

# Another Perspective on Planned Obsolescence: Is there really too much innovation?\*

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## Abstract

Models of durable goods with network externalities that set instantaneously have emphasized that a monopolist selling those goods has too high an incentive to introduce new vintages of the durable good, to make previous vintages (already bought by consumers) obsolete. This is referred to as planned obsolescence. We examine the robustness of planned obsolescence to the inclusion of network externalities that *set in with a lag*. If externalities set in with a lag (however small), consumers have an incentive to wait for other consumers to adopt the new vintage first, and in the absence of any change in prices, that leads to inefficient delay in adoption.

Combining the two types of incentives we show that the monopolist is able to overcome consumer's inertia and still generate planned obsolescence through both intratemporal and intertemporal price discrimination. However, if monopoly power is "short lived" (for example due to copying), we show that, depending on the parameters of the model, we could have both types of inefficiencies: planned obsolescence or *delay*. Delay is brought about because copying limits the ability of the monopolist to increase prices in the future and therefore gives consumers an incentive to wait for both the onset of the (lagged) externality effect and the reduction in price caused by copiers. Delay appears mainly when the externality effect is strong and the new vintage is a significant improvement over the existing durable good.

**Keywords:** Planned obsolescence, durable goods, lagged network externalities, monopoly, delay.

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# 1 Introduction

Models of durable goods with network externalities that set instantaneously have emphasized that a monopolist selling those goods has too high an incentive to introduce new vintages of the durable good, to make previous vintages (already bought by consumers) obsolete. This is referred to as planned obsolescence. We examine the robustness of planned obsolescence to the inclusion of network externalities that *set in with a lag* (lagging externalities). Combining the two types of incentives we show that the monopolist is able to overcome consumer's inertia and still generate planned obsolescence through both intratemporal and intertemporal price discrimination. However, if monopoly power is "short lived" (for example due to copying), we show that, depending on the parameters of the model, we could have both types of inefficiencies: planned obsolescence or *delay*. Delay is brought about because copying limits the ability of the monopolist to increase prices in the future and therefore gives consumers an incentive to wait for both the onset of the (lagged) externality effect and the reduction in price caused by copiers. Delay appears mainly when the externality effect is strong and the new vintage is a significant improvement over the existing durable good.

We introduce lagging externalities (instead of the more commonly used *contemporaneous* externalities) when analyzing durable goods with two motivations in mind.<sup>1</sup> First, we can think of network externalities arising in durable goods not because of the presence of any physical interconnection (as in the case of the choice of a telecommunications standard), but because of the need of a support base for those durables, that becomes more accessible as the number of users of a particular model increases. If we think that there are some frictions or inertia in this support industry, then the result is a lagged adjustment to the movements in the user base of the downstream durable good. These kind of indirect network externalities arise, for example, in car sales and their corresponding network of repair shops; or computer platforms are the corresponding availability of software titles compatible with them, or in microprocessors and the availability of software that can use all its capabilities.<sup>2</sup>

Another motivation for the use of lagging externalities considers the importance for many consumer products and capital goods on the availability of information and experience of use (learning by using). A product with a bigger user base would make it easier for someone using a particular model to find someone else with the same model and be able to get some information about his experience of use of that product. If we consider that the development and transmission of knowledge takes some time, then it seems appropriate to consider lagging instead of contemporaneous externalities. An example of this kind of indirect network externality is the use of a particular word processing or spreadsheet software: the utility one gets out of a particular brand of word processor depends not only on the number of users to which one can exchange files (a contemporaneous effect), but also on how easy one can find a fellow user to work out some "bugs" in the

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<sup>1</sup>We are not the first to use the notion of lagged externalities. They appear, for example, in Durlauf (1991, 1993), Gale (1995) and Adserà and Ray (1998) although in a different context.

<sup>2</sup>An example from the computer industry is the appearance of the Pentium MMX processor, which included a special set of instructions to improve multimedia applications. A general complaint at the time of its appearance was the lack of software that could use all the new capabilities of the MMX chip: "The last time Intel added new instructions for its major PC microprocessor line was two years ago, when it introduced the Pentium MMX chip. MMX stands for multimedia extensions, and software developers *eventually* took advantage of the new hooks to develop software with better graphics and sound." [New York Times, Jan. 14th, 1999]

The same convenience of having new software was observed at the time of the appearance of the Pentium III processor: "If you are running the same old software it doesn't do much for you. It's a little faster than the Pentium II 450". Linley Gwennap (Microprocessor Report) on the Pentium III chip. [New York Times, Feb. 24, 1999]. "[People] know that to take full advantage of a Pentium III machine, they will have to buy new software." [New York Times, Jan. 14th, 1999]. However, industry analysts recognized the usual delay for developers of new software: "[T]he software that will fully exploit the parallel processing power of the [Pentium III's] new instructions is going to dribble out over the next year. By the time such software is widely available, perhaps six months from now, and by the time a critical mass of Internet Web sites have set up new features to take advantage of the added power, Intel will have switched to a new manufacturing process that will greatly increase the speed of future Pentium III's well beyond the current levels." [New York Times, March 4th, 1999].

program (which depends on the whole history of use of that brand).

However, as Rosenberg (1982) has pointed out, this learning effect can also arise in the feedback provided from users to producers, and can take significant time to develop.<sup>3</sup> This effect not only influences the development of new products, but also affects the utility derived from the use of existing durable goods, for example, because of cost savings in maintenance or the discovery of “bugs” that force product recalls. These cost savings in maintenance are apparently very important in the aircraft industry<sup>4</sup>, but also in electricity production in steam power plants.<sup>5</sup> In the case of the computer industry, this effect is also important and can benefit existing users of a software through free “add-ons” that correct for bugs in earlier versions.<sup>6</sup>

The second justification for the use of lagging externalities is more theoretical. We can consider externalities arising not only from the current size of the user base for a particular durable good, but also from the history of use of that product. Thus, to model the importance of the externality associated with a particular model, we can consider a general distributed scheme of its present and past use. We view the use of contemporaneous externalities as an assumption that puts all the weight in one of the extremes: the history of use is of no importance at all. What we propose here is to take the alternative view and test how sensitive the results of the typical model are to the choice of the timing of the externalities one uses in the model.

The main question of the paper will then be: Does a monopoly producing durable goods introduces new models too fast to make vintages already in the market obsolete? This question derives from casual observation about the reliability of durable goods, or the sometimes too fast perceived pace of introduction of “improved” versions of the good.<sup>7</sup>

As we will see in the next section, the literature focused first on the question of optimal durability and the incentives of a monopolist supplying durable goods. The question of planned obsolescence is slightly

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<sup>3</sup> “[...] in an economy with complex new technologies, there are essential aspects of learning that are a function not of the experience involved in producing the product, but of its *utilization* by the final user. This is particularly important in that case of capital goods. [...] in more general terms, the performance characteristics of a durable capital good often cannot be understood until after prolonged experience with it. [...] Closely related to this is the fact that optimal servicing and maintenance characteristics can be determined only after extensive use—in many cases, only after many years.” [Rosenberg (1982), pp. 122–123]

<sup>4</sup> “During the operation of a new aircraft, operating cost reductions depend heavily upon learning more about the performance characteristics of the system and components, and therefore upon understanding more clearly the full potential (as well as potential bugs), of a new design. For example, it is only through extensive usage that detailed knowledge is gained about engine operation, maintenance needs, minimum servicing and overhaul requirements, and so on.” [Rosenberg (1982) p. 125]

“In the case of the jet engine, [the interval between complete overhauls] was originally based on the experience with reciprocating engines and was extended as experience was gained. [...] whereas the airlines had overhauled piston engines after 2,000 to 2,500 hours of service, after some years of experience with the jet engines the time interval between overhauls was as high as 8,000 hours.” [p. 130]

<sup>5</sup> See the discussion in Rosenberg (1982), p. 136.

<sup>6</sup> “The development of effective software is highly dependent upon user experience. The modification of software systems in response to this experience is now intrinsic to software engineering. This is so because most software products permit wide variations in inputs and processing options. These options cannot possibly be tested completely prior to the release of software. Thus, the optimal design of software depends upon a flow of information from its customers. Furthermore, many computer companies routinely provide extensive software support that involves software modification when bugs are discovered by customers—as they inevitably are—when the software is used. The effectiveness of support services in improving the product after its release appears to be very important to the competitive success of computer firms.” [Rosenberg (1982), p. 139]. This type of user feedback is explicitly used for example by Netscape and its “Netscape feedback agent” that automatically prompts the user to send feedback to Netscape’s web site after any unexpected malfunction in the program.

<sup>7</sup> “They don’t make autos, [...] which last as well as they did in father’s day!” is a phrase cited in Swan (1972) as an example of the conventional wisdom on the subject. Bulow (1986) also cites John Kaplan about book revisions: “If an intelligent person is revising his textbook, do you think he’s going to redo it in such a way that you can use the old version.?” Notice however, that there is a qualitative difference between the two statements. In the first one, there is an issue of *reduced durability*, in the sense that the new durable to be introduced in the market will face no competition from the older version. In the second statement, the problem is *planned obsolescence*: when the supplier decides to introduce a new version of the durable, the old model is still around in the market, and in principle can still be used (maybe less efficiently) to perform the task for which it was sold.

different, since at the time the monopolist is deciding on introducing a new model into the market, the old version is still being used, and in effect, the monopolist competes with himself. With contemporaneous externalities, Waldman (1993) and Choi (1994) show that a monopolist will have an incentive to introduce new models in the market too fast, in order to make the existing models obsolete and induce old consumers in the market to scrap their old durables and buy again the new models. Thus, in equilibrium, there is planned obsolescence.

With lagging externalities, the situation is significantly different. The monopolist still has incentives to introduce new models very fast, to extract the old consumer's surplus again. However, the presence of lagging externalities, as in Ruiz (1997), creates an incentive for consumers to delay their adoption of a new model. The intuition is the following: since it takes one period for any change in the number of users to be reflected in the profitability of using a model, then when old consumers are considering a switch to a new model they have an incentive to let others go first and build up the externality associated with this new model<sup>8</sup>. We have two opposing forces on the two sides of the market, and in principle we could have any outcome depending on the parameters of the model. However, as we show in the first part, for all parameter configurations, the effect of the consumer side (delay) is weaker than the effect of the supply side (too early introduction of a new vintage). Thus, in equilibrium, there is never delay, and in some cases there is still planned obsolescence as in the previous literature with contemporaneous externalities.

As stated before, the situation changes if we allow a competitive fringe to copy the new vintage and supply it at marginal cost one period after its introduction by the monopolist. In that case, consumers will exercise their option to wait since now prices will never increase in the following periods. This generates both planned obsolescence or inefficient delay depending on the parameters of the model, something that cannot occur if externalities are contemporaneous.

The remainder of the paper is structured as follows. The next section reviews the previous literature on optimum durability and planned obsolescence. Section three presents the basic model. Section four presents the equilibrium outcome in the case of a monopolist, depending on the parameters of the model. Section five shows the socially optimum outcome and compares it with the market solution. Section six introduces the competitive fringe that can copy and sell any vintage one period after it has been introduced and again compares the equilibrium outcome with the planners solution, which is the same as in section five. Section seven concludes with some general comments

## 2 Related Literature

The literature on planned obsolescence as such is relatively new, although it is closely related to the literature on the optimal durability of a durable good produced by a monopolist. One of the first formal models of planned durability was Swan (1972). In that model, Swan argued that since the monopolist can control two

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<sup>8</sup>In the example of the new brand of car, an old consumer doesn't want to be the first one to give up the well developed repair shop network for his old model in order to buy a new model that still doesn't have that network today (although he knows it will have it in the future). Further, even if everyone else switches to a new vintage, he might as well wait for one period with his old model (still enjoying a good service) and then switch one period later than the rest of buyers, when the repair shops for the new model have had time to develop.

Rosenberg (1976), for example, states that "[o]n the purely technological level, innovations in their early stages are usually exceedingly ill-adapted to the wide range of more specialized uses to which they are eventually put. Potential buyers may postpone purchase to await the elimination of "bugs" or the inevitable flow of improvements in product performance or characteristics."

The same kind of reasoning is put forward in the case of microprocessors and software: "Experienced PC users have also learned that it is wise to wait a few months after any new technology, hardware or software, is first put on sale. [...] Remember, the early bird gets the bugs as well as the worm" [New York Times, March 4th, 1999]

variables, price and durability<sup>9</sup>, price will be used to extract the maximum consumer surplus, and durability will be set at the optimum level to minimize the cost of any flow of services from a stock of durable goods. This apparently closed the discussion about the existence of too low durability coming from a monopolist producing durable goods.

However, Bulow (1986) noted that the monopolist's pricing policy faces a time inconsistency problem: rational consumers anticipating that the monopolist will lower the price tomorrow to capture part of the residual demand from today's monopolistic price will restrain from buying today. The basic problem of the monopolist is that it cannot credibly commit to a constant price over time, and that lack of commitment hurts him.

In particular, one of the problems of the monopolist producing a durable good is the presence of a proportion of the goods sold in the first period that are carried over to the second. The monopolist, when making his decisions about whether to reduce his price or not in the second period, does not take into account the effect on the buyers in the first period, thus "flooding the market" and lowering the price too much, something the first-period consumers rationally anticipate.

However, one of the mechanisms the monopolist can use to escape from this "time inconsistency" trap is to manipulate the durability of the good away from the optimal (cost-minimizing) level: by reducing the durability of the good, the monopolist reduces the volume of the used goods present in the second period, and thus the negative externality on consumers is lower (and so is the time inconsistency problem). A similar point is made by Rust (1986) in the context of a second-hand market for durables and endogenous scrappage values for old durables.

All these models, while trying to incorporate the decision of killing the stock of the used good, focus on the durability dimension of the product. As Bulow (1986) already pointed out:

"... planned obsolescence is much more than a matter of durability; it is also and perhaps primarily about how often a firm will introduce a new product, and how compatible the new product will be with older versions."

With this in mind, Waldman (1993) develops a model where the monopolist does not choose durability, but whether or not to introduce a new product that is incompatible with the good already used in the market. Compatibility is important because there exists *contemporaneous* network externalities attached to each version of the product. A model by Choi (1994) addresses the same issues, but it also allows the monopolist to choose whether or not the new product will be compatible with the old one. The basic result in these two papers is very similar to the previous result on durability: the monopolist has an incentive to introduce new versions of the good too fast in order to make the existing versions obsolete, and thus induce consumers who already bought the product to buy again and so extract their consumer surplus twice.

These two papers, however, use contemporaneous externalities, and also allow consumers to coordinate on the Pareto superior equilibrium of the game, which tends to bias their result towards new products being taken by consumers very rapidly. We believe these models tend to suffer from the appearance of equilibria that are sustained by self-fulfilling expectations, as in Krugman (1991). The introduction of lagged externalities seem in the context of durable goods not only natural (as emphasized in the previous

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<sup>9</sup>It was not until the model by Choi (1994) and Waldman (1993) that a formal distinction is made between durability and obsolescence, although Swan (1972) and later Bulow (1986) already hinted that the two concepts were not exactly equivalent.

section) but it also rescues the importance of history in conditioning the equilibrium outcome of a game with payoff externalities, as emphasized by Adsera and Ray (1994) and Ruiz (1997).

### 3 Model

Consider a model of three periods  $t \in \{0, 1, 2\}$ . There exists a monopolist with constant marginal costs  $c$  of producing any of two vintages of a durable good, indexed by  $x \in \{A, B\}$ . Up to period 0 this monopolist could only produce and sell vintage  $A$ , but starting from period 1 he can also produce and sell vintage  $B$  at the same time. In period 0, there is a continuum of consumers with mass  $N$  who own vintage  $A$  of the durable good. Henceforth these will be referred to as “old consumers,” who live until period 2. In period 1, a mass  $N$  of new consumers arrive without the good and live also until period 2.

We assume that externalities *set in with a lag* (as in Ruiz 1997). In particular, it takes one period for a change in the number of users of a particular vintage to affect the utility of using that vintage. Formally, if  $n_{xt}$  is the mass of consumers using vintage  $x \in \{A, B\}$  in period  $t \in \{0, 1, 2\}$ , then the payoff of using a durable good of vintage  $x$  in period  $t$  is given by  $f(x) + n_{x,t-1}$ . The first term is just the “stand-alone” utility  $f(\cdot)$  derived from that vintage, whereas the second term is the lagged externality effect. Consumers without a durable good get a utility of zero.

To make the analysis simple, we assume that the two vintages  $A$  and  $B$  are incompatible. This means that we are assuming away any compatibility decisions, although the monopolist can always decide to continue producing the old vintage  $A$ , which is completely compatible with that currently used by consumers in period 0. We assume no discounting and

$$c < f(A) < f(B) \tag{1}$$

which has two direct implications. First, from the social point of view it is always better to produce at least one of the vintages of the durable good, since the stand-alone benefit of having the good outweighs its production costs. Second, (1) implies that the new vintage  $B$  coming on the market is intrinsically better than the older one.

#### 3.1 Timing and Strategies

In periods 1 and 2, the monopolist decides which prices to set for each vintage during that period. We allow the monopolist to discriminate between current users of different vintages, for example by letting consumers of vintage  $A$  trade their used durable goods if they decide to buy model  $B$ . Thus, for each period and model, the monopolist specifies two (possibly equal) prices.<sup>10</sup> We will therefore denote by  $p_{sx}$  the regular price for vintage  $x \in \{A, B\}$  in period  $s \in \{1, 2\}$ . Accordingly,  $p_{sx}^t$  will be the trade-in price for vintage  $x$  in period  $s$ . Notice, however, that in period 1, consumers still do not have model  $B$  so there is no possibility of trading model  $B$  for model  $A$ . This eliminates  $p_{2A}^t$  from the strategy set of the monopolist. Each period, after observing the monopolist pricing scheme for that period, consumers decide which vintage to adopt. The sequence of moves is summarized in figure (1)

We will look for the subgame perfect equilibria of this game. However, given the presence of externalities at the consumer level, there is a (potential) problem of multiplicity of equilibria, driven by expectations about

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<sup>10</sup>Of course, the monopolist has to take into account the possibility of arbitrage between the two different groups charged different prices. Note also that the monopolist can set prices so that a particular model will not be bought in equilibrium.

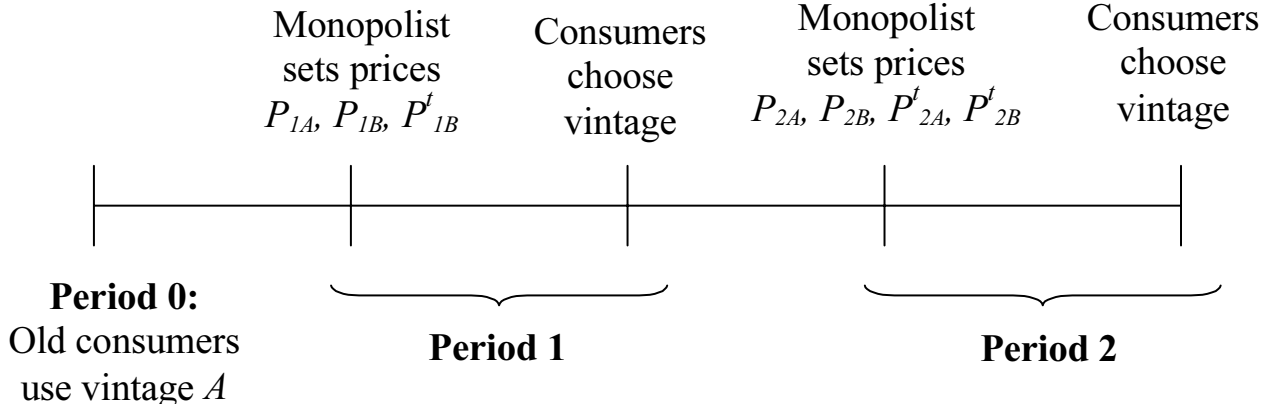


Figure 1: Sequence of moves

other consumer's actions. Where this problem arises, we will assume, as in Katz and Shapiro (1986) and most of the literature thereafter, that agents can coordinate on their Pareto-optimal equilibrium strategy<sup>11</sup>.

## 4 The Monopolist's Problem

As stated before, for each period the monopolist has set prices for each vintage. Since we are looking for subgame-perfect Nash equilibria, we will work backwards from the second period, taking into account any possible histories up to period 1.

The history of the game up to period 1 can be expressed in terms of prices set by the monopolist or in terms of vintages used by old and new consumers. For the remainder of the paper it will be more useful to take these second approach. At the beginning of period 1, the history of the game is just the tuple  $\{A, \emptyset\}$  where the first component is the vintage used by old consumers and  $\emptyset$  in the second component represents that no vintage was used by the old consumers. At the beginning of period 2, there are six possible histories:  $\{\{A, \emptyset\}, \{w, y\}\}$  where  $w \in \{A, B\}$  is the vintage used by old consumers in period 1 and  $y \in \{A, B, \emptyset\}$  is the vintage used by new customers in that first period.<sup>12</sup>

### 4.1 Second Period Decision

There are six possible histories in the second period. Since the common component  $\{A, \emptyset\}$  will play no role in the second period, we will just denote each of the possible six histories by the outcome of the first period:  $\{w, y\}$ , where  $w \in \{A, B\}$  and  $y \in \{A, B, \emptyset\}$ .

Table 1 summarizes the possible histories in period 1 and the corresponding maximum second period monopoly prices that induce a purchase by consumers, extracting the maximum surplus from them. Of course, since this is the last period, the monopolist will never try to induce a purchase setting second period

<sup>11</sup>Note however, that as in Adsera and Ray (1998), the use of lagging externalities significantly reduces the set of equilibria as compared with the case of contemporaneous externalities. In fact, multiplicity is eliminated in consumer's choice in period 2.

<sup>12</sup>Note that even with assumption (1) the monopolist may successfully prevent the newcomers from buying any model in period 1 by the appropriate choice of trade-in prices targeted to old consumers. In particular, he can always set a ridiculously high regular price and an "affordable" trade-in price.

prices below marginal cost  $c$ . Therefore  $c$  constitutes the lower bound on any equilibrium second period price set by the monopolist. The maximum price the monopolist can charge to each group depends on each consumer's outside option, which in most cases is equal to the payoff they would have received by keeping the same vintage they had in period 1. The only exception to this are histories  $\{A, \emptyset\}$  and  $\{B, \emptyset\}$  where new consumers have not used any vintage yet, and therefore have zero reservation utility. Note that in each case the prices that can be charged each group of consumers are independent of the actions of the other group (or even of the actions of other consumers of the same group). The reason is that externalities are lagging, so any change in the number of users today will not affect the utility derived from the use of a particular vintage today, and since this is the last period, it will not matter at all.<sup>13</sup>

Note that given the strategy of the monopolist in the second period, consumers are indifferent between buying that vintage or not. These prices represent the limit of prices set to make consumers strictly prefer that particular vintage. The monopolist's price strategy also includes prices which do not strictly correspond with the history of the game. For example, if in the first period both types of consumers are using model  $A$  (i.e. history  $(A, A)$  in the table), the monopolist still has to specify a regular price  $p_{2B}$  and a trade-in price  $p_{2B}^t$  for vintage  $B$ . This takes into account out-of-equilibrium strategies on the part of consumers in period 1.

In order to understand the contents of table 1, it would be useful to review some of its entries. For example, if old consumers kept their durable good  $A$  in period 1 and new consumers acquired also that vintage we are in the first row of table 1 (outcome  $(A, A)$  in the first period). In the second period, the monopolist will extract the maximum surplus from all consumers. In period 2, vintage  $A$  enjoys an externality effect of  $2N$  (the number of users of that vintage in period 1), so if period 1 customers keep vintage  $A$  they will get a utility flow in period 2 equal to  $f(A) + 2N$ . Even if all decide to change to vintage  $B$  each one would only get utility  $f(B)$  in that period. Therefore, the maximum trade-in price the monopolist can set for vintage  $B$  in this subgame is  $f(B) - f(A) - 2N$ . Of course, it needs to be checked that this price is not below marginal cost, and therefore  $p_{2B}^t = \text{Max}\{f(B) - f(A) - 2N, c\}$ . Notice that if  $p_{2B}^t = c > f(B) - f(A) - 2N$  then consumers will not trade-in their vintage  $A$  in exchange for vintage  $B$ . If a consumer shows up without any vintage to trade in and wants to buy vintage  $A$ , the monopolist can extract all the surplus generated by that vintage in period 2 since that consumer has no outside option. Therefore  $p_{2A} = \text{Max}\{f(A) + 2N, c\} = f(A) + 2N$  from assumption (1). If that consumer wants to buy vintage  $B$ , it will be willing to pay only up to  $p_{2B} = \text{Max}\{f(B), c\} = f(B)$ , again keeping no consumer surplus in period 2. If there is a consumer in period 2 that already has model  $B$  and wants to have model  $A$ , it will not be willing to pay  $p_{2A} = f(A) + 2N$  since now his outside option is to keep model  $B$  and enjoy a second period consumer surplus equal to  $f(B)$ . Therefore, the monopolist will have to offer a trade-in price  $p_{2A}^t = \text{Max}\{f(A) - f(B) + 2N, c\}$ . Therefore, the monopolist has two possible second-period outcomes to choose from for this subgame:  $(B, B)$  if he sells vintage  $B$  to both groups or  $(A, A)$  if he decides not to sell to anyone. The same type of restriction applies to the case where both groups own vintage  $B$  in the second period (fourth line in table 1).

Notice that in the derivation of optimal prices and the equilibrium behavior of consumers we have not assumed that consumers coordinate their purchases. This assumption, which is common in the literature, is needed in the case of contemporaneous externalities. With contemporaneous externalities a change in the number of users today affects the utility of the durable good in the same period. With lagging externalities, a change in the user base today does not affect today's payoffs, and therefore there is a unique equilibrium

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<sup>13</sup>Note however, that when we analyze the pricing scheme for the first period, actions in the current period still do not affect the utility derived from a particular model in that same period, but will affect the utility derived from keeping that model in the *next* period.

Period 1 outcome (old, new)	Monopolist's second period prices that induce purchase by consumers	Possible second period outcomes
$A, A$	$p_{2A} = f(A) + 2N$ $p_{2A}^t = \text{Max}\{f(A) - f(B) + 2N, c\}$ $p_{2B} = f(B)$ $p_{2B}^t = \text{Max}\{f(B) - f(A) - 2N, c\}$	$(A, A)$ $(B, B)$
$A, B$	$p_{2A} = f(A) + N$ $p_{2A}^t = \text{Max}\{f(A) - f(B), c\} = c$ $p_{2B} = f(B) + N$ $p_{2B}^t = \text{Max}\{f(B) - f(A), c\}$	$(A, B)$ $(B, B)$
$B, A$	$p_{2A} = f(A) + N$ $p_{2A}^t = \text{Max}\{f(A) - f(B), c\} = c$ $p_{2B} = f(B) + N$ $p_{2B}^t = \text{Max}\{f(B) - f(A), c\}$	$(B, A)$ $(B, B)$
$B, B$	$p_{2A} = f(A)$ $p_{2A}^t = \text{Max}\{f(A) - f(B) - 2N, c\} = c$ $p_{2B} = f(B) + 2N$ $p_{2B}^t = \text{Max}\{f(B) - f(A) + 2N, c\}$	$(B, B)$
$A, \emptyset$	$p_{2A} = f(A) + N$ $p_{2A}^t = \text{Max}\{f(A) - f(B) + N, c\}$ $p_{2B} = f(B)$ $p_{2B}^t = \text{Max}\{f(B) - f(A) - N, c\}$	$(A, A)$ $(A, B)$ $(B, A)$ $(B, B)$
$B, \emptyset$	$p_{2A} = f(A)$ $p_{2A}^t = \text{Max}\{f(A) - f(B) - N, c\} = c$ $p_{2B} = f(B) + N$ $p_{2B}^t = \text{Max}\{f(B) - f(A) + N, c\}$	$(B, A)$ $(B, B)$

Table 1: Feasible actions in the second period and associated profit-maximizing prices

in the second period.

Consumers will buy a vintage different than the one they are using only if the price is less than or equal than the utility derived in period 2 from that vintage. Looking at table 1 we can see that in histories  $\{A, B\}$ ,  $\{B, A\}$ ,  $\{B, B\}$  and  $\{B, \emptyset\}$  consumers will not trade in model  $A$  for model  $B$  as the price necessary for consumers to be willing to exchange vintage is below marginal cost  $c$ <sup>14</sup>.

After eliminating cases where trade-in prices would have to be below marginal cost, the possible alternatives for the monopolist in the second period and their associated profits for that period ( $\pi_2$ ) are given in table 2. The only difference between second period outcomes in table 2 and those of table 1 is in history  $\{B, \emptyset\}$ . Notice that if the monopolist arrives to the second period without selling to new consumers, then it is more profitable to sell them model  $B$  than model  $A$  ( $p_{2B} = f(B) + N > p_{2A} = f(A)$ ) so the monopolist will induce outcome  $(B, B)$  in the second period, rather than  $(B, A)$ . Of course the specific choice of (behavioral) strategy for this period will depend on the relationship between the parameters  $c$ ,  $N$ ,  $f(A)$  and  $f(B)$ . Note in particular that if the monopolist has already sold any vintage to new consumers in the first period (the first four histories), it may be optimal not to sell again to any group in the second period, specially if the improvement embodied in the new vintage (the difference  $f(B) - f(A)$ ) is low relative to the marginal cost of production  $c$ .

If we are to establish the monopolist's equilibrium strategy for period 2, we need to take into account

<sup>14</sup>Actually, given that we have made no assumptions about the relationship between  $c$ ,  $N$  and  $f(B) - f(A)$ , we can only eliminate those cases that yield negative prices in the second period. Note however, that later on we will allow prices in the first period to be below marginal cost  $c$ , as a way to promote one of the networks and increase its value.

Period 1 outcome (old, new)	Second period outcomes (old cons., new cons.) and associated second period profits
$A, A$	$(B, B) \quad \pi_2 = 2(f(B) - f(A) - 2N - c)N$ $(A, A) \quad \pi_2 = 0$
$A, B$	$(A, B) \quad \pi_2 = 0$ $(B, B) \quad \pi_2 = (f(B) - f(A) - c)N$
$B, A$	$(B, A) \quad \pi_2 = 0$ $(B, B) \quad \pi_2 = (f(B) - f(A) - c)N$
$B, B$	$(B, B) \quad \pi_2 = 0$
$A, \emptyset$	$(A, A) \quad \pi_2 = (f(A) + N - c)N$ $(A, B) \quad \pi_2 = (f(B) - c)N > 0$ $(B, A) \quad \pi_2 = (f(B) - 2c)N$ $(B, B) \quad \pi_2 = (2f(B) - f(A) - N - 2c)N$
$B, \emptyset$	$(B, B) \quad \pi_2 = (f(B) + N - c)N > 0$

Table 2: Undominated outcomes in second period and associated second period monopoly profits

not only prices for consumers on the equilibrium path, but also optimal prices off the equilibrium path (for example, if a consumer arrives without having bought a vintage in period 1). The discussion in this section can be summarized in the following

**Proposition 1** *Given a history in period 2 :  $\{\{A, \emptyset\}, \{w, y\}\}$  where  $w \in \{A, B\}$  and  $y \in \{A, B, \emptyset\}$ , there exists a unique equilibrium in the subgame starting in period 2. Equilibrium prices and outcomes are given by table 3. Equilibrium prices are such that the monopolist is able to extract all consumer surplus (except for each consumer's outside option of keeping the vintage they used in period 1).*

**Proof.** Straightforward computations from table 2. ■

## 4.2 First Period Decision

In the first period, the monopolist has six possible strategies to choose from, depending on whether it wants to sell vintage  $B$  to the old consumers or not, and whether it wants to sell vintage  $A$ ,  $B$ , or he does not want to sell any durable good to the newcomers. We analyze in detail each of the possible subgames that can be opened in period 1.

For this first period, we still have the same multiplicity of equilibria that arises in models with contemporaneous externalities.<sup>15</sup> We will therefore assume that, given the prices set by the monopolist, consumers

<sup>15</sup>Suppose for example that prices in the first period are  $p_{1A}$  and  $p_{1B}$  for vintages  $A$  and  $B$ , respectively. If all consumers (new and old) decide to use vintage  $A$ , we will have outcome  $(A, A)$  in the first period (first line in table 3). A new consumer who goes along with all the rest of his cohort and buys  $A$  gets total utility in the two periods equal to  $2f(A) + 3N - p_{1A}$ , irrespective of the two possible equilibrium outcomes in period 2. If he deviates and buys  $B$  he would get total utility  $2f(B) - p_{1B}$  in the two periods. Thus, he would buy  $A$  as long as

$$p_{1B} - p_{1A} > 2(f(B) - f(A)) - 3N \quad (2)$$

On the other hand, if everyone decides to buy vintage  $B$  in the first period (outcome  $(B, B)$ , fourth line in table 3), then a new consumer that follows his cohort gets total utility  $2f(B) + 2N - p_{1B}$ . Deviating and buying  $A$  in the first period gives utility  $2f(A) - p_{1A}$ . He would therefore buy  $A$  as long as

$$p_{1B} - p_{1A} > 2(f(B) - f(A)) + 2N \quad (3)$$

Comparing (2) and (3) we can see that there is a range of prices for which we have at least two equilibria, depending on the beliefs about other consumers' choice of vintage.

Period 1 outcome (w, y)	Period 2 equilibrium prices, period 2 equilibrium outcome	
A, A	iff $f(B) - f(A) > 2N + c$	$\left. \begin{array}{l} p_{2A} > f(A) + 2N \\ p_{2A}^t \geq c \\ p_{2B} = f(B) \\ p_{2B}^t = f(B) - f(A) - 2N \end{array} \right\} (B, B)$
	iff $2N < f(B) - f(A) < 2N + c$	$\left. \begin{array}{l} p_{2A} > f(A) + 2N \\ p_{2A}^t = \text{Max}\{f(A) - f(B) + 2N, c\} \\ p_{2B} = f(B) \\ p_{2B}^t \geq c \end{array} \right\} (A, A)$
	iff $f(B) - f(A) < 2N$	$\left. \begin{array}{l} p_{2A} = f(A) + 2N \\ p_{2A}^t = \text{Max}\{f(A) - f(B) + 2N, c\} \\ p_{2B} > f(B) \\ p_{2B}^t \geq c \end{array} \right\} (A, A)$
A, B	iff $f(B) - f(A) < c$	$\left. \begin{array}{l} p_{2A} > f(A) + N \\ p_{2A}^t \geq c \\ p_{2B} = f(B) + N \\ p_{2B}^t \geq c \end{array} \right\} (A, B)$
	iff $f(B) - f(A) > c$	$\left. \begin{array}{l} p_{2A} > f(A) + N \\ p_{2A}^t \geq c \\ p_{2B} = f(B) + N \\ p_{2B}^t = f(B) - f(A) \end{array} \right\} (B, B)$
B, A	iff $f(B) - f(A) < c$	$\left. \begin{array}{l} p_{2A} > f(A) + N \\ p_{2A}^t \geq c \\ p_{2B} = f(B) + N \\ p_{2B}^t \geq c \end{array} \right\} (B, A)$
	iff $f(B) - f(A) > c$	$\left. \begin{array}{l} p_{2A} > f(A) + N \\ p_{2A}^t \geq c \\ p_{2B} = f(B) + N \\ p_{2B}^t = f(B) - f(A) \end{array} \right\} (B, B)$
B, B	iff $f(B) - f(A) + 2N < c$	$\left. \begin{array}{l} p_{2A} > f(A) \\ p_{2A}^t \geq c \\ p_{2B} = f(B) + 2N \\ p_{2B}^t \geq c \end{array} \right\} (B, B)$
	iff $f(B) - f(A) + 2N > c$	$\left. \begin{array}{l} p_{2A} > f(A) \\ p_{2A}^t \geq c \\ p_{2B} = f(B) + 2N \\ p_{2B}^t = f(B) - f(A) + 2N \end{array} \right\} (B, B)$
A, $\emptyset$	iff $f(B) - f(A) < N$	$\left. \begin{array}{l} p_{2A} = f(A) + N \\ p_{2A}^t = \text{Max}\{f(A) - f(B) + N, c\} \\ p_{2B} > f(B) \\ p_{2B}^t \geq c \end{array} \right\} (A, A)$
	iff $N < f(B) - f(A) < N + c$	$\left. \begin{array}{l} p_{2A} > f(A) + N \\ p_{2A}^t \geq c \\ p_{2B} = f(B) \\ p_{2B}^t \geq c \end{array} \right\} (A, B)$
	iff $N + c < f(B) - f(A)$	$\left. \begin{array}{l} p_{2A} > f(A) + N \\ p_{2A}^t \geq c \\ p_{2B} = f(B) \\ p_{2B}^t = f(B) - f(A) - N \end{array} \right\} (B, B)$
B, $\emptyset$	iff $f(B) - f(A) < c - N$	$\left. \begin{array}{l} p_{2A} > f(A) \\ p_{2A}^t \geq c \\ p_{2B} = f(B) + N \\ p_{2B}^t \geq c \end{array} \right\} (B, B)$
	iff $f(B) - f(A) > c - N$	$\left. \begin{array}{l} p_{2A} > f(A) \\ p_{2A}^t \geq c \\ p_{2B} = f(B) + N \\ p_{2B}^t = f(B) - f(A) + N \end{array} \right\} (B, B)$

Table 3: Equilibrium in period 2

coordinate on their Pareto-preferred equilibrium.

Notice that from the previous proposition, the monopolist in period 2 is able to extract any additional surplus that a consumer may obtain by an exchange of vintage. Therefore, a consumer in period 2 will get the utility of keeping the same vintage he has in period 1, irrespective of the exchange made with the monopolist in that period. We proceed to analyze each particular subgame that opens in period 1.

#### 4.2.1 Subgame $(A, A)$ : Sell vintage $A$ to new consumers

The monopolist decides to sell vintage  $A$  to the new customers in the first period. In the second period he cannot tell old and new consumers apart by means of trade-in prices since they start with the same vintage. From table 3, the monopolist will sell vintage  $B$  to both types of consumers if and only if

$$f(B) - f(A) > 2N + c \quad (4)$$

otherwise, in equilibrium, he won't sell any product in the second period.

If (4) is satisfied then consumers know that in the second period they will be induced to abandon good  $A$  and switch to  $B$ . When trading vintage  $B$  for vintage  $A$  in the second period, they will keep a consumer surplus in that period equal to  $f(B) - p_{2B}^t = f(A) + 2N$ . The monopolist can charge in the first period a price for vintage  $A$  equal to the consumer surplus of both periods, which is equal to the utility of using vintage  $A$  during the two periods. Note that in the second period these consumers realize the externalities of a group of size  $2N$ , which means  $p_{1A} = 2f(A) + 3N > c$ . Total profits for the whole subgame are thus equal to  $(p_{1A} - c)N + \pi_2 = (2f(B) - N - 3c)N$ .

On the other hand, if (4) is not satisfied, then in the second period the monopolist will choose not to sell vintage  $B$  to those consumers who already have vintage  $A$  (the necessary trade-in price to induce a purchase would be below marginal cost). The price that he can set for vintage  $A$  in the first period is still the utility generated by that vintage in the two periods  $p_{1A} = 2f(A) + 3N$ . In this case, total profits in the two periods will be equal to profits generated in the first period:  $(2f(A) + 3N - c)N$ .

Finally, in order to prevent new consumers from buying vintage  $B$ , the monopolist will have to set a price in the first period  $p_{1B}$  bigger than  $2f(B) + N$ , which is the total consumer surplus obtained from vintage  $B$  in the two periods if *all* new consumers coordinate on buying that vintage in period 1.<sup>16</sup>

Therefore, total profits in the two periods are:

$$\pi_{AA} = \begin{cases} (2f(B) - N - 3c)N. & \text{if } f(B) - f(A) > 2N + c \quad (\text{outcome } \{(A, A), (B, B)\}) \\ (2f(A) + 3N - c)N & \text{otherwise} \quad (\text{outcome } \{(A, A), (A, A)\}) \end{cases} \quad (5)$$

In order to sustain this subgame in equilibrium, we also need to set up regular and trade-in prices for vintage  $B$  to discourage consumers from switching to it:

$$\begin{aligned} p_{1A} &= 2f(A) + 3N \\ p_{1B} &> 2f(B) + 2N \\ p_{1B}^t &> 2f(B) - 2f(A) \end{aligned} \quad (6)$$

Notice that with these prices, an old consumer does not want to trade vintage  $B$  for  $A$  since any surplus gain will be appropriated by the monopolist through  $p_{1B}^t$ . If *all* consumers decide to switch to vintage  $B$ , then

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<sup>16</sup>Recall that in the second period, this consumer will only get a surplus equal to  $f(B) + N$  if all new consumers decided to adopt vintage  $B$  in period 1. Any trade in the second period will have the monopolist extracting any consumer surplus above this level.

old and new consumers would get a surplus in the two periods equal to  $2f(B) + 2N$ . If old consumers keep vintage  $A$  they are assured a consumer surplus of  $2f(A) + 2N$ . If  $p_{1B}^t \geq 2f(B) - 2f(A)$  then old consumers are better off keeping vintage  $A$  even if all consumers coordinate a switch to vintage  $B$ . New consumers weakly prefer to buy vintage  $A$  and get zero surplus rather than buying vintage  $B$  and obtaining negative surplus. The regular prices  $p_{1A}$  and  $p_{1B}$  ensure that new consumers will choose to buy vintage  $A$ . Note also that if new consumers want to acquire vintage  $B$  buying vintage  $A$  first and then trading it in, they will pay a total price  $p_{1A} + p_{1B}^t > 2f(B) + 3N > p_{1B}$ . Thus new consumers do not want to acquire vintage  $B$  through this double buying.

#### 4.2.2 Subgame $(A, B)$ : Sell vintage $B$ to new consumers

In this subgame the monopolist sells the improved version of the durable ( $B$ ) only to new consumers in the first period, but sets a price that prevents old consumers from upgrading to  $B$ . In the second period, the monopolist will also sell vintage  $B$  to old consumers if and only if

$$f(B) - f(A) > c \quad (7)$$

New consumers know that they will never be induced to change to vintage  $A$  in period 2 (see table 3). Given that they will keep vintage  $B$ , new consumers in the first period will be willing to pay up to  $p_{1B} = 2f(B) + N$ . This is true regardless of what old consumers do: since externalities are lagging, what happens in period 2 does not affect the utility derived from using vintage  $B$  in period 2.

Therefore profits in the first period are always equal to  $(2f(B) + N - c)N$ . On top of that, if the monopolist sells vintage  $B$  to old consumers in period 2 then he will get extra profits  $\pi_2 = (f(B) - f(A) - c)N$ . Total profits for this subgame are therefore given by

$$\pi_{AB} = \begin{cases} (3f(B) - f(A) + N - 2c)N. & \text{if } f(B) - f(A) > c \quad (\text{outcome } \{(A, B), (B, B)\}) \\ (2f(B) + N - c)N & \text{otherwise} \quad (\text{outcome } \{(A, B), (A, B)\}) \end{cases} \quad (8)$$

To sustain this subgame, we need to make sure that new consumers do not want to buy vintage  $A$  and old consumers do not want to buy (or trade-in) vintage  $B$ . Therefore prices satisfy the following conditions:

$$\begin{aligned} p_{1A} &> 2f(A) + 3N \\ p_{1B} &= 2f(B) + N \\ p_{1B}^t &> 2f(B) - 2f(A) \end{aligned} \quad (9)$$

Note that at price  $p_{1B} = 2f(B) + N$ , old consumers will not be willing to buy vintage  $B$ . If old consumers keep vintage  $A$  they will get total utility equal to  $2f(A) + 2N$  in the two periods. Even if all consumers buy vintage  $B$  in the first period, that would give old consumers utility  $2f(B) + 2N - p_{1B} = N$  over the two periods, which is lower than keeping  $A$ . Therefore, this first period price for  $B$  effectively excludes old consumers from buying that vintage. By the same argument, the trade in price for vintage  $B$  that prevents old consumers buying it satisfies  $2f(B) + 2N - p_{1B}^t < 2f(A) + 2N$ . To prevent new consumers from buying vintage  $A$  we have to make sure that even in the best scenario for vintage  $A$  (i.e. *all* consumers switching to  $A$  in period 1), new consumers prefer to coordinate on vintage  $B$ . This implies  $0 = 2f(B) + N - p_{1B} > 2f(A) + 3N - p_{1A}$  which results in the price stated above.

#### 4.2.3 Subgame $(B, A)$ : Sell vintage $B$ to old consumers and $A$ to newcomers

In this subgame the monopolist sells the improved version of the durable ( $B$ ) to old consumers and *at the same time* he sells the old vintage to newcomers. The monopolist uses trade-in prices for old consumers to

prevent arbitrage between groups. In the second period, the monopolist will also sell vintage  $B$  to the new consumers if and only if

$$f(B) - f(A) > c \quad (10)$$

and will not sell any vintage otherwise. By a similar argument as in the previous case, old consumers' gross surplus is not affected whether or not newcomers decide to adopt vintage  $B$  in the second period, since the externality associated with  $B$  will not change for period 2. The same is true for their opportunity cost of giving up vintage  $A$  : if *every* old consumer keeps vintage  $A$ , he would get utility  $2f(A) + 3N$  over two periods. Therefore, they are willing to pay up to  $p_{1B}^t = 2f(B) - 2f(A) - 2N$ .

Newcomers face a similar situation: If (10) is satisfied, then they know they will be induced to change to vintage  $B$  in the second period. In the second period they can obtain a consumer surplus equal to  $f(B) + N - p_{2B}^t = f(A) + N$ . Therefore, in the first period, they are willing to pay up to their consumer surplus in the two periods:  $p_{1A} = 2f(A) + 2N$ . If (10) is not satisfied, they will still use vintage  $A$  in the second period, and will be willing to pay the same period-one price as before.

Combining the two types of consumers we get that first period profits for this subgame are:<sup>17</sup>

$$\pi_{BA} = \begin{cases} (3f(B) - f(A) - 3c)N. & \text{if } f(B) - f(A) > c \quad (\text{outcome } \{(B, A), (B, B)\}) \\ (2f(B) - 2c)N & \text{otherwise} \quad (\text{outcome } \{(B, A), (B, A)\}) \end{cases} \quad (11)$$

Note that new consumers in any case get a net utility of zero after the two periods. In order to prevent new consumers buying vintage  $B$  we need to set  $p_{1B}$  high enough. In particular, if all new consumers were to adopt vintage  $B$  together with the old consumers, they would get two period utility equal to  $2f(B) + 2N$ . Therefore, the regular price for vintage  $B$  has to make sure that new consumers get a higher utility buying vintage  $A$  (zero in this case):  $0 = 2f(A) + 2N - p_{1A} > 2f(B) + 2N - p_{1B}$ . Therefore, prices that induce this subgame are:

$$\begin{aligned} p_{1A} &= 2f(A) + 2N \\ p_{1B} &> 2f(B) + 2N \\ p_{1B}^t &= 2f(B) - 2f(A) - 2N \end{aligned} \quad (12)$$

In particular, new consumers interested in vintage  $B$  could purchase vintage  $A$  first and immediately trade it for vintage  $B$  at the prices set for old consumers. Following this strategy the total price they would pay for vintage  $B$  is  $p_{1A} + p_{1B}^t = 2f(B)$ . If every old consumer decides to pursue this double-buying strategy, they would get utility  $2f(B) + 2N$  which is bigger than the price they are paying. We will therefore assume that the monopolist can prevent this double-buying strategy (for example by selling vintage  $B$  before vintage  $A$  is offered in the market in period 1), and so can sustain subgame  $(B, A)$  if he so wishes.<sup>18</sup>

#### 4.2.4 Subgame $(B, B)$ : Immediate switch to vintage $B$

The monopolist sells vintage  $B$  to all consumers in the first period. As in the first case, the monopolist can price-discriminate in the first period but not in the second, as trade-in prices are of no use then (everyone has

<sup>17</sup>Note that new consumers in any case get a net utility of zero after the two periods. We are implicitly assuming that the monopolist can differentiate between old and new consumers when setting trade-in prices for the former. In particular, new consumers interested in vintage  $B$  could purchase vintage  $A$  first and immediately trade it for vintage  $B$  at the prices set for old consumers. Following this strategy the total price they would pay for vintage  $B$  is  $2f(B)$ . If all old consumers decide to pursue this arbitrage strategy, they would get utility  $2f(B) + 2N$  which is bigger than the price they are paying.

This means that if the monopolist cannot prevent new consumers from buying twice in period 1, he will not be able to sustain outcome  $(B, A)$  in the first period. However, this does not change the results in this paper, since we will see later that the monopolist will never want to induce subgame  $(B, A)$  even if he can prevent arbitrage from new consumers.

<sup>18</sup>If the monopolist cannot prevent new consumers buying twice in period 1, he will not be able to sustain outcome  $(B, A)$  in the first period. However, this does not change the results in this paper, since we will see later that the monopolist will never want to induce subgame  $(B, A)$  even if he can prevent the double-buying strategy from new consumers.

the same good to be traded). There is no profitability in selling again the old vintage in the second period, and therefore consumers know they will keep their new vintage for two periods. However, old consumers have a higher opportunity cost than new consumers, since they could all continue using vintage  $A$  and get two-period utility of  $2f(A) + 2N$ . Therefore, the monopolist will use trade in prices:<sup>19</sup>  $p_{1B}^t = 2f(B) - 2f(A) < p_{1B} = 2f(B) + 2N$ . Total profits for this subgame are equal to

$$\pi_{BB} = (4f(B) - 2f(A) + 2N - 2c)N \quad (\text{outcome } \{(B, B), (B, B)\}) \quad (13)$$

Note that, if new consumers were to acquire vintage  $A$  and old consumers keep vintage  $A$ , then new consumers would get total utility  $2f(A) + 3N$ . To discourage new consumers buying vintage  $A$  (and therefore, also discouraging old consumers from *keeping* vintage  $A$ ) we need to set  $p_{1A} > 2f(A) + 3N$ . Therefore, prices that induce this subgame are:

$$\begin{aligned} p_{1A} &> 2f(A) + 3N \\ p_{1B} &= 2f(B) + 2N \\ p_{1B}^t &= 2f(B) - 2f(A) \end{aligned} \quad (14)$$

#### 4.2.5 Subgame $(A, \emptyset)$ : No sale in first period

In this subgame the monopolist waits during the first period and only introduces the durable good again in the second. Therefore, total profits will depend on which of the four options with non-negative profits for the second period is more profitable.

From proposition 1, and after some manipulation, total profits are:

$$\pi_{A\emptyset} = \begin{cases} (f(A) + N - c)N & \text{if } f(B) - f(A) < N & (\text{outcome } \{(A, \emptyset), (A, A)\}) \\ (f(B) - c)N & \text{if } N < f(B) - f(A) < N + c & (\text{outcome } \{(A, \emptyset), (A, B)\}) \\ (2f(B) - f(A) - N - 2c)N & \text{if } N + c < f(B) - f(A) & (\text{outcome } \{(A, \emptyset), (B, B)\}) \end{cases} \quad (15)$$

To induce this subgame, the monopolist would need to set prices  $p_{1A}$ ,  $p_{1B}$  and  $p_{1B}^t$  high enough. In particular, to prevent new consumers buying vintage  $A$ , it will need to set prices higher than  $2f(A) + 3N$ , which is new consumer's utility if everyone has vintage  $A$  in period 1. To prevent that new consumers buy vintage  $B$  then it is enough to set prices for vintage  $B$  higher than the best scenario for that vintage, which is both groups adopting it in period 1. New consumers would increase their utility by  $2f(B) + 2N$ , whereas old consumers

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<sup>19</sup>If *every* old consumer decides to keep his old vintage  $A$ , then he gets a total utility in the two periods equal to  $2f(A) + 2N$ , which is exactly the maximum amount a new consumer would be willing to pay for vintage  $A$  (if all of them decide to buy  $A$  secondhand). There is no possibility of arbitrage in this case if we assume that trading-in vintage  $B$  for  $A$  implies returning  $A$  to the monopolist. Another interpretation is that old consumers are offered just an upgrade that converts  $A$  into  $B$ , whereas new consumers are given the whole good with the characteristics of  $B$ .

would increase their utility by  $2f(B) - 2f(A)$ . Accordingly, prices that sustain this subgame are:<sup>20</sup>

$$\begin{aligned} p_{1A} &> 2f(A) + 3N \\ p_{1B} &> 2f(B) + 2N \\ p_{1B}^t &> 2f(B) - 2f(A) \end{aligned} \tag{16}$$

#### 4.2.6 Subgame $(B, \emptyset)$ : Sell $B$ only to old consumers

The monopolist only sells vintage  $B$  to old consumers and denies it to newcomers, for example by only allowing trading vintage  $B$  for  $A$  in the first period.<sup>21</sup> In the second period, the monopolist will sell vintage  $B$  also to new consumers. In the first period, old consumers will be willing to pay up to  $p_{1B}^t = 2f(B) - 2f(A) - N$  which represents the gross surplus from using vintage  $B$  for two periods minus what they forgo by abandoning vintage  $A$ . Total profits for this subgame are therefore equal to

$$\pi_{B\emptyset} = (3f(B) - 2f(A) - 2c)N \quad (\text{outcome } \{(B, \emptyset), (B, B)\}) \tag{17}$$

To induce this subgame, the monopolist needs to ensure that new consumers will not buy any model. If all consumers (new and old) use vintage  $A$  in period 1, then new consumers would get utility  $2f(A) + 3N$ . Thus the monopolist needs to set a regular price for vintage  $A$  higher than that. If new consumers buy vintage  $B$ , they would get utility  $2f(B) + 2N$ , so the regular price for vintage  $B$  also has to be higher than that. Therefore, prices necessary to induce this subgame are:

$$\begin{aligned} p_{1A} &> 2f(A) + 3N \\ p_{1B} &> 2f(B) + 2N \\ p_{1B}^t &= 2f(B) - 2f(A) - N \end{aligned} \tag{18}$$

### 4.3 Partial, Full or No Introduction of a new vintage

Since the monopolist can, by his choice of prices in the first period, induce any of the six subgames we analyzed before, we can think of him choosing which subgame he wants to play in the first period. Looking at the total profits associated with each particular subgame in the previous subsection, we can easily see that a monopolist would never pursue subgames  $(A, \emptyset)$ ,  $(B, \emptyset)$  or  $(B, A)$ , since it is more profitable to introduce the new vintage ( $B$ ) to all consumers in the market in the first period (subgame  $(B, B)$ ). The intuition for this dominance in the first two cases is clear: the monopolist gains nothing by delaying the adoption of any

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<sup>20</sup>Note that these prices for vintage  $B$  are just sufficient to sustain this subgame. But lower prices may also work. In particular, one could set the following prices:

$$\begin{aligned} p_{1A} &> 2f(A) + 3N \\ p_{1B} &> 2f(B) + 2N \\ p_{1B}^t &> 2f(B) - 2f(A) - N \end{aligned}$$

These prices ensure that new consumers will never buy vintage  $B$  (not even if old consumers buy it). Old consumers, aware of this, know that they won't be followed in case they all adopt vintage  $B$ . Adopting  $b$  therefore gives them utility  $2f(B) + N$ , whereas keeping vintage  $A$  would bring them utility  $2f(A) + 2N$ . The price  $p_{1B}^t$  chosen above makes sure old consumers don't want to switch.

The same reasoning can be applied now by switching old and new consumers. Suppose now we make sure that  $p_{1B}^t$  is high enough that old consumers never switch to  $B$  (even if new consumers follow suit). New consumers, aware of this, need now a lower price to be discouraged from buying vintage  $B$ . These prices are:

$$\begin{aligned} p_{1A} &> 2f(A) + 3N \\ p_{1B} &> 2f(B) + N \\ p_{1B}^t &> 2f(B) - 2f(A) \end{aligned}$$

<sup>21</sup>As in the subgame  $(B, B)$  there is no possibility of arbitrage between old and new consumers if trading-in vintage  $A$  requires returning the good to the monopolist. Alternatively we can again think that the monopolist only sells an upgrade to  $A$  that transforms it into  $B$ .

vintage by the newcomers, since he can sell any of the two vintages above marginal cost in that period. In the last case (subgame  $(B, A)$ ), the monopolist knows that he will be able to extract all consumer surplus from both types of consumers, and so he doesn't want to reduce it by forcing old consumers to buy the new product while offering the old vintage to newcomers. He could get the same distribution of use of vintages  $A$  and  $B$  (and thus the same effect on willingness to pay) by pursuing subgame  $(A, B)$  with the obvious cost saving of not having to force old consumers to buy twice.

We are left with subgames  $(A, A)$ ,  $(A, B)$  and  $(B, B)$ . By further examination of the payoff associated with the first two, we can conclude that the outcome  $\{(B, B), (B, B)\}$  dominates both  $\{(A, A), (B, B)\}$  and  $\{(A, B), (B, B)\}$ . The intuition for this is also straightforward: the size of the network in period 1 determines the surplus generated in the second period, which the monopolist can extract by his choice of prices. The monopolist is interested in building up a sizeable network (be it in vintage  $A$  or  $B$ ) as soon as possible. Thus if the monopolist is going to arrive to period two with a network of size  $2N$  for vintage  $B$ , he might as well build it from period 1 and have time for the externality to be fully developed in period 2. In this way, he will be able to extract that increased surplus by the appropriate choice of prices in period 1.

Therefore, the only undominated outcomes are  $\{(A, A), (A, A)\}$ ,  $\{(A, B), (A, B)\}$  and  $\{(B, B), (B, B)\}$ . By looking at the extent to which the new vintage ( $B$ ) is introduced in the market, we will call them the **no introduction (NI)**, **partial introduction (PI)**<sup>22</sup> and **full introduction (FI)** outcomes, respectively. Let  $\gamma \equiv f(B) - f(A) > 0$  be the intrinsic difference in utility provided by the two vintages of the durable good. We can also interpret  $\gamma$  as the degree of improvement of the new vintage over the existing one. Which type of outcome arises in equilibrium depends on the relationship between  $\gamma$ , the marginal cost of production  $c$ , and the size of the (potential) network  $2N$ .

To summarize, the monopolist will maximize profits by inducing the equilibrium outcomes specified in the following proposition.

**Proposition 2** *Assuming that consumers coordinate on their Pareto-preferred strategy in the first period, the equilibrium outcome in the monopoly case is:*

1.  $\{(A, A), (A, A)\}$  [*No Introduction (NI)*]: if  $\gamma \equiv f(B) - f(A) < \text{Min} \{N, \frac{c+N}{4}\}$   
with price  $p_{1A} = 2f(A) + 3N > c$   
and total profits  $\pi_{AA} = (2f(A) + 3N - c)N$
2.  $\{(A, B), (A, B)\}$  [*Partial Introduction (PI)*]: if  $N < \gamma \equiv f(B) - f(A) < \frac{c-N}{2}$   
with price  $p_{1B} = 2f(B) + N > c$   
and total profits  $\pi_{AB} = (2f(B) + N - c)N$
3.  $\{(B, B), (B, B)\}$  [*Full Introduction (FI)*]: if  $\text{Max} \{ \frac{c+N}{4}, \frac{c-N}{2} \} < \gamma \equiv f(B) - f(A)$   
with prices  $p_{1B}^t = 2f(B) - 2f(A)$  and  $p_{1B} = 2f(B) + 2N > c$   
and total profits  $\pi_{BB} = (4f(B) - 2f(A) + 3N - 2c)N$

**Proof.** The previous analysis showed that the three outcomes are the only ones that do not yield lower profits than the  $\{(B, B), (B, B)\}$  outcome. The bounds for each outcome arise from straightforward manipulation of (5), (8) and (13). Prices and profits are computed in the previous subsections. ■

<sup>22</sup>Fudenberg and Tirole (1998) call this the "leapfrogging" case, as new consumers, who started without any good, end up in equilibrium adopting an improved version of the product, whereas old consumers keep their old vintage.

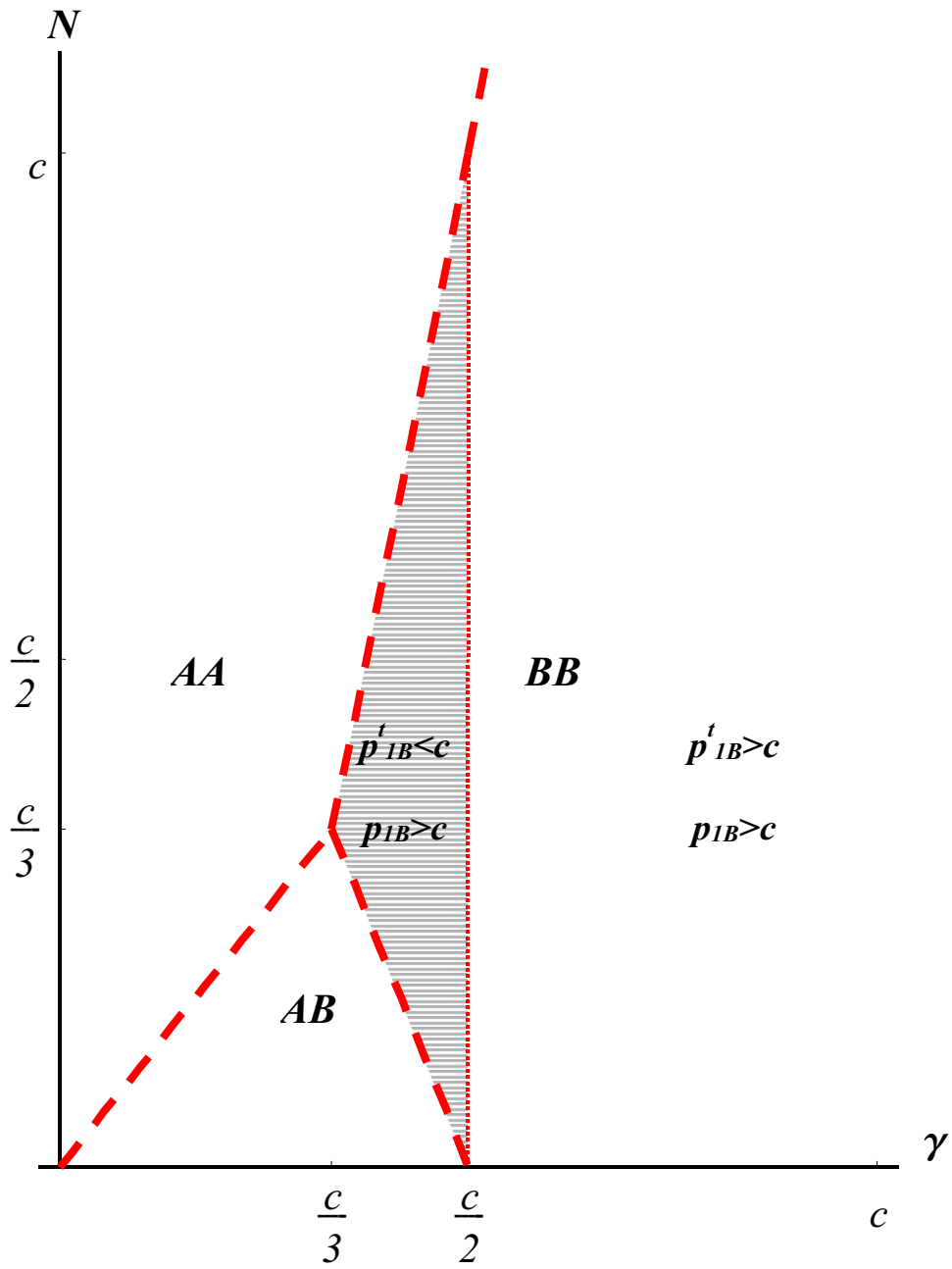


Figure 2: Monopolist and lagged externalities: equilibrium outcome

The results of the proposition are reproduced in figure 2, where the full introduction equilibrium includes also the shaded area. We can distinguish two different regimes depending on the size of the market relative to marginal costs. In all these cases the issue is the same: in order to introduce the new vintage to old consumers, the monopolist has to compensate them for the loss of the network they already enjoy by using vintage  $A$ . That can be done either by offering a new vintage  $B$  that increases their stand-alone utility (a high value of  $\gamma$ ) or by promising that they will enjoy a bigger network with  $B$ . In order to convince new consumers to buy vintage  $B$  the monopolist also has to offer the same advantages, although the task is easier than with old consumers, since they are not locked-in with an existing network. Newcomers are easier to convince to adopt  $B$  than old consumers, since the former have a lower reservation utility. That makes it possible to sustain the partial introduction equilibrium  $\{(A, B), (A, B)\}$ .

#### 4.3.1 Case I: Small market size $N < \frac{c}{3}$

If production costs are high relative to the size of the market (and therefore the strength of network externalities), then depending on the stand-alone utility differential  $\gamma$  between the two vintages one of the three types of equilibrium outcomes will arise: no introduction (NI), partial introduction (PI) or full introduction (FI). If the utility differential is low ( $\gamma < N$ ), then the monopolist prefers not to introduce the new vintage in the market, since it cannot compensate old consumers for the loss of their established network in good  $A$ . If, on the other hand, the utility differential is high ( $\gamma > \frac{c-N}{2}$ ) then the monopolist can induce old and new consumers to adopt  $B$ , since the differential will compensate the production costs of providing the good for  $2N$  consumers.

Partial introduction of the new vintage  $B$  can also be sustained as an equilibrium if the productivity differential lies between these two extremes ( $N < \gamma < \frac{c-N}{2}$ ). In this case the utility differential is not big enough to induce old consumers to give up their established network. Also, network effects are not very strong, so even if all consumers move to the new vintage  $B$ , the remaining consumer will prefer to keep vintage  $A$ . However, new consumers are not “locked” into any network yet, so they can get a higher benefit by adopting the newest technology. The basic intuition here is the same as in Ruiz (1997): old consumers have an incentive to delay the timing of their adoption of a new vintage because externalities take one period to “set in” and one period to disappear.

#### 4.3.2 Case II: Big market size: $N > \frac{c}{4}$

If the size of the potential network is big compared with the marginal cost of production of the good, then we have complete networks as the only possible type of equilibria in this game. If the utility gain  $\gamma$  brought up by the new vintage  $B$  is small (even though it is positive), then the monopolist will induce all consumers to end up with the old vintage  $A$ , since he can charge a high price for the network effect. On the other hand, if vintage  $B$  represents a sufficiently big improvement ( $\gamma > \frac{c+N}{4}$ ), then the monopolist will induce a complete switch to the new vintage. In this case consumers’ willingness to pay for the advanced vintage compensates the cost of losing the externality effect in vintage  $A$  for one period.

Notice that in the full introduction (FI) equilibrium, the monopolist uses trade-in prices, offering a discount equal to  $f(A) + N$  to old consumers when they trade in vintage  $B$  for  $A$ . Actually, for some parameters, the monopolist actually sells the new vintage  $B$  to old consumers at a price below marginal cost:<sup>23</sup>

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<sup>23</sup>Notice, however, that total profits are still positive, since  $f(B) > f(A) > c$ .

**Lemma 3** *If  $\text{Max} \left\{ \frac{c+N}{4}, \frac{c-N}{2} \right\} < \gamma \equiv f(B) - f(A) < \frac{c}{2}$  then in equilibrium the monopolist introduces the new vintage  $B$  to all consumers and uses trade-in prices below marginal cost:  $p_{1B}^t < c < p_{1B}$*

**Proof.** Direct consequence of proposition 2. ■

Note that in the case described in the lemma, the monopolist is willing to make a loss in old consumers in order to increase the value of the network  $B$ , and therefore extract a higher price from new consumers. The shaded area in figure 2 represents the values of  $\gamma$  and  $N$  for which the monopolist uses trade-in prices below marginal cost in period 1. This effect of introductory prices below marginal costs is a recurrent feature in many industries (think, for example, of free updates policies for some software, especially antivirus programs). However, as we show in the appendix, a model with *contemporaneous* network externalities never generates prices below marginal cost.

## 5 Efficiency properties of the equilibrium outcome

In order to assess whether the market solution exhibits planned obsolescence or too much delay in adopting the improved version of the good, we need to derive the social optimum for this case. Suppose that we have a planner interested in maximizing social welfare  $S$ , defined as gross consumer surplus minus production costs.

As in the monopolist's problem, there are six possible outcomes in the first period. However, since we have assumed in (1) that the stand-alone utility of any vintage of the good is higher than the marginal cost of production, then we can discard as socially suboptimal the first period outcomes  $(A, \emptyset)$  and  $(B, \emptyset)$ . The planner could, in these cases, allocate any vintage to new consumers in the first period and increase social welfare since  $f(B) > f(A) > c$ .

For the rest of possible histories in the first period, there are four possible outcomes in the second period. Total surplus  $S$  associated with each particular option is given by table 4.<sup>24</sup>

After comparing social surplus for each possible outcome we can obtain the following lemma:

**Lemma 4** *For given values of  $\gamma$ ,  $N$  and  $c$ , the socially optimal outcome is*

1.  $\{(A, A), (A, A)\}$  [*No Introduction (NI)*]: if  $\gamma \equiv f(B) - f(A) < \text{Min} \left\{ \frac{3N}{2}, \frac{c}{4} + \frac{N}{2} \right\}$
2.  $\{(A, B), (A, B)\}$  [*Partial Introduction (PI)*]: if  $\frac{3N}{2} < \gamma \equiv f(B) - f(A) < \frac{c-N}{2}$
3.  $\{(B, B), (B, B)\}$  [*Full Introduction (FI)*]: if  $\text{Max} \left\{ \frac{c}{4} + \frac{N}{2}, \frac{c-N}{2} \right\} < \gamma \equiv f(B) - f(A)$

**Proof.** Note from table 4 that the full introduction (**FI**) outcome  $\{(B, B), (B, B)\}$  yields a higher surplus than any other outcome except for the no introduction (**NI**) and the partial introduction (**PI**) outcomes. The bounds in the lemma are obtained by direct comparison of the surplus found for each of these three cases. ■

Graph 3 depicts the bounds for the three possible outcomes in the planner's choice of introduction of a new vintage. We should compare these bounds with those of the monopoly outcome in the previous

<sup>24</sup>Note in particular that all options yield positive surplus given assumption (1)

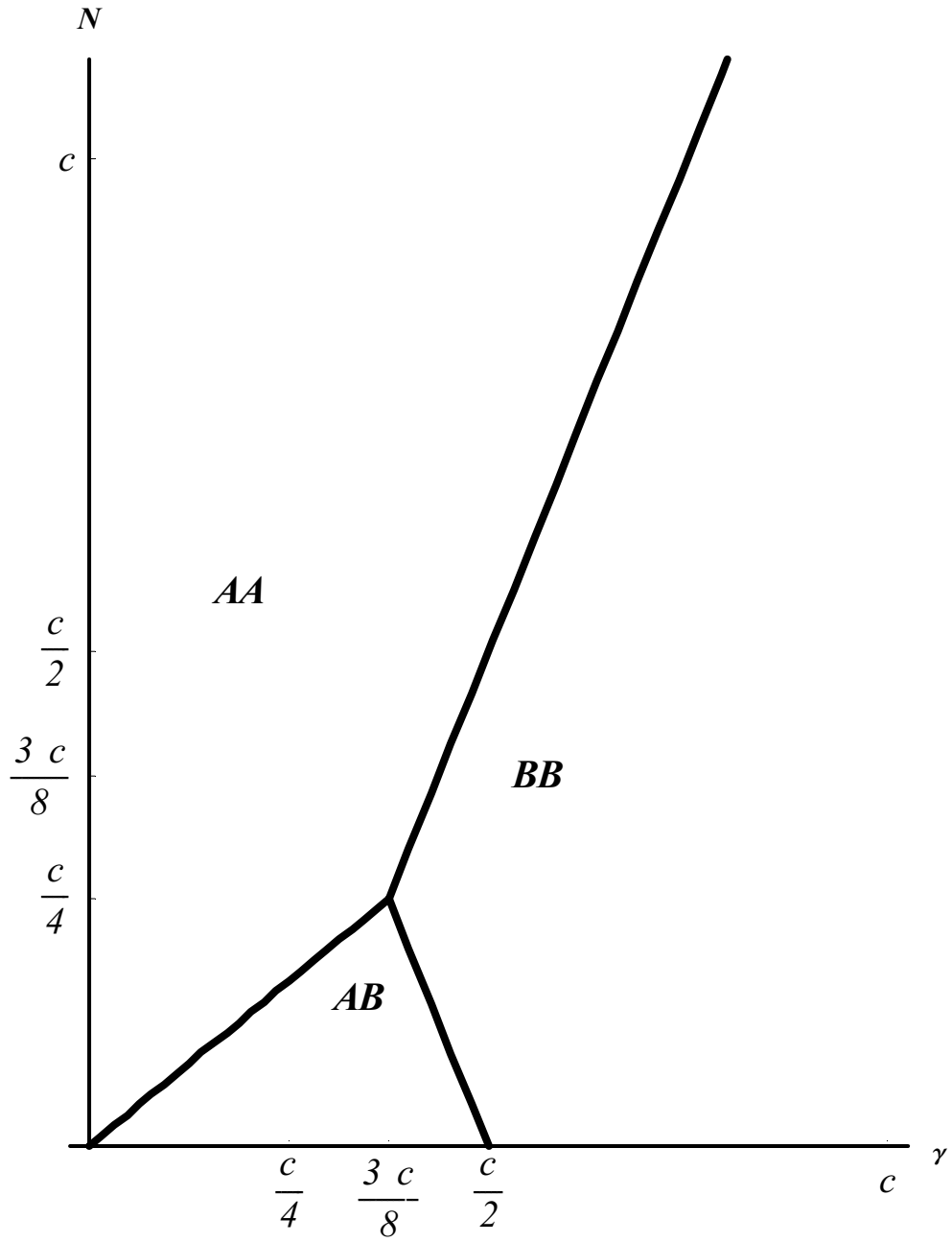


Figure 3: Social Optimum with lagged externalities

First period outcome	Second period outcome	Total Surplus
(A, A)	(A, A) [NI]	$S = (4f(A) + 6N - c)N$
	(A, B)	$S = (f(B) + 3f(A) + 4N - 2c)N$
	(B, A)	$S = (f(B) + 3f(A) + 4N - 2c)N$
	(B, B)	$S = (2f(B) + 2f(A) + 2N - 3c)N$
(A, B)	(A, A)	$S = (f(B) + 3f(A) + 3N - 2c)N$
	(A, B) [PI]	$S = (2f(B) + 2f(A) + 3N - c)N$
	(B, A)	$S = (2f(B) + 2f(A) + 3N - 3c)N$
	(B, B)	$S = (3f(B) + f(A) + 3N - 2c)N$
(B, A)	(A, A)	$S = (f(B) + 3f(A) + 3N - 3c)N$
	(A, B)	$S = (2f(B) + 2f(A) + 3N - 4c)N$
	(B, A)	$S = (2f(B) + 2f(A) + 3N - 2c)N$
	(B, B)	$S = (3f(B) + f(A) + 3N - 3c)N$
(B, B)	(A, A)	$S = (2f(B) + 2f(A) - 4c)N$
	(A, B)	$S = (3f(B) + f(A) + 2N - 3c)N$
	(B, A)	$S = (3f(B) + f(A) + 2N - 3c)N$
	(B, B) [FI]	$S = (4f(B) + 4N - 2c)N$

Table 4: Total surplus per outcome

section. In order to find out if there is planned obsolescence in this context, then we should examine what happens with old and new consumers. We will define planned obsolescence as a situation in which the social optimum involves a consumer keeping (or buying) model  $A$ , and yet the monopolist is able in equilibrium to induce them to buy vintage  $B$  in period 1<sup>25</sup>. Following Waldman (1993), we will define two types of obsolescence. If only new consumers are induced to buy vintage  $B$  when they should acquire  $A$ , but old consumers (efficiently) keep vintage  $A$  then we have a case of “weak obsolescence.” On the other hand, if both types of consumers are induced to switch to  $B$  when they should use  $A$ , then we have a case of “strong obsolescence.”<sup>26</sup> The following proposition summarizes the type of inefficiency found in the case of a monopolist with lagged externalities.

**Proposition 5 (Monopolist and Planned Obsolescence)** *In equilibrium, there is planned obsolescence for some combinations of  $\gamma$  and  $N$ . In some cases, old consumers are induced to switch to vintage  $B$  when they should keep vintage  $A$  (“strong obsolescence”). In other cases, new consumers are induced to buy vintage  $B$  when they should be buying vintage  $A$  instead (“weak obsolescence”)*

**Proof.** Immediate from comparison of graphs 3 and 2. See graph 4. ■

The statement of the proposition can be seen graphically in figure 4 where the marked regions represent the combinations of  $\gamma$  and  $N$  that would imply an equilibrium outcome different from the social optimum. The intuition of the proposition is the following: old consumers have an incentive to delay adoption of a new vintage because externalities are lagging, and therefore, they can enjoy the network effect for one period even after every other consumer has switched to a new vintage. Therefore, no single consumer wants to “move first”, and that would result in inefficient delay in switching to the new vintage  $B$  in cases where it would be socially optimal for them to do so. However, this is true if prices for vintage  $B$  are held constant between periods 1 and 2. However, the monopolist is able to price discriminate not only intratemporally, but also intertemporally. If a consumer decides to wait and let others adopt vintage  $B$  in period 1, then when period

<sup>25</sup>Notice that both in the market outcome and the social optimum consumers only switch to model  $B$  in period 1, since switching only in period 2 is not an equilibrium in the former and suboptimal in the latter.

<sup>26</sup>Notice that, in equilibrium, it will never be the case that old consumers switch to vintage  $B$  and new consumers buy vintage  $A$ . Therefore, if old consumers suffer planned obsolescence, then so do new consumers.

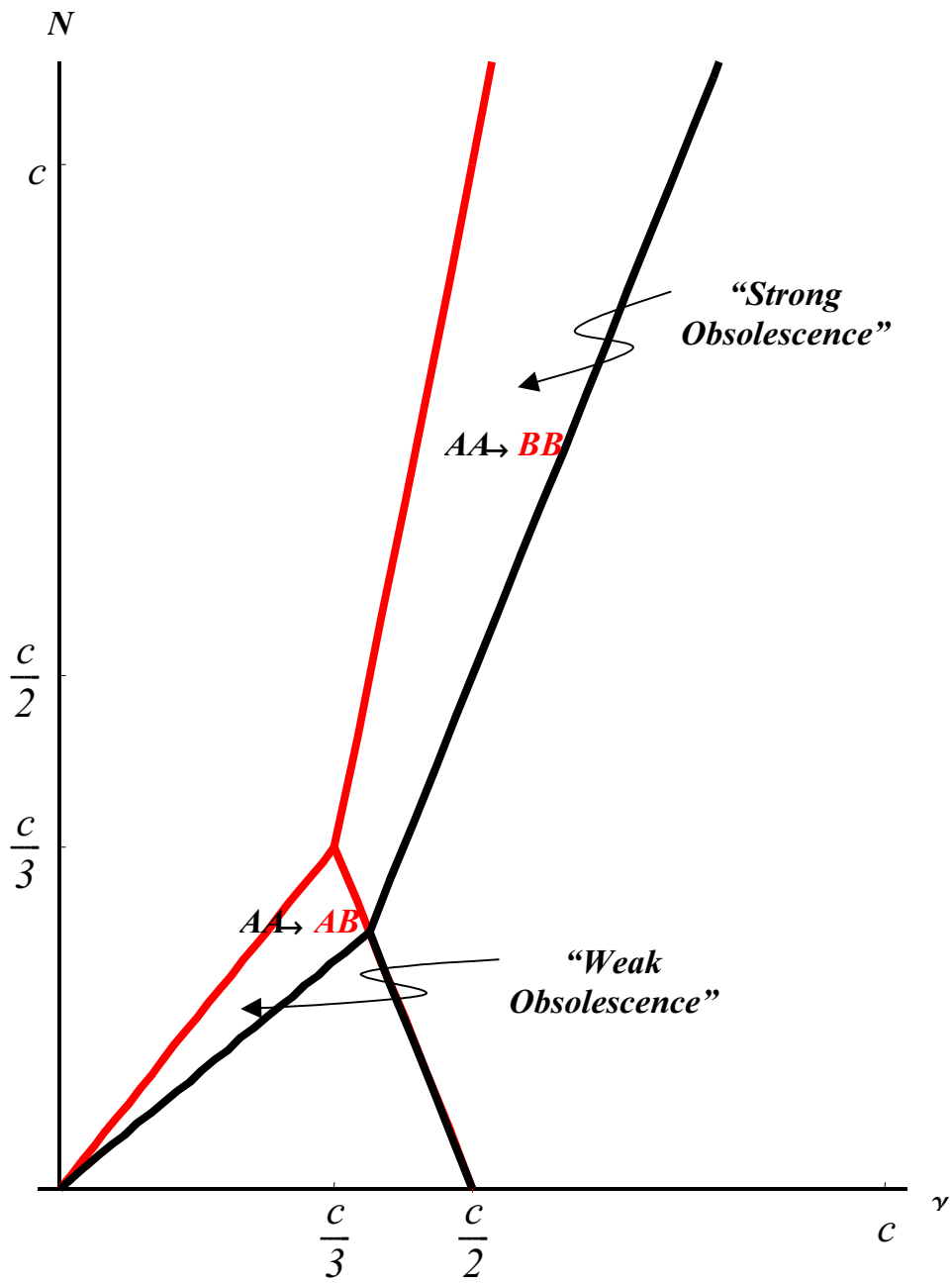


Figure 4: Monopolist: Equilibrium (red) vs. socially optimal (black) outcomes.

2 comes, the monopolist will be able to extract all its consumer surplus by the appropriate choice of trade-in prices. In some cases, the monopolist can even close the market of a particular vintage, by offering suitable regular or trade-in prices for the other vintage. Therefore, the monopolist can control and break consumer's inertia generated by lagged externalities by its control over prices in the second period. Summarizing, any gains that consumers may generate by waiting in period 1 will be appropriated by the monopolist in period 2, and therefore that eliminates any incentive to delay adoption. Thus, only the monopolist's incentive to introduce new vintages too fast remains in place, and therefore planned obsolescence results.

This proposition, therefore, confirms the robustness of results in Waldman (1993) and Choi (1994) to the introduction of incentives for consumer's inertia. In that respect, having lagged externalities does not introduce a qualitative change of results from these papers.

Notice that there is nothing in the reasoning above that requires having cohorts of consumers of the same size. The presence of lagged externalities provides all the incentives for old consumers to delay their adoption of a new vintage of the durable good. However, the introduction of a new parameter into the model prevents the graphical analysis performed for the case of equal sized cohorts. The following proposition, formalizes this idea.<sup>27</sup>

**Proposition 6** *Suppose that there is a mass  $N_1$  of old consumers and  $N_2$  of new consumers. In equilibrium, there is planned obsolescence for some combination of  $\gamma$ ,  $N_1$  and  $N_2$ . Depending on the parameters, planned obsolescence can be of the “strong” or the “weak” type.*

## 6 The effect of copying

The previous result highlights the fact that even in the case where consumers have an incentive to wait, the monopolist can manipulate prices in the second period to induce consumers to buy the good immediately. Suppose, however, that monopoly rights are short lived, in the sense that there is a competitive industry that can provide a copy of a new vintage one period after its appearance in the market. This particular setup can be found, for example in Shleifer (1986). As a motivation, one can think of successive versions of a software provided by the monopolist. Once the new version is introduced in the market, it is easy to find copied versions of it after some time.

Formally, suppose that a competitive fringe is able to supply each vintage at marginal cost  $c$  one period after it has been introduced by the monopolist. In particular, in period 1, the competitive fringe can offer vintage  $A$  at marginal cost  $c$  (since it has been around since period 0) and in period 2 the same competitive fringe can offer vintage  $B$  at marginal cost if it has been sold by the monopolist in period 1.

The first thing to notice is that the existence or not of the competitive fringe does not change the socially optimal outcomes described in lemma 4 since marginal costs of production are the same irrespective of the producer of the vintage. The only thing left to be analyzed is the equilibrium outcome in the presence of this competitive fringe. Following a similar approach to the one used in section 4, one can derive the following proposition.

**Proposition 7 (Monopolist with copying competitive fringe)** *Suppose there is a competitive fringe copying each vintage one period after its introduction. For given values of  $\gamma$ ,  $N$  and  $c$ , and assuming that*

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<sup>27</sup>Proof of this proposition can be obtained from the author upon request.

consumers coordinate on their Pareto-preferred strategy in the first period, one of the following equilibrium outcomes will arise:

1. If  $\gamma < c - 2N$  :

- (a)  $\{(A, A), (A, A)\}$  [No Introduction (**NI**): if  $\gamma \equiv f(B) - f(A) < \text{Min}\{\frac{N}{2}, \frac{c}{4} - \frac{N}{2}\}$
- (b)  $\{(A, B), (A, B)\}$  [Partial Introduction (**PI**): if  $N < \gamma \equiv f(B) - f(A) < \frac{c-3N}{2}$
- (c)  $\{(B, B), (B, B)\}$  [Full Introduction (**FI**): if  $\text{Max}\{\frac{c}{4} - \frac{N}{2}, \frac{c-3N}{2}\} < \gamma \equiv f(B) - f(A) < c - 2N$

2. If  $c - 2N < \gamma < c$  :

- (a)  $\{(A, A), (A, A)\}$  [No Introduction (**NI**): if  $\gamma \equiv f(B) - f(A) < \text{Min}\{\frac{N}{2}, N - \frac{c}{2}\}$
- (b)  $\{(A, B), (A, B)\}$  [Partial Introduction (**PI**): if  $\frac{c}{2} < \frac{N}{2} < \gamma \equiv f(B) - f(A) < c$
- (c)  $\{(B, B), (B, B)\}$  [Full Introduction (**FI**): if  $\text{Max}\{c - 2N, N - \frac{c}{2}\} < \gamma \equiv f(B) - f(A) < c$  and  $N < c$

3. If  $c < \gamma < c + 2N$  :

- (a)  $\{(A, A), (A, A)\}$  [No Introduction (**NI**): if  $c < \gamma \equiv f(B) - f(A) < N - c$
- (b)  $\{(A, B), (B, B)\}$  [Slow Diffusion (**SD**): if  $\text{Max}\{N - c, c\} < \gamma \equiv f(B) - f(A) < N$
- (c)  $\{(B, B), (B, B)\}$  [Full Introduction (**FI**): if  $\text{Max}\{N, c\} < \gamma \equiv f(B) - f(A) < 2N + c$

4. If  $c + 2N < \gamma$  :

- (a)  $\{(B, B), (B, B)\}$  [Full Introduction (**FI**)].

**Proof.** See Appendix B ■

The results of this proposition can be summarized in graph 5. Notice that the possible outcomes include now the possibility that the new vintage may diffuse slowly across different groups of consumers (outcome  $\{(A, B), (B, B)\}$ ). In this particular case, old consumers decide to wait in period 1 and not buy the new vintage  $B$ . The monopolist finds it profitable to sell vintage  $B$  to new consumers (vintage  $A$  can no longer be sold by the monopolist since it has already been copied by the competitive fringe). In period 2, the competitive fringe can sell vintage  $B$  to old consumers who are willing to pay up to  $f(B) - f(A)$  which for that area is bigger than the marginal cost of providing the good. As in the previous case, for some parameter values, the monopolist finds it optimal to offer trade-in prices for the new vintage  $B$  below cost, in order to increase the value of the network for new consumers and induce them to buy vintage  $B$  as well. That is represented by the shaded area in figure 5 and expressed formally in the following lemma.

**Lemma 8** If  $\text{Max}\{\frac{c}{4} - \frac{N}{2}, \frac{c-3N}{2}, N - \frac{c}{2}\} < \gamma \equiv f(B) - f(A) < \text{Min}\{\frac{c-N}{2}, N\}$  then in equilibrium the monopolist introduces the new vintage  $B$  to all consumers and uses trade-in prices below marginal cost:  $p_{1B}^t < c < p_{1B}$

Finally, in order to assess the efficiency properties of this market structure, we need to compare this outcome with the social optimum found in lemma 4. We can define two types of inefficient delay. The first situation, which we will call “weak delay,” has the equilibrium outcome  $\{(A, B), (B, B)\}$ , whereas the

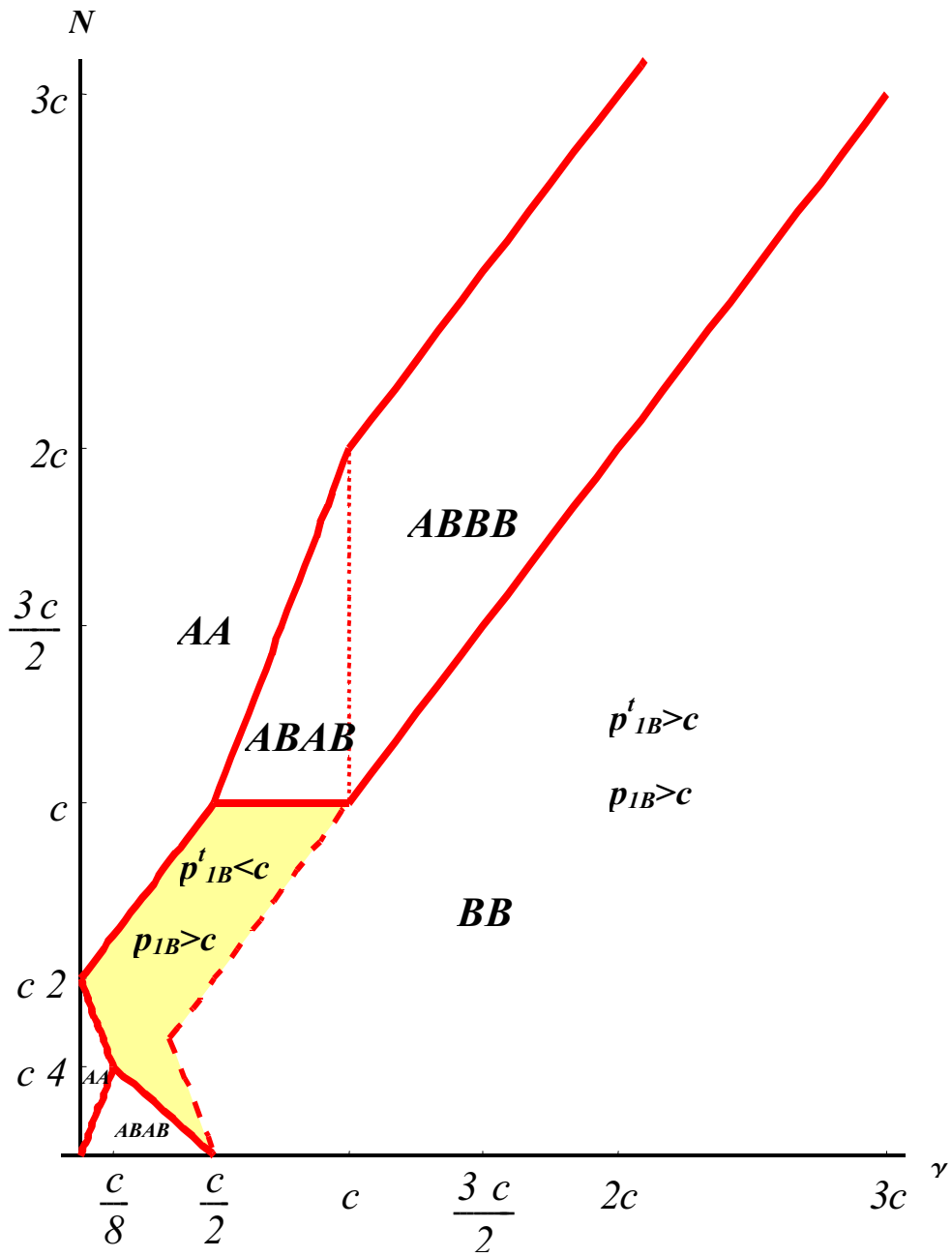


Figure 5: Monopolist with competitive fringe: equilibrium outcome

social optimum implies that the new vintage  $B$  should be adopted immediately:  $\{(B, B), (B, B)\}$ . Therefore, there is an inefficiency in the sense that old consumers take too long to adopt the new vintage. This is the inertia brought about by the lagging externality plus the onset of a reduction in price by the competitive fringe. The second type of delay, which we call “strong delay,” involves no introduction in equilibrium (outcome  $\{(A, A), (A, A)\}$ ) whereas the social optimum involves immediate switch to vintage  $B$  (outcome  $\{(B, B), (B, B)\}$ ).

Direct comparison of the bounds in the equilibrium outcome and the social optimum yields the following proposition, represented by figure 6. Note that the darkly shaded area represents the region where planned obsolescence still occurs, whereas the faintly shadowed area represents areas where delay appears.

**Proposition 9 (Obsolescence and delay with competitive fringe)** *If the monopolist faces a competitive fringe that copies the durable good after one period, then, depending on the parameters of the model the equilibrium outcome may involve planned obsolescence (weak or strong) or delay (weak or strong).*

Notice that inefficient delay appears mainly when the externality effect  $N$  is strong and when the new vintage  $B$  represents a significant improvement over the existing vintage  $A$ . (i.e  $\gamma$  big). The intuition is clear. Precisely if the new vintage  $B$  is a significant improvement over the existing vintage, a planner would like all consumers to switch to that vintage. However, if the externality effect is strong enough (big  $N$ ), then consumers have a stronger incentive to let other consumers adopt the new vintage first and develop the externality associated with it. That is, consumer inertia is stronger for  $N$  big. The combination of these two factors generates the no adoption of the new vintage when it would be socially optimal (inefficient delay). The following lemma (represented in figure 7) highlights the fact that inefficient delay is the most important feature in this type of market structure.

**Lemma 10** *If  $c = 0$  and there is a competitive fringe copying the new vintage one period after its introduction by the monopolist, then there is never planned obsolescence in equilibrium. For some parameter values, there is inefficient delay.*

## 7 Concluding Remarks

We have analyzed in this paper a monopolist selling durable goods that exhibit lagging network externalities. The main question we wanted to answer was whether we could still see planned obsolescence in this context, given that now consumers have an incentive to delay the adoption of a new vintage of the durable good. We have seen that the answer depends on how much can the monopolist’s property rights be maintained. If the monopolist can retain monopoly power over the new vintages, then in can price discriminate both intratemporally and intertemporally to break consumer’s inertia by the appropriate choice of prices in the second period. However, if the monopolist faces a competitive fringe that can copy and sell the new version of the good one period after its introduction, that we still have planned obsolescence for some parameter values, but for others we see inefficient delay emerging in equilibrium. The fact that we have incentive to wait and non-increasing prices generates situations where the new vintage is not introduced (“strong delay”) or is adopted too slowly (“weak delay”).

It is important to emphasize that the emergence of delay is not due solely to declining prices brought by the competitive fringe. If we had a competitive fringe but with contemporaneous externalities as in the

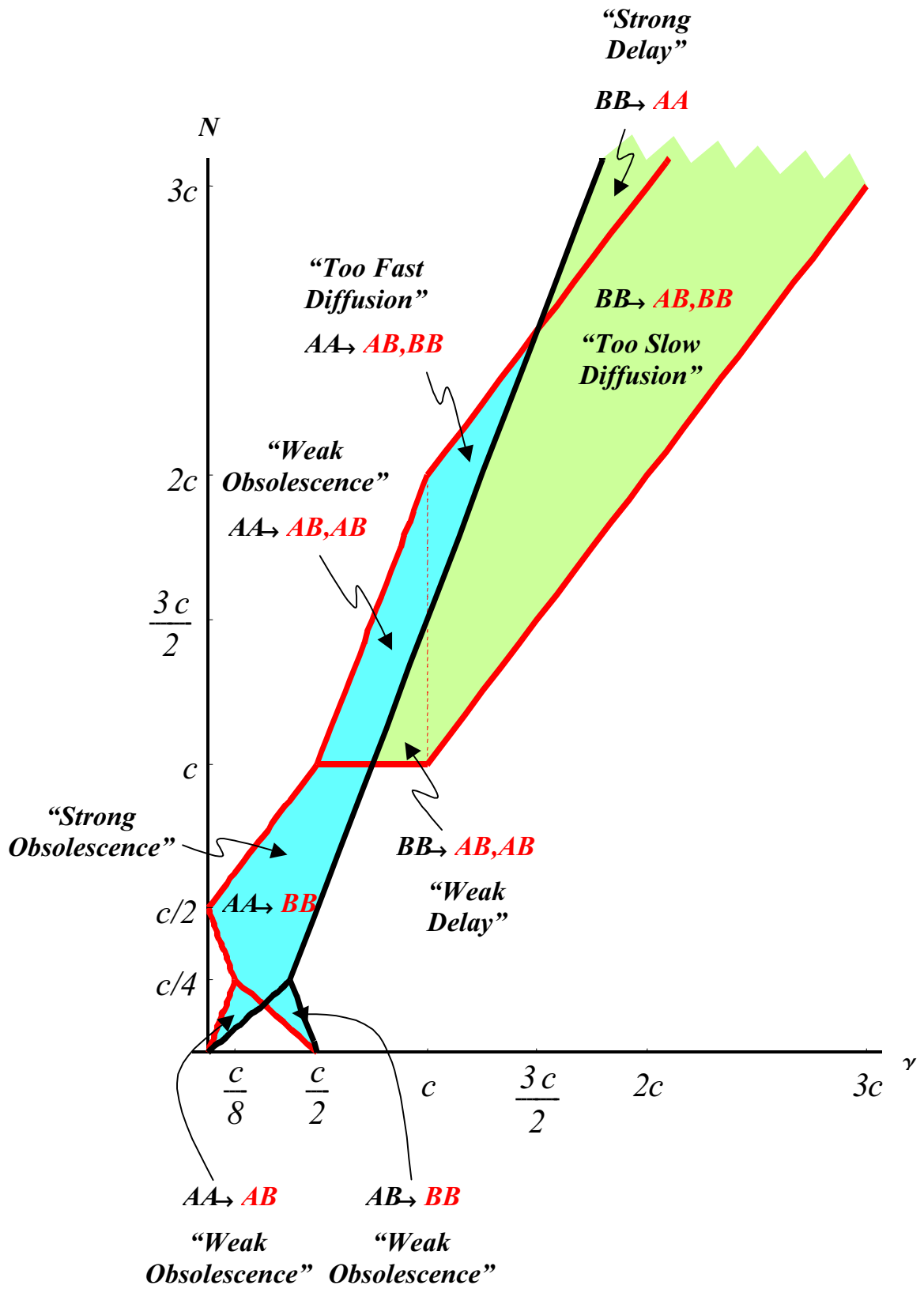


Figure 6: Monopolist with competitive fringe (red) versus Socially optimal outcome (black).

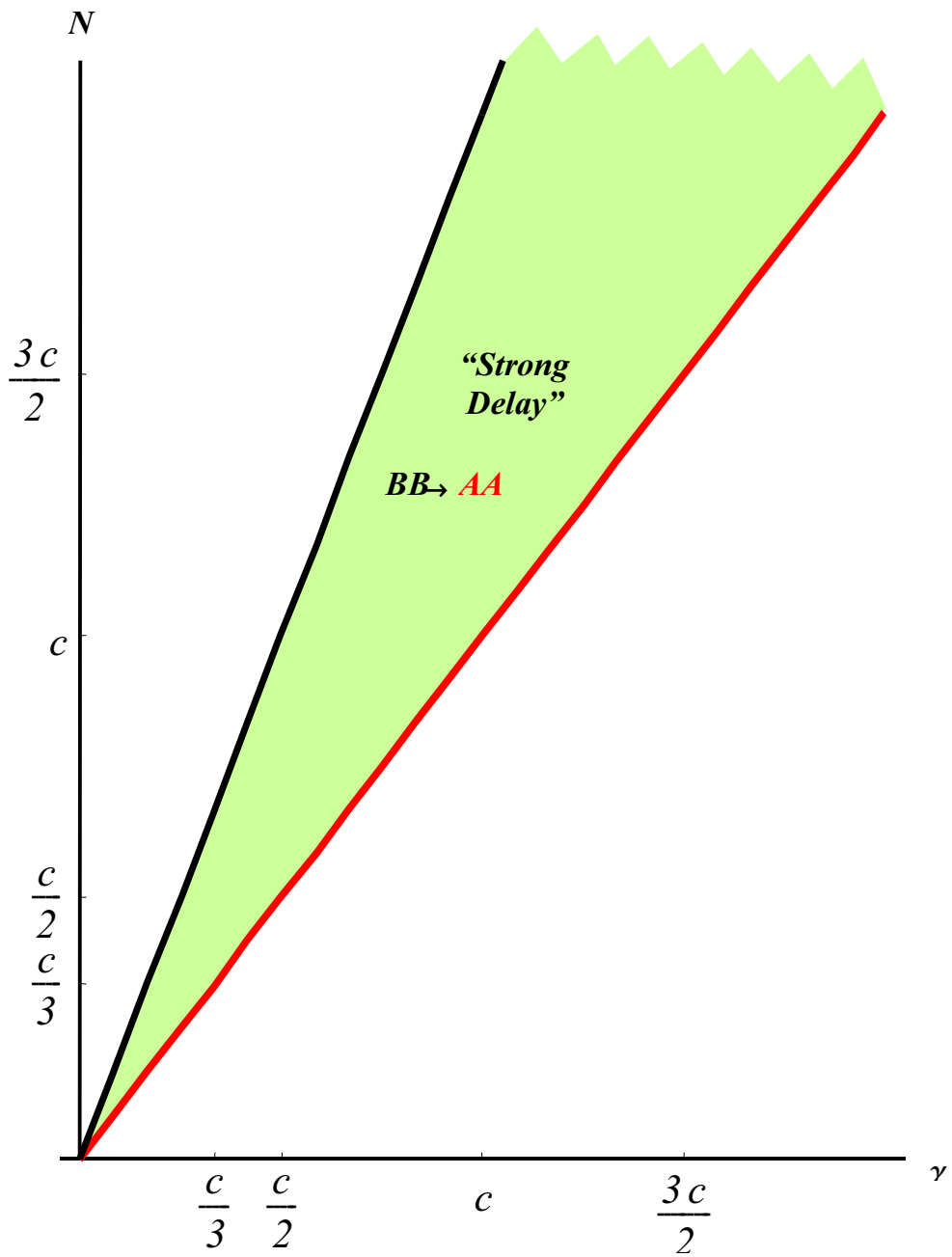


Figure 7: Monopoly with copying competitive fringe: Equilibrium (red) vs. Socially optimal outcome (black) for zero marginal costs ( $c = 0$ ).

literature, then we will still have planned obsolescence and no delay, as one can see in figure 8. Therefore, key to the emergence of delay is the fact that lagged externalities generate the incentive on consumers to free ride on other consumers adopting the new vintage first.

This result has some important implications for antitrust policy: it is no longer true that we may have to take care of a monopolist trying to introduce new versions too fast, but, depending on the timing of the externalities, we may have to give incentives to old consumers to switch to a new standard to prevent some inefficient “lock-in” to an old technology. The same question applies for policies that subsidize the scrapping of old consumer durables (e.g. subsidies for automobile replacement in France or Spain).

One possible interesting extension of this model would be to allow the monopolist to control the timing of the externality. Notice that in this model, one of the reasons why a consumer may want to delay a switch to a new vintage is the possibility of enjoying the externality associated with the old vintage even one period after it has been discarded by everyone else. If the indirect network externality is derived from a support industry and the monopolist can control it, then he may want to stop providing service to old vintages at the moment it introduces a new one, thus reducing the incentives to delay. This suggests that vertical integration may prove efficient in markets with this type of externalities, although the possibility of controlling the extent and timing of the externality seems less compelling in the case of learning effects. We leave this extension for future research.

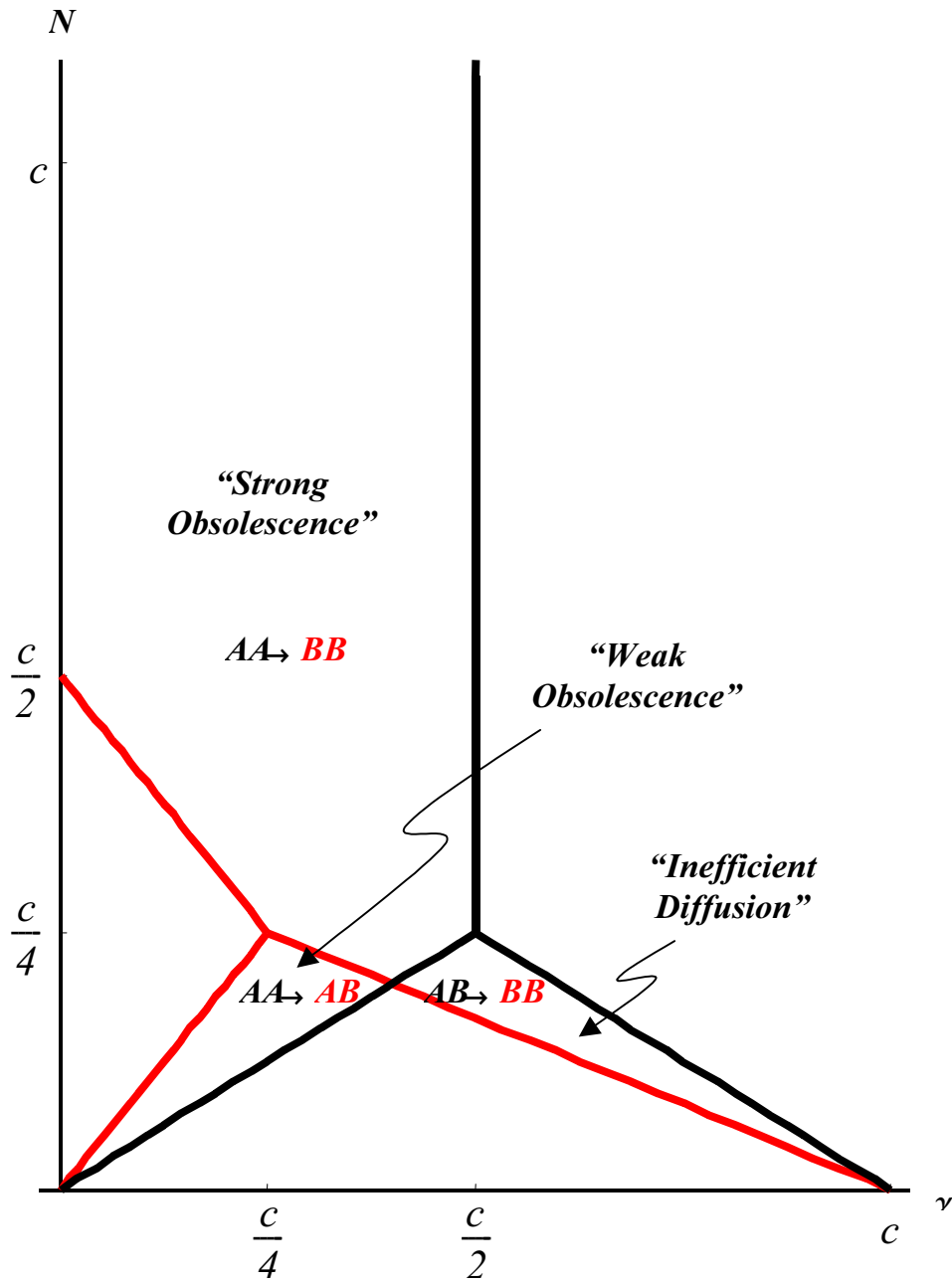


Figure 8: Monopolist with competitive fringe: Equilibrium (red) versus socially optimal outcome (black) with contemporaneous externalities.

# Appendix

## A The Case of Perfect Competition with lagging externalities

To illustrate the incentives of consumers to delay their switching to a new vintage, we can examine the case of perfect competition with lagging externalities. Consider that there is a competitive industry supplying vintage  $A$  and  $B$  at marginal cost  $c$  in periods 1 and 2. Obviously, since  $f(B) > f(A) > c$ , then new consumers always buy some vintage in period 1. Therefore, in period 2, we only have four possible histories. We solve the equilibrium of this game by backward induction.

### A.1 Period 2

In period 2, both vintages are sold at marginal cost  $c$ . If in period 1 both types of consumers used vintage  $A$  (history  $(A, A)$ ), then in period 2 a consumer would only switch to vintage  $B$  (outcome  $\{(A, A), (B, B)\}$ ) if and only if

$$\gamma \equiv f(B) - f(A) > 2N + c$$

since vintage  $B$  has no externalities associated with it (has not been used in period 1) and vintage  $A$  has an externality effect of  $2N$  in period 2. Both types of consumers get a surplus in period 2 equal to  $f(A) + 2N + \text{Max}\{\gamma - 2N - c, 0\}$

If vintage  $B$  has been used by only one group in period 1 (history  $(A, B)$  or  $(B, A)$ ), then in period 2 both vintages enjoy the same externality effect, and so a user of vintage  $A$  will switch to vintage  $B$  (outcome  $\{(A, B), (B, B)\}$  or  $\{(B, A), (B, B)\}$ ) if and only if

$$\gamma \equiv f(B) - f(A) > c$$

Notice that a consumer that uses vintage  $B$  in period 1 does not want to switch to vintage  $A$  in period 2 since  $f(B) > f(A)$  and both vintages have the same externality effect in period 2. Consumers with vintage  $A$  in period 1 get surplus in period 2 equal to  $f(A) + N + \text{Max}\{\gamma - c, 0\}$ . Consumers with vintage  $B$  in period 1 get surplus in period 2 equal to  $f(B) + N$ .

Finally, if both consumers used vintage  $B$  in period 1, then they will never want to switch to vintage  $A$  in period 2, since  $f(B) > f(A)$  and on top of that, vintage  $B$  also enjoys a bigger externality effect in period 2. Both types of consumers will get surplus in period 2 equal to  $f(B) + 2N$ .

In table 5 we summarize the surplus obtained in period 2 as a function of period 1's outcome and the vintage used in period 1 by that consumer. Notice that we are also considering cases in which an agent deviates in period 1 from the specified outcome. Since each agent is infinitesimal, we assume that such a deviation does not change the externality effect in period 2.

### A.2 Equilibria in Period 1

In period 1, we have to analyze four possible subgames.

First Period Outcome	Consumer Surplus in second period	
	user of vintage $A$ in period 1	user of vintage $B$ in period 1
$(A, A)$	$f(A) + 2N + \text{Max}\{\gamma - 2N - c, 0\}$	$f(B) + \text{Max}\{2N - \gamma - c, 0\}$
$(A, B)$	$f(A) + N + \text{Max}\{\gamma - c, 0\}$	$f(B) + N$
$(B, A)$	$f(A) + N + \text{Max}\{\gamma - c, 0\}$	$f(B) + N$
$(B, B)$	$f(A) + \text{Max}\{\gamma + 2N - c, 0\}$	$f(B) + 2N$

Table 5: Perfect Competition and Lagging Externalities: Second period consumer surplus

### A.2.1 Subgame $(A, A)$ :

If new consumers buy vintage  $A$  at marginal cost in period 1 and old consumers keep vintage  $A$ , then new consumers would get utility  $2f(A) + 3N - c + \text{Max}\{\gamma - 2N - c, 0\}$  and old consumers get  $2f(A) + 3N + \text{Max}\{\gamma - 2N - c, 0\}$ .

To check the range of values of  $\gamma$  and  $N$  for which this can be sustained as an equilibrium, we need to check individual deviations by old or new consumers. Start with new consumers. If one of them deviates in period 1 and buys vintage  $B$ , then he will get  $f(B) - c$  in period 1 and  $f(B) + \text{Max}\{2N - \gamma - c, 0\}$  in period 2 (see table 5) for a total of  $2f(B) - c + \text{Max}\{2N - \gamma - c, 0\}$ . If an old consumer deviates and buys vintage  $B$  in period 1 he will also get the same surplus. To prevent this deviation from new and old consumers we need to satisfy, respectively

$$2f(A) + 3N - c + \text{Max}\{\gamma - 2N - c, 0\} > 2f(B) - c + \text{Max}\{2N - \gamma - c, 0\} \quad (19)$$

$$2f(A) + 3N + \text{Max}\{\gamma - 2N - c, 0\} > 2f(B) - c + \text{Max}\{2N - \gamma - c, 0\} \quad (20)$$

Notice that only condition (19) is binding, since, if new consumers do not want to deviate, then neither old consumers, as they save the adoption cost  $c$  by keeping vintage  $A$ . Condition (19) is equivalent to

$$\gamma < \text{Min}\left\{N + c, \frac{3N}{2}\right\} \quad (21)$$

which constitutes the range of values of  $\gamma$  and  $N$  where equilibrium  $(A, A)$  is possible in the first period.

### A.2.2 Subgame $(A, B)$ :

If new consumers buy  $B$  while old consumers keep  $A$ , then new consumers would get utility in the two periods equal to  $2f(B) + N - c$ , while old consumers get utility  $2f(A) + 2N + \text{Max}\{\gamma - c, 0\}$ .

If a new consumer decides to deviate and buy vintage  $A$  in period 1, he would get utility  $f(A) + N - c + \text{Max}\{f(B) + N - c, f(A) + N\} = 2f(A) + 2N - c + \text{Max}\{\gamma - c, 0\}$ . If an old consumer deviates in period 1 and buys  $B$ , he will get total utility  $2f(B) - c + N$ . To ensure that  $(A, B)$  is an equilibrium outcome in the first period, we need to make sure these two deviations are not profitable, that is

$$2f(A) + 2N - c + \text{Max}\{\gamma - c, 0\} < 2f(B) + N - c < 2f(A) + 2N + \text{Max}\{\gamma - c, 0\} \quad (22)$$

or equivalently

$$\text{Max}\left\{\frac{N}{2}, N - c\right\} < \gamma < \text{Max}\left\{N, \frac{N}{2} + \frac{c}{2}\right\} \quad (23)$$

Intuitively, for the same price  $c$  and the same level of externality  $N$  (which affects both vintages equally in the second period) new consumers are more willing to buy the new technology than old consumers, since the

latter have an outside option which is to keep their existing vintage  $A$ . Since new consumers need to buy in period 1, they have a lower opportunity cost of buying  $B$  (lower in the amount  $c$ ). Thus the equilibrium can be sustained.

### A.2.3 Subgame $(B, A)$ :

In this subgame, new consumers get utility in two periods equal to  $2f(A) + 2N - c + \text{Max}\{\gamma - c, 0\}$ , whereas old consumers, who buy  $B$ , get total utility  $2f(B) + N - c$ . If a new consumer deviates in period 1 and buys  $B$  he will get utility  $2f(B) + N - c$ . If an old consumer deviates and buys  $A$  in period 1, then he would get utility  $2f(A) + 2N + \text{Max}\{\gamma - c, 0\}$ . To sustain  $(B, A)$  as an equilibrium we would need

$$2f(A) + 2N + \text{Max}\{\gamma - c, 0\} < 2f(B) + N - c < 2f(A) + 2N - c + \text{Max}\{\gamma - c, 0\} \quad (24)$$

which is clearly a contradiction for  $c > 0$ . Therefore  $(B, A)$  cannot be sustained as an equilibrium. In words, whenever a new consumer is willing to buy  $A$  and does not want to deviate and behave like an old consumer, then obviously an old consumer, with lower opportunity cost of using  $A$  in period 1, would want to deviate from the prescribed strategy in this subgame (i.e. using  $b$  in period 1).

### A.2.4 Subgame $(B, B)$ :

If everyone switches to vintage  $B$  in period 1, then new and old consumers get utility  $2f(B) + 2N - c$ . If a new consumer deviates in period 1 and keeps vintage  $A$ , then he would get utility  $2f(A) + N - c + \text{Max}\{\gamma + 2N - c, 0\}$ . An old consumer trying the same deviation would get utility  $2f(A) + N + \text{Max}\{\gamma + 2N - c, 0\}$ . To ensure that these deviations are not profitable, we need

$$2f(B) + 2N - c > 2f(A) + N - c + \text{Max}\{\gamma + 2N - c, 0\} \quad (25)$$

$$2f(B) + 2N - c > 2f(A) + N + \text{Max}\{\gamma + 2N - c, 0\} \quad (26)$$

Notice that only condition (26) is binding. That is, if we can ensure that old consumers, with a lower opportunity cost of keeping  $A$ , do not want to deviate from buying  $B$ , then neither do new consumers. Condition (26) can also be expressed as

$$\gamma > \text{Max}\left\{\frac{c}{2} - \frac{N}{2}, N\right\} \quad (27)$$

### A.2.5 Equilibrium Selection

Notice that given the three types of equilibria found above, there are ranges of values of  $\gamma$  and  $N$  for which there will be multiple equilibria. In those cases, we select the Pareto optimal equilibrium assuming that consumers can coordinate their choices in order to attain it. Table 6 summarizes the total utility attained over the two periods by old and new consumers in each equilibrium. Notice that in equilibrium  $(A, A)$ , old or new consumers will never switch to vintage  $B$  in the second period for the range of  $\gamma$  for which that is an equilibrium outcome. In general, in equilibrium, we will have that second-period outcomes will be the same as first period outcomes, except for equilibrium  $(A, B)$ , where old consumers may decide to switch to  $B$  only in period 2, after the externalities associated with that vintage have been developed.

For the ranges of values of  $\gamma$  for which there is the possibility of multiple equilibria, we select the Pareto optimal one. In order to do that we just need to compare the utility obtained by each group in each of the possible cases.

Equilibrium First Period	Range	Utility	
		Old consumer	New Consumer
(A, A)	$\gamma < \text{Min}\{N + c, \frac{3N}{2}\}$	$2f(A) + 3N$	$2f(A) + 3N - c$
(A, B)	$\text{Max}\{\frac{N}{2}, N - c\} < \gamma < \text{Max}\{N, \frac{N}{2} + \frac{c}{2}\}$	$2f(A) + 2N + \text{Max}\{\gamma - c, 0\}$	$2f(B) + N - c$
(B, B)	$\text{Max}\{\frac{c}{2} - \frac{N}{2}, N\} < \gamma$	$2f(B) + 2N - c$	$2f(B) + 2N - c$

Table 6: Perfect Competition and Lagged Externalities: two-period utility in each equilibrium

**Equilibrium (A, A) :** We may have first period outcomes (A, A) or (A, B) in equilibrium for the following range:

$$\text{Max}\{\frac{N}{2}, N - c\} < \gamma < \text{Max}\{N, \frac{N}{2} + \frac{c}{2}\} \text{ and } \gamma > \frac{3N}{2}$$

From table 6, new consumers would prefer to coordinate on subgame (A, A) instead of (A, B) if  $2f(A) + 3N - c > 2f(B) + N - c$ , that is

$$\gamma < N \tag{28}$$

We may have first period outcomes (A, A) and (B, B) in equilibrium for the following range:

$$\text{Max}\{\frac{c}{2} - \frac{N}{2}, N\} < \gamma < \text{Min}\{N + c, \frac{3N}{2}\}$$

From table 6, old and new consumers would prefer to coordinate on subgame (A, A) instead of (B, B) if  $2f(A) + 3N \geq 2f(B) + 2N - c$  (old consumers) and  $2f(A) + 3N - c \geq 2f(B) + 2N - c$  (new consumers), that is

$$\gamma < \frac{N}{2} \tag{29}$$

Therefore, equilibrium  $\{(A, A), (A, A)\}$  will be the unique equilibrium for  $\gamma < \text{Min}\{\frac{N}{2}, N - c\}$  and the Pareto preferred equilibrium for  $\text{Min}\{\frac{N}{2}, N - c\} < \gamma < N$ . Therefore,  $\{(A, A), (A, A)\}$  will be played in equilibrium in the range  $\gamma < N$

**Equilibrium (A, B) :** We have already seen that in the range  $\text{Max}\{\frac{N}{2}, N - c\} < \gamma < \text{Max}\{N, \frac{N}{2} + \frac{c}{2}\}$  and  $\gamma > \frac{3N}{2}$ , (A, A) and (A, B) are two possible equilibrium outcomes in the first period. Following the previous analysis, (A, B) will be Pareto preferred on the part of this range where

$$\gamma > N$$

On the other hand, we may have first period outcomes (A, B) and (B, B) in equilibrium for the following range:

$$\text{Max}\{\frac{c}{2} - \frac{N}{2}, N\} < \gamma < \frac{N}{2} + \frac{c}{2}$$

Using similar arguments, to sustain (A, B) as an equilibrium in the first period we need to satisfy  $2f(B) + N - c \geq 2f(A) + 3N - c$  for new consumers and  $2f(A) + 2N + \text{Max}\{\gamma - c, 0\} \geq 2f(B) + 2N - c$  for old consumers. That is

$$N \leq \gamma \leq \frac{c}{2} \tag{30}$$

Examining the two cases, we can easily see that  $\{(A, B), (A, B)\}$  will be the unique equilibrium for  $\frac{3N}{2} < \gamma < \frac{c}{2} - \frac{N}{2}$  and the Pareto preferred equilibrium for  $\text{Max}\{\frac{c}{2} - \frac{N}{2}, N\} < \gamma < \frac{c}{2}$ . Therefore,  $\{(A, B), (A, B)\}$  will be played in equilibrium in the range  $N \leq \gamma \leq \frac{c}{2}$ .

Period 1 outcome (old, new)	Monopolist's second period prices that induce purchase by consumers	Second Period outcomes
$A, A$	$p_{2A} = p_{2A}^t = c$ $p_{2B} = \text{Max}\{f(B) - f(A) - 2N + c, c\}$ $p_{2B}^t = \text{Max}\{f(B) - f(A) - 2N, c\}$	$(A, A)$ if $f(B) - f(A) < 2N + c$ $(B, B)$ if $f(B) - f(A) > 2N + c$
$A, B$	$p_{2A} = p_{2A}^t = p_{2B} = p_{2B}^t = c$	$(A, B)$ if $f(B) - f(A) < c$ $(B, B)$ if $f(B) - f(A) > c$
$B, A$	$p_{2A} = p_{2A}^t = p_{2B} = p_{2B}^t = c$	$(B, A)$ if $f(B) - f(A) < c$ $(B, B)$ if $f(B) - f(A) > c$
$B, B$	$p_{2A} = p_{2A}^t = p_{2B} = p_{2B}^t = c$	$(B, B)$

Table 7: Monopolist with competitive fringe: feasible actions in the second period and associated profit-maximizing prices

**Equilibrium  $(B, B)$ :** Finally, from the previous discussion, we can conclude that  $\{(B, B), (B, B)\}$  will be played in the range  $\gamma > \text{Max}\{\frac{c}{2}, N\}$

We can summarize the previous results in the following

**Proposition 11** *Under perfect competition and lagged externalities, the Pareto optimal equilibrium outcome is*

$$\begin{cases} \{(A, A), (A, A)\} & \text{if } \gamma < N \\ \{(A, B), (A, B)\} & \text{if } N \leq \gamma \leq \frac{c}{2} \\ \{(B, B), (B, B)\} & \text{if } \gamma > \text{Max}\{\frac{c}{2}, N\} \end{cases}$$

## B Proof of Proposition 7

To prove proposition 7, we need to start with period 2 and then solve backwards to period 1. Given that in period 1 vintage  $A$  is sold at marginal cost by the competitive fringe and  $f(A) > c$ , in equilibrium, we will never observe histories  $\{A, \emptyset\}$  or  $\{B, \emptyset\}$  in the first period, since a new consumer can always get positive utility in period 1 buying vintage  $A$  at marginal cost, without affecting his chances of getting any other vintage in period 2. Therefore, we just concentrate on the other four possible histories in period 1:  $\{A, A\}, \{A, B\}, \{B, A\}$  and  $\{B, B\}$ .

### B.1 Period 2

Table 7 summarizes the possible histories and maximum prices paid by consumers in each of them. Notice that since there is a competitive fringe copying each vintage one period after its introduction, in period 2, vintage  $A$  is always sold at marginal cost and vintage  $B$  is also sold at marginal cost in any history where vintage  $B$  has been introduced in period 1 (lines 2, 3 and 4 in table 7). In the case of histories where vintage  $B$  was not introduced in period 1, the maximum price charged for vintage  $B$  has to take into account the outside option of buying vintage  $A$  at marginal cost  $c$ . Given that, in period 1 vintage  $A$  is sold at marginal cost by the competitive fringe and  $f(A) > c$ , in equilibrium, we will never observe histories  $\{A, \emptyset\}$  or  $\{B, \emptyset\}$ , since a new consumer can always get positive utility in period 1 buying vintage  $A$  at marginal cost, without affecting his chances of getting any other vintage in period 2.

As in the case of the monopolist, in the second period we do not need to assume that consumers coordinate

their behavior to obtain a unique equilibrium. Given that this is the last period, consumers still disregard other consumer's actions when deciding the adoption of a new vintage. This determines the outcome in the second period as stated in the third column of table 7.

For histories  $\{A, B\}$ ,  $\{B, A\}$  and  $\{B, B\}$ , where vintage  $B$  has been introduced already in period 1, the monopolist cannot set prices above marginal cost in period 2 and therefore its second period profits will be  $\pi_2 = 0$ . The only non-trivial decision for the monopolist is in the case of history  $\{A, A\}$ , where the monopolist still retains monopoly rights over vintage  $B$ . The monopolist will choose to introduce vintage  $B$  if and only if  $p_{2B}^t = f(B) - f(A) - 2N > c$  in which case  $\pi_2 = f(B) - f(A) - 2N - c$ .

## B.2 Period 1

Due to the same problem of multiplicity of equilibria, we still need to select among the many equilibria in period 1. We again assume that consumers can coordinate to choose their Pareto-optimal outcome. In this first period, vintage  $A$  is already supplied by the competitive fringe (it has been available since period 0) so  $p_{1A} = c$ .

### B.2.1 Subgame $(A, A)$ :

The monopolist decides not to sell in the first period. In the first period new consumers buy vintage  $A$  from the competitive fringe at marginal cost. In the second all consumers switch to vintage  $B$  if and only if  $f(B) - f(A) > 2N + c$ . Total profits for this subgame are therefore:

$$\pi_{AA} = \begin{cases} (f(B) - f(A) - 2N - c)2N, & \text{if } f(B) - f(A) > 2N + c \quad (\text{outcome } \{(A, A), (B, B)\}) \\ 0 & \text{if } f(B) - f(A) < 2N + c \quad (\text{outcome } \{(A, A), (A, A)\}) \end{cases}$$

In order to sustain this subgame in equilibrium, prices for vintage  $B$  need to be high enough to deter purchases from either group. If *all* consumers decide to use vintage  $B$  in period 1, they would get total utility over the two periods equal to  $2f(B) + 2N$ . If old consumers keep vintage  $A$  they can get utility  $2f(A) + 2N$ .<sup>28</sup> If  $p_{1B}^t > 2f(B) - 2f(A)$  then old consumers will never buy vintage  $B$ . If new consumers buy vintage  $B$  instead of  $A$ , they would also get extra utility  $2f(B) - p_{1B} - (2f(A) - c)$ , therefore, the following prices ensure that even in the best scenario for vintage  $B$  (i.e. the expectation that everyone would buy  $B$ ) no consumer is willing to buy that vintage.

$$\begin{aligned} p_{1B} &> 2f(B) - 2f(A) + c \\ p_{1B}^t &> 2f(B) - 2f(A) \end{aligned}$$

Notice that in this subgame there is no incentive to wait, since the monopolist is not introducing vintage  $B$  in the first period, as in the following subgames.

### B.2.2 Subgame $(A, B)$ :

The monopolist only introduces vintage  $B$  to new consumers. Old consumers keep using vintage  $A$ . In the second period, all prices drop to marginal cost (see table 7) and old consumers switch to vintage  $B$  if and only if  $f(B) - f(A) > c$ , that is, if vintage  $B$  is sufficiently better than vintage  $A$  to justify paying the price  $c$  to switch.

<sup>28</sup>Notice that as in the case of the monopolist without a competitive fringe, in this subgame, if the monopolist sells vintage  $B$  in the second period, then it will be able to extract all surplus from buyers.

Notice that since the monopolist sells vintage  $B$  in period 1, it will become available at marginal cost in period 2. Moreover, the externality effect of vintage  $B$  will only develop in period 2, and this gives incentives for consumers to deviate and let other consumers buy  $B$  in period 1, while they try to buy it only in period 2. This forces the monopolist to lower prices for vintage  $B$  in period 1 in order to be able to sell it.

In particular, by adopting vintage  $B$  in period 1, a new consumer would get total utility over the next two periods equal to  $2f(B) + N - p_{1B}$ . On the other hand, if every new consumer buys vintage  $B$  in period 1, a new consumer could deviate and buy vintage  $A$  in period 1 and then buy vintage  $B$  only in period 2, getting total surplus equal to  $f(B) + f(A) + 2N - 2c$ . Another deviation for a new consumer could be to buy vintage  $A$  and keep it both periods, to get total utility  $2f(A) + 2N - c$ . Therefore, the price charged for vintage  $B$  that prevents these individual deviations has to satisfy

$$p_{1B} \leq \text{Min}\{f(B) - f(A) - N + 2c, 2f(B) - 2f(A) - N + c\} \quad (31)$$

On the other hand, we also have to make sure that a coordinated deviation by all new consumers in period 1 does not leave them with a higher utility. If all new consumers decide to buy vintage  $A$  at marginal cost in period 1, in period 2 they can keep vintage  $A$  or switch to  $B$  in which case we would return to subgame  $(A, A)$  and any extra surplus generated by a switch to  $B$  would be appropriated by the monopolist. Therefore, they would get total utility of  $2f(A) + 3N - c$ . The price charged for vintage  $B$  in period 1 has to ensure that this deviation is not profitable:

$$p_{1B} \leq 2f(B) - 2f(A) - 2N + c \quad (32)$$

From (31) and (32) we obtain an upper bound on the regular price of vintage  $B$  in period 1:

$$p_{1B} = \text{Min}\{\gamma - N + 2c, 2\gamma - 2N + c\} = \gamma - N + c + \text{Min}\{c, \gamma - N\}$$

Notice in particular that the price for vintage  $B$  in this subgame is lower than in the equivalent subgame when there is no competitive fringe (see eq. (9) in the main text).

If every old consumer switches to  $B$  in period 1, he would get total utility  $2f(B) + 2N$  minus the price it pays for that vintage. Keeping vintage  $A$  they get  $2f(A) + 2N + \text{Max}\{\gamma - c, 0\}$  since in the second period they have the option of buying vintage  $B$  if the productivity difference is higher than the price  $c$  of vintage  $B$  in period 2. Thus, to discourage old consumers buying vintage  $B$ , we need to satisfy:

$$\begin{aligned} p_{1B} &> 2f(B) - 2f(A) - \text{Max}\{\gamma - c, 0\} = 2\gamma - \text{Max}\{\gamma - c, 0\} = 2\gamma + \text{Min}\{\gamma, c\} \\ p_{1B}^t &> 2f(B) - 2f(A) - \text{Max}\{\gamma - c, 0\} = 2\gamma - \text{Max}\{\gamma - c, 0\} = 2\gamma + \text{Min}\{\gamma, c\} \end{aligned}$$

This, together with the previous condition for new consumers, implies that this subgame can only be induced if  $\gamma - N + c + \text{Min}\{c, \gamma - N\} > 2\gamma + \text{Min}\{\gamma, c\}$ , that is if

$$2N + \gamma < c \quad (33)$$

The monopolist does not make any sales above marginal cost in the second period. Actually, given condition (33), old consumers will not be willing to buy vintage  $B$  in the second period from the competitive fringe, and therefore the only outcome in this subgame is  $\{(A, B), (A, B)\}$ . Also, for that range,  $p_{1B} = \gamma - N + c + \text{Min}\{c, \gamma - N\} = 2\gamma - 2N + c$  Total profits in this subgame are just profits obtained in the first period.

$$\pi_{AB} = (p_{1B} - c)N = (2\gamma - 2N)N, \quad \text{if } 2N + \gamma < c$$

### B.2.3 Subgame $(B, A)$ :

The monopolist only introduces vintage  $B$  to old consumers. New consumers buy vintage  $A$  at marginal cost. In the second period, as prices for all vintages drop to  $c$ , new consumers switch to vintage  $B$  if and only if  $f(B) - f(A) > c$  (see table 7). Adopting vintage  $B$  in period 1 gives an old consumer utility  $2f(B) + N - p_{1B}^t$ . On the other hand, a single old consumer can deviate and keep vintage  $A$  in the first period and buy vintage  $B$  only in the second period, getting total utility  $f(B) + f(A) + 2N - c$ . Another option is to keep vintage  $A$  both periods getting total utility  $2f(A) + 2N$ . Therefore, with these individual deviations, old consumers get utility equal to  $2f(A) + 2N + \text{Max}\{\gamma - c, 0\}$ . The trade-in price  $p_{1B}^t$  that prevents these individual deviations therefore has to satisfy

$$\begin{aligned} p_{1B}^t &\leq 2f(B) + N - (2f(A) + 2N + \text{Max}\{\gamma - c, 0\}) \\ &= \text{Min}\{f(B) - f(A) - N + c, 2f(B) - 2f(A) - N\} \end{aligned} \quad (34)$$

We also need to check that old consumers do not want to coordinate on keeping vintage  $A$  in period 1. As in the previous case, this coordinated deviation would return us to history  $(A, A)$  for period 2, where any additional gain from switching will be appropriated by the monopolist. Therefore, old consumers would get utility  $2f(A) + 3N$  by coordinately sticking to vintage  $A$  in period 1. The trade-in price charged for vintage  $B$  has to ensure that this collective “deviation” is not profitable:

$$p_{1B}^t \leq 2f(B) - 2f(A) - 2N \quad (35)$$

From (34) and (35) we obtain an upper bound on  $p_{1B}^t$ :

$$p_{1B}^t \leq \gamma - N + \text{Min}\{c, \gamma - N\} \quad (36)$$

We also have to ensure that new consumers do not want to buy vintage  $B$ . If all of them get vintage  $B$  in the first period, then they would get utility  $2f(B) + 2N$  minus the price paid for  $B$ . Buying vintage  $A$  at marginal cost they get utility  $2f(A) + 2N - c + \text{Max}\{\gamma - c, 0\}$  since they have the option of buying vintage  $B$  in the second period. Therefore, to prevent new consumers buying vintage  $B$ , we need:

$$\begin{aligned} p_{1B} &> 2f(B) - 2f(A) + c - \text{Max}\{\gamma - c, 0\} = 2\gamma + c - \text{Max}\{\gamma - c, 0\} \\ p_{1B}^t &> 2f(B) - 2f(A) - \text{Max}\{\gamma - c, 0\} = 2\gamma - \text{Max}\{\gamma - c, 0\} \end{aligned}$$

where the condition on  $p_{1B}^t$  takes into account that new consumers can also get vintage  $B$  by acquiring vintage  $A$  at marginal cost and then trading it in for  $B$  at price  $p_{1B}^t$ . From condition (36) on old consumers, this results in  $p_{1B}^t = \gamma - N + \text{Min}\{c, \gamma - N\} = 2\gamma - 2N - \text{Max}\{\gamma - c - N, 0\} > 2\gamma - \text{Max}\{\gamma - c, 0\}$ , which cannot be satisfied for non-negative values of  $N$ .

Therefore, subgame  $(B, A)$  cannot be induced in equilibrium: any price that makes old consumers willing to buy vintage  $B$  in the first period will also make new consumers willing to acquire it. The intuition is simple: an old consumer has always the option of keeping  $A$  and that assures them a surplus of  $2f(A) + 2N + \text{Max}\{\gamma - c, 0\}$  when buying  $B$ . But notice that new consumers can also acquire  $B$  if they are willing to pay  $p_{1B}^t + c$ , that is, buying  $A$  first and then exchanging it with  $B$ . If every new consumer does that, he would obtain a surplus equal to  $2f(B) + 2N - p_{1B}^t - c = f(B) + f(A) + 3N - c - \text{Min}\{c, \gamma\} = 2f(A) + 3N - c + \text{Max}\{\gamma - c, 0\}$ , which is bigger than  $2f(A) + 2N - c + \text{Max}\{\gamma - c, 0\}$ , the surplus obtained from buying vintage  $A$ . In words, if every new consumer decides to buy vintage  $B$  then that increases the network effect of vintage  $B$ , and so, if old consumers were willing to switch, then new consumers would also be willing to switch after buying vintage  $A$  in the first period.

#### B.2.4 Subgame $(B, B)$ :

In this subgame, the monopolist introduces vintage  $B$  to both consumers. Since new consumers can always acquire vintage  $A$  at marginal cost and then exchange it to get  $B$  at the regular trade-in price, then  $p_{1B} = p_{1B}^t + c$ . Old consumers would get utility  $2f(B) + 2N - p_{1B}^t$  and new consumers would get utility  $2f(B) + 2N - p_{1B} = 2f(B) + 2N - p_{1B}^t - c$ .

In period 2, vintage  $B$  could be acquired at marginal cost. Therefore, an old consumer could wait one period and switch to vintage  $B$  in period 2. That would give him utility  $f(B) + f(A) + 3N - c$ . If only one old consumer decides to keep vintage  $A$  in both periods, he would get  $2f(A) + N$ . Therefore, old consumers are sure to obtain a surplus of  $2f(A) + N + \text{Max}\{\gamma - c + 2N, 0\}$ . To prevent individual deviations from old consumers, we need to satisfy

$$2f(B) + 2N - p_{1B}^t > 2f(A) + N + \text{Max}\{\gamma - c + 2N, 0\} \quad (37)$$

On the other hand, if old consumers coordinate and use the same strategy, then, by keeping vintage  $A$  in period 1, then each of them can get utility  $2f(A) + 2N + \text{Max}\{\gamma - c, 0\}$ . Therefore, to sustain old consumers willing to switch collectively, we also need to ensure that

$$2f(B) + 2N - p_{1B}^t > 2f(A) + 2N + \text{Max}\{\gamma - c, 0\} \quad (38)$$

New consumers confront the same conditions (37) and (38). Even though they have to pay  $c$  if they want to deviate and acquire  $A$ , they also have to pay a price higher than old consumers in exactly the same amount  $c$ .

The last thing to check is that both type of consumers (old and new) do not want to coordinate and use vintage  $A$  in the first period. That would give old consumers a utility equal to  $f(A) + N$  in the first period. In the second period, as vintage  $B$  has not been sold, the monopolist would still retain his rights over that vintage, and so consumers would be extracted any additional surplus from switching to  $B$  (see first line of table 7). Therefore, total utility from buying vintage  $A$  in period 1 would be  $2f(A) + 3N$ . To ensure that this deviation is not profitable, we need to satisfy

$$2f(B) + 2N - p_{1B}^t > 2f(A) + 3N \quad (39)$$

Combining (37), (38) and (39) we have:

$$\begin{aligned} p_{1B}^t &= p_{1B} - c \\ &= 2f(B) - 2f(A) + 2N - \text{Max}\{\gamma - c + 3N, 3N\} \\ &= 2\gamma - N - \text{Max}\{\gamma - c, 0\} \\ &= \gamma - N + \text{Min}\{\gamma, c\} \end{aligned}$$

Total profits for this subgame are

$$\begin{aligned} \pi_{BB} &= (p_{1B}^t - c)N + (p_{1B} - c)N \\ &= [2(\gamma - N + \text{Min}\{\gamma, c\}) - c]N \end{aligned}$$

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