QUALITY OF INFORMATION AND OLIGOPOLISTIC PRICE DISCRIMINATION*

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Abstract

Recent developments in information technology (IT) have resulted in the collection of a vast amount of customer specific data. As the IT advances the quality of such information improves. We analyze a sequential spatial model of oligopolistic third degree price discrimination where the firms use the available information to classify the consumers into segments and charge each consumer group a different price. Higher information quality increases the number of identifiable consumer groups. Among our findings: i) when the information quality is low, a unilateral commitment not to price discriminate arises in equilibrium, but for high information precision such a commitment is a dominated strategy and the game becomes a prisoners' dilemma and ii) equilibrium profits exhibit a U-shape relationship with the information quality.

JEL Classification Codes: D43, L13, O30.

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1 Introduction

The rapid development of the internet as a medium of communication and commerce has enhanced the firms' ability to accumulate and store a vast amount of customer specific information. Recent

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advances in information technology (IT) and software tools, coupled with all this accumulated data, has taken price discrimination to a new level. The practice of dynamic pricing, where consumers pay different prices depending upon their demographics, purchasing history and income, is one prominent example.[1] Another example is coupon targeting. Many retailers ask the consumers when they visit their web page to register by divulging personal information such as their name, e-mail, address of residence (or zip code), age and family size. This, in turn, can facilitate the distribution of targeted coupons via e-mail with different face values depending upon each customer's willingness to pay as this is implied by his frequency of past coupon redemptions combined with his personal characteristics.[2] The IT also allows the sellers to keep track of consumers' purchase behavior over time and merge an increasing number of seemingly unrelated databases[3] which implies that the available customer data can be easily updated and refined. We posit a sequential spatial (Hotelling type) model of oligopolistic third degree price discrimination where the firms use the available information (acquired in stage 1 and modeled as a partition of the unit interval) to classify the consumers into segments by imperfectly estimating their *degree* of brand loyalty and charge each consumer group a different price (stages 2 & 3). Higher information quality leads to a refinement of the information partition. We focus on the firms' incentives to acquire customer specific information of a given level of quality as well as on the evolution of these incentives, profits, prices and welfare as the quality improves.

Economists' interest in price discrimination dates back to the work of Robinson (1933) who studied the issue in a monopolistic environment. Later work on the subject was still confined to a one seller market, [e.g. Schmalensee (1981) and Varian (1985)]. In a monopolistic situation, price discrimination leads to higher profits for the monopolist and to ambiguous consumer welfare

[1] All one has to do is type in google.com the phrase *dynamic pricing* and a plethora of relevant links will appear. Also, see the article "On the Web, Price Tags Blur: What You Pay Could Depend on Who You Are," washingtonpost.com, September 27, 2000. In the same article it is stated that: *Amazon, the largest and most potent force in e-commerce, was recently revealed to be selling the same DVD movies for different prices to different customers.* Bailey (1998) offers another example of behavior-based price discrimination: Books.com - a books retailer - adopted in early 1998 a price discrimination strategy where different buyers were paying different prices for the same item depending on their shopping behavior.

[2] See Rossi, McCulloch and Allenby (1996) and Allenby and Rossi (1999) who model consumer heterogeneity and develop statistical procedures, based on panel data on household purchase behavior, in order to estimate consumers' sensitivity on (among other things) targeted price promotions.

[3] One example is the *Abacus Catalog Alliance*, a database that contains transactional data with detailed information on consumer and business-to-business purchasing and spending behavior. It is a blind alliance of 1,800 merchants offering shared data representing over 90 million households and is the largest proprietary database of consumer transactions used for target marketing purposes [see http://www.doubleclick.com/us/].

results. This prediction does not necessarily carry over to imperfectly competitive markets. In an oligopolistic price discrimination model with symmetric firms Holmes (1989) shows that the consumer welfare result is still ambiguous. However, Corts (1998) and Shaffer and Zhang (2000) by relaxing the assumption of symmetric demand made by Holmes show that unambiguous price and welfare results may arise, where all consumers become better off as a result of being charged a lower price, while firms' profits decrease. Bester and Petrakis (1996) in a duopoly model with price discrimination obtain similar results, that is, the ability to charge different consumer segments different prices increases competition and reduces profits. When decisions about whether to discriminate and what prices to charge are made simultaneously, price discrimination is a dominant strategy and the game a prisoners' dilemma [e.g. Chen (1997), Corts and Shaffer and Zhang (1995)].[4] If, on the other hand, we consider a sequential game where firms can commit to a uniform price prior to the price setting stage, Corts proved that a unilateral commitment not to price discriminate arises in equilibrium.

The literature on price discrimination in spatial models has mainly assumed that firms either: i) have the ability to identify the location of each consumer *perfectly* [e.g. Anderson and de Palma (1988), Bhaskar and To (2001), Lederer and Hurter (1986), Shaffer and Zhang (2001), Thisse and Vives (1988) and Ulph and Vulkan (2000)], or ii) discriminate between only *two* groups of consumers [e.g. Bester and Petrakis, Chen, Fudenberg and Tirole (2000) and Shaffer and Zhang (1995 & 2000)]. Our main innovation is in extending the literature to analyze all the intermediate cases (including the two extreme ones). By varying the quality of information, we can obtain all levels of price discrimination and in the limit (as the number of identifiable consumer segments goes to infinity) we approximate the case of perfect discrimination. This modeling approach provides a more complete picture of the information acquisition incentives and the transition of the equilibrium variables and more importantly fits reality better, where the quality of consumer information that firms are using to develop their pricing strategies is far from perfect, but is constantly improving due to advances in IT.

In our model the information provides a credible commitment technology. If a firm does not

[4] Firms do not necessarily become worse off when they engage in price discrimination. Borenstein (1985) analyzes a circular model with free entry and shows that price discrimination, for a fixed number of firms, always leads to higher profits if consumers are sorted by their reservation prices. Shaffer and Zhang (2001) look at one-to-one promotions based on perfect information under the assumption that firms are *asymmetric*, and they show that the game (relative to the one price strategy) is *not* a prisoners' dilemma.

acquire information in stage 1, then it is not feasible to price discriminate. We show that for very low levels of information quality the game is not a prisoners' dilemma. Unilateral commitments to a uniform price arise in equilibrium [even with a zero cost of information], consistent with Cort's result. For high information quality, however, such a commitment is not an equilibrium. Acquiring information becomes each firm's dominant strategy resulting in lower profits than the ones obtained under a uniform pricing rule [prisoners' dilemma]. Consequently, we should not expect the firms to adopt policies which aim at limiting the practice of price discrimination, such as every day low pricing or no-haggle policy, if the firms possess fine enough information about their customers' preferences. Interestingly, the profits when both firms price discriminate are non-monotonic as a function of the information precision. In particular, they exhibit a U-shape pattern. This implies that initially better information intensifies the competition between the sellers, but eventually the surplus extraction effect prevails and firms become relatively better off when the information is refined. Our non-monotonicity result stands in stark contrast with the one obtained by Chen, Narasimhan and Zhang (2001) who, by employing a model à la Varian (1980) and Narasimhan (1988), showed that the firms' equilibrium profits exhibit an *inverted* U-shape as a function of the information accuracy (targetability). We elaborate more on this comparison in section 3.1.

Policymakers, and regulators have raised concerns that the increasing collection of information about consumers' shopping behavior may have detrimental effects on consumer welfare. Consumer groups and organizations are also concerned about the way that personal information collected from consumers or about consumers by third parties is used.[5] This paper attempts to highlight some of these issues.[6] We show that the consumer welfare has an inverted U-relationship with the information precision, implying that moderate information is the most beneficial for the consumers. After the peak of the consumer welfare, as the information quality improves consumers start paying (relatively speaking) higher prices. Hence, as far as the consumers' welfare is concerned, the regulatory authorities should create an environment which allows and fosters *only a limited* collection

[5] See, for instance, the Electronic Privacy Information Center's [www.epic.org, August 10, 2001] complaint against Microsoft, concerning Windows XP and Microsoft's ability to collect a huge amount of personal information which is allegedly unfair and leads to deceptive trade practices. See also the investigation that was launched by FTC against DoubleClick Inc., an internet advertising company, about whether, in collecting and maintaining information concerning Internet users, the firm has engaged in unfair or deceptive practices, ["DoubleClick target of FTC inquiry," USATODAY.com, June 7, 2000].

[6] Of course, the whole debate about consumer privacy and the issue of the proprietary rights of consumer information and how this information should be used is far more general than the specific approach we have taken in this paper.

and application of consumer information.

The rest of the paper is organized as follows. The model and the three stage game is presented in section 2. The game is analyzed in section 3 and section 4 offers a discussion on the main results derived in the previous section. We summarize in section 5. All proofs can be found in the appendix.

2 The description of the model

Two firms - 1 and 2 - located at the two endpoints of a unit interval sell competing brands to a continuum of customers who have unit demands and are uniformly distributed on $[0, 1]$. We assume that each consumer derives a benefit equal to V if he buys a product from either one of the firms. Let p_1 and p_2 be the prices that firm 1 and 2 charge respectively. Both firms' marginal costs are normalized to zero. In addition, each consumer incurs a linear unit transportation cost denoted by $t > 0$. Therefore a consumer who is located at point $x \in [0, 1]$ and buys from firm 1 enjoys a surplus of $V - tx - p_1$. Likewise, if he buys from firm 2 his surplus is $V - t(1 - x) - p_2$. Each consumer buys the product which gives him the highest positive surplus. We assume that V is sufficiently high, ensuring that each consumer will buy.[7]

So far we have implicitly assumed that firms have no information regarding the location of each consumer. All they know is that consumers are uniformly distributed on the unit interval. Now assume that (some) information regarding the location of each consumer becomes available. In practice, firms can obtain such information from a number of different sources, such as: i) directly through repeated past transactions with the customers, ii) via a telemarketing or direct-mail survey, iii) from credit card reports, or iv) from a marketing firm [see Shaffer and Zhang (2000 & 2001) for a more extensive discussion and more references on this issue]. The information partitions the interval into N sub-intervals (indexed by m, $m = 1, ..., N$) of equal distance and the firm who acquires this information has a better idea about the location of the consumers and their willingness to pay. In this case, a firm can charge different prices to different groups of consumers (by, say, distributing

[7] This is a standard assumption, made by most of the cited authors in this paper, which facilitates the exposition and the analysis.

targeted coupons via e-mail), though the price is the same within each group. Arbitrage between consumers is not feasible.

For simplicity and tractability, we further assume that $N = 2^k$, $k = 0, 1, 2....$ Hence, k will parameterize the information quality, with higher k's being associated with higher information precision (information refinement). We assume that an information of quality k is available to both firms at a cost equal to zero[8] and that the current state of technology dictates the quality of information k which the firms take as exogenously given.

One way to motivate our information structure is to view it as the outcome of a repeated interaction between buyers and sellers where the firms (or an information vendor who records these interactions) can classify the consumers into different groups based on their past purchasing behavior [e.g. purchasers and non-purchasers as in Chen, Fudenberg and Tirole and Villas-Boas (1999)]. The partition of consumers in period $t+1$ is a refinement of that in period t. In the above mentioned papers, however, the refinement of the information stops quickly as either there are only two periods (Chen and Fudenberg and Tirole) or the horizon is infinite but each consumer lives only for two periods (Villas-Boas) and hence the number of consumer segments is at most two. Our modeling approach can be viewed as a contribution to this strand of literature where the number of identifiable consumer groups is any number between one and infinity. On the other hand, for tractability, our model sidesteps any dynamic considerations (present in the above two papers) and assumes a very specific information refinement which is treated as exogenously given.

The three stage sequential game we consider unfolds as follows:

Stage 1: Given the quality of information k firms decide, simultaneously and independently, whether to acquire information or not.

[8] Of course, in practice the information "price" is not zero. For example, as Video Business in February 21, 2000 writes: "*Video retailers are looking to use their databases of customer information as an additional revenue stream, by selling those names to marketers. According to some analysts each name in those databases could be worth as much as $700.*" Our model determines how much each firm is willing to pay for the information and can be augmented to incorporate a stage where the information price is endogenously determined. This, however, will critically depend upon the assumptions we make about the information provision market structure (monopoly versus oligopoly information vendors) as well as the method by which the information is sold (fixed price versus royalties, exclusive versus non-exclusive provision etc.) and will add considerably to the length and complexity of our paper. We reserve this interesting extension for future research.

Stage 2: Firms simultaneously and independently choose their regular prices.

Stage 3: Firms (if they have acquired information) simultaneously and independently distribute targeted price promotions (discounts) to the consumer segments.

Our set up parallels the multistage games that have been examined in the literature [e.g. Banks and Moorthy (1999), Rao (1991) and Shaffer and Zhang (1995 & 2001)] where firms choose their promotional strategies *after* they have chosen their regular prices. This assumption is consistent with the common view that a firm's regular price can be adjusted slower than the choice of targeted coupons. In addition, if both decisions are made simultaneously no pure strategy equilibrium exists in the subgames where only one firm has information [see also Shaffer and Zhang (1995, footnote 11)].[9]

In the next section, we look for a subgame perfect equilibrium of this game.

3 Analysis

We solve the game backwards starting from stage 3 and proceeding to stage 1. We begin by analyzing the four subgames after the firms' information acquisition decisions in stage 1. The first subgame is when neither firm acquires information and they both set their regular prices in stage 2. Stage 3 is never reached in this case. The second subgame occurs after both firms have acquired information. In stage 2, they both choose their regular prices and promotions take place in stage 3. Finally, the third and the fourth subgames emerge when only one firm possesses information. Both firms set their regular prices in stage 2 and the firm with the information, after it observes the regular price of its rival, in stage 3 offers discounts. In Shaffer and Zhang (2001) firms have an incentive to choose a regular price in order to shelter their loyal customers from competitive poaching. This strategic role of the regular price is absent in our model. The reason behind this discrepancy is that in our model the firms incur the information cost (if any) in stage 1 and when they choose their promotional strategies this cost is irrelevant. This is not the case in Shaffer and Zhang (2001) where the targeting cost is not sunk when the firms make their targeting decisions.

[9] Proof is available upon request.

In light of the above discussion, it is equivalent (and reduces the notational burden) if we assume that a firm with information does not choose a regular price in stage 2. After having solved all four subgames, we proceed to stage 1 where the firms choose whether to acquire information or not.

3.1 Pricing decisions (Stages 2 & 3)

In this subsection, we solve for the equilibrium in each of the four subgames.

Subgame 1: Neither firm has information (NI,NI)

This is the standard Hotelling model under a uniform price. The firms choose their regular prices in stage 2. The demand of each of the two firms' products is given by,

$$d_1 = \frac{p_2 - p_1 + t}{2t} \text{ and } d_2 = \frac{p_1 - p_2 + t}{2t}.$$

It can be easily shown [e.g. Tirole (1988), p. 280] that in equilibrium the regular prices and profits are: $p_1 = p_2 = t$, and

$$\pi_1^{NI,NI} = \pi_2^{NI,NI} = \frac{t}{2}. \quad (1)$$

Subgame 2: Both firms have information (I,I)

Without loss of generality, we assume that the firms do not choose any regular prices in stage 2 and all the pricing decisions are made in stage 3. Since both firms have consumer information, they know in which of the $N = 2^k$ segments each consumer is located and therefore they are able to charge different prices for different segments. The interval $[0, 1]$ is equally divided into 2^k segments, each one having length of $1/2^k$. Segment m can be expressed as the interval $[(m-1)/2^k, m/2^k]$, where m is an integer between 1 and 2^k [see figure 1 below].

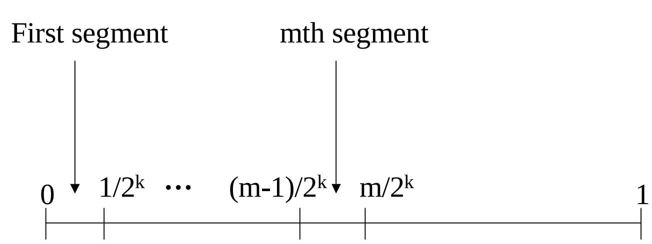

Figure 1: Partition of the unit interval

In segment m, firm 1 and 2 charge prices p_{1m} and p_{2m}, the demands of their products are,

$$d_{1m} = \frac{p_{2m} - p_{1m} + t}{2t} - \frac{m-1}{2^k} \text{ and } d_{2m} = \frac{m}{2^k} - \frac{p_{2m} - p_{1m} + t}{2t},$$

with d_{1m} and d_{2m} in $\left[0, 1/2^k\right]$ and their profits are,

$$\pi_{1m}(p_{1m}, p_{2m}) = p_{1m}d_{1m}, \text{ and } \pi_{2m}(p_{1m}, p_{2m}) = p_{2m}d_{2m}.$$

Firm i's, problem is,

$$\max_{p_{im} \geq 0} \pi_{im}(p_{1m}, p_{2m}), \text{ for each } m, \; m = 1, ..., N, \text{ and } i = 1, 2.$$

Let $\pi_1^{I,I}(k)$ and $\pi_2^{I,I}(k)$ denote the equilibrium profits, when both firms have information, as a function of the information quality. The next proposition summarizes the solution to the above problem.

Proposition 1 *Assume that both firms acquire information. Then, for each k ($k \geq 1$), there exist two thresholds (integers) m_1 and m_2 (with $2^k + 1 \geq m_2 > m_1 \geq 0$) where,*

$$m_2 = 2^{(k-1)} + 2 \text{ and } m_1 = 2^{(k-1)} - 1 \geq 0$$

such that:

i) [This case is valid only when $m_1 \geq 1$]. Firm 1's equilibrium demand is equal to $1/2^k$ in all segments from 1 to m_1, i.e., firm 1 is a constrained monopolist in these segments. Firm 2's equilibrium demand in these segments is zero. Moreover, firm 1's prices are: $p_{1m}^* = t(2^k - 2m)/2^k$, while firm 2 sets $p_{2m}^* = 0$, $m = 1, ..., m_1$.

ii) Both firms share[10] *the demand in the segments from $m_1 + 1$ to $m_2 - 1$. Moreover, firm 1's prices are: $p_{1m}^* = t\left(2^k - 2m + 4\right) / \left(3 \times 2^k\right)$, and firm 2's prices are: $p_{2m}^* = t\left(2m - 2^k + 2\right) / \left(3 \times 2^k\right)$, $m = m_1 + 1, ..., m_2 - 1$.*

iii) [This case is valid only when $m_2 \leq 2^k$]. Firm 2's equilibrium demand is equal to $1/2^k$ in all segments from m_2 to 2^k, i.e., firm 2 is a constrained monopolist in these segments. Firm 1's

[10] Here and in the remaining of the paper when we say "share" we *do not* necessarily mean a 50-50 division of the segment demand.

equilibrium demand in these segments is zero. Moreover, firm 2's prices are: $p^*_{2m} = t(2m - 2^k - 2)/2^k$*, while firm 1 sets* $p^*_{1m} = 0$*,* $m = m_2, ..., 2^k$*.*

Finally, the equilibrium profits of each firm as a function of k are:

$$\pi_i^{I,I}(k) = \frac{t(9 - 18 \times 2^{-k} + 40 \times 4^{-k})}{36}, \ i = 1, 2. \tag{2}$$

Proof. See appendix. ■

Two-way brand switching - where each firm poaches some of the other firm's loyal customers - occurs in equilibrium in the middle two consumer segments [that is, in segments $2^{(k-1)}$ and $2^{(k-1)} + 1$], independent of k. Consumers located on the left of these two segments buy exclusively from firm 1 and those on the right from firm 2. As $k \to \infty$ brand switching vanishes.

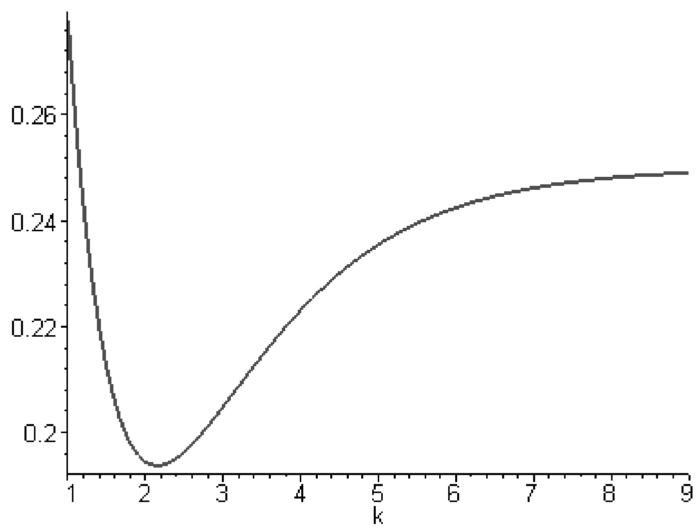

Figure 2: Profits in the subgame where both firms have information

The equilibrium profits exhibit a U-shape as a function of k [figure 2]. The intuition behind the non-monotonicity result will be best understood if we explore the movements of the reaction functions as the quality improves [see figure 3]. Let's begin by assuming that no information is available, i.e., $k = 0$. Firm 1's reaction function is, $p_1 = p_2/2 + t/2$, while firm 2's reaction function (after having solved for p_1) is: $p_1 = 2p_2 - t$. Both reaction functions are increasing and they intersect at the symmetric equilibrium price vector (t, t). The firms through their pricing strategies try to strike an optimal balance between gaining (losing) marginal consumers and losing (gaining) inframarginal rents. Now suppose that we move to the next information refinement, i.e., $k = 1$ and let's look at the first segment, i.e., $m = 1$. It can be easily seen that the reaction function of firm 1 remains unchanged, but that of firm 2 becomes: $p_1 = 2p_2$, that is, it shifts to the left. This is because the trade off between marginal consumers and inframarginal rents has not changed for firm 1, but it has for firm 2. The complete separation between markets allows firm 2 to charge two different prices, which makes it less concerned about sacrificing its inframarginal rents when it pursues a more aggressive pricing strategy in firm 1's own turf in an attempt to poach some of firm 1's loyal customers [the same is true for firm 1 in firm 2's territory]. Since the reaction functions are upward sloping, firm 1 reacts by lowering its price to induce the customers to stay, resulting in lower profits for both firms. When $k = 2$ firm 2's reaction function becomes: $p_1 = 2p_2 + t/2$. Firm 2, by applying the same logic, becomes even more aggressive in the first segment and prices fall further together with both firms' profits. For this k firm 2's equilibrium price is zero (marginal cost) and sells to no consumer in the first segment, while firm 1 chooses a price equal to $t/2$. From this particular point on the prices that firm 1 charges start to increase with k since it is now quite clear that firm 2 cannot attract any customer from the first segment. When $k = 3$, the equilibrium price vector is $(0, 3t/4)$. So if we look at the consumers who are located very close to firm 1, they initially face a decreasing sequence of prices but after a certain threshold the prices they pay increase with k, resulting in profits that are U-shaped. This line of reasoning can be extended to any segment (of course in the interior segments firm 1's reaction function changes as well) and the aggregation of profits over all segments yields a total profit function that also exhibits a U-shape. To sum up, there are two forces at work: i) the intensified competition effect and ii) the surplus extraction effect. When a segment of consumers is shared between the two firms an information refinement intensifies the competition and prices fall. This occurs for low levels of information quality. For high precision this fighting over consumers ceases as it now becomes more apparent which brand the consumers in a given segment prefer and the other firm cannot attract them even if it offers its product at marginal cost. Any further information improvements allow the firms to extract more surplus by raising their prices.

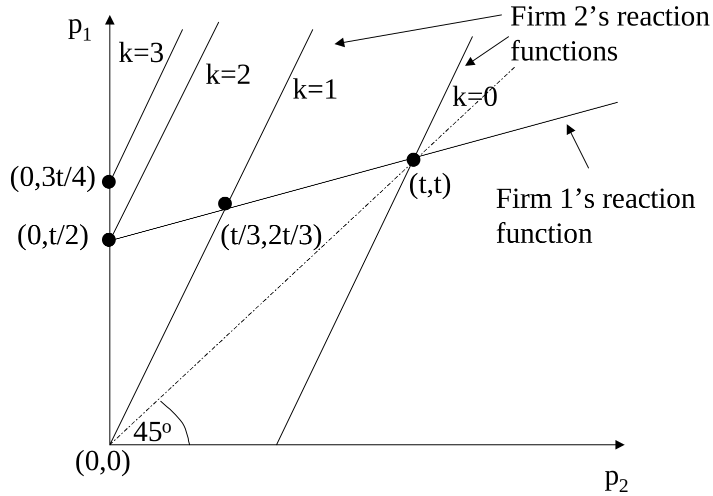

Figure 3: Reaction functions for the first segment ($m = 1$)

Comparison with Chen, Narasimhan and Zhang (2001)

Chen *et al.* predict that the equilibrium profits are inverse U-shaped as a function of the information accuracy. In their model there are only three groups of consumers: loyal to each firm consumers who do not compare prices and switchers who buy from the cheaper firm. Firms receive imperfect signals about the loyalty of each customer. At low levels of targetability profits increase as the accuracy improves. This is due to two factors: first the firms are able to extract more surplus from their loyal customers as now they can identify them better and second the competition between the two firms for the switchers is very soft since they cannot be clearly separated from the loyal customers. As the level of targetability improves profits eventually decrease because the consumers can be segmented more accurately and the ensuing intense competition for the switchers outweighs the surplus extraction benefits. In our model the relative strength of these two effects work in the opposite direction, as was illustrated in the preceding paragraph.

It is worth exploring these striking differences between the partition (that we assume) and the signal (Chen *et al.*) approaches a bit more. In a common value auction with n bidders Mattews (1984, p.191-192) showed that bidders' profits are inverse U-shaped as a function of the signal accuracy, similar to the result in Chen *et al.* Our conjecture is that the common value auction model can be extended to encompass spatial differentiation models with a continuum of consumers. The firms are the bidders who receive private signals about the location of each customer. This opens up new avenues for future research into the specific way information enters a market and how the different information formats affect the transition of the equilibrium profits. For instance, if consumers are categorized (by a marketing firm say) into groups and all firms acquire the same consumer databases which are updated regularly, then the partition approach seems more reasonable. If, on the other hand, firms conduct their own marketing research and receive potentially different signals about each consumer's willingness to pay and then charge him accordingly, then the signal approach is more plausible.

Next, we continue with our model.

<u>Numerical example 1</u>

Suppose $k = 3$, i.e., $N = 8$. Then $m_1 = 3$ and $m_2 = 6$, implying that firm 1 and 2 are constrained monopolists in the segments 1,2,3 and 6,7,8 respectively. In segments 4 and 5 both firms have strictly positive demands. The prices that firm 1 charges, starting from segment 1 are: $p_{11}^* = 3t/4$, $p_{12}^* = t/2$, $p_{13}^* = t/4$, $p_{14}^* = t/6$, $p_{15}^* = t/12$ and $p_{16}^* = p_{17}^* = p_{18}^* = 0$. Firm 2's prices are symmetric with the highest price in segment 8 and price equal to zero in segment 1. The equilibrium profits are $\pi_1^{I,I} = \pi_2^{I,I} = .2049t$.

<u>Perfect discrimination</u>

We digress from our approach so far and we assume that firms have perfect information about each consumer and consequently can charge each one a different price. We then use these results to compare the perfect discrimination case with the profits in (2) as $k \to \infty$.[11]

Each consumer is paying a different price, depending on his location. Consider the consumer who is located at $x < 1/2$. Both firms know his exact location and the equilibrium prices are $p_1^* = t(1 - 2x)$ and $p_2^* = 0$. Therefore, the consumer who is located at $x = 0$ pays $p_1^* = t$ for the product of firm 1, while firm 2 charges a zero price. The consumer who is located at $x = 1/2$ pays a zero price. Hence the average price that firm 1 charges is $t/2$ and firm 1's profits are equal to $t/4$. Since the problem is symmetric the same holds for firm 2.

[11] Technically speaking, our limit (as the number of subintervals goes to infinity) is not the same as the perfect price discrimination, where each one of the continuum of consumers pays a different price. In our limit we can generate all rational numbers in $[0, 1]$, but it is not the case that each consumer pays a different price. We do however, obtain the same equilibrium variables as in the perfect discrimination paradigm.

Now it can be easily checked that,

$$\lim_{k\to\infty} \pi_i^{I,I}(k) = \lim_{k\to\infty} \left[\frac{t(9 - 18 \times 2^{-k} + 40 \times 4^{-k})}{36}\right] = \frac{t}{4}.$$

Not surprisingly, our game approaches the perfect discrimination one as the number of identifiable consumer segments goes to infinity.

Subgames 3 & 4: Only one firm has information (I,NI) or (NI,I)

Due to symmetry let's assume that firm 1 is the firm who has information. Firm 2 chooses its regular price p_2 in stage 2 and firm 1 chooses its promotional prices p_{1m}, $m = 1, ..., 2^k$ in stage 3. We first solve firm 1's problem. Its demand in each segment is,

$$d_{1m} = \frac{p_2 - p_{1m} + t}{2t} - \frac{m-1}{2^k}, \text{ for } m = 1, ..., 2^k.$$

Given p_2 firm 1 chooses the depth of the discount that it offers to maximize,

$$\pi_1^{I,NI} = \sum_{m=1}^{2^k} p_{1m} d_{1m}.$$

Let $p_{1m}^*(k, p_2)$, $m = 1, ..., 2^k$ denote the solution to the firm's maximization problem. Now let's turn to firm 2's problem. Its demand in each segment is,

$$d_{2m} = \frac{m}{2^k} - \frac{p_2 - p_{1m}^*(k, p_2) + t}{2t}, \text{ for } m = 1, ..., 2^k.$$

Given the reaction function of firm 1, firm 2 chooses its regular price p_2 to maximize,

$$\pi_2^{I,NI} = p_2 \sum_{m=1}^{2^k} d_{2m}.$$

Let $\pi_1^{I,NI}(k)$ and $\pi_2^{I,NI}(k)$ denote the equilibrium profits when only firm 1 has information as a function of the information quality. The next proposition summarizes the properties of the solution. In the proof of this proposition we can no longer solve for the equilibrium in each segment separately, as we did in proposition 1, because firm 2 cannot treat the consumer segments independently due to its inability to charge more than one price. This adds to the difficulty and length of the proof significantly.

Proposition 2 *Assume that only firm 1 has acquired information. Then, for each* k $(k \geq 1)$*, there exist two thresholds (integers)* m_1 *and* m_2 *with (with* $2^k + 1 \geq m_2 > m_1 \geq 0$*) where,*

$$m_2 = 3 \times 2^{(k-2)} + 2 \text{ and } m_1 = 3 \times 2^{(k-2)} - 1, \text{ for } k \geq 2,$$

$$\text{and } m_1 = 0,\ m_2 = 3 \text{ for } k = 1$$

such that:

i) Firm 2's regular price is: $p_2 = t\left(1/2 + 2^{-(k+1)}\right)$.

ii) [This case is valid only when $m_1 \geq 1$*]. Firm 1's equilibrium demand is equal to* $1/2^k$ *in all segments from 1 to* m_1*, i.e., firm 1 is a constrained monopolist in these segments. Firm 2's equilibrium demand in these segments is zero. Moreover, firm 1's prices are:* $p^*_{1m} = t\left(3/2 + 2^{-(k+1)} - 2^{(1-k)}m\right)$, $m = 1, ..., m_1$.

iii) Both firms share the demand in the segments from $m_1 + 1$ *to* $m_2 - 1$*. Moreover, firm 1's prices are:* $p^*_{1m} = t\left(3/4 + (5/4) \times 2^{-k} - 2^{-k}m\right)$, $m = m_1 + 1, ..., m_2 - 1$.

iv) [This case is valid only when $m_2 \leq 2^k$*]. Firm 2's equilibrium demand is equal to* $1/2^k$ *in all segments from* m_2 *to* 2^k*, i.e., firm 2 is a constrained monopolist in these segments. Firm 1's equilibrium demand and prices in these segments are zero i.e.,* $p^*_{1m} = 0$, $m = m_2, ..., 2^k$.

Finally, the equilibrium profits of each firm as a function of k *are:*

$$\pi_1^{I,NI}(k) = \frac{t(9 - 6 \times 2^{-k} + 5 \times 4^{-k})}{16} \text{ and } \pi_2^{I,NI}(k) = \frac{t(1 + 2 \times 2^{-k} + 4^{-k})}{8}. \tag{3}$$

Proof. See appendix. ■

Here, unlike the case where both firms have information, two-way brand switching occurs in equilibrium only when $k = 1$, i.e., two identifiable consumer groups. For any $k \geq 2$, the market experiences a one-way poaching where only firm 1 steals some of firm 2's loyal customers, a situation that persists as $k \to \infty$.

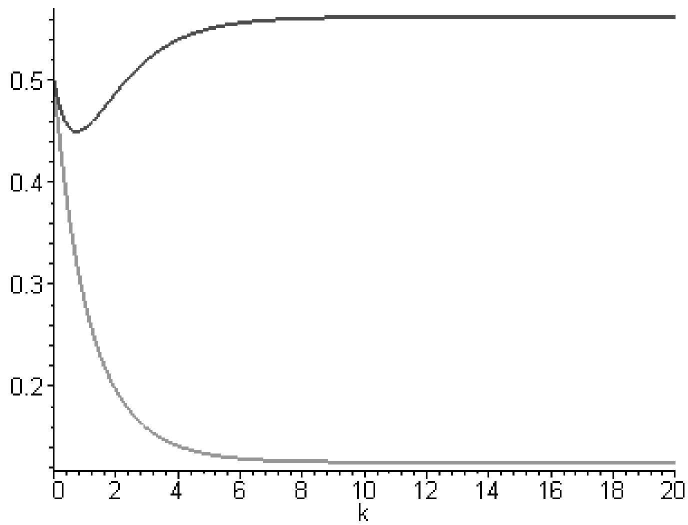

Figure 4: Profits in the subgame where only one firm has information

The profits of both firms are the same ($\pi_1 = \pi_2 = .5t$) when $k = 0$ (no information is available). Then, as k increases firms 2's profits monotonically decrease and they approach $.125t$ as the information tends to become perfect (i.e., $k \to \infty$). Firm 1's profits, however, are non-monotonic in k. For low information quality (that is, only for $k = 1, 2$) firm 1's profits with information are lower than the ones without information (even with a zero cost of information). This is because firm 2 is aware of the fact that firm 1 has credibly committed to price discriminate which forces firm 2 to follow a more defensive pricing strategy by lowering its regular price significantly. On the other hand, the quality of information is initially quite low and therefore firm 1 cannot take a full advantage of the information benefits. When the precision increases the surplus extraction effect becomes more dominant and firm 1's profits increase and they approach $.5625t > .5t$ as $k \to \infty$.

Numerical example 2

Suppose $k = 3$, i.e., $N = 8$. Then $m_1 = 5$ and $m_2 = 8$, implying that firm 1 and 2 are constrained monopolists in the segments 1,2,3,4,5 and 8 respectively. In segments 6 and 7 both firms share the demand. Firm 2's regular price is $p_2 = .5625t$. The prices that firm 1 charges, starting from segment 1 are: $p^*_{11} = 1.3125t$, $p^*_{12} = 1.0625t$, $p^*_{13} = .8125t$, $p^*_{14} = .5625t$, $p^*_{15} = .3125t$, $p^*_{16} = .15625t$, $p^*_{17} = .03125t$ and $p^*_{18} = 0$. The equilibrium profits are $\pi_1^{I,NI} = .5205t$ and $\pi_2^{I,NI} = .1582t$.

3.2 Information acquisition decisions (Stage 1)

The game played between the two firms in the first stage can be summarized in the following 2×2 matrix,

$F\ 1 \backslash F\ 2$	NI	I
NI	$\left(\frac{t}{2}, \frac{t}{2}\right)$	$\left(\frac{t(1+2\times 2^{-k}+4^{-k})}{8}, \frac{t(9-6\times 2^{-k}+5\times 4^{-k})}{16}\right)$
I	$\left(\frac{t(9-6\times 2^{-k}+5\times 4^{-k})}{16}, \frac{t(1+2\times 2^{-k}+4^{-k})}{8}\right)$	$\left(\frac{t(9-18\times 2^{-k}+40\times 4^{-k})}{36}, \frac{t(9-18\times 2^{-k}+40\times 4^{-k})}{36}\right)$

where the profits in the cells above have been taken from Eqs.(1), (2) and (3). As we have mentioned in section 2, we assume that the information is costless. The following proposition summarizes the equilibrium in the game.

Proposition 3 *When $k < 3$, "not to acquire information" (NI) is each firm's dominant strategy, while for $k \geq 3$, "to acquire information" (I) becomes the dominant strategy.*

The proof of the above proposition can be easily seen by comparing the profit functions and therefore is omitted.

When k is low both firms charge a uniform price to their customers. For high k's both firms engage in price discrimination facilitated by the consumer information.

4 Discussion

In this section, we evaluate the implications of information improvements for the firms' profits and consumer and social welfare.

Implications for firms

From proposition 3, firm i's equilibrium profits in the three-stage game are,

$$\pi_i(k) = \begin{cases} \frac{t}{2}, & \text{if } k < 3 \\ \frac{t(9-18\times 2^{-k}+40\times 4^{-k})}{36}, & \text{if } k \geq 3 \end{cases}.$$

Firms' profits exhibit a U-shape as a function of k [see figure 5]. More specifically, as k increases, profits are initially flat but at $k = 3$ they drop and then they increase with k, approaching asymptotically $.25t$. Thus, for $k \geq 3$, the game becomes a prisoners' dilemma. Despite the fact that a commitment device not to deviate from a uniform price (by not acquiring information) is technologically feasible, firms *do not* find it in their best interest to unilaterally utilize such a restraint if its quality is high. This complements Cort's finding [proposition 7, p.320] who showed that when firms can discriminate only between two consumer groups ($k = 1$ in our model), such unilateral commitments in stage 1 *do* constitute an equilibrium.

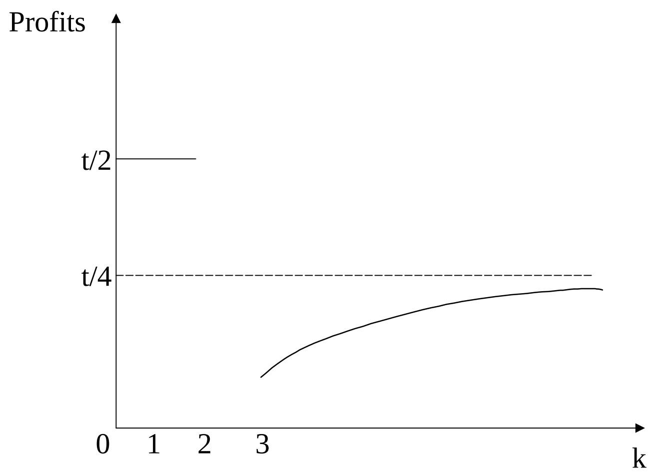

Figure 5 : Equilibrium profits in the three stage game

In practice, firms may make an attempt to soften the intensity of competition through various commitments such as the adoption of every day low pricing or a no-haggle policy [see Corts, section 4]. Our game can be slightly modified in order to provide new insights on this issue. Suppose that firms already possess consumer information of quality k. In stage 1 they decide whether to commit not to use the available information, by unilaterally announcing publicly the adoption of one of the above mentioned policies; and stages 2 and 3 are the same as in our game. Then, from our analysis so far, it follows easily that such commitments will prevail in equilibrium only for low information quality ($k < 3$). In any other case firms simply find it in their best interest to utilize their information and price discriminate.

Implications for consumers

The welfare implications for the consumers are as follows. Ignoring for the moment the transportation cost, consumers become better off compared to the no discrimination case, as the prices that each firm charges are uniformly below the non-discriminatory price t for any k. To see this consider for example firm 1 (the same holds for firm 2). When firm 1 is a constrained monopolist it charges a price equal to $t(2^k - 2m)/2^k \leq t$, $m = 1, ..., m_1$. A similar result holds in the segments where the two firms share the demand. The reason is that markets are treated asymmetrically by the two firms. Firm 1's strongest market (i.e., the group of consumers closest to firm 1) is firm 2's weakest market and so on. This is the so-called *best-response asymmetry* which is a necessary condition for price discrimination to yield unambiguous price effects [see Corts and Shaffer and Zhang (2000)].

The average price of each firm over the consumers who buy from this firm is 2 times its profit, i.e.,

$$AVP(k) = \frac{t(9 \times 2^{2k} - 18 \times 2^k + 40)}{18 \times 2^{2k}}.$$

Therefore, the average price that consumers have to pay also exhibits a U-shape pattern similar to that in figure 2. That is, even though all consumers pay lower prices when the firms price discriminate, the average price (after a given threshold) increases as the quality of information increases. In addition to the price, consumers also incur the transportation cost, which is calculated below.

Transportation cost: Due to symmetry we only calculate the average transportation cost of the consumers who buy from firm 1. The calculation is done as follows. In each segment m we find the transportation cost of the middle consumer among the ones who buy from firm 1, we multiply it by firm 1's demand in that segment, and we sum up over all segments that firm 1 has a strictly positive demand. By combining this with the average transportation cost incurred by the consumers who buy from firm 2, we obtain,[12]

$$AVTC(k) = \frac{t\left(9 \times 2^{2k} + 8\right)}{36 \times 2^{2k}}.$$

Hence as $k \to \infty$ the average transportation cost approaches $.25t$ which is the same as that in the (NI, NI) subgame. That is, in the limit every consumer buys from the closest firm.

Combining the average transportation cost with the average price $AVP(k)$ we can compute the average consumer surplus (given a sufficiently high V), which is,

$$AVCS(k) = V - \frac{t(27 \times 2^{2k} - 36 \times 2^{k} + 88)}{36 \times 2^{2k}}.$$

Moreover, when no information is available the average consumer surplus is $V - 1.25t$, which can be easily seen that it is always less than that when information is available for any k. The average consumer surplus for the entire game is,

$$AVCS(k) = \begin{cases} V - 1.25t, & \text{if } k < 3 \\ V - \frac{t(27\times 2^{2k} - 36\times 2^{k} + 88)}{36\times 2^{2k}}, & \text{if } k \geq 3 \end{cases}.$$

Consumer surplus exhibits an inverse U-shape as a function of k, implying that moderate levels of information yield the highest surplus. Therefore, consumers are better off when firms use information than when no information is used, but given that information is used there is an optimal (for the consumer welfare) level which is finite. After the peak of the consumer welfare, some consumers pay higher prices as information quality increases [see figure 6] and consumer welfare approaches asymptotically $V - .75t$.

[12] The file with the calculations is available upon request.

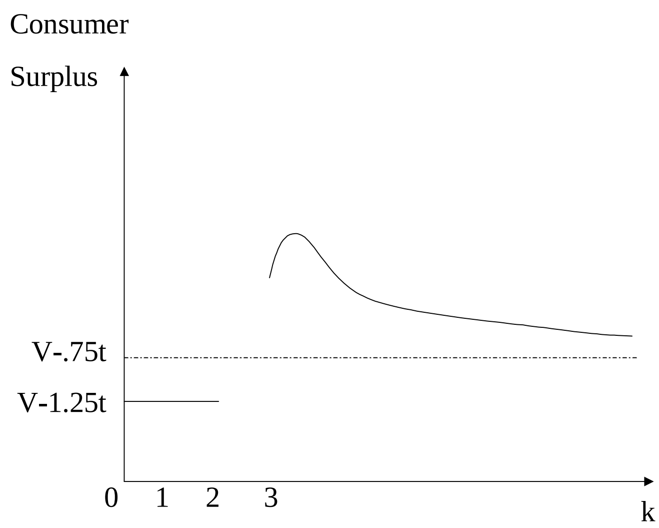

Figure 6: Consumer surplus

Social welfare

The average social welfare is $V - AVTC$, i.e., the value that consumers derive from buying a good minus the average transportation cost. The market outcome when no information is available and firms do not price discriminate is Pareto optimal (given the locations of the firms).[13] Hence, the availability of information cannot improve upon this outcome. Actually, for any $k > 0$ total welfare is lower than when $k = 0$. The reason is that the transportation cost is not minimized because as we show in proposition 1 the firms share the demand in the middle two segments, that is, there are consumers who do not buy from the firm they like most. For example, a consumer who is located at $x < 1/2$ (i.e., in firm 1's "back yard"), but in the right side of segment $m_1 + 1$, now is purchasing from firm 2 resulting in an unnecessary increase in transportation cost, which creates a dead-weight loss to the society. This is due to customer poaching, which monotonically decreases with k. To sum up, social welfare starts at $V - .25t$ when $k = 0$, it falls when k is 3 and then it increases approaching asymptotically $V - .25t$.

[13] An outcome is Pareto optimal when the transportation cost is minimized. When the locations of the two firms are exogenously fixed at 0 and 1 respectively, and no information is available, then each consumer buys from the nearest firm.

5 Conclusion

This paper evaluates the role of an improving information quality on the firms': i) incentives to acquire it, ii) equilibrium profits and iii) consumer and social welfare. We do so by analyzing a three stage game of spatial price discrimination. In the first stage the firms make their information acquisition decisions; in the second stage they choose their regular prices; and in the third stage they choose their promotional prices. The information (which is assumed to be costless) allows the sellers to group the customers into different segments according to their brand preferences and charge each segment a different price. An information improvement increases the number of identifiable segments. Our main results can be summarized as follows:

1. *Information acquisition decisions*: When the information quality is low, it is each firm's dominant strategy to credibly commit not to price discriminate. The game is not a prisoners' dilemma. After a certain threshold of information precision acquiring information with the intention to price discriminate becomes a dominant strategy resulting in a prisoners' dilemma.

2. *Equilibrium profits*: When both firms engage in price discrimination, their profits are U-shaped with respect to information improvements and always lower than the ones under a uniform price.

3. *Consumer and social welfare:* Consumer welfare exhibits an inverse U-shape as a function of the information quality, but it is always higher than the one under no price discrimination. Moderate information quality is the most beneficial for the consumers. Social welfare, on the other hand, is always lower when the firms price discriminate compared to the one price case, but it increases as the quality advances.

There are at least a couple of interesting extensions to the present set-up. First, we can endogenize the cost of information which in this paper is assumed to be zero. We can augment our game by introducing an information selling stage aiming at addressing two important issues: i) the determination of the information price as a function of its quality and ii) the level of quality that is optimally chosen. We can explore different possibilities beginning with a monopoly information vendor who chooses the optimal selling mechanism for his information of an endogenously chosen quality. We can also investigate the consequences of allowing for competition between vendors by introducing a second one who can also generate information which can either substitute or complement the one possessed by the first vendor. Second, the presence of information provides the firms with incentives to customize their products along with their prices. These incentives clearly should depend on the quality of the available information. The main structure of the present model can be used to address this issue.

APPENDIX

Proof of proposition 1

We conjecture the following structure. There exist two integers m_1 and m_2 with $0 \leq m_1 < m_2 \leq 2^k + 1$, such that: i) [*left segments*] firm 1 is a constrained monopolist in all segments from 1 to m_1 (if $m_1 = 0$, then firm 1 is never a constrained monopolist), ii) [*middle segments*] in all segments from $m_1 + 1$ to $m_2 - 1$ the two firms share (in some way) the segment demand and iii) [*right segments*] in all segments from m_2 to 2^k firm 2 is a constrained monopolist (again if $m_2 = 2^k + 1$) firm 2 is never a constraint monopolist. Next, we set out to prove that this structure indeed holds.

♦ Both firms charge strictly positive prices (middle segments).

Ignoring the nonnegativity constraints and setting $\partial\pi_{im}/\partial p_{im} = 0$, $i = 1, 2$, we obtain following solutions for the prices,

$$p_{1m} = \frac{t(2^k - 2m + 4)}{3 \times 2^k} \text{ and } p_{2m} = \frac{t(2m - 2^k + 2)}{3 \times 2^k}.$$

Using these prices we obtain the demands,

$$d_{1m} = \frac{-2m + 2^k + 4}{6 \times 2^k} \text{ and } d_{2m} = \frac{2m - 2^k + 2}{6 \times 2^k}.$$

We can see that d_{1m} is decreasing in m, and d_{2m} is increasing in m. This means that firm 1 may decide to charge a zero price and give the entire segment demand to firm 2 for segments that are in firm 2's territory. The same holds for firm 2 in segments that are in firm 1's territory. For segments in the middle of the interval both firms charge positive prices. Observe that $d_{2m} = \left(2m - 2^k + 2\right) / \left(6 \times 2^k\right) \leq 0$ for any $m \leq 2^k/2 - 1$ and $d_{1m} = \left(2^k - 2m + 4\right) / \left(6 \times 2^k\right) \leq 0$ for any $m \geq 2^k/2+2$. Now define $m_1(k)$ to be the highest integer that is less than equal to $2^k/2-1$, and $m_2(k)$ to be the lowest integer that is higher than or equal to $2^k/2+2$. Obviously,

$$m_1 = 2^{(k-1)} - 1 \text{ and } m_2 = 2^{(k-1)} + 2.$$

This will be used later in the proof. Hence, for any $m = m_1+1, ..., m_2-1$, both firms charge strictly positive prices and have strictly positive segment demand.

♦ Firm 1 charges strictly positive prices while firm 2 charges a zero price (left segments).

Following the analysis above, this case is valid for $m \leq m_1$. Then $d_{2m} \leq 0$. This implies that $d_{2m} = 0$ and $d_{1m} = 1/2^k$. This further implies that $p_{2m} = 0$, and p_{1m} is the solution to $d_{1m}(p_{2m} = 0) = 1/2^k$, which yields $p_{1m} = t(2^k - 2m)/2^k$.

♦ Firm 2 charges strictly positive prices while firm 1 charges a zero price (right segments).

This case is valid for $m \geq m_2$. This case is symmetric to case 2. Firm 2's prices in these segments are: $p_{2m} = t(2m - 2^k - 2)/2^k$. Below we summarize the results:

The equilibrium prices and profits are,

i) if $m_1 + 1 \leq m \leq m_2 - 1$ (middle segments),

$$\begin{aligned}
p_{1m} &= \frac{t(2^k - 2m + 4)}{3 \times 2^k} \text{ and } p_{2m} = \frac{t(2m - 2^k + 2)}{3 \times 2^k} \\
d_{1m} &= \frac{-2m + 2^k + 4}{6 \times 2^k} \text{ and } d_{2m} = \frac{2m - 2^k + 2}{6 \times 2^k} \\
\pi_{1m}^{I,I}(k) &= \frac{t(2m - 2^k - 4)^2}{18 \times 2^{2k}} \text{ and } \pi_{2m}^{I,I}(k) = \frac{t(2m - 2^k + 2)^2}{18 \times 2^{2k}},
\end{aligned}$$

ii) if $m \leq m_1$ (left segments),

$$\begin{aligned}
p_{1m} &= \frac{t(2^k - 2m)}{2^k} \text{ and } p_{2m} = 0 \\
d_{1m} &= \frac{1}{2^k} \text{ and } d_{2m} = 0 \\
\pi_{1m}^{I,I}(k) &= \frac{t(2^k - 2m)}{2^{2k}} \text{ and } \pi_{2m}^{I,I}(k) = 0,
\end{aligned}$$

iii) and if $m \geq m_2$ (right segments),

$$\begin{aligned} p_{1m} &= 0 \text{ and } p_{2m} = \frac{t(2m - 2^k - 2)}{2^k} \\ d_{1m} &= 0 \text{ and } d_{2m} = \frac{1}{2^k} \\ \pi^{I,I}_{1m}(k) &= 0 \text{ and } \pi^{I,I}_{2m}(k) = \frac{t(2m - 2^k - 2)}{2^{2k}}. \end{aligned}$$

Therefore, firms' profits for each k are,

$$\begin{aligned} \pi^{I,I}_1(k) &= \sum_{m=1}^{m_1} \frac{t(2^k - 2m)}{2^{2k}} + \sum_{m=m_1+1}^{m_2-1} \frac{t(2m - 2^k - 4)^2}{18 \times 2^{2k}}, \\ \pi^{I,I}_2(k) &= \sum_{m=m_1+1}^{m_2-1} \frac{t(2m - 2^k + 2)^2}{18 \times 2^{2k}} + \sum_{m=m_2}^{2^k} \frac{t(2m - 2^k - 2)}{2^{2k}}. \end{aligned}$$

By performing the summation $\left(\text{and using } m_1 = 2^{(k-1)} - 1 \text{ and } m_2 = 2^{(k-1)} + 2\right)$ we obtain,

$$\pi^{I,I}_i(k) = \frac{t(9 - 18 \times 2^{-k} + 40 \times 4^{-k})}{36}, \ i = 1, 2.$$

■

Proof of proposition 2

As in the proof of proposition 1, we conjecture the following structure. There exist two integers m_1 and m_2 with $0 \leq m_1 < m_2 \leq 2^k + 1$, such that: i) [*left segments*] firm 1 is a constrained monopolist in all segments from 1 to m_1 (if $m_1 = 0$, then firm 1 is never a constrained monopolist), ii) [*middle segments*] in all segments from $m_1 + 1$ to $m_2 - 1$ the two firms share (in some way) the segment demand and iii) [*right segments*] in all segments from m_2 to 2^k firm 2 is a constrained monopolist (again if $m_2 = 2^k + 1$) firm 2 is never a constraint monopolist. Next, we set out to prove that this structure indeed holds.

Let's begin by analyzing firm 1's problem given the regular price p_2 of the other firm. Firm 1's demand and profit function in segment m are,

$$d_{1m} = \frac{p_2 - p_{1m} + t}{2t} - \frac{m - 1}{2^k}$$

and

$$\pi_{1m}(p_{1m}; p_2, k) = p_{1m} d_{1m}.$$

♦ Left segments. For each $m = 1, ..., m_1$ firm 1's demand is: $1/2^k$ and the price in each one of these segments is,

$$p_{1mL} = p_2 + t - \frac{2tm}{2^k}.$$

For this to be firm 1's best response it must be that,

$$\left.\frac{\partial \pi_{1m}}{\partial p_{1m}}\right|_{p_{1m}=p_2+t-\frac{2tm}{2^k}} = \frac{2tm + 2t - 2^k p_2 - 2^k t}{2t2^k} < 0. \tag{A1}$$

By setting the above partial derivative equal to zero and solving with respect to m we obtain,

$$m^* = \frac{2^k p_2 + 2^k t - 2t}{2t}. \tag{A2}$$

For any $m < m^*$, (A1) is strictly negative. m^* however is not an integer (except for a specific p_2). Let $m_1(p_2, k)$ be the first integer that is less than m^*. This will be the last segment where firm 1 is a constrained monopolist.

♦ Right segments. In the right segments firm 1's demand and price is zero, i.e., $p_{1mR} = 0$. For this to be a best response it must be that,

$$\left.\frac{\partial \pi_{1m}}{\partial p_{1m}}\right|_{p_{1mR}=0} = \frac{2^k p_2 + 2^k t - 2tm + 2t}{2t2^k} < 0. \tag{A3}$$

By setting the above partial derivative equal to zero and solving with respect to m we obtain,

$$m^{**} = \frac{2^k p_2 + 2^k t + 2t}{2t}. \tag{A4}$$

For any $m > m^{**}$, (A3) is strictly negative. m^{**} however is not an integer (except for a specific p_2). Let $m_2(p_2, k)$ be the first integer that is greater than m^{**}. This will be the first segment where firm 2 is a constrained monopolist.

♦ Middle segments. The demands of both firms are strictly positive. Firm 1's best response is,

$$\frac{\partial \pi_m}{\partial p_{1m}} = 0 \Rightarrow p_{1mM} = \frac{2^k p_2 + 2^k t - 2tm + 2t}{2 \times 2^k}.$$

By considering all segments together, firm 1's profit function is,

$$\pi_1(p_2,k) = \sum_{m=1}^{m_1(p_2,k)} \frac{1}{2^k} p_{1mL} + \sum_{m=m_1(p_2,k)+1}^{m_2(p_2,k)-1} \left[\frac{p_2 - p_{1mM} + t}{2t} - \frac{m-1}{2^k} \right] p_{1mM} + \sum_{m=m_2(p_2,k)}^{2^k} 0. \quad \text{(A5)}$$

We continue by studying firm 2's problem. Its demand function in each segment is,

$$d_{2m} = \frac{m}{2^k} - \frac{p_2 - p_{1m}(p_2,k) + t}{2t}.$$

Firm 2 chooses p_2 to maximize,

$$\pi_2(p_2,k) = \left[\sum_{m=1}^{m_1(p_2,k)} 0 + \sum_{m=m_1(p_2,k)+1}^{m_2(p_2,k)-1} \frac{m}{2^k} - \frac{p_2 - p_{1mM}(p_2,k) + t}{2t} + \sum_{m=m_2(p_2,k)}^{2^k} \frac{1}{2^k} \right] p_2. \quad \text{(A6)}$$

We cannot solve for the optimal p_2 directly because we cannot obtain a closed form expression for $\pi_2(p_2,k)$. This is due to the fact that we do not have closed form expressions for $m_1(p_2,k)$ and $m_2(p_2,k)$. Recall that these two thresholds were found by integerizing (A2) and (A4) and they are actually step functions (as a function of p_2) which complicates the analysis. Nevertheless, we were able to circumvent this problem and obtain a closed form solution for p_2, p_{1m}, π_1 and π_2. We describe our approach next.

First, we solved the problem by assigning k a couple of specific numerical values, i.e., $k = 1, 2, 3$. This can be easily done using a standard software package (we used Maple). It cannot, however, be replicated for a general k. Given the solutions for p_2 we obtained, we then guessed the general form of p_2 as a function of k. Finally, we verified that this is indeed the (unique) solution.

Here we present in detail the steps outlined in the above paragraph. We set $t = 1$ and $k = 1, 2, 3$ and we maximized (A6) with respect to p_2. This yields, $p_2 = .75(k = 1)$, $p_2 = .625(k = 2)$ and $p_2 = .5625(k = 3)$ respectively. Our guess for the general form was,

$$p_2 = t\left(\frac{1}{2} + 2^{-(k+1)}\right). \quad \text{(A7)}$$

Given the general function for firm 2's price, we can also guess the general functions for m_1 and m_2. These are,

$$m_1 = 3 \times 2^{(k-2)} - 1 \text{ and } m_2 = 3 \times 2^{(k-2)} + 2, \text{ for } k \geq 2. \quad \text{(A8)}$$

When $k = 1$, $m_1 = 0$ and $m_2 = 3$. We plug m_1 and m_2 into (A6) and we perform the summation. This yields,

$$\pi_2(p_2, k) = \frac{p_2\left(2t + 2^{(1-k)}t - 2p_2\right)}{4t}. \tag{A9}$$

Notice that in (A9) m_1 and m_2 are not allowed to vary when p_2 varies. In other words, we assume that the structure does not change with p_2. This might be true for local changes of p_2. We differentiate (A9) with respect to p_2 and then we plug in (A7) to obtain,

$$\left.\frac{\partial \pi_2(p_2, k)}{\partial p_2}\right|_{p_2 = t\left(\frac{1}{2} + 2^{-(k+1)}\right)} = 0. \tag{A10}$$

If, i) local changes of p_2 (around the guessed solution) do not change the m_1 and m_2 and ii) $\pi_2(p_2)$ is strictly concave in p_2, then (A10) implies that (A7) is indeed the unique solution. Next, we prove these two conjectures.

By plugging p_2 (as given by (A7)) into m^* and m^{**} [as they are given by (A2) and (A4) respectively], we obtain,

$$m^* = \frac{\left(3 \times 2^k - 1\right)}{4} \text{ and } m^{**} = \frac{\left(3 \times 2^k + 5\right)}{4},$$

which are clearly never integers. Therefore, a small change in p_2 will not change m_1 and m_2. Finally, we prove that $\pi_2(p_2, k)$ is strictly concave in p_2 for any k.

We begin this proof by first noting (easy to see) that for any k, p_2 should always be within $[0, t]$. The $[0, t]$ interval can be divided into a number of subintervals with the following properties: i) any change of p_2 within any subinterval keeps the m_1 and m_2 unchanged and ii) a jump of p_2 from one subinterval to the next one leads to an increase of both m_1 and m_2 by 1 [see figure A1 where we have plotted m^*, m^{**} (the straight lines) and m_1, m_2 (the step functions)]. First, we show that $\pi_2(p_2, k)$ as given in (A6) is strictly concave, assuming that we vary p_2 without changing $m_1(p_2, k)$ and $m_2(p_2, k)$. The left segments ($m \leq m_1$) are irrelevant (since firm 2's demand is zero) and the right segments ($m \geq m_2$) are linear in p_2 [see (A6) with m_2 fixed]. Let's then look at the middle segments ($m_1 + 1 \leq m \leq m_2 - 1$) where,

$$\pi_{2m} = \frac{p_2\left(2tm - 2^k p_2 - 2^k t + 2t\right)}{4 \times 2^k t}$$

is the profit function in segment m which is clearly strictly concave. This is true for any m in the middle segments and therefore starting from any given m_1 and m_2, $\pi_2(p_2, k)$ is strictly concave provided that the structure does not change. Of course, the structure changes at certain values of p_2 [as figure A1 clearly illustrates]. It might be true that at these p_2's the profit function is not concave. Therefore, it remains to show that $\pi_2(p_2, k)$ is concave even when changes of p_2 alter the structure.

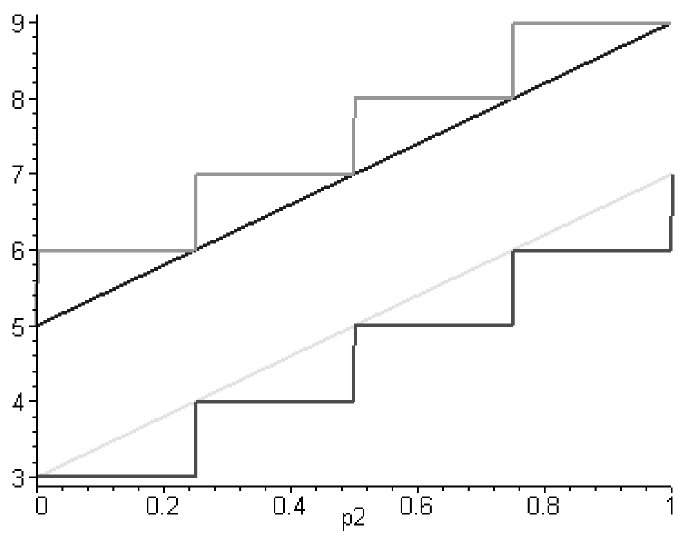

Figure A1: t=1, k=3

Choose a p_2 (say $\bar{p}$) such that m^* and m^{**} are integers (observe that m^* and m^{**} become integers simultaneously). Thus, $m^* = \tilde{m}_1$ and $m^{**} = \tilde{m}_2$. If p_2 increases slightly, then we move to $(\tilde{m}_1, \tilde{m}_2 + 1)$ and if p_2 decreases slightly, then we move to $(\tilde{m}_1 - 1, \tilde{m}_2)$ [consult the above graph again]. At $p_2 = \bar{p}$ the profit function is,

$$\pi_2(\bar{p}, k) = \left[\sum_{m=\tilde{m}_1+1}^{\tilde{m}_2-1} \frac{m}{2^k} - \frac{\bar{p} - p_{1mM}(\bar{p}, k) + t}{2t} + \sum_{m=\tilde{m}_2}^{2^k} \frac{1}{2^k} \right] \bar{p}.$$

Suppose p_2 increases by ε, i.e., $p_2 = \bar{p} + \varepsilon$. The profit function can now be written as,

$$\pi_2(\bar{p} + \varepsilon, k) = \left[\sum_{m=\tilde{m}_1+1}^{\tilde{m}_2} \frac{m}{2^k} - \frac{(\bar{p} + \varepsilon) - p_{1mM}((\bar{p} + \varepsilon), k) + t}{2t} + \sum_{m=\tilde{m}_2+1}^{2^k} \frac{1}{2^k} \right] (\bar{p} + \varepsilon).$$

Observe that the limit of the summation has changed after the price increase. The right derivative of the profit function at $p_2 = \bar{p}$ is,

$$\frac{\partial \pi_2(\bar{p}+)}{\partial p_2} = \lim_{\varepsilon \to 0+} \frac{\pi_2(\bar{p}+\varepsilon, k) - \pi_2(\bar{p}, k)}{\varepsilon} = \lim_{\varepsilon \to 0+} \frac{3 \times 2^{-k} t + t - 2\bar{p} - \varepsilon}{2t}.$$

Analogously, the left derivative is,

$$\frac{\partial \pi_2(\bar{p}-)}{\partial p_2} = \lim_{\varepsilon \to 0-} \frac{\pi_2(\bar{p}, k) - \pi_2(\bar{p}-\varepsilon, k)}{\varepsilon} = \lim_{\varepsilon \to 0-} \frac{3 \times 2^{-k} t + t - 2\bar{p} + \varepsilon}{2t}.$$

The left and right first derivatives are the same at any p_2 such that m^* and m^{**} are integers, i.e., $\partial \pi_2(p+)/\partial p = \partial \pi_2(p-)/\partial p$. This result coupled with the fact that π_2 is strictly concave for fixed m_1 and m_2 implies that firm 2's profit function is strictly concave in p_2.

By performing the summation in (A6), using (A8), we obtain a closed form expression for firm 2's profit function,

$$\pi_2(k) = \frac{t(1 + 2 \times 2^{-k} + 4^{-k})}{8}.$$

Firm 1's profit function from (A5) and using (A8) again is,

$$\pi_1(k) = \frac{t(9 - 6 \times 2^{-k} + 5 \times 4^{-k})}{16}.$$

■

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