Mediating Market Power in Electricity Networks*

Richard Gilbert — University of California
Karsten Neuhoff — University of Cambridge
David Newbery — University of Cambridge

August 7, 2002

Abstract

We ask under what conditions transmission contracts increase or mitigate market power. We show that the allocation process of transmission rights is crucial. In an efficiently arbitraged uniform price auction generators will only obtain contracts that mitigate their market power. However, if generators inherit transmission contracts or buy them in a 'pay-as-bid' auction, then these contracts can enhance market power. In the two-node network case banning generators from holding transmission contracts that do not correspond to delivery of their own energy mitigates market power. Meshed networks differ in important ways as constrained links no longer isolate prices in competitive markets from market manipulation. The paper suggests ways of minimising market power considerations when designing transmission contracts

1 Introduction

Transmission capacity has been an impediment to electricity market liberalisation in Europe and the United States. In many cases the interconnection capacity between regions or countries was developed to provide security rather than to facilitate energy trade. In liberalised markets where consumers are free to buy from out-of-area generators, this capacity is often inadequate and must be rationed or priced. Increasing interconnector capacity has an additional advantage as it reduces local market power. Transmission constraints isolate electricity markets and limit the number of generators competing to supply local consumers. In many European countries, the concentration in generation is considerable, and the dominant producer is required to produce for a large fraction of the year to make up any shortfall between demand and available supply

*gilbert@econ.berkeley.edu, karsten.neuhoff@econ.cam.ac.uk, david.newbery@econ.cam.ac.uk.

Research support from Gottlieb-Daimler Foundation and Economic and Social Research Council (award R42200034307) is gratefully acknowledged.

Sidgwick Avenue, Cambridge CB3 9DE, England Phone 44 1223 335200, Fax 44 1223 335299

from imports and other producers. As the short-run elasticity of electricity demand is extremely low, dominant producers have very substantial market power when they are the required residual supplier. The Californian electricity crisis that started in December 2000 provides stark evidence that market power can be a serious problem in tight markets even when they appear unconcentrated (Joskow and Kahn).

Mitigating market power is therefore high on the agenda for regulators on both sides of the Atlantic. In the longer run, increased interconnection will reduce effective concentration, as should entry by new generating companies, and possibly regulatory pressure encouraging divestment, as in Britain. In the short run, though, interconnection capacity is limited and fixed. When the transmission constraints are binding then transmission capacity becomes valuable and revenues from financial transmission rights or exercise of physical transmission rights influence generators' production decisions. The key policy question that this paper addresses is how such scarce interconnection capacity should be made available to minimise the damaging effects of market power that current levels of generation concentration afford. We are specifically concerned with the design of auctions and markets (both spot and contract) to mitigate market power.

In a nodal pricing system, or where interconnection connects neighbouring areas facing different spot markets, transmission constraints expose market participants to locational price differences. Transmission contracts provide access to scarce transmission lines and provide financial hedges against price risk. In addition, prices provide information to guide expansion of transmission capacity (Hogan 1992). One immediate question confronting regulators is whether, and if so, under what conditions, transmission contracts increase or mitigate the market power of electricity generators.

The literature on the analysis of market power in transmission networks has developed in a series of papers addressing particular problems. These provide considerable insight, though the robustness of their findings and the implications for network management and market design are rather scattered and hard to assess. The paper therefore aims to present a systematic analysis of the best way to mitigate market power in constrained networks. This question provides a natural and important organising principle for investigating the effects of different arrangements on generator behaviour and consumer welfare.

Borenstein, Bushnell and Stoft (2000) show that it can be profitable for generators to withhold output in order to constrain a transmission line that would not have been constrained under perfect competition. Borenstein et al (1996) cite empirical evidence for Northern California to this effect. Neuhoff (2002, DAE Mimeo) models generators' abilities to constrain transmission links and concludes that market power is mitigated if the separate energy spot markets and markets for physical transmission contracts are integrated in a nodal design or market-splitting approach. Harvey and Hogan (2000) compare market power under nodal and zonal congestion management and conclude that the impact of market power is always weakly lower under nodal pricing than if both nodes are aggregated into a single zone.

Whilst the previously quoted papers evaluate situations when market power influences whether transmission constraints are binding, this paper is solely concerned with cases when constraints are binding irrespective of the exercise of market power. Stoft (1999) notes that transmission contracts may curb market power in a two-node network, while leaving open the question whether this result is robust to the location of market power and the extension to more complex meshed networks. The most impressive and exhaustive treatment is to be found in Joskow and Tirole (2000), on which much of this paper is built. European regulators pay considerable attention to the distinction between physical and financial transmission contracts, and the importance of ensuring that scarce transmission capacity is not with-held, by imposing 'use-it-or-lose-it' conditions.[1] This last point is almost universally accepted and practised by regulators, and therefore allows us to apply our models equally to financial and physical transmission contracts with the 'use-it-or-lose-it' condition. The implications of their other results for market design and regulatory policy are less clear. Our paper focuses more directly on these issues, attempts to provide a more comprehensive and robust range of cases, and concentrates on issues of practical market design, informed by current debates in Europe, the source of most of the experience with cross-border transmission auctions and markets.

Joskow and Tirole concentrate on the two-node case with a single monopolist at one node facing competitive generators at the other node, and give examples in which contracts either enhance or mitigate market power. In our interpretation transmission contracts serve the same function as forward contracts for energy. A transmission contract links the value of generation at one node to the price at another node. If the price at the other node is given (or hedged in local spot markets), then a transmission contract is a forward contract for energy.

If a dominant generator at an importing node imports energy with a transmission contract and sells it in the local spot market, this will increase the total volume of energy he sells at the spot market price and increases his incentive to withhold domestic output to increase spot prices. Market power is therefore enhanced if importing generators hold transmission contracts. In contrast, in a two-node network if an exporting generator holds transmission contracts he can effectively pre-commit that part of his output, in the same way that selling in a forward market would pre-commit output. As with other forms of contracting generator output, this is pro-competitive as it reduces the fraction sold at the spot market price and hence the incentive

[1] For example, the Dutch electricity regulator has taken steps against companies that did not use all of the transmission capacity allocated to them. See http://www.nma-dte.nl/en/news/2002/ pr0201.htm

to influence the spot price. Transmission contracts held by exporting generators mitigate market power. In two-node markets like Scotland-England or Germany-Netherlands the market power can therefore be mitigated by restricting generators from buying transmission contracts that do not correspond to delivery of energy from their power plants.

Joskow and Tirole also address the three-node network but conclude that “The difference between the two-and three-node networks is, we feel, more quantitative than qualitative; ...” (Joskow and Tirole, 2000, p. 479). However, loop flows in a meshed network make prices at different nodes interdependent even if transmission constraints are binding. If a generator holds a transmission contract to a node with a price that increases more due to withholding than the price at the production node, then the incentive to withhold output is increased. In a meshed network transmission contracts held by exporting generators can enhance market power whereas the same contracts in the two-node network always mitigate market power. Our main result for meshed networks (which includes the two-node case) is that a perfectly arbitraged single-price auction ensures that contracts never enhance and may mitigate market power. Traders value transmission contracts more than generators because in their possession the contracts do not create additional production inefficiencies in the energy market. As a result traders will secure all the contracts.

Joskow and Tirole contrast pay-as-bid and single price (or discriminatory and tender bid) auctions for the monopoly case. They show that generators play a mixed strategy in the pay-as-bid auction. We solve this case explicitly and extend it to the more commonly encountered oligopoly case, which differs in some important respects. Traders know only the expected distribution of generators’ bids. This lack of information implies that traders can only arbitrage the price of transmission contracts in expectation and therefore bid less aggressively than in a uniform price auction. Generators then obtain variable amounts of transmission contracts that increase their market power.

Uniform price auctions seem preferable to pay-as-bid auctions as they allow traders to arbitrage away the extra market power that transmission contracts offer generators. However, one should be cautious before accepting that uniform price auctions suffice to address market power problems. The result only applies to the full information case and it remains an open question whether it would still apply with asymmetric information or uncertainty. The result also depends on the assumption that the energy spot market equilibrium price is predictable, as it is with Cournot competition and information about cost and demand characteristics (of the kind normally available to informed participants in electricity markets). If the energy market is modelled as a supply function equilibrium as in Green and Newbery (1992), then the Nash equilibrium in bidding strategies is no longer unique. As a result the value of transmission contracts to traders is uncertain. In that case the outcome may share some of the unfavorable outcomes of the pay-as-bid auction: traders bid less aggressively, allowing generators to obtain market power-enhancing contracts. The working assumption of Cournot competition in that sense is most favourable for a regulatory minimalist approach to market intervention. Whilst careful auction design can reveal private information and therefore reduce these information asymmetries it may be more direct to use a simple auction design and explicitly ban generators from obtaining certain types of transmission contracts.

Our analysis suggests that one possible solution to the risk that contracts may enhance market power could be to identify the reference network node whose price is least influenced by any generator's output decision, and to define all transmission contracts towards that node. Generators would be restricted to buying transmission contracts towards this reference node. If such a node can be identified (and to be useful its identity would need to be stable over defined time periods), then all contracts should mitigate market power and our aim would have been achieved. Consumers can buy transmission contracts from the reference node to their off-take node such that the entire transmission risk is eliminated.

The paper also examines another related and practically important issue of trading in contracts. Industry restructuring resulted in various legacy interconnection contracts held by generators, which Joskow and Tirole demonstrate influence market power. An important question is whether subsequent trade in these contracts would resolve that problem. We show that a monopolist would not sell contracts that enhance his market power. Whether oligopolists sell contracts depends both on the trading structure and on the initial allocation of contracts. If a well-defined final trading period exists then oligopolists will always sell some of their market power-enhancing contracts. If their initial contract holding is symmetric and sufficiently small they will even sell all these contracts. However, in the case of continuous trading lacking a well-defined last trading period, generators with symmetric holdings of contracts will not sell any of these contracts. Neither will symmetric generators sell contracts that they obtained in a discriminatory auction. We therefore conclude that contract trading does not in general resolve the problem of an inappropriate initial allocation of transmission contracts. Regulators should therefore consider the market power implications when grandfathering contracts.

2 Two-node networks with importer market power

We assume that perfectly competitive generators with constant marginal costs c_1 are net exporters of electricity from node one. At node two, a total of $i = 1, ..., n$ identical generators with constant marginal costs c_2 act as Nash-Cournot competitors facing linear demand. A transmission link with capacity K connects the nodes. (Figure 1).

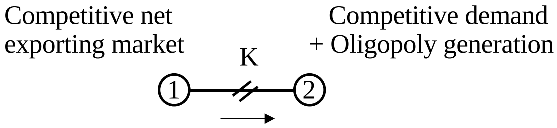

Figure 1:

At stage one of the game, transmission contracts are allocated in an auction or inherited. At stage two generators determine their outputs in the energy spot market. This replicates the current market structure in European countries where transmission auctions close before the spot market closes. In section 2.3 we insert an additional stage before the energy spot market to assess what would happen if generators could retrade transmission contracts after the initial allocation of these contracts.

We solve the model backwards, starting with the energy market at stage two. We assume that the transmission link is always used at full capacity so output decisions at node two do not influence prices at node one, which stay constant at marginal cost c_1. Demand at node two is linear, therefore the price at node two decreases linearly with available energy from imports K and local production $Q = \sum_i q_i$:

$$p_2 = \frac{A - Q - K}{\alpha}, \qquad p_{2,comp} = c_2 > c_1. \tag{1}$$

The generators chose output to maximise the profits, π_i, they obtain from selling energy in the local spot market, plus the revenue from their transmission contracts k_i:

$$\pi_i^{spot}(q_i, k_i) = (p_2 - c_2)\, q_i + k_i\,(p_2 - c_1)\,. \tag{2}$$

Substituting (1) in (2), we use the first order conditions (FOC) for profit maximization to obtain the equilibrium outputs and the price at node two:

$$\begin{aligned} q_i &= \frac{A - K - \alpha c_2 + \sum_{j=1}^{n} k_j}{n+1} - k_i, \quad Q = \frac{n\,(A - K - \alpha c_2) - \sum_{j=1}^{n} k_j}{n+1}, \quad \left|\sum_{j=1}^{n} k_j\right| \leq K, \\ p_2 &= \frac{A - K + n\alpha c_2 + \sum_{j=1}^{n} k_j}{\alpha\,(n+1)}, \quad Q_{comp} = A - K - \alpha c_2. \end{aligned} \tag{3}$$

The competitive output is $Q_{comp} = Lim_{n\to\infty} Q$. Transmission capacity increases a generator's exposure to the price at node two. A generator with transmission capacity k_i will reduce output by $\frac{n}{n+1} k_i$ compared to $k_i = 0$, holding k_j constant. However, if other generators hold positive quantities of transmission contracts, then they also commit to lower output in the energy market. The residual market left for our generator is bigger, hence he increases his output. Combining both effects, the holding of transmission contracts reduces total output by $\frac{1}{n+1}\sum k_i$. As generators with market power already reduce output relative to a competitive scenario by the factor $\frac{1}{n+1}$, the reduction of output due to transmission contracts induces a further deviation from the competitive (welfare optimal) equilibrium. This result is summarised in:

Proposition 1 *In a two-node capacity-constrained network, transmission contracts enhance the market power of importing generators in the spot market.*

Generator i's profit is

$$\pi_i^{spot}(k_i) = (p_2 - c_2)q_i + k_i(p_2 - c_1)$$

and

$$\frac{d\pi_i^{spot}(k_i)}{dk_i} = (p_2 - c_1) + \frac{d}{dk_i}\left[(p_2 - c_2)q_i\right].$$

Let v_t and v_g be the marginal value of transmission capacity for the trader and the generator. Noting that $v_t = p_2 - c_1$ and using (3) we have

$$\frac{d\pi_i^{spot}(k_i)}{dk_i} = v_g = v_t - (p_2 - c_2)\left[\frac{n-1}{n+1}\right]. \tag{4}$$

Equation (4) shows that an oligopoly generator values the marginal transmission contract less than a trader. If a generator owns more transmission contracts, then he will withhold more output and thereby increase prices, benefiting both the trader and generator. However, the generator foregoes the revenue on the marginal unit of output and therefore values the transmission contract less then a trader ($v_g < v_t$).

2.1 Allocation in the Uniform Price Auction

In a single or uniform price auction with no uncertainty all bidders pay the market clearing price. The market clearing price equals the predicted price difference between markets and therefore generators cannot make profits on their transmission contracts.

Each generator submits a bid schedule defining the capacity $k_i(\eta)$ he is willing to buy at price η and so do traders, represented by their aggregate bid schedule $k_t(\eta)$. The auctioneer determines the market clearing price:

$$\eta^* = \max \eta \quad \text{satisfying} \quad \sum_i k_i(\eta) + k_t(\eta) \geq K. \tag{5}$$

2.1.1 Trader's bid schedule

We assume a perfectly contestable market with new traders entering if any arbitrage opportunity exists. Therefore traders makes zero profits and pay the auction price that corresponds to the value contracts will have in the subsequent energy market :

$$\eta = p_2\left(k_t\left(\eta\right)\right) - c_1$$

Substituting p_2 from (3) and using the market clearing condition for transmission rights (5) gives the aggregate bid function of traders:

$$k_t\left(\eta\right) = Q_{comp} + K - \alpha\left(n+1\right)\left(\eta - c_2 + c_1\right). \tag{6}$$

2.1.2 Generator's bid schedule

The firm's profit is the profit in the energy market (2) less the cost for obtaining transmission contracts. Due to arbitrage the price paid for transmission contracts equals the spot price difference between the nodes:

$$\begin{aligned}\pi_i^{auction} &= \left(p_2 - c_2\right) q_i + k_i(p_2 - c_1 - \eta) = \left(p_2 - c_2\right) q_i \\ &= \frac{\left(Q_{\text{comp}} + \sum_{j=1}^{n} k_j\right)^2}{\alpha\left(n+1\right)^2} - \frac{k_i}{\alpha}\frac{\left(Q_{\text{comp}} + \sum_{j=1}^{n} k_j\right)}{n+1}.\end{aligned} \tag{7}$$

The first order condition of (7) with respect to k_i and symmetry between generators gives the optimal quantity $\bar{k}$ of transmission contracts to be bought by a generator:[2]

$$\overline{k} = -\frac{n-1}{n^2+1} Q_{comp}. \tag{8}$$

A monopolist ($n = 1$) would not buy any transmission contracts. Oligopolists ($n > 1$) would buy a negative quantity of transmission contracts. A negative quantity of transmission contracts corresponds to an energy delivery in the opposite direction of the constraint. As energy flows superimpose in electricity networks a reverse flow relieves congestion and is therefore valuable. Buying a negative quantity of transmission contracts therefore means receiving money in exchange for, in the case of physical transmission contracts, the obligation to deliver energy, or in the case of financial contracts, the obligation to pay the price difference in the energy spot markets.

[2] The generator's profit is a concave function of k_i, so the FOC is sufficient for proft maximization. The boundary conditions are that total allocated transmission contracts do not exceed transmission capacity: $\left|\sum_j k_j\right| \leq 1$. It can be shown that the only feasible generator bid schedule is Cournot, independent of η.

If importing generators hold negative quantities of transmission contracts, corresponding to flows against the direction in which the constraint is congested, then they increase, according to (3), their output towards the competitive equilibrium output. Market power is mitigated. Netting negative flows against positive flows is therefore valuable, but as of 2001, most European transmission auctions did not offer this facility, though it was under discussion. A financial transmission contract should not face this difficulty. In the absence of financial transmission contracts and if negative physical quantities cannot be issued, then the non-negativity constraint implies that $k_i \geq 0$ and the optimal choice of importing generators is not to participate in a uniform price auction: $k_i = 0$.

The equilibrium bidding strategy of traders is to submit an increasing bid schedule such that they buy $K - nk$ contracts at a price $\eta(k) = p_2(k) - c_1$. Whatever quantity an oligopolist intends to acquire he would have to pay at the arbitrage price. Hence he declines to participate in the auction. The result is summarised in:

Proposition 2 *Assume transmission is sold in a uniform price auction in a two-node capacity-constrained network with no uncertainty about future equilibrium spot prices and traders perfectly arbitrage transmission prices.*

(i) With no financial contracts and no netting of transmission flows, $k_i = 0$ for all i. Arbitraging traders outbit importing generators with market power.

(ii) With financial contracts and/or netting, symmetric importing generators will offer negative import capacity or sell transmission contracts in amounts given by (8).

2.2 Pay-as-bid auction

Joskow and Tirole (2000) analysed a 'pay-as-bid' auction for transmission contracts. In Europe all electricity transmission auctions are uniform, but access contracts to the UK gas transmission network are auctioned pay-as-bid. In period one generators and traders submit sealed bid schedules specifying the quantities they are prepared to buy at different prices. The auctioneer determines the successful bids and allocates transmission capacity and the spot market for energy clears in period two. Joskow and Tirole show that a pure strategy equilibrium does not exist for this game. A monopoly generator bids with a mixed strategy. Below we explore the mixed strategy equilibrium for the monopoly case and derive the equilibrium with n Nash-Cournot oligopolists in Appendix A.

As before, we assume that traders perfectly arbitrage any profit opportunity. Let $k_m(\eta)$ be the monopolist's bid schedule. Following Joskow and Tirole, the mixed strategy Nash equilibrium which satisfies the condition that traders make on expectation non-negative profits is defined by two functions. The first is the distribution function $H(.)$ from which the monopolist draws his bid schedule $\eta \epsilon (\underline{\eta}, \overline{\eta})$ with $Prob(\eta \leq x) = H(x)$. The second is the aggregate bid function of traders, $k_t(\eta)$, where $k_t(\eta) + k_m(\eta) = K$. The generator secures $max(0, K - k_t(\eta))$. The distribution function $H(.)$ has to ensure that traders make zero expected profit and the equilibrium bid schedule of traders has to satisfy the condition that the monopolist is indifferent between choosing any η from the support $(\underline{\eta}, \overline{\eta})$ of $H(.)$.

2.2.1 Equilibrium bid schedule of traders

The profit of the monopolist consists of profit in the energy market minus the cost of buying transmission contracts. Because the monopolist's capacity purchase is always the difference between total capacity and the traders' bid schedule, we have:

$$\pi_m^{auction}(K - k_t(\eta)) = \frac{(Q_{\text{comp}} + K - k_t(\eta))^2}{4\alpha} + (K - k_t(\eta))(c_2 - c_1 - \eta). \tag{9}$$

For the monopolist to be indifferent between any η, his profit must be constant as η changes. Setting the first derivative to zero gives

$$\frac{\partial k_t(\eta)}{\partial \eta} = -\left[\frac{K - k_t(\eta)}{\frac{Q_{\text{comp}} + 2\alpha(c_2 - c_1)}{2\alpha} + \frac{k_m(\eta)}{2\alpha} - \eta}\right] = -\left[\frac{K - k_t(\eta)}{\eta_0 + \frac{1}{2\alpha}k_m - \eta}\right], \tag{10}$$

where η_0 is the value of transmission contracts for traders when the monopolist does not own contracts. The solution of the differential equation (10) is

$$k_t(\eta) = K - 2\alpha\left(\eta - \eta_0 + \sqrt{(\eta - \eta_0)^2 - \frac{K}{2}const_1}\right). \tag{11}$$

2.2.2 Equilibrium monopoly bid distribution function

The distribution $H(\cdot)$ is such that traders make zero profit if and only if their aggregate bid schedule is the one calculated in (11). The expected value of a marginal bid equals the integral over all bids by the monopolist that are lower than the bid price η of the trader, weighted with their probability:

$$E[v_t(\eta)] = \int_{\underline{\eta}}^{\eta} H\prime(\widetilde{\eta})(p_2(\widetilde{\eta}) - p_1 - \eta)\, d\widetilde{\eta}.$$

As profit has to be zero for all η, the change in profit also has to be zero:

$$\frac{\partial}{\partial \eta}E[v_t(\eta)] = H\prime(\eta)(p_2(\eta) - p_1 - \eta) - H(\eta) \equiv 0. \tag{12}$$

The upper support of the bids is $\overline{\eta}$ with $H(\overline{\eta}) = 1$. Solving (12) gives

$$H(\eta) = \exp\left[-\int_{\eta}^{\overline{\eta}} \frac{1}{p_2(\widetilde{\eta}) - p_1 - \widetilde{\eta}} d\widetilde{\eta}\right]. \tag{13}$$

Substituting p_2 from (1) and $k_m(\eta) = K - k_t(\eta)$ from (11) into (13) we obtain

$$H(\eta) = \exp\left[-\int_{\eta}^{\overline{\eta}} \frac{1}{\sqrt{(\widetilde{\eta} - \eta_0)^2 - \frac{K}{2} const_1}} d\widetilde{\eta}\right] = \frac{\eta - \eta_0 + \sqrt{(\eta - \eta_0)^2 - \frac{K}{2} const_1}}{\overline{\eta} - \eta_0 + \sqrt{(\overline{\eta} - \eta_0)^2 - \frac{K}{2} const_1}}. \tag{14}$$

At the lower support $\underline{\eta}$, $H(\underline{\eta}) = 0$. This gives us $\underline{\eta} - \eta_0 + \sqrt{(\underline{\eta} - \eta_0)^2 - \frac{K}{2} const_1} = 0$, which is only satisfied for $\underline{\eta} = \eta_0$ and $const_1 = 0$. We therefore obtain:

$$k(\eta) = 4\alpha(\eta - \eta_0), \quad \overline{\eta} = \eta_0 + \frac{K}{4\alpha}, \quad H\prime(\eta) = \frac{4\alpha}{K}. \tag{15}$$

2.2.3 Result of the pay-as-bid auction

The results are summarised in Figure 2.[3] The monopolist bid schedule η is drawn with uniform

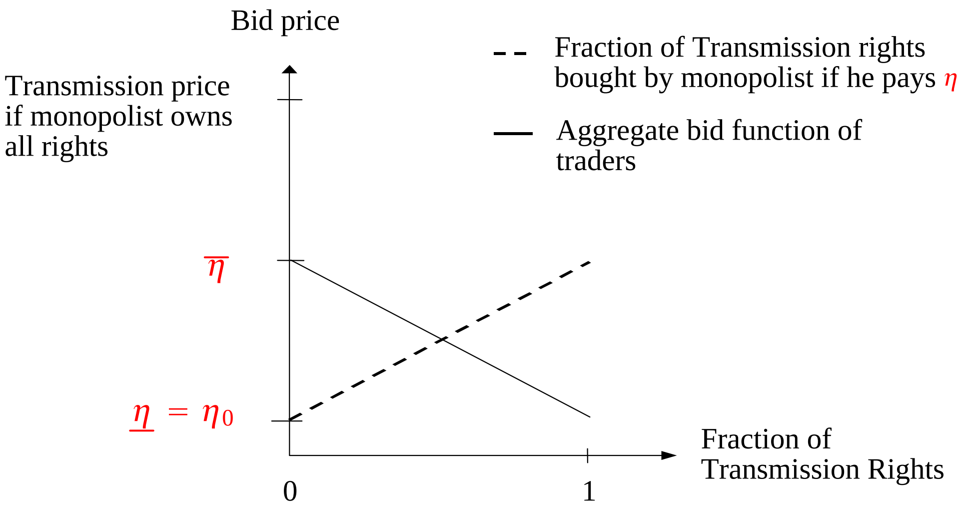

Figure 2:

probability over the interval ranging from the value transmission contracts take if the generators owns no contracts to the value transmission contracts take if he owns all contracts. On average the monopolist obtains half of the contracts. In the pay-as-bid auction with asymmetric information traders sometimes bid too high and therefore risk losses. These losses are compensated by the bids that are accepted below the value of the transmission contract. In expectation traders therefore make zero profit. The marginal bid does not arbitrage prices in a pay-as-bid auction,

[3]Using parameter values $n = 1$, $\alpha = 1$, $K = 3$ and $Q = 3/2$ we obtain profits from transmission rights if traders do not anticipate the generator to buy them $\Delta\pi_\lambda = 9\lambda^2$. These profits shrink to 0 in the Nash equilibrium bidding strategy, because generators profits equal the profits when they do not obtain transmission rights. $\eta_0 = \frac{Q_{comp}}{2\alpha} + c_2 - c_1 = \frac{3}{4} + c_2 - c_1$, $\overline{\eta} = \frac{3}{2} + c_2 - c_1$, $H\prime(\eta) = \frac{4}{3}$, $\lambda(\eta) = \frac{4}{3}(\eta - \eta_0)$.

so the monopolist obtains transmission contracts below its market value to compensate for the lost revenues on additional withheld output.

In the monopoly case the generator does not profit from obtaining transmission contracts because traders increase their bids until the generator's profits when obtaining transmission contracts equal profits when he does not obtain transmission contracts. Appendix A shows that in the oligopoly case, generators profit in expectation from the auction of transmission contracts. This is because generators' profits are increased relative to a situation without transmission contracts if other generators obtain transmission contracts in the auction and reduce output. Traders bid such that generators are indifferent between participating and not participating in the auction. We summarize these results in the following proposition.

Proposition 3 *If transmission is sold in a pay-as-bid auction in a two-node capacity-constrained network where import capacity exceeds $(n-1)$ times domestic competitive output, and if there is no opportunity for post-auction trading in transmission, then importing generators with market power will play a mixed strategy and secure some fraction of transmission contracts, enhancing their market power.*

2.3 Retrading Transmission Contracts

If, as is common in Europe after the restructuring of the electricity supply industry, generators inherit (legacy) contracts, equation (3) shows that they will reduce output relative to the situation in which they have no transmission contracts. Total welfare losses are increased as production decisions are further distorted. If, however, they can resell transmission contracts before the energy spot market opens, then it is important to see whether arbitrage again eliminates this additional inefficiency.

2.3.1 One period of retrading

If an oligopolist sells Δ_i of its contracts, the extra profit is :

$$\begin{aligned} \Delta\pi^{sell}_{\Delta_i} &= \pi^{spot}_{k_i-\Delta_i} - \pi^{spot}_{k_i} + \Delta_i \left(p_{2,k-\Delta} - c_1\right) \\ &= \frac{Q_{\text{comp}} + \sum\limits_{j=1}^{n}\left(k_j - \Delta_j\right)}{\alpha\left(n+1\right)}\left(\Delta_i - \frac{2}{n+1}\sum_{j=1}^{n}\Delta_j\right) - \frac{\left(\sum\limits_{j=1}^{n}\Delta_j\right)^2}{\alpha\left(n+1\right)^2}. \end{aligned} \tag{16}$$

Using the FOC of (16) with respect to Δ_i we obtain the total amount of transmission contracts Δ oligopolists would wish to sell assuming that only one trading opportunity exists:

$$\Delta = \frac{n^2-n}{1+n^2}\left(Q_{comp} + \sum_{j=1}^{n} k_j\right) \geq 0. \tag{17}$$

If generators have an initial symmetric holding of transmission contracts ($\forall j$, $k_i = k_j$) then they will sell all their transmission contracts if $\Delta > \sum_{j=1}^{n} k_j$. Symmetric generators will sell all their transmission contracts if only one trading period exists and

$$\sum_{j=1}^{n} k_j < n\frac{n-1}{n+1}Q_{comp}. \tag{18}$$

Summarising we can say:

Proposition 4 *If only one trading period exist then oligopolists ($n > 1$) will sell transmission contracts. If they have initially symmetric holdings, then they will sell all their contracts if $k_j < \frac{n-1}{n+1}Q_{comp}$. A monopolist will not sell any contracts.*

2.3.2 Continuous retrading possible

If continuous trading possibilities exists, then it appears that generators will always want to sell additional transmission contracts at successively lower prices, effectively extracting all the surplus under the demand schedule in the energy market down to the point of zero transmission contracts. This, however, is not a rational expectations equilibrium, as it requires that traders fail to anticipate subsequent sales.

We will establish, that in a rational expectations equilibrium, symmetric generators with symmetric holding of transmission contracts will not sell any of these contracts. The corresponding Lemmas are in appendix B.

Proposition 5 *Assume continuous trading over an open time interval with each generator owning the same amount of transmission contracts. No generator will sell his contracts to a trader.*

Proof. Lemma 11 shows that traders will only pay the price these contracts take after all generators have sold their contracts and that generators will either sell all or no contracts. Setting $\Delta_i = k_i = k$ in (16) the resulting change of profits for generator i is:[4]

$$\Delta\pi_i^{\text{all sell all}} = -\frac{k}{\alpha\,(n+1)^2}\left((n-1)\,Q_{\text{comp}} + n^2 k\right). \tag{19}$$

The change in profit is negative, suggesting that generators lose from the sale of transmission contracts. Generators therefore watch whether other generators sell their contracts and only sell

[4] Compare (19) to a situation where all but generator i sell their transmission contracts

$$\Delta\pi_i^{\text{only one keeping}} = -\frac{k}{\alpha\,(n+1)^2}\left((2n-2)\,Q_{\text{comp}} + \left(n^2-1\right)k\right).$$

Generator i makes losses in either situation, but if $k < (n-1)\,Q_{\text{comp}}$, or assuming the generators hold all contracts $K < \frac{n-1}{n}Q_{\text{comp}}$, it is preferable sell as well, if the others already sold. This explains why in the one-trading period setting all generators sell transmission contracts as long as $k_j < \frac{n-1}{n+1}Q_{comp}$ (Proposition 4).

if others do so. This waiting has no cost because the price obtained for selling contracts does not change, but stays at the price when no generator owns contracts (Lemma 11). Whilst waiting has no cost it has the advantage that at the end no generator sells contracts and therefore all generators earn higher profits. The only Nash equilibrium strategy is therefore for all players to wait for others to sell. In equilibrium no generator sells contracts. ■

2.3.3 Retrading after pay-as-bid auction

Whilst it is possible for several generators to inherit legacy transmission contracts, the outcome of a pay-as-bid auction is that only one generator will secure contracts (the probability of two independently randomizing bidders choosing the same constant bid price for all capacity is zero). The same question arises whether this generator will resell the transmission contracts acquired in the auction.

Setting $k_j = \Delta_j = 0$, $\forall j \neq i$ in (16) it follows that the generator only profits from a sale ($\pi^{sell}_{\Delta_i} > 0$) if he sells fewer transmission contracts than

$$\Delta_i < \frac{n-1}{n}\left(Q_{comp} + k_i\right). \tag{20}$$

If several trading periods exist then traders buying transmission contracts from the oligopolist fear that the oligopolist will sell additional transmission contracts in subsequent periods, reducing the value of the transmission contracts. Again, the rational expectations equilibrium with no obvious final selling period is that traders will only buy at the price these contracts take if the generator sold all of them. It is profitable for the generator to sell all transmission contracts at once $\Delta_i = k_i$ in (20) if and only if

$$k_i < (n-1)Q_{comp}.$$

This is the opposite condition to that for participating in the pay-as-bid auction described in (36), so either generators will not bid and hence have nothing to sell, or will bid and not subsequently sell. This is summarised in:

Proposition 6 *If an oligopoly importer acquires transmission contracts in a pay-as-bid auction, then it is not profitable to resell any contracts if there is no final trading period.*

3 Exporter market power in a two-node network

We briefly consider the case in which the oligopoly is located at the exporting node of a two-node network (Figure 3). The importing node is assumed perfectly competitive with price $p_2 = c_2$. Demand is linear at node one, with

$$p_1 = \frac{A - Q + K}{\alpha}. \tag{21}$$

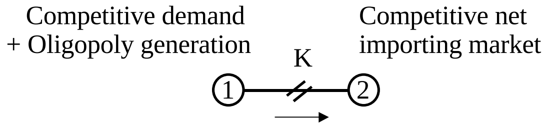

Figure 3:

In the energy market the oligopolist maximises the profit function

$$\pi_i^{spot}(q_i, k_i) = (p_1 - c_1)\, q_i + k_i\, (c_2 - p_1)\,.$$

The FOC for optimal output choice gives:

$$q_i = \frac{(A + K - \alpha c_1) - \sum_{j=1}^{n} k_j}{n+1} + k_i, \qquad Q = \frac{n\,(A + K - \alpha c_1) + \sum_{j=1}^{n} k_j}{n+1}. \tag{22}$$

3.1 Uniform price auction

As in Section 2.1 the optimal contract volume k_i for each of the n symmetric exporting oligopolistic generators in a uniform price auction can be found:

$$k_i = \frac{n-1}{n^2+1}\,(A + K - \alpha c_1) = \frac{n^2-1}{n^2+1} q_{i,k=0}, \tag{23}$$

provided that no more transmission contracts are demanded than are available, $\sum_i k_i \leq K$. If (23) implies $\sum k_j > 1$ then generators effectively foreclose the market and we set $\sum k_j = 1$. In any case allowing generators to participate in the auction and even foreclose the market is welfare improving.

Given the equilibrium contract quantity from (23), or when the capacity constraint binds, the output $Q_{uniform}$ or $Q_{foreclose}$ in the subsequent energy market (22) can be calculated. Again, the competitive output is denoted Q_{comp} and output without transmission contracts $Q_{nocontracts}$:[5]

$$\begin{aligned} Q_{comp} &= A + K - \alpha c_1 > Q_{uniform} = \frac{n^2}{n^2+1} Q_{comp} \qquad (24)\\ &\geq Q_{foreclose} = \frac{n}{n+1} Q_{comp} + \frac{K}{n+1} \geq Q_{nocontracts} = \frac{n}{n+1} Q_{comp}, \qquad n > 1. \end{aligned}$$

For $n > 1$, and therefore not for monopolists because they do not participate in the market, we obtain $Q_{uniform} > Q_{nocontracts}$, showing that the transmission contracts mitigate market power and reduce withholding of output. The price at node two is independent of production at node

[5] If the constraint $\sum \lambda_j \leq 1$ binds, the equilibrium output would be $Q_{pre-empt} = \frac{n}{n+1} Q_{comp} + \frac{K}{n+1} < Q_{uniform}$

one as the transmission link is fully used. Transmission contracts towards node correspond to contracts for difference and so generators use the transmission contracts to sell energy in the forward market. This corresponds to a second contracting stage and makes the outcome more competitive, as in Newbery (1998). In that model, long-term contracts are signed in period one, and the remaining energy is traded in the spot market in period two. The more energy is covered by long-term contracts, the lower the exposure of generators to the spot-market price. Therefore they increase their output to obtain revenue on the marginal unit at the cost of overall lower prices. Rational expectations imply that the expected spot-market prices feed back to the long-term contract prices. The result shows that allowing access to the transmission line decreases market power by serving as an initial contracting stage. This general Cournot result confirms Stoft's (1999) special case for zero demand elasticity at the import node .

Provided there is complete information about costs and demand and no uncertainty, the pay-as-bid auction has the same equilibrium, because the oligopolistic generators use a pure strategy. Therefore there is no information asymmetry about bidding and traders can arbitrage the market in every situation. We can summarise the result as:

Proposition 7 *In a two-node capacity-constrained network, allowing exporting generators with market power to buy transmission contracts is always welfare-enhancing.*

This result fails for meshed networks (see Section 4.2).

3.2 Re-trading of transmission contracts

The equilibrium just computed was for a one-shot transmission auction. The fact that transmission contracts act as a commitment device raises the question whether it might be profitable for generators to increase their holding of transmission contracts after the initial auction. As in Section 2.3 the change in profits if a generator buys an additional Δ_i transmission contracts is:

$$\begin{aligned} \Delta\pi_i^{\text{buy}} &= \pi_{k_i+\Delta_i}^{spot} - \pi_{k_i}^{spot} - \Delta_i \left(c_2 - p_{1,k_i+\Delta_i}\right), \\ &= \frac{\Delta_i}{n+1}\left((n-1)\left(p_{1,k_i} - c_1\right) - \frac{n}{\alpha}\frac{\Delta_i}{n+1}\right). \end{aligned} \tag{25}$$

Buying additional transmission contracts is unprofitable for a monopolist ($n = 1$), because he has to pay the higher ex-post price. However, in an oligopoly setting ($n > 1$) buying additional small quantities of contracts $\Delta_i < \frac{\alpha}{n}\left(n^2 - 1\right)\left(p_{1,k_i} - c_1\right)$ is profitable for all generators as long as the price-cost margin $p_{1,k_i} - c_1$ is positive. Oligopolists have the advantage that by buying contracts they commit to higher output, inducing competitors to reduce their output. The reduction of competitors' output has a positive impact on profits, which is not present in the monopoly case.

If several retrading periods exist, then traders anticipate that oligopolists will buy additional transmission contracts in subsequent periods. The value of transmission contracts rises with the total amount held by generators and traders therefore charge a high price starting with the first sale. Consider the change of profits for a generator buying Δ_i contracts when the remaining contracts are bought by other generators and when traders charge the price corresponding to all capacity held by generators:

$$\Delta\pi_{\text{buy } \Delta_i \text{ at final price}} = \frac{1}{\alpha (n+1)^2} \left[K(K - 2Q_{comp}) + (n+1)\,\Delta_i(Q_{comp} - K) \right]. \qquad (26)$$

If $K < Q_{comp}$ then profit increases in the amount of transmission contracts a generator buys and he will therefore buy as many contracts as possible. Participation of generators is guaranteed, because if any generator obtains a large enough fraction of all contracts, then his profits are higher than in a no-trade situation. As a result generators buy all contracts from traders, even so their aggregate profit is reduced: $\sum \Delta\pi = K\left((1-n)\,Q_{comp} - K^2\right) / \left(\alpha\,(n+1)^2\right)$.

The situation is similar to a Coasian durable goods monopolist. Generators find themselves in the unenviable position of buying additional transmission contracts that mitigate their market power (similar to the Coasian durable goods monopolist selling to traders thereby reducing his exposure to spot prices and incentive to exercise market power by withholding output). The only equilibrium will be that the generators buy all available transmission contracts at the price difference that corresponds to their holding all contracts, even in the original auction. If they attempt to pay less in that auction, traders will anticipate profitable sales in the aftermarket and will bid up the price to the final equilibrium price. Generators are therefore forced to commit to the higher output associated with selling all transmission output forward. This outcome corresponds to Allaz and Vilas' (1993) result that with additional periods of trading of forward contracts the equilibrium outcome in the energy market gets closer to the competitive outcome.

For $K > Q_{comp}$ a generator's profit (26) decreases with the amount of transmission contracts Δ_i he buys. Therefore no generator buys contracts at the ex-post price and traders have to offer lower prices. Traders still benefit relative to a no-trade situation, because they capture some of the profits (25) from generators. If ownership of contracts is disperse, then each trader waits for other traders to sell their contracts expecting to subsequently receive higher prices, but this deadlock prevents all trades. If ownership of transmission contracts is sufficiently concentrated then traders sell all contracts to generators.

The example shows that allowing generators with market power to secure transmission export contracts mitigates market power in a two-node network. As prices at the importing node are fixed at full utilisation of transmission, the transmission market acts just like a long-term contract market and induces generators to sell more energy than in a one-stage Cournot model.

4 Market power in a three-node network

Joskow and Tirole (2000) conclude that "the effects of transmission contracts holding on market power on a three-node network are conceptually similar to those on a two-node network". This cannot be quite right as the two-node network allows a very simple characterisation of policy on transmission contracts to mitigate market power. Exporting generators should be encouraged to trade for transmission (as often as possible) while importing generators should be discouraged from obtaining transmission. The reason for the simplicity is that the market price in a competitive region connected to an oligopolistic region by a constrained transmission link is independent of who secures rights to that transmission link, so that the entire impact of generator transmission contracts is in the region with the market power. If the fraction of generator controlled supply sold into the oligopolistic market is increased by transmission contracts, market power is enhanced and vice versa.

Matters are more complex in meshed networks where single link constraints do not necessarily isolate other competitive markets from market power at a node. A uniform auction for transmission contracts in the presence of complete information still ensures that transmission contracts cannot enhance market power. If, however, transmission contracts are inherited or secured in a 'pay-as-bid' auction, then the policy conclusion from the two-node network that only exporting generators should hold transmission contracts is no longer sufficient to prevent the enhancement of market power.

4.1 Loop flow considerations

In a simple two-node network with a single link there is no ambiguity about the concept of transmission capacity, as all power from one node must flow along the single link to the other. In a meshed network with more than one possible path from one node to another, electricity will flow over all links, distributed according to Kirchoff's Laws (Bohn, Caramanis, and Schweppe 1984). A generator at one node may sign a contract to deliver power to a consumer at another node, and then seek to sign a contract with the transmission operator of the most direct link, but only some of the power will actually flow along this link, with the balance creating 'loop flows' along all other paths connecting the source (the generator) to the sink (final consumer). Dealing with these loop flows bedevils the management of federal transmission systems, in which various sub-grids of the interconnected system are under the jurisdiction of separate Transmission System Operators (TSOs). One direct consequence of these loop flows is that a transmission constraint on one link impacts on the flows that are possible on every electricity transmission link in the network.

Two different approaches have been proposed to explicitly manage transmission constraints in a liberalised electricity market: property rights and nodal prices. First, property rights are created and auctioned for all links that might potentially be constrained. The fundamental entity is called a flow gate right. The system operator calculates proportionality factors γ_{ij}^{k} to determine what proportion of energy flow between injection node i and offtake node j will pass over link k. The proportionality factor γ_{ij}^{k} is negative if the energy flow goes in the opposite direction to the defined orientation of the link.[6] Subsequently market participants multiply the power volume (positive amount of MW)[7] they want to transmit between two nodes with the corresponding proportionality factor. This determines how many flow gate rights they have to obtain for each link. The system operator can issue a net amount of flow gates up to the capacity of the link.

In the second approach, nodal pricing, the system operator determines the market clearing price at each node that respects the transmission and other constraints. Generators receive the nodal price of their injection point, and consumers pay the nodal price at the off-take point. Financial transmission contracts then allow market participants to hedge against the risk that the nodal prices may differ substantially over space. The system operator de-facto simulates a market of property rights for scarce transmission assets to determine the nodal prices. Nodal pricing can therefore be interpreted as an interface to simplify the underlying market structure and reduce transaction costs to match physical transmission contracts to energy delivery. In a perfectly competitive environment nodal prices are therefore identical to physical transmission contracts, because any design of physical transmission contracts can subsequently be implemented by the system operator when simulating property rights to determine nodal prices. In a situation with market power nodal pricing differs from physical transmission contracts because the market for transmission and energy clears simultaneously rather than first for transmission contracts and then for energy under physical contracts. The allocation of physical transmission contracts determines the net import/export capacity to each node or zone. In nodal pricing the physical transmission capacity is allocated as a function of the prices in the spot energy market and thereby creates additional demand elasticity and mitigates market power. Physical transmission contracts are allocate ex-ante and therefore can not be adjusted in the energy spot market. Therefore spot market prices do not influence import/export capacity and net-demand elasticity is reduced, enhancing market power relative to nodal pricing.[8] The pre-commitment

[6] The orientations are determined arbitrarily, and these will determine the signs of the factors γ_{ij}^{k} and hence the consistency of the flow analysis.

[7] Energy is measured in MWh, while capacity is measured in MW. We define a unit of time during which flows are constant, and the energy is then MW multipled by the time interval, taken here as 1 unit.

[8] Mimeo Karsten Neuhoff, DAE (2002) "Demand Elasticity in Constraint Networks."

effect that allows physical transmission contracts to mitigate market power should be equally provided by financial transmission contracts. Therefore ceteris paribus nodal pricing with financial transmission contracts should mitigate market power better than physical transmission contracts.

4.2 Exporting generator holding transmission contracts

The following example demonstrates that in meshed networks exporting oligopolists may use transmission contracts to enhance their market power. As before, the game is solved backwards, first determining equilibrium prices in the energy market, and then determining the bidding strategy of generators and traders in stage one. As in Joskow and Tirole (4), a competitive net exporting market is located at node one, an oligopoly at node two and competitive net demand at node three (Figure 4).

As described in the previous section, market participants have to obtain flow gate rights for the constrained link $\overline{13}$, if they want to transmit energy between two nodes in the network. Due to loop flows $\gamma_{13} = 2/3$ of electricity exports Q_1 from node one and $\gamma_{23} = 1/3$ of exports Q_2 from node two pass along the constrained link $\overline{13}$. To sell one unit of energy from node two to three, an oligopoly exporter therefore needs 1/3 of a unit of flow-gate on $\overline{13}$. To sell one unit of energy from node one to node three 2/3 of a unit of flow gate rights on $\overline{13}$ is required. In this

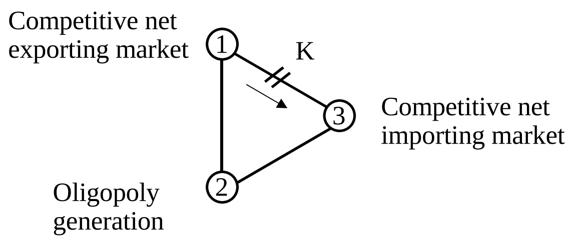

Figure 4:

particular network total exports are therefore constrained by:

$$2Q_1 + Q_2 \leq 3K. \tag{27}$$

The value η of transmission contracts to the flow gate is proportional to the price difference between nodes times the inverse of the proportion of flows between the nodes that goes along the flow gate, and this value defines the efficient arbitrage condition:

$$\eta = \frac{3}{2}(p_3 - p_1) = 3(p_3 - p_2). \tag{28}$$

Assuming competitive constant marginal cost c_1 at node one and linear demand $p_3 = \frac{A - Q_1 - Q_2}{\alpha}$ at node three and using binding constraint (27), (28) can be solved for p_2 as function of p_1 and

p_3:

$$p_3 = \frac{2A - 3K - Q_2}{2\alpha}; \qquad p_2 = \frac{c_1}{2} + \frac{2A - 3K - Q_2}{4\alpha}. \tag{29}$$

In the energy market oligopolist i at node 2 who produces q_i with constant marginal costs c_2 and who owns k_i transmission contracts (flow-gate rights on $\overline{13}$, each of which allows him to sell 3 units from node 2 to 3) maximises the following profit function:

$$\pi_i^{spot} = (p_2 - c_2)\, q_i + 3\,(p_3 - p_2)\, k_i. \tag{30}$$

Substituting p_2 from (29) in (30) the FOC gives the optimal output choice:

$$\begin{aligned} q_i &= \frac{2\alpha c_1 - 4\alpha c_2 + 2A - 3K + \sum_j 3k_j}{n+1} - 3k_i\ ; \qquad (31) \\ \pi^{spot}(k_i) &= \frac{\left(Q_{comp} + \sum_j 3k_j\right)^2}{4\alpha\,(n+1)^2} + 3k_i\,(c_2 - c_1)\,; \\ Q_2 &= \frac{n\,(2\alpha c_1 - 4\alpha c_2 + 2A - 3K) - \sum_j 3k_j}{n+1}. \end{aligned}$$

The competitive output would be $Q_{comp} = Lim_{n\to\infty} Q_2 = 2\alpha c_1 - 4\alpha c_2 + 2A - 3K$. Equation (31) shows that ownership of transmission contracts k_i causes the oligopolist to decrease output q_i. To evaluate the effect of transmission contracts we can apply the First Welfare theorem according to which the competitive equilibrium is efficient. The output in a situation with market power and without transmission contracts is reduced relative to the competitive output by the factor $\frac{n}{n+1}$. If generators own transmission contracts, then their market power is enhanced and they reduce the output by an additional $\frac{1}{n+1}\sum_j 3k_j$. As all other nodes are competitive, increased deviation from the efficient level at the distorted node increases inefficiency. It follows that ownership of transmission contracts decreases welfare even though the oligopolists are exporting.

4.3 Effect of single price auction

In contrast to the two node case, if an oligopoly exporter possesses transmission contracts, then the outcome may be inferior to banning him from owning such contracts, the next step is to see whether there is any danger of his acquiring such contracts in an auction. As before, there are no problems with a uniform price auction, which will reveal all information and therefore result in arbitrage of prices. Generators chose the proportion of transmission contracts to buy k_i in order to maximise total profits $\pi_i^{auction} = \pi^{spot}(k_i) - 3k_i\,(p_3 - p_2)$. The first order condition gives the total quantity to be obtained:

$$3k_i = \frac{1-n}{1+n^2} Q_{comp} \leq 0. \tag{32}$$

In a uniform price auction the oligopolist buys in equilibrium a negative quantity of transmission contracts, therefore market power is mitigated to the same degree as calculated for the two-node network in section (3.1). Note that this requires the System Operator to ensure that the oligopolists indeed acts to relieve the constraint, and allows additional flow-gate rights to be issued to other participants (the arithmetical or directional sum over all contracts is the binding constraint). If negative quantities of flow-gate rights are not available, then generators will buy no transmission contracts and the result that market power is not enhanced if transmission contracts are auctioned in a uniform price auction survives. The following theorem summarising these more general results is proved in appendix (C).

Theorem 8 *If constrained transmission capacity in a meshed network is sold in a single price auction that is efficiently arbitraged by traders who can accurately predict future equilibrium spot prices, then oligopolists will only acquire contracts that mitigate market power. Marginal costs of generation should not increase by more than* $\left(\sqrt{2}-1\right)$ *times demand slope. If generators are asymmetric, then marginal costs of each generator should not increase by more than* $1/n$ *times demand slope (lower bounds due to approximations).*

These conditions should be easily satisfied, because electricity demand is very inelastic (demand slope high) while marginal costs of generators that can alter their output decision are comparatively flat.

If transmission contracts are formulated as options and not as obligations, then generators cannot acquire a negative quantity of transmission contracts. If negative quantities would mitigate market power, then generators will end up not acquiring any transmission contracts. This is the reason for the weak formulation in theorem (8) that transmission contracts will not enhance market power. If transmission contracts are formulated as obligations than they will mitigate market power if allocated in a uniform price auction with full information.

4.4 Pay-as-bid auction

If oligopolists only buy negative transmission contracts in a single price auction, would a pay-as-bid auction allow them to secure transmission and enhance market power by playing a mixed strategy, as in the two-node example? As before, the first step is to see if an oligopolist would increase profits by buying transmission contracts at the value the contracts have if traders do not expect oligopolists to obtain contracts:

$$\begin{aligned}\Delta\pi_k^{auction} &= \pi_{k_i}^{spot} - \pi_{k_i=0}^{spot} - 3k_i\left(p_{3,k=0} - p_{2,k=0}\right), \\ &= \frac{1}{4\alpha}\frac{3k_i}{(n+1)^2}\left(3k_i - (n-1)\,Q_{comp}\right). \qquad (33)\end{aligned}$$

As in the two-node case (36), the ratio between transmission capacity $3k_i$ and $(n-1)\,Q_{comp}$ determines whether generators can profitably bid for market power-enhancing contracts when information asymmetry prevents perfect arbitrage. However, in the three-node case oligopolists

($n > 1$) are more likely to play a mixed strategy equilibrium than in the two node case. This is because in the meshed network export capacity from node two is three times the transmission capacity of the line $\overline{13}$. It is reflected in the factor $3k_i$ instead of k_i in the brackets. Mixed strategy equilibria with their market power enhancing implications are therefore more likely in meshed networks than in the two node case. In appendix (D) the mixed strategy equilibrium of a monopolist in the three node network is calculated analogously to the situation in a two node network.

4.5 Selling inherited contracts

Finally, it is important to see whether inherited transmission contracts enhance market power or these additional distortions can be traded away. The oligopolist's total profit function is

$$\begin{aligned} \Delta\pi^{sell}_{\Delta k_i} &= \pi^{spot}_{k_i-\Delta k_i} - \pi^{spot}_{k_i} + 3\Delta k_i \left(p_{3,k_i-\Delta k_i} - p_{2k_i-\Delta k_i}\right), \\ &= \frac{3\Delta k_i}{(n+1)^2}\left(\left(n^2-1\right)\left(p_{2,k_i}-c_2\right) - \frac{n}{4\alpha}3\Delta k_i\right). \end{aligned} \tag{34}$$

The situation corresponds exactly to the two node network with an importing oligopolist inheriting transmission contracts. The oligopolist could profitably sell small quantities of transmission contracts at the ex-post price. However, traders will not buy contracts as they anticipate that further sales by oligopolists would further reduce the value of their contracts and we return to the same discussion we presented for the two node network.

5 Conclusions

Allowing generators with market power access to transmission auctions when transmission capacity is constrained may amplify or mitigate their market power. Regulators may therefore wish to consider under what circumstances it would be desirable to prevent such generators from securing or retaining transmission contracts. We find that in the two-node case if the generators with market power are located at an import constrained node, it is always undesirable to allow them to retain existing transmission contracts. If they are to be compensated, it should be by a formula that does not depend on the subsequently realised spot price, to make sure that they have no additional reasons for influencing that price. It is also undesirable to allow them access to the transmission auction where this is pay-as-bid, or where there is likely to be asymmetric information favouring the generators. Even under the ideal circumstances of perfectly informed arbitrageurs and a single-price auction, while there is no harm in allowing generators to bid, there is also no benefit. Nor do the generators benefit from bidding even in the pay-as-bid case, as all the additional distortionary revenue is secured by the transmission company.[9] On the other hand, if the generators with market power are located at an export constrained node, their bids for transmission capacity allow them to pre-commit to additional output and reduce prices at the exporting node, to the benefit of consumers there.

In a three-node network (and with more nodes), transmission contracts can enhance market power even when they correspond to own production (and not just imports), at least if loop flows imply that output changes have a bigger impact on the price at the delivery node than at the origin node. The solution is to define transmission contracts to the reference node that has a price least influenced by any generator's output decision. Generators should then be restricted to transmission contracts with this reference node. Consumers and their representatives should then obtain transmission contracts from the reference node onward. Such a policy minimises the risk that transmission contracts enhance market power while ensuring that they provide risk hedging services and provide information for network expansion.

We analysed the effect of transmission contracts on output decisions when transmission constraints are permanently binding. However, links between several regions are only constrained part of the time with intermediate periods where the constraint is 'just' binding or 'just' not binding. While there are several models showing that market power can increase the period when constraints are binding, the effect of transmission contracts on whether or not constraints are binding is still an open question.

A Mixed strategy Equilibrium in Oligopoly Case

An oligopoly with $n > 1$ generators is located at the importing node. In a "pay-as-bid" auction any generator will either not participate in the auction or buy all transmission contracts available at price η. Bids are chosen from a continuous distribution, therefore no more than one generator obtains transmission contracts.

A.1 Energy market

To simplify our subsequent calculations we define η as the margin paid above the value transmission contracts take if generators own no contracts. We know from (3) that if a generator owns k

[9] In some European countries, generators still have an ownership interest in transmission, and this would provide an additional motive for market manipulation, even with "functional" unbundling and separation agreements (as required by the EU Electricity Directive). This may provide additional reasons for banning the acquisition of such transmission rights.

contracts then the value of contracts is increased due to the decrease in output:

$$\Delta p_{2,k} = \frac{k}{\alpha\,(n+1)}. \tag{35}$$

Furthermore by substituting (3) in (2) we obtain the change in profits $\Delta\pi_k^{change}$ in the energy market for a generator owning transmission contracts k and similarly the change in profits $\Delta\pi_{-k}^{change}$ for a generator when a competing generator obtains contracts:

$$\Delta\pi_k^{change} = \frac{1}{\alpha}\frac{k}{(n+1)^2}\left(k + (1-n)\,Q_{\text{comp}}\right), \tag{36}$$

$$\Delta\pi_{-k}^{change} = \frac{1}{\alpha}\frac{k}{(n+1)^2}\left(k + 2Q_{\text{comp}}\right). \tag{37}$$

A.2 Game structure

The game with an oligopoly of generators is the same as with a monopolist. Generators and traders submit their bid schedule and the auctioneer accepts the highest bids. Generators either submit no bid or bid for all transmission contracts available at price η. Generators draw the bid price η from the interval $[\underline{\eta}, \overline{\eta}]$ with probability $Prob(\eta < x) = H(x)$. As the interval is continuous the probability that more than one generator submits a bid of the same price is zero.

A.3 Distribution from which generators draw their bids

We now calculate the aggregate density function $H(\eta)$ describing the probability $H'(\eta)$ with which a generators chooses a bid η. The generator decides on the distribution function $H(\eta)$ such that he can ensure that all traders make zero profits if and only if the aggregate bid schedule of traders is $1 - k\,(\eta)$.

Generators can only profitable bid if they obtain a significant positive quantity of transmission contracts and hence $H(0) > 0$ and $H(\eta) = H(0)$ for $\eta\epsilon[0, \underline{\eta}]$.

The expected value $v_t\,(\eta)$ of marginal bid by a trader is zero if the bid is not accepted. If the bid is accepted, then $v_t\,(\eta)$ is the weighted integral over price increase minus bid price:

$$E\left[v_t\,(\eta)\right] = H(0)^n\left(\Delta p_{2,0} - \eta\right) + \int_{\underline{\eta}}^{\eta} nH(\widetilde{\eta})^{n-1}H'(\widetilde{\eta})\left(\Delta p_{2,k} - \eta\right)d\widetilde{\eta}. \tag{38}$$

After substituting $\Delta p_{2,k}$ from (35) we differentiate the right hand side with respect to η. It has to be zero because profits for all trades are zero:

$$\frac{H'(\eta)}{H(\eta)} = \frac{1}{n}\frac{\alpha\,(n+1)}{k - \eta\alpha\,(n+1)}. \tag{39}$$

Setting $\eta = \underline{\eta}$ in (38) gives $E\left[v_t\left(\underline{\eta}\right)\right] = -H(0)^n\underline{\eta}$. Traders submitting bids at $\underline{\eta} > 0$ would make losses, therefore we conclude that $\underline{\eta} = 0$.

A.4 Traders' aggregate bid schedule

We now calculate which distribution of bid function $1-k(\eta)$ of traders ensures that generators will be indifferent between not bidding or bidding any value on the interval $[\underline{\eta},\overline{\eta}]$. Generator's expected profits when submitting a bid (η, K) consist of the profits of winning, given by the probability that other generators bid lower $H(\eta)^{n-1}$ times profits minus costs for transmission rights. Furthermore we add the weighted profits made when other generators obtain transmission rights and therefore reduce their output and push up prices. All these profits have to equal the profits from not participating in the auction:

$$H(\eta)^{n-1}\left[\Delta\pi_{k(\eta)}-\eta k(\eta)\right]+\int_{\eta}^{\overline{\eta}}(n-1)\,H(\widetilde{\eta})^{n-2}H'(\widetilde{\eta})\Delta\pi_{-k(\widetilde{\eta})}d\widetilde{\eta}$$

$$=\int_{\underline{\eta}}^{\overline{\eta}}(n-1)\,H(\widetilde{\eta})^{n-2}H'(\widetilde{\eta})\Delta\pi_{-k(\widetilde{\eta})}d\widetilde{\eta}. \tag{40}$$

Differentiation (40) with respect to η and then substituting $H'()$ from (**??**) and $\Delta\pi$'s from (36) and (37) gives:

$$\frac{\partial k}{\partial\eta}=k\frac{\alpha(n+1)^2+(n-1)\frac{H'(\eta)}{H(\eta)}\left((1+n)\,Q_{\text{comp}}+\alpha(n+1)^2\eta\right)}{2k-\alpha(n+1)^2\eta+(1-n)\,Q_{\text{comp}}}$$

Substituting $\frac{H'(\eta)}{H(\eta)}$ from (39) provided us with a second differential equation:

$$\frac{\partial k}{\partial\eta}=\alpha(n+1)^2k\frac{1+\frac{n-1}{n}\frac{Q_{\text{comp}}+\alpha(n+1)\eta}{k-\eta\alpha(n+1)}}{2k-\alpha(n+1)^2\eta+(1-n)\,Q_{\text{comp}}}. \tag{41}$$

Using parameter values $n=2$, $\alpha=1$, $K=3$ and $Q=3/2$ the differential equations (**??**) and (41) can be written as

$$\frac{\partial k(\eta)}{\partial\eta}=\frac{9}{2}k\frac{4k-6\eta+3}{(k-3\eta)(4k-18\eta-3)},\quad \frac{\partial H(\eta)}{\partial\eta}=\frac{1}{2}\frac{H(\eta)}{\frac{1}{3}k-\eta}. \tag{42}$$

A.5 Start values

In order to numerically integrate (42) we still have to determine two unknown parameters $\underline{\lambda}$ and $H(0)$. The requirement that generators are indifferent between participating and not participating in the auction can be expressed by setting $\eta=\underline{\eta}$ in generators' profit function (40): $\Delta\pi_{k(\eta)}=\eta k(\eta)$. Using (36) and the numeric values for the parameters gives then $\underline{\lambda}=\frac{1}{2}K=\frac{3}{2}$.

The choice of $H(0)$ is best illustrated with the help of the final result presented in Figure (5). The probability that any player does not bid is $H(0)=0.91$. This is the value which ensures that $H(\overline{\eta})=k(\overline{\eta})=1$ for $\overline{\eta}=0.127$. If we would chose a different $H(0)$ such that $k(\overline{\eta})>K$ then generators could already obtain all transmission rights for a lower η' and therefore they will not submit any bids above η' and therefore $H(0)$ would increase. If on the other hand $k(\overline{\eta})<K$,

then a generator increasing his bid to $\overline{\eta} + \varepsilon$ would obtain all transmission rights. This would be a profitable deviation suggesting that $H(0)$ would decrease. The unique solution is therefore $H(0) = 0.91$.

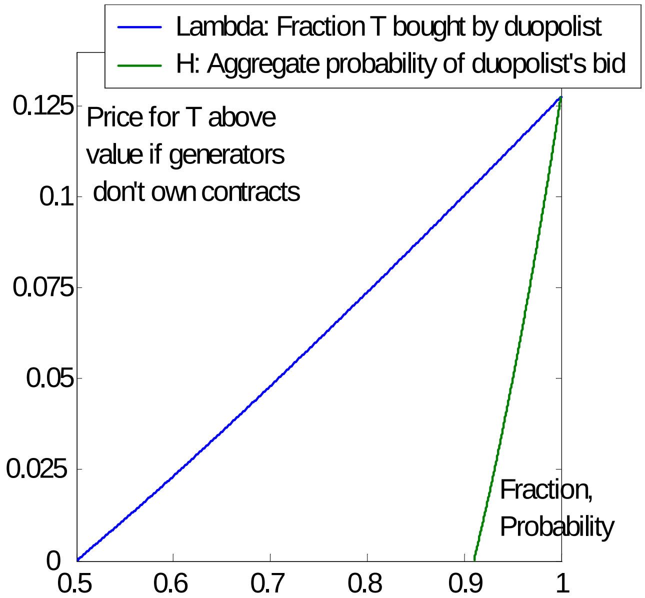

Figure 5:

A.6 Stability

We calculated a Nash equilibrium, which ensures stability given other players' bid functions. To check for stability or existence of this equilibrium it should not be profitable for a generator to announce not to participate in the auction. An announced deviation could be profitable even if the previous analysis determined a Nash equilibrium, because other players change their bid functions following the announcement.

The intuitive argument why such a deviation is not profitable goes as follows. To first order all generators are indifferent between bidding and not bidding, therefore the remaining generators increase their bidding probability by 1/n. Traders do not observe any difference because the aggregate distribution of bids stays the same and therefore maintain their bidding strategy.

Generators however observe a second order effect. If they do not bid, then the probability that someone else will bid has been reduced by $1/n^2$ and therefore the expected profit has been reduced. If they bid a low price, then the probability that someone else bids a higher price has likewise been reduced by $1/n^2$, once again resulting in a reduction of expected revenue, but by less than $1/n^2$. Only if generators bid the maximum price could profit stay the same as before. Therefore generators will submit high bids more frequently. As a result the profits for traders increase. However, we assume perfect arbitrage, therefore additional traders will submit bids. This additional bids reduce the amount of transmission contracts generators can obtain for a given price and therefore the profits of generators. As a result the high bids become less profitable for generators. So generators reaction to the second order effect pushes up revenue for low bids while at the same time traders' reaction reduces revenue for the high bids until generators are indifferent between no, low and high bids. Overall profits for generators are reduced, but by not more than $1/n^2$. This shows that the deviation strategy is not profitable for generators.

B Lemmas to understand continuous retrading

Lemma 9 *Assume continuous trading over an open time interval with traders offering to buy transmission contracts at* p_x*. A generator can not profitably sell transmission contracts, if profitability requires the assumption that he is the last to sell transmission contracts.*

Proof. The profit for a generator to sell at p_x assuming that no other generator will subsequently sell transmission contracts is:

$$\begin{aligned} \Delta\pi_{\Delta_i}^{\text{sell alone at p}_r} &= \pi_{k_i-\Delta_i}^{spot} - \pi_{k_i}^{spot} + \Delta_i\left(p_x - c_1\right) \\ &= \frac{\Delta_i}{\alpha\left(n+1\right)^2}\left(\alpha\left(n+1\right)^2\left(p_x - c_2\right) + \Delta_i - 2k_i - 2Q_{\text{comp}} - 2\sum_{j\neq i} k_j\right). \end{aligned} \tag{43}$$

Profits are increasing in Δ_i. It follows that if a generator sells, then he will sell all transmission contracts $\Delta_i = k_i$.

Profits increase if the total sum of transmission contracts held by generators is reduced. Therefore if it was profitable for one generator to sell at p_r assuming that no other generator sells, then it will be profitable for all subsequent generators to sell at p_r assuming that no other generators sells. ■

Lemma 10 *Assume continuous trading over an open time interval with symmetric allocation of transmission contracts and traders offering to buy transmission contracts at* p_y*. If one generator profitably sells his transmission contracts assuming others will follow, then all others will follow.*

Proof. Assume a generator believes that all other generators will subsequently sell their transmission contracts. His expected profits from selling are:

$$\begin{aligned}
\Delta\pi^{\text{all sell at p}_r} &= \pi_0^{spot} - \pi_k^{spot} + \Delta_i \left(p_y - c_1\right) \\
&= \frac{k_i}{\alpha\left(n+1\right)^2}\left(\alpha\left(n+1\right)^2\left(p_y - c_2\right) - 2\left(Q_{\text{comp}} + \sum_{j=1}^{n} k_j\right)\sum_{j=1}^{n}\frac{k_j}{k_i}\right). \quad (44)
\end{aligned}$$

Initially each generators owned k transmission contracts, and then s generators sold all their transmission contracts. Differentiating the profits obtained from selling transmission contracts (44) with respect to how many generators s already sold their contracts gives:

$$\begin{aligned}
\frac{\partial}{\partial s}\Delta\pi_s^{\text{all sell at p}_r} &= \frac{\partial}{\partial s}\frac{k}{\alpha\left(n+1\right)^2}\left[\alpha\left(n+1\right)^2\left(p_y - c_2\right) - 2\left(Q_{\text{comp}} + \left(n-s\right)K\right)\left(n-s\right)\right] \\
&= \frac{k}{\alpha\left(n+1\right)^2} 2\left(Q_{\text{comp}} + 2\left(n-s\right)K\right) > 0.
\end{aligned}$$

If it was profitable for the first generator to sell it's contracts, then it will be so for all subsequent generators. ■

Lemma 11 *Assume continuous trading over an open time interval with each generator owning k transmission contracts. Traders will not offer to buy contracts at price p_r above the price p_0 that contracts have when no generator holds any contracts.*

Proof. We first show that traders will not offer to buy an unlimited amount of contracts at $p_r > p_0$. Lemma 10 establishes that if the first generator profitably sells his contracts knowing that others will follow, then all generators will sell their contracts. The result is that the value of contracts falls to p_0 and traders make losses. Therefore traders cannot pay a price which would allow all traders to profitably sell their transmission contracts. To prevent all generators selling their transmission contracts by making profits in (44) negative, the traders have to offer less than

$$p_x = c_2 + \frac{2nQ_{\text{comp}} + 2n^2 k}{\alpha\left(n+1\right)^2}.$$

If traders offer to buy transmission contracts at $p < p_x$ then generators can only profitably sell contracts, if they anticipate that not all other generators will sell their contracts. Lemma 9 shows that if a generator can only profitably sell if he assumes he will be the last to sell, then another generator can profitably sell subsequently if he assumes he will be the last to sell. This cannot be a rational expectations equilibrium and no generator can buy at $p < p_x$.

Now assume traders would offer to buy a limited amount of contracts at $p_r > p_0$. The profit from selling contract for generators increases with the amount of contracts already sold, both for generators assuming that all contracts will eventually be sold, (44), and for generators that believe that only a fraction of all contracts will be sold to traders, (43). Increasing profitability of selling implies that generators would profit from selling additional contracts and the limit can only be imposed from the traders' side. New traders can always enter and offer to buy at p_r, so the limit could only be that buying at p_r is unprofitable. But given that generators sell profitably at p_r they will also sell profitably at $p_r - \varepsilon$. A trader that has not bought any transmission contracts could offer this deal. The additional sales reduce the value of contracts below p_r and induce losses on all traders that bought contracts at p_r. Therefore traders cannot rationally offer to only buy a limited amount of contracts. ■

C Uniform price auction in meshed networks

We first give the direct proof for a symmetric generators with constant marginal costs (Lemma 12) and then the extended version for asymmetric players with increasing marginal costs (Lemma 13 and following).

Lemma 12 *If constrained transmission capacity in a meshed network is sold on a single price auction that is efficiently arbitraged by traders who can accurately predict future equilibrium spot prices, then symmetric oligopolists with constant marginal costs will only acquire contracts that mitigate market power.*

Proof. Assume that the set of transmission constraints that is binding does not change with the allocation of transmission contracts and that market power is only exercised at one node. As all transmission constraints stay binding and demand is linear, prices are some linear function of demand. Assume the oligopoly generators are located at node 2 with constant marginal costs c_2. Oligoplist i produces q_i, and total production at node 2 is Q. Oligopolists sell to node 3, where the price is p_3 (which can be several nodes, in which case p_3 is a linear combination of prices of several nodes). Prices at node 3 depend on constant competitive production costs, subsumed into A_3, and on output at node 2, where α_3 can be positive, zero or negative:

$$p_2 = A_2 - \alpha_2 Q; \qquad p_3 = A_3 - \alpha_3 Q. \tag{45}$$

In the energy market the oligopolist owning k_i transmission contracts maximises the profit function:

$$\pi_i^{spot} = (p_2 - c_2)\, q_i + (p_3 - p_2)\, k_i \theta, \tag{46}$$

where θ is the ratio of the amount that can be sold to node 3 to the underlying flow-gate rights ($\theta = 3$ in (30)). The FOC gives the optimal output choices in the energy market:

$$q_i = \frac{A_2 - c_2 - (\alpha_2 - \alpha_3) \sum k_i \theta}{\alpha_2 (n+1)} + \frac{\alpha_2 - \alpha_3}{\alpha_2} k_i \theta; \qquad Q = \frac{n (A_2 - c_2) + (\alpha_2 - \alpha_3) \sum k_i \theta}{(n+1)\, \alpha_2}\ . \tag{47}$$

This allows the determination of energy market profits as a function of transmission contracts. In a uniform price auction competitive arbitrageurs will ensure that the market clearing price for transmission contracts will equal the value of these rights in the energy auction $(p_3 - p_2)$. A generator chooses the number of transmission contracts he bids for in the auction to maximise profits in the energy auction π_{energy} minus the costs he incurs to buy the rights, $(p_3 - p_2)\, k_i\theta$.

$$\begin{aligned} \pi_i^{auction} &= \pi^{spot}(k_i) - (p_3 - p_2)\, k_i\theta \\ &= \frac{A_2 - c_2 - (\alpha_2 - \alpha_3)\sum k_i\theta}{(n+1)} \left(\frac{A_2 - c_2 - (\alpha_2 - \alpha_3)\sum k_i\theta}{\alpha_2\,(n+1)} + \frac{\alpha_2 - \alpha_3}{\alpha_2} k_i\theta \right). \end{aligned}$$

Calculating the first order condition with respect k_i, using symmetry between all generators and resubstituting in 47 gives:[10]

$$\sum k_i\theta = \frac{n^2 - n}{1+n^2}\frac{A_2 - c_2}{\alpha_2 - \alpha_3}; \qquad Q_{transmission\_rights} = \frac{n^2}{1+n^2}\frac{A_2 - c_2}{\alpha_2}.$$

If output reductions of generators increase prices at the original node more than at the destination node of the transmission contract ($\alpha_2 > \alpha_3$), then generators will buy a positive quantity of transmission contracts (corresponding to an export contract), otherwise a negative quantity.

The result can be compared with the output choice $Q_{comp} = \frac{A_2 - c_2}{\alpha_2}$ in a competitive scenario ($n \to \infty$), and with the output Q_{no_tr} in a situation without transmission contracts ($k_i = 0\ \forall i$).

$$Q_{no\_tr} = \frac{n}{n+1} Q_{comp} \;<\; Q_{transmission\_rights} = \frac{n^2}{1+n^2} Q_{comp} \;<\; Q_{comp}.$$

As all nodes but node two are competitive, the only deadweight loss will occur due to withholding at node two. Deadweight losses are reduced if generators' market power is mitigated and they withhold less output, as demonstrated. Therefore transmission contracts allocated through a uniform auction are efficiency improving. ■

To generalise the previous lemma to allow for asymmetric generators and increasing marginal costs we require an indirect proof. Lemma 14 calculates output choice in the energy market. Aggregate output is an increasing function of the weighted sum of transmission contract holding. We therefore have to show that this weighed sum is positive to prove that market power is mitigated. Lemma 15 gives the FOC for the transmission contracts auction. Lemma 16 shows that the LHS of the FOC is positive and Lemma 17 proves that the weighted sum of transmission contracts holdings is positive given conditions on the slope the marginal cost curve. These conditions should be easily satisfied, because electricity demand is very inelastic (demand slope high) while marginal costs of generators that can alter their output decision are comparatively flat.

[10] We assume that less transmission contracts than available capacity are obtained by generators ($|\sum \lambda_i| < K$). Otherwise the argumentation parallel to section (3.1) shows that generators pre-empt all available transmission capacity mitigating market power.

Lemma 13 *If constrained transmission capacity in a meshed network is sold in a single price auction that is efficiently arbitraged by traders who can accurately predict future equilibrium spot prices, then oligopolists will only acquire contracts that mitigate market power. Marginal costs of generation should not increase by more than* $\left(\sqrt{2}-1\right)$ *times demand slope. If generators are asymmetric, then marginal costs of each generator should not increase by more than* $1/n$ *times demand slope (lower bounds due to approximations).*

Proof. We maintain the demand structure (45) but change the production cost incurred by generators from $C(q_i) = cq_i$ to $C(q_i) = c_i q_i - \frac{\beta_i}{2} q_i^2$. This allows the representation of the local shape of any cost curve.

The proof strategy is to show that the LHS of equation (49), which follows below, is positive if generators are symmetric (because $\overline{c} = c_i$) or if generators are asymmetric and $\forall i\ \beta_i \leq \alpha_2/n$ (Lemma 16). From Lemma (15) follows that the RHS of (49) is positive and using Lemma (49) and $\forall i\ \beta_i \leq \sqrt{2}-1$ it follows that $(\alpha_2 - \alpha_3) \sum \frac{k_i}{\alpha_2+\beta_i}$ is positive. Finally it follows from Lemma (14) that aggregate output is increased relative to a scenario without transmission contracts and therefore market power mitigated. ■

Lemma 14 *Total output of generators is increasing with* $(\alpha_2 - \alpha_3)\,\theta \sum \frac{k_i}{\alpha_2+\beta_i}$.

Proof. In the energy market the oligopolist owning k_i transmission contracts maximises the profit function

$$\pi_i^{spot} = \left(A_2 - \alpha_2 Q - c_i - \frac{\beta_i}{2} q_i\right) q_i + \left(A_3 - A_2 + (\alpha_2 - \alpha_3)\, Q\right) k_i \theta.$$

The FOC gives the optimal output choice for a given allocation of transmission contracts k_i:

$$\begin{aligned} Q &= \frac{A_2 \sum \frac{1}{\alpha_2+\beta_i} - \sum \frac{c_i}{\alpha_2+\beta_i} + (\alpha_2 - \alpha_3)\,\theta \sum \frac{k_i}{\alpha_2+\beta_i}}{1 + \sum \frac{\alpha_2}{\alpha_2+\beta_i}}, \\ q_i &= \frac{A_2 - \alpha_2 Q - c_i + (\alpha_2 - \alpha_3)\, k_i \theta}{\beta_i + \alpha_2}. \end{aligned} \tag{48}$$

Iff

$$(\alpha_2 - \alpha_3)\,\theta \sum \frac{k_i}{\alpha_2 + \beta_i} > 0$$

then aggregate output Q is increased according to (48) and therefore market power mitigated. ■

Lemma 15 *The FOC in the transmission contracts auction, defining* $A = \sum \frac{\alpha_2}{\alpha_2+\beta_i}$ *and* $\overline{c} =$

$\sum \frac{c_i}{n}$, *is*

$$\frac{A}{1+nA}(n-1)(A_2-\overline{c})+\sum_i \frac{\beta_i}{\alpha_2+\beta_i}(\overline{c}-c_i)$$
$$= (\alpha_2-\alpha_3)\theta K\left(\sum \frac{(1+A)^2}{1+nA}\frac{\beta_i}{\alpha_2}k_i+\sum \frac{\alpha_2}{\alpha_2+\beta_i}k_i\right). \tag{49}$$

Proof. If transmission contracts are allocated in a uniform price auction, then prices are arbitraged and ownership of transmission contracts only influences the output decision $q_i(k_i)$ of generators but does not directly influence the total profit function

$$\pi_i^{total} = \left(A_2-\alpha_2 Q-c_i-\frac{\beta_i}{2}q_i\right)q_i. \tag{50}$$

Substituting (48) in (50) and calculating the first order condition with respect to k_i for optimal transmission ownership gives:

$$\left(A_2+\sum_j \frac{\alpha_2(c_j-c_i)}{\alpha_2+\beta_j}-(\alpha_2-\alpha_3)\theta\sum \frac{\alpha_2 k_i}{\alpha_2+\beta_i}-c_i\right)$$
$$\left(\frac{\alpha_2}{2\alpha_2+\beta_i}-\frac{\frac{\alpha_2}{\alpha_2+\beta_i}}{1+\sum \frac{\alpha_2}{\alpha_2+\beta_i}}\right)$$
$$= \left(\beta_i\sum \frac{\alpha_2}{\alpha_2+\beta_i}+\beta_i+\alpha_2\frac{\alpha_2}{\alpha_2+\beta_i}\right)\frac{\alpha_2-\alpha_3}{2\alpha_2+\beta_i}k_i\theta. \tag{51}$$

Summing (51) over i and defining $A=\sum \frac{\alpha_2}{\alpha_2+\beta_i}$ and $\overline{c}=\sum \frac{c_i}{n}$ we obtain (49). ■

Lemma 16 *The LHS of FOC (49) is positive if* $\forall i \quad \frac{\beta_i}{\alpha_2}\leq 1/n$.

Proof. Define $f=\max_i(\frac{\beta_i}{\alpha_2})$. Given that $n\geq 2$ and $f\leq 1$ it follows that $A=\sum \frac{\alpha_2}{\alpha_2+\beta_i}\geq \frac{n}{1+f}\geq 1$. Using this lower bound on A we obtain that the LHS of FOC (49) is positive if

$$\frac{n-1}{n+1}(A_2-\overline{c})>\sum_i \frac{\beta_i}{\alpha_2+\beta_i}(c_i-\overline{c}). \tag{52}$$

The worst case scenario is that m small generators have high marginal costs $c_i=c_0+c_d$ which are increasing, with $\beta_i=f\alpha_2$, while the remaining $n-m$ generators have constant and lower marginal costs $\beta_i=0$ and $c_i=c_0$. Then (52) takes the form

$$A_2-c_0>\frac{\frac{n+1}{n-1}m(n-m)\frac{f}{1+f}+m}{n}c_d. \tag{53}$$

For small generators to participate their marginal costs c_0+c_d have to be such that demand is positive at that price $A_2-c_0-c_d>0$. Therefore we require in (53) that

$$\frac{n+1}{n-1}m(n-m)\frac{f}{1+f}+m<n. \tag{54}$$

The LHS of (54) takes the maximum for $m = n/2 + \frac{n-1}{n+1}\frac{1+f}{2f}$. If we assume $f \leq 1/n$ then the m for which the LHS takes the maximum is $m \geq n/2\,(1 + (n-1)/n) \geq n - 1$ (second inequality follows for $n \geq 2$). Substituting $m = n - 1$ for the worst possible mix of generators in (54) gives our requirement $f \leq \frac{1}{n}$. ■

Lemma 17 *$(\alpha_2 - \alpha_3)\,\theta \sum \frac{\alpha_2}{\alpha_2+\beta_i} k_i$ is positive if LHS of FOC (49) is positive and $\forall_i\ \frac{\beta_i}{\alpha_2} \leq \sqrt{2}-1$.*

Proof. (by contradiction). Assume that

$$(\alpha_2 - \alpha_3)\,\theta \sum \frac{\alpha_2}{\alpha_2 + \beta_i} k_i < 0. \tag{55}$$

We know that the LHS of FOC (49) is positive, therefore the RHS is positive:

$$(\alpha_2 - \alpha_3)\,\theta \left(\sum \frac{(A+1)^2}{1+nA} \frac{\beta_i}{\alpha_2} k_i + \sum \frac{\alpha_2}{\alpha_2 + \beta_i} k_i \right) > 0. \tag{56}$$

Inequality (55) and (56) can only be satisfied simultaneously if for some i with $k_i > 0$ the coefficient of k_i in the first sum exceeds the coefficient in the second sum:

$$\frac{(A+1)^2}{1+nA} \frac{\beta_i}{\alpha_2} > \frac{\alpha_2}{\alpha_2 + \beta_i} \quad \text{or} \quad \frac{\beta_i}{\alpha_2} \frac{\alpha_2 + \beta_i}{\alpha_2} > \frac{1+nA}{(A+1)^2}. \tag{57}$$

Use $f = \max_i(\frac{\beta_i}{\alpha_2})$ to obtain a lower bound for $A = \sum_i \frac{\alpha_2}{\alpha_2+\beta_i} > \frac{n}{1+f}$. Apply the lower bound to the RHS of (57), which is increasing in A, and the upper bound for β_i to the LHS to obtain:

$$f\,(f+1) > \frac{1 + nn\frac{1}{1+f}}{\left(n\frac{1}{1+f} + 1\right)^2} \quad \text{or} \quad f\,(n+f+1)^2 > f + 1 + nn. \tag{58}$$

For our assumption to be true (58) requires that f is bigger than

n	2	3	4	6	∞
f	$\sqrt{2}-1$	.52	.57	.61	1

.

If we assume that $f < \sqrt{2}-1$ then inequality (55) and (56) can not be satisfied simultaneously when $(\alpha_2 - \alpha_3)\,\theta \sum \frac{\alpha_2}{\alpha_2+\beta_i} k_i$ is positive. ■

D Mixed-strategy equilibrium in three-node network

The result for the monopoly case is given here. The monopolist will offer to buy all contracts available at price η. He draws η with from the distribution $H()$ with probability $H'(\eta)$. The aggregate bid function of traders does not change in equilibrium because additional bids make losses while fewer bids provide an arbitrage opportunity. The monopolist therefore anticipates the proportion of contracts $k(\eta)$ he can obtain when bidding η in the auction. In Nash equilibrium the monopolist should be indifferent between all prices and therefore indifferent between changing the price of his bid. Subtract the costs for buying transmission contracts ηk from the profits in the energy market (30) to obtain total profits:[11]

$$\Delta\pi_{total}(k_i) = \frac{(Q_{\text{comp}} + 3k)^2}{4\alpha(n+1)^2} + (c_2 - c_1)\,3k - \eta k. \tag{59}$$

The first order condition (FOC) with respect to the bid price η gives

$$\frac{\partial k(\eta)}{\partial \eta} = \frac{k(\eta)}{\frac{3}{8\alpha}Q_{\text{comp}} + 3(c_2 - c_1) + \frac{9}{8\alpha}k(\eta) - \eta} = \frac{k(\eta)}{\frac{\eta_0}{3} + \frac{9}{8\alpha}k(\eta) - \eta}. \tag{60}$$

In (60), η_0 is the value of transmission contracts for traders when the monopolist does not own contracts. The solution of the differential equation is

$$k(\eta) = \frac{\eta - \frac{\eta_0}{3} + \sqrt{\left(\eta - \frac{\eta_0}{3}\right)^2 - \frac{9K}{8}const_1}}{\frac{9}{8\alpha}}. \tag{61}$$

The traders' strategy is determined from the no-arbitrage condition that requires all bids to make zero expected profit. A bid is only accepted if the price is at least as high as the monopolist's bid. The expected profit from a bid equals the integral over all bids by the monopolist that are lower than the bid price η of the trader, weighted with their probability:

$$E[\pi_t(\eta)] = \int_{\underline{\eta}}^{\eta} H\prime(\widetilde{\eta})\,(3p_3(\widetilde{\eta}) - 3p_2(\widetilde{\eta}) - \eta)\,d\widetilde{\eta}.$$

As profit has to be zero for all η, the change in profit also has to be zero:

$$\frac{\partial}{\partial \eta}E[\pi_t(\eta)] = H\prime(\eta)\,(3p_3(\eta) - 3p_2(\eta) - \eta) - H(\eta) \equiv 0. \tag{62}$$

Differential equation (62) determines the probability with which the monopolist chooses bids for a mixed strategy equilibrium to exist. The upper support of the bids is $\overline{\eta}$ with $H(\overline{\eta}) = 1$. Solving (62) gives

$$H(\eta) = \exp\left[-\int_{\eta}^{\overline{\eta}} \frac{1}{3p_3(\widetilde{\eta}) - 3p_2(\widetilde{\eta}) - \widetilde{\eta}}d\widetilde{\eta}\right]. \tag{63}$$

Substituting p_2 and p_3 from (29) and $k()$ from (61) into (63) we obtain

$$H(\eta) = \exp\left[-\int_{\eta}^{\overline{\eta}} \frac{1}{\sqrt{\left(\widetilde{\eta} - \frac{\eta_0}{3}\right)^2 - \frac{9K}{8}const_1}}d\widetilde{\eta}\right] = \frac{\eta - \frac{\eta_0}{3} + \sqrt{\left(\eta - \frac{\eta_0}{3}\right)^2 - \frac{9K}{8}const_1}}{\overline{\eta} - \frac{\eta_0}{3} + \sqrt{\left(\overline{\eta} - \frac{\eta_0}{3}\right)^2 - \frac{9K}{8}const_1}} \tag{64}$$

At the lower support $\underline{\eta}$, $H(\underline{\eta}) = 0$. This gives $\underline{\eta} - \frac{\eta_0}{3} + \sqrt{\left(\underline{\eta} - \frac{\eta_0}{3}\right)^2 - \frac{9K}{8}const_1} = 0$, which is only satisfied for $\underline{\eta} = \frac{\eta_0}{3}$ and $const_1 = 0$. We therefore obtain:

$$k(\eta) = \frac{16\alpha}{9}\left(\eta - \frac{\eta_0}{3}\right), \qquad \overline{\eta} = \frac{\eta_0}{3} + \frac{9K}{16\alpha}, \quad H\prime(\eta) = \frac{16\alpha}{9K}. \tag{65}$$

The mixed-strategy equilibrium is thereby described and looks similar to the one presented in figure (2).

[11] Oligopolist: $\Delta\pi_{total}(\lambda_i) = \frac{1}{4\alpha}\left(\frac{2\alpha c_1 - 4\alpha c_2 + 2A - 3K + \sum_j \lambda_j 3K}{n+1}\right)^2 + (c_2 - c_1)\,\lambda_i 3K - \eta(\lambda)\,\lambda_i K$

References

Allaz, B. and J.-L. Vila (1993). Cournot competition, forward markets and efficiency. *Journal of Economic Theory 59*, 1–16.

Bohn, R. E., M. C. Caramanis, and F. C. Schweppe (1984). Optimal pricing in electrical networks over space and time. *Rand Journal of Economics 15, No 3*, 360–376.

Borenstein, S., J. Bushnell, E. Kahn, and S. Stoft (1996). Market power in california electricity markets. *Utilities Policy 5*, 1–21.

Borenstein, S., J. Bushnell, and S. Stoft (2000). The competitive effects of transmission capacity in a deregulated electricity industry. *RAND Journal of Economics 31, No. 2*, 294–325.

Green., R. J. and D. M. Newbery (1992). Competition in the british electricity spot market. *Journal of Political Economy 100 (5)*, 929–953.

Harvey, S. M. and W. W. Hogan (2000). Nodal and zonal congestion management and the exercise of market power. Harvard University, http://ksghome.harvard.edu/ .whogan.cbg.ksg/.

Hogan, W. W. (1992). Contract networks for electric power transmission. *Journal of Regulatory Economics (4)*, 211–242.

Joskow, P. and E. Kahn (2002). A quantitative analysis of pricing behavior in california's wholesale electricity market during summer 2000: The final word. *Working paper*.

Joskow, P. and J. Tirole (2000). Transmission rights and market power on electric power networks. *RAND Journal of Economics 31 No. 3*, 450–487.

Newbery, D. M. (1998). Competition, contracts, and entry in the electricity spot market. *RAND Journal of Economics 29 (4)*, 726–749.

Stoft, S. (1999). Financial transmission rights meet cournot: How tccs curb market power. *Energy Journal 20 (1).*