

An Analysis of Advertising Wars*

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Abstract

Comparative advertising by one brand against another showcases its merits versus the demerits of the other. In a two-stage game among finitely many firms, firms decide first on how much to advertise against whom. In the second stage, given the advertising configuration, firms compete as Cournot oligopolists. In the symmetric case, equilibrium advertising expenses constitute a clear welfare loss. Equilibrium advertising levels and advertising expenditures decline with rising advertising costs. Whereas equilibrium levels of advertising decrease in the number of firms, aggregate advertising expenditures increase. We further relate effectiveness of advertising to proximity in product space. With two firms, comparative advertising and quality choice have similar effects. In a three-stage game, where firms choose first locations (variety), then advertising levels (quality), and then quantities, we obtain maximum horizontal product differentiation and minimum vertical product differentiation.

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1 Introduction

In the present paper we address a fairly common phenomenon in today's advertising world that has been largely ignored in the literature, namely advertising wars where one brand compares itself favorably with a competing brand in various kinds of media, especially television. The reader is reminded of the cola wars and similar episodes of casual empiricism. We are mainly interested in the economic aspects of advertising and advertising wars and less in the details of the craft which are the subject of marketing research. The economics literature on advertising has focused on two separate yet related issues: First, the competitive or anti-competitive effects of advertising. Second, the question whether advertising is too little or too much from the perspective of social welfare.

On the issue of competitive effects of advertising, Kaldor in his seminal 1950 contribution suggests a "concentration-effect" which, depending on the circumstances, may operate at the manufacturing, wholesale or retail level. Advertising may facilitate or simply accelerate industry concentration through the creation of brands, differentiated products and "goodwill". Much of the later debate centers around the question whether and why incumbent firms have an advantage in advertising and can use it to put up barriers to entry.¹ In a very influential empirical study, Telser (1964) concludes that "there is little empirical support for an inverse association between advertising and competition, despite some plausible theorizing to the contrary". Porter's 1974 study suggests that competitive effects of advertising are likely to be found in a small group of "convenience goods" industries with particular characteristics. On theoretical and empirical grounds, Sutton (1991) suggests that within advertising-intensive industries, there can coexist a small number of small advertisers and a competitive fringe of firms with little or no advertising.

On purely theoretical grounds, it may be difficult to separate causes and effects of advertising, if the degree and effectiveness of advertising depend on market conditions which in turn are modified by advertising.² In the sequel, we shall assume an oligopolistic industry structure. This assumption permits

¹See Bain (1956), Schmalensee (1974), Comanor and Wilson (1979), among others.

²According to Dorfman and Steiner (1954), there will be no advertising under perfect competition and heavy advertising under imperfect competition. The specific conclusion of zero advertising under perfect competition need not obtain in other models; see e.g., Stigler and Becker (1977), Stegeman (1991).

comparative statics of the equilibrium levels of advertising with respect to the number of oligopolists. Thus to some extent, the effect of the market structure on the intensity of advertising can be studied in our model. The converse question of the causes of the prevailing market structure — and of advertising as a potential cause — is beyond the scope of our investigation.

Regarding the question whether firms buy the socially optimal amount of advertising, the direct costs and benefits to the individual firm are evident. As a rule, it pays for its own advertisements. Its benefits derive from the fact that *ceteris paribus* its advertising effort affects the demand for its product positively. This may occur through a gain of market share at the expense of other firms or through an increase of demand for the entire industry.³ In the first case, in the absence of greater general demand, it is possible that individual demand shifts — which each are profitable for the respective firm — neutralize each other. Then the aggregate effect of advertising may be zero or insignificant whereas the firms incur substantial advertising costs. Such a Prisoner-Dilemma-like situation can arise in our model.

As for consumer welfare, the literature distinguishes between “informative” and “persuasive” advertising. As Kaldor (1950) has noted, this distinction is a matter of degree; whereas all advertising is persuasive in intention and informative in character, the motive of persuasion can be very strong in some cases and relatively weak in others. Informative advertising can be beneficial to consumers to the extent that it reduces search costs.⁴ When used to signal quality, informative advertising can be conducive to the profitability and provision of high quality products, another benefit to consumers. Despite these positive consequences, the net benefit of informative advertising can be negative for many consumers. Take for example the viewer of “free” network television news which is bundled with advertising. Even if the ads are informative, the information may be quite useless to a viewer who is not or very infrequently interested in the particular products. This viewer could spend some of his time in a more productive or more pleasant way, if the news were available without the ads.

The implications of persuasive advertising for consumer welfare are much more controversial. For instance, consider the partial equilibrium model of oligopolistic competition that we are going to analyze. Suppose one ignores

³Advertising may also help deter entry as noted earlier.

⁴It may inform consumers about the existence of a product, its characteristics, its price or price distribution, the location of its vendors, etc.

the usual concern whether consumer surplus is an adequate measure of welfare and compares consumer surpluses for different levels of advertising. Are these valid welfare comparisons, if the shift of demand curves is brought about by a shift of consumer tastes? How should one compare welfare, if advertising changes preferences? For their model with explicit utility functions which have advertising as an argument, Dixit and Norman (1978) argue that different market outcomes should be compared on the basis of constant preferences, say pre-advertising or post-advertising tastes. They find excessive advertising in terms of pre-advertising and post-advertising tastes. Fisher and McGowan (1979) argue that each outcome should be evaluated on the basis of the preferences that brought it about. See also the reply by Dixit and Norman (1979).

Fisher and McGowan's argument against Dixit and Norman's "shifting tastes" approach hints at the main alternative, the "stable tastes" approach pioneered by Stigler and Becker (1977) and others, applied by Nichols (1985) and further developed by Becker and Murphy (1993).⁵ Apart from some striking conclusions, the appeal of the stable tastes approach lies in the fact that it can rely on standard methods of economic analysis without resorting to explanations from other social sciences.

Although consumer preferences are not explicitly modelled in our partial equilibrium setting of oligopolistic competition, the distinction between shifting and stable tastes remains relevant. Namely, one can follow either Dixit and Norman or Fisher and McGowan when comparing total (consumer plus producer) surpluses. For some symmetric versions of our model, such a choice is unnecessary. We side with Fisher and McGowan, if we have to choose. Let us add that the prevailing taxonomy, shifting versus stable tastes, is neither exclusive nor exhaustive. As a conceivable third alternative, consider the case of two chemically and physically identical laundry detergents. Suppose consumers are well aware of this fact and indifferent between the two. If prices are significantly different, a consumer will choose the cheaper alternative. Otherwise, the consumer chooses at random or, as a repeat buyer, sticks with the previously chosen brand, with occasional experimentation. All that advertising does is to raise consumer awareness of a brand so that consumers choose it or experiment with it with higher probability. But

⁵Stigler and Becker assume that consumers do not care about goods per se, but their attributes — or characteristics in the tradition of Lancaster. Becker and Murphy postulate that advertisements and the goods advertised are complements in stable metautility functions.

advertising does not make the brand more valuable to consumers. If the advertised brand happens to be unavailable in the store while the other brand is available, consumers are not disappointed, since they still assess the two brands as equally good. Last but not least, there are indirect welfare effects of advertising beyond the scope of this paper and most of the literature. For example, if for whatever reasons advertising could boost general consumer demand, then it might be helpful in preventing, mitigating or shortening economic recessions. As another example, if in fact advertising happens to have a “concentration-effect”, then one has to deal with the pros and cons of industry concentration as well.

Our subject are advertising wars where explicitly or by implication one brand takes on a particular competing brand and vice versa. Such targeted ad campaigns are comparative in nature and suggest the superiority of one’s own brand in some dimension(s). They may turn negative and stress the inferiority of the competing brand. Because of its surge in recent political campaigns and its rise in commercial campaigns, comparative advertising has received increased attention in both the popular and the specialized media. However, the tactical details of comparative advertising such as the optimal framing of ads are of secondary concern to us here — despite some potentially exciting economic, ethical, legal or marketing issues.⁶ For our purposes, we need not and do not specify whether the content of a firm’s ads is positive or negative, though we implicitly assume that the firm chooses whatever format works best for it. Thus in our reduced form model of advertising wars, a firm simply determines its amount of advertising against each of the other firms.

We develop a simple game theoretic model wherein targeted comparative advertising will have a positive impact on the demand for the advertised brand and a negative impact on the demand for the targeted competing brand. The player set consists of a finite number of firms. Our static game has two stages. In the first stage, firms decide whom to advertise against. In the second stage, given the advertising configuration determined in the first stage, they compete as Cournot oligopolists. In section 2, we introduce the second-stage Cournot model. In section 3, we develop the static two-stage model of comparative advertising and provide sufficient conditions for the existence of a subgame perfect Nash equilibrium. Under these conditions,

⁶What exactly constitutes “competitive bashing” or comparative advertising is a matter of degree and perception. For attempts to define negative advertising and assess its effectiveness, see James and Hensel (1991) and Sorescu and Gelb (2000).

the equilibrium advertising efforts are unique and positive. Later in the section, we revisit some of the welfare issues raised above.

In section 4, we consider the perfectly symmetric case where one can explicitly solve for the subgame perfect Nash equilibrium. In this particular case, equilibrium prices and quantities are the same as in the Cournot model without advertising. Hence each firm would gain if they all refrained from advertising. We go on to study the sustainability of collusion with respect to advertising in the infinitely repeated two-stage game. Obviously, the firms could make further gains by colluding in advertising and output decisions. It turns out that under certain conditions, collusion in advertising and output can be supported as a subgame perfect equilibrium outcome of the infinitely repeated game whenever collusion in advertising alone can be supported. In the symmetric static model, we further find, among other things, that individual advertising expenditures increase with the number of firms. Hence a firm gains more from collusion in advertising when there are more firms.

In section 5, we explore the possible links between the degree of product differentiation and the effectiveness of comparative advertising. Section 6 contains concluding comments. Section 7 contains proofs and derivations.

2 The Cournot Model

In this section, we present the Cournot model that will constitute the second stage of our two-stage advertising model. There is a finite number $n \geq 2$ of firms belonging to the set $N = \{1, 2, \dots, n\}$. Generic firms are denoted i , j , or k . p_i denotes firm i 's price and q_i denotes its quantity. Firms produce imperfect substitutes so that the inverse demand function for firm i assumes the form

$$p_i = \alpha_i - q_i - \varepsilon \cdot \sum_{k \neq i} q_k \quad (1)$$

where $\alpha_i > 0$ and $0 < \varepsilon < 1$.

For each firm i , we assume a constant marginal cost c_i and a fixed cost C_i^F resulting from the first-stage advertising decision so that its total costs are

$$C_i = c_i \cdot q_i + C_i^F. \quad (2)$$

With profits given by $\pi_i = p_i \cdot q_i - C_i$, we solve for the Cournot equilibrium in subsection 7.1 where we also derive the necessary and sufficient conditions

(19) for positive equilibrium quantities. In the latter case, Cournot equilibrium quantities and profits are given by:

$$q_i = \frac{1}{2-\varepsilon} [\alpha_i - c_i] - \frac{\varepsilon}{2-\varepsilon} \cdot \frac{1}{2+(n-1)\varepsilon} \sum_{k \in N} [\alpha_k - c_k]; \quad (3)$$

$$\pi_i = q_i^2 - C_i^F. \quad (4)$$

From (3), we obtain $\partial q_i / \partial \alpha_i = \frac{1}{2-\varepsilon} \cdot (1 - \frac{\varepsilon}{2+(n-1)\varepsilon})$ and $\partial q_i / \partial \alpha_j = -\frac{1}{2-\varepsilon} \cdot \frac{\varepsilon}{2+(n-1)\varepsilon}$ for $i \neq j$. Since $\varepsilon < 2$, $\partial q_i / \partial \alpha_i > 0$ and $\partial q_i / \partial \alpha_j < 0$ for $i \neq j$.

For later reference, let us also report the collusive outcome. If the firms collude to maximize joint profits, the quantity chosen and profits earned are given by

$$q_i = \frac{1}{2(1-\varepsilon)} [\alpha_i - c_i] - \frac{\varepsilon}{2(1-\varepsilon)} \cdot \frac{1}{1+(n-1)\varepsilon} \sum_{k \in N} [\alpha_k - c_k]; \quad (5)$$

$$\pi_i = \frac{1}{2} [\alpha_i - c_i] q_i - C_i^F. \quad (6)$$

3 The Two-Stage Model of Advertising

In this section, we consider a two-stage game played by the n firms where in the second stage, the firms engage in Cournot competition described by the model of the previous section.

In the first stage of the game a (pure) strategy for firm i is a vector $s_i = (s_{i1}, \dots, s_{i,i-1}, s_{i,i+1}, \dots, s_{in})$ where $s_{ij} \in \mathbb{R}_+$ for each $j \in N \setminus \{i\}$. Throughout the paper we restrict our attention to pure strategies. The set of first-period (pure) strategies of firm i is denoted by S_i . Since i has the option of advertising against each of the other $n-1$ firms, $S_i = \mathbb{R}_+^{n-1}$. The set $S = S_1 \times \dots \times S_n$ is the joint strategy space of all firms. A strategy profile $s = (s_1, \dots, s_n) \in S$ defines an advertising outcome. Also denote $S_{-i} = \prod_{j \neq i} S_j$, the set of strategy profiles for all firms but i .

To describe the effects of advertising, we introduce numbers $\theta_{ij}^i > 0$ and $\theta_{ij}^j > 0$ for all i and $j \neq i$. If firm i chooses the advertising level $s_{ij} \geq 0$ against firm j , then there is a positive gain for firm i in the sense that its demand increases by $s_{ij} \cdot \theta_{ij}^i$. If conversely firm j chooses an advertising level s_{ji} against firm i , then the demand of the latter falls by $s_{ji} \cdot \theta_{ji}^i$. To

be precise, advertising affects the intercepts α_i of the second-stage inverse demand functions (1) as follows:

$$\alpha_i = \beta_i + \sum_{j \neq i} s_{ij} \cdot \theta_{ij}^i - \sum_{j \neq i} s_{ji} \cdot \theta_{ji}^i \quad (7)$$

where $\beta_i > 0$ is exogenously given. Notice that (1) can be viewed as a demand relation where i 's demand depends on its own price and the quantities produced by others,

$$q_i = \alpha_i - p_i - \varepsilon \cdot \sum_{k \neq i} q_k. \quad (8)$$

We have stated that the second-stage fixed costs in (2) are advertising expenses determined by first-stage decisions. Specifically, we assume numbers $\phi_{ij} > 0$ for all i and $j \neq i$. Then for firm i the cost of advertising is given by

$$C_i^F = \sum_{j \neq i} s_{ij}^2 \cdot \phi_{ij}. \quad (9)$$

In the second stage, the firms maximize profits, given their cost functions, the (inverse) demand functions and the strategy profile of the first stage. The Cournot outcome is unique and is given by (3) and (4) in case it is positive. It depends on the first-stage profile $s \in S$ via (7) and (9). Hence for every strategy profile s of the first stage, there will be a unique Cournot Nash equilibrium in the second stage. Let $q_i(s)$ denote the equilibrium quantity chosen by firm i given the strategy profile s of the first stage and $\pi_i(s)$ be the resultant equilibrium profits. In the remainder of this section, we utilize the explicit expressions for $q_i(s)$ and $\pi_i(s)$ to determine gains and losses from advertising (subsection 3.1), present sufficient conditions for the existence of the subgame perfect equilibrium of the two-stage game (subsection 3.2), and assess equilibrium welfare (subsection 3.3).

3.1 Gains and Losses from Advertising

If one disregards advertising costs, then as a rule, an advertiser gains from comparative advertising and the targeted firm loses. Moreover, the profits of third parties tend to be affected as well. Each effect has two components. By (3), (4), (7), and (9) firm i 's advertising impacts on its own profits as follows:

$$\begin{aligned}
\frac{\partial \pi_i(s)}{\partial s_{ij}} &= 2q_i(s) \cdot \left[\frac{\partial q_i(s)}{\partial \alpha_i} \cdot \frac{\partial \alpha_i}{\partial s_{ij}} + \frac{\partial q_i(s)}{\partial \alpha_j} \cdot \frac{\partial \alpha_j}{\partial s_{ij}} \right] - \frac{\partial C_i^F}{\partial s_{ij}} \\
&= 2q_i(s) \cdot \frac{1}{2-\varepsilon} \cdot \left[\left(1 - \frac{\varepsilon}{2+(n-1)\varepsilon} \right) \theta_{ij}^i + \frac{\varepsilon}{2+(n-1)\varepsilon} \theta_{ij}^j \right] \\
&\quad - 2s_{ij} \cdot \phi_{ij}
\end{aligned}$$

Hence there are two sources of gains from strategic advertising, namely the fact that the advertising firm's demand is increasing and the fact that the demand of the firm advertised against is decreasing. The loss of course stems from the cost of advertising. Hence as long as the cost of advertising is sufficiently small, a firm always gains by advertising.

If firm i advertises against firm j , then the profits of the latter are affected as follows:

$$\begin{aligned}
\frac{\partial \pi_j(s)}{\partial s_{ij}} &= 2q_j(s) \cdot \left[\frac{\partial q_j(s)}{\partial \alpha_j} \cdot \frac{\partial \alpha_j}{\partial s_{ij}} + \frac{\partial q_j(s)}{\partial \alpha_i} \cdot \frac{\partial \alpha_i}{\partial s_{ij}} \right] \\
&= -2q_j(s) \cdot \frac{1}{2-\varepsilon} \cdot \left[\left(1 - \frac{\varepsilon}{2+(n-1)\varepsilon} \right) \theta_{ij}^j + \frac{\varepsilon}{2+(n-1)\varepsilon} \theta_{ij}^i \right]
\end{aligned}$$

There are also two sources of loss for the firm being advertised against, namely a decline of its own demand and a rise of its rival's demand.

Finally consider third parties, that is firms k different from i and j . Such a firm's profit is affected by i 's advertising against j as follows:

$$\begin{aligned}
\frac{\partial \pi_k(s)}{\partial s_{ij}} &= 2q_k(s) \cdot \left[\frac{\partial q_k(s)}{\partial \alpha_j} \cdot \frac{\partial \alpha_j}{\partial s_{ij}} + \frac{\partial q_k(s)}{\partial \alpha_i} \cdot \frac{\partial \alpha_i}{\partial s_{ij}} \right] \\
&= 2q_k(s) \cdot \frac{1}{2-\varepsilon} \cdot \frac{\varepsilon}{2+(n-1)\varepsilon} [\theta_{ij}^j - \theta_{ij}^i]
\end{aligned}$$

Thus third parties stand to gain if the loss incurred by the firm advertised against is greater than the gain of the advertising firm (where advertising costs are ignored).

3.2 Subgame Perfect Nash Equilibrium

We have solved for the Cournot Nash equilibrium in the second stage, obtaining quantities $q_i(s)$ and profits $\pi_i(s)$ as functions of the first-period strategy

profile. To obtain the Subgame Perfect Nash Equilibrium (SPNE) of the two-stage game, we can use backward induction and solve for a Nash equilibrium $s^* \in S$ of the one-stage game based on continuation payoffs $\pi_i(s)$. To this end, put

$$V_{ij} = \frac{1}{2 - \varepsilon} \cdot \left[\left(1 - \frac{\varepsilon}{2 + (n-1)\varepsilon} \right) \theta_{ij}^i + \frac{\varepsilon}{2 + (n-1)\varepsilon} \theta_{ij}^j \right]$$

for $i \neq j$. Then the previous expression for $\partial\pi_i(s)/\partial s_{ij}$ reduces to

$$\frac{\partial\pi_i(s)}{\partial s_{ij}} = 2q_i(s)V_{ij} - 2s_{ij} \cdot \phi_{ij}.$$

Assuming $V_{ij} > 0$, the first order condition for maximization with respect to s_{ij} becomes

$$s_{ij} = q_i(s)V_{ij}/\phi_{ij} \tag{10}$$

where $q_i(s)$ is given by (3) and (7). If the resulting system of linear equations in the $n(n-1)$ variables $s_{ij}, i \neq j$, has a strictly positive solution s^* , then $(s^*, q(\cdot)) = (s_i^*, q_i(\cdot))_{i \in N}$ is a candidate for a SPNE of the two-stage game. For a wide range of parameter values, such an s^* exists and is unique and $(s^*, q(\cdot))$ is a SPNE indeed.

Proposition 1 *Suppose $V_{ij} > 0$ holds for all $i \neq j$ and (20) holds for all $i \in N$. Then for sufficiently large $\phi_{ij}, i \neq j$, there exists a subgame perfect equilibrium $(s^*, q(\cdot))$ of the two-stage game. Moreover, s^* is unique and satisfies $0 < s_{ij}^* < 1$ for all $i \neq j$.*

The proof can be found in subsection 7.2. In essence, the proposition says that if the cost of advertising is sufficiently large, then there will be a positive, but limited amount of advertising.

3.3 Equilibrium Welfare

Under certain conditions, like in the special case considered in the next section, the equilibrium advertising efforts of firm i against j and vice versa will offset each other. Hence advertising expenses constitute a net loss from a social welfare perspective. Neither producers nor consumers are ultimately affected by the possibility of comparative advertising, except for advertising

costs. Still, each individual firm has an incentive to advertise up to a certain point. Such a scenario is reminiscent of the Prisoner’s Dilemma and has been described verbally already by Pigou (1924, p. 176): “It may happen that the expenditures on advertising made by competing monopolists simply neutralize one another, and leave the industrial position exactly as it would have been if neither had expended anything. For clearly, if each of two rivals makes equal efforts to attract the favour of the public away from the other, the total result is the same as it would have been if neither had made any effort at all.” Consumers may have gained, of course, if the advertising efforts have created the perception of higher product quality. With comparative advertising, however, the utility effects may also neutralize each other and Pigou’s verdict of wasteful advertising may well be justified even if consumer welfare is taken into account.

In general, straightforward welfare conclusions may prove impossible. First, our existence result relies on the quadratic form of the cost terms $s_{ij}^2 \cdot \phi_{ij}$ in (9). This gives rise to second order terms

$$\partial^2 \pi_i(s) / \partial s_{ij}^2 = 2V_{ij}^2 - 2\phi_{ij}$$

which become negative for sufficiently large ϕ_{ij} . On the other hand, linear cost terms $s_{ij} \cdot \phi_{ij}$ would yield second order terms

$$\partial^2 \pi_i(s) / \partial s_{ij}^2 = 2V_{ij}^2 > 0.$$

Consequently, best responses and a SPNE would fail to exist. Secondly, advertising efforts need not be offsetting across firms and, therefore, equilibrium quantities and prices may be affected by advertising. In that case, the welfare of firms can still be evaluated. But as explained in the introduction, the assessment of consumer welfare tends to be more problematic if consumers respond to comparative advertising. To the extent that comparative advertising is uninformative and merely persuasive, in other words is an attempt to manipulate consumer preferences, the question arises how to account for the part of the change in consumer surplus that is attributable to a shift of the consumers’ willingness to pay. Thirdly, as a rule, the SPNE is only implicitly given which makes it difficult to draw firm conclusions. For these reasons, we specialize in the next section.

Before we turn to this special case, let us report (without the straightforward but lengthy analytic details) some general facts. In the spirit of Stigler and Becker (1977), Fisher and McGowan (1979), and Becker and Murphy

(1993), let us take preferences as stable.⁷ In concrete terms, let us measure welfare by means of total surplus that is the sum of producer surplus (industry profits) and consumer surplus. Obviously one source of inefficiency is the very fact that the firms behave as Cournot oligopolists and hence do not choose quantities at levels where price equals marginal cost. But that is a well known fact and we are primarily interested in inefficiencies related to strategic advertising. So let us suppose that the firms continue to behave as Cournot oligopolists but the social planner can now select the levels of strategic advertising in a bid to maximize welfare. Then as a rule, the first order conditions for welfare maximizing advertising levels differ from the system (10). In the special perfectly symmetric case of the next section, the welfare maximizing advertising levels turn out to be zero so that Pigou's verdict holds even if consumer welfare is taken into account. However, the welfare maximizing advertising levels are not always zero. The latter occurs for an almost symmetric model with differential impact of advertising as follows. There are constants $\alpha > 0, c > 0, \theta > 0$, and $\phi > 0$ such that for all i and $j \neq i$: $\beta_i = \alpha, c_i = c, \theta_{ij}^i = \theta_{ji}^j = 2\theta, \theta_{ij}^j = \theta_{ji}^i = \theta, \phi_{ij} = \phi$. Provided that the existence and uniqueness result of Proposition 1 holds, we find that for sufficiently large ϕ , the welfare maximizing advertising levels are positive but less than the SPNE levels. We conjecture that for large, but not too large ϕ , the welfare maximizing advertising levels may exceed the SPNE levels.

4 The Symmetric Case

In the perfectly symmetric case, one can explicitly solve for the SPNE. This permits detailed welfare analysis and a study of the effects and sustainability of collusion. The perfectly symmetric case is given by constants $\alpha > 0, c > 0, \theta > 0$, and $\phi > 0$ such that for all i and $j \neq i$: $\beta_i = \alpha, c_i = c, \phi_{ij} = \phi, \theta_{ij}^i = \theta_{ij}^j = \theta_{ji}^i = \theta_{ji}^j = \theta$. Because of the last identities, advertising has an offsetting impact, if firms choose equal advertising levels.

⁷For lack of an explicit description of preferences, this is an implicit assumption, though.

4.1 Subgame Perfect Nash Equilibrium

As an immediate consequence of Proposition 1, we obtain

Corollary 1 *Suppose $\alpha > c + (n - 1)\theta$. Then for sufficiently large ϕ , there exists a unique subgame perfect equilibrium $(s^*, q(\cdot))$ of the two-stage game with the property that $0 < s_{ij}^* < 1$ for all $i \neq j$.*

Under the symmetry assumptions, the SPNE can be easily found. Namely, we obtain $V_{ij} = \theta/(2 - \epsilon)$ for all $i \neq j$. Hence (10) reduces to $s_{ij} = q_i(s) \cdot \theta/((2 - \epsilon)\phi)$. Now set

$$s_{ij}^* = \frac{1}{2 - \epsilon} \frac{\alpha - c}{2 + (n - 1)\epsilon} \frac{\theta}{\phi} \quad (11)$$

for $i \neq j$. With that choice, (7) and (3) become $\alpha_i = \alpha$ for all i and

$$q_i(s^*) = \frac{\alpha - c}{2 + (n - 1)\epsilon} \quad (12)$$

for all i , respectively, and the first order conditions (10) are satisfied. Therefore:

Corollary 2 *Suppose $\alpha > c + (n - 1)\theta$. Then for sufficiently large ϕ , the SPNE advertising levels are given by (11) and the equilibrium quantities are given by (12).*

This SPNE possesses several interesting properties. First and foremost, as expected the equilibrium quantities are unaffected by the benefit and cost parameters θ and ϕ and the corresponding equilibrium levels of advertising. The pairwise levels of advertising in (11) are linear in the benefit-cost ratio θ/ϕ . Individual advertising expenditures are given by $e = (\frac{1}{2 - \epsilon} \frac{\alpha - c}{2 + (n - 1)\epsilon} \theta)^2 / \phi$. Therefore, a rise in the cost parameter ϕ leads to a reduction of advertising levels and a decline of advertising expenditures. Whereas levels of advertising decrease in the number of firms, aggregate advertising expenditures increase with the number of firms.

4.2 Beneficial Collusion

Firms have an incentive to collude with respect to advertising or output decisions, or both. Let us focus on advertising first. The equilibrium advertising levels (11) amount to a total advertising expenditure $E = n(n - 1)e$.

Since the equilibrium quantities (12) are unaffected by advertising, consumer welfare is unaffected by advertising. Hence the advertising expenditure E incurred by firms is a clear welfare loss. This would be a justification for a ban on comparative advertising. Germany, for instance, has traditionally banned comparative advertising, because it was considered a form of “improper competition”. Another way to avoid the welfare loss would be collusion among firms — which might be tolerated by regulators as long as consumer protection is not at issue. Compared to the SPNE outcome, each firm can gain

$$g = (n - 1)e$$

if they refrain from advertising, assuming that second-stage Cournot competition persists. But given that nobody else advertises, a firm has an incentive to deviate from the collusive outcome. If it does not advertise, its payoff is given by (4) and (12) with $C_i^F = 0$. Let h denote the additional payoff the firm receives, if it chooses the optimal levels of advertising. Since $\partial\pi_i(s)/\partial s_{ij} = 2q_i(0, \dots, 0)V_{ij} = 2q_i(s^*)V_{ij} > 0$ at $s = (0, \dots, 0)$, $h > 0$ holds. In fact, $h > g$ is possible.

4.3 Sustainable Collusion

While there exist incentives to deviate from the collusive outcome in the two-stage game, collusion may be sustainable if the two-stage game is infinitely repeated. Suppose there are periods $t = 0, 1, 2, \dots$. In each period, the two-stage game is played. Suppose firm i 's time preferences are given by a discount factor $\delta_i \in (0, 1)$. We assume that in each period, firms choose advertising levels in the first stage and play a Cournot equilibrium in the second stage given these advertising levels. By a standard argument, there exists a subgame perfect Nash equilibrium in trigger strategies for the repeated game, without advertising along the equilibrium path, provided $g \cdot \frac{\delta_i}{1-\delta_i} > h$ or $\delta_i > \frac{h/g}{1+h/g}$ for all i . This means that the collusive outcome is sustainable for sufficiently large discount factors.

The firms have an incentive to collude with respect to advertising and output levels, since without advertising, the collusive payoffs given by (5) and (6) exceed the Cournot payoffs given by (3) and (4), say by $g' > 0$. Then $G = g + g'$ is a firm's payoff gain from two-fold collusion relative to its SPNE payoff in the two-stage game. Clearly, there exist stronger incentives for two-fold collusion than collusion in advertising only. But one might suspect that

the incentives for deviation are also stronger. It turns out that in the infinitely repeated two-stage game, the incentives to deviate from two-fold collusion can be less than the incentives to deviate from collusion in advertising only. Let h' denote the additional payoff the firm receives, if it deviates optimally from collusion in the Cournot model without advertising. The key observation is that in each period, the two-stage game is played. Therefore, if in any period, a player deviates in the first stage, the other players have the opportunity for instantaneous retaliation in the second stage of the same period. To be concise, let us state

Proposition 2 *Suppose $h' \leq h$. If for some discount factors $\delta_i, i \in N$, collusion in advertising can be supported by a subgame game perfect equilibrium in trigger strategies, then there exist discount factors $\delta'_i < \delta_i, i \in N$, such that collusion in advertising and output can be supported by a subgame perfect equilibrium in trigger strategies.*

The essence of the argument is as follows. Suppose that a firm considers a first deviation in the first stage of some period. Then the trigger strategies can be such that play reverts from collusion in output to Cournot equilibrium play in the second stage of that period and from collusion in advertising and output to the SPNE of the two-stage game in all subsequent periods. Hence the one-time undiscounted gain from such a deviation is less than h , the one-time undiscounted gain from such a deviation when only collusion in advertising was implemented, whereas the subsequent foregone benefits from collusion are higher. Suppose next that in some period, a firm considers a first deviation in the second stage of that period. Then h' , the one-time undiscounted gain from such a deviation is at most h . But the potential deviator foregoes the (undiscounted) collusive benefits G in every subsequent period when play reverts from collusion in advertising and output to the SPNE of the two-stage game. Since G is higher than g , the forfeited benefits from collusion in advertising only, it follows that smaller discount factors suffice to sustain collusion in advertising and output.

The condition $h' \leq h$ is satisfied, for example, in case $n = 2, \varepsilon = 1/2, \alpha = 2, c = 1, \theta = 1, \phi = 2$. One can replace the condition $h' \leq h$ by the weaker condition $h' < h + g'$, if $\delta_i > 1/2$ is assumed for all i .

5 Product Differentiation and Advertising

In this section, we explore two plausible premises about the link between the degree of product differentiation and the effectiveness of comparative advertising. One assumption is that less horizontal product differentiation makes comparative advertising more effective. The other assumption is that comparative advertising is an attempt to create or alter consumer perception of vertical product differentiation.

5.1 Horizontal Differentiation and Advertising

To convey the general idea, let us assume that each firm places itself (its product) in an abstract product space represented by a metric space. If two firms i and k are close in the product space, then their products are close substitutes and comparative advertising between them is very effective.

Formally, the inverse demand functions (1) are replaced by the slightly more general form

$$p_i = \alpha_i - q_i - \sum_{k \neq i} \varepsilon_{ik} q_k. \quad (13)$$

Let d_{ik} denote the distance between firms i and k . Then the assumption that less product differentiation makes products closer substitutes and comparative advertising more effective amounts to the following two conditions:

- (a) $\varepsilon_{ik} = \varepsilon_{ki}$ is strictly decreasing in d_{ik} .
- (b) θ_{ik}^i and θ_{ik}^k are strictly decreasing in d_{ik} .

Notice that $d_{ki} = d_{ik}$ and by symmetry the effect of k 's advertising against i decreases as well if they locate further apart. One would expect that *ceteris paribus* a firm will advertise more against close than against distant competitors. Ford's Lincoln might be pitted primarily against General Motor's Cadillac and not against Fiat's Uno. This could even happen when only (a) holds while the θ 's are independent of distance so that the impact of horizontal product differentiation is only transmitted through one channel. Consider, as a numerical example, the situation of three firms where firms 1 and 2 sell (almost) identical products whereas firm 3 sells a product quite different from the other two.

Example. There are three firms. Let us assume that there exists a number $\alpha > 0$ such that $\beta_i - c_i = \alpha$ for each firm i . We postulate that $d_{12} = 0$ or $d_{12} \approx 0$ whereas d_{13} and d_{23} are very large. Let us further assume that this implies $\varepsilon_{12} \approx 1$, $\varepsilon_{13} \approx 0$, $\varepsilon_{23} \approx 0$. For simplicity we set $\varepsilon_{12} = 1$ and $\varepsilon_{13} = \varepsilon_{23} = 0$. Hence at the second stage, the market is divided into a segment served by firms 1 and 2 and a segment served by firm 3. However, we allow for advertising spillovers from one segment of the market to the other, assuming $\theta_{ij}^i = \theta_{ij}^j = 1$ for each pair of firms i and j . Let the advertising cost parameters be given as $\phi_{ij} = 8$ for each pair of firms i and j . Then they are sufficiently large to satisfy the second order conditions with respect to advertising.

In the second-stage Cournot equilibrium of the duopoly formed by firms 1 and 2, the equilibrium profits are

$$\pi_1 = \frac{1}{9} \cdot (2\alpha_1 - \alpha_2)^2 - C_1^F, \quad \pi_2 = \frac{1}{9} \cdot (2\alpha_2 - \alpha_1)^2 - C_2^F \quad (14)$$

where $\alpha_1 = \alpha + s_{12} - s_{21} + s_{13} - s_{31}$ and $\alpha_2 = \alpha + s_{21} - s_{12} + s_{23} - s_{32}$. The second-stage monopoly profit for firm 3 is

$$\pi_3 = \frac{1}{4} \cdot \alpha_3^2 - C_3^F \quad (15)$$

where $\alpha_3 = \alpha + s_{31} - s_{13} + s_{32} - s_{23}$. To determine equilibrium advertising levels, we exploit symmetry and set $s_{12} = s_{21}$, $s_{13} = s_{23}$, $s_{31} = s_{32}$. This reduces the first order conditions (10) to the following three equations immediately derived from (14) and (15):

$$\begin{aligned} 8s_{12} &= \frac{3}{9}(\alpha + s_{13} - s_{31}); \\ 8s_{13} &= \frac{2}{9}(\alpha + s_{13} - s_{31}); \\ 8s_{31} &= \frac{1}{4}(\alpha + 2s_{31} - 2s_{13}). \end{aligned}$$

The solution is

$$s_{12}^* = \frac{43.5}{1048}\alpha, \quad s_{13}^* = \frac{29}{1048}\alpha, \quad s_{31}^* = \frac{33}{1048}\alpha.$$

Thus, indeed, advertising between the two duopolists is fiercer than across market boundaries. Interestingly enough, the monopolist advertises somewhat more against a duopolist than vice versa. The reason is that for equal

advertising levels, the α_i are equal whereas the monopolist's profits and marginal benefits from advertising are clearly higher as a comparison of (14) and (15) shows.

Now suppose that in the example, the impact of horizontal product differentiation gets transmitted through both channels; that is both (a) and (b) hold. We can achieve this by setting $\theta_{12}^1 = \theta_{12}^2 = \theta_{21}^1 = \theta_{21}^2 = \hat{\theta} > 1$ while keeping all other parameters fixed at their previous levels. Then only the first of the first order conditions changes and becomes

$$8s_{12} = \frac{3}{9}\hat{\theta}(\alpha + s_{13} - s_{31}).$$

Therefore, the new equilibrium advertising levels are $\hat{s}_{12} = \hat{\theta}s_{12}^*$, $\hat{s}_{13} = s_{13}^*$, $\hat{s}_{31} = s_{31}^*$. The advertising war among the duopolists has intensified as to be expected while advertising across market boundaries has not changed.

Symmetric Case. For arbitrary fixed locations, the analysis becomes very complicated. In contrast, the situation becomes fully transparent in the symmetric case with two firms to which we turn next. Let us assume $n = 2$ and make the symmetry assumptions of the previous section. Moreover, we assume (a) and (b). Since there are only two firms, we can drop the subscript from ε and θ . In the absence of advertising, (3) and (4) become $q_i = (\alpha - c)/(2 + \varepsilon)$ and $\pi_i = (q_i)^2$, respectively. Consequently, a firm's Cournot equilibrium profit is decreasing in ε and, by (a), increasing with its distance from the other firm. In a two-stage game where firms first choose locations on the unit interval and then choose quantities, these comparative statics yield maximum product differentiation:

Proposition 3 *Suppose the product space is the unit interval. In a subgame perfect equilibrium of the two-stage game where firms first choose locations and then choose quantities, one firm locates at one endpoint and the other firm locates at the opposite endpoint.*

Next we introduce advertising and consider a three-stage game. In the first stage, firms choose locations in product space. In the second stage, they choose advertising levels. In the final stage, they choose quantities. Notice that when locations are fixed, ε and θ are determined and the two firms face a symmetric situation as in the previous section. Hence the subgame perfect

equilibrium outcomes of stages two and three are given by (11) and (12). For $n = 2$, these expressions simplify to $s_i = (\alpha - c)/(4 - \varepsilon^2) \cdot \theta/\phi$ and, as before, $q_i = (\alpha - c)/(2 + \varepsilon)$. We have already seen that a firm favors maximum product differentiation if we ignore advertising. The comparative statics of the equilibrium advertising levels s_i reinforce this conclusion: s_i is increasing in ε and θ . Hence the firm spends less on advertising, if it is located further away from the other firm. Therefore, q_i increases and $C_i^F = s_i^2 \cdot \phi$ decreases, if firm i moves further away from the other firm. Consequently, i 's profit $\pi_i = q_i^2 - C_i^F$ is increasing with its distance from the other firm. In the three-stage game, these comparative statics yield again maximum product differentiation:

Proposition 4 *Suppose the product space is the unit interval. In a subgame perfect equilibrium of the three-stage game where firms choose first locations, next advertising levels, and finally quantities, one firm locates at one endpoint and the other firm locates at the opposite endpoint.*

In the classical Hotelling model of a linear city, firms first choose locations on the interval and then compete as Bertrand duopolists for consumers uniformly distributed on the interval. If a firm moves closer to its competitor, it experiences two opposing effects on its profits. The strategic effect refers to the loss the firm incurs because of fiercer price competition. The market share effect refers to the increased demand for the firm's product. D'Aspremont, Gabszewicz and Thisse (1979) have shown that the strategic effect dominates the market share effect so that maximum product differentiation results in the subgame perfect equilibrium. Here we have obtained a similar result in a very different context: the losses from moving closer to one's competitor outweigh the gains. In particular, if the firm reduces its distance from its competitor, then its advertising becomes more effective. But the competitor's advertising becomes also more effective. Consequently, the ensuing advertising war intensifies to the detriment of both firms' profits.

5.2 Advertising as Perceived Product Differentiation

In this section, we briefly elaborate on the possibility that functionally equivalent products may be perceived as different as a consequence of persuasive advertising. If advertising is merely aimed at brand recognition, otherwise

identical products may be perceived as horizontally differentiated. If advertising is comparative, otherwise identical products may be perceived as vertically differentiated. This kind of virtual product differentiation can coexist with actual product differentiation and amplify or alleviate the effects of the latter. To be more specific, let us reconsider a modified inverse demand function of the form (13). In our previous interpretation, we had assumed that the degree of actual horizontal product differentiation determines the magnitude of the substitution coefficients ε_{ik} and of the marginal advertising effect parameters θ_{ik}^i and θ_{ik}^k in (7). Now let us make the opposite assumption that actual product differentiation is non-existent or ineffective. By postulating (7) and (1) or (13), we have always presumed that comparative advertising makes the own product more desirable and the product of the firm advertised against less desirable. In that sense, the effect of comparative advertising is similar to the effect of raising the quality of one's own product or lowering the quality of the other product. In addition to affecting the intercepts (constant terms) α_i of the inverse demand functions, advertising could influence the substitutability of products in a more direct way, for example as follows:

$$\varepsilon_{ij} = \varepsilon - s_{ij} \cdot \theta_{ij}^i + s_{ji} \cdot \theta_{ji}^i. \quad (16)$$

This formulation reflects the intuition that if a firm advertises against another firm, then its own product becomes more of a substitute for the other firm's product whereas the other firm's product becomes less of a substitute for one's own product. This specification allows for the possibility of $\varepsilon_{ij} \neq \varepsilon_{ji}$. In other words, substitutability is no longer a symmetric property — as should be the case with vertical product differentiation. One can reanalyze the symmetric two-stage game of section 4 under new assumptions. In particular, let us assume $n = 2$, $c = 0$, $\phi_{ij} = \phi$, and $\alpha_i = \beta_i = \alpha$ so that α_i does not respond to advertising. On the other hand, let us assume (16) with $\theta_{ij}^i = \theta_{ij}^j = \theta_{ji}^i = \theta_{ji}^j = \theta$. Then once more, (12) obtains for the SPNE quantities. In contrast, the SPNE advertising levels are now given by

$$s_{ij}^* = \frac{1}{2 - \varepsilon} \left(\frac{\alpha}{2 + \varepsilon} \right)^2 \frac{\theta}{\phi}. \quad (17)$$

Although (11) and (17) are not directly comparable because of the different roles of θ in (7) and (16), they share several qualitative features. In particular, in both instances equilibrium advertising is positive and linear in the benefit-cost ratio θ/ϕ .

5.3 Two-dimensional Product Differentiation: a Reinterpretation of the Model

So far we have argued that comparative advertising creates the perception of vertical product differentiation. It takes only a minor conceptual step to say that comparative advertising “is” vertical product differentiation, at least in the case of symmetric duopoly. In that case, our formal model can be converted into a model with vertical product differentiation by means of the following reinterpretation: $s_1 = s_{12}$ stands for the quality choice of firm 1 and $s_2 = s_{21}$ stands for the quality choice of firm 2. As a consequence of Corollary 2, one obtains

Corollary 3 *For $\alpha - c > \theta$ and ϕ sufficiently large, in the SPNE of the two-stage game where firms make first quality and then quantity choices, the duopolists choose identical positive qualities, that is minimum vertical product differentiation.*

By (17), a similar conclusion holds under the alternative specification (16). Next suppose that the spectrum of product variety (horizontal product differentiation) is given by the unit interval and the range of product quality (vertical product differentiation) is given by the nonnegative real numbers. Then the analogue of Proposition 4 holds:

Corollary 4 *In a subgame perfect equilibrium of the three-stage game where the two firms decide first on variety, next on quality, and finally on quantities, maximum horizontal product differentiation and minimum vertical product differentiation (with a positive quality level) result.*

Thus as a by-product, our analysis yields an interesting result on two-dimensional product differentiation. Our model is of a special reduced form in that individual consumer choice is not explicitly specified. Fully specified models of multi-dimensional product differentiation prove hard to analyze. Economides (1989) considers quality variations in a duopoly of locationally differentiated products with linear transportation costs. In his two-stage game, variety is chosen in the first stage and quality and prices are chosen in the second stage. In his three-stage game, variety choice occurs in the first stage, quality choice in the second stage, and price choice in the last stage. Equilibria in pure strategies need not exist. They do exist for certain parameter ranges in which case the SPNE outcome for both models is

maximum horizontal product differentiation combined with minimal quality differentiation. Neven and Thisse (1990) consider quality variations in a duopoly of locationally differentiated products with quadratic transportation costs. Potential qualities are restricted to an interval $[\underline{q}, \bar{q}]$. In their two-stage game, variety and quality are chosen in the first stage and prices are chosen in the second stage. They identify numbers $K_h > K_v > 0$ such that for $\bar{q} - \underline{q} \geq K_v$, there exists an equilibrium with maximum horizontal product differentiation and minimum vertical product differentiation (at quality \bar{q}) and for $\bar{q} - \underline{q} \leq K_h$, there exists an equilibrium with minimum horizontal product differentiation (at location $1/2$) and maximum vertical product differentiation. In particular, for $\bar{q} - \underline{q} \in [K_v, K_h]$, both types of equilibria exist. As Neven and Thisse (1990) have noted, their findings as well as those of Economides and ours suggest that “firms will have a tendency to select similar strategies with respect to some characteristics, if at the same time they are sufficiently differentiated along the remaining dimensions.” Neven and Thisse further assert that a similar result could be established with two vertical characteristics. Indeed, Vandenbosch and Weinberg (1995) study a two-stage game of product competition in two vertical dimensions followed by price competition and show existence of a subgame perfect equilibrium with MaxMin differentiation, that is maximum differentiation in one dimension and minimum differentiation in the other dimension. However, sometimes an equilibrium with maximum differentiation in both dimensions (or with maximum differentiation in one dimension and intermediate differentiation in the other) may also exist. Irmen and Thisse (1998) find that in a location game with $n \geq 2$ characteristics, firms choose to maximize differentiation in the dominant characteristic and to minimize in the others when the salience coefficient of the former is sufficiently large, corroborating findings by Tabuchi (1994) for $n = 2$ and Ansari, Economides and Steckel (1998) for $n = 2, 3$.

6 Concluding Comments

In this paper, we perform a theoretical analysis of advertising wars where firms engage in comparative advertising against each other. This practice has received a fair amount of media attention in recent years. Any rendition of comparative advertising episodes would be incomplete without mentioning the cola wars; see, e.g., Prince (2000). In 1975, Pepsi launched a widely publicized taste test called the “Pepsi Challenge” in which customers were

asked to sample both Pepsi and Coke side by side without being aware of the labels. The alleged superior performance of Pepsi in this test was widely advertised and led to an impressive increase in sales. Over the years, both soft drink giants have launched numerous spot ads against each other.

The basic premise of our analysis is that disregarding costs, a firm's advertising against another firm benefits the advertiser and harms the target. James and Hensel (1991) summarize a number of studies in the marketing literature on the impact of negative advertising on brand perception and induced brand demand based on consumer surveys. Several authors find that comparative advertising is particularly beneficial to new brands. Comparative advertising can also be very effective if it makes undisputed claims like ads by Visa against American Express or for Aleve against Tylenol. Comparative advertisements invite retaliation with potentially devastating effects for both sides and likely benefits to third parties. Examples are AT&T versus MCI, Pizza Hut versus Papa John's, Tylenol versus Advil. In our static model, we obtain advertising wars, namely positive levels of advertising, provided that advertising is not too expensive. The asymmetric version of the model allows for (positive or negative) side-effects on third parties, a feature notably absent from most of the literature.

In the perfectly symmetric version of the model, we obtain that advertising levels are positive in equilibrium, but second-stage quantities and prices are the same as with zero advertising. Indeed, zero advertising would be socially optimal. This result supports the time honored contention — dating back to Pigou (1924) at least — that advertising efforts by competitors might just neutralize each other and prove wasteful. Netter (1982) reports indirect empirical evidence for the mutual cancelling of advertising efforts. Indeed, some firms “have chosen disarmament after years of ad warfare proved fruitless — such as Unilever's Ragu and Campbell Soup Co.'s Prego” as Neff (1999) reports. But other long-lasting feuds keep going. Even as we were writing this paragraph, we have seen television ads for Advil against Tylenol, for Aleve against Tylenol and for Pine-Sol against Lysol.

If in fact advertising efforts by close competitors neutralize each other, then there are potential gains from (explicit or tacit) collusive agreements to refrain from comparative advertising. This possibility motivated us to study the sustainability of collusion in a dynamic version of our model. Empirically, collusion may be hard to identify and to our knowledge has not

been systematically investigated at the brand level.⁸ However, the research of Alston *et al.* (2001) hints at potentially huge gains from collusion at the more aggregate level. They start from the premise that profits from generic advertising by a producer group often come partly at the expense of producers of closely related commodities. They compare a scenario where different producer groups cooperate and choose their advertising expenditures jointly to maximize the sum of profits across the groups, and a scenario where they optimize independently. They calibrate an example using 1998 data for U.S. beef and pork and find that the noncooperatively chosen expenditure on beef and pork is more than three times the cooperative optimum.

Collusion is one possible explanation why comparative advertising is not predominant. Other reasons are regulatory restrictions and their legal repercussions. Moreover, comparative advertising may just not be very effective in some markets. While it may not be predominant, comparative advertising seems far from negligible. According to Neff (1999), among the ads in the ARS data base used in a 1997 study, 12% had direct comparisons and 16% used indirect comparisons.

In section 6, we relate product differentiation and comparative advertising. We first explore the assumption that less horizontal product differentiation makes comparative advertising more effective. We illustrate the implications of this assumption in an asymmetric example with three firms. We then consider the symmetric case with two firms and the unit interval as the product space. In a subgame perfect equilibrium of the three-stage game where firms choose first locations, next advertising levels, and finally quantities, one firm locates at one endpoint and the other firm locates at the opposite endpoint.⁹ Next we discuss the assumption that comparative advertising is an attempt to create or alter consumer perception of vertical product differentiation. Finally, we identify comparative advertising effort with quality choice and obtain a result on two-dimensional product differentiation. Prior to us, Economides (1989) also considers two-dimensional product differentiation and mentions advertising effort as an example of a quality feature whose cost is independent of output. Research effort falls

⁸Though Scherer and Ross (1990, pp. 596f) mention a few overt collusive attempts by cigarette and automobile manufacturers which failed with one exception.

⁹Some of the literature on persuasive advertising, e.g., von der Fehr and Stevik (1998), Bloch and Manceau (1999) adopts Hotelling's linear city model with exogenously given firm locations, because it provides a convenient way to model individual consumer responses to advertising.

into this category as well. But other quality features like the use of better material, say stainless steel, often contribute to the unit cost of output. There is also a crucial difference between true and perceived quality attributes of consumer durables – which brings us back to the earlier debate on the effect of advertising on consumer welfare. True high quality attributes like reliability, size, speed are embodied in the product and tend to last whereas perceived quality is subject to change, in particular if the ads creating the perception are discontinued or countered by the competition.

7 Proofs and Derivations

7.1 Cournot Equilibrium

Let $Q \equiv \sum_i q_i$. Firm i chooses a positive best response if and only if

$$\alpha_i - c_i > \varepsilon \cdot \sum_{k \neq i} q_k. \quad (18)$$

In this case, the best response of firm i is given by the first order condition

$$2q_i = \alpha_i - c_i - \varepsilon \cdot \sum_{k \neq i} q_k.$$

Therefore,

$$q_i = \frac{1}{2 - \varepsilon} [\alpha_i - c_i] - \frac{\varepsilon}{2 - \varepsilon} Q.$$

Summation over all firms yields:

$$\begin{aligned} Q &= \frac{1}{2 - \varepsilon} \sum_{k \in N} (\alpha_k - c_k) - \frac{n\varepsilon}{2 - \varepsilon} Q; \\ Q &= \frac{1}{2 - \varepsilon + n\varepsilon} \sum_{k \in N} (\alpha_k - c_k). \end{aligned}$$

(3) and (4) follow. Now replace the summation index k in (18) by j . Next replace each q_j by the corresponding right-hand expression in (3). This results in the following necessary and sufficient conditions for positive equilibrium quantities:

$$\left[(2 - \varepsilon) + \frac{(n - 1)\varepsilon^2}{2 + (n - 1)\varepsilon} \right] (\alpha_i - c_i) > \left[\varepsilon - \frac{(n - 1)\varepsilon^2}{2 + (n - 1)\varepsilon} \right] \sum_{k \neq i} (\alpha_k - c_k) \quad (19)$$

for all i . The conditions (19) are satisfied if the terms $\alpha_i - c_i$ are identical or not too different across firms. When the conditions are not met, one can distinguish between the set F of firms who choose a positive output in equilibrium and the set $N \setminus F$ of firms who choose zero output in equilibrium. Then the counterparts of (3) and (19) hold for firms i and k restricted to F .

7.2 Existence

We are going to prove the existence claim of Proposition 1 by means of a fixed point argument. We assume $V_{ij} > 0$ for all $i \neq j$ and (20) for all $i \in N$.

An SPNE has the form $(s^*, q(\cdot))$. Among other things, we have to associate a Cournot equilibrium $q(s) = (q_1(s), \dots, q_n(s))$ to each strategy profile s of the first stage. It is straightforward to show that given any strategy profile s of the first stage, a Cournot equilibrium of the subsequent subgame exists. It is also clear from the previous subsection that at most one Cournot equilibrium exists at which all firms produce positive amounts. Now take any Cournot equilibrium given s . By keeping the equilibrium quantities of all but two firms fixed and analyzing the reduced game between the remaining two firms, say i and j , one finds that if $\alpha_i - c_i \leq \alpha_j - c_j$, then the corresponding equilibrium outputs satisfy $q_i \leq q_j$. Therefore, applying the analysis of the previous subsection to the subset of firms who choose positive equilibrium quantities, we can select a Cournot equilibrium $q(s)$ by requiring that the number of players choosing positive quantities be maximal. Let us proceed with this selection $q(s), s \in S$.

The next step is to establish existence of a strategy profile $s^* \in S$ that satisfies the first order conditions (10). In two further steps, we show that the pair $(s^*, q(\cdot))$ is an SPNE and that s^* is unique. The arguments of all three steps require that the coefficients $\phi_{ij}, i \neq j$, be sufficiently large. Given the assumption that $V_{ij} > 0$ for all $i \neq j$ and (20) for all i , we shall show existence of a number $\varphi > 0$ such that if $\phi_{ij} \geq \varphi$ for all $i \neq j$, then the conclusion of Proposition 1 holds.

Without advertising, the conditions (19) become

$$\left[(2 - \varepsilon) + \frac{(n-1)\varepsilon^2}{2 + (n-1)\varepsilon} \right] (\beta_i - c_i) > \left[\varepsilon - \frac{(n-1)\varepsilon^2}{2 + (n-1)\varepsilon} \right] \sum_{k \neq i} (\beta_k - c_k) \quad (20)$$

for all i . The conditions (20) are satisfied if the terms $\beta_i - c_i$ are identical or not too different across firms. Now suppose that they are satisfied and each

V_{ij} is positive. Then because of (7), there exists a number $b \in (0, 1)$ such that if $0 \leq s_{ij} \leq b$ for all $i \neq j$, then the conditions (19) hold. Choose $b > 0$ with this property and let $B = [0, b]^{n(n-1)} \subset S$. Then for $s \in B$ and $i \in N$, $q_i(s)$ is given by (3) and positive. Since each $q_i(s)$ depends continuously on $s \in B$ and B is compact, there exists $K > 0$ such that $q_i(s) \leq K$ for all $i \in N$ and $s \in B$. Therefore, there exists $\varphi_1 > 0$ such that if $\phi_{ij} \geq \varphi_1$ for all $i \neq j$, then $0 < q_i(s)V_{ij}/\phi_{ij} \leq b$ for all $s \in B$ and $i \neq j$. Let us choose such a $\varphi_1 > 0$ and suppose $\phi_{ij} \geq \varphi_1$ for all $i \neq j$. Then we can define a continuous mapping $\sigma : B \rightarrow B$ by setting $\sigma_{ij}(s) = q_i(s)V_{ij}/\phi_{ij}$ for all $s \in B$ and $i \neq j$. Since B is nonempty, compact and convex, σ has a fixed point, by Brouwer's fixed point theorem. A fixed point s^* of σ satisfies $s^* = \sigma(s^*)$, hence for any $i \neq j$: $s_{ij}^* = \sigma_{ij}(s^*) = q_i(s^*)V_{ij}/\phi_{ij}$. That is, s^* satisfies the first order conditions (10). Hence $(s^*, (q(\cdot)))$ is an SPNE, provided that for each firm i , $\pi_i(s_i, s_{-i}^*)$ is maximized at s_i^* . We distinguish two cases.

Case 1: $s_{ij} \leq b$ for all $j \neq i$. Recall that (3) and (4) hold for any choice of $s = (s_i, s_{-i}^*)$ such that $s_{ij} \leq b$ for all $j \neq i$. Because of (7) and (9), there exists $\psi_i > 0$ such that if $\phi_{ij} \geq \psi_i$ for all $j \neq i$, then firm i 's profit function $\pi_i(s_i, s_{-i}^*)$ has a negative definite Hessian matrix and, therefore, is a strictly concave function of $s_i \in [0, b]^{(n-1)}$. Choose such a $\psi_i > 0$ for each i and set $\varphi_2 = \varphi_1 + \sum_i \psi_i$. Suppose $\phi_{ij} \geq \varphi_2$ for all $i \neq j$. Then because of the first order conditions (10), s_i^* is the unique best response against s_{-i}^* in $[0, b]^{(n-1)}$.

Case 2: $s_{ij} > b$ for some $i \neq j$. Let $\|\cdot\|$ denote the Euclidean norm on \mathbb{R}^{n-1} . The best that can happen to firm i is that all other firms choose zero advertising and zero output. In that case, its monopoly profit is

$$\Pi_i(s_i) = (\beta_i - c_i + \sum_{j \neq i} s_{ij} \theta_{ij}^i)^2 / 4 - \sum_{j \neq i} s_{ij}^2 \phi_{ij}$$

if firm i chooses $s_i \in S_i$ in the first stage and its profit maximizing quantity given s_i in the second stage. There exists $\vartheta_i > 0$ such that: If $\phi_{ij} \geq \vartheta_i$ for all $j \neq i$, then $\Pi_i(s_i) < 0$ for $\|s_i\| > b$. Choose such a $\vartheta_i > 0$ for all i . Set $\varphi_3 = \varphi_2 + \sum_i \vartheta_i$. Suppose $\phi_{ij} \geq \varphi_3$ for all $i \neq j$. Then $s_{ij} > b$ for some $i \neq j$ implies $\|s_i\| > b$ and $\pi_i(s_i, s_{-i}^*) \leq \Pi_i(s_i) < 0 < \pi_i(s^*)$. Combined with Case 1 this means that s_i^* is the unique best response against s_{-i}^* in S_i . Since this holds true for any i , $(s^*, (q(\cdot)))$ is an SPNE, indeed. Moreover, $0 < s_{ij}^* < 1$ for all $i \neq j$ as asserted.

It remains to be shown that s^* is unique. To this end, consider the system of linear equations in s given by (10), (3), and (7). It can be summarized in the form $s = y + As$ or $(I - A)s = y$ where I is the $n(n-1) \times n(n-1)$ identity

matrix and $y + As$ is the right-hand side of the system (10). The constant vector y and the square matrix A are determined by the model parameters. *Ceteris paribus*, the entries of A can be made arbitrarily small in absolute value by setting the advertising cost parameters ϕ_{ij} , $i \neq j$, sufficiently large. Choose $\varphi_4 \geq \varphi_3$ so that this holds true if $\phi_{ij} \geq \varphi_4$ for all $i \neq j$. Suppose that in fact $\phi_{ij} \geq \varphi_4$ for all $i \neq j$. In that case, the matrix $I - A$ has a dominant diagonal and, consequently, is non-singular [by Theorem 1 in McKenzie (1960)]. Therefore, the system $(I - A)s = y$ has a unique solution s^* . Hence there exists only one SPNE where all firms choose positive quantities and advertising levels.

Under the hypothesis that $V_{ij} > 0$ for all $i \neq j$ and (20) for all i , we have shown existence of a number $\varphi_4 > 0$ such that if $\phi_{ij} \geq \varphi_4$ for all $i \neq j$, then the conclusion of Proposition 1 holds. This completes the proof.

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