Prediction Ability and Investment Under Uncertainty (This is an updated version of Discussion Paper Series 518, University of Essex, November, 2000)

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Abstract

This paper provides a framework for analyzing one of the most important intangible assets in a firm: the ability to predict profitable investment opportunities. This paper shows theoretically how to measure the accuracy of information used to predict opportunities, and estimates the value of information in the context of a firm's investment decision problem. Empirical study

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confirms the theoretical results of the model: (1) prediction ability has a large positive impact on firm's expected profits; and (2) prediction ability increases the mean and the variance of the growth rate of a firm's capital stock.

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1 Introduction

The age of talent has arrived. Many companies, like Microsoft and Yahoo, have raised their market value without having many physical assets. Instead, human capital has become highly valued in the labor market. For example, according to Murphy (1998), between 1992 and 1996, the median pay level for CEOs in financial services in the United States increased 53% to $4.6 million. The popular press stresses the importance of intangible assets in a firm, but what is the real effect of intangible assets? In particular, suppose that a firm can predict where profitable investment opportunities exist. How much does this ability raise its market value? This paper provides a theoretical framework addressing this question.

In this paper, prediction ability is defined as the ability to observe informative signals in order to predict the unknown profitability of investment opportunities. Hence, the notion of prediction ability is naturally linked to that of informativeness, analyzed by Blackwell (1953). The previous empirical studies on the value of information assume that agents will make inferences based on publicly observable variables [O'Brien (1981) and Antonovitz and Roe (1986)]. Hence, the results of previous research did not capture the economic value of unobserved local information, the importance of which has been emphasized by Hayek (1945) and Kirzner (1973). This paper attempts to estimate the value of information allowing for the fact that agents can process unobserved local information as well as publicly observable information in the context of a firm's investment decision problem.

How do I estimate prediction ability?: This paper shows that a firm's prediction ability can be estimated from the squared correlation coefficient between the profitability of investment and investment when a firm is endowed with a quadratic adjustment cost function of investment.

It is shown that the correlation can distinguish the impact of information from that of adjustment costs. Intuitively, large adjustment costs lower not only the covariance of profitability and investment, but also the variance of investment. With a quadratic adjustment cost function, the two effects are canceled out. Hence, the correlation succeeds in separating the impact of information from that of adjustment costs.

I apply this intuition to Tobin's Q type dynamic investment model, which is established by Lucas and Prescott (1971) and Hayashi (1982). Since Tobin's Q is known to reflect the future profitability of capital, the correlation between a firm's future Q and its current growth rate is expected to contain information about the prediction ability of the firm. This is the feature that allows me to estimate prediction ability at the firm level.

One technical difficulty is that Tobin's Q is an endogenous variable. In order to take into account the endogeneity of Tobin's Q, I modify the original measure. Then, it is shown that the squared correlation coefficient times the variance ratio of Tobin's Q and the random shock, which is defined below, can capture prediction ability. The intuition behind the variance ratio term is that if a firm has more accurate information, it can more aggressively change its investment decisions. This behavior increases the variance of Tobin's Q.

What is the impact of prediction ability?: Using the measure of prediction ability, the COMPUSTAT data set confirms the theory: expected Tobin's Q is an increasing function of the prediction ability of the firm. The quantitative impact of prediction ability is not small. Doubling prediction ability raises expected Tobin's Q by 18 %, which is expected to be worth 74 (1045) million dollars for median (average) firms in 1999.

In addition, the model predicts that the variance of the growth rate of capital stock is higher when a firm's prediction ability is large. Since the firm has better prediction, it can change its investment decisions based on its accurate information. Moreover, it is also shown that if an increase in capital stock reduces the firm's adjustment cost of investment, which is implicitly assumed in Tobin's Q type investment model, then the firm's prediction ability has a positive impact on the expected growth rate of capital stock. The COMPUSTAT data set supports both results: prediction ability increases the mean and the variance of the growth rate of capital. This evidence supports the relevance of the model.

The organization of the paper: The next section explains intuitions using a static investment model. Section 3 sets up an infinite horizon investment model. Section 4 analyzes the model under independently and identically distributed (i.i.d.) random shocks. Section 5 extends the results under an i.i.d. sequence to a Markov process with stationary transitions. Section 6 offers some empirical evidence. The last section concludes the main results and discusses the directions for future research.

2 The Static Model[1]

This section develops a static investment model, which assists in understanding the intuitions of a dynamic model. Suppose that a representative firm solves:

$$\pi\left(E\left(z|s\right)\right)=\max_{I}\left[\int zdG\left(z|s\right)I-\frac{AI^{2}}{2k}\right], \tag{1}$$

where A is an adjustment cost parameter, I is the amount of investment, k is capital stock, z measures the profitability of investment, s is a signal and $G\left(z|s\right)$ is the conditional distribution of z given s. Managers observe the signal s, infer z and decide how much they invest.

In order to analyze the impact of the accuracy of the signal on the expected profits, I define the following measure of prediction ability.

Definition 1 *The basic measure of prediction ability, h, is defined by:*

$$h\equiv 1-\frac{\int Var\left(z|s\right)dG_{s}\left(s\right)}{\sigma_{z}^{2}}, \tag{2}$$

where $\sigma_z^2=\int\left(z-\int zdG_z\left(z\right)\right)^2 G_z\left(z\right)$, $Var\left(z|s\right)=\int\left\{z-\int zdG\left(z|s\right)\right\}^2 dG\left(z|s\right)$, and $G_z\left(z\right)$ and $G_s\left(s\right)$ are the marginal distributions of z and s, respectively.

If managers can lower the variance of z on average after observing the signal, their prediction ability raises h. If managers can perfectly predict z, then $h=1$; if the signal is useless for prediction, then $h=0$.

The following theorem is easily established. As the proof is similar to the proof of theorem 5, I do not repeat it here.

[1]Although their emphases are different from this paper, Nelson (1961) and Takii (2003) also establish some of the results in this section. To minimize repetition, the explanation of the static model is brief, but readers can find more intuitive discussions in Takii (2003).

Theorem 1 *Expected profits, the variance of investment and the marginal profitability of the capital stock are an increasing function of h:*

$$\Pi \equiv \int \pi\left(E\left(z|s\right)\right) dG_s\left(s\right) = \left[\frac{\left(z^e\right)^2 + \sigma_z^2 h}{2A}\right] k, \tag{3}$$

$$\frac{d\Pi}{dk} = \left[\frac{\left(z^e\right)^2 + \sigma_z^2 h}{2A}\right],$$

where $z^e = \int zG_z\left(z\right)$, *and*

$$\sigma_I^2 = \frac{\sigma_z^2 k^2 h}{A^2} \tag{4}$$

where $\sigma_I^2 = \int \left[I\left(s\right) - I^e\right]^2 dG_s\left(s\right)$ *and* $I^e = \int I\left(s\right) dG_s\left(s\right)$. *It can be estimated by*

$$h = \left(\rho_{zI}\right)^2, \ \rho_{zI} \geq 0 \tag{5}$$

where $\rho_{zI} = \frac{\int [z-z^e][I(s)-I^e]dG(z,\, s)}{\sqrt{\int [z-z^e]^2 dG_z(z) \int [I(s)-I^e]^2 dG_s(s)}}$.

Four remarks are worthy of attention. First, an information structure increases the expected profit if and only if it raises h. Hence, if an information structure is more informative than others in the sense of Blackwell (1953), it raises prediction ability, h, though the opposite direction is not true. Second, prediction ability increases the variance of investment. The managers who can accurately predict opportunities have an incentive to change their investment decisions based on their information. This behavior increases the variance of investment. Third, h increases the marginal profitability of the capital stock. Note that adjustment costs are decreasing in the capital stock. The small adjustment cost means that managers have a large amount of discretion. The benefit from large amounts of discretion would be larger when the manager's decisions are correct. Hence, the marginal productivity of the capital stock increases in h. This is why prediction ability can increase the expected growth rate in the dynamic model below.

Finally, the measure h can be estimated by the squared correlation between the profitability of investment z and investment I. Managers endowed with prediction ability increase investment when profitability is large, and reduce it when profitability is small. This behavior increases the correlation between profitability and investment. Note that equation (5) does not depend on A. Hence, this measure can distinguish the impact of information from that of adjustment costs. Intuitively, large adjustment costs lower not only the covariance of profitability and investment, but also the variance of investment. With a quadratic adjustment cost function, the two effects are canceled out[2].

3 The Dynamic Model

I extend the static model to a dynamic investment model in order to construct an empirically operational model. All intuitions in the static model will be maintained in the dynamic model. Moreover, it is shown that prediction ability can increase the expected growth rate.

Environment: Assume that a production function is linear in capital stock:

$$y_t = z_t k_t,$$

where y_t is output, $z_t \in \left[\underline{z},\ \bar{z}\right]$ is a random shock and k_t is the capital stock at date

[2]More generally, if the cost function is not a quadratic, it is shown that the variance of the marginal cost of investment is high when h is large and h can be estimated by the squared correlation between z and the marginal cost of investment. With a quadratic adjustment cost function, the marginal cost is proportional to I. Hence, to the extent that the marginal cost is increasing in I, the result in this paper can be considered the first order approximation.

t. The random shock, z_t, can be thought of as the profitability of the capital stock at date t.

Assume that a manager observes a signal, s_t and a realized shock, z_t, at date t, and infers the stream of future profitability, $\{z_s\}_{s=t+1}^{\infty}$. I define a vector $u_t = (z_t, s_t)$. Assume that $\{u_t\}$ follows a Markov process with a stationary transition function $F(u_{t+1}|u_t)$, and let $F_u(u)$ denote the marginal distribution of u. In reality, the economy might be non-stationary. However, as explained by Stokey and Lucas (1989), a Markov process with stationary transitions captures a wide range of important economic issues. In this paper, I follow Stokey and Lucas's strategy.

Assume that adjustment costs, x_t, take the following form:

$$x_t = \frac{AI_t^2}{2k_t}, \tag{6}$$

where I_t is investment at date t, and A is the adjustment cost parameter. Adjustment costs are evaluated by the investment price, which is assumed to be 1 over time to simplify the analysis. This adjustment cost function has two properties that are common in empirical studies of investment: it is convex in I_t and it exhibits constant to return to scale in k_t and I_t.

The Firm's Problem: The firm's profit maximization problem is:

$$\begin{aligned} V(u_0,\ k_0) = \max_{\{k_{t+1}\}_{t=0}^{\infty}} E\left[\textstyle\sum_{t=0}^{\infty}\left(\frac{1}{1+r}\right)^t [z_t k_t - I_t - x_t]\,|u_0\right] \\ s.t.\ x_t = \tfrac{AI_t^2}{2k_t} \\ I_t = k_{t+1} - k_t, \end{aligned} \tag{7}$$

where r is a constant interest rate. For simplicity, I assume that the depreciation rate is 0. This assumption does not affect the results at all.

Since the production function and adjustment cost function exhibit constant returns to scale in k_t and I_t, I can divide both sides of (7) by k_0:

$$\frac{V(u_0,\ k_0)}{k_0} = \max_{\{g_t\}_{t=0}^{\infty}} \left\{ E\left[\sum_{t=0}^{\infty} \left(\Pi_{s=0}^{t}\beta_s\right)\left[z_t - g_t - \frac{A}{2}g_t^2\right]|u_0\right]\right\}, \tag{8}$$

where $\beta_s = \frac{1+g_{s-1}}{1+r}\ \ if\ s \geq 1,\ \beta_0 = 1$, and $g_t = \frac{k_{t+1}-k_t}{k_t}$.

I can define the Bellman equation, which will be shown to be equivalent to (8):

$$Q(u) = \max_{g}\left[z - g - \frac{A}{2}g^2 + \beta\int Q(u')\,dF(u'|u)\right], \tag{9}$$

where $z \in \left[\underline{z},\ \bar{z}\right]$, and $\beta = \frac{1+g}{1+r}$. I will show that $Q(u)$ is equivalent to Tobin's Q.

The Maximization Conditions: Now, I present two basic equations which characterize the investment decision. In order to do so, I need two technical conditions:

$$g_t \in [0,\ \alpha],\ \alpha < r,\ and \tag{10}$$

$$\underline{z} > r,\ \bar{z} < \frac{Ar^2}{2} + r. \tag{11}$$

The upper bound of equation (10) prevents the optimal solution from exploding. The lower bound of equation (10) does not need to be 0, but for simplicity this is assumed. Equation (11) guarantees that the solution is interior.

The following well known theorem simply restates the results of Lucas and Prescott (1971) and Hayashi (1982) in this special formulation. Since the result is known, I omit a formal proof, which can be found in Takii (2000).

Theorem 2 [3]*Suppose that equation (10) is satisfied. Then equation (9) has a unique*

[3] It is implicitly assumed that the space of u, U, is a compact Borel set, that $F(u'|u)$ has the Feller property and that $Q(\cdot) \in C$ where C is the space of bounded measurable continuous functions with the sup norm.

solution $Q(\cdot)$ *and:*

$$Q(u) = \frac{V(u,\ k)}{k} \ \ for\ any\ \ (u,\ k)\,. \tag{12}$$

Moreover, suppose that assumption (11) is also satisfied. Then $Q(\cdot)$ *and the associated unique policy function* $g(\cdot)$ *satisfy:*

$$g(u) = \frac{1}{A}\left[\frac{1}{1+r}\int Q(u')\,dF(u'|u) - 1\right], \tag{13}$$

$$Q(u) = \left[z + 1 + Ag(u) + \frac{A}{2}g^2(u)\right]. \tag{14}$$

This $Q(u)$ is nothing more than Tobin's average Q.[4] Equation (13) is the first order condition, which says that investment depends on managers' expectations about Tobin's Q. Equation (14) is derived from the Bellman equation, which shows how Q is determined.

4 The Case When $(z_{t+1},\ s_t)$ is I.I.D.

Let me assume that a sequence $\{(z_{t+1},\ s_t)\}$ is i.i.d.. Then the transition function is rewritten as $F(u_{t+1}|u_t) = G(z_{t+1}|s_t)\,G_s(s_{t+1})$, where $G(z|s)$ is the conditional distribution of z given s and $G_s(s)$ is the marginal distribution of s. In this case, given s_t and z_t, the managers can only predict z_{t+1}. Hence, the basic measure of prediction ability in the static model can be directly applied to this i.i.d. case. As the proof is similar to the proof of theorem 5, I do not repeat it here.

[4] Tobin's Q is usually defined as $Q^{\#}(u) = \frac{1}{1+r}\int Q(u')\,dF(u'|u)$. That is, $Q^{\#}(u)$ is evaluated before the random shock is realized; $Q(u)$ is evaluated after the shock is realized. This definition allows me to explicitly examine the impact of expectation. Readers can also refer to Ueda and Yoshikawa (1986), who show that investment is positively related to the expectation of future Tobin's Q rather than current Tobin's Q when there are time-to-build or delivery lags.

Theorem 3 *Suppose* $\{(z_{t+1}, s_t)\}$ *is an i.i.d. sequence. The present value of expected profits, the variance of the growth rate, and the expected growth rate are increasing in* h :

$$\int\int V(z,\ s,\ k_0)\, dG_z(z)\, G_s(s) = Q^e k_0, \tag{15}$$

$$Q^e = (1+r)\,[Ar+1] - \sqrt{(1+r)^2 A\left[Ar^2 + 2(r - z^e)\right] - \sigma_z^2 h},$$

$$\sigma_g^2 = \frac{\sigma_z^2 h}{A^2 (1+r)^2},$$

$$g^e = r - \sqrt{r^2 + 2\frac{r - z^e}{A} - \frac{\sigma_z^2 h}{A^2 (1+r)^2}}, \tag{16}$$

where $Q^e = \int\int Q(z,\ s)\, dG_z(z)\, G_s(s)$, *and* $g^e = \int g(s_t)\, dG_s(s)$. *Moreover, prediction ability can be estimated by:*

$$h = (\rho_{zg})^2\ ,\ \rho_{zg} \geq 0 \tag{17}$$

where $\rho_{zg} = \frac{\int [z_{t+1} - z^e][g(s_t) - g^e]\, dG(z_{t+1},\ s_t)}{\sqrt{\int (z - z^e)^2 dG_z(z) \int [g(s_t) - g^e]^2 dG_s(s_t)}}$.

Theorem 3 shows that every result in the static model is maintained in the dynamic model: an information structure increases the present value of the expected profit if and only if it raises h, h increases the variance of investment, and h can be estimated by the squared correlation between z and g.

The main difference is equation (16): prediction ability has a positive impact on the expected growth rate. Current investment not only increases future profits but also reduces future adjustment costs. Since flexibility increases the value of information, a superior manager has more incentive to invest today in order to establish a flexible position in the future.

5 The Case When u_t is a Markov Process

The generalized measure of prediction ability: If a random process $\{u_t\}$ follows a Markov process with stationary transitions, then predicting next period profitability is not enough to make investment decisions. Managers must predict the whole path of future profitability. Since Theorem 2 suggests that the entire path of future profitability is captured by only one variable, Tobin's Q, it is natural to define a measure of the ability to predict Tobin's Q in a fashion similar to the way I define h.

One difficulty arises from the fact that Tobin's Q is an endogenous variable. A signal not only helps predicting Tobin's Q, but also affects its movement. In order to take care of this problem, I need one more definition.

Definition 2 *The benchmark Q, $Q^*(z)$, is defined by the $Q(u)$ that solves equation (9) along with technical conditions (10) and (11) without observing any signal. That is, $Q^*(.)$ is the unique function that satisfies:*

$$g^* = \frac{1}{A}\left[\beta \int Q^*(z)\, dF_z(z) - 1\right], \text{ and} \tag{18}$$

$$Q^*(z) = z + 1 + Ag^* + \frac{A}{2}(g^*)^2. \tag{19}$$

Using this benchmark Q, I can define the generalized measure of prediction ability:

Definition 3 *The generalized measure of a manager's ability to predict Q, h_Q, is defined by:*

$$h_Q = \frac{\sigma_Q^2}{\sigma_{Q^*}^2} - \frac{\int Var(Q(u')\,|u)\, dF_u(u)}{\sigma_{Q^*}^2}, \tag{20}$$

$$\begin{aligned} \textit{where } \sigma_Q^2 &= \int \left[Q(u) - \int Q(u)\, dF_u(u) \right]^2 dF_u(u), \\ \sigma_{Q^*}^2 &= \int \left[Q^*(z) - \int Q^*(z)\, dF_z(z) \right]^2 dF_z(z) \ \textit{and} \\ Var\left(Q(u')\,|u\right) &= \int \left[Q(u') - \int Q(u')\, dF(u'|u) \right]^2 dF(u'|u). \end{aligned}$$

The crucial difference between this and the previous measure is that here I use the unconditional variance of the benchmark Q instead of the observable Q as an adjustment factor. The reason for this is that Tobin's Q is an endogenous variable and the unconditional variance of the observable Q already reflects the effect of the signal. In order to separate the effect of the signal from the adjustment factor, I use the benchmark Q. As a result, the maximum value of $\left[\int Var\left(Q(u')\,|u\right) dF_u(u)\right]/\sigma_{Q^*}^2$ is not 1, but $\sigma_Q^2/\sigma_{Q^*}^2$. For this reason, I subtract $\left[\int Var\left(Q(u')\,|u\right) dF_u(u)\right]/\sigma_{Q^*}^2$ from $\sigma_Q^2/\sigma_{Q^*}^2$.

The identity of h_Q and h: I want to show that h_Q is a natural extension of h. The proof is established in Appendix 1.

Theorem 4 *If the random sequence $\{(z_{t+1},\ s_t)\}$ is i.i.d., then $h_Q = h$.*

The theorem says that if the random sequence is i.i.d., the value of the two measures coincides. Hence, it is fair to say that the generalized measure is a natural extension of the basic measure.[5]

[5] I must emphasize that this is just one possible practical treatment. In fact, I cannot claim that the generalized measure is adjustment cost parameter free. Since Tobin's Q is an endogenous variable, a change in the adjustment cost parameter varies the value of the generalized measure. Hence, I must assume that every firm has the same adjustment cost function.

The effects of prediction ability: The following theorem summarizes the main results of this paper. The proof is established in Appendix 1.

Theorem 5 *Suppose that a random shock and a signal follow a Markov process with stationary transitions. The present value of expected profits, the variance of the growth rate and the expected growth rate are increasing in h:*

$$\int V(u,\ k_0)\, dF_u(u) = Q^e k_0$$

$$Q^e = (1+r)[Ar+1] - \sqrt{(1+r)^2 A[Ar^2 + 2(r-z^e)] - \sigma_z^2 h_Q},$$

$$\sigma_g^2 = \frac{\sigma_z^2 h_Q}{A^2(1+r)^2},$$

$$g^e = r - \sqrt{r^2 + 2\frac{r-z^e}{A} - \frac{\sigma_z^2 h_Q}{A^2(1+r)^2}},$$

Moreover h_Q *is estimated by:*

$$h_Q = a(\rho_{Qg})^2,\ a = \frac{\sigma_Q^2}{\sigma_z^2},\ \rho_{Qg} \geq 0, \tag{21}$$

where $\rho_{Qg} = \frac{\int [Q(u_{t+1})-Q^e][g(u_t)-g^e]dF(u_{t+1},\ u_t)}{\sqrt{\sigma_Q^2\sigma_g^2}}$.

The results are the same as those in the i.i.d. case, except that h_Q is estimated by the squared correlation coefficient between future Tobin's Q and the current growth rate times the variance ratio of Tobin's Q and the random shock, as opposed to the

I have three comments on this problem. First, empirical studies in the investment literature usually assume that every firm has the same adjustment cost parameter (Summers 1981, Salinger and Summers 1983, Fazzari, Hubbard and Petersen 1988 and Cummins, Hassett and Hubbard 1994). Second, if I assume a quadratic adjustment cost function, $x_t = \frac{A}{2}I_t^2$, then I can prove that the marginal Q does not depend on the parameter A. In this case, the generalized measure assesses the value of information for any adjustment cost parameter A. Third, as Theorem 5 shows, it is still true that more accurate information in the Blackwell sense has a larger value of h.

simple correlation coefficient. The variance ratio reflects the fact that Tobin's Q is an endogenous variable. Since good managers can aggressively change their investment decisions based on their own signals, they will also change the value of Tobin's Q. This ratio reflects this effect.

6 Empirical Evidence

I would like to investigate the impact of prediction ability on a firm's expected profit. The main question is how much such ability is worth to the firm. My theory also predicts that a firm that can predict the profitability of investment frequently changes its investment decisions and increases the firm's average investment. I would like to test this hypothesis as well[6].

I can derive the following empirical equations by taking a Taylor approximation in $\log z^e$, $\log \sigma_z^2$ and $\log h_Q$ for the logarithm of equations in Theorem 5:

[6]For this empirical study, it is implicitly assumed that the value of h_Q is known. Hence, everybody knows the true distribution functions of z and s. In addition, it is assumed that bankruptcy does not occur for any realization of z_t and that managers act in the interest of all shareholders, which means we can avoid the issues of adverse selection raised by Stiglitz and Weiss (1981) and Myers and Majluf (1984). The presence of adverse selection increases financial costs, as shown by Fazzari, Hubbard and Petersen (1988). Potential distortions due to a liquidity constraint are discussed below. Finally, the random shocks include firm-specific shocks and there is a cost involved in observing other managers' investment decisions. These assumptions prevent managers from mimicking or learning from other managers' behavior.

$$\log Q^e = a_1 + a_2 \log z^e + a_3 \log \sigma_z^2 + a_4 \log h_Q + \varepsilon,$$

$$\log g^e = b_1 + b_2 \log z^e + b_3 \log \sigma_z^2 + b_4 \log h_Q + u, \tag{22}$$

$$\log \sigma_g^2 = c_1 + c_2 \log \sigma_z^2 + c_3 \log h_Q + \upsilon, \tag{23}$$

where a_i, b_i and c_i are parameters, and ε, u and υ are error terms. The error terms are assumed to be normally distributed. The theory predicts that all parameters should be positive. I am especially interested in the parameters a_4, $b_4$and c_3, which show the impact of prediction ability.

The data consist of a 40-year (1960-1999) unbalanced panel of firms from the COMPUSTAT data base. We do not utilize the data from the 1950s because they do not contain many samples. The choice of periods does not change the main results. Appendix 2 shows how I construct variables Q, g and z. Using these three variables I can estimate Q^e, g^e, z^e, σ_z^2 and h_Q by calculating sample means, variances and correlation coefficients over time for each firm and each decade (1960-1969, 1970-1979, 1980-1989 and 1990-1999). Decadal averages are chosen to balance two requirements: the need to construct reliable estimates and the need to increase the number of observations in the regression. This provides panel data for my regression analysis.

Econometric issues

Simultaneous Equation Bias and Measurement Error: Since my measure of prediction ability is constructed using two endogenous variables - Tobin's Q and the growth rate of capital stock - the ordinary least square (OLS) may be subjected to the simultaneous equation bias. In order to deal with the bias, I need instruments. I use my estimates of each variable constructed by data from the previous decade as the instruments of the corresponding variables ($\log z^e$, $\log \sigma_z^2$ and $\log h_Q$).

The instruments are necessary for a further reason. Since the sample mean is constructed by at most 10 observations, they may not be accurate proxies of the expected values. Note that although the correlation coefficient is a non-linear function of the covariance and the standard error, the logarithm of estimates can separate error terms from regressors. For example, equation (22) implies:

$$\log Q^e = a_1 + a_2 \log z^e + a_3 \log \sigma_z^2 + a_4 \left[2 \log Cov\left(Q, g\right) - \log \sigma_g^2 - \log \sigma_z^2\right] + \varepsilon$$

Note that the sample mean of z, the sample variances of z and g, and the sample covariance of Q and g are the unbiased estimators of z^e, σ_z^2, σ_g^2 and $Cov\left(Q, g\right)$. Hence I can assume that:

$$\hat{x} = xu$$

where x is the true value of z^e, σ_z^2, σ_g^2 or $Cov\left(Q, g\right)$, $\hat{x}$ is the unbiased estimate of x and u is a measurement error, which is independent of x and $E\left(u\right) = 1$. Given this assumption, the regression equation can be rewritten as:

$$\begin{aligned} \log Q^e &= a_1 + a_2 \log z^e + a_3 \log \sigma_z^2 + a_4 \log h_Q + v, \\ v &= \varepsilon + a_2 \log u_z + a_3 \log u_{\sigma z} + a_4 \left[2 \log u_{cov} - \log u_g - \log u_{\sigma z}\right], \end{aligned}$$

where u_z, $u_{\sigma z}$, u_{cov}, and u_g are measurement errors corresponding to the estimates of z^e, σ_z^2, $Cov\left(Q, g\right)$ and σ_g^2. Hence, I can separate error terms from regressors and apply a common instrumental variable technique for this estimation.

Unobserved Heterogeneity: In order to control unobserved heterogeneity, three-digit industry dummies and decade dummies are controlled in my regression. I also conduct a fixed effect regression, to observe a possible bias due to unobserved heterogeneity at a firm level, which three-digit industry dummies fail to capture. However, since the fixed effect regression uses the deviation from the mean as independent variables, it makes it impossible to use the previous value as instruments. Hence, I report the results of both regressions to collect the maximum information from data.

Because some firms grow and others decline over time, some researchers have questioned whether the measure of predictive ability may be affected by the heterogeneity of a firm's growth rate. An advantage of the measure is that a large trend does not change its value. If all variables grow at the same rate, not only is the covariance of Q and g large, but the variance of Q, g and z is also large. Hence, the measure would be the same. As a result, it is unlikely that the results below are significantly affected by the heterogeneity of the firm's growth rate.

Results

Summary statistics: Table 1 shows the summary statistics of Q^e, g^e,z^e, σ_z^2, h_Q and $\rho_{Qg} = \left[\int \left[Q\left(u_{t+1}\right) - Q^e\right]\left[g\left(u_t\right) - g^e\right] dF\left(u_{t+1}, u_t\right)\right] / \sqrt{\sigma_Q^2 \sigma_g^2}$ (the simple correlation between future Q and the current growth rate). My estimates of prediction ability h_Q have a mean of 18, a standard deviation of 421, and a median of 0.25. The large standard deviation indicates the large variation of prediction ability in my sample. Moreover, the large deviation between the mean and the median suggests that the distribution of prediction ability may be highly skewed.

However, this result is not confirmed by the simple correlation coefficient: the mean is 0.12, the standard deviation is 0.4 and the median is 0.15. This is expected since the simple correlation lies between -1 and 1. Hence, the large variation and skewness of the prediction ability measure is generated from the variance ratio term.

Note that the number of observations for h_Q is lower than that for ρ_{Qg}. Constructing h_Q requires a positive correlation coefficient between the current growth rate and future Tobin's Q. But 38% of the firms in the sample do not satisfy this

Table 1: Summary Statistics

	Q^e	g^e	σ_g^2	z^e	σ_z^2	h_Q	ρ_{Qg}
mean	1.624	0.279	0.832	1.486	3217	18.22	0.119
standard deviation	1.179	0.652	70.32	26.93	4×10^5	421.6	0.403
median	1.282	0.234	0.009	0.487	0.033	0.253	0.154
# of observations	17688	17688	17688	17688	17688	10941	17688

Q^e is expected Tobin's Q. g^e is the expected value of investment over capital stock. z^e is the expected random shock (measure of profitability). σ_z^2 is the variance of the random shock (measure of risk). h_Q is the measure of prediction ability. ρ_{Qg} is the simple correlation between future Q and the current growth rate.

condition[7]. To check the robustness of the results, I also investigate the regression using ρ_{Qg}. Although this measure does not have any direct connection with the theory, it has the benefit of using every observation. Moreover, the simple correlation is intuitively appealing. Hence, the simple correlation measure can also be used to check the relevance of my arguments below.

There may be a concern that the measure of prediction ability simply reflects the availability of internal funds. If a liquidity constraint is severe, a firm may not be able to invest even if its managers know of investment opportunities. Hence, the difference in available internal funds may affect the prediction measure. This point is briefly considered. Because z is estimated by operating income over the net capital stock, $\log z^e$ can be considered as a proxy of available internal funds[8]. There is a negative

[7]There are several possible reasons why this might happen. Average Q might be a noisy indicator of investment opportunities (Cummins, Hassett and Hubbard 1994). Another possible scenario is that many firms are still learning the potential profitability. To investigate these possibilities, an alternative model is required. This topic is left for future research.

[8]A similar measure is used for a proxy of available internal funds by Fazzari, Hubbard and Petersen (1988). Although they use cash flow over the capital stock as the proxy, the use of cash

Table 2: The Effect of Prediction Ability on $\log Q^e$:

The dependent variable is $\log Q^e$.

	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)
	OLS	OLS	$Fixed$	$Fixed$	$2SLS$	$2SLS$	OLS	OLS
ρ_{Qg}	0.017^{**}		0.021^{**}		0.100		0.018^{*}	
	(0.007)		(0.009)		(0.342)		(0.010)	
$\log(h_Q)$		0.068^{***}		0.046^{***}		0.182^{***}		0.050^{***}
		(0.001)		(0.002)		(0.033)		(0.002)
$Adj-R^2$	0.344	0.513	0.085	0.333	.	.	0.364	0.523
obs	17688	10941	17688	10941	7086	2664	7086	2664

h_Q is the measure of prediction ability. ρ_{Qg} is the simple correlation between future Q and the current growth rate. Every regression equation controls the constant term, $\log(z^e)$ and $\log(\sigma_z^2)$, and all but the fixed effect regressions control the decade dummies and the three-digit industry dummies. * indicates significance at the 10 % level. ** indicates significance at the 5 % level. *** indicates significance at the 0.5% level. The standard error is reported in parentheses.

correlation between $\log z^e$ and $\log h_Q$ (-0.36), and no correlation between $\log z^e$ and ρ_{Qg}. This evidence suggests that a liquidity constraint is less likely to be a major factor affecting the measure of prediction ability.

Prediction ability raises expected Tobin's Q: Table 2 reports the effect of prediction ability on expected Tobin's Q. I report only the coefficients of prediction ability, but all regression equations control the constant term, $\log(z^e)$ and $\log(\sigma_z^2)$, and all but the fixed effect regressions control decade dummies and three-digit industry dummies.

flow does not change the results.

Regression equations (a) and (b) report the results of simple regressions. Regression equation (a) uses a simple correlation and equation (b) uses the logarithm of h_Q as a regressor. Both equations show that prediction ability has a significant positive impact on expected Tobin's Q.

Equations (c) and (d) report fixed effect regressions. Both equations show positive and significant effects. The coefficient is similar to that of the simple OLS when the simple correlation is a regressor; it drops mildly when $\log(h_Q)$ is a regressor. The interpretation of the mild reduction is difficult. On the one hand, since productive firms are likely to have high prediction ability, the unobserved productivity which three-digit industry dummies fail to capture may cause the upper bias of the OLS result. On the other hand, since the fixed effect regression increases bias due to measurement errors, the coefficient of the fixed effect regression may be underestimated. Fortunately, the difference is not too large. Hence, the importance of the bias due to unobserved heterogeneity at a firm level would be relatively small.

The more significant problems would be the simultaneous equation bias or measurement errors. The coefficients of prediction ability by the two stage least square (2SLS) estimations [equations (e) and (f)] are much larger than those estimated by the OLS methods[9]. This indicates that the simple OLS is likely to underestimate the impact of prediction ability. Since the number of observations for the 2SLSs is much smaller than that of the OLS estimations, sample selection may be an alternative reason that the 2SLS estimations have large coefficients. I conduct the OLS estimations using the same sample as the 2SLSs to examine this possibility, and

[9]When I use a simple correlation as the measure of prediction ability, it is difficult to separate error terms from regressors, and there is no theoretical proof that the instrumental variable technique can recover true values. Hence, when the simple correlation is used, the results of two stage least square (the 2SLS) estimations are reported only for reference.

equations (g) and (h) report the results. The coefficients of equations (g) and (h) do not differ much from those of equations (a) and (b). Hence, measurement errors or the endogeneity of independent variables is likely to be the main problem.

Let me ask another question: how much is prediction ability worth? The coefficient of $\log(h_Q)$ has the natural interpretation of elasticity. The 2SLS result shows that doubling h_Q increases expected Tobin's Q by 18 %. Suppose that prediction ability does not affect the book value of capital. Then the percentage increase in a firm's Tobin's Q can be interpreted as the increase in the firm's market value. That is, doubling h_Q increases the expected market value by 18 %. Since median (average) firms were valued at 411 (5806) million dollars by the market in 1999, doubling prediction ability is expected to be worth 74 (1045) million dollars for median (average) firms in 1999.

In sum, my empirical study shows that prediction ability has a robust, positive and significant impact on expected Tobin's Q, and the impact is not small.

Prediction ability raises the mean and the variance of the growth rate: The theory also suggests that prediction ability increases the mean and the variance of the growth rate. I would like to test this prediction by data.

Table 3 discloses the impact of prediction ability on the mean and variance of a firm's growth rate. Again, I report only the coefficients of prediction ability, although every regression controls the same variables as before.

The first table reports the impact on the mean of a firm's growth rate. All results are positive, which is expected, and most of the results are significant. Prediction ability has a robust and positive impact on the expected growth rate. Again, the coefficients under the fixed effect regression are smaller than those of the OLS estimations. Once more, the potential explanations are mixed: unobserved productivity,

Table 3: The Impact of Prediction Ability on $\log \mathbf{g}^e$ and $\log \boldsymbol{\sigma}_g^2$:

The dependent variable is $\log \mathbf{g}^e$

	(*i*)	(*j*)	(*k*)	(*l*)	(*m*)	(*n*)	(*o*)	(*p*)
	OLS	*OLS*	*Fixed*	*Fixed*	*2SLS*	*2SLS*	*OLS*	*OLS*
ρ_{Qg}	0.074***		0.055***		0.843		0.035***	
	(0.009)		(0.011)		(0.552)		(0.012)	
$\log(h_Q)$		0.032***		0.018***		0.046*		0.012***
		(0.002)		(0.002)		(0.027)		(0.003)
$Adj - R^2$	0.365	0.389	0.204	0.237	.	.	0.427	0.434
obs	17688	10941	17688	10941	7086	2664	7086	2664

The dependent variable is $\log \boldsymbol{\sigma}_g^2$

	(*q*)	(*r*)	(*s*)	(*t*)	(*u*)	(*v*)	(*w*)	(*x*)
	OLS	*OLS*	*Fixed*	*Fixed*	*2SLS*	*2SLS*	*OLS*	*OLS*
ρ_{Qg}	0.349***		0.219***		3.450*		0.200***	
	(0.024)		(0.034)		(1.946)		(0.035)	
$\log(h_Q)$		0.080***		0.053***		0.138*		0.039***
		(0.004)		(0.008)		(0.077)		(0.008)
$Adj - R^2$	0.373	0.373	0.302	0.303	.	.	0.415	0.385
obs	17688	10941	17688	10941	7086	2664	7086	2664

h_Q is the measure of prediction ability. ρ_{Qg} is the simple correlation between future Q and the current growth rate. All regressions control the constant term and $\log(\sigma_z^2)$, and all but the fixed regressions control the decade dummies and the three-digit industry dummies. Additionally, $\log(z^e)$ are controlled for the regressions on the expected growth rate. * indicates significance at the 10 % level. ** indicates significance at the 5 % level. *** indicates significance at the 0.5% level. The standard error is reported in parentheses.

which three-digit industry dummies fail to capture, may be positively correlated with prediction ability, while the fixed effect regressions may increase the impact of measurement errors. The coefficients of the 2SLSs are also larger than those of the OLSs. Comparing equations (j) and (n), the coefficient increases from 0.032 to 0.046. It appears to be a mild increase. However, note that the OLS based on the same sample as the 2SLS has the coefficient of 0.012 [equation (p)]. Hence, it seems that the simultaneous equation bias or measurement errors fairly underestimate the coefficients of the simple OLSs.

Note that every regression controls $\log(z^e)$. While $\log(z^e)$ is considered as a proxy of investment opportunities in this paper, it can also be interpreted as a proxy of available internal funds. If the availability of internal funds is a major factor affecting the prediction measure, after controlling $\log z^e$ we should not expect the measure to have any significant impacts on the mean of a firm's growth rate. Evidence confirms the previously stated result, that the availability of internal funds is less likely to be the main component of h_Q and ρ_{Qg}.

I take 0.046 from the coefficient of the 2SLS [equation (n)] for a quantitative exercise. Since the median firm has a growth rate of 0.23, doubling prediction ability increases the expected growth rate of the median firm by 1%. This exercise shows the impact of prediction ability on the mean of the growth rate is small.

The second table reports the effect of prediction ability on the variance of the growth rate. All results show significant and positive coefficients. It confirms the hypothesis: prediction ability increases the variance of the growth rate. The coefficients under the fixed effect regressions are smaller than those of the OLSs. However, it is not clear why large unobserved productivity must increase the variance of the growth rate. Hence, the reduction in coefficients of the fixed effect regressions are more likely to be due to measurement errors: the fixed effect regression increases a bias due to measurement errors. In fact, the coefficients of the 2SLSs are much larger than those of the OLSs. Both sets of evidence indicate the potential importance of the measurement errors. The coefficient of the 2SLS [equation (v)] shows that doubling prediction ability increases the variance of the growth rate by 14%.

7 Conclusion and Extensions

This paper provides a framework for analyzing one of the most important intangible assets in a firm: the ability to predict profitable investment opportunities. My theory and empirical study show that prediction ability has a large positive impact on a firm's expected profits. The evidence also confirms the hypothesis that prediction ability increases the mean and the variance of the growth rate of a firm's capital stock.

Although I provide a method for estimating the prediction ability of a firm in this paper, obviously there are several unresolved issues. The evidence in this paper suggests that a liquidity constraint is unlikely to be a major factor affecting the measure of prediction ability. However, more detailed investigation based on a structural model will be important. The measurement errors of investment opportunities are another difficult problem. If the production function or the adjustment cost function do not exhibit constant returns to scale, average Q cannot be a suitable measure of investment opportunities.

Both of these problems may require a measure that can be applied to a more general adjustment cost function. Recently, Lehmann (1988), Persico (2000) and Athey and Levin (2001) extend the analysis of Blackwell (1953) to a specific class of utility functions. Their research suggests a new research direction for the value of information. Empirical investigation of their conditions would be a challenging subsequent research agenda.

An equally important future question is how a firm can increase prediction ability. Hiring talented managers or analysts, an increase in R&D and investing in information technology are possible answers. I hope that finding the source of prediction ability will increase the understanding of how a firm can increase its own intangible assets.

Appendix 1: Proofs of Theorems

Proof of theorem 4: In the i.i.d. environment, z_t does not include any information with which to predict z_{t+1}. Hence, the growth rate, g, depends only on the signal s_t. Hence, it is derived from (13) and (14) that:

$$\begin{aligned} E\left[Q\left(u'\right)|s\right] &= E\left(z'|s\right) + 1 + AE\left[g\left(s'\right)\right] + \frac{A}{2}E\left[g^2\left(s'\right)\right], \\ E\left[Q\left(u'\right)\right] &= E\left(z'\right) + 1 + AE\left[g\left(s'\right)\right] + \frac{A}{2}E\left[g^2\left(s'\right)\right]. \end{aligned}$$

Hence:

$$Var\left[E\left(Q\left(u'\right)|s\right)\right] = Var\left[E\left(z|s\right)\right].$$

The following lemma completes the proof.

Lemma 1 *The generalized measure can be rewritten as:*

$$h_Q = \frac{Var\left(E\left(Q\left(u_{t+1}|u_t\right)\right)\right)}{\sigma_z^2}. \tag{24}$$

Proof. Using the identity equation:

$$\sigma_Q^2 = E\left(Var\left(Q\left(u_{t+1}\right)|u_t\right)\right) + Var\left(E\left(Q\left(u_{t+1}\right)|u_t\right)\right), \tag{25}$$

I can rewrite equation (20) to be:

$$h_Q = \frac{Var\left(E\left(Q\left(u_{t+1}|u_t\right)\right)\right)}{\sigma_{Q^*}^2}.$$

To complete the proof of Lemma 1, I need to show that $\sigma_{Q^*}^2 = \sigma_z^2$. Taking expectations on both sides of (19):

$$\int Q^*(z)\, dF_z(z) = \int z dF_z(z) + 1 + Ag^* + \frac{A}{2}(g^*)^2.$$

Combining (19) and this equation, I can calculate the unconditional variance of Q^*. Then it is easy to see that $\sigma_{Q^*}^2 = \sigma_z^2$. ■■

Proof of Theorem 5: First, I show that prediction ability positively affects the variance of the growth rate. It is derived from equation (13) that:

$$A^2\sigma_g^2 = \frac{Var(E(Q(u'|u)))}{(1+r)^2}.$$

By equation (24),

$$\sigma_g^2 = \frac{\sigma_z^2 h_Q}{A^2(1+r)^2}. \tag{26}$$

Next, I prove the effect on the expected growth rate. It is derived from (13) and (14) that:

$$1 + Ag(u) = \frac{1}{1+r}\left\{ \begin{array}{c} E[z'|u] + 1 + AE[g(u')|u] \\ +\frac{A}{2}E[g^2(u')|u] \end{array} \right\}.$$

Taking expectations on both sides,

$$0 = g^{e2} - 2rg^e + 2(A)^{-1}[z^e - r] + \sigma_g^2.$$

Hence,

$$g^e = r \pm \sqrt{r^2 + 2\frac{r - z^e}{A} - \sigma_g^2}.$$

Since $g \leq \alpha - 1 < r$,

$$g^e = r - \sqrt{r^2 + 2\frac{r - z^e}{A} - \sigma_g^2}. \tag{27}$$

Third, I prove the effect on expected Tobin's Q. It is derived from (13), (26) and (27) that:

$$\begin{aligned}\int Q(u)\,dF_u(u) &= (1+r)\left(A\int g(u)\,dF_u(u)+1\right)\\ &= (1+r)[Ar+1]-\sqrt{(1+r)^2 A\left[Ar^2+2(r-z^e)\right]-\sigma_z^2 h_Q}.\end{aligned}$$

Moreover,

$$E[V^*(u_0,\ k_0)] = E[Q(u_0)k_0] = Q^e k_0.$$

Finally, I show that the prediction measure can be estimated by the correlation. By multiplying (13) by $g(u_t)$ and taking expectations on both sides,

$$g^e + A\int g^2(u_t)\,F_u(u_t) = \frac{1}{1+r}\int Q(u_{t+1})\,g(u_t)\,dF(u_{t+1},u_t). \tag{28}$$

By taking expectations on both sides of (13) and multiplying the result by g^e,

$$g^e + A(g^e)^2 = \frac{1}{1+r}\int Q(u)\,dF_u(u)\,g^e. \tag{29}$$

Then subtracting (29) from (28) gives:

$$Cov(g(u_t)Q(u_{t+1})) = (1+r)A\sigma_g^2.$$

Hence,

$$\rho_{gQ} = \frac{Cov(g(u_t)Q(u_{t+1}))}{\sqrt{\sigma_g^2\sigma_Q^2}} = \frac{(1+r)A\sigma_g^2}{\sqrt{\sigma_g^2\sigma_Q^2}} = \sqrt{\frac{\sigma_z^2 h_Q}{\sigma_Q^2}}.$$

The desired result is immediate. ■

Appendix 2: Data Constructions

Tobin's Q: $Q(u_{t+1})$...the market value of assets at the end of year t, divided by the book value of assets at the end of year t, $(\#6)_t$:

$$Q(u_{t+1}) = \frac{(\#6)_t + (\#25)_t \times (\#199)_t - (\#60)_t}{(\#6)_t}$$

where $(\#X)_t$ implies COMPUSTAT number X in year t, and the market value of assets equals the book value of assets, $(\#6)_t$, plus the market value of common equity, $(\#25)_t \times (\#199)_t$, less the book value of common equity, $(\#60)_t$. I assume that the value at the beginning of period $t+1$ can be estimated by the value at the end of the previous period. A similar definition of Tobin's Q is used by Kplan and Zingales (1997). I also try my exercise with the different definition of Tobin's Q by estimating the replacement cost of capital. Estimating replacement cost does not change my results. If readers wish to see the result from using replacement cost, refer to Takii (2000).

The Growth Rate: g...capital expenditures during period t, $(\#128)_t$, divided by net capital stock at the end of year t, $(\#8)_t$:

$$g_t = \frac{(\#128)_t}{(\#8)_t}$$

Random Shock: z...operating income during period t, $(\#13)_t$, divided by net capital stock at the end of year t, $(\#8)_t$:

$$z_t = \frac{(\#13)_t}{(\#8)_t}$$

In estimating g and z, it would be more consistent with the theory to use net capital stock at the beginning of year t. However, some of the results are more robust when I use net capital stock at the end of period t. Hence, I decided to define the variables as explained above.

Some details: First, for the maximum use of data, I delete only samples that have fewer than three observations in each period. Second, in order to take a log, I delete observations if the sample means of Q, g and z are non-positive.

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