\bar{q}_1,0}$.*

(b) *Suppose firm 1 exits first when it has no investment option and low capacity, i.e. $\frac{D(\underline{q}_1)}{D(\bar{q}_2)} < \frac{b_1\bar{q}_2}{b_2\underline{q}_1}$. Moreover, assume that the revenue increase from investment is small relative to the gain from firm 2 leaving, i.e. $g > \left(\frac{D(\bar{q}_1+\bar{q}_2)\cdot\bar{q}_1}{D\left(\underline{q}_1+\bar{q}_2\right)\cdot\underline{q}_1}\right)^{1-\beta_2}$. Then there exist investment costs C_1, so that (21) has multiple solutions.*

(c) *Suppose $\frac{D\left(\underline{q}_1\right)}{D(\bar{q}_2)} > \frac{b_1\bar{q}_2}{b_2\underline{q}_1}$, so that firm 2 exits first at low capacity without investment option. Moreover, assume firm 1's value at high capacity decreases faster in Y than it decreases if firm 1 is at low capacity. Expressed formally, this condition is*

$$\frac{\partial\widetilde{V}_1}{\partial P}\left(\Delta_{\underline{q}_1+\bar{q}_2,\bar{q}_1+\bar{q}_2}{}^{-1}P^{exit,2}_{\bar{q}_2,0}, \underline{q}_1, \bar{q}_2\right) > \frac{\partial V_1}{\partial P^+}\left(P^{exit,2}_{\bar{q}_2,0}, \bar{q}_1, \bar{q}_2\right)\Delta_{\underline{q}_1+\bar{q}_2,\bar{q}_1+\bar{q}_2}.$$

Then there exist investment cost C_1 so that predatory investment occurs. $\frac{\partial V}{\partial P^+}$ is the right-hand partial derivative.

Proof. *See appendix.* ■

The case studied in part (c) of the proposition is relatively restrictive. It requires that firm 1 benefits from a change in the likelihood of firm 2's exit more strongly at high capacity than it does at low capacity. This occurs, if firm 1 is small at low capacity, but is relatively large at high capacity. Unfortunately, the condition cannot be boiled down to an expressions similar to those in part (b) of the proposition.

The proposition, however, does not clarify whether we may arrive in a situation like in figure 6(a) or (b). For the figure 6(b)-type situation the later analysis quickly becomes very complex. We can however avoid complications even in this situation and keep the following analysis focused and tractable by assuming firm 2 does not invest at very low prices. Then predatory investment of firm 1 only occurs when prices fall:

Assumption 3: The investment cost of firm 2, C_2, are large enough so that firm 2 will not invest at prices below $\Delta^{-1}_{\underline{q}_1+\underline{q}_2,\overline{q}_1+\overline{q}_2} P^{exit,2}_{\overline{q}_2,\underline{q}_1}$.

Given this assumption, state $\left(\underline{q}_1,\overline{q}_2\right)$ can only be initially reached at prices larger than $\Delta^{-1}_{\underline{q}_1+\overline{q}_2,\overline{q}_1+\overline{q}_2} P^{exit,2}_{\overline{q}_2,\underline{q}_1}$. To see this, suppose S has three elements (P^{*}, P^{**}, P^{***}); from Lemma 2, we know that at most two elements of S are larger than $\Delta^{-1}_{\underline{q}_1+\overline{q}_2,\overline{q}_1+\overline{q}_2} P^{exit,2}_{\overline{q}_2,\underline{q}_1}$. Therefore, the smallest element of S, P^{*}, is smaller than the price after investment. Hence, it *cannot* be the case that firm 1 finds predatory investment profitable after an increase in price, but not initially. See figure 6(b).

Of course, predatory investment does not always occur. When prices react weakly to changes in supply, then firm 1 does not gain much from firm 2's exit. This gives the following

Proposition 5 *If demand is sufficiently elastic, i.e.* $\forall Q_1, Q_2 : \Delta_{Q_1,Q_2} \approx 1$, *or if demand is not too inelastic and the costs of investment* C_1 *are sufficiently large, then predatory investment never occurs.*

Proof. *See appendix.* ■

In case predatory investment does occur, the exit value-matching condition for firm 1 has to be modified to

$$V_1(P^{pred,1}_{\underline{q}_1,\overline{q}_2}, \underline{q}_1, \overline{q}_2) = V_1(\Delta_{\underline{q}_1+\overline{q}_2,\overline{q}_1} P^{pred,1}_{\underline{q}_1,\overline{q}_2}, \overline{q}_1, \overline{q}_2) - C_1 \quad . \tag{25}$$

This value matching condition requires value before and after investment to be equal at the time of predatory investment. On the timing for its investment, firm 1 decides without any constraint. For regular investment, it uses a smooth-pasting condition to optimally determine the price trigger. For predatory investment, firm 1 will always wait until the price falls to the point at which investment forces the other firm to leave immediately afterwards. This is a result of the following Lemma.

Proposition 6 *If a firm invests predatorily (but does not invest at any price), it prefers to invest not before price falls to* $\Delta^{-1}_{\underline{q}_1+q_{-i},\bar{q}_1+q_{-i}}P^{exit,-i}_{q_{-i},\bar{q}_i}$. *At this price the competitor leaves directly after investment is realized. For example in state* $\left(\underline{q}_1,\bar{q}_2\right)$ *firm 1 prefers to invest predatorily at* $\Delta^{-1}_{\underline{q}_1+\bar{q}_2,\bar{q}_1+\bar{q}_2}P^{exit,2}_{\bar{q}_2,0}$ *if it invests predatorily.*

Proof. See appendix. ■

At $\Delta^{-1}_{\underline{q}_1+\bar{q}_2,\bar{q}_1+\bar{q}_2}P^{exit,2}_{\bar{q}_2,0}$ the value function $V_1(\Delta_{\underline{q}_1+\bar{q}_2,\bar{q}_1+\bar{q}_2}P,\bar{q}_1,\bar{q}_2)$ has a kink, so there is no need for a smooth-pasting condition to hold–which is may be a simplified alternative way to express the above Lemma.

Economically, this means firm 1 will wait with predatory investment until investment will force firm 2 to leave directly afterwards with probability 1. Before, the increase in earnings does not cover the costs of investment and also firm i does not loose any monopoly gain by waiting for a further price decrease. It can still make sure to obtain that gain by investing later at $\Delta_{\underline{q}_1+\bar{q}_2,\bar{q}_1+\bar{q}_2}{}^{-1}P^{exit,2}_{\bar{q}_2,0}$. Waiting longer is suboptimal, firm 1 would forego the monopoly profits. In summary:

Corollary 1 *If firm 1 invests predatorily in state* $\left(\underline{q}_1,\bar{q}_2\right)$, *then it will use* $P^{pred,1}_{\underline{q}_1,\bar{q}_2} = \Delta^{-1}_{\underline{q}_1+\bar{q}_2,\bar{q}_1+\bar{q}_2}P^{exit,2}_{\bar{q}_2,0}$ *as price trigger for predatory investment.*

4.3 Investment decisions in duopoly when no firm has invested yet

When neither of the firms has invested, yet, both firms compete for being the first investor (leader) timing investment strategically. Suppose firm i becomes the leader. While q_{-i} stays low, firm i has invested and increased its own capacity and now can only decide on exit. However, firm i is still influenced by firm -i's investment decisions. Therefore, firm $i's$ value function is determined by the value-matching condition for the price at which firm $-i$ invests and by the equilibrium of the exit game. So we have

$$V_i(P^{inv,-i}_{\underline{q}_{-i},\bar{q}_i},\bar{q}_i,\underline{q}_{-i}) = V_i(\Delta_{\underline{q}_1+\bar{q}_2,\bar{q}_1+\bar{q}_2}P^{inv,-i}_{\underline{q}_{-i},\bar{q}_i},\bar{q}_1,\bar{q}_2), \tag{26}$$

and if firm i expects to leave first, value matching for exit yields

$$V_i(P^{exit,i}_{\bar{q}_i,\underline{q}_{-i}},\bar{q}_i,\underline{q}_{-i}) = 0, \tag{27}$$

or else, when firm i expects to leave second, value matching yields

$$V_i(P^{exit,-i}_{\underline{q}_{-i},\bar{q}_i},\bar{q}_i,\underline{q}_{-i}) = V_i(\Delta_{\bar{q}_i+\underline{q}_{-i},\bar{q}_i}P^{exit,-i}_{\underline{q}_{-i},\bar{q}_i},\bar{q}_i,0). \tag{28}$$

These conditions determine firm $i's$ valuation of the leader's position. This valuation differs from the valuation of the follower's position and the value difference between both positions determines firm $i's$ incentive to preempt on firm $-i$ and take the lead. Following Huisman and Kort (1999), we define a function $\Phi_i(P)$ that represents the advantage of taking the role of the leader at price P. This means $\Phi_i(P)$ measures the difference in value of investing at P instead of becoming the follower when the other firm invests at price P.

$$\Phi_i(P) := \underbrace{V_i(\Delta_{\underline{q}_i+\underline{q}_{-i},\overline{q}_i+\underline{q}_{-i}}P, \overline{q}_i, \underline{q}_{-i}) - C}_{\text{Value as leader}} - \underbrace{V_i(\Delta_{\underline{q}_i+\underline{q}_{-i},\underline{q}_i+\overline{q}_{-i}}P, \underline{q}_i, \overline{q}_{-i})}_{\text{Value as follower}}$$

Whenever $\Phi_i(P) = 0$ firm i is indifferent between taking the lead at price P and following. In order to describe the root-behavior of $\Phi_i(P)$ some more (short-hand) notation is necessary. We denote the state-$\left(\underline{q}_i, \underline{q}_{-i}\right)$ price at which firm i invests after becoming the follower by $\overline{P}_i$, i.e. $\overline{P}_i = \Delta^{-1}_{\underline{q}_i+\underline{q}_{-i},\underline{q}_i+\overline{q}_{-i}} P^{inv,i}_{\underline{q}_i,\overline{q}_{-i}}$. Analogously, $\underline{P}_i$ is the price at which firm i quits as follower, invests predatorily, or firm $-i$ quits, whichever happens first:

$$\underline{P}_i := \Delta^{-1}_{\underline{q}_i+\underline{q}_{-i},\underline{q}_i+\overline{q}_{-i}} \cdot \max\left\{P^{exit,i}_{\underline{q}_i,\overline{q}_{-i}}, P^{pred,i}_{\underline{q}_i,\overline{q}_{-i}}, P^{exit,-i}_{\overline{q}_{-i},\underline{q}_i}\right\}.$$

The following proposition gives the maximum number of indifference points, i.e. solutions to $\Phi_i(P) = 0$.

Lemma 3 **(a)** *On* $M := \left[\max_{j=1,2}\{\underline{P}_j\}, \min_{j=1,2}\{\overline{P}_j\}\right]$

$$\Phi_i(P) = 0 \tag{29}$$

has at most three solutions.

(b) *Suppose* $\overline{P}_i \leq \overline{P}_{-i}$, *so firm* i *as follower invests earlier than firm* $-i$ *would do in that position. Then* $\Phi_i(P) = 0$ *has at most two solutions on* M. *In addition to solutions on* M, $\overline{P}_{-i}$ *is also a solution and* $\frac{\partial \Phi_i(\overline{P}_{-i})}{\partial P} < 0$.

(c) *If* $\overline{P}_i > \overline{P}_{-i}$, *then* $\Phi_i(\overline{P}_i) = 0$ *and* $\Phi_i(P) < 0$ *for all* $P \in \left]\overline{P}_{-i}, \overline{P}_i\right[$.

(d) *If* $\Phi_i(P) = 0$ *has two solutions on* M *and* $\overline{P}_i \leq \overline{P}_{-i}$ *as in (b), or* $\Phi_i(P) = 0$ *has three solutions and* $\overline{P}_i > \overline{P}_{-i}$ *as in (c), then* $\Phi_i(\max\{\underline{P}_i\}) > 0$. *Moreover, there can only exist one additional solution on* $\min_{j=1,2}\{\underline{P}_j\} < P < \max_{j=1,2}\{\underline{P}_j\}$ *if* $\underline{P}_i < \underline{P}_{-i}$.

Proof. See appendix. ■

If there are two solutions to $\Phi_i(P) = 0$ and $\overline{P}_i \leq \overline{P}_{-i}$ (or three solutions and $\overline{P}_i > \overline{P}_{-i}$), the smallest one is the preemption threshold for predatory investment and the (next) larger one is the preemption threshold for non-predatory investment. These thresholds are denoted by $P^{pred,i}_{pre}$ and $P^{inv,i}_{pre}$ respectively. They are represented by the dotted lines in figure 7(a) and represent a price at which firm i is indifferent between being the leader and being the follower. For (an environment of) prices below $P^{pred,i}_{pre}$ and above $P^{inv,i}_{pre}$ firm i prefers the leader's position.

Figure 7: Preemption thresholds for

(a) both, predatory and regular investment (b) only regular investment

If there is only one solution for $\Phi_i(P) = 0$ on M, there is only a non-predatory preemption threshold, see figure 7(b).

If $\overline{P}_i \leq \overline{P}_{-i}$, then $\Phi_i(\bar{P}_i) > 0$. At $\bar{P}_i$ firm i follows, i.e. it invests and has to pay investment costs C_i. The game now enters state $(\bar{q}_i, \bar{q}_{-i})$. This is worse for firm i than being the leader at the same aggregate state of demand Y. The leader's position implies a higher price, since firm $-i$'s investment is still to come.

In the extreme the leader's position can be so valuable for firm i that $\Phi_i(P) = 0$ may have no solution on M. Suppose, for example, that the price reacts very strong to quantity changes and only firm i taking the lead can force firm $-i$ to exit first. In such a case firm i has a strong incentive to invest both, predatorily and non-predatorily.

When there are no solutions to $\Phi_i(P) = 0$, obviously no solution can serve as a preemption threshold. Instead we set $P_{pre}^{inv,i} = \min(M)$ and $P_{pre}^{pred,i} = \overline{P}_i$. This is the only case in which $P_{pre}^{inv,i} > P_{pre}^{pred,i}$.

Also, the other extreme may be attained. If $\overline{P}_i > \overline{P}_{-i}$, firm i might prefer to be the second mover at any price, so that $\Phi_i(P) < 0$ for all $P < \overline{P}_i$. If the value of waiting is very large for firm i due to large overhead costs, such a situation may for example occur. However, by definition $\Phi_i(\bar{P}_i) = 0$ and hence we set $P_{pre}^{inv,i} = \overline{P}_i$.

Now, these thresholds and $\Phi_i(P)$ only describe the incentive of firm i to take the lead and invest, given that it expects its competitor to do the same otherwise. It can well be that none of the firms would unilaterally like to invest below $\max \bar{P}_j$, although preemption thresholds from $\Phi_i(P) = 0$ exist. The function Φ_i only describes conditional incentives to invest earlier, and this conditionality applies to both regular and predatory investment. For predation, there may be a situation where none of the firms gains from predatory investment unless it expects the other firm to prey in the future. For regular investment, both firms may prefer the gains from tacit-collusive delay (or even inactivity) to the leader's position at all prices.

This leads to the well studied typical problem of non-unique equilibria in timing games of (dis-)investment which is associated with Fudenberg and Tirole´s (1985) notion of perfect timing game equilibria. Equilibria that only base on the relative valuation of the leaders position are not renegotiation proof.[10] Hence, for non-predatory investment we need to compare the valuation of tacit collusion, $V_i^C\left(P, \underline{q}_i, \underline{q}_{-i}\right)$, with the valuation of the leader's position $V_i(\Delta_{\underline{q}_i+\underline{q}_{-i},\overline{q}_i+\underline{q}_{-i}}P, \overline{q}_i, \underline{q}_{-i}) - C_i$. We do not derive the collusive value function explicitly but refer to Huisman and Kort (1999), Sparla (2001) or Weeds (2003) for a detailed discussion and derivation of V_i^C in similar cases.[11] If there exists a price such that one firm prefers the leader's position over simultaneous investment, only the preemption equilibrium studied in the proposition below prevails:

Proposition 7 *If there exists a P such that for one of the firms $V_i(\Delta_{\underline{q}_i+\underline{q}_{-i},\overline{q}_i+\underline{q}_{-i}}P, \overline{q}_i, \underline{q}_{-i}) - C_i > V_i^C\left(P, \underline{q}_i, \underline{q}_{-i}\right)$, then the only Markov-perfect equilibrium for the preemption game for non-predatory investment is the following: Let L be the firm for which $P_{pre}^{inv,L} < P_{pre}^{inv,-L}$. Firm L takes the lead and chooses an investment-price trigger so that the competitor is at least indifferent between becoming leader or follower. Therefore, either $P_{\underline{q}_L,\underline{q}_{-L}}^{inv,L} = P_{pre}^{inv,-L}$ or $P_{\underline{q}_L,\underline{q}_{-L}}^{inv,L}$ is set unconstrained optimal and solves a smooth pasting condition, if this yields a smaller price than $P_{pre}^{inv,-L}$.*

Proof. *See appendix.* ■

As outlined before, that preemption thresholds exist for predatory investment does not necessarily imply that predatory investment occurs in a renegotiation-proof Markov-perfect equilibrium. Again, we need to define an auxiliary value function $\hat{V}$. This function is based on assumed exit price-triggers and the non-predatory investment equilibrium as described before. Just as in the case when firm 1 is the follower, only if at least for one firm

$$\hat{V}_i\left(\Delta^{-1}_{\underline{q}_i+\underline{q}_{-i},\overline{q}_i+\underline{q}_{-i}}P^{exit,-i}_{\underline{q}_{-i},\overline{q}_i}, \underline{q}_i, \underline{q}_{-i}\right) + C < V_i(\Delta_{\underline{q}_i+\underline{q}_{-i},\overline{q}_i+\underline{q}_{-i}}P^{exit,-i}_{\underline{q}_{-i},\overline{q}_i}, \overline{q}_i, \underline{q}_{-i}) \tag{30}$$

there will be predation in equilibrium. If in addition predatory preemption thresholds are defined for both firms—i.e.$\Phi_i(P) = 0$ has multiple solutions—then there will be preemption for predation and the following proposition describes the resulting equilibria.

Proposition 8 *Suppose there is a preemption game for predatory investment. In addition,*

(a) *suppose $P_{pre}^{pred,i} < P_{pre}^{inv,-i}$ for both firms. Then the only Markov-perfect equilibrium (outcome) is that the firm with the higher $P_{pre}^{pred,i}$ takes the lead for predatory investment. It invests when the price falls to $P_{\underline{q}_i,\underline{q}_{-i}}^{pred,i}$. This price trigger is $P_{\underline{q}_i,\underline{q}_{-i}}^{pred,i} = P_{pre}^{pred,-i}$*

[10] Indeed, in some numerical simulations (not reported) we found non-renegotiation-proof predatory equilibria. Renegotiation-prooveness is but a strong assumption on rationality.

Therefore, we might in reality observe a circular situation of the following form: One agent takes preemptive, predatory action in threat of a predatory action of the other agent. This other agent however has no incentive to undertake that action as long as the first agent does not take action. The first agent however takes action as she is threatened. Triggered off by a sunspot, the situation escalates.

[11] The appendix contains a description of how this function can be derived.

FOR THE REFEREE: This appendix could be dropped to shorten the paper.

if preempting is a constraint. If preemption poses no constraint, i.e. if $P_{pre}^{pred,-i} < \Delta^{-1}_{\underline{q}_i+\underline{q}_{-i},\bar{q}_i+\underline{q}_{-i}} P^{exit,-i}_{\underline{q}_{-i},\bar{q}_i}$, *the trigger is* $P^{pred,i}_{\underline{q}_i,\underline{q}_{-i}} = \Delta^{-1}_{\underline{q}_i+\underline{q}_{-i},\bar{q}_i+\underline{q}_{-i}} P^{exit,-i}_{\underline{q}_{-i},\bar{q}_i}$.

(b) *Suppose* $P_{pre}^{pred,i} > P_{pre}^{inv,-i}$ *for firm* i *and one of the firms has an unilateral (unconstrained) incentive to invest on* $\left[P_{pre}^{inv,-i}, P_{pre}^{pred,i}\right]$. *If* $P_0 > P_{pre}^{pred,i}$, *then in all Markov-perfect equilibria firm* $-i$ *invests predatorily at* $P_{pre}^{pred,i}$.

(c) *Again suppose* $P_{pre}^{pred,i} > P_{pre}^{inv,-i}$ *for one of the firms, but none of the firms has an unilateral incentive to invest on* $\left[P_{pre}^{inv,-i}, P_{pre}^{pred,i}\right]$. *Then in all renegotiation-proof Markov-perfect equilibria firm* i *invests predatorily at its unconstrained optimal predatory investment price-trigger or at* $P_{pre}^{pred,-i}$, *whichever is the higher price.*

Proof. *See appendix.* ■

If there is no preemption for predation but firm i has an unilateral incentive to prey, i.e. (30) holds for firm i, then firm i will invest predatorily at $\Delta^{-1}_{\underline{q}_i+\underline{q}_{-i},\bar{q}_i+\underline{q}_{-i}} P^{exit,-i}_{\underline{q}_{-i},\bar{q}_i}$. To avoid further complication for determining the exit price-triggers, and avoid strategic situations of the "gap-equilibrium" type studied by Murto (2004), we make the following assumption according to the price-level (and investment costs) in $t = 0$:

Assumption 4: At the initial price-level P_0 at least one firm finds it unprofitable to invest and both firms do not find it profitable to exit. Moreover, P_0 lies between the preemption thresholds $\left(P_{pre}^{pred,i}, P_{pre}^{inv,i}\right)$, i.e.

$$\min_{i=1,2}\{P_{pre}^{pred,i}\} < P_0 < \max_{i=1,2}\{P_{pre}^{inv,i}\} \quad .$$

4.4 Equilibrium exit-strategies

So far, we have taken the price-triggers which both firms use for their exit-decision as given. However, exit strategies have to be determined as an equilibrium of both firms competing to monopolize the market. We already studied their exit decisions, when firm 2 is at low, but firm 1 is at high capacity, when both firms are at high capacity, or when one of them is a monopolist. We will now generalize the exit equilibrium considerations to any state (q_i, q_{-i}). This includes the states studied before just as well as the situation when only firm 1 or both firms operate at low capacity.[12]

Given that firm i expects to exit first in state (q_i, q_{-i}), firm i will use the value-matching and smooth pasting condition

$$V_i(P^{exit,i}_{q_i,q_{-i}}, q_i, q_{-i}) = \frac{\partial V_i}{\partial P}(P^{exit,i}_{q_i,q_{-i}}, q_i, q_{-i}) \overset{!}{=} 0 \tag{31}$$

[12] Pure exit decisions have been studied by Lambrecht (2001) and Murto (2004) in more detail. Essentially, their analysis carries over, so we can be relatively brief.

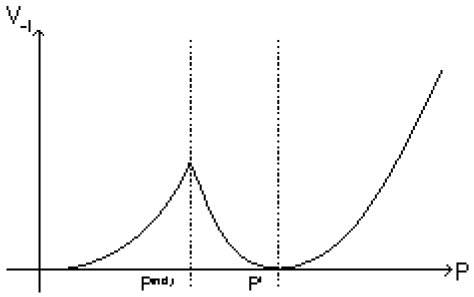

Figure 8: Construction of $P^{ind,i}_{q_i,q_{-i}}$

to determine its own exit price-trigger. Firm i can ignore the exit price of the competitor once it expects to leave first. Being the first to exit is worse than being the second and thus, firm i will never make an effort meeting the constraint that its exit-price trigger must be larger than firm $-i$'s exit price. Hence, the exit price-trigger that is determined according to conditions (31) is myopic and will be denoted by $P^{m\_exit,i}_{q_i,q_{-i}}$.

If firm i expects to leave second in state (q_i, q_{-i}), $q_{-i} \neq 0$, its value function is locally independent from its own exit price-trigger $P^{exit,i}_{q_i,q_{-i}}$. Only being second to leave matters. Thus, firm i is indifferent about the level of its own exit price-trigger on the margin. Therefore, there are always multiple equilibria which only differ with respect to the (virtual) exit price-trigger of the firm which exits second.

However, which firm actually exits second we still need to determine. Here, the limited liability constraint of Definition 2 is crucial. Denote by $P^{ind,i}_{q_i,q_{-i}}$ the largest exit-price-trigger firm i can use, so that the limited liability constraint (in Definition 2) of firm $-i$ holds with equality for some P' given that firm $-i$ expects to leave second. Expressed formally this is:

$$P^{ind,i}_{q_i,q_{-i}} := \sup \left\{ P^{exit,i}_{q_i,q_{-i}} \;\middle|\; \begin{array}{c} \underbrace{\forall P^{exit,-i}_{q_{-i},q_i} :}_{\text{even if firm -i leaves second,}} \qquad \underbrace{\exists P' > P^{exit,i}_{q_i,q_{-i}} :}_{\text{before firm i leaves}} \\ \underbrace{V_{-i}\left[P', q_{-i}, q_i | P^{\#}_{-i}, \left(P^{\#}_{i}, P^{exit,i}_{q_i,q_{-i}}\right)\right] = 0}_{\text{firm -i prefers to exit}} \end{array} \right\} \tag{32}$$

If firm i chooses an exit price trigger below $P^{ind,i}_{q_i,q_{-i}}$, firm $-i$ cannot credibly threaten to stay longer than firm i, because the limited liability constraint would become binding otherwise, see figure 8.

Thus, firm $-i$ leaving at the myopic price trigger and the other firm choosing an exit price-trigger smaller than $P^{ind,i}_{q_i,q_{-i}}$ will be an equilibrium if $P^{ind,i}_{q_i,q_{-i}}$ fulfills the other conditions of Definition 2.

Note that $P^{ind,i}_{q_i,q_{-i}}$ always exists and $P^{ind,i}_{q_i,q_{-i}} \geq \Delta^{-1}_{q_i+q_{-i},q_{-i}} P^{exit,-i}_{q_{-i},0}$ always holds, since firm $-i$ would immediately exit in state $(q_{-i},0)$ at any price below $P^{exit,-i}_{q_{-i},0}$, so that at $\Delta^{-1}_{q_i+q_{-i},q_{-i}}P^{exit,-i}_{q_{-i},0}$ the limited liability constraint definitely binds for firm $-i$. The same argument also implies that firm i cannot threaten to exit below $\Delta^{-1}_{q_i+q_{-i},q_i}P^{exit,i}_{q_i,0}$. This time, firm i would immediately exit after firm $-i$ had left. Therefore, $P^{ind,i}_{q_i,q_{-i}} \geq \Delta^{-1}_{q_i+q_{-i},q_i}P^{exit,i}_{q_i,0}$ is necessary to make $P^{ind,i}_{q_i,q_{-i}}$ a credible threat.

The following proposition describes all equilibria of the exit-game. What type the equilibrium actually attains, depends on the parameters of the environment.

Proposition 9 **(a)** *Suppose that for firm i the myopic exit price-trigger $P^{m\_exit,i}_{q_i,\overline{q}_{-i}}$, obtained from* (31) *, is smaller than $P^{ind,i}_{q_i,q_{-i}}$. Then the only equilibrium of the (q_i,q_{-i}) stage is that firm $-i$ chooses $P^{m\_exit,-i}_{q_{-i},q_i}$ and firm i chooses a price-trigger lower than or equal to $P^{ind,i}_{q_i,q_{-i}}$.*

(b) *If $\left[\Delta_{q_i+q_{-i},q_i}{}^{-1}P^{exit,i}_{q_{-i},0}, P^{ind,i}_{q_i,q_{-i}}\right] = \emptyset$, then firm i chooses its myopic exit price-trigger $P^{m\_exit,i}_{q_i,\overline{q}_{-i}}$ in all equilibria of the (q_i,q_{-i}) stage.*

(c) *If $P^{m\_exit,-i}_{q_{-i},q_i} > P^{ind,i}_{q_i,q_{-i}}$ and $\left[\Delta_{q_i+q_{-i},q_i}{}^{-1}P^{exit,i}_{q_{-i},0}, P^{ind,i}_{q_i,q_{-i}}\right] \neq \emptyset$, then firm i choosing some $P^{exit,i}_{q_i,q_{-i}} \in \left[\Delta_{q_i+q_{-i},q_i}{}^{-1}P^{exit,i}_{q_{-i},0}, P^{ind,i}_{q_i,q_{-i}}\right]$ and firm $-i$ choosing $P^{m\_exit,-i}_{q_{-i},q_i}$ is an equilibrium of the (q_i,q_{-i}) stage,* ***but only if this yields no incentive for firm $-i$ to invest predatorily****.*

(d) *If in any possible equilibrium given in* (c) *some firm has an incentive to invest predatorily, then both firms preempt on predatory investment.*

Proof. *See appendix.* ■

A few explanatory remarks that give an idea of the proof and reasoning of this proposition seem appropriate: The idea behind (a) can be summarized relatively simple. Even if firm i plans to leave first, it will choose $P^{m\_exit,i}_{q_i,\overline{q}_{-i}}$ and consequently forces firm -*i* to leave first. Hence i can only exit second. This line of argument is discussed in detail in Lambrecht (2001).

Case (b) has already been analyzed to the most extent. More interesting is (c). Firstly, it gives rise to the problem of equilibria that are even non-unique in the sequence of exit.[13] It may be the case that both firms can force the other firm to leave first. In the numerical analysis we tackle this problem by assuming that the firm with the larger $P^{ind,i}_{q_i,q_{-i}}$ is selected as the one who leaves second.[14]

[13] As explained before, equilibria always are non-unique in the exit-price of the firm leaving second. These exit price-triggers are only effective out of equilibrium.

[14] This selection can be motivated by the following idea: Suppose firm $-i$ chooses an exit price-trigger marginally larger than $P^{ind,i}_{q_i,q_{-i}}$. Is the choice of $P^{ind,i}_{q_i,q_{-i}}$ still credible then? Of course not for the firm with the lower $P^{ind,i}_{q_i,q_{-i}}$.
Conversely, this rule can be interpreted as a notion of conservativeness in the following sense: Suppose the shareholders of firm i imagine the worst case, i.e. firm $-i$ defaults just one logical second before $i's$ (proposed) trigger price is reached. Then only if the proposed exit price trigger is larger than the own

Secondly, under a credible threat of predation, firm i can never expect to leave second and monopolize the market. Thus, firm $-i$ will choose an exit price trigger below $P^{ind,-i}_{q_{-i};q_i}$ (if this is possible) and exit second. Firm i planing to leave first may or may not prevent the predatory investment. Whether predatory investment still occurs is determined as outlined before.

5 Numerical Examples

On the basis of the theoretical discussion in the last sections we can calculate the equilibrium for any given parameter constellation numerically. This section now presents some examples based on some variations in parameter values. Table 1 contains the values for the non firm-specific parameters. In all calculations an isoelastic inverse demand function is used. Three cases are considered, in two of them investment costs are homogeneous, in the other one investment costs differ among firms.

Table 1: General Parameter Values

Discount Rate ρ	0.05
Drift μ	0.03
Variance σ^2	0.1
Inverse-Demand	$Q^{-\xi}$

Tables 2 and 3 report the results for these three cases. First of all, from these examples we see that investment price-triggers for the duopoly and the monopoly differ significantly.[15]

For the follower, this is just the standard result of (Cournot) competition–the residual demand is less elastic. For the preemption threshold of the leader, the low price-trigger is a result of strong first-mover advantages of the Stackelberg-leader. The usual first-mover advantages are amplified in the presence of predatory behavior and / or a reversed order of exit upon investment.[16] These additional first-mover advantages can be very strong which can be seen by comparing the price-triggers with the "naive" or "Marshallian" net-present-value price trigger for investment. This naive price trigger can be calculated

indifference price trigger, i has no incentive to exit before.
Another motivation for this rule would be that firms step by step and sequentially undercut each other's exit price-triggers before the actual game commences.

[15] This is also true for the trigger values of Y, which can be obtained by rescaling the price triggers by 1.86607 for the leader and 1.95912 for the follower, setting $D(\overline{q}) = 1$.

[16] The effect of the reversed order of exit can also be seen by comparing the follower investment price-triggers: For firm 1 investment is more valuable as follower compared to firm 2. It invests earlier to become the firm that later monopolizes the market.

as

$$NPV_{Rule\_1} = \frac{\rho C_i}{\left(\overline{q}_i \Delta_{\underline{q}_i+\underline{q}_{-i}, \overline{q}_i+\underline{q}_{-i}} - \underline{q}_i\right)}, \text{ or} \tag{33}$$

$$NPV_{Rule\_2} = \frac{\rho C_i}{\left(\overline{q}_i \Delta_{\underline{q}_i+\underline{q}_{-i}, \overline{q}_i+\underline{q}_{-i}} - \underline{q}_i \Delta_{\underline{q}_i+\underline{q}_{-i}, \underline{q}_i+\overline{q}_{-i}}\right)}. \tag{34}$$

In the three cases, the price triggers are 8.2, 7.6, and 10.9 respectively assuming the other firm never invests (rule 1). If one assumes the other firm invests at $t = 0$ unless firm i invests (rule 2), then the net-present-value rule yields a price trigger of 4.5 in case A, which is only slightly below the equilibrium investment-price trigger of firm 1.

The "true" naive net-present-value rule investment-price trigger uses the equilibrium behavior of firm 2 and hence falls in between the two extremes. Therefore, the equilibrium investment threshold price trigger is extremely close to or lower than this "naive" price trigger. Hence, we can conclude that the gains from waiting are completely outweighed in some cases by the threat of being forced to exit first. This strong strategic value of investment is present in all three exemplary cases.

The main differences (in the parameters) between cases A, B, and C may be summarized as follows:

Case A Firm 1 has lower leverage before and after investment.

Case B Firm 1 has a higher leverage before, but lower leverage after investment.

Case C The size of the investment projects differs between Firm 1 and 2.

Comparing the equilibrium outcomes of the three cases (see tables 2 and 3) shows that the firm with the higher initial, the firm with the higher post-investment leverage, and the firm with the lower leverage in both states can prey in equilibrium. Therefore, interpreting the overhead costs as debt-service, our model includes not only the cases studied by Busse (2001) but also cases in which the financially healthier firm preys. Figure 9 shows the Φ_i functions for both firms corresponding to Case A. Recall, these functions represent the gain of becoming the leader.

Table 4 reports the effects of a change in the fixed costs of firm 1 (relative to Case A). When investment does not change the ordering of exit price triggers the effect of fixed costs on investment-price triggers is rather minor. As firm 1 becomes leader in equilibrium, investment in duopoly is delayed by an intermediate increase in fixed costs.

However, if both firms become more similar, overhead costs starkly influence investment decisions. If firm 2 can expect that firm 1 leaves the market first when firm 2 becomes the leader then first-mover advantages become very strong. As we have seen, the first-mover advantages can be strong enough to induce firm 1 to invest below the simple net-present-value price trigger in equilibrium.

Table 2: Case A,B: $\xi = 0.9$

	Firm 1 (A)	Firm 1 (B)	Firm 2
Production before Investment	18	17.9	18
Production after Investment	20	20	20
Investment-Costs	171.43	171.43	171.43
Fixed-Costs [Overhead]	49.9	49.9	50
MONOPOLY			
Exit-Price Trigger after investment	0.4496	0.4496	0.4505
Exit-Price Trigger before investment	0.4980	0.4980	0.4990
Investment-Price Trigger	99.844	95.337	99.844
DUOPOLY			
Both Invested, Exit-Price Trigger	Firm 2 exits	Firm 2 exits	0.45049
Firm 1 Follower: Exit-Price Trigger	0.4867	0.4888	Firm 1 exits
Firm 1 Follower: Investment-Price Trigger	15.872	15.061	Firm 1 inv.
Firm 2 Follower: Exit-Price Trigger	Firm 2 exits	Firm 2 exits	0.4881
Firm 2 Follower: Investment-Price Trigger	Firm 2 inv.	Firm 2 inv.	17.48
Preemption Threshold Non-Predatory Investment	4.57881	4.36741	4.895 (4.873)
Unilateral Incentive to Predatorily Invest as Leader	No	No	Yes
Firm 1: Predatory Investment-Price Trigger	0.611624	0.6206	Firm 1 inv.

Table 3: Case C: $\xi = 0.8$

	Firm 1	Firm 2
Production before Investment	17	18
Production after Investment	21	20
Investment-Costs	228.57	171.43
Investment-Costs per unit	57.14	85.72
Fixed-Costs [Overhead]	70	70
MONOPOLY		
Exit-Price Trigger after investment	0.601	0.631
Exit-Price Trigger before investment	0.7296	0.695
Investment-Price Trigger	34.75	49.77
DUOPOLY		
Both Invested, Exit-Price Trigger	Firm 2 exits	0.631
Firm 1 Follower: Exit-Price Trigger	0.693	Firm 1 exits
Firm 1 Follower: Investment-Price Trigger	9.293	Firm 1 inv.
Firm 2 Follower: Exit-Price Trigger	Firm 2 exits	0.680
Firm 2 Follower: Investment-Price Trigger	Firm 2 inv.	15.924
Preemption Threshold Non-Predatory Investment	3.579	3.109
Unilateral Incentive to Predatorily Invest as Leader	No	Yes
Firm 1: Predatory Investment-Price Trigger	Firm 2 inv.	0.786

Table 4: Effects of firm 1's fixed costs on investment-price triggers

fixed costs	$P^{inv,1}_{\underline{q}_1,0}$	$P^{inv,1}_{\underline{q}_1,\overline{q}_2}$	$P^{inv,1}_{pre}$	$P^{inv,2}_{pre}$
49.9*	99.8436	15.8715	4.57881	4.89525
49	99.8422	17.2948	6.0871	6.52665
48	99.8407	17.2942	6.08645	6.52679
47	99.8392	17.2935	6.08581	6.52694
40	99.8294	17.2894	6.08161	6.5279
35	99.8232	17.2868	6.07892	6.52851

* Firm 1 exits first as follower and also predatorily invests as leader.

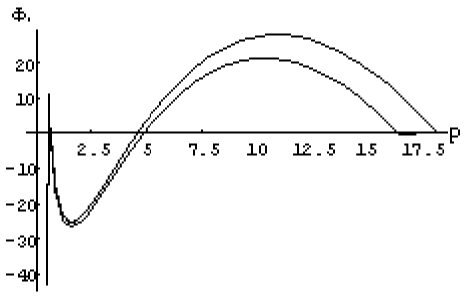

Figure 9: Gain, Φ_i, from becoming the leader, Case A

Although only results for firms that are relatively symmetric are reported, solutions have been calculated for cases that are more asymmetric in production or investment costs. Qualitatively, the results do not change much moving to asymmetric cases; only predatory outcomes become more likely, i.e. we obtain predatory equilibria also for cases with a greater difference in fixed costs or less pronounced reactions in price. Generally speaking, predatory outcomes are likely if firm 1 is small compared to firm 2 and both have large overhead costs.

6 Conclusions

In this paper the analysis of real options in duopoly has been extended to allow for simultaneous irreversible investment and exit decisions. The duopoly has been modelled in continuous time, and firms could default on their obligations at no costs.

We have found that allowing for endogenous exit decisions alters the strategic situation significantly. Firms may invest not because investment is fundamentally profitable, but because this makes the exit of the competitor more likely ("predatory investment"). Therefore, in the model presented fixed costs have a negative strategic effect. This may for example explain why companies are willing to spend much lump-sum money and effort to cut back on overhead costs.

However, the fixed costs are found to have a discontinuous effect on investment incentives. For moderate costs, investment tends to decrease when fixed costs increase. However, if cost levels get large enough, investment-incentives become very strong. Then both firms seek to become the leader and subsequently monopolize the market as soon as revenues drop due to adverse shocks to aggregate demand.

This reasoning gives an explanation for predatory behavior in a dynamic setting, but is neither relying on asymmetric information among competitors nor on learning-curve or network effects. The numerical examples show that in equilibrium both, the firm with the larger and the firm with the smaller overhead, may invest predatorily. Moreover, we have found that the outlined strategic incentive can have a substantial influence also for investment not directly aimed at crowding out the competitor. The strategic motive may even completely offset the value of waiting. Then, in equilibrium the first firm times investment as if it would follow a "naive" net-present value rule for investment. Hence, the interaction of exit and investment decisions becomes economicly important when irreversibility of decisions is taken into account.

The irreversibility framework also partly shields our model of predatory investment against the usual critique that a predator could just acquire its competitor. Whether or not acquisition is a possible alternative depends on the kind of fixed costs that drive the competitor into exit. If, for example, the fixed costs come in the form of financial obligations, then the remaining firm may not necessarily be able to default on them separately after the acquisition. Consequently, buying the firm means loosing one exit option and capacity cannot be reduced saving on fixed costs. Yet, aggregate capacity reduction is exactly the reason for predatory investment. In such a case, the predator may prefer to actually spend investment costs over the acquisition of its competitor.

For further research several extensions can be made: First of all, overhead-costs may be chosen endogenously. Moreover, collusive behavior could be studied. Other possible extensions include market entry and technological choice. Moreover, welfare issues and issues of competition policy have not been analyzed.

7 Appendix

In the following appendix, we first derive the functional form of the value function used in our model. Thereafter, the proofs which were omitted in the main text are presented.

7.1 Deriving the value functions

The expected capital gain is calculated treating $V_i(P, q_i, q_{-i})$ as an asset value and using (2). This yields according to Itô´s Lemma:

$$\mathbb{E}\left[\frac{dV_i(P, q_i, q_{-i})}{dt}\right] = \frac{1}{2}\sigma^2 P^2 \frac{\partial^2 V_i(P, q_i, q_{-i})}{\partial P^2} + \mu P \frac{\partial V_i(P, q_i, q_{-i})}{\partial P} \tag{35}$$

This expected capital gain plus the dividend, $q_i P - b_i$, must be equal to the normal return $\rho V_i(P, b_i, q_i, q_{-i})$ to eliminate arbitrage. Thus, we obtain the following differential equation

$$\rho V_i(P, q_i, q_{-i}) = \frac{\sigma^2}{2} P^2 \frac{\partial^2 V_i(P, q_i, q_{-i})}{\partial P^2} + \mu P \frac{\partial V_i(P, q_i, q_{-i})}{\partial P} + q_i P - b_i \tag{36}$$

A particular solution to this equation is

$$V_i(P, q_i, q_{-i}) = q_i \frac{P}{\rho - \mu} - \frac{b_i}{\rho} \tag{37}$$

The complementary solution as in (6) involves terms in the form of P^β, for each solution β to the fundamental quadratic equation

$$\beta^2 \frac{\sigma^2}{2} + \beta\left(\mu - \frac{\sigma^2}{2}\right) - \rho = 0 \tag{38}$$

as given in (5). (See Dixit and Pindyck (1994) for details.) Hence, the value function V_i takes the form

$$V_i(P, q_i, q_{-i}) = q_i \tfrac{P}{\rho-\mu} - \tfrac{b_i}{\rho} + a_{i1}(q_i, q_{-i})P^{\beta_1} + a_{i2}(q_i, q_{-i})P^{\beta_2}. \tag{39}$$

7.2 Proofs of the propositions of the main text

7.2.1 Proof of Proposition 2

For notational convenience, we introduce some short-hand notation. We denote by $\Delta_1 := \Delta_{\overline{q}_1+\overline{q}_2, \overline{q}_1}$, by $\Delta_2 := \Delta_{\overline{q}_1+\overline{q}_2, \overline{q}_2}$,and by $r := \frac{\overline{q}_1 b_2}{\overline{q}_2 b_1}$. Closely related to r is the ratio of relative fixed costs, $\frac{l_2}{l_1}$, since $r = \frac{l_2}{l_1}\frac{\Delta_2}{\Delta 1}$.

Lemma 4 *Under the assumptions of our model and with g as in the main text*

$$g \geq h := (\Delta_1)^{\beta_2} - \beta_2(\Delta_1 - 1)\left(\frac{r}{\Delta_2}\right)^{1-\beta_2} \geq 1 \tag{40}$$

holds for all $\Delta_1 \geq 1 \geq \Delta_2^{-(1-\beta_2)}$.

Proof. *We have* $1 \geq \Delta_2^{-(1-\beta_2)}$ *and so* h *must be smaller than* g :

$$\begin{aligned} h(\Delta_1) &= \left(\frac{1}{\Delta_1}\right)^{-\beta_2} \underbrace{-\beta_2}_{+}\underbrace{(\Delta_1-1)}_{+}\underbrace{\left(\frac{r}{\Delta_2}\right)^{1-\beta_2}}_{<r^{1-\beta_2}} \\ &\leq \left(\frac{1}{\Delta_1}\right)^{-\beta_2} - \beta_2\left(\Delta_1-1\right)r^{1-\beta_2} = g\left(\Delta_2\right). \end{aligned} \tag{41}$$

Now, taking the first derivative of h *with respect to* Δ_1 *yields*

$$\frac{\partial h(\Delta_1)}{\partial \Delta_1} = \beta_2 \Delta_1^{\beta_2-1} - \beta_2\left(\frac{r}{\Delta_2}\right)^{1-\beta_2}$$

Using $r = \frac{l_2}{l_1}\frac{\Delta_2}{\Delta 1}$, *we obtain*

$$\frac{\partial h(\Delta_1)}{\partial \Delta_1} = \beta_2 \Delta_1^{\beta_2-1} - \beta_2\left(\frac{l_2}{l_1}\frac{1}{\Delta 1}\right)^{1-\beta_2} = -\beta_2\Delta_1^{\beta_2-1}\left[\left(\frac{l_2}{l_1}\right)^{1-\beta_2} - 1\right] > 0$$

Assumption 1 yields $\frac{l_2}{l_1} > 1$. *Now, note that* $h(1) = 1$ *which completes the proof.* ■

Lemma 5 *For any* $P^{exit,2}_{\bar{q}_2,\bar{q}_1}$ *and* $P \geq P^{exit,2}_{\bar{q}_2,\bar{q}_1} \geq \Delta^{-1}_{\bar{q}_1+\bar{q}_2,\bar{q}_2} P^{exit,2}_{\bar{q}_2,0} > 0$ *value of firm 1 is positive in state* $(\bar{q}_1,\bar{q}_2)$.

Proof. *The later firm 2 exits, the lower will be firm 1's value, since firm 1 must wait longer to monopolize the market. Hence, we only need to check, whether firm 1's value is positive for all* $P \geq P^{exit,2}_{\bar{q}_2,\bar{q}_1}$ *when* $P^{exit,2}_{\bar{q}_2,\bar{q}_1} = \Delta^{-1}_{\bar{q}_1+\bar{q}_2,\bar{q}_2} P^{exit,2}_{\bar{q}_2,0} = \Delta_2^{-1} r P^{exit,1}_{\bar{q}_1,0}$. *This is the lowest price at which firm 2 would not obtain negative value if firm 1 leaves at firm 2's exit price-trigger. Expecting that firm 1 leaves second, the corresponding value matching condition for firm 1 yields*

$$\begin{aligned} &\frac{\bar{q}_1}{\rho-\mu}\Delta_2^{-1} r P^{exit,1}_{\bar{q}_1,0} - \frac{b_1}{\rho} + \alpha\left(\Delta_2^{-1} r P^{exit,1}_{\bar{q}_1,0}\right)^{\beta_2} \\ = &\frac{\bar{q}_1\Delta_1}{\rho-\mu}\Delta_2^{-1} r P^{exit,1}_{\bar{q}_1,0} - \frac{b_1}{\rho} + \frac{1}{1-\beta_2}\frac{b_1}{\rho}\left(\Delta_1\frac{\Delta_2^{-1} r P^{exit,1}_{\bar{q}_1,0}}{P^{exit,1}_{\bar{q}_1,0}}\right)^{\beta_2} \end{aligned}$$

α *represents the option-value term,* a_{i2}. *Substituting in* $\frac{\beta_2}{\beta_2-1}\frac{\rho-\mu}{\rho}\frac{b_1}{\bar{q}_1}$ *for* $P^{exit,1}_{\bar{q}_1,0}$, *we obtain*

$$\begin{aligned} &\frac{\bar{q}_1}{\rho-\mu}\Delta_2^{-1} r \frac{\beta_2}{\beta_2-1}\frac{\rho-\mu}{\rho}\frac{b_1}{\bar{q}_1} + \alpha\left(\Delta_2^{-1} r P^{exit,1}_{\bar{q}_1,0}\right)^{\beta_2} \\ = &\frac{\bar{q}_1}{\rho-\mu}\Delta_1\Delta_2^{-1} r \frac{\beta_2}{\beta_2-1}\frac{\rho-\mu}{\rho}\frac{b_1}{\bar{q}_1} + \frac{1}{1-\beta_2}\frac{b_1}{\rho}\left(\Delta_1\Delta_2^{-1} r\right)^{\beta_2}. \end{aligned}$$

This can be rewritten as

$$\Delta_2^{-1} r \frac{\beta_2}{\beta_2-1}\frac{b_1}{\rho} + \alpha\left(\Delta_2^{-1} r P^{exit,1}_{\bar{q}_1,0}\right)^{\beta_2} = \Delta_1\Delta_2^{-1} r \frac{\beta_2}{\beta_2-1}\frac{b_1}{\rho} + \frac{1}{1-\beta_2}\frac{b_1}{\rho}\left(\Delta_1\Delta_2^{-1} r\right)^{\beta_2}.$$

Subtracting $\Delta_2^{-1} r \frac{\beta_2}{\beta_2-1}\frac{b_1}{\rho}$ *and dividing by* $\left(\Delta_2^{-1} r\right)^{\beta_2}$ *we obtain*

$$\begin{aligned}\alpha\left(P_{\bar{q}_1,0}^{exit,1}\right)^{\beta_2} &= \frac{b_1}{\rho}\frac{1}{1-\beta_2}\left(\left(1-\Delta_1\right)\Delta_2^{-1} r\beta_2+\left(\Delta_1\Delta_2^{-1} r\right)^{\beta_2}\right)\left(\Delta_2^{-1} r\right)^{-\beta_2} \\ &= \frac{b_1}{\rho}\frac{1}{1-\beta_2}\left(\Delta_1^{\beta_2}-\beta_2\left(\Delta_1-1\right)\left(\frac{r}{\Delta_2}\right)^{1-\beta_2}\right)=h\cdot\frac{b_1}{\rho}\frac{1}{1-\beta_2}.\end{aligned}$$

Substitution α *back in our general formulation of firm 1's value function, we obtain for the hypothetical value* H *of firm 1–given that firm 2 exits at* $\Delta_{\bar{q}_1+\bar{q}_2,\bar{q}_2}^{-1} P_{\bar{q}_2,0}^{exit,2}$ *:*

$$H\left(P\right)=\frac{\bar{q}_1 P}{\rho-\mu}-\frac{b_1}{\rho}+h\cdot\frac{b_1}{\rho}\frac{1}{1-\beta_2}\left(\frac{P}{P_{\bar{q}_1,0}^{exit,1}}\right)^{\beta_2}, P\geq\Delta_{\bar{q}_1+\bar{q}_2,\bar{q}_2}^{-1} P_{\bar{q}_2,0}^{exit,2} \tag{42}$$

This is of the same form as V*. Ignoring for the moment that* V *changes its form at* $P_{\bar{q}_1,0}^{exit,1}$*, we rewrite* H *as*

$$H\left(P\right)=V\left(P,\bar{q}_1,0\right)+\left(h-1\right)\cdot\underbrace{\frac{b_1}{\rho}\frac{1}{1-\beta_2}\left(\frac{P}{P_{\bar{q}_1,0}^{exit,1}}\right)^{\beta_2}}_{>0}. \tag{43}$$

By construction, we know that V *obtains its minimum at* $P_{\bar{q}_1,0}^{exit,1}$ *with* $V\left(P_{\bar{q}_1,0}^{exit,1},\bar{q}_1,0\right)=0$*. Moreover, we know* $h\geq 1$ *from the above Lemma. Hence,* $H\left(P\right)\geq 0$ *follows.* ■

Proof of Proposition 2. First note that under the proposed equilibrium strategy firm 2 never becomes a monopolist. Therefore, only the actual price and not the quantity of the competitor matter for firm 2. Thus, firm 2 behaves myopically. Therefore, the value of firm 2 under the proposed strategy is zero at $P_{\bar{q}_1,\bar{q}_2}^{exit,2}$, which then is indeed the optimal trigger price.

Secondly, we have to show that firm 2 cannot profitably choose an exit price trigger smaller than $P_{\bar{q}_1,\bar{q}_2}^{exit,1}$. Suppose firm 2 chooses a lower price trigger. Then firm 2 becomes monopolist after firm 1 exits. However, the price after firm 1 has left the market is still below firm 2´s monopoly exit price-trigger, and hence the value associated with this strategy must be negative. Hence, firm 2 has no incentive to deviate.

Firm 1 also has no incentive to exit at a price different to $P_{\bar{q}_1,\bar{q}_2}^{exit,1}$. If firm 2 chooses $P_{\bar{q}_2,\bar{q}_1}^{exit,2}$, all price triggers below $P_{\bar{q}_2,\bar{q}_1}^{exit,2}$ yield the same payoff given P. According to Lemmas 4 and 5 this payoff is positive and larger than the value of the firm that leaves first. So for firm 1, leaving second is credible and profitable.

Last, we need to show, that there can be no other equilibrium in which firm 1 exits second: If firm 1 chooses an price-trigger larger than $\Delta_{\bar{q}_1+\bar{q}_2,\bar{q}_2}{}^{-1} P_{\bar{q}_2,0}^{exit,2}$, firm 2 can profitably deviate and set a price slightly smaller, leave second, and obtain monopoly profits.

That $\left]\Delta_{\bar{q}_1+\bar{q}_2,\bar{q}_1}{}^{-1} P_{\bar{q}_1,0}^{exit,1}, \Delta_{\bar{q}_1+\bar{q}_2,\bar{q}_2}{}^{-1} P_{\bar{q}_2,0}^{exit,2}\right[$ is non-empty follows straightforward from assumption 1, the assumption on $\frac{l_1}{l_2}$. ■

7.2.2 Proofs of Proposition 3 to 6 and Lemma 1 and 2

Proof of Proposition 3. Investment causes no continuous costs but only a lump sum payment. Thus, earnings after investment are strictly larger than before investment. Because of this, firm-value at low capacity is strictly smaller than it is at high capacity. Therefore at low capacity, firm 2 must leave before $P^{exit,2}_{\bar{q}_2,0}$ has been reached—both, in duopoly and monopoly. Otherwise its value would become negative before exit, see figure 3. Due to the same reasoning as in proposition 2, firm 2 can again not credibly threaten to exit second. Hence, we obtain $P^{exit,2}_{\underline{q}_2,\bar{q}_1} > P^{exit,2}_{\bar{q}_2,\bar{q}_1} = P^{exit,2}_{\bar{q}_2,0}$.
As before, this leads to myopic behavior of firm 2, so that $P^{exit,2}_{\underline{q}_2,\bar{q}_1} = P^{exit,2}_{\underline{q}_2,0}$ and $P^{inv,2}_{\underline{q}_2,\bar{q}_1} = P^{inv,2}_{\underline{q}_2,0}$. ■

Proof of Lemma 1. (a) Potential investment of firm 1 decreases the value of firm 2. In monopoly or in state $(\bar{q}_i, \bar{q}_{-i})$ there is no such potential investment of a competitor. In both cases, at $P^{exit,2}_{\bar{q}_2,0}$ firm-value is zero. Thus, if firm 2 has not left in state $\left(\bar{q}_2, \underline{q}_1\right)$ when prices are as low as $P^{exit,2}_{\bar{q}_2,0}$, the potential investment will force firm-value to be negative. Therefore, the stated inequality must hold.
(b) Upon investment of firm 1, prices drop by factor $\Delta_{\underline{q}_1+\bar{q}_2,\bar{q}_1+\bar{q}_2}$. Therefore, if firm 1 actually invests at $\Delta^{-1}_{\underline{q}_1+\bar{q}_2,\bar{q}_1+\bar{q}_2} P^{exit,2}_{\bar{q}_2,\bar{q}_1}$ firm 2 would immediately leave and obtain zero value. However, potential investment decreases the value of firm 2 only by the expected value of the loss upon investment. This is less than the drop in value caused by actual investment. Therefore, firm 2's value at $\Delta^{-1}_{\underline{q}_1+\bar{q}_2,\bar{q}_1+\bar{q}_2} P^{exit,2}_{\bar{q}_2,\bar{q}_1}$ must be positive and the stated inequality follows. ■

Proof of Lemma 2. If firm 2 exits first, $\widetilde{V}_1(P, \underline{q}_1, \bar{q}_2)$ exhibits a kink. Due to Lemma 1, this kink must be at a smaller P than the kink in $V_1(\Delta_{\underline{q}_1+\bar{q}_2,\bar{q}_1+\bar{q}_2} P, \bar{q}_1, \bar{q}_2)$. Define the continuous function

$$f(P) := \widetilde{V}_1(P, \underline{q}_1, \bar{q}_2) + C_1 - V_1(\Delta_{\underline{q}_1+\bar{q}_2,\bar{q}_1+\bar{q}_2} P, \bar{q}_1, \bar{q}_2). \tag{44}$$

If $\Delta_{\underline{q}^1+\bar{q}^2,\bar{q}^1+\bar{q}^2} P^{exit,1}_{\underline{q}_1,\bar{q}_2} < P^{exit,2}_{\bar{q}_2,0}$, then $f(P)$ has the following functional form for $P > \max\{P^{exit,2}_{\bar{q}_2,\underline{q}_1}, P^{exit,1}_{\underline{q}_1,\bar{q}_2}\}$:

$$f(P) = \begin{cases} x_{11}P + x_{12}P^{\beta_2} + C & \text{if } \Delta_{\underline{q}^1+\bar{q}^2,\bar{q}^1+\bar{q}^2} P < P^{exit,2}_{\bar{q}_2,0} \\ x_{21}P + x_{22}P^{\beta_2} + C & \text{if } \Delta_{\underline{q}^1+\bar{q}^2,\bar{q}^1+\bar{q}^2} P \geq P^{exit,2}_{\bar{q}_2,0} \end{cases}. \tag{45}$$

Otherwise,

$$f(P) = x_{31}P + x_{32}P^{\beta_2}. \tag{46}$$

Hence, f must be either concave or convex on each subset, but is possibly concave or convex on both subsets. Consequently $f(P) = 0$ can have at most four solutions.
By assumption 2(a), after investment a company profits stronger from a demand-increase, so $\lim_{P\to+\infty} f(P) = -\infty$. Moreover, neglect for the moment the change in the functional

form of f at $\max\{P^{exit,2}_{\bar{q}_2,\underline{q}_1}, P^{exit,1}_{\underline{q}_1,\bar{q}_2}\}$, but suppose f keeps the form of (45) for all $P \geq \Delta^{-1}_{\underline{q}_1+\bar{q}_2,\bar{q}_1} P^{exit,1}_{\bar{q}_1,0}$. This is the price when firm 1 exits in monopoly after investment normalized to state $\left(\underline{q}_1, \bar{q}_2\right)$-prices. As $\widetilde{V}_1(P, \underline{q}_1, \bar{q}_2) \geq 0$ by construction, and $V_1(\Delta^{-1}_{\bar{q}_1+\bar{q}_2,\bar{q}_1} P^{exit,1}_{\bar{q}_1,0}, \bar{q}_1, \bar{q}_2) = 0$, we can conclude $f(\Delta^{-1}_{\underline{q}_1+\bar{q}_2,\bar{q}_1+\bar{q}_2} \Delta^{-1}_{\bar{q}_1+\bar{q}_2,\bar{q}_1} P^{exit,1}_{\bar{q}_1,0}) > 0$. Therefore, the number of solutions to $f(P) = 0$ must be odd on the set of price-levels that are larger than the monopoly exit-price. Now note that $\left\{P \,\middle|\, P > \max\{P^{exit,2}_{\bar{q}_2,\underline{q}_1}, P^{exit,1}_{\underline{q}_1,\bar{q}_2}\}\right\}$ is subset of this set. This completes the proof. ■

Proof of Proposition 4. (a) First note that $V_1\left(P, \bar{q}_1, \bar{q}_2\right)$ is convex (and decreasing at $P = P^{exit,2}_{\bar{q}_2,0}$), so that $\frac{\partial V_1}{\partial P}\left(P, \bar{q}_1, \bar{q}_2\right) = 0$ is a sufficient condition for a minimum. We have

$$\frac{\partial V_1}{\partial P}\left(P^{\min}, \bar{q}_1, \bar{q}_2\right) = 0 \Leftrightarrow \frac{\bar{q}_1}{\rho - \mu} + g \cdot \frac{b_1}{\rho} \frac{\beta_2}{1-\beta_2} \left[\frac{P^{\min}}{P^{exit,1}_{\bar{q}_1,0}}\right]^{\beta_2 - 1} \frac{1}{P^{exit,1}_{\bar{q}_1,0}} = 0 \Leftrightarrow$$

$$\frac{1}{P^{exit,1}_{\bar{q}_1,0}} \left[\frac{P^{\min}}{P^{exit,1}_{\bar{q}_1,0}}\right]^{\beta_2 - 1} = g^{-1} \cdot \frac{\bar{q}_1}{\rho - \mu} \frac{\rho}{b_1} \frac{\beta_2}{\beta_2 - 1} \Leftrightarrow \left[\frac{P^{\min}}{P^{exit,1}_{\bar{q}_1,0}}\right]^{\beta_2 - 1} = g^{-1} \Leftrightarrow P^{\min} = g^{\frac{1}{1-\beta_2}} P^{exit,1}_{\bar{q}_1,0}$$

(b) If firm 1 exits first when not equipped with an investment option, it will behave myopically and exit at $\frac{\bar{q}_1}{\underline{q}_1} P^{exit,1}_{\bar{q}_1,0}$. Comparing this exit price with $\Delta^{-1}_{\underline{q}_1+\bar{q}_2,\bar{q}_1+\bar{q}_2} P^{\min}$, where $V_1(\Delta_{\underline{q}_1+\bar{q}_2,\bar{q}_1+\bar{q}_2} P, \bar{q}_1, \bar{q}_2)$ obtains its minimum, we find that $g > \left(\frac{D(\bar{q}_1+\bar{q}_2)}{D(\underline{q}_1+\bar{q}_2)} \cdot \frac{\bar{q}_1}{\underline{q}_1}\right)^{1-\beta_2}$ ensures that firm 1 will not exit at a price larger than $\Delta^{-1}_{\underline{q}_1+\bar{q}_2,\bar{q}_1+\bar{q}_2} P^{\min}$. Thus, at $C_1 = V_1(P^{\min}, \bar{q}_1, \bar{q}_2) - \tilde{V}_1\left(\Delta^{-1}_{\underline{q}_1+\bar{q}_2,\bar{q}_1+\bar{q}_2} P^{\min}, \underline{q}_1, \bar{q}_2\right)$ both functions are tangentially. Increasing C_1 by some small amount yields multiple solutions to (21).
(c) If firm 2 exits first, the peak in $V_1\left(\Delta_{\underline{q}_1+\bar{q}_2,\bar{q}_1+\bar{q}_2} P, \bar{q}_1, \bar{q}_2\right)$ lies at $P = \Delta_{\underline{q}_1+\bar{q}_2,\bar{q}_1+\bar{q}_2}{}^{-1} P^{exit,2}_{\bar{q}_2,0}$. According to Lemma 1, $\Delta_{\underline{q}_1+\bar{q}_2,\bar{q}_1+\bar{q}_2}{}^{-1} P^{exit,2}_{\bar{q}_2,0} > P^{exit,2}_{\bar{q}_2,\underline{q}_1}$. Thus, the peak in $\widetilde{V}_1\left(P, \underline{q}_1, \bar{q}_2\right)$ is at a price lower than $\Delta_{\underline{q}_1+\bar{q}_2,\bar{q}_1+\bar{q}_2}{}^{-1} P^{exit,2}_{\bar{q}_2,0}$. Consider the costs C' that give

$$\widetilde{V}_1\left(P, \underline{q}_1, \bar{q}_2\right) + C' < V_1\left(\Delta_{\underline{q}_1+\bar{q}_2,\bar{q}_1+\bar{q}_2} P, \bar{q}_1, \bar{q}_2\right) \tag{47}$$

for all $P \geq \Delta_{\underline{q}_1+\bar{q}_2,\bar{q}_1+\bar{q}_2}{}^{-1} P^{exit,2}_{\bar{q}_2,0}$ except for one point P'(see figure **??**). At this point either both functions are tangentially or $P' = \Delta_{\underline{q}_1+\bar{q}_2,\bar{q}_1+\bar{q}_2}{}^{-1} P^{exit,2}_{\bar{q}_2,0}$, which is ruled out by the assumption on the derivatives.
Therefore, at costs C' firm 1 invests for all prices $P \geq \Delta_{\underline{q}_1+\bar{q}_2,\bar{q}_1+\bar{q}_2}{}^{-1} P^{exit,2}_{\bar{q}_2,0}$ and

$$\hat{V}_1\left(P, \underline{q}_1, \bar{q}_2\right) \equiv V_1\left(\Delta_{\underline{q}_1+\bar{q}_2,\bar{q}_1+\bar{q}_2} P, \bar{q}_1, \bar{q}_2\right) - C'.$$

Now take costs to be equal to $C' + \varepsilon$, $\varepsilon > 0$. Assuming that there is only one price trigger for investment P^{inv}, will lead to a contradiction: For this trigger $P' \leq P^{inv}$ holds.

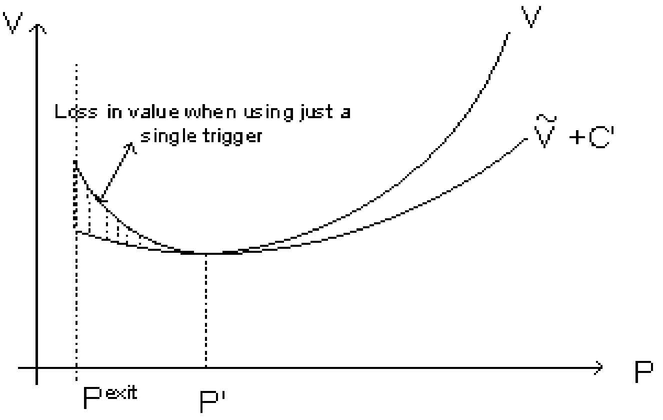

Figure 10: Loss in value from a single trigger

Define the stopping-time $\tau(p) := \inf\{t \in R | P_t = p\}$, then the difference in value for $\Delta_{\underline{q}_1+\overline{q}_2,\overline{q}_1+\overline{q}_2}{}^{-1} P^{exit,2}_{\overline{q}_2,0} < P < P'$ evaluates as

$$
\begin{aligned}
& \hat{V}_1\left(P, \underline{q}_1, \overline{q}_2 | C'\right) - \hat{V}_1\left(P, \underline{q}_1, \overline{q}_2 | C' + \varepsilon\right) \\
= \;& \mathbb{E}\left[\int_0^{\tau\left(P^{inv}\right)} \left(V_1\left(\Delta_{\underline{q}_1+\overline{q}_2,\overline{q}_1+\overline{q}_2} P_t, \overline{q}_1, \overline{q}_2\right) - C' - \widetilde{V}_1\left(P_t, \overline{q}_1, \overline{q}_2\right)\right) e^{-\rho t} dt \,\middle|\, P_0 = P\right] \\
\geq \;& \mathbb{E}\left[\int_0^{\tau(P')} \left(V_1\left(\Delta_{\underline{q}_1+\overline{q}_2,\overline{q}_1+\overline{q}_2} P_t, \overline{q}_1, \overline{q}_2\right) - C' - \widetilde{V}_1\left(P_t, \overline{q}_1, \overline{q}_2\right)\right) e^{-\rho t} dt \,\middle|\, P_0 = P\right] > 0.
\end{aligned}
$$

Both inequalities follow from (47) (and $\tau(P^{inv}) \geq \tau(P')$): Thus, a marginal change in costs would lead to a non-marginal drop in value, since the last integral does not depend on ε. However, if firm 1 uses a system of two price-triggers of investment depending on ε, the value drop is only marginal and hence for ε small enough two price-triggers must be optimal. ■

Proof of Proposition 5. If $\Delta_{\overline{q}_1+\overline{q}_2,\overline{q}_1} \to 1$, we have $g \to 1$ and firm 1 does not gain from firm 2 leaving. Moreover, the value of firm 1 as a monopolist and its value as a duopolist converge. Therefore, firm 1 cannot gain anything from firm 2 leaving.

If demand is not completely inelastic, firm 1's value at the exit price of firm 2 is bounded. Now suppose the costs of investment exceed this value. Then there can be no solution to (21) for prices smaller than $\Delta^{-1}_{\overline{q}_1+\overline{q}_2,\overline{q}_1+\overline{q}_2} P^{exit,2}_{\overline{q}_2,\underline{q}_1}$. Moreover, the number of solutions to (21) must be odd: At $\Delta^{-1}_{\underline{q}_1+\overline{q}_2,\overline{q}_1+\overline{q}_2} P^{exit,2}_{\overline{q}_2,\underline{q}_1}$ the left hand side of (21) is larger than the right hand side, while for $P \to \infty$ the reverse holds true. So the first and the last time both functions cross, the value function at low capacity crosses from above. Hence, the number of solutions is odd (V is continuous). However, from Lemma 2 we now know there can be at most two solutions. Thus, both value functions can only cross once. ■

Proof of Proposition 6. Suppose it is optimal for firm i to invest at a price P' strictly larger than $\Delta_{\underline{q}_i+q_{-i},\bar{q}_i+q_{-i}}{}^{-1}P^{exit,-i}_{q_{-i},\bar{q}_i}$. Then the only cost of a marginal delay, i.e. a marginal decrease in the predatory investment price trigger, is the expected foregone increase in earnings. This marginal loss is–expressed a bit more formally:

$$L\left(X\right):=\left.\frac{\partial\mathbb{E}\int_{\tau_0(X)}^{\tau_1}e^{-\rho t}\left(\bar{q}_i\Delta_{\underline{q}_i+q_{-i},\bar{q}_i+q_{-i}}-\underline{q}_i\right)P_t dt}{\partial X}\right|_{P_0=X}. \tag{48}$$

In this term τ_1 denotes the time of exit of the competitor and $\tau_0:=\inf\{t|P_t\leq X\vee P_t\geq P^{inv,i}_{\underline{q}_i,q_{-i}}\}$ is the time of investment of firm i. Both τ_0 and τ_1 are stochastic variables (stopping times). Note that this derivative gets larger, the larger X is: While the change in τ_0 remains the same, the integrand increases. The time of the competitor leaving, τ_1, is unchanged by altering the predatory investment price-trigger X; yet, only as long as X is strictly smaller than $\Delta_{\underline{q}_i+q_{-i},\bar{q}_i+q_{-i}}{}^{-1}P^{exit,-i}_{q_{-i},\bar{q}_i}$.
The expected gain from delay is constant in X and given by

$$G\left(X\right):=\left.\frac{\partial\mathbb{E}\left[e^{-\rho\tau_0(X)}C_i\right]}{\partial X}\right|_{P_0=X}. \tag{49}$$

To be optimal, at P' marginal gain and marginal loss from waiting must be equal. So $L\left(P'\right)=G\left(P'\right)$ needs to hold. But then, at any price P''strictly larger than P' firm i would prefer to have invested earlier, as $L\left(P''\right)>G\left(P''\right)=G\left(P'\right)$. Therefore, P' can only be a locally optimal investment price trigger if firm i invests at all prices larger than P'. Hence, it cannot be a predatory investment price-trigger.
However, at $\Delta_{\underline{q}_i+q_{-i},\bar{q}_i+q_{-i}}{}^{-1}P^{exit,-i}_{q_{-i},\bar{q}_i}$ the exit time of the competitor, τ_1, also changes in the predatory price-trigger. Therefore, the same argument does not apply to $P'=\Delta_{\underline{q}_i+q_{-i},\bar{q}_i+q_{-i}}{}^{-1}P^{exit,-i}_{q_{-i},\bar{q}_i}$. Hence, $\Delta_{\underline{q}_1+\bar{q}_2,\bar{q}_1+\bar{q}_2}{}^{-1}P^{exit,2}_{\bar{q}_2,0}$ is the only candidate for a predatory investment price-trigger. ■

7.2.3 Proof of Lemma 3

Lemma 6 $\Phi_i(P)$ *can be represented by* $\Phi_i(P)=x_{0i}+x_{1i}P+x_{2i}P^{\beta_1}+x_{3i}P^{\beta_2}$ *for* $P\in M:=[\max_{j=1,2}\{\underline{P}_j\},\min_{j=1,2}\{\overline{P}_j\}]$*; with* $x_{1i}>0$, $x_{2i}<0$*. Moreover,* $\Phi_i(P)$ *is also continuous on* $[\min_{j=1,2}\{\underline{P}_j\},\max_{j=1,2}\{\overline{P}_j\}]$

Proof. *First note that* $\Phi_i(P)$ *has the stated functional form since it is a difference of functions of the type given in* (5) *(which are analytic on* M*). It is clear that the follower's firm value must be a convex function. Moreover, the leader´s value decreases by the potential entry of the follower, therefore* $x_2<0$*. Sales are increased by investment; this implies* $x_1>0$*. Continuity follows from the value-matching conditions* ■

Lemma 7 *Let* $f(P) = x_0 + x_1 P + x_2 P^{\beta_1} + x_3 P^{\beta_2}$*;* $P > 0$ *and* $x_1 > 0$*,* $x_2 < 0$*, then* f *has at most three roots. Moreover, if at* P' $f(P') > 0$*, there can only be two roots of* f *for* $P < P'$*.*

Proof. *We have to consider two cases:*
Case 1: $x_3 \leq 0$*, then* f *is concave and therefore has at most two roots.*
Case 2: $x_3 > 0$*. Firstly, note that the second derivative changes its sign at most once: Suppose* $f''(P^*) = P^{*-2}\left[\beta_1(\beta_1 - 1)x_2 P^{*\beta_1} + \beta_2(\beta_2 - 1)x_3 P^{*\beta_2}\right] = 0$*. Then*

$$f'''(P^*) = -2P^{*-3}\left[\beta_1(\beta_1 - 1)x_2 P^{*\beta_1} + \beta_2(\beta_2 - 1)x_3 P^{*\beta_2}\right] + P^{*-2}\left[\beta_1(\beta_1 - 1)\beta_1 x_2 P^{*\beta_1} + \beta_2(\beta_2 - 1)\beta_2 x_3 P^{*\beta_2}\right]. \quad (50)$$

However, the first term equals $-2\frac{f''(P^*)}{P^*}$ *and thus, is zero and therefore:*

$$f'''(P^*) = P^{*-2}[\underbrace{\beta_1(\beta_1 - 1)\beta_1}_{>0}\underbrace{x_2 P^{*\beta_1}}_{<0} + \underbrace{\beta_2(\beta_2 - 1)\beta_2}_{<0}\underbrace{x_3 P^{*\beta_2}}_{>0}] < 0\ . \quad (51)$$

This implies that the second derivative changes its sign at most once.
Therefore, the turning point P^* *divides* f *in a convex and a concave part. If* $f(P^*) < 0$*, then on the concave part of* f*, i.e.* $P > P^*$*, there may be two roots. Since* $f(P^*) < 0$ *function* f *crosses 0 from above at any* $P < P^*$*, such that* $f(P) = 0$*, so that* $f'(P)$ *must be negative. Yet,* f *is convex on this subset and so there can be no additional roots to this third one. The case* $f(P^*) \geq 0$ *follows analogously.* ■

Proof of Lemma 3. (a) follows straightforward from the last two Lemmata.
(b) At $\overline{P}_i$ firm i invests as follower, therefore

$$V_i(\Delta_{\underline{q}_i+\underline{q}_{-i},\underline{q}_i+\overline{q}_{-i}}P, \underline{q}_i, \overline{q}_{-i}) = V_i(\Delta_{\underline{q}_i+\underline{q}_{-i},\overline{q}_i+\overline{q}_{-i}}P, \overline{q}_i, \overline{q}_{-i}) - C \quad \forall P \geq \overline{P}_i. \quad (52)$$

This implies $\Phi_i(P) > 0$ if $\overline{P}_i \leq P < \overline{P}_{-i}$ and $\Phi_i(\overline{P}_{-i}) = 0$, since sales of the leader are larger before the follower has invested and

$$V_i(\Delta_{\underline{q}_i+\underline{q}_{-i},\overline{q}_i+\underline{q}_{-i}}\overline{P}_{-i}, \overline{q}_i, \underline{q}_{-i}) - C = V_i(\Delta_{\underline{q}_i+\underline{q}_{-i},\overline{q}_i+\overline{q}_{-i}}\overline{P}_{-i}, \overline{q}_i, \overline{q}_{-i}) - C. \quad (53)$$

(c) At prices larger than $\overline{P}_{-i}$ firm $-i$ invests as follower, therefore firm i can only trigger joint investment and

$$\forall P \geq \overline{P}_{-i} : V_i(\Delta_{\underline{q}_i+\underline{q}_{-i},\overline{q}_i+\underline{q}_{-i}}P, \overline{q}_i, \underline{q}_{-i}) - C = V_i(\Delta_{\underline{q}_i+\underline{q}_{-i},\overline{q}_i+\overline{q}_{-i}}P, \overline{q}_i, \overline{q}_{-i}) - C. \quad (54)$$

As long as $P < \overline{P}_i$, it is not profitable for firm i to invest as follower and obtain $V_i(\Delta_{\underline{q}_i+\underline{q}_{-i},\overline{q}_i+\overline{q}_{-i}}P, \overline{q}_i, \overline{q}_{-i}) - C$. Hence, the right-hand term must be smaller than the value of i being the follower when $P < \overline{P}_i$.
(d) $\Phi_i(\max_{j=1,2}\{\underline{P}_j\}) > 0$ follows from the continuity of Φ_i and in case (b) from $\Phi_i\left(\bar{P}_i\right) > 0$, respectively $\Phi_i\left(\bar{P}_{-i}\right) < 0$ in case (c). Define $f(P)$ as stated in Lemma 7. Now suppose

that firm i exits at $\underline{P}_i \geq \underline{P}_{-i}$, then $\Phi_i(P) > f(P)$ for $P < \underline{P}_i$, since the value of firm i as follower is a convex function with derivative zero at $\underline{P}_i$. Therefore, for $\Phi_i(P)$ to have an additional root, $f(P)$ must have an additional root, too. However, due to Lemma 7, $f(P)$ cannot have that additional root.
If firm i invests predatorily at $\underline{P}_i$, then for all $\min_{j=1,2}\{\underline{P}_j\} < P < \max_{j=1,2}\{\underline{P}_j\}$

$$\Phi_i(P) = V_i(\Delta_{\underline{q}_i+\underline{q}_{-i},\overline{q}_i+\underline{q}_{-i}}P, \overline{q}_i, \underline{q}_{-i}) - V_i(\Delta_{\underline{q}_i+\underline{q}_{-i},\overline{q}_i+\overline{q}_{-i}}P, \overline{q}_i, \overline{q}_{-i}) > 0 \tag{55}$$

because the sales of the follower are lower, and firm $-i$ exits later. ■

7.2.4 Proofs of Proposition 7 to 9

Proof of Proposition 7. First note that at $P = P^{inv,L}_{\underline{q}_L,\underline{q}_{-L}}$ firm L indeed prefers to be the leader, because $\Phi_i(P) > 0$. Moreover because of the assumption on the valuation for collusion we have a preemption game for non-predatory investment: Suppose τ denotes the stopping-time associated with the optimal investment-price trigger of firm i. Then firm $-i$ has an incentive to invest at time $\tau - \epsilon$, as long as the price is above its preemption threshold. Therefore, when $P_t \in [P^{pred,-L}_{pre}, P^{inv,-L}_{pre}]$ firm $-L$ prefers to be the follower, while at $P^{inv,L}_{\underline{q}_L,\underline{q}_{-L}}$ firm L profitable invests. ■

Proof of Proposition 8. (a) Suppose firm i wishes to invest at a price $P' < P^{pred,-i}_{pre}$. Define τ to be the corresponding stopping time. Then firm $-i$ would have an incentive to preempt and invest at a smaller price at time $\tau - \varepsilon$. Therefore, investing predatorily below $P^{pred,-i}_{pre}$ cannot be part of an equilibrium. However, at prices between $P^{pred,-i}_{pre}$ and $P^{inv,-i}_{pre}$ firm $-i$ wishes to become follower and will therefore not preempt. Moreover, at prices below $P^{pred,i}_{pre}$ firm i wishes to become leader, so that the solution described is indeed optimal for firm i, given firm $-i$ would invest as soon as prices hit the preemption thresholds.
(b) If $\Delta_{\underline{q}_{-j}+q_j,\overline{q}_{-j}+q_j}{}^{-1}P^{exit,j}_{q_j,\overline{q}_{-i}}$ falls between $P^{pred,i}_{pre}$and $P^{inv,-i}_{pre}$ and firm $-j$ has a unilateral incentive to invest at a price P' from this interval, this establishes a credible threat of firm $-j$ investing at P'. Thus, the firms wish to preempt until $P^{pred,i}_{pre}$ is reached. At this price firm i is indifferent between becoming leader or follower. As investment-price trigger, firm $-i$ will choose its unconstrained optimal predatory investment-price trigger (if this is possible). If the constraint binds, $P^{pred,i}_{pre}$ is chosen as investment price-trigger.
(c) We firstly need to argue that investing on $[P^{inv,-i}_{pre}, P^{pred,i}_{pre}]$ cannot be renegotiation-proof. Suppose one firm would invest at $P^* \in [P^{inv,-i}_{pre}, P^{pred,i}_{pre}]$. Then, since neither firm has an unilateral incentive to invest at some price $P \in [P^{inv,-i}_{pre}, P^{pred,i}_{pre}]$, both firms would find it profitable to renegotiate and sign an incentive compatible contract that investment should be carried out at the proposed price-triggers for predatory and non-predatory investment.
The result for predatory investment follows from the same line of argument. ■

Proof of Proposition 9. (a) Firm value is increasing in the exit price of the competitor. If undercutting the price-trigger of firm i is not credible even when i chooses the myopic price-trigger, then $-i$ cannot threaten to exit second.
(b) $\left[\Delta_{q_i+q_{-i},q_i}{}^{-1}P^{exit,i}_{q_{-i},0}, P^{ind,i}_{q_i,q_{-i}}\right] = \emptyset$ implies that leaving second always yields positive equity value for all credible exit price triggers of the competitor i. Therefore, in this case firm $-i$ will leave second, as this increases value at the myopic exit price-trigger. Note that since $P^{ind,i}_{q_i,q_{-i}} \geq \Delta^{-1}_{q_i+q_{-i},q_{-i}}P^{exit,-i}_{q_{-i},0}$, the interval can never be empty for both firms.
(c) This has been already mostly discussed in the main text. It remains to be mentioned that if firm $-i$ invests predatorily, this decreases value below the value obtained by behaving myopically, since the competitor will only invest predatorily (if not preempting) if she expects to leave second after investment.
(d) See main text. ■

7.3 Construction of the collusive value

Once the price has reached the price-trigger at which the follower invests, but the leader has not invested yet, both firms can only invest simultaneously. For simultaneous investment, we need to distinguish two cases. In the first case, simultaneous investment decreases revenues of both firms, and in the other case investment increases revenues for at least one firm.

In the former case, both firms will rather abstain from investing at all, than to invest simultaneously. In the latter case, we need to determine the investment price-trigger at which each of the firms prefers to invest simultaneously. If simultaneous investment only increases revenues for one firm, the other firm will simply set the "collusive investment price-trigger" to infinity and $a_{i1} = 0$.

To determine the preferred collusive investment price-trigger for firm i, $P^{c\_inv,i}_{\underline{q}_i,\underline{q}_{-i}}$, we have to solve the smooth pasting and value matching conditions for investment for the this price-trigger.

$$\begin{aligned} V_i^C\left(P^{c\_inv,i}_{\underline{q}_i,\underline{q}_{-i}}, \underline{q}_i, \underline{q}_{-i}\right) &= V_i\left(\Delta_{\underline{q}_1+\underline{q}_2,\bar{q}_1+\bar{q}_2}P^{c\_inv,i}_{\underline{q}_i,\underline{q}_{-i}}, \bar{q}_i, \bar{q}_{-i}\right) - C_i \\ \left.\frac{\partial V_i^C\left(P, \underline{q}_i, \underline{q}_{-i}\right)}{\partial P}\right|_{P=P^{c\_inv,i}_{\underline{q}_i,\underline{q}_{-i}}} &= \left.\frac{\partial V_i\left(\Delta_{\underline{q}_1+\underline{q}_2,\bar{q}_1+\bar{q}_2}\cdot P, \bar{q}_i, \bar{q}_{-i}\right)}{\partial P}\right|_{P=P^{c\_inv,i}_{\underline{q}_i,\underline{q}_{-i}}} \end{aligned}$$

In this calculation the value matching conditions for exit have to be taken into account. If firm i expects to leave first this is $V_i(P^{exit,i}_{\underline{q}_i,\underline{q}_{-i}}, \underline{q}_i, \underline{q}_{-i}) = 0$, and the value matching condition becomes the following, if firm i expects to leave second:

$$V_i(P^{exit,-i}_{\underline{q}_{-i},\underline{q}_i}, \underline{q}_i, \underline{q}_{-i}) = V_i(\Delta_{\underline{q}_i+\underline{q}_{-i},\bar{q}_i}P^{exit,-i}_{\underline{q}_{-i},\underline{q}_i}, \underline{q}_i, 0). \quad (56)$$

Now any price-trigger is a candidate for collusive investment that falls between the largest price-trigger for the follower to invest, normalized to $\left(\underline{q}_1, \underline{q}_2\right)$-prices, and the lowest of the preferred collusive price-triggers. The upper bound of this interval is the pareto-optimal equilibrium and will be re-negotiated as collusive investment price-trigger. Formally, we determine the equilibrium collusive investment price-trigger $P^{c\_inv,*}_{\underline{q}_i,\underline{q}_{-i}}$ as $\min_{i=1,2}\left(P^{c\_inv,i}_{\underline{q}_i,\underline{q}_{-i}}\right)$. The constants a_{i1} and a_{i2} are then solved from the value matching conditions for investment

$$V_i^C\left(P^{c\_inv,*}_{\underline{q}_i,\underline{q}_{-i}},\underline{q}_i,\underline{q}_{-i}\right)=V_i\left(\Delta_{\underline{q}_1+\underline{q}_2,\bar{q}_1+\bar{q}_2}P^{c\_inv,*}_{\underline{q}_i,\underline{q}_{-i}},\bar{q}_i,\bar{q}_{-i}\right)-C_i \tag{57}$$

and exit.

References

ATHEY, SUSAN and ARMIN SCHMUTZLER (2001): "Investment and market dominance", *RAND Journal of Economics*, **32 (1)**, 1-26.

BUSSE, MEGHAN (2002): "Firm financial condition and airline price wars", *RAND Journal of Economics*, **33 (2)**, 298-318.

BOLTON, PATRICK and DAVID S. SCHARFSTEIN (1990): "A Theory of Predation Based on Agency Problems in Financial Contracting", *American Economic Review,* **80,** 93-106.

BRANDER, J.A. and T.R. LEWIS (1986): "Oligopoly and the financial Structure: The limited Liability Effect", *American Economic Review*, **76**, 956-70.

CABRAL, LUIS and MICHAEL RIORDAN (1994): "The Learning Curve, Market Dominance, And Predatory Pricing", *Econometrica*, **62(5)**, 1115-1140.

CABRAL, LUIS and MICHAEL RIORDAN (1997): "The Learning Curve, Predation, Antitrust, and Welfare", *Journal of Industrial Economics*, **XLV**, 155-169.

DIXIT, AVINASH K. and ROBERT S. PINDYCK (1994): "Investment under Uncertainty", Princeton University Press, Princeton, New Jersey.

FERSHTMAN, CHAIM and ARIEL PAKES (2000): "A dynamic oligopoly with collusion and price wars", *RAND Journal of Economics*, **31 (2)**, 207-236.

FUDENBERG, DREW and JEAN TIROLE (1985): "Preemption and Rent Equalization in the Adoption of New Technology", *Review of Economic Studies,* **LII,** 383-401.

GRENADIER, STEVEN R. (1996): "The Strategic Exercise of Options: Development Cascades and Overbuilding in Real Estate Markets", *Journal of Finance*, **LI**, 1653-1679.

HUISMAN, KUNO J.M. and PETER M. KORT (1999): "Effects of Strategic Interactions on the Option Value of Waiting", CentER Discussion Paper No. 9992, Tilburg.

JOAQUIN, DOMINGO CASTELO and NAVEEN KHANNA (2001): "Investment timing under threat of potential competition: Why firm size matters", *Quarterly Review of Economics and Finance,* **41,** 1-17.

JOU, JYH-BANG (2001): "Entry financing, and bankruptcy decisions: The limited liability effect", *Quarterly Review of Economics and Finance*, **41**, 69-88.

KREPS, DAVID M. and ROBERT WILSON (1982): "Reputation and Imperfect Information", *Journal of Economic Theory,* **27**, 253-79.

LAMBRECHT, BART (2001): "The Impact of Debt Financing on Entry and Exit in a Duopoly", *Review of Financial Studies*, **14**, 765-804.

MILGROM, PAUL, and JOHN ROBERTS (1982): "Predation, Reputation, and Entry Deterrence", *Journal of Economic Theory*, **27,** 280-312.

MURTO, PAULI (2004): "Exit in Duopoly under Uncertainty", *RAND Journal of Economics,* **17,** 111-27.

ORDOVER, JANUSZ. A. and GARTH SALONER (1989): "Predation, Monopolization, and Antitrust" in: *The Handbook of Industrial Organization*, R. Schmalensee and R. Willig (eds.)., North-Holland, 537-596.

SELTEN, REINHARD (1978): "The chain store paradox", *Theory and Decision,* **9,** 127-59.

SPARLA, THOMAS (2001): "Closure Options in Duopoly", *Ph.D. Thesis University of Dortmund,* Dortmund.

WEEDS, HELEN (2003): "Strategic Delay in a Real Options Model of R&D Competition", *Review of Economic Studies,* **69**, 729-47