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Increasing returns and strategic behavior: the worker-firm ratio

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This article presents a model of an increasing returns economy in which each agent is allowed to choose his occupation: he can be a worker or an employer. It is shown that as the number of agents increases to infinity, the proportion of employers in the population approaches zero. A large economy can be a competitive economy, a natural oligopoly, or a natural monopoly, depending upon the asymptotic significance of scale economies. Replication does not eliminate the per capita *welfare loss due to imperfect competition in the natural oligopoly case. The asymptotic behavior of income per head and its functional distribution are also discussed.*

1. Introduction

■ Empirical research in industrialized countries has documented the following long-term trends.[1]

(1) The worker-firm ratio has increased over time.

(2) Output per head, labor productivity, and the real wage have increased over time.

(3) Labor's share of output has increased over time, while the profit rate has declined over time.

In this article, I present a general equilibrium model of an economy whose behavior is described by (1) through (3) above. I argue that the occurrence of these trends is mainly due to (a) increases in the size of the economy as measured by the total population, (b) the existence of increasing returns to scale in production, and (c) the use of strategic behavior both in production-investment decisions and in occupational choice decisions. The other major concern of this article is to describe the conditions under which a Cournot-Nash equilibrium exists in an economy with increasing returns to scale and the asymptotic efficiency properties of this equilibrium.

The principal motivation for studying these issues is that there currently is no model that can explain (1) through (3) simultaneously. The division of agents into employers and

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[1] For the evidence on the worker-firm ratio, see Kuznets (1966, pp. 186–193). For the magnitudes in (2), see Kuznets (1966, pp. 63–68 and 160–195). For the evidence on labor's share of output, see Kuznets (1966, pp. 160–195) and Organization for Economic Cooperation and Development (1984, pp. 74–75). The evidence on the profit rate is less conclusive, mainly because of measurement difficulties, and can be found in Gillman (1957), Hill (1979), and Wallich (1978). Samuelson (1978) discusses the reasons for a falling profit rate, of which his "capital deepening" is probably the most relevant to the approach of this article.

workers has been explained by postulating that (i) agents are price takers, (ii) that agents differ either in their ability to manage (Lucas, 1978) or in their willingness to bear risk (Kihlstrom and Laffont, 1979) or in their endowments (Roemer, 1982), and (iii) that the markets for managerial services, risk, or credit are missing. The explanation offered in this article differs from earlier ones in that agents are not price takers, agents are identical, and markets are complete.

The question of the existence and efficiency of a Cournot-Nash equilibrium is typically asked in a model in which returns to scale are postulated to be small (in various senses) relative to demand and in which agents are exogenously divided into a finite set of consumers and a countably infinite set of potential firms. (The number of active firms is determined by a zero-profit condition.) The typical result is that if production sets are bounded (unbounded), then the total (*per capita*) welfare loss associated with imperfect competition converges to zero as the economy replicates. In contrast, I assume that production sets exhibit the strongest possible form of global increasing returns (in a sense made precise later) and that each agent is allowed to choose his occupation. An existence result is presented, and production sets are classified according to the value of a parameter that measures the asymptotic significance of scale economies; as the value of this parameter decreases from infinity to zero, the asymptotic economy changes from a Pareto-efficient natural monopoly to an inefficient (in *per capita* terms) natural oligopoly. When the parameter is zero, the number of firms is infinite, and the *per capita* welfare loss is zero.

The main idea is to model the economic process as a two-stage game. In the first stage, each agent decides whether to be an employer or a worker; in the second stage, he decides on his production level, his production technique, his purchases, and his sales, taking his first-stage decision as given. The basic variable of the model is the worker-firm ratio, which is determined by the interaction of two opposing forces: increasing returns to scale, which favors production by only one agent, so the average production cost is minimized, and strategic behavior, which favors production by every agent and, therefore, minimizes the monopolistic or monopsonistic exploitation associated with non-price-taking behavior. In equilibrium, the gains from running one's own firm (avoiding monopolistic exploitation) are balanced by the efficiency losses that occur due to the fact that with many firms, the unit cost and, therefore, the price at any given output level are not as low as they could be. One can consider this mechanism as a general equilibrium extension of the result "excessive profits invite entry."

2. Description of the economy

■ The economy consists of n identical agents, one producible good, y, and labor, x. Each agent is endowed with a unit of labor. Labor is assumed to have no disutility,[2] so that the total labor supply is always equal to the number of agents, n, and each agent's utility function, u_i: $\mathcal{R}_+ \to \mathcal{R}$, can be set equal to the identity function without loss of generality. The technology of the economy is equally accessible to all agents and is described by the following menu of techniques:

$$x = \begin{cases} 0 & \text{if} \quad y = 0 \\ F + \Psi(F)y & \text{if} \quad y > 0, \end{cases} \tag{1}$$

where F ($1 < F < \infty$) represents the capital intensity, or plant size, under which y is produced. For any given F, the unit variable cost, $\Psi(F)$, is fixed and independent of the output level, y. A higher F implies a lower unit variable cost, or $\Psi'(F) < 0$. (Otherwise, firms would never consider increasing F.)

[2] Kihlstrom and Laffont (1979), Lucas (1978), and Roemer (1982) make this assumption, too.

The following restrictions on Ψ are assumed throughout the article. The first assumption excludes self-employment, i.e., ensures that producers always hire some labor.

Assumption 1. (a) $\Psi(F) = +\infty$ on $[0, 1]$

and (b) $\lim_{F \to 1+} \Psi(F) = +\infty$.

The following three assumptions guarantee that the minimization problem that defines the cost function, namely, $C(y) = \min\{F + \Psi(F)y: F \geq 0\}$, has a unique interior solution for all $y > 0$.

Assumption 2. $\Psi''(F) > 0, \quad 1 < F < \infty$.

Assumption 3. $\lim_{F \to 1+} \Psi'(F) = -\infty$.

Assumption 4. $\lim_{F \to \infty} \Psi'(F) = 0$.

The problem of maximizing total output subject to a labor constraint, namely, $\max\{y: F + \Psi(F)y \leq n, F \geq 0\}$, has an interior solution for all $n > 1$ only if the next assumption holds.

Assumption 5. $$\lim_{F \to 1+} \left(-F\frac{\Psi'(F)}{\Psi(F)}\right) = +\infty.$$

Finally, assume that the benefits from further division of labor are finite for large F.

Assumption 6. $$\lim_{F \to \infty} \left(-F\frac{\Psi'(F)}{\Psi(F)}\right) = \alpha < \infty, \text{ and } \lim_{F \to \infty} \Psi(F) = \Psi_\infty \geq 0.$$

It will be shown later that all the relevant information about the asymptotic properties of the production set is summarized by α. (In fact, $\alpha > 0$ implies that $\Psi_\infty = 0$.)

Note that the cost function, C, exhibits declining marginal costs everywhere. In fact, the degree of increasing returns assumed is the strongest possible given the regularity assumptions; a violation of Assumption 2 for some range of F would imply nonexistence of the cost function for some range of y. In particular, increasing returns here are stronger than those in Guesnerie and Hart (1985): they assumed that

$$\lim_{y \to \infty} [C(y) - yC'(y)] < \infty,$$

while here this limit is infinity.

Examples of functions that satisfy Assumptions 1 through 6 are, for $F > 1$,

$$\Psi(F) = (F-1)^{-\alpha} \qquad (\alpha > 0, \Psi_\infty = 0),$$

$$\Psi(F) = (\log F)^{-1} \qquad (\alpha = 0 = \Psi_\infty),$$

$$\Psi(F) = \Psi_\infty \frac{F}{F-1} \qquad (\alpha = 0, \Psi_\infty > 0),$$

with $\Psi(F) = +\infty$ for $F \in [0, 1]$.

3. The economic game and existence of equilibrium

■ The economic process is modelled as a game in two stages. The game will be solved via backwards induction, finding a Nash equilibrium for each subgame and letting the equilibrium payoff functions of the second stage be the payoff functions of the first stage.

□ **Second stage.** I need to determine the agents' payoffs as functions of the number of workers, m. To this end, let $\sigma : I \rightarrow \{1, 2\}$ be a function from the set of agents, $I = \{1, \ldots, n\}$, to the set of pure first-stage strategies, $\{1, 2\}$, where 1 is the strategy "be a worker" and 2, the strategy "be an employer"; σ assigns an occupation to each agent and is taken as given in this second stage. The number of workers is

$$m = m(\sigma) = \{\text{the number of } i \text{ such that } i \in I \text{ and } \sigma(i) = 1\}.$$

Markets are assumed to work as in the trading post model of Dubey and Shubik (1977). The supply of labor is always equal to the number of agents, n, because labor has no disutility. Each employer, j, makes a nominal bid for labor (including his own), l_j, and the nominal wage rate (paid by each firm), w, clears the market, where

$$w = \frac{\sum l_j}{n}. \tag{2}$$

The market for the final good works in exactly the same way. Each worker i makes a nominal bid for the final good, b_i, while each firm j makes an offer to produce q_j units of output. The actual price (paid by each consumer), p, clears the market, where

$$p = \frac{\sum_I b_i}{\sum q_j}. \tag{3}$$

Strategy sets are given by

$$S_i = \begin{cases} \mathcal{R}_+ & \text{if} \quad \sigma(i) = 1 \\ \mathcal{R}_+^3 & \text{if} \quad \sigma(i) = 2. \end{cases} \tag{4}$$

A typical worker chooses his bid for goods, b, while a typical firm (employer) chooses a bid for labor, l, a technique of production, F, his level of supply, q, and implicitly his consumption, c. (Firms do not buy the final good because they gain nothing by entering both sides of the same market; see Mas-Colell (1982a).)

Let $S = \prod_{i=1}^{n} S_i$ be the aggregate strategy set at stage 2, and denote by

$$s = (s_1, \ldots, s_i, \ldots, s_n)$$

a typical element of S; then, $s_i = b_i$ if $\sigma(i) = 1$, and $s_i = (l_i, F_i, q_i)$ if $\sigma(i) = 2$. For each strategy vector $s \in S$, those agents who have satisfied their budget constraints receive the real value of their choices (at market-clearing prices), while those who have violated their budget constraints receive nothing.

The utility function of a worker ($\sigma(i) = 1$) is

$$U_i(s) = \begin{cases} \dfrac{b_i}{p} & \text{if} \quad b_i \leq w \\ 0 & \text{otherwise,} \end{cases} \tag{5}$$

while that of an employer ($\sigma(i) = 2$) is

$$U_i(s) = \begin{cases} \dfrac{1}{\Psi(F_i)} \left[\dfrac{l_i}{w} - F_i \right] - q_i, & \text{if} \quad l_i \leq w + pq_i \\ 0 \quad \text{otherwise.} \end{cases} \tag{6}$$

Note that an employer earns revenue by selling both the final good (pq_i) and labor (w), while the worker earns only labor income (w). The employer's constraint dictates that he should not spend (l) more than he earns ($w + pq$).

An equilibrium is a strategy vector, $s^* \in S$, such that for any i and any $s_i \in S_i$

$$U_i(s^*) \geq U_i(s_i, s^*_{-i})$$

$$\text{where} \qquad s^*_{-i} = (s^*_1, \ldots, s^*_{i-1}, s^*_{i+1}, \ldots, s^*_n). \tag{7}$$

The only equilibria examined here are symmetric equilibria, in which agents of the same occupation play identical strategies. Symmetric equilibria are called trivial if all employers earn zero payoffs and are nontrivial otherwise.

Proposition 1. If s^* is a second-stage, nontrivial symmetric equilibrium (NSE) with m workers, then it is uniquely determined by the following equations:

(i) If $\sigma(i) = 1$, then $s^*_i = b^*_i$ and

$$\frac{b^*_i}{p^*} = \frac{n}{n-1}\frac{1}{\Psi(F_n(m))}\left[\frac{n-m-1}{n-m}\right]^2. \tag{8}$$

(ii) If $\sigma(i) = 2$, then $s^*_i = (l^*_i, q^*_i, F^*_i)$,

$$\frac{l^*_i}{w^*} = \frac{n}{n-m}, \tag{9}$$

$$q^*_i = \frac{n}{n-1}\frac{1}{\Psi(F_n(m))}\frac{m(n-m-1)^2}{(n-m)^3}, \tag{10}$$

and

$$\frac{p^*}{w^*\Psi(F_n(m))} = \frac{n-1}{n}\left[\frac{n-m}{n-m-1}\right]^2. \tag{11}$$

(iii) $F^*_i = F_n(m)$, where $F_n(m)$ is the unique solution of

$$h(F) \equiv F - \frac{\Psi(F)}{\Psi'(F)} = \frac{n}{n-m}. \tag{12}$$

Proof. See the Appendix.

The strategy described by Equations (8) through (12) will henceforth be denoted by (*). By (9), firms share the labor supply equally. By (11) and (12), increases in the market size of each firm, $n/(n-m)$, increase both the fixed cost, F, and the price-marginal cost ratio, $(p/w\Psi)$. By (11) again, the effect of $m/(n-m)$ on the relative price, p/w, is ambiguous: the (p/MC) ratio increases, but the unit variable cost, Ψ, decreases. This explains why, by (8) and (10), the effect of $n/(n-m)$ on the workers' payoffs and on supply is ambiguous.

Let $\omega_n(m)$ and $\pi_n(m)$ be the symmetric equilibrium payoffs of the workers and of the employers, respectively. By Proposition 1, $\omega_n(m) = \pi_n(m) = 0$ if a nontrivial, symmetric equilibrium does not exist. If one does exist, then $\omega_n(m) = \omega^*_n(m)$ and $\pi_n(m) = \pi^*_n(m)$, where

$$\omega^*_n(m) = \frac{n}{n-1}\frac{1}{\Psi(F_n(m))}\left[\frac{n-m-1}{n-m}\right]^2 \tag{13}$$

and

$$\pi^*_n(m) = \frac{1}{\Psi(F_n(m))}\left\{\frac{n}{n-m} - F_n(m) - \frac{n}{n-1}\frac{m(n-m-1)^2}{(n-m)^3}\right\}. \tag{14}$$

The following lemma characterizes those m for which an NSE exists.

Lemma 1. A second-stage, nontrivial, symmetric equilibrium with m workers exists if and only if $\pi_n^*(m)$ is strictly positive.

Proof. See the Appendix.

Sufficient conditions for existence are given by the following proposition.

Proposition 2.
(i) If $\alpha = 0$, then for each integer $k \geq 1$, there exists another integer, $N(k)$, such that $n \geq N(k)$ implies that a nontrivial, symmetric equilibrium with m workers exists for $n - k \leq m \leq n - 1$.
(ii) There exist functions $M(\alpha)$ and $K(\alpha)$ such that if $\alpha > 0$, a second-stage, nontrivial, symmetric equilibrium with m workers exists, provided that $n \geq M(\alpha)$ and $n - K(\alpha) \leq m \leq n - 1$. In fact, $K(\alpha)$ is the greatest integer strictly smaller than $(1 + \alpha)^{1/2}/[(1 + \alpha)^{1/2} - 1]$, and $M(\alpha)$ is an increasing function of α.

Proof. See the Appendix.

Lemma 1 shows that a NSE with m workers exists if all $n - m$ employers earn positive payoffs. Proposition 2 provides the sufficient conditions for this to happen: the size of the market must be sufficiently large and there must be sufficiently many workers. Note that if scale economies are asymptotically significant ($\alpha > 0$), then there is an upper bound on the number of employers that can be supported in equilibrium; this bound depends only on α. One would expect that any number of employers could earn positive payoffs if the market size were sufficiently large; this is indeed the case under asymptotically constant returns to scale ($\alpha = 0$). But, when $\alpha > 0$, increases in market size cause employers to increase their fixed costs, $F_n(m)$, at a rate that is proportional to the rate of increase of the market's size, and if there are "too many" employers, fixed costs can never become small enough relative to the market's size to allow positive payoffs.

To avoid trivialities, assume that a sufficiently large market can accommodate at least two firms.

Assumption 7. $\quad 0 \leq \alpha < \alpha_0 \equiv \sup \{\alpha \geq 0: K(\alpha) \geq 2\}.$

Assumption 7 means that scale economies are small asymptotically.

□ **First stage.** Each agent takes the functions $\omega_n(\cdot)$ and $\pi_n(\cdot)$ as given and decides whether to be an employer or a worker. Strategy spaces are denoted by $\Sigma_i = \{1, 2\}$, $i = 1, \ldots, n$. Let $\Sigma = X_{i=1}^n \Sigma_i$ be the aggregate strategy set, and let σ be a typical element of Σ. Each agent's payoff function is defined by

$$U_i(\sigma) = \begin{cases} \omega_n(m(\sigma)) & \text{if} \quad \sigma(i) = 1 \\ \pi_n(m(\sigma)) & \text{if} \quad \sigma(i) = 2. \end{cases} \tag{15}$$

An equilibrium is a strategy vector $\sigma^* \in \Sigma$ such that for any i and any $\sigma_i \in \Sigma_i$

$$U_i(\sigma^*) \geqq U_i(\sigma_1^*, \ldots, \sigma_{i-1}^*, \sigma_i, \sigma_{i+1}^*, \ldots, \sigma_n^*). \tag{16}$$

An equilibrium, σ^*, is nontrivial if $U_i(\sigma^*) > 0$ for all i. Define A_n: $[0, n) \rightarrow \mathcal{R}$ and A_n^*: $[0, n) \rightarrow \mathcal{R}$ by

$$A_n(x) = \pi_n(x) - \omega_n(x + 1) \tag{17}$$

and

$$A_n^*(x) = \pi_n^*(x) - \omega_n^*(x+1). \tag{18}$$

$A_n(x)$ is the incentive to become an employer when there are x workers. An agent who chooses to employ labor earns $\pi_n(x)$, while an agent who chooses to sell labor increases the number of workers by one and earns $\omega_n(x+1)$.

Hence, σ is first-stage equilibrium if and only if $m = m(\sigma)$ satisfies the following two conditions: (i) $A_n(m) \geq 0$ (no employer wants to become a worker) and (ii) $A_n(m-1) \leq 0$ (no worker wants to become an employer). The next proposition proves the existence of a nontrivial equilibrium using the following regularity assumption.

Assumption 8. A_n^* has a nonzero derivative at each and every one of its roots. (Roots are not critical points.)

Proposition 3. Suppose Assumptions 1 through 8 hold. Then, there is an increasing, integer-valued function, $N(\alpha)$, such that if $n \geq N(\alpha)$, a nontrivial first-stage equilibrium exists. There are at least two firms in any equilibrium.

Proof. See the Appendix.

The monopoly equilibrium ($m = n - 1$) occurs if and only if $\alpha > \alpha_0$. An economy with $\alpha > \alpha_0$ is called a natural monopoly.[3]

A first-stage equilibrium exists because of the opposing forces of increasing returns, which favor production by one firm, and strategic behavior (avoidance of monopolistic exploitation), which favors production by as many firms as possible.

4. Asymptotic properties of nontrivial equilibria

■ The propositions in this section are the theoretical counterparts of the trends cited in the introduction, provided that one thinks of the dynamic economy as a sequence of static, nonoverlapping economies that differ only with respect to the sizes of their populations. Another possibility is to consider an overlapping generations structure, restricting attention to equilibria in Markov strategies.[4]

To avoid pathologies, assume that if $n - m$ firms can earn positive payoffs at an NSE, a reduction in the number of firms will not reduce the payoffs below zero.

Assumption 9. If $\pi_n^*(m) > 0$, then $\pi_n^*(m+1) > 0$.

Assumption 9 holds if the equation $\pi_n^*(m) = 0$, $1 \leq m \leq n-1$, has at most one solution or if $\pi_n^*(m)$ is increasing in m; the latter is true if

$$\frac{(\Psi')^2}{\Psi\Psi''} < \frac{(n-m)(n-1) + 3m(n-m-1)}{(n-m)^2(n-1)}, \qquad 1 \leq m \leq n-1.$$

The left-hand side of this inequality equals $dF/d(n/n-m)$. Hence, Assumption 9 should be understood as an upper bound on the slope of the optimal technique, F, as a function of the firm's market size, $n/n-m$; ultimately, Assumption 9 bounds the speed of unit cost reductions caused by increases in fixed cost.

Proposition 4. The following hold true as $n \to \infty$.
(i) The ratio of workers to total population, m/n, converges to unity; or equivalently, the absolute size of each firm, $n/(n-m)$, diverges to infinity.
(ii) The fixed cost parameter of each firm, F, diverges to infinity.

[3] My use of this term is different than that of Baumol *et al.* (1982).
[4] This remark is due to an Associate Editor.

Proof. See the Appendix.

As market size, n, increases, the extent of the average cost reductions that are associated with the fuller exploitation of scale economies increases too. This strengthens the incentive to concentrate relative to the incentive to avoid monopolistic exploitation. Hence, the absolute size of each firm increases. Faced with a larger market, each firm chooses a technique involving a higher fixed cost and a lower unit variable cost.

In what follows, I need the following definition.

Definition 1. The profit rate is the ratio of money profits to money outlays of a firm; that is,

$$\frac{p\pi_n(m)}{l} \equiv \frac{\pi_n(m)}{w_n(m)} \cdot \frac{n-m}{n}.$$

Also, I will consider only those values of α for which

$$t(\alpha) = \frac{(1+\alpha)^{1/2}}{(1+\alpha)^{1/2}-1}$$

is not an integer. The (countably many) omitted values of α generate pathologies of little economic interest.

Proposition 5. The following hold true as $n \to \infty$.

(i) The number of firms converges to an integer, $x(\alpha) \geq 2$; $x(\alpha)$ is finite when $\alpha > 0$, and $x(0) = \infty = \lim_{\alpha \to 0} x(\alpha)$.

(ii) The fixed cost of each firm relative to the market size, (F/n), converges to $\frac{\alpha}{(1+\alpha)x(\alpha)}$; this limit is zero if and only if $\alpha = 0$.

(iii) The product per head converges to $\frac{1}{(1+\alpha)\Psi_\infty}$; this limit is infinity if $\alpha > 0$ because $\alpha > 0$ implies that $\Psi_\infty = 0$.

(iv) The real wage, $w_n(m)$, converges to $\frac{1}{\Psi_\infty}\left(\frac{x(\alpha)-1}{x(\alpha)}\right)^2$; this limit is infinity if $\alpha > 0$.

(v) The employer's payoff, $\pi_n(m)$, increases to infinity if $\alpha > 0$.

(vi) Labor's share of the total product converges to a (appropriately defined) number, $\lambda(\alpha) > 0$, such that $\lambda(\alpha) = 1$ if and only if $\alpha = 0$. The profit rate converges to $1/\lambda(\alpha) - 1$.

(vii) The price over marginal cost ratio, $\frac{p}{w\Psi(F)}$, converges to $\left(\frac{x(\alpha)-1}{x(\alpha)}\right)^2$; this limit is unity if and only if $\alpha = 0$.

Proof. See the Appendix.

Proposition 5 suggests the following terminology. If scale economies are asymptotically zero ($\alpha = 0$), the large economy is competitive: as market size increases to infinity, the number of firms increases to infinity; the size of the firm relative to the market's size, $(n-m)^{-1}$, converges to zero; the price over the marginal cost ratio converges to one; and the profit rate converges to zero. Hence, this is a case similar to those already discussed in the literature. (See Mas-Colell (1982a and 1982b) or Guesnerie and Hart (1985).) If the scale economies are asymptotically significant but not too large ($0 < \alpha < \alpha_0$), the economy is a natural oligopoly; as the market's size increases to infinity, the number of firms converges to a finite integer that is not less than two, the size of the firm relative to the market's size and the profit rate remain bounded away from zero, and the price over marginal cost ratio remains bounded away from one. Finally, Proposition 6 will show that the *per capita* welfare loss due to imperfect competition is bounded away from zero if $0 < \alpha < \alpha_0$.

To examine the asymptotic efficiency property of equilibria, I need the following definitions.

Definition 2. $Q_n = m\omega(m) + (n - m)\pi(m)$ is the total output in equilibrium.

Definition 3. $\bar{Q}_n = \max \{Q: F + \Psi(F)Q \leq n\}$ is the potential output.

Since labor has no disutility, an equilibrium is Pareto efficient if and only if $Q_n = \bar{Q}_n$. Let $S_n = \bar{Q}_n - Q_n \geq 0$ be a measure of the waste due to imperfect competition.

Proposition 6.
(i) If $\alpha > 0$, then $\lim_{n\to\infty} \sup Q_n/\bar{Q}_n < 1$; hence, the *per capita* waste, S_n/n, increases to infinity with the market's size, n.
(ii) If $\alpha = 0$, $\Psi_\infty > 0$, then $\lim_{n\to\infty} Q_n/\bar{Q}_n = 1$; hence, the *per capita* waste, S_n/n, converges to zero.

Proof. See the Appendix.

5. Concluding remarks

■ In this article, I have demonstrated that the trends presented in Section 1 can be generated as equilibrium outcomes in a model with identical agents who face complete markets but are not price takers. These equilibrium outcomes are all caused by the basic tension between increasing returns and strategic behavior. I have also shown that Cournot-Nash equilibria exist under stronger (in fact, the strongest possible) forms of increasing returns than those hitherto assumed in the literature and that if the benefits from division of labor are asymptotically significant but not too large and if each agent is allowed to choose his occupation, then Cournot-Nash outcomes are not asymptotically efficient in *per capita* terms.

Further research in the same direction would require extensions to multiproduct economies and more general market mechanisms. The issues of cooperation and coalition formation as determinants of the size of firms also needs to be explored.

Appendix

■ The proofs of Propositions 1, 2, 3, 4, 5, 6, and of Lemma 1 follow.

Proof of Proposition 1. Let s^* be an NSE. If i is a worker, $s_i^* = b_i^* = w^*$. If i is an employer, $s_i^* = (l_i^*, q_i^*, F_i^*)$ must solve

$$\max \frac{1}{\Psi(F_i)}\left[\frac{l_i}{w} - F_i\right] - q_i$$

subject to $l_i \leq w + pq_i$, $l_i, q_i, F_i \geq 0$.

At an NSE, employers earn positive payoffs. Hence, $l_i^* > 0$, $q_i^* > 0$, and $F_i^* > 1$. By substituting Equations (2) and (3) into the maximization problem, the Kuhn-Tucker conditions with respect to l_i, q_i, F_i are given by Equations (A1), (A2), and (A3), respectively. Equation (A4) is the budget constraint.

$$\frac{n}{\Psi(F_i^*)}\frac{\sum_{j\neq i} l_j^*}{(\sum_j l_j^*)^2} = \lambda_i \frac{n-1}{n}. \tag{A1}$$

$$\lambda_i \frac{(\sum_j b_j^*)(\sum_{j\neq i} q_j^*)}{(\sum_j q_j^*)^2} = 1. \tag{A2}$$

$$n = \frac{l_i^*}{\sum_j l_j^*} = F_i^* - \frac{\Psi(F_i^*)}{\Psi'(F_i^*)} \equiv h(F_i^*). \tag{A3}$$

$$l_i^* = \frac{\sum_j l_j^*}{n} + q_i^* \frac{\sum_j b_j^*}{\sum_j q_j^*}. \tag{A4}$$

Eliminating λ_i and exploiting symmetry, I obtain Equations (8) through (12). For example, (A3) yields

$$\frac{n}{n-m} = h(F_i^*),$$

which is Equation (12). As $h' > 0$, $F_n(m)$ is the unique solution of (A3). By (A4),

$$b^* = l^* \frac{n-m}{n}, \tag{A5}$$

and by (A1) and (A2),

$$q^* = \frac{n^2}{n-1} \frac{1}{\Psi(F^*)} \frac{m(n-m-1)^2}{(n-m)^4} \frac{b^*}{l^*}. \tag{A6}$$

From (A5) and (A6), I obtain (10); q^* is of course unique. The rest of the proof is left to the reader. *Q.E.D.*

Proof of Lemma 1. Suppose $\pi_n^*(m) > 0$, and let all agents except i play strategy (∗), described by Equations (8) through (12). It is shown that the unique best reply of firm i to (∗) is (∗) itself. Let $\Delta_i = \{(l, q, F) \in K_+^3 : F > 1, l \leq w + pq, 1/\Psi(F)\,[l/w - F] - q \geq 0\}$. By the assumption that $\pi_n^*(m) > 0$, Δ_i contains (∗). As Δ_i is bounded, the closure of Δ_i is a nonempty, compact set. Hence, the employer's maximization problem has a solution, (l, q, F), and that solution must yield a positive payoff, since (∗) does so. There exists, therefore, $\lambda_i > 0$ such that (A1) through (A4) hold with (l_i^*, q_i^*, F_i^*) replaced by (l, q, F). Consider (A3) for any $j \neq i$; j plays (∗) by assumption, so

$$n \frac{l^*}{l + (n-m-1)l^*} = h(F^*) = \frac{n}{n-m},$$

which clearly has the unique solution $l = l^*$. The unique solution of (A3) for i, then, is $F = F^*$. The rest is routine. *Q.E.D.*

Proof of Proposition 2. First, note that by (14), the condition $\pi_n^*(m) > 0$ is equivalent to

$$\frac{n}{n-m}\left[1 - \frac{m(n-m-1)^2}{(n-1)(n-m)^2}\right] > F_n(m).$$

Using (12), this inequality can be written as

$$e(F_n(m)) < \frac{(n-1)(n-m)^2}{m(n-m-1)^2} - 1, \tag{A7}$$

where $e(F) \equiv -F\Psi'(F)/\Psi(F)$ is the elasticity of the unit variable cost.

(i) If $\alpha = 0$, fix $k \geq 1$, and let the number of workers, m, satisfy $n - k \leq m \leq n - 1$. By Lemma 1, it suffices to show $\pi_n^*(m) > 0$ for all n sufficiently large. First, note that $n/(n-m) \geq n/k$ for all n, so as $n \to \infty$, $F_n(m) \to \infty$ and $e(F_n(m)) \to 0$. On the other hand, the right-hand side of (A7) is, for all n, greater than $(k/(k-1))^2 - 1 > 0$. Hence, there exists a function, $N(k)$, such that for all $n \geq N(k)$, (A7) holds true.

(ii) Let $K(\alpha)$ be the greatest integer strictly smaller than $(1+\alpha)^{1/2}/(1+\alpha)^{1/2} - 1$. By assumption,

$$n - K(\alpha) \leq m \leq n - 1.$$

Then, $n/(n-m) \geq n/K(\alpha)$ for all n; as in Part (i), $e(F_n(m)) \to \alpha$ as $n \to \infty$. On the other hand, the right-hand side of (A7) exceeds $(K(\alpha))/(K(\alpha)-1)^2 - 1$ for all n. From the construction of $K(\alpha)$,

$$\alpha < (K(\alpha))/(K(\alpha)-1)^2 - 1.$$

There is, therefore, an integer, $M(\alpha)$, such that for all $n \geq M(\alpha)$, (A7) holds. *Q.E.D.*

Proof of Proposition 3. The proof proceeds in several steps.

Step 1. There exists $N_1(\alpha)$ such that for each $n \geq N_1(\alpha)$, there is a number of workers, $2 \leq m \leq n-2$, such that $A_n^*(m-1) \geq 0$, $\pi_n^*(m) > 0$, and $A_n^*(m) \geq 0$.

Proof. Notice that, by (14), (13), and (18),

$$A_n^*(n-2) = \pi_n^*(n-2) - \omega_n^*(n-1) = \pi_n^*(n-2)$$
$$= \frac{F_n(n-2)}{\Psi(F_n(n-2))}\left[\frac{n}{2F_n(n-2)} - 1 - \frac{n}{8(n-1)F_n(n-2)}\right], \tag{A8}$$

where $F_n(n-2)$ is defined by (12) as the unique solution of

$$h(F) \equiv F - \frac{\Psi(F)}{\Psi'(F)} = \frac{n}{2}.$$

The term in brackets in (A8) converges to $1/\alpha$ as $n \to \infty$ because, by (12) and the fact that $h'(F) > 0$,

$$\lim_{n\to\infty} F_n(n-2) = \infty,$$

while, by (12) and Assumption 6,

$$\lim_{n\to\infty} \frac{n}{2F_n(n-2)} = 1 + \frac{1}{\alpha}.$$

Hence, there exists $M_i(\alpha)$ such that $n \geq M_i(\alpha)$ implies $A_n^*(n-1) > 0$.

On the other hand, by (13), (14), and (18),

$$A_n^*(2) = \pi_n^*(2) - \omega_n^*(3)$$
$$= \frac{F_n(2)}{\Psi(F_n(2))}\left\{\frac{n}{(n-2)F_n(2)} - 1 - \frac{n}{n-1}\frac{2(n-3)^2}{F_n(2)(n-2)^3} - \frac{n}{(n-1)F_n(2)}\left(\frac{n-4}{n-3}\right)^2\right\}, \tag{A9}$$

where, by (12), $h(F_n(2)) = n/(n-2)$. Given that $h'(F) > 1$ and that $h(1) = 1$, we have that $\lim_{n\to\infty} F_n(2) = 1$. But then the term in brackets in (A9) converges to -1 as $n \to \infty$. There exists, therefore, an integer, $M_2(\alpha)$, such that $n \geq M_2(\alpha)$ implies $A_n^*(2) < 0$. Let $N_1(\alpha) = \max\{M_1(\alpha), M_2(\alpha)\}$, and let $n \geq N_1(\alpha)$. Then, Assumption 8 implies that A_n^* has an odd number of roots in the interval $[2, n-2]$ and that A_n^* has positive slope at the greatest and the smallest of these roots because $A_n^*(n-2) > 0$, $A_n^*(2) < 0$, and the roots of A_n^* alternate in sign. Hence, for some m, $2 \leq m \leq n-2$, the interval $[m-1, m]$ contains an odd number of the roots of A_n^*, for otherwise A_n^* would have an even number of roots in $[2, n-2]$. For this m, $A_n^*(m) > 0$ and $A_n^*(m-1) \leq 0$. Finally, if this particular m equals $n-2$, $\pi_n^*(m) > 0$ by construction; if $m < n-2$, then

$$\pi_n^* = A_n^*(m) + \omega_n^*(m+1) \geq \omega_n^*(m+1) > 0.$$

Step 2. If m, where $m \in [2, n-2]$, is such that $A_n^*(m) \geq 0$, $A_n^*(m-1) \leq 0$, and $\pi_n^*(m) > 0$, then any $\alpha \in \Sigma$ with $m(\alpha) = m$ is a nontrivial, first-stage equilibrium.

Proof. Remember that the second-stage equilibrium payoffs, $\omega_n(m)$ and $\pi_n(m)$, equal $\omega_n^*(m)$ and $\pi_n^*(m)$ if a nontrivial equilibrium exists and that both equal zero if not. By assumption, $\pi_n^*(m) > 0$. By the same reasoning, we have

$$A_n(m) = \begin{cases} A_n^*(m) & \text{if} \quad \pi_n^*(m+1) > 0 \\ \pi_n^*(m) & \text{if} \quad \pi_n^*(m+1) \leq 0, \end{cases}$$

and

$$A_n^*(m-1) = \begin{cases} A_n^*(m-1) & \text{if} \quad \pi_n^*(m-1) > 0 \\ -\omega_n^*(m) & \text{if} \quad \pi_n^*(m-1) \leq 0. \end{cases}$$

In all cases, $A_n(m) \geq 0$ and $A_n(m-1) \leq 0$; so, any α with $m(\alpha) = m$ is a first-stage equilibrium.

Step 3. If $m = 0$ or $m = n$, then m cannot be the number of workers in a nontrivial equilibrium.

Proof. $\pi_n(0) = \omega_n(0) = 0 = \pi_n(n) = \omega_n(n)$.

Step 4. There exists $N(\alpha)$ such that for each $n \geq N(\alpha)$, the number of workers, m, at a nontrivial equilibrium cannot be $n-1$.

Proof. Let $N(\alpha)$ be such that $n \geq N(\alpha)$ implies $\pi_n^*(n-2) > 0$. Then,

$$A_n(n-1) = \pi_n^*(n-1) - \omega_n(n) = \pi_n^*(n-2) > 0$$

and $$A_n(n-2) = \pi_n^*(n-2) - \omega_n^*(n-1) = \pi_n^*(n-2) > 0;$$

so, $m = n - 1$ cannot be an equilibrium because this would imply $A_n(n-2) \leq 0$. *Q.E.D.*

Proof of Proposition 4. For each n, choose any equilibrium number of workers and call it $m(n)$. To show $\lim_{n\to\infty} m(n)/n = 1$, it suffices to show that any convergent subsequence $\langle y_n \rangle$ of the (bounded) sequence $\langle m(n)/n \rangle$ converges to one. By Lemma 1, $\pi_n^*(m(n)) > 0$; by Assumption 9, $\pi_n^*(m(n)+1) > 0$; so, $\pi_n(m(n)) = \pi_n^*(m(n))$, and $\omega_n(m(n)) = \omega_n^*(m(n))$. Hence, $A_n(m(n)) = \tau_n^*(m(n)) - \omega_n^*(m(n)+1)$. By the definition of equilibrium, $A_n(m(n)) \geq 0$; using the definition $y_n = n \cdot m(n)$, Assumption 1, (13), (14), (18), and passing to the appropriate subsequence, the condition $A_n(m(n)) \geq 0$ can be written as

$$\frac{1}{1-y_n} - F_n(m(n)) - \frac{n}{n-1}\frac{y_n}{1-y_n}\left[\frac{n(1-y_n)-1}{n(1-y_n)}\right]^2 \geq \frac{n}{n-1}\frac{\Psi(F_n(m(n)))}{\Psi(F_n(m(n)+1))}\left[\frac{n(1-y_n)-2}{n(1-y_n)-1}\right]^2;$$

by (12),

$$h(F_n(m(n))) = \frac{1}{1-y_n}$$

and $$h(F_n(m(n)+1)) = \frac{1}{1-y_n-\frac{1}{n}}.$$

Suppose, instead, that $y_n \to y < 1$ as $n \to \infty$. Then, $\lim_{n\to\infty} F_n(m(n)) = F = \lim_{n\to\infty} F_n(m(n)+1) < \infty$. Hence, by the inequality $\lim_{n\to\infty} A_n(m(n)) \geq 0$, I obtain $\frac{1}{1-y} - F - \frac{y}{1-y} \geq 1$, which is a contradiction. *Q.E.D.*

Proof of Proposition 5. We will only prove Part (i); the remaining results can be obtained from (i) and Equations (8) through (12). The proof proceeds in several steps.

Step 1. $A_n(m-1) \leq 0$ implies that $A_n^*(m-1) \leq 0$.

Proof.

$$A_n(m-1) = \begin{cases} \pi_n^*(m-1) - \omega_n^*(m) & \text{if} \quad \pi_n^*(m-1) > 0 \\ -\omega_n^*(m) & \text{if} \quad \pi_n^*(m-1) \leq 0. \end{cases}$$

Hence, in all cases, $A_n^*(m-1) \leq A_n(m-1)$.

Step 2. Let $\langle m(n) \rangle_{n=1}^{\infty}$ be any sequence of solutions of the inequalities $A_n(m) \geq 0$, $A_n(m-1) \leq 0$, and $\pi_n^*(m) > 0$ as n increases. Then,

$$A_n^*(m(n)-1) \leq 0 \tag{A10}$$

and $$\lim_{n\to\infty} \frac{m(n)}{n} = 1. \tag{A11}$$

Proof. (A10) is a direct consequence of Step 1; (A11) is proved in the proof of Proposition 4.

Step 3. If $\langle m(n) \rangle_{n=1}^{\infty}$ is a sequence that satisfies (A10) and (A11) and if $\alpha = 0$, then,

$$\lim_{n\to\infty} \langle n - m(n) \rangle = \infty.$$

Proof. Suppose not; then, $\langle n - m(n) \rangle$ contains a subsequence that converges to $t < \infty$. Without loss of generality, $\lim_{n\to\infty} (n - m(n)) = t$. Then, $A_n^*(m(n)-1) \leq 0$ yields, by (14), (13), and (18),

$$\frac{n}{(n-m(n)+1)F_n(m(n)-1)}\left\{1 - \frac{m(n)-1}{n-1}\left[\frac{n-m(n)}{n-m(n)+1}\right]^2\right\}$$
$$\leq 1 + \frac{n}{(n-1)F_n(m(n))}\left[\frac{n-m(n)-1}{n-m(n)}\right]^2. \tag{A12}$$

By (12), the definition of $F_n(m)$, and (A11), $\lim_{n\to\infty} F_n(m(n)-1) = \infty = \lim_{n\to\infty} F_n(m(n))$, and

$$\lim_{n\to\infty} \frac{n}{[n-m(n)+1]F_n(m(n)-1)} = \lim_{F\to\infty}\left(1 - \frac{\Psi(F)}{F\Psi'(F)}\right)$$
$$= 1 + 1/\alpha = \infty \qquad (\text{since } \alpha = 0).$$

Hence, by the above and the contradiction hypothesis, the left-hand side of (A12) converges to ∞, while the right-hand side converges to one, as $n \to \infty$.

Step 4. If $\langle m(n)\rangle_{n=1}^{\infty}$ is a sequence that satisfies (A10), (A11), and $\pi_n^*(m(n)) > 0$ and if $\alpha > 0$, then $\lim_{n\to\infty} (n - m(n)) = x(\alpha)$, where $x(\alpha)$ is the greatest integer smaller than $t(\alpha)$.

Proof. Assume, instead, that $\langle n - m(n)\rangle$ is unbounded. Then, it contains a subsequence that diverges to infinity. Without loss of generality, $\lim_{n\to\infty} (n - m(n)) = \infty$. By $\pi_n^*(m(n)) > 0$ and (14),

$$\frac{n}{(n-m(n))F_n(m(n))}\left\{1 - \frac{m(n)}{n-1}\left[\frac{n-m(n)-1}{n-m(n)}\right]^2\right\} > 1. \tag{A13}$$

By (A11) and the contradiction hypothesis, the term in brackets in (A13) tends to zero. By (A11), (12), and the definition of $F_n(m(n))$,

$$\lim_{n\to\infty} F_n(m(n)) = \infty \tag{A14}$$

and

$$\lim_{n\to\infty} \frac{n}{(n-m(n))F_n(m(n))} = \lim_{n\to\infty}\left(1 - \frac{\Psi(F)}{F\Psi'(F)}\right) = 1 + 1/\alpha < \infty \qquad (\text{since } \alpha > 0). \tag{A15}$$

Hence, the left-hand side of (A13) converges to 0, which is a contradiction; $\langle n - m(n)\rangle_{n=1}^{\infty}$ is therefore bounded. Consider any convergent subsequence of $\langle n - m(n)\rangle_{n=1}^{\infty}$, and let t be its limit. By assumption, $\langle m(n)\rangle_{n=1}^{\infty}$ satisfies (A10) and, therefore, (A12). By (A12), (A14), (A15), and by passing to the appropriate subsequence, I obtain, as $n \to \infty$,

$$(1 + 1/\alpha)\left(1 - \left(\frac{t}{t+1}\right)^2\right) \le 1, \tag{A16}$$

while by (A13), (A14), (A15), and passing to the appropriate subsequence I obtain, as $n \to \infty$,

$$(1 + 1/\alpha)\left(1 - \left(\frac{t-1}{t}\right)^2\right) \ge 1. \tag{A17}$$

By (A16) and (A17),

$$\left(\frac{t-1}{t}\right)^2 \le (1+\alpha)^{-1} \le \left(\frac{t}{t+1}\right)^2, t \ge 2, \qquad \text{or}$$

$$t(\alpha) - 1 \le t \le t(\alpha) \equiv \frac{(1+\alpha)^{1/2}}{(1+\alpha)^{1/2} - 1}. \tag{A18}$$

(A18) uniquely determines t because t is an integer while, by assumption, $t(\alpha)$ is not: $t = x(\alpha) \equiv$ the greatest integer smaller than $t(\alpha)$. I have shown that every convergent subsequence of the (bounded) sequence $\langle n - m(n)\rangle_{n=1}^{\infty}$ converges to $x(\alpha)$. This completes the proof of Part (i) of Proposition 5. *Q.E.D.*

For the rest, I need the following.

Lemma A1. If $\alpha > 0$, then $\Psi_\infty = 0$.

Proof. Let $\alpha > 0$; suppose that $\Psi_\infty > 0$. By the convexity of Ψ, for each $F > 1$ and $x > 1$,

$$\frac{\Psi(F)}{\Psi(x)} \ge 1 + \frac{x(-\Psi'(x))}{\Psi(x)}\left[1 - \frac{F}{x}\right].$$

Letting $x \to \infty$, I obtain $\Psi(F)/\Psi_\infty \ge 1 + \alpha$ for all $F > 1$. Letting $F \to \infty$, I obtain a contradiction. *Q.E.D.*

Proof of Proposition 6. Let $\alpha > 0$, and suppose that $(\Gamma_n, \bar{Q}_n)$ is a solution of society's optimization problem:

$$\max Q$$

$$\text{subject to } \gamma + \Psi(\gamma)Q \leq n.$$

Then, Γ_n and $\bar{Q}_n$ are uniquely determined by

$$h(\Gamma_n) = n \tag{A19}$$

$$\text{and} \quad \bar{Q}_n = \frac{n - \Gamma_n}{\Psi(\Gamma_n)}. \tag{A20}$$

Q_n is the total product in equilibrium. Hence, $Q_n = m(n)\omega_n(m(n)) + (n - m(n))\pi_n(m(n))$; or, by (13), (14), and (12),

$$Q_n = \frac{n - (n - m(n))F_n}{\Psi(F_n)} \tag{A21}$$

$$\text{and} \quad h(F_n) = \frac{n}{n - m(n)}. \tag{A22}$$

By (A20) and (A21),

$$\frac{Q_n}{\bar{Q}_n} = \frac{n - (n - m(n))F_n}{n - \Gamma_n} \frac{\Psi(\Gamma_n)}{\Psi(F_n)}. \tag{A23}$$

By (A19) and (A22), $\lim_{n\to\infty} \Gamma_n = \infty = \lim_{n\to\infty} F_n$.

By (A19),

$$\frac{\Gamma_n}{n}(1 + e(\Gamma_n)) = 1,$$

where $e(\Gamma) = -\Gamma \dfrac{\Psi'(\Gamma)}{\Psi(\Gamma)}$. Hence,

$$\lim_{n\to\infty} \frac{\Gamma_n}{n} = \frac{1}{1 + \alpha}.$$

Similarly, $\lim_{n\to\infty} \dfrac{F_n}{n} = \dfrac{1}{(1 + \alpha)x(\alpha)}$. Hence,

$$\lim_{n\to\infty} \frac{n - (n - m(n))F_n}{n - \Gamma_n} = 1. \tag{A24}$$

By the convexity of Ψ, for each n, $\Psi(F_n) \geq \Psi(\Gamma_n) + \Psi'(\Gamma_n)(F_n - \Gamma_n)$, or

$$\frac{\Psi(\Gamma_n)}{\Psi(F_n)} \leq \left[1 + \frac{\Gamma_n(-\Psi'(\Gamma_n))}{\Psi(\Gamma_n)}\left[1 - \frac{F_n}{\Gamma_n}\right]\right]^{-1}. \tag{A25}$$

By (A20) and (A22),

$$\lim_{n\to\infty} \frac{F_n}{\Gamma_n} = \frac{1}{x(\alpha)} \leq \frac{1}{\alpha}. \tag{A26}$$

By (A25) and (A26),

$$\limsup_{n\to\infty} \frac{\Psi(\Gamma_n)}{\Psi(F_n)} \leq \left[1 + \alpha\left[1 - \frac{1}{x(\alpha)}\right]\right]^{-1} \leq \left[1 + \frac{\alpha}{2}\right]^{-1} < 1. \tag{A27}$$

By (A23), (A24), and (A27),

$$\limsup_{n\to\infty} \frac{Q_n}{\bar{Q}_n} \equiv \delta < 1. \tag{A28}$$

By (A28), there exists K such that $n \geq K$ implies $Q_n \leq \delta\bar{Q}_n$; therefore, as $S_n = \bar{Q}_n - Q_n$,

$$\frac{S_n}{n} \geq (1 - \delta)\frac{\bar{Q}_n}{n}.$$

From (A19), (A20), and the fact that $\alpha > 0$ implies $\Psi_\infty = 0$ and $\lim_{n\to\infty} \frac{\Gamma_n}{n} = \frac{1}{1+\alpha}$, I get

$$\lim_{n\to\infty} \frac{\bar{Q}_n}{n} = \infty.$$

(ii) By (A23) and the fact that $F_n \to \infty$ as $n \to \infty$, $\Gamma_n \to \infty$ as $n \to \infty$. *Q.E.D.*

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