

OPTIMAL DIVISIONALIZATION OF CABLE TELEVISION OPERATORS NETWORKS

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Abstract: In this article, a condition for the optimal division's number is presented, for a market with two cable operators who offer a network service. First, a reason is presented to justify a partial covering of the national market from the cable operators. Second, a problem of moral hazard is revealed, which is able to appear through the implementation of franchising schemes with independent divisions.

Key Words: Optimal Divisionalization, Franchising, Vertical Integration.

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1. INTRODUCTION

The industries whose functioning is based on the creation of structuralized networks (as for example, the cable nets), and who use digital platforms of distribution, have come to assume an increasing importance in the development of the national economies.

These bi-directional networks, that allow upload and download of information flows, need further investigations, that, on the one hand, explore the producer' strategy (designated by operators) in the determination of the optimal number of selling divisions, and, on the other hand, the choices in terms of the type of legal and business relationship, to establish between the agents.

The present article conciliates two research lines, that praise, respectively, the study of the Economic of Networks (which includes the systems whose constitution is based on products or services that present a complementary and interconnectable nature), and the determination of the optimal dimension of the networks (that is, the number of selling divisions that guarantee the maximization of the operator profit).

The main innovation of the present analysis is, the inclusion of the demand curve for network services, proposed by Economides and Himmelberg (1995), and Economides (1996), into the analysis of the optimal dimension of the networks. Moreover, the influence of the characteristics of the network services is explored, which may reveal some complementarities, expressed by network externalities (in production, and consumption) that may influence the strategic decision concerned with the (total or partial) covering of a national market of consumers (designated by subscribers).

Taking into consideration the installed network, the operators must proceed to the optimisation of network externalities in consumption, that are expressed by the service valuation attributed by the consumers, which increases with the number of consumers who subscribe this service.

In the present analysis, one considers a national market with two cable television operators, who offer network services, under the form of integrated packages, a re-evaluation of the incentives for the creation of selling divisions networks, is made. Later, the results of a game that considers the implementation of distinct royalties' modalities are analysed.

This analysis aims to extend the scope of application of literature about the creation of selling divisions networks, to present a condition for the determination of the optimal number of selling divisions applied to the case of a cable television distribution network, and to evaluate the impact of distinct royalties modalities on the profit earned by the operator.

In the first section, a model for the determination of the optimal number of selling divisions is presented, based in a game of two phases, in which participate two cable television operators. In the second section, a game expanded with three phases is developed, taking into consideration the possibility of celebrating franchising contracts.

Finally, the main conclusions related with the development of the model are presented, and the reasons for the necessity of regulation in cable television industry are revealed.

2. THE MODEL

In the works developed by Corchón (1991), Baye, Crocker and Ju (1996), Corchón and González-Maestre (2000), and Bru, Faulí-Oller and Haro (2001) is presented the problematic of the incentives for the companies to create selling divisions networks (which interact in a independent way, in the market). It is also advocated that the costs of creation of these networks tend for zero, considering that Cournot competition scheme leads to a perfectly competitive result, given the dissipation of the oligopoly results.

Nevertheless, the operators can delegate to the administrators of the selling divisions, the production decisions, and modify its behaviour through the implementation of incentives schemes (González-Maestre, 2000).

Instead of the Cournot scheme that is usually considered in literature about the optimal divisionalization, the companies can still enter in a competitive war (Huck, Konrade and Müller, 2001).

The network owner can have a predatory behaviour to its direct competitors, as far as the effect of property of the network generates inter-temporal incomes (Farrel and Katz, 2001).

In the literature concerning to the creation of selling divisions networks, the companies have a strategic incentive to create independent franchisee divisions, assuming that this procedure leads to a more aggressive behaviour in order to increase the market share of the Mother Company (Warren-Boulton, 1974; O'Brien and Shaffer, 1992; Baye et al, 1996; Bru et al, 2001; Dana and Spier, 2001).

In the game presented at the last section of this article, the main economic fact that characterizes the franchising contracts is that, the incentives of the contractual parts do not always coincide (Klein, 1995).

Taking as starting point the pioneering work of Baye et al (1996), a simple model of duopoly applied to the cable television industry is presented, in which it is considered the co-existence of two upstream operators, who sell packages of cable television services, and for the purpose of distribution, use a downstream selling divisions network, or alternatively, an independent franchisee divisions network.

To determine the optimal number of selling divisions, we consider the establishment of competing independent units, as being the process of creation of the selling divisions network (in the company internal environment), even so the model is also applicable equally to the cases where the franchising option is followed (in the company external environment).

For simplification, we consider two identical cable operators, which offer a homogeneous cable television service, and support a constant marginal cost (c). In this formalization is assumed that the selling divisions of the operator support the same marginal cost, in the distribution of the television service.

In this analysis, we consider the transactions of a network service, so the externalities generated by this kind of service allow to explore the possibility of offering complementary services, by each one of the operators, under the form of integrated packages, as far as the number of subscribers who subscribe the same cable television service increases (Economides and Himmelberg, 1995; Economides, 1996; Cabral, Salant, Woroch, 1997; Yang, 1997; Yannelis, 2001).

2.1. PHASE 1: OPTIMAL NUMBER OF SUBSCRIBERS

In phase 1, all the selling divisions have *a la Cournot* behaviour, as independent players, in a simultaneous game, and each operator takes the decision about the number of subscribers to reach, but considering that the market price will depend on the number of subscribers obtained by both operators.

For a certain cable operator, when we consider that the number of subscribers of the potential base (N^e) is equal to the number of subscribers of the installed base (N), the following one is observed:

Proposition 1: Given the configuration in form of inverted U of the demand curve with realized expectations, the maximum profit of a cable operator is obtained in $N = 2/3$. For $2/3 < N < 1$, decreasing prices are observed that do not assure the profit maximization of the cable operator.

Proof: See the appendix ■

To proceed to the resolution of this game, we consider that competition is initiated in two phases. Having n_{ij} , as being the amount of subscriptions obtained by the i th division of operator j , where: $i = 1, \dots, n_j$; e $j = 1, 2$. Additionally, N_{-ij} is considered as being the total amount of subscriptions obtained by all the selling divisions, except the number of subscriptions reached by the i th selling division of operator j .

The profit of the selling division (π_{ij}), can be enunciated as:

$$\pi_{ij}(N_{ij}, N_{-ij}) = N^e(1 - N)n_{ij} - cn_{ij} \quad \text{Eq. (1)}$$

Where: $N = \sum_{i=1}^{n_j} \sum_{j=1}^2 n_{ij}$ = Total number of subscribers in the market of cable television.

We consider that, the i th selling division of operator j , chooses n_{ij} , in order to maximize its profit. This requires that, the condition of profit maximization be respected, given the equality between the marginal revenue (MR) and the marginal cost (MC). This implies that, for any selling division (n_{ij}^*) the optimal number of subscribers must satisfy the following condition:

$$N^e - N^e N_{-ij} - 2N^e n_{ij} = c \quad \text{Eq. (2)}$$

To simplify the model, we consider that all the selling divisions are identical, and then all must choose, in equilibrium, the same optimal number of subscribers, that is, $\forall i, j, n^* = n_{ij}^*$.

Taking into consideration that the total number of selling divisions is $(\eta_1 + \eta_2)$, so N_{-ij} must be equal to: $(\eta_1 + \eta_2 - 1) \cdot n^*$. Substituting the conditions presented in the Eq. (2), the following one is obtained:

Proposition 2: The optimal number of subscribers for each operator is given by:

$$n^* = \frac{N^e - c}{N^e (\eta_1 + \eta_2 + 1)} \quad \text{Eq. (3)}$$

Proof: See the appendix ■

Follows that, the total number of subscribers in the cable television market (N), and the price (p), are obtained in the following way:

Proposition 3: The total number of subscribers (N) is given by the product between the total of selling divisions of the two cable operators, and the optimal number of subscribers for operator, and is expressed by:

$$N = (\eta_1 + \eta_2) \cdot n^* \Leftrightarrow N = \frac{(\eta_1 + \eta_2)}{(\eta_1 + \eta_2 + 1)} \cdot \frac{(N^e - c)}{N^e} \quad \text{Eq. (4)}$$

Proposition 4: The price (p) practised by each operator is given by:

$$p = N^e(1 - N) \Leftrightarrow p = \frac{N^e + (\eta_1 + \eta_2) \cdot c}{(\eta_1 + \eta_2 + 1)} \quad \text{Eq. (5)}$$

Proof: See the appendix ■

2.2. PHASE 2: OPTIMAL NUMBER OF DIVISIONS

In phase 2, each one of the two operators chooses the number of selling divisions in order to operate in the downstream market, having, η_1 e η_2 as being the number of downstream selling divisions, chosen by operators 1 and 2, respectively. Additionally, a company incurs into a sunk cost K , in the creation of the local network (in the specific localization of the selling division) for the distribution of the cable television service. In this phase of the game, each selling division, when establishes a price (p), gets a profit (π_{ij}) given by:

Proposition 5: The profit of each operator (π_{ij}) is given by the following expression:

$$\pi_{ij} = p \cdot n^* - c \cdot n^* \Leftrightarrow \pi_{ij} = \frac{(N^e - c)^2}{N^e \cdot (\eta_1 + \eta_2 + 1)^2} \quad \text{Eq. (6)}$$

Proof: See the appendix ■

The two operators who anticipate the competition between the selling divisions in phase 2, and that do not intend to cover the totality of the national market of cable television services ¹, have that to decide, in phase 1, which the number of selling divisions η_1, η_2 to implement.

¹ Remind the result enunciated by Proposition 1.

The profit of operator 1, can be written in the following way:

$$\pi_1 = \sum_{i=1}^{\eta_1} \pi_{i1} - K \cdot \eta_1 \quad \text{Eq. (7)}$$

Where: π_{i1} = Profit of i th selling division of operator 1, in phase 2.

Reminding the Eq. (6), which expresses the profit obtained by each selling division of operator 1, in phase 2, the total profit of operator 1 can be rewritten as follows:

$$\pi_1(\eta_1, \eta_2) = \eta_1 \cdot \frac{(N^e - c)^2}{N^e \cdot (\eta_1 + \eta_2 + 1)^2} - K \cdot \eta_1 \quad \text{Eq. (8)}$$

Therefore, operator 1, in order to maximize the profit, chooses the total number of selling divisions (η_1^*) that will constitute its downstream distribution network, considering that operator 2 has (η_2) selling divisions. Therefore, operator 1 chooses the best response function (η_1^*), in face of the number of divisions that operator 2 has (η_2). By the calculation of the first-order condition ($\partial\pi_1 / \partial\eta_1 = 0$), we obtain the following expression:

Proposition 6: The best response function of operator 1, is given by:

$$K = \frac{(N^e - c)^2}{N^e \cdot (\eta_1 + \eta_2 + 1)^2} \cdot \left[1 - \frac{2\eta_1}{(\eta_1 + \eta_2 + 1)} \right] \quad \text{Eq. (9)}$$

Proof: See the appendix ■

In this game, when we consider that operator 2 is identical to operator 1, a symmetrical condition for η_2^* can be met. For such, having that: $\eta^* = \eta_1^* = \eta_2^*$, it can be recognized that this symmetry implies that: $2\eta^* = \eta_1^* + \eta_2^*$. Solving the Eq. (9), in order to η^* , the following one is obtained:

Proposition 7: The optimal number of selling divisions is dependent of the differential between the number of subscribers of the potential base and the marginal cost, as well as of the sunk cost supported in the creation of cable television network, and is expressed by:

$$\eta^* = \frac{1}{2} \left[\frac{(N^e - c)^2}{K \cdot N^e} \right]^{\frac{1}{3}} - 1 \quad \text{Eq. (10)}$$

Proof: See the appendix ■

3. FRANCHISING OPTIONS

The theoretical framework related with franchising was developed from the seminal work of Coase (1937), which originated distinct research lines, such as, the theory of agency (Ross, 1973, Arrow, 1985, Rees 1985.a, 1985.b), and the theory of specific assets and opportunism (Klein, Crawford and Alchian, 1978; Williamson, 1985).

Franchising can be considered as a form of economic relationship that is amongst the extremities of the hierarchies and the markets (Williamson, 1985).

This relationship signals the principal that grants, and the agent who receives the license or the concession for using a product, service, technology or trade mark. This organizational form also combines the creation of a mechanism for a good coordination of the company activities, and the decentralization of the decision taking process.

To modelling this situation, after deducting the optimal number of selling divisions that guarantees the profit maximization for the cable operator, an extension of the game presented in section 2 is made, that equates the implementation of two distinct franchising options, which can be implemented by each one of the operators.

To extend the model presented in section 2, a game with the following phases is considered:

- Phase 1: The operator chooses the optimal number of divisions, in franchising.
- Phase 2: The operator selects the royalty modality.
- Phase 3: The selling divisions (or the franchisees) take the decision on the number of subscribers to reach (according to the royalty modality selected by the operator).

3.1. OPTIMAL NUMBER OF DIVISIONS, IN FRANCHISING

In phase 1, and taking into consideration the result founded in section 2.2. (Eq. (10)), each operator chooses the number of selling divisions to operate in the downstream market.

Lemma 1: Considering that, operators 1 and 2 are identical ($\eta_1^* = \eta_2^*$), the optimal number

of selling divisions, in franchising, is given by: $\eta^* = \frac{1}{2} \left[\frac{(N^e - c)^2}{K \cdot N^e} \right]^{\frac{1}{3}} - 1$.

3.2. ROYALTIES MODALITIES

In phase 2, the operator can apply two alternative royalties' modalities, to the franchisee.

3.2.1. MODALITY 1

In modality 1, we assume that the contract celebrated between the operator and the franchisee, establishes a price for the cable television service equal to a cost c , as well as a royalty payment, which corresponds to a fraction (α) of the total revenue.

Celebrating this kind of contract, the profit (π_1^F)² will be equal to total revenue after-royalty, less the corresponding cost, and is enunciated as follows:

$$\pi_1^F = (1 - \alpha) \cdot P \cdot n_{ij} - c \cdot n_{ij} \quad \text{Eq. (11)}$$

² Consider Franchisee Profit (F), with modality 1.

Lemma 2: For any positive royalty ($\alpha > 0$), the number of subscribers obtained through the application of modality 1, is smaller than the number of subscribers desired by the upstream operator.

Proof: See the appendix ■

3.2.2. MODALITY 2

In modality 2, the contract between the operator and the franchisee establishes a price for the cable television service equal to a cost c , and the royalty payment corresponds to a fraction (α) of the total profit obtained by the franchisee. Therefore, the profit of the franchisee (π_2^F)³ can be enunciated in the following way:

$$\pi_2^F = (1 - \alpha) \cdot [P \cdot n_{ij} - c \cdot n_{ij}] \quad \text{Eq. (12)}$$

In an analogous way to modality 1, this payment system causes a reduction on the MR of the franchisee. However, such as it is easily observable in the Eq. (12), the modality now presented also provokes a reduction of the MC supported by franchisee one.

3.3. OPTIMAL NUMBER OF SUBSCRIBERS

3.3.1. MODALITY 1

In phase 3, applying the modality 1, and considering the equality between the net MR of the franchisee, and the MC supported by the operator, we can derive the optimal level of the franchisee subscribers, which is given by:

$$n_{ij} = \frac{N^e(1 - \alpha) - c}{N^e(\eta_1 + \eta_2 + 1)(1 - \alpha)} \quad \text{Eq. (13)}$$

³ Consider Franchisee Profit (F), with modality 2.

Nevertheless, the operator can try to increase the profit level, through the application of a higher royalty.

Observing the Eq. (13), we find out that this procedure would result in a decrease of the number of subscribers, that is, in practice, the royalty would be charged over a smaller volume of subscribers.

Alternatively, the operator could establish one royalty equal to zero, however this option would result in selling the cable television service to the franchisee, at the marginal cost. This situation would cause an optimal joint profit, but the totality of this profit would be earned by the downstream franchisee.

The imposition of a royalty different of zero is the only available way for the operator to get a positive profit. However, any increase observed in α , would cause a reduction of the number of subscribers, which would not guarantees the maximization of the two companies joint profit.

Under the franchisee point of view, the imposition of a royalty over the total revenue is similar to the application of a rate over the total revenue. Therefore, this provides the reduction of its MR, as well as the reduction of the global incentives to reach new subscribers.

As a result of this, the royalties' scheme shows an imperfection revealed by the fact that franchisee reach a smaller number of subscribers, using a higher price. In this situation, we expect that, the franchisee is more interested in the profit maximization, in detriment of the maximization of the number of subscribers.

This situation describes the coordination problems originated by the implementation of this royalty modality, which may arise between the interests of the upstream companies, and the downstream ones.

3.3.2. MODALITY 2

In phase 3, applying the modality 2, and considering the equality between the MR and the MC, we can derive the optimal level of subscribers for the franchisee, expressed by the following:

$$n^* = n_{ij} = \frac{N^e - c}{N^e \cdot (\eta_1 + \eta_2 + 1)} \quad \text{Eq. (14)}$$

Lemma 3: For any positive royalty ($\alpha > 0$), the number of subscribers obtained through the application of modality 2, is equal to the optimal number of subscribers desired by an integrated company.

Proof: See the appendix ■

3.4. INFORMATION ASYMMETRY

The joint profit maximization is provided by the Eq. (12), which reveals the optimal level of subscribers that is obtained. However, this option, sometimes is not successful in the real world, because contrarily to the presuppositions used in this analysis, the upstream operator can have an insufficient level of information about the net profit obtained by the franchisee, due to lack of information concerning the formation of the costs supported by the franchisee.

The existence of this information asymmetry means that, the upstream operator faces a problem of moral hazard in the achievement of its commercial relations with the downstream franchisee.

For example, considering that a franchisee supports a fixed cost (F), in the development of its activity, known by the franchisee, but not by the operator. For such, the net profit obtained by the franchisee (π_2^F)⁴ would be equal to:

$$\pi_2^F = (1 - \alpha) \cdot [P \cdot n_{ij} - c \cdot n_{ij} - F] \quad \text{Eq. (15)}$$

In this situation, the franchisee can communicate a higher value of F , or transmit an inflated value of the total costs, in order to appropriate the totality of the profit.

In short, the liquidation of royalties based on the total profit (or, simply in the total revenue) does not provide a satisfactory result for the upstream operators, since harmonization of interests between the operators and the franchisees, can effectively fail.

⁴ Consider Franchisee Profit (F), incorporating a fixed cost, with modality 2.

4. CONCLUSIONS

The main contribution of the model is revealed by the Eq. (10), which shows that how much bigger will be the difference between the number of subscribers of the potential base (N^e) and the marginal cost (c), and how much smaller it will be the sunk cost of the cable network installation (K), greater will be the number of selling divisions chosen by the two operators, in phase 1 of the game.

However, the denominator of the Eq. (10), it is also affected by N^e , and then if we observe an increase in $N^e K$, the number of selling divisions chosen by the operators will be smaller.

This result is in contrast with the conclusions of Baye et al (1996), where is defended that how bigger will be the differential price-cost, greater will be the incentive for the creation of selling divisions, and for such reason the optimal number of selling divisions (η^*) more will be raised.

However, it must be remembered that the present analysis incorporates the demand curve of a network service, in form of inverted U , having this procedure as main implication the determination of a new η^* , that guarantees the maximization of the operators profit, using selling division networks that guarantee a partial market covering, and taking into consideration the sunk cost supported in the creation of the cable network.

The determination of η^* allows that the two operators (duopolists, in the national market) do not come close to the so-called competitive balance.

This type of analysis can help to explain the reason for the existence of a partial covering of the national territory, in terms of the cable television service.

The situation described above suggests that, the vertical relations established with the creation of an integrated selling divisions network, or alternatively, a franchisees network, can be harmful, in terms of total welfare.

This type of strategic conduct by the operators and the franchisees needs special attention, on the part of the regulating agencies. Furthermore, the cable television industry presents a complex technical composition that must be regulated, since it congregates the broadcasting and the circuits' areas, which make possible the access to integrated packages tie-in sales of differentiated services (basic, premium, internet, telephone, etc).

Finally, the regulating agencies must still have in consideration the effect of this type of vertical relations, on the total welfare, as well as inquiring the conditions of entry, safeguarding the inexistence of some kind of foreclosure, by the established operators against the entrants in the cable television industry.

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6. APPENDIX

Proof of Proposition 1:

$$\pi = p \cdot N \Leftrightarrow \pi = N^2 - N^3$$

From the first order condition, we obtain that:

$$\frac{\partial \pi}{\partial N} = 0 \Leftrightarrow 2N - 3N^2 = 0 \Leftrightarrow N(2 - 3N) = 0 \Leftrightarrow N = 0 \vee N = \frac{2}{3} \blacksquare$$

Proof of Proposition 2:

Considering that: $p = N^e(1 - N)$

Having: $N = N_{-ij} + n_{ij}$

$$N_{-ij} = (\eta_1 + \eta_2 - 1)n_{ij} \Leftrightarrow N_{-ij} = an_{ij}$$

The total profit is given by:

$$\pi_{ij} = N^e(1 - N)n_{ij} - cn_{ij} \Leftrightarrow \pi_{ij} = N^e(1 - N_{-ij} - n_{ij})n_{ij} - cn_{ij} \Leftrightarrow$$

$$\Leftrightarrow \pi_{ij} = N^e n_{ij} - N^e N_{-ij} n_{ij} - N^e n_{ij}^2 - cn_{ij}$$

The marginal revenue (MR) is given by the following expression:

$$MR = \frac{\partial TR}{\partial n_{ij}} = N^e - N^e N_{-ij} - 2N^e n_{ij}$$

From the condition of profit maximization, we derive the following:

$$MR = MC \Leftrightarrow N^e - N^e N_{-ij} - 2N^e n_{ij} = c \Leftrightarrow N^e - N^e a n_{ij} - c = 2N^e n_{ij} \Leftrightarrow$$

$$\Leftrightarrow N^e - c = (2N^e + N^e a)n_{ij} \Leftrightarrow n_{ij} = \frac{N^e - c}{N^e(2+a)} \Leftrightarrow n_{ij} = \frac{N^e - c}{N^e(\eta_1 + \eta_2 + 1)} \blacksquare$$

Proof of Proposition 4:

$$p = N^e(1-N) \Leftrightarrow p = N^e \cdot \left[1 - \frac{(\eta_1 + \eta_2)}{(\eta_1 + \eta_2 + 1)} \cdot \frac{(N^e - c)}{N^e} \right] \Leftrightarrow$$

$$\Leftrightarrow p = N^e \cdot \left[\frac{(\eta_1 + \eta_2 + 1) \cdot N^e - (\eta_1 + \eta_2) \cdot (N^e - c)}{(\eta_1 + \eta_2 + 1) \cdot N^e} \right] \Leftrightarrow$$

$$\Leftrightarrow p = N^e \cdot \left[\frac{N^e + (\eta_1 + \eta_2) \cdot c}{(\eta_1 + \eta_2 + 1)} \right] \blacksquare$$

Proof of Proposition 5:

$$\pi_{ij} = p \cdot n_{ij} - c \cdot n_{ij} \Leftrightarrow \pi_{ij} = \left[\frac{N^e + (\eta_1 + \eta_2) \cdot c}{(\eta_1 + \eta_2 + 1)} \right] \cdot \left[\frac{N^e - c}{N^e(\eta_1 + \eta_2 + 1)} \right] - c \cdot \left[\frac{N^e - c}{N^e(\eta_1 + \eta_2 + 1)} \right] \Leftrightarrow$$

$$\Leftrightarrow \pi_{ij} = \left[\frac{N^e + (\eta_1 + \eta_2) \cdot c}{(\eta_1 + \eta_2 + 1)} - c \right] \cdot \left[\frac{N^e - c}{N^e(\eta_1 + \eta_2 + 1)} \right] \Leftrightarrow$$

$$\Leftrightarrow \pi_{ij} = \left[\frac{(N^e - c)^2}{N^e(\eta_1 + \eta_2 + 1)^2} \right] \blacksquare$$

Proof of Proposition 6:

The total profit function of operator 1 is given by:

$$\pi_1(\eta_1, \eta_2) = \eta_1 \cdot \frac{(N^e - c)^2}{N^e \cdot (\eta_1 + \eta_2 + 1)^2} - K \cdot \eta_1$$

From the first order condition, we get the following:

$$\frac{\partial \pi_1}{\partial \eta_1} = 0 \Leftrightarrow \frac{(N^e - c)^2}{N^e \cdot (\eta_1 + \eta_2 + 1)^2} - \frac{2\eta_1 \cdot (N^e - c)^2}{N^e \cdot (\eta_1 + \eta_2 + 1)^3} - K = 0 \Leftrightarrow$$

$$\Leftrightarrow K = \frac{(N^e - c)^2}{N^e \cdot (\eta_1 + \eta_2 + 1)^2} \cdot \left[1 - \frac{2\eta_1}{(\eta_1 + \eta_2 + 1)} \right] \blacksquare$$

Proof of Proposition 7:

Taking into consideration the result enunciated by the Proposition 6, that is:

$$K = \frac{(N^e - c)^2}{N^e \cdot (\eta_1 + \eta_2 + 1)^2} \cdot \left[1 - \frac{2\eta_1}{N^e \cdot (\eta_1 + \eta_2 + 1)} \right]$$

Since $\eta_1^* = \eta_2^* = \eta^*$, we can make the correspondent substitution, and get the following:

$$K = \frac{(N^e - c)^2}{N^e \cdot (1 + 2\eta^*)^2} \cdot \left[1 - \frac{2\eta^*}{(1 + 2\eta^*)} \right] \Leftrightarrow$$
$$\Leftrightarrow K \cdot N^e \cdot (1 + 2\eta^*)^2 = (N^e - c)^2 \cdot \left[1 - \frac{2\eta^*}{(1 + 2\eta^*)} \right] \Leftrightarrow$$

$$\begin{aligned}
&\Leftrightarrow K \cdot N^e \cdot (1 + 2\eta^*)^2 = (N^e - c)^2 \cdot \left[\frac{1}{(1 + 2\eta^*)} \right] \Leftrightarrow K \cdot N^e \cdot (1 + 2\eta^*)^3 = (N^e - c)^2 \Leftrightarrow \\
&\Leftrightarrow (1 + 2\eta^*)^3 = \frac{(N^e - c)^2}{K \cdot N^e} \Leftrightarrow 1 + 2\eta^* = \left[\frac{(N^e - c)^2}{K \cdot N^e} \right]^{\frac{1}{3}} \Leftrightarrow 2\eta^* = \left[\frac{(N^e - c)^2}{K \cdot N^e} \right]^{\frac{1}{3}} - 1 \Leftrightarrow \\
&\Leftrightarrow \eta^* = \frac{1}{2} \cdot \left[\frac{(N^e - c)^2}{K \cdot N^e} \right]^{\frac{1}{3}} - 1 \blacksquare
\end{aligned}$$

Proof of Lemma 2:

With modality 1 (that is, payment of a royalty over the total revenue), the franchisee profit is the following:

$$\pi_1^F = (1 - \alpha) \cdot P \cdot n_{ij} - c \cdot n_{ij} \Leftrightarrow \pi_1^F = (1 - \alpha) \cdot (N^e n_{ij} - N^e N_{-ij} n_{ij} - N^e n_{ij}^2) - c \cdot n_{ij}$$

From the maximization profit condition (MR = MC), we obtain the optimal level of subscribers with the application of modality 1:

$$MR = MC \Leftrightarrow [N^e - N^e N_{-ij} - 2N^e n_{ij}] \cdot (1 - \alpha) = c \Leftrightarrow$$

$$\Leftrightarrow [N^e - N^e (\eta_1 + \eta_2 - 1) n_{ij} - 2N^e n_{ij}] \cdot (1 - \alpha) = c \Leftrightarrow [N^e - N^e a n_{ij} - 2N^e n_{ij}] \cdot (1 - \alpha) = c \Leftrightarrow$$

$$\Leftrightarrow N^e - N^e a n_{ij} - 2N^e n_{ij} - \alpha N^e + \alpha N^e a n_{ij} + 2\alpha N^e n_{ij} = c \Leftrightarrow$$

$$\Leftrightarrow N^e - \alpha N^e - c = (N^e a + 2N^e - \alpha N^e a - 2\alpha N^e) n_{ij} \Leftrightarrow$$

$$\Leftrightarrow N^e (1 - \alpha) - c = n_{ij} N^e (a + 2)(1 - \alpha) \Leftrightarrow n_{ij} = \frac{N^e (1 - \alpha) - c}{N^e (\eta_1 + \eta_2 + 1)(1 - \alpha)} \blacksquare$$

Proof of Lemma 3:

Under the modality 2 (that is, payment of a royalty over the total profit), the franchisee profit is the following:

$$\pi_2^F = (1 - \alpha) \cdot [P \cdot n_{ij} - c \cdot n_{ij}] \Leftrightarrow \pi_2^F = (1 - \alpha) \cdot [N^e n_{ij} - N^e N_{-ij} n_{ij} - N^e n_{ij}^2 - c \cdot n_{ij}]$$

From the maximization profit condition (MR = MC), we obtain the optimal level of subscribers with the application of modality 2:

$$MR = MC \Leftrightarrow [N^e - N^e N_{-ij} - 2N^e n_{ij}] \cdot (1 - \alpha) = c \cdot (1 - \alpha) \Leftrightarrow$$

$$\Leftrightarrow [N^e - N^e a n_{ij} - 2N^e n_{ij}] = c \Leftrightarrow N^e - c = (2N^e + aN^e) n_{ij} \Leftrightarrow$$

$$\Leftrightarrow n_{ij} = \frac{N^e - c}{N^e (\eta_1 + \eta_2 + 1)} \blacksquare$$