Open Source vs Closed Source Software: Public Policies in the Software Market *

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Abstract

This paper analyses the impact of public policies supporting open source software (OSS). Users can be divided between those who know about the existence of OSS, the "informed" adopters, and the "uninformed" ones; the presence of uniformed users yields to market failures that justify government intervention. We study three policies: *i)* mandatory adoption, when government forces public agencies, schools and universities to adopt OSS, *ii)* information campaign, when the government informs the uninformed users about the existence and the characteristics of OSS and, *iii)* subsidisation, when consumers are payed a subsidy when adopting OSS. We show that the second policy enhances welfare, the third is always welfare decreasing while mandatory adoption can be either good or bad for society depending on the number of informed and uninformed adopters. We extend the model to the presence of network effects and we show that strong externalities require "drastic" policies.

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1 Introduction

Open source software is currently one of the most debated phenomena in the software industry, both theoretically and empirically. At the most basic level, the term open source software simply means software for which the source code is *open* and *available*. The source code is the program in which a software is originally written. A software is said "open" when its source code can be read (seen) and written (modified) by everybody. Availability implies that anybody can acquire the code either free of charge or for a nominal fee (usually media and shipping charges or online connection charges).[1]

In recent years, the growth and development of the open source movement has been boosted by the Internet: today, making a source code available can be as simple as posting it on the World Wide Web or in an online newsgroup. Furthermore, making the software open is also extremely simple, i.e. place no restrictions on how the software is actually used or by whom.

Typically, open source software has been extremely successful in those segments of the market where the potential purchasers are "sophisticated users", i.e. system and server administrators or more generally those that are experienced in handling computers and that, for this reason, are well aware of all various packages available. Just to take a relevant example, the open source software Apache is currently the most popular software for web servers; its market share is about 60% of the total, more than two times larger than Microsoft, its "closed source" commercial rival.[2] Other examples of well established open source softwares are Sendmail, the dominant messaging service program for routing and handling email by email servers and Linux, an operating system which is probably the best-known example of the emerging open source software movement, which has a current market share of about 30%.[3]

Open source software has recently attracted a great amount of attention. Many researchers have focused on explaining where contributors find their motivations to develop new software or to improve the existing one, provided that open source software and its further developments are usually made available at zero price. Any software improvement is a costly activity, and supplying it for free does not reflect these costs; other benefits related to career concerns and/or ego gratification must be taken into account when analysing motivations for open source software developers. Lerner and Tirole (2002) and

[1] All the details related to the open source software movement can be found at the web site `www.opensource.org/`.

[2] Since 2000 Netcraft.com is counting the active web servers. See `www.netcraft.com`.

[3] There are many others very well known open source software packages such as the programming language PERL, the standard for secure communications over the Internet OpenSSL, the world famous browser Mozilla or the database MySQL.

Lakhani and von Hippel (2000) go in this direction. A different explanation has been offered by Raymond (1999) which stresses on the idea that open source is a form of gift economy based on altruistic motives.[4]

The open source software is attracting a lot of attention not just in the computing and in the academic communities but, recently, also in the political arena. Many governments around the world are actually supporting open source software or are in the process of supporting it, thus tilting the playing field towards open source.[5]

Various rationales have been put forward to justify active government policies. Supporters of such interventions argue that open source is technically superior with respect to the closed one, that it is more stable and secure since it allows adopters to check and fix on their own any possible bug and to optimise systems against viruses. Moreover, the possibility of accessing the source code, allows adopters to tailor the software to their specific needs. Also, from a more dynamic perspective, some argue that open source software provides a better "environment" to spur innovation. The availability of the source code makes it easier for software developers to improve upon the already existing programs.

Even though it is not our aim to discuss and evaluate the alleged superiority of open source, we observe that this cannot be considered as a justification for active policy interventions. After all, absent market failures, the market is the natural place where different products compete. If it is technically superior, then more consumers will decide to adopt the open source software rather than the closed source alternative and the market will standardise on the open source without any external intervention. From an economic point of view, governments should intervene only when market forces do not yield a socially efficient allocation.

We believe that an important aspect that has to be taken into account to understand the software market and its possible failures relates to the presence of different typologies of users. As pointed out above, the open source software has been particularly successful in those segments of the market where the potential demand is made of sophisticated users. On the contrary, in the mass-diffusion segments, where the demand typically comes from less sophisticated consumers who are generally not informed about the existence and/or the characteristics of the various packages, open source softwares have hardly

[4]The organisational literature surrounding open source is now quite well developed. A part from the mentioned papers based on the altruistic/ego gratification motives, others strands of literature focus on different motivations for open source software developers. A complete survey of this literature can be found in Casadesus-Masanell and Ghemawat (2003).

[5]A clear, non technical and comprehensive discussion of government policies towards open source software is in Hahn (2002).

gained relevant market shares. Many open source packages that have been developed to target the mass-diffusion segments have failed in this scope. The typical example is OpenOffice, a suite which has been created in direct competition with Microsoft Office in the market for office applications. According to a recent review by the Washington Post, OpenOffice has not gained a significant share of the market in spite of the fact that its performances are comparable with those of the rival software[6] and that it is supplied for free.

The presence of a large fraction of unsophisticated or "uninformed" users, especially in the mass-diffusion segments, is somehow intrinsic in the software industry. While closed source software is produced and sold by commercial firms who have strong incentives to advertise their products and to inform potential adopters about their packages' features, the open source software is usually distributed by individual developers or by news groups, who have different motivations rather than profit maximisation.

Starting from this simple observation, the aim of this paper is to study the effects on social welfare of the various forms of interventions that have been envisaged by many governments around the world. The analysis moves from a recent contribution by Schmidt and Schnitzer (2002); the set up is based on a very simple static model in which two software products are offered. The closed source software is supplied by a commercial firm, while the open source is sold at marginal cost by a fringe of independent developers.

Absent quality differences between products, we assume that potential adopters can be segmented into two distinct categories: the informed users who know about the existence of both the closed and the open source software and the uninformed ones who ignore the existence and/or characteristics of the open source alternative. Clearly, competition between open and closed source software occurs only in the market for informed users and we model it according to a standard Hotelling-like framework.

In the first part of the paper we look at the pure market equilibrium, that is the equilibrium when the government does not intervene; from this analysis we are able to characterise the presence of market failures. The remaining sections of the paper are devoted to present the impact on social welfare of the most commonly proposed government policies intended to correct these market failures. We show that while welfare increases when the government supports open source software through an information campaign aimed at

[6] Rob Pegoraro in an article appeared in the Washington Post ("The office suite and lets you see past Redmond, Washington Post, May 12, 2002, p.H07") states that "After using the Windows version of OpenOffice for the past week and a half, I can attest that it either matches or beats Microsoft Office in features and ease of use, at the cost of slower performance on older computers and the occasional slight garbling of complicated Microsoft Office documents."

informing the uninformed users about the existence of the open source software, a more intrusive and direct policy in support of the open source based on direct subsidisation of those who adopt this software always harms society. Finally, when the policy consists of mandating public agencies, bureaus, public schools or universities to adopt open source software then welfare may increase or not according on how the market is segmented between informed and uninformed users.

In the last section of the paper we extend the model by introducing network externalities, a commonly recognised feature of software markets. Our main message is that when network effects are pervasive enough, the policies work only if they are sufficiently "drastic" and induce the market to standardise on one of the two products. Perfect standardisation allows network effects to be at their maximum level, thus increasing welfare.

Our analysis is essentially static: it does not contemplate neither R&D activity from software producers nor firms' entry and compatibility strategies, which have both dynamic nature.[7] In particular, one of the reasons that has been put forward to justify government intervention in favor of the open source movement is that open source developers have more incentives to innovate than those working for a commercial producer. This is a very interesting and controversial issue: both Smith (2002) and Schmidt and Schnitzer (2002) contrast this argument and argue that it is the marketplace rather than the policy arena to provide the right incentives to ensure continuous innovation.

Bessen (2002) argues in favor of open source; he contends that open source addresses market failures associated with incomplete contracts and asymmetric information: the government should remove the impediments it has imposed to developers through software patents, which tilt the market in favor of proprietary developers.

As it is clear from this concise discussion, the issue of government intervention in the software market is still very much debated. The vast majority of the literature does not provide a sufficiently developed analytical framework able to analyse the complex intricacies of the various issues at stake; our contribution is a first, although simple, effort to try to fill this gap.

1.1 Government policies towards open source

To understand current governments thinking on open source it is useful to survey the major public initiatives to support open source software (OSS hereafter).[8]

[7]Note that in this static framework, the main and only characteristic of open source software is that it is offered free of charge.

[8]We follow Hahn (2002).

Governments are great purchasers of computer software. As reported in Evans and Reddy (2002), the US government alone spent $3.7 billion on software in 2000. A first and direct public intervention in the software market is therefore very simple: whatever its merits and characteristics, mandate adoption of OSS in public administration and at schools or universities.

Many governments have already decided to support OSS in this way: in Latin America, the Brazilian government passed a legislation that mandates OSS to be adopted in municipal governments; in Germany, the Bundestag mandated all public administrations servers to run Linux. In other countries, pending legislations are waiting for parliamentary approval: in Europe, the French parliament is actually discussing a bill that forbids to use anything but OSS. In Italy too a similar legislation is currently under discussion.

Other governments, such as Germany and Singapore, have also adopted a different approach. Singapore is offering tax breaks to companies that use the open source Linux operating system instead of Windows; in Germany, an agreement has been reached between the government and IBM that offers discounts on IBM machines with preinstalled Linux. In both these examples, the adoption of OSS rather than the commercial alternative is subsidised either through a tax saving or a hardware discount.

A third policy, certainly less intrusive than the first two and for this reason also very popular around the world, is based on supporting OSS through promotional campaigns. In the US, many public education consortiums are promoting OSS. Just to take an example, in North Mississippi, in August 2002 the local Education Consortium has adopted a pilot program called "freedom to learn" aimed at campaigning OSS in public schools: students will be introduced to OSS through 2002 and 2003 school years. It must be noted that all the initiatives based on mandating public schools to adopt OSS also fall in this third category: by using OSS at schools, students learn about OSS and this certainly constitutes a form of promotional campaign. In Spain, the parliament is discussing a bill which requires regional governments to take initiatives to promote open source products.

Following these examples, it is possible to distinguish the various public initiatives towards OSS in the following three broad categories: *i)* mandatory adoption, *ii)* subsidisation and *iii)* information campaign.

2 The model

2.1 Firm's behavior

Competition occurs between a closed source software (CSS hereafter) and an open source software. CSS is supplied by a single commercial firm while OSS is offered by a fringe of independent developers.

The two products are horizontally differentiated and we model competition using a standard Hotelling framework; the two products are located at the extremes of a unit length segment: CSS at 0 and OSS at 1.

Open source software is sold at marginal cost while CSS is priced at p by its producer. We assume zero marginal production cost for the two products and that the commercial vendor is not able to discriminate consumers according to their location or to their typology and it therefore charges only one price.

2.2 Consumers

There are two typologies of consumers: *a*) the "informed" users, i.e. those who know about the existence of both CSS and OSS and that take their adoption decision comparing the utility given by each alternative, and *b*) the "uniformed" users, i.e. those who ignore the existence of OSS and therefore when taking their adoption decision consider only the closed source software.

We assume for simplicity that the population of consumers is of mass 1: a portion η are the uninformed and the remaining $1 - \eta$ are the informed ones. Irrespectively on their type, consumers are uniformly distributed on a unit length segment. A consumer located at $x \in [0, 1]$ gets a net utility from buying the closed source software of

$$U_c = v - tx - p, \tag{1}$$

where v is the gross utility from adopting the software, t is a transportation cost and p is the price charged by the CSS producer. t may be interpreted in many ways: the cost of learning how to use the software, the installation cost or the cost of adapting other software applications.

Similarly, the consumer's net utility from adopting OSS is

$$U_o = v - t(1 - x). \tag{2}$$

3 Pure market equilibrium, market failures and government policies

In order to highlight the presence and the characteristics of market failures, and therefore the scope for a possible government intervention, we need to determine the pure market equilibrium, that is, the outcome that arises when no public policy is in place; we then compare this equilibrium with the social optimum. Let us start with the pure market equilibrium.[9]

3.1 Pure market equilibrium

The two software packages compete only in the market of the informed users. In this case, given the price p charged by the producer of the closed source, the indifferent consumer is the one who gets the same net utility from the two versions of the software. Formally, the indifferent consumer is located at x_i, where x_i solves the following:[10]

$$v - tx - p = v - t(1-x) \qquad \Rightarrow \qquad x_i = \frac{t-p}{2t}. \tag{3}$$

Uninformed users ignore that an open source software is available and confront the utility that they receive when adopting CSS with the zero utility of not buying any software. Therefore the indifferent uniformed consumer is the one located at x_u, where x_u is the solution of:[11]

$$v - tx - p = 0 \qquad \Rightarrow \qquad x_u = \frac{v-p}{t}. \tag{4}$$

Given x_i, x_u and the distribution of consumers among informed and uninformed types, the profits for the CSS producer are

$$\pi = p((1-\eta)x_i + \eta x_u). \tag{5}$$

From the first order condition it is easy to derive the profit maximising price:[12]

$$p^* = \frac{t(1-\eta) + 2\eta v}{2(1+\eta)}. \tag{6}$$

[9] The presence of market failures drives our model away from Evans (2002), where it is claimed that the market is "per-se" efficient.

[10] The subscript i stays for "informed".

[11] The subscript u stays for "uninformed".

[12] The second order condition is

$$\frac{d^2\pi}{dp^2} = -\frac{1+\eta}{t} < 0,$$

which is clearly always satisfied.

Given its inability to price discriminate, the firm charges a price p^* which is a weighted average between the optimal prices that it would charge if it were able to discriminate consumers according to their typology; formally, it is easy to rewrite expression (6) as follows:

$$p^* = \frac{(1-\eta)}{(1-\eta)+2\eta}\, p_i^* + \frac{2\eta}{(1-\eta)+2\eta}\, p_u^*, \tag{7}$$

where $p_i^* = \frac{t}{2}$ and $p_u^* = \frac{v}{2}$ are the optimal prices that the firm would charge to the informed and uninformed consumer respectively and where the weights of these two prices depend on the distribution of consumers in the two segments of the market. Clearly, the larger (resp. smaller) the mass of uninformed consumers (η high/resp. low), the closer p^* is to p_u^* (resp. p_i^*). In other words, when setting p^* the closed source software producer cares more of one segment of the market or the other depending on its relative magnitude.

Equilibrium market shares are obtained by replacing expression (7) into x_i and x_u:[13]

$$x_i^* = \frac{t + 3\,\eta\, t - 2\, v\eta}{4t\,(1+\eta)}, \tag{8}$$

$$x_u^* = \frac{2\, v - t + \eta\, t}{2t\,(1+\eta)}. \tag{9}$$

Figure 1 gives a graphical representation of the pure market equilibrium.

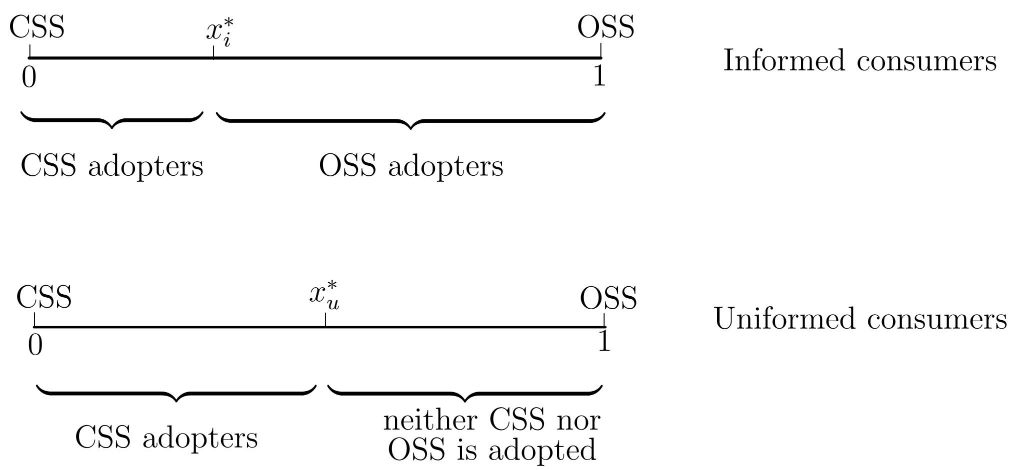

Figure 1: pure market equilibrium

[13]Note that we need to impose restrictions on the parameters in order to ensure the existence of the model. See the Appendix.

It is worth noting that in the segment of the informed consumers, OSS market share is larger than that of CSS. This is an obvious consequence of the fact that the former is given away for free, while the latter is priced at $p^* > 0$. In the segment of uninformed users, CSS is adopted by all those consumers who are located to the left of x_u^*. Consumers located further away from the CSS do not adopt any software at all.

Finally, having defined the equilibrium market allocations and the price, we can derive the consumers surplus of uninformed and informed consumers respectively:

$$S_u = \eta \int_0^{x_u^*} (v - t\mu - p^*) d\mu, \tag{10}$$

$$S_i = (1 - \eta) \left[\int_0^{x_i^*} (v - t\mu - p^*) d\mu + \int_{x_i^*}^1 (v - t(1 - \mu)) d\mu \right]. \tag{11}$$

Total welfare is defined as the sum of consumers surplus and firm's profit: $W = S_u + S_i + \pi$.

3.2 First and second best allocations

Let us now consider the socially optimal solution, namely the allocation that a welfare maximising planner would obtain by assigning consumers with one of the two software packages.

All through the paper we assume that $v > \frac{t}{2}$; this condition ensures that it is efficient to assign each informed consumer with one of the two softwares (either OSS or CSS). It is worth distinguishing between two cases: second best and first best.

3.2.1 Second best

We define second best as the welfare maximising allocation under the constraint that uninformed consumers can only adopt CSS; this is equivalent to say that the regulator decides "who is going to adopt and what" but she takes as given the distribution of consumers between informed and uninformed.

In this case, given that the two softwares have the same gross valuation, v, the social optimum requires the minimization of the transportation costs in the segment of informed consumers; conversely, in the segment of uninformed consumers, the social surplus is maximised when all the individuals for which the gross valuation v is larger than the transportation cost adopt the closed source software.

Remark 1 *(Second best allocation).* The second best is achieved when i) the informed consumers adopt CSS for $x \leq 1/2$ and OSS otherwise and ii) the uniformed consumers adopt the CSS for $x \leq min\{v/t, 1\}$.

3.2.2 First best

By first best we mean the situation in which the planner can also determine the typology of the consumers. This amounts to say that the government is also able to decide the distribution of consumers between informed and uninformed.

Remark 2 *(First best allocation).* The first best is achieved when i) all consumers are informed ($\eta = 0$) and ii) those located at $x \leq 1/2$ adopt CSS and OSS otherwise.

By choosing $\eta = 0$, the planner is able to replicate any possible allocation that it can be obtained when $\eta > 0$. Therefore, when there are no uninformed consumers, the social welfare is at least as large as when $\eta > 0$. Moreover, it is socially optimal to allocate all the (informed) consumers in a way that minimises the transportation costs. As for the second best, this occurs when consumers are equally split between the two products: those to the left of 1/2 adopt CSS, while the remaining adopt OSS. Clearly, the second best allocation converges to the first best as η goes to zero: welfare tends to increase the smaller the mass of uninformed consumers.

3.2.3 Market failures

Market failures can be easily determined by simple comparison of the social optimum with the pure market equilibrium given in expressions (8) and (9); our set-up encompasses three different failures, which are summarised in the following remarks.

Remark 3 *(Market failure 1).* Too few informed consumers adopt CSS: $x_i^* < 1/2$.

As already discussed, in the segment of informed consumers, social optimality would require to split consumers equally between the two products in order to minimise transportation costs. However, market forces do not yield this outcome: since CSS is supplied at p^*, while OSS is available for free, the latter is adopted by "too many" informed consumers.

Remark 4 *(Market failure 2).* Conditional on the presence of uninformed consumers, $\eta > 0$, too few adopt CSS, $x_u^* < min\{v/t, 1\}$.

Remark 5 *(Market failure 3).* Uninformed consumers do not make social optimal adoption decisions.

Failures 2 and 3 are related to the monopolistic segment of the industry, that is that of uninformed consumers. As in a standard monopoly setting, a too little amount of CSS is sold: there is a deadweight loss due to the presence of a monopolistic producer (market failure 2). Furthermore also those consumers who would have optimally adopted OSS either do not buy any software at all or do adopt the wrong one. This means that due to their ignorance about the existence of OSS, some uninformed consumers do not make the socially optimal adoption decision (market failure 3).

3.3 Government policies towards OSS

As we have neatly discussed in the introduction, government policies towards OSS may take essentially three forms. On the one side, governments may force agencies, schools or universities to adopt OSS. Alternatively, a less intrusive policy may be based on informing people about the existence and characteristics of OSS. Finally, the government can subsidise the adoption of the software open source through monetary incentives. We define the first policy as *mandatory adoption*, the second as *information campaign* and the third as *subsidisation*.

Clearly, the government may actually decide to undertake a sort of "policy mix" by combining two or more of these policies. Furthermore, also without an explicit policy mix, both subsidisation and mandatory adoption have intrinsically an informative nature. Never the less, the aim of the paper is to study each policy in isolation from the others, in order to disentangle their "true" welfare effect. The evaluation of a policy mix is therefore a simple combination of the results that we obtain for each policy and it can be easily derived.

The first two policies have the effect of changing the masses of consumers belonging to the two typologies of informed and uninformed, while the third one has the effect of increasing consumers valuation for OSS. Formally, we model these policies in the following way:

1. *mandatory adoption*: the government randomly selects an amount β of consumers and forces them to adopt OSS. As a consequence, the mass of informed consumers reduces to $(1-\beta)(1-\eta)$, while the mass of uninformed consumers reduces to $(1-\beta)\eta$.

2. *information campaign*: the government informs a share α of the uninformed consumers about the existence and characteristics of the OSS. As a consequence, the mass of uninformed consumers becomes $\eta - \alpha$, while that of the informed ones increases to $1 - \eta + \alpha$.

3. *subsidisation*: the government pays a subsidy s to every individual who adopts OSS. Therefore, the gross consumer's valuation of OSS becomes $v + s$.

4 Mandatory adoption

The first policy that we analyse is *mandatory adoption*. With this policy the government forces some or all the public agencies, universities and schools to adopt OSS. More precisely, we consider the case in which the government imposes to these consumers either to adopt OSS or not to adopt any software at all.[14]

It is realistic to assume that, as for the closed source producer, also the regulator ignores the location (i.e. the preferences) of each consumer and also that she cannot distinguish between informed and uninformed users. As a consequence, with this policy the government randomly extracts an amount $\beta \in (0, 1)$ of consumers from the entire population and forces them to adopt OSS. The mass of informed consumers reduces to $(1 - \eta)(1 - \beta)$, that of uninformed consumers reduces $\eta (1 - \beta)$, while β represents the mass of mandated consumers.

Clearly, this policy has the effect of reducing the mass of potential CSS adopters. However, the proportion of informed to uninformed consumers remains unchanged. This fact implies that the weights in expression (7) do not change and that the policy does not alter the equilibrium price p^* nor the equilibrium indifferent consumers defined in expressions (8) and (9).

The impact of the mandatory adoption on total welfare is summarised in the following Proposition.

Proposition 1. *Only if the mass of uniformed consumers is sufficiently large, it is optimal to mandate the adoption of the open source software. Formally there exists a unique* $\bar{\eta} \in (0, 1)$ *such that:*

$$\frac{dW}{d\beta} > 0 \quad \text{if and only if} \quad \eta > \bar{\eta}.$$

[14] Mandated consumers adopt OSS only if $v - t(1 - x) \geq 0$, while they do not adopt any software if $v - t(1 - x) < 0$.

The effect of the policy on the surplus of the various agents can be easily understood. Since the mass of CSS potential adopters reduces, profits for the commercial firm decline. Conversely, the effect on consumers' surpluses depends on the typology of users. Given that p^* does not change, the surplus of those consumers who are not mandated to adopt OSS is not altered. The informed consumers who are now mandated to adopt OSS are, overall, worse-off: those who shift from CSS to OSS as a consequence of the policy obtain a strictly smaller level of utility, while those who were already adopting the OSS are not affected by the policy. On the contrary, the impact on the uninformed consumers who are now mandated by the government depends on their location: while the policy harms those uninformed that have strong preferences towards CSS (formally, those located closed to the origin in figure 1b), the uniformed that without the policy were not buying any software are clearly better-off. On average, the latter positive effect dominates the former.

From this discussion, it is clear that only the uniformed users who are mandated by the government to adopt OSS may benefit from the policy. Therefore, according to Proposition 1, mandatory adoption is welfare increasing only when the mass of uninformed users is sufficiently large.[15]

In terms of market failures, since x_i^* and x_u^* remain unchanged, the policy does not affect market failures 1 and 2: the allocation of informed consumers does not minimise the transportation costs and there is still the deadweight loss in the segment of uninformed users. On the contrary, mandatory adoption has a positive effect on type 3 market failure since some uninformed consumers that previously were not adopting any software, now buy OSS, while others efficiently move from CSS to OSS.

5 Information campaign

Through an *information campaign*, some uninformed consumers receive information about the existence and the characteristics of OSS. Formally, this policy consists in moving a fraction $\alpha \in [0, \eta]$ of consumers from the uninformed to the informed segment of the market. As a consequence, the mass of consumers that remain uninformed reduces to $\eta - \alpha$, while that of informed becomes $1-\eta+\alpha$. In our analysis we assume that the government implements such a policy at no cost.

[15] This result contrasts with Schmidt and Schnitzer (2003) where mandatory adoption always hurts society; this is due to the strong assumption that while the firm is not able to discriminate between informed and uninformed consumers, the government actually discriminates consumers and selects only the informed ones when mandating the adoption of the open source software.

The study of this case closely resembles the one conducted when presenting the pure market equilibrium. However, as the proportion of informed to uninformed consumers changes, the optimal price charged by the commercial producer is also altered. As it is shown in the appendix:[16]

$$\frac{dp^*}{d\alpha} > 0 \quad \text{if and only if} \quad t > v.$$

Consider the expression of the pure market optimal price (7). Since the information campaign increases the proportion of informed users, then the optimal price converges towards p_i^*. It is easy to verify that when $t > v$, then $p_i^* > p_u^*$ and therefore in this case the optimal price increases with α.

The next Proposition characterises the effect on total welfare of the information campaign:

Proposition 2. *Supporting the open source software through a costless information campaign always increases welfare:*

$$\frac{dW}{d\alpha} > 0 \quad \forall \quad \alpha \in [0, \eta].$$

As in the case of mandatory adoption, the effect of the policy on producer surplus is straightforward: since the policy shrinks the captive market of uninformed users, it undoubtedly harms the firm.

To understand the effect of the information campaign on consumer's surplus we need to distinguish between two cases depending on whether the price charged for the closed source software decreases (this happens when $t \leq v$) or increases ($t > v$) with α. Suppose that p^* decreases with α; in this case all the consumers are better-off. More precisely, while the informed consumers who adopt CSS benefit from a lower price, the policy does not have any impact on those who adopt OSS. Uninformed consumers who adopt CSS benefit from a reduction in p^* while those who do not adopt any software are unaffected. Finally, all the uninformed consumers who become informed as a consequence of the campaign, are better-off. If they adopt CSS they are charged a lower price, while if they adopt OSS they benefit from a choice that was not available before.

From this discussion it is clear that when the policy induces the firm to charge a lower price, the information campaign makes the market working better in all respects; each market failure is lessened by the policy: it increases

[16]In the proof of Proposition 2, we show that the firm's optimal price in this case is:

$$p^* = \frac{t\,(1-\eta+\alpha) + 2v\,(\eta-\alpha)}{2\,(1+\eta-\alpha)}. \tag{12}$$

It easy to verify that $\frac{dp^*}{d\alpha} > 0$ if $t > v$.

the share of consumers having adopted CSS both in the segment of informed and in the segment of uniformed consumers (failures 1 and 2) and, obviously, it induces some previously uninformed users to take a more appropriate decision (failure 3). Therefore, it should not be a surprise that the total welfare increases with α.

However, as stated by Proposition 2, the information campaign always increases welfare, also when it induces an increase in the price of the closed source software. In this case, market failures 1 and 2 become more severe but these negative effects are still dominated by the positive effect on market failure 3. In other words, the fact that an additional fraction α of consumers is now able to make a more informed adoption decision makes the proposed policy always welfare enhancing.

6 Subsidisation

We end this first part of the paper with the analysis of the last policy: government subsidisation of OSS.

We model this policy by assuming that the government pays a monetary transfer s to each individual who adopt the open source software; formally, this amounts to assuming that the gross utility from adopting OSS becomes $v + s$. Moreover, we also assume that the cost of the subsidy is entirely borne by the society through lump sum taxation.[17] The following proposition can be easily proved:

Proposition 3. *Subsidising the open source software always reduces welfare:*

$$\frac{dW}{ds} < 0.$$

The policy clearly induces the CSS producer to charge a different price for its product. Nevertheless, to get more intuition of Proposition 3, consider first what happens if p^* does not change. The utility of all those consumers who adopt OSS increases of an amount s. However, this positive effect is exactly offset by the increase in government expenditures and, therefore, for these consumers there is no net welfare effect. Similarly, also for those consumers who adopt CSS there is no effect on social welfare: their utility does not change nor the government pays any subsidy to them. On the contrary, for those consumers who move from CSS to OSS as a consequence of the policy, the net effect on welfare is strictly negative. The increase in the utility of these consumers does not compensate the increase in government expenditure.

[17]For simplicity, we assume that taxation does not have additional distortionary effects.

Without subsidies, CSS is their preferred software, and when moving to OSS they get higher utility but the increase is certainly smaller than s.

Finally, the policy obviously hurts the producer of the closed source software; it is possible to show that the firm has to cut its price in order to protect its market. While consumers benefit from this price decrease, the overall welfare effect of a subsidy is negative.

In terms of market failures, while subsidising OSS does not impact type 3 market failure, it mitigates failure 2 and it worsens failure 1. Given that the subsidy induces the commercial software producer to lower its price then the deadweight loss in the uninformed segment is reduced. However, it is possible to verify that the reduction in p^* does not entirely compensate the subsidy payed to OSS adopters and, as a consequence, OSS market shares enlarges. This clearly accentuates market failure 1.

7 Public policies with network externalities

A relevant feature of software markets is the presence of network externalities: the individual benefit from adopting a certain software is positively affected by the number of other individuals having adopted the same software or a compatible one. This effect lies on the simple observation that the more widespread a package, the easier is to exchange files and information with other users and, therefore, the higher the utility from adopting that particular software.[18]

It is interesting to extend our analysis by introducing network externalities into the model and to verify how our results change when these effects are taken into account. The simplest way to model network effects at the individual level is to introduce an additive term in the utility functions (1) and (2):[19]

$$U_c = v + \theta N_c - tx - p, \qquad U_o = v + \theta N_o - t(1-x).$$

where N_i represents the number of consumers adopting software $i = open$, *closed* and $\theta \geq 0$ measures the strength of network effects:[20] the greater θ, the larger the individual utility from joining a network with other N_i users. The analysis goes in the same way as in the previous section: we start by

[18] Despite the relevance of network externalities has been commonly accepted in software markets, there are only few econometric papers that try to prove empirically the role of network effects in the software industry. These works mainly focus on the spreadsheets segment of the market; see Gandal (1994) and Brynjolfsson and Kemerer (1996).

[19] We follow a standard way to model network externalities, see Shy (2001).

[20] We are also assuming that closed and open source software are incompatible. This amounts to saying that network externalities are based only on the number of users adopting the same software. The case of compatible software is less relevant in practice and also of less theoretical interest.

characterising the socially optimal allocation and we use this as a reference point for the following discussion.

The most interesting case is when network effects are not too strong; in this case it is possible to prove the following proposition:

Proposition 4 (Second best with network effects). *With low network externalities, $\theta \leq \frac{t-v}{2}$, the second best is achieved when i) informed consumers adopt CSS for*

$$x \leq x_i^{sb} = \frac{2\,\theta\,(t - 2\,\theta\,\eta\,(1-\eta) - v\eta) - t^2}{2\,[2\,\theta\,(t - \theta\,\eta\,(1-\eta)) - t^2]}, \tag{13}$$

and OSS otherwise and ii) uninformed users adopt CSS for

$$x \leq x_u^{sb} = \frac{(\theta\,(1-\eta) + v)\,(2\,\theta\,(1-\eta) - t)}{2\,\theta\,(t - \theta\,\eta\,(1-\eta)) - t^2}. \tag{14}$$

From Proposition 4 an interesting consequence follows immediately:

Corollary 1. *Second best CSS market shares increase with the strength of network effects:*

$$\frac{dx_i^{sb}}{d\theta} > 0, \qquad \frac{dx_u^{sb}}{d\theta} > 0.$$

This corollary has a clear explanation. In the second best scenario, the planner takes the mass of uninformed consumers as given; therefore, since these consumers can only be assigned with CSS, the closed source software has a larger potential installed base. A software with a wider installed base induces larger network effects which translate into higher welfare levels: with network externalities, the government tends to prefer CSS to OSS and this preference is stronger the larger the externalities. According with Corollary 1, the stronger the network externalities, the larger the socially efficient closed source software market share.

Corollary 1 provides also the intuition for what happens when θ is large. When network effects are very strong, the second best requires standardisation towards CSS.[21]

Unfortunately, even in this extremely simplified framework, we are not able to formally characterise the impact of the various policies on social welfare. For this reason, we proceed as follows; in the first part, we restrict the analysis

[21] Note that the first best is achieved when $\eta = 0$ and, for θ large, with full standardisation towards either CSS or OSS. Indeed, in the first best scenario the potential installed base of the two softwares is the same: 1. However, this is a trivial result, which is also of little empirical evidence given that network effects do not have such a pervasive role.

to a neighborhood of $\theta = 0$; this allows us to evaluate analytically how the effectiveness of the policies changes with small network effects. In the last part of the paper we extend the model to the analysis of larger values of θ through numerical simulations.

7.1 A local analysis

A first and insightful discussion about the effectiveness of the proposed government policies in the presence of network externalities can be conducted locally, by analysing the model in a neighborhood of $\theta = 0$. Formally, we consider the derivatives with respect to θ of the previous policies and we evaluate their sign at θ approaching zero: $sign\left\{d\left(\frac{dW}{di}\right)/d\theta\right\}\big|_{\theta\to 0}$, where i=β, α and s; when the sign is positive, the externality reinforces the impact of the policy when this has a positive impact, and it makes it less damaging when it has a negative welfare effect.

Since the role of network effects is similar in all the three policies presented in this paper, we focus only on the case of mandatory adoption.[22]

Proposition 5. *The presence of small network effects, makes the policy more effective (or, equivalently, makes it less damaging) if there are few uninformed users; with larger shares of uninformed consumers, this happens only if the government mandates a sufficiently large number of consumers to adopt OSS; formally:* $\left.\frac{d\left(\frac{dW}{d\beta}\right)}{d\theta}\right|_{\theta\to 0} > 0$ *if i)* $\eta < 0.139$ *and ii) if* $\eta > 0.37$ *provided that* $\beta > \hat{\beta}$*, with* $\hat{\beta} \in (0,1)$.

From the analysis of the second best, we know that with network externalities, welfare increases as long as the market tips towards standardisation on one software: in this case network effects, and therefore the welfare, are at their maximum level.

When η is small, the vast majority of consumers is informed and since OSS is provided for free, it gains a larger users' installed base than CSS. By mandating additional consumers to adopt OSS, the government tilts the market even further towards open source software; this reinforces the positive welfare effect of a larger installed base and improves the performance of the policy. Therefore, as stated in Proposition 5, when η is small the presence of network effects makes mandatory adoption more effective (or less damaging in case the policy has an overall negative effect on welfare) in enhancing social welfare. On the contrary, when the number of uninformed consumers is large enough,

[22] The analyses of the information campaign and of the subsidisation policy go exactly in the same way and we make them available on request.

then OSS can no longer enjoy of a larger installed base. As a consequence, the presence of network effects improves the effectiveness of the policy only if the government is able to influence the adoption decision of a great amount of consumers thus making OSS the software with the largest installed base.

Similar arguments can be applied when the government acts via an information campaign or through subsidisation.

7.2 Some simulations

The main message emerging from the previous section is that the presence of network effects makes the policies in support of OSS more effective (or less damaging) when the open source software has the largest installed base or when it is able to gain it thanks to the policies themselves. However, this analysis does not allow us to fully appreciate the role of network effects, that is to discuss whether the various policies increase or decrease welfare in the presence of network externalities. In this final section we provide a more global analysis on the role of network effects; we do so by resorting to numerical simulations.

Consider the case of mandatory adoption. Figure 2 plots different cut-off functions $\overline{\eta}(\beta)$: these functions give for any value of β the corresponding value of $\overline{\eta}$ such that $dW/d\beta > 0$ for $\eta > \overline{\eta}(\beta)$. The plot given in figure 2 has been drawn for $v = 1$ and $t = 1.3$ and for five values of the externality: $\theta = 0, 0.05, 0.1, 0.15, 0.2$.[23]

When $\theta = 0$, $\overline{\eta}(\beta)$ is constant: $\overline{\eta}(\beta) = \overline{\eta} \approx 0.8$; as we already know from Proposition 1, the policy increases social welfare if and only if $\eta > \overline{\eta}$. Figure 2 shows that for larger values of θ, the cut-off function $\overline{\eta}(\beta)$ becomes negatively sloped.

This implies that *i*) when the policy mandates few consumers, i.e. β close to the origin, then the policy has a positive welfare effect only if there are few uninformed consumers, fewer than without network effects ($\eta > \overline{\eta}(\beta) > \overline{\eta}$) and that *ii*) when the policy mandates a large number of users to adopt OSS, then it is good for society also if there are many uninformed users ($\eta < \overline{\eta}(\beta) < \overline{\eta}$).

Moreover, since $\overline{\eta}(\beta)$ becomes steeper as θ increases, impacts *i*) and *ii*) become more relevant as the externalities gets stronger.

[23] These are reasonable values of the parameters; we have run a complete scan of the parameters space and we have obtained plots with the same characteristics.

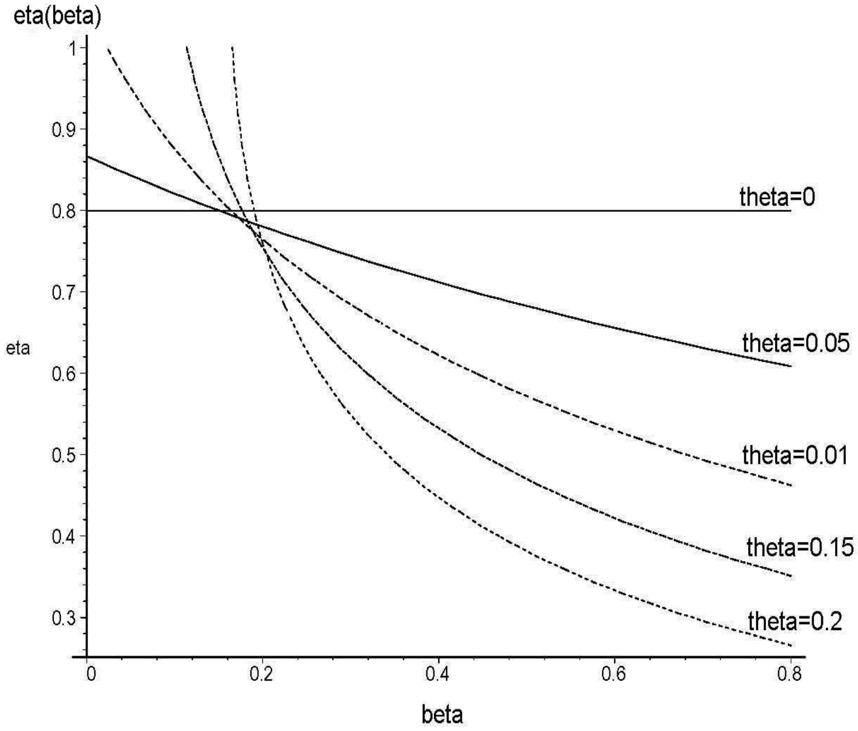

Figure 2: the function $\eta(\beta)$

From this analysis we conclude that with significant network externalities, the impact on social welfare of a policy based on mandatory adoption depends on the amount of consumers that the government is able influence. Given that with network externalities welfare tends to be maximised when there is standardisation towards one software, then mandatory adoption is more likely to be welfare increasing when the government is able to mandate the adoption of a large number of consumers, thus tilting the market towards OSS.

Similar conclusions can be derived in case of an information campaign. From Proposition 2 we know that without network externalities this policy always improves social welfare. This remains true also for small values of θ. However, with stronger network effects things may change radically. In Figure 3 we plot different welfare functions $W(\alpha)$ for five values of $\theta = 0, 0.2, 0.4, 0.6, 0.8$ and 1.[24]

[24]Again, we have plotted these functions for $v = 1$ and $t = 1.3$; furthermore, we have assumed $\eta = 0.5$. Similar results are obtained with different values.

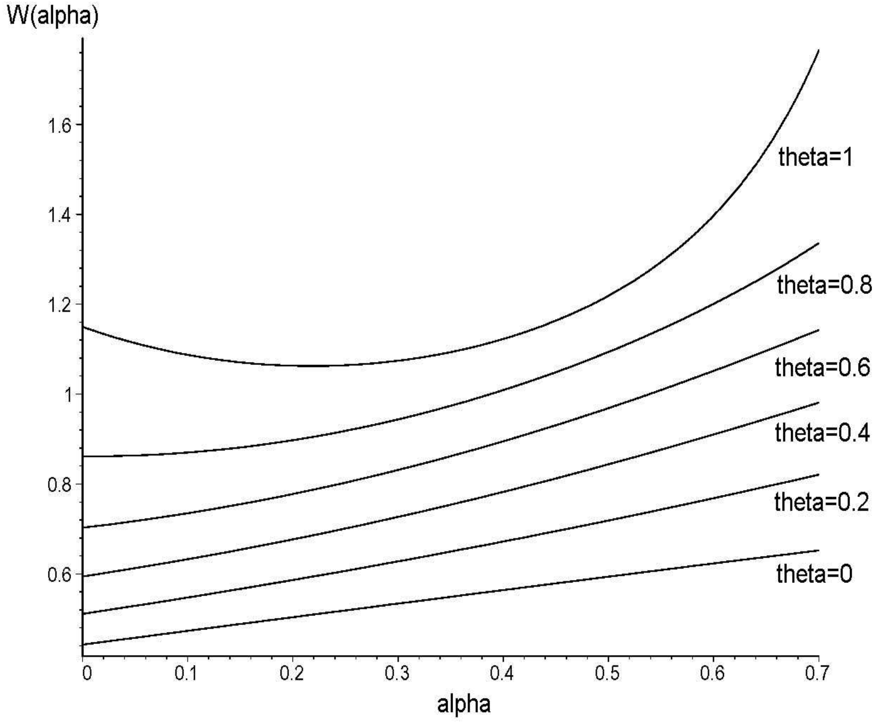

Figure 3: the function $W(\alpha)$

As expected, $W(\alpha)$ shifts upwards as θ enlarges: for given α, welfare is higher the stronger the externality. As shown in the figure, for low values of θ, $W(\alpha)$ is always positively sloped. However, when θ is large (i.e. $\theta = 1$), $W(\alpha)$ becomes U-shaped: for low values of α the information campaign reduces welfare. Again, the intuition for this result relies on the presence of network externalities. When θ is large, welfare tends to be maximised when there is one software that prevails on the other. However the presence of a sufficiently large mass of uninformed consumers makes CSS the software with the largest installed base; this implies that market forces themselves move towards standardisation on CSS and this is efficient in terms of welfare. Therefore, in this case, informing few consumers (α low) about the existence of OSS can only induce a deterioration in the level of welfare. Some of these consumers are induced to shift to OSS but they do not internalise the negative effect that this decision produces on those consumers who continue to adopt CSS.

On the contrary, as in the case of mandatory adoption, the information campaign has a positive effect on welfare when the government informs many consumers (α large) since in this case the regulator helps the market to standardise on OSS.

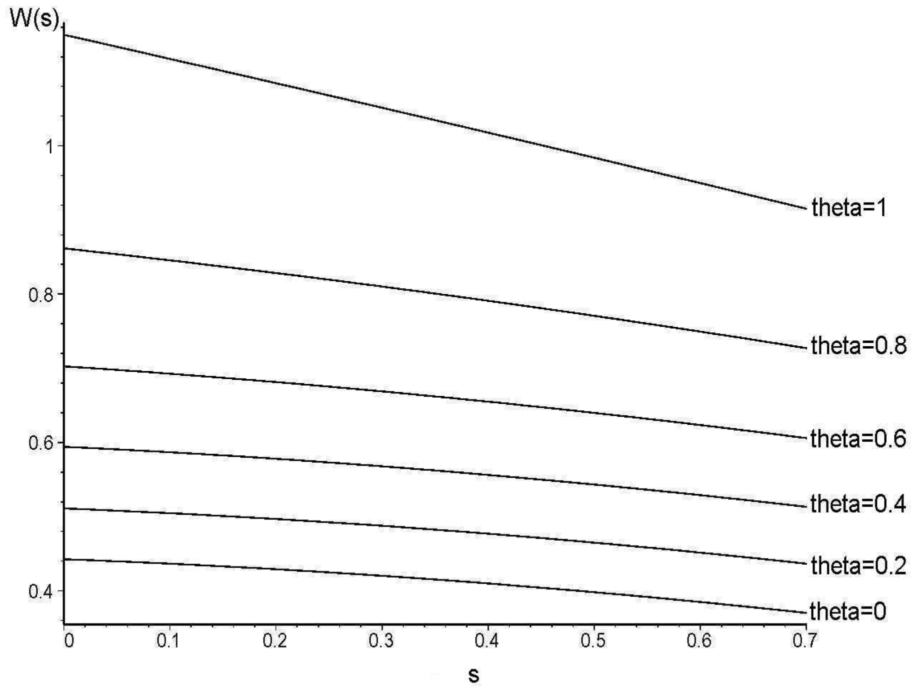

Figure 4: the function $W(s)$

Finally, consider the case of a public subsidy payed to OSS adopters. Figure 4 shows total welfare as a function of the subsidy when $\theta = 0, 0.2, 0.4, 0.6, 0.8$ and 1.[25]

Again, as θ gets larger, $W(s)$ shifts upwards. In this case $W(s)$ is always negatively sloped irrespective on the strength of network effects: subsidising OSS always harms society.

8 Conclusion

In this paper we have analysed the impact on social welfare of the three most popular government policies in support of the open source software that have been proposed in many countries. The analysis has been conducted using a

[25]Again, we have plotted these functions for $v = 1$ and $t = 1.3$ and $\eta = 0.5$ but similar plots can be obtained with different parameters' values.

static model in which an open source and a closed source/commercial software compete. According to the observation of what happens in the reality, we divide consumers according to their knowledge of the open source: the more experienced consumers usually know the existence and the characteristics of the software open source while the others, less experienced users, may ignore it; in the first case they are said "informed", while they are "uninformed" otherwise. Competition occurs only in the segment of informed consumers.

We have shown that while the promotion of OSS through public subsidies paid to OSS adopters always induces a decline in the level of social welfare, implementing a less intrusive policy such as information campaign may actually increase welfare. Finally, the policy aimed at mandating OSS adoption in public agencies, schools and universities improves welfare only if the market is characterised by a large mass of uniformed consumers.

In the last part of the paper, we have extended the model in order to consider the case of network externalities, a situation which is extremely relevant in the software market. As expected, with significant network effects mandatory adoption and information campaign increases welfare only when they are "drastic": when the government is able to influence a large mass of consumers then the market tilts towards the open source software and network effects are at the maximum level thus pushing welfare up. On the contrary, subsidisation always lower social welfare no matter the magnitude of network externalities.

References

Bessen, J. (2002). *What Good is Free Software?* in Hahn (2002).

Brynjolfsson, E. and Kemerer, C. (1995). Network Externalities in Microcomputer Software: An Econometric Analysis of the Spreadsheet Market. MIT Center for Information Systems Research, working paper no. 265.

Casadesus-Masanell, R. and Ghemawat, P. (2003). Linux vs. Windows: Modeling Competition between Open-Source and Closed Software. Harvard Business School, mimeo.

Evans, D. S. (2002). *Politics and Programming: Government Preferences for Promoting Open Source Software.* in Hahn (2002).

Evans, D. S. and Reddy, B. (2002). Government Preferences for Promoting Open-Source Software: a Solution in Search of a Problem. N.E.R.A. working paper.

Gandal, N. (1994). Hedonic Price Indexes for Spreadsheets and an Empirical Test of the Network Externalities Hypothesis. *RAND Journal of Economics*, 25:160–170.

Hahn, R. W. (2002). *Government Policy toward Open Source Software.* AEI-Brooking Joint Center for Regulatory Studies, Washington D.C.

Lakhani, K. R. and von Hippel, E. How Open Source Software Works: "Free" User-to-Assistance.

Lerner, J. and Tirole, J. (2002). Some Simple Economics of Open Source. *Journal of Industrial Economics*, 50:197–234.

Raymond, E. (1999). *The Cathedral and the Bazaar: Musings on Linux and Open Source from an Accidental Revolutionary.* Sebastapol: CA: O'Reilly and Associates.

Schmidt, K. and Schnitzer, M. (2003). Public Subsidies for Open Source? Some Economic Policy Issues of the Software Market. CEPR Discussion Paper, No. 3793.

Shy, O. (2001). *The Economics of Network Industries.* Cambridge University Press.

Smith, B. L. (2002). *The Future of Software: Enabling the Marketplace to Decide.* in Hahn (2002).

Appendix

Pure market equilibrium: restrictions on the parameters. In order to ensure the existence of the model, we need to check when the parameters satisfy the following conditions:

$$0 \leq x_i^* \leq 1, \qquad 0 \leq x_u^* \leq 1, \qquad U_c^* = v - tx_i^* - p^* \geq 0.$$

where p^*, x_i^* and x_u^* are given in expressions (7), (8) and (9).
The first two conditions guarantee that at the market equilibrium the indifferent consumers lie on the $[0, 1]$ segment; the third condition ensures that the indifferent informed consumer gets non negative utility.
From the first condition derives that:

$$v \leq \frac{t(1+3\eta)}{2\eta},$$

while from the second one:

$$\frac{t(1-\eta)}{2} \leq v \leq \frac{t(3+\eta)}{2}.$$

Finally, the indifferent informed consumer enjoys a non negative utility if:

$$v \geq \frac{t(3+\eta)}{2(2+\eta)}.$$

Putting all these conditions together it is easy to verify that the model exists for:

$$v \in (\underline{v}, \overline{v}), \qquad \text{where} \quad \underline{v} = \frac{t(3+\eta)}{2(2+\eta)}, \quad \text{and} \quad \overline{v} = \frac{t(3+\eta)}{2}.$$

Proof of Proposition 1. When the government extracts β consumers form the entire population and forces them to adopt OSS, firm's equilibrium profits are simply $(1-\beta)$ times the profits given in (5):

$$\pi = (1-\beta)\, p \left((1-\eta)\frac{t-p}{2t} + \eta \frac{v-p}{t} \right).$$

and firm's optimal price is the same as in the pure market equilibrium. Moreover, the range of existence of the model is the same as in the pure market equilibrium case.
Two cases must be considered: *i*) the gross utility v is not big enough and among the mandated consumers, only those located "non too far" from the

right end of the segment adopt OSS, and *ii*) v is so high that, irrespective of their location, all the selected consumers buy the OSS. Formally: *i*) $v < t$ such that only those located in $(\frac{t-v}{t}, 1)$ adopt OSS, and *ii*) $v \geq t$. Note that $t \in (\underline{v}, \overline{v})$ so that case *i*) is defined only for $v \in (\underline{v}, t)$ while case *ii*) is defined for $v \in (t, \overline{v})$.

The net consumer surplus of the informed, the uninformed and the mandated consumers are respectively:

$$S_i = (1-\eta)(1-\beta)\left[\int_0^{x_i^*} (v - t\mu - p^*)\, d\mu + \int_{x_i^*}^1 (v - t(1-\mu))\ d\mu\right],$$

$$S_u = \eta(1-\beta)\int_0^{x_u^*} (v - t\mu - p^*)\, d\mu,$$

$$S_m = \beta \int_{max\{0, \frac{t-v}{t}\}}^1 (v - t(1-\mu))\, d\mu.$$

where in the expression for S_m we have incorporated the two possible cases *i*) and *ii*). Let us prove the proposition for case *i*); the proof for case *ii*) goes in the same way and it is omitted for brevity.[26]

In this case, total welfare which is defined as the sum of consumers' and producer's surplus, $W = S_i + S_u + S_m + \pi$, is:

$$W = \frac{((4(1-\beta)\eta^2 + 8(\eta+\beta))v^2 + t(1-\beta)(1-\eta)(4(3\eta+4)v - t(3\eta+5)))}{16(1+\eta)t}.$$

Differentiating this expression with respect to β:

$$\frac{dW}{d\beta} = \frac{-(3t^2 - 12vt + 4v^2)\eta^2 - 2t(t-2v)\eta - 16vt + 8v^2 + 5t^2}{16\,(1+\eta)\,t}.$$

The sign of this expression is given by the sign of the numerator which is a parabola in η, with $\eta \in (0,1)$. It is easy to verify that at the extremes $\eta = 0$ and $\eta = 1$:

$$sign\left\{\frac{dW}{d\beta} sign\right\}\Bigg|_{\eta=0} = sign\{8v^2 - 16vt + 5t^2\}, \qquad sign\left\{\frac{dW}{d\beta}\right\}\Bigg|_{\eta=1} = sign\{4v^2\}.$$

From simple inspection, it is immediate to check that at $\eta = 0$, $dW/d\beta < 0$ for any $v \in (\underline{v}, t)$ while at $\eta = 1$, $dW/d\beta > 0$. This is sufficient to prove that there exists a unique value $\overline{\eta} \in (0,1)$ such that $dW/d\beta < 0$ for $\eta < \overline{\eta}$ and $dW/d\beta > 0$ otherwise. □

[26] Proof available on request.

Proof of Proposition 2. Firm's profits are:

$$\pi = p\left((1-\eta+\alpha)\frac{t-p}{2t} + (\eta+\alpha)\frac{v-p}{t}\right).$$

Solving the first order condition, firm's optimal price is:[27]

$$p^* = \frac{t\,(1-\eta+\alpha) + 2v\,(\eta-\alpha)}{2\,(1+\eta-\alpha)}. \tag{15}$$

Unlike for the case of mandatory adoption, with an information campaign firm's optimal price changes with respect to the pure market case: we need to find the new range of parameters for the existence of the model. It is possible to show that in this case, the model exists for $v \in [\underline{v}, \overline{v}]$, where $\underline{v} = \frac{t(3+\eta-\alpha)}{2(2+\eta-\alpha)}$, $\overline{v} = \frac{t(3+\eta-\alpha)}{2}$.

Replacing (15) into the expressions for the indifferent consumers (3) and (4) and into the consumers surpluses given in (10) and(11), we finally derive the total welfare:

$$W = \frac{4\,(\alpha-\eta)\,(\alpha-2-\eta)\,v^2 - 4t\left(\eta-4-\alpha-6\eta\alpha+3\eta^2+3\alpha^2\right)v + t^2\,(1-\eta+\alpha)\,(3\alpha-3\eta-5)}{16t\,(1+\eta-\alpha)}. \tag{16}$$

Differentiate with respect to α and get:

$$\frac{dW}{d\alpha} = \frac{(3\,t^2+4\,v^2-12\,vt)\,(\alpha-\eta)\,(\alpha-2-\eta) + 8\,v^2 + 7\,t^2 - 20\,vt}{16t\,(1+\eta-\alpha)^2}. \tag{17}$$

This function decreases with alpha; to see this, take the second order derivative

$$\frac{d^2W}{d\alpha^2} = \frac{(v-t)^2}{2t\,(1+\eta-\alpha)^3},$$

which is always negative for $\alpha \in (0,\eta)$: $dW/d\alpha$ is monotonically decreasing in α. In order to verify that the welfare increases with α it is enough to show that $dW/d\alpha > 0$ at $\alpha = \eta$. From expression (17):

$$\left.\frac{dW}{d\alpha}\right|_{\alpha=\eta} = \frac{20\,vt - 8\,v^2 - 7\,t^2}{16t}.$$

The sign of this expression is given by the sign of the numerator which is concave in v. It is easy to verify that the numerator is positive at $v = \underline{v}$ and at $v = \overline{v}$, which is sufficient to guarantee that it is positive for any admissible value of v. □

[27] The second order condition is $\frac{d^2\pi}{dp^2} = -\frac{1+\eta-\alpha}{t} < 0$ and it is always satisfied.

Proof of Proposition 3. Consider the market for the informed consumers. When the government pays a subsidy s to all those individuals who adopt OSS, the indifferent consumer is located at x_i, where x_i solves the following condition:

$$v - p - tx_i = v + s - t(1 - x_i), \Longrightarrow x_i^* = \frac{t - p - s}{2t}. \tag{18}$$

The location of the indifferent uniformed consumer does not change with respect the pure market equilibrium and it is defined in (4). Firm's profits are therefore:

$$\pi = p\left((1-\eta)\frac{t-p-s}{2t} + \eta\frac{v-p}{t}\right).$$

Solving the first order condition, the optimal price is:[28]

$$p^* = \frac{(t-s)(1-\eta) + 2\eta v}{2 + 2\eta}.$$

Replacing this price into the expressions for the indifferent consumers (18) and (4) and into the consumers surpluses given in (11) and (10), we can derive the welfare as a function of the policy parameter s. Moreover it is possible to verify that:

$$\frac{dW}{ds} = -\frac{(1-\eta)(s + t(1-\eta) + 7s\eta + 2\eta v)}{8t(1+\eta)} < 0.$$

□

Proof of Proposition 4. In the second best scenario the planner takes η as given and chooses the allocation (x_i, x_u) in order to maximise total welfare; denoting with GS_i and GS_u the gross surplus of informed and uninformed consumers respectively, the second best allocation can be derived by solving the following maximisation problem:

$$max_{x_i, x_u} W(x_i, x_u) = GS_i + GS_u,$$

where:

$$GS_i = (1-\eta)\left[\int_0^{x_i} (v + \theta((1-\eta)x_i + \eta x_u) - t\mu)\, d\mu + \int_{x_i}^1 (v + \theta(1-\eta)(1-x_i) - t(1-\mu))\, d\mu\right],$$

[28]The second order condition is $\frac{d^2\pi}{dp^2} = -\frac{1+\eta}{t} < 0$ and it is always satisfied. Moreover, the model exists for $v \in [\underline{v}, \overline{v}]$, where in this case:

$$\underline{v} = \frac{(t-s)(3+\eta)}{2(2+\eta)}, \quad \overline{v} = \frac{t(3+\eta) - s(1-\eta)}{2}.$$

$$GS_u = \eta \left[\int_0^{x_u} (v + \theta((1-\eta)x_i + \eta(x_u)) - t\mu)\, d\mu \right].$$

Total welfare in terms of x_i and x_u is therefore:

$$\begin{aligned} W(x_i, x_u) &= \left(2\theta\,(1-\eta)^2 - t\,(1-\eta)\right) x_i{}^2 + \left(t\,(1-\eta) - 2\theta\,\left((1+x_u)\,\eta^2 - (2+x_u)\,\eta + 1\right)\right) x_i \\ &+ \left(1 - 2\eta + \eta^2 + x_u{}^2\eta^2\right)\theta + \left(v x_u - \frac{t}{2} x_u{}^2 + \frac{t}{2} - v\right)\eta + v - \frac{t}{2}. \end{aligned} \quad (19)$$

Let us define $\overline{\theta} = \frac{t-v}{2}$; in what follows we characterise the second best allocation for $\theta < \overline{\theta}$. $W(x_i, x_u)$ is concave for $\theta < \frac{t}{2}\frac{1-\sqrt{2\eta^2-2\eta+1}}{\eta(1-\eta)}$, with $\frac{t}{2}\frac{1-\sqrt{2\eta^2-2\eta+1}}{\eta(1-\eta)} < \overline{\theta}$.[29] Differentiating expression (19) the second best allocation can be easily derived:

$$x_i^{sb} = \frac{2\,\theta\,\left(-2\,\theta\,\eta\,\left(1-\eta\right) + t - v\eta\right) - t^2}{2\left[2\,\theta\,\left(-\theta\,\eta\,\left(1-\eta\right) + t\right) - t^2\right]},$$

$$x_u^{sb} = \frac{\left(\theta\,\left(1-\eta\right) + v\right)\left(2\,\theta\,\left(1-\eta\right) - t\right)}{2\,\theta\,\left(-\theta\,\eta\,\left(1-\eta\right) + t\right) - t^2}.$$

To complete the proof, we need to verify that $0 \leq x_i^{sb}, x_u^{sb} \leq 1$. By simple algebraic manipulation, one can note that: *i*) a sufficient condition for $x_i^{sb} \geq 0$ is that $\theta < min\{\frac{t}{2(1-\eta)}, \frac{t}{2\eta}\}$; since $\overline{\theta} < min\{\frac{t}{2(1-\eta)}, \frac{t}{2\eta}\}$, then $x_i^{sb} \geq 0$; *ii*) $x_i^{sb} \leq 1$ for $\theta \leq \frac{t^2}{2(t+v\eta)}$, with $\overline{\theta} < \frac{t^2}{2(t+v\eta)}$; *iii*) $x_u^{sb} \geq 0$ for $\theta \leq \frac{t}{2(1-\eta)}$, with $\overline{\theta} < \frac{t}{2(1-\eta)}$ and finally; *iv*) $x_u^{sb} \leq 1$ for

$$2\,\left(1-\eta\right)\theta^2 + \left(2\,v\,\left(1-\eta\right) - t\,\left(3-\eta\right)\right)\theta + t\,\left(t-v\right) \geq 0.$$

This last expression decreases with η.[30] It is possible to verify that for any $\theta \leq \overline{\theta}$, the above inequality holds at $\eta = 1$, which is enough to prove that $x_i^{sb} \leq 1$. This completes the proof. □

Proof of Corollary 1. Taking the derivative of (13) with respect to θ it follows that:

$$sign\left\{\frac{dx_i^{sb}}{d\theta}\right\} = sign\{\eta\,\left(-2\,\left(1-\eta\right)\left(t+\eta\,v\right)\theta^2 + 2\,t^2\left(1-\eta\right)\theta + vt^2\right)\}.$$

[29] Proof available on request.

[30] More precisely, it decreases with η for $\theta > \frac{t-2v}{2}$; given that $v > t/2$, $\frac{t-2v}{2} < 0$ and this expression is decreasing with η for any $\theta \geq 0$.

The above expression is a concave function of θ, it is positive at $\theta = 0$ and at $\theta = \frac{t^2}{2(t+v\eta)} > \overline{\theta}$. Therefore, x_i^{sb} increases with θ for any $\theta \leq \overline{\theta}$. Similarly, taking the derivative of (14) with respect to θ and rearranging it follows that:

$$sign\left\{\frac{dx_u^{sb}}{d\theta}\right\} = sign\{2\,\left((2\,v-t)\,\eta^3 - 4\,(v-t)\,\eta^2 + (2\,v-5\,t)\,\eta + 2\,t\right)\theta^2$$
$$-4\,t\left((t-v)\,\eta^2 - (2\,t-v)\,\eta + t\right)\theta + t^2\,(t\,(1-\eta) + 2\,\eta\,v)\}.$$

This expression does not have real roots in θ and it is always positive for $\theta \leq \overline{\theta}$. □

Proof of Proposition 5. [31] In order to prove the proposition, we need to determine the market equilibrium. We focus on the case with $v > t$ so that all the consumers that are mandated by the government do actually adopt OSS. A similar reasoning applies for $v < t$.
With network effects the location of the indifferent informed and uninformed consumers are obtained by solving the following system of equations:

$$\begin{cases} v + \theta(x_i(1-\eta)(1-\beta) + x_u\eta(1-\beta)) - tx_i - p = v + \theta((1-x_i)(1-\eta)(1-\beta) + \beta) - t(1-x_i), \\ v + \theta(x_i(1-\eta)(1-\beta) + x_u\eta(1-\beta)) - tx_u - p = 0 \end{cases}$$

where the first equation defines the indifferent informed consumer while the second one the indifferent uninformed consumer. Let the locations of the indifferent informed/uninformed consumer when the firm charges the price p, $x_i^{**}(p)$ and $x_u^{**}(p)$, be solution to this system of equation,[32] then the firm sets p to maximize its profit:

$$\pi = p(1-\beta)((1-\eta)x_i^{**}(p) + \eta x_u^{**}(p)).$$

From the first order condition:[33]

$$p^* = \frac{\left(t\left((1-\eta)^2 + \eta\,\beta\,(1-\eta)\right) + (1-\eta)\,v\eta\,(1-\beta)\right)\theta - t^2\,(1-\eta) - 2\,\eta\,tv}{-2\,t\,(1+\eta) + 2\,\theta\,\eta\,(1-\beta)\,(1-\eta)}.$$

The equilibrium indifferent consumers x_i^{**} and x_u^{**} can be obtained replacing this expression into $x_i^{**}(p)$ and $x_u^{**}(p)$. Equilibrium profits and the surplus enjoyed by informed and uniformed consumers are respectively:

$$\pi = p^*(1-\beta)((1-\eta)x_i^{**} + \eta x_u^{**}),$$

[31] This is a sketch of the proof. Full details are available on request.

[32] For the sake of brevity, we omit to present x_i^{**} and x_u^{**} and we make them available on request.

[33] Note that in Proposition 5 we restrict our analysis in a neighborhood of $\theta = 0$. In this case, the second order condition $\left.\frac{d^2\pi}{dp^2}\right|_{\theta=0} = -\frac{(1-\beta)(1+\eta)}{t} < 0$ is verified.

$$S_i = (1-\eta)(1-\beta)\left[\int_0^{x_i^{**}} (v+\theta((1-\eta)(1-\beta)x_i^{**}+\eta(1-\beta)x_u^{**})-p^*-t\mu)\,d\mu \right.$$
$$\left. + \int_{x_i^{**}}^1 (v+\theta((1-\eta)(1-\beta)(1-x_i^{**})+\beta)-t(1-\mu))\,d\mu\right],$$

$$S_u = \eta(1-\beta)\left[\int_0^{x_u^{**}} (v+\theta((1-\eta)(1-\beta)x_i^{**}+\eta(1-\beta)(x_u^{**}))-p^*-t\mu)\,d\mu\right],$$

while the net consumer surplus of those who are mandated to adopt OSS is:

$$S_m = \beta\left[\int_0^1 (v+\theta((1-\eta)(1-\beta)(1-x_i^{**})+\beta)-t\mu)\,d\mu\right].$$

Total welfare is: $W = S_i + S_u + S_m + \pi$.

We can now prove the proposition. By differentiating twice the equilibrium welfare function, it is possible to show that:

$$sign\left\{\frac{d\left(\frac{dW}{d\beta}\right)}{d\theta}\right\}\Bigg|_{\theta\to 0} = sign\left\{A\beta - B\right\}, \tag{20}$$

where

$$A = 4t^2\left(1+\eta\left(3+3\eta^2+\eta\left(8+\eta^2\right)\right)\right) + 2t\left(5\eta^2 v(\eta-1)+3v\eta\left(1-\eta^3\right)\right)+4v^2\eta^2\left(3\eta^2+2\eta+3\right)$$

and

$$B = 2\left(2t^2+6v^2-3tv\right)\eta^4+\left(8v^2+4tv+7t^2\right)\eta^3+\left(-10tv+5t^2+12v^2\right)\eta^2+3\left(4tv-5t^2\right)\eta-t^2.$$

The term A is always positive; therefore $\frac{d\left(\frac{dW}{d\beta}\right)}{d\theta}\Big|_{\theta\to 0} > 0$ if and only if $\beta > \frac{B}{A} \equiv \widehat{\beta}$.

Next, we need to verify when $\widehat{\beta} \in (0,1)$; we do so by using the fact that the model is defined for $v \in (t, \overline{v})$, where $\overline{v} = \frac{t(3+\eta)}{2}$.[34]

It is possible to prove that $\widehat{\beta} < 1$ if and only if $v < \frac{t(5\eta^2+22\eta+5)}{6\eta(1-\eta)}$; since $\overline{v} < \frac{t(5\eta^2+22\eta+5)}{6\eta(1-\eta)}$, then $\widehat{\beta} < 1$. Moreover, $\widehat{\beta} > 0$ if and only if $v > \widetilde{v}$, where $\widetilde{v} = \frac{t\left(\eta\left(5-2\eta+3\eta^2\right)-6+\sqrt{(1-\eta)(\eta(39\eta^2+89\eta+80)+48)(1+\eta)^2}\right)}{4\eta(\eta(2+3\eta)+3)}$. It can be verified that

[34]Remember that we are analysing the model in a neighborhood of $\theta = 0$: the model exists for any $v \in (t, \frac{t(3+\eta)}{2},)$ as in case *ii*) in the proof of Proposition 1.

$\widetilde{v} < t$ for $\eta > 0.37$, while $\overline{v} > \widetilde{v}$ for $\eta > 0.139$.
Summing up, we have proved that *i)* $\widehat{\beta} < 0$, and therefore expression (20) is always positive, for $\eta < 0.139$, and *ii)* $\widehat{\beta} \in (0, 1)$, and therefore (20) is positive for $\beta > \widehat{\beta}$, when $\eta > 0.37$. Finally, when $\eta \in (0.139, 0.37)$, $\widehat{\beta} < 0$ for any $v \in (t, \widetilde{v})$ and $\widehat{\beta} \in (0, 1)$ for any $v \in (\widetilde{v}, \overline{v})$. Therefore, in the former case (20) is always positive, while in the latter it is positive provided that $\beta > \widehat{\beta}$. $\square$