ENDOGENOUS FIRM OBJECTIVES

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Abstract. We analyze the behavior of a monopolistic firm in general equilibrium when the firm's decision are taken through shareholder voting. We show that, depending on the underlying distribution, rational voting may imply overproduction as well as underproduction, relative to the efficient level. Any initial distribution of shares is an equilibrium, if individuals do not recognize their influence on voting when trading shares. However, when they do, and there are no short-selling constraints the only equilibrium is the efficient one. With short-selling constraints typically underproduction occurs. It is not market power itself causing underproduction, but the inability to perfectly trade the rights to market power.

1. Introduction

Under perfect competition, profit or net market value maximization of firms are derived from the goals of the shareholders, since it maximizes their wealth at a given price system. Moreover, the price normalization problem does not occur, since a complete system of relative prices is taken as given and it suffices to compare the values of different production plans.

Under imperfect competition, however, questions about the suitability and appropriateness of profit or net market value maximization arose early on. As for suitability, the lack of fairly general equilibrium existence results was a concern. Standard techniques turned out to have little impact in many instances, while non-existence was established in some other instances. As for appropriateness, the objective of profit or net market value maximization is questionable if firms exercise market power. In certain models, even the definition of profits is dubious because of the *price normalization* or *numéraire* problem. Moreover, shareholders often tend to disagree about

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the objectives the firm should pursue, and none of them may favor profit maximization.

Recently, advances have occurred in both suitability and appropriateness. The existence problem has been mitigated by novel results on the aggregation of demand. New insights regarding the proper objectives of firms have been gained by looking at the problem from different angles.

When firms exercise market power and maximize nominal profits, the price normalization has real effects as first pointed out by Gabszewicz and Vial (1972). Different real outcomes would then be obtained under different price normalization rules; see Grodal (1996), Haller (1986). Lately, Böhm (1994) and Dierker and Grodal (1996) have attempted to address or resolve this issue. A further issue is that when a firm has market power, net market value maximization may not be supported by the shareholders who often disagree on the objectives the firm should undertake. Thus the need to reconcile or aggregate shareholder interests arises. Shareholder voting may be the solution. This paper therefore introduces shareholder voting instead of postulating profit maximization.

The direction the literature has taken is to focus on the existence of shareholder voting equilibria. Sadanand and Williamson (1991) established existence of equilibria with shareholders voting in stock markets. DeMarzo (1993) has shown that in some cases where a voting equilibrium exist, the firm's production plan is optimal for the largest shareholder of the firm. In a general equilibrium model with certain externalities between production and consumption, Kelsey and Milne (1996) show the existence of a simultaneous equilibrium with competitive exchange in markets where consumers and producers are price-takers, but each firm's production decisions are determined by an internal collective choice criterion.

In this paper we analyze the impact of a distribution of share ownership on the behavior and efficiency of a monopolist in a general equilibrium framework, when the firm's decisions are taken through shareholder voting. Since the firm's decisions are taken through voting among its owners, we ensure consistency between preferences of the shareholders and the objective of the firm. In other words the objective function is endogenized. Therefore, the price normalization issue is never a question since an imperfectly competitive firm would by no means maximize profits. Moreover, imperfect competition generates *bad* outcome[1] if firms are profit maximizing. However, if firms are not profit maximizing, imperfect competition need not be *bad* as such. Whether imperfectly competitive firms need to be regulated

[1]For instance, a firm exercising monopoly power can raise its price above marginal cost. Such behavior leads to a price that is too high and to a dead-weight loss for society.

would depend on the distribution of shares in the economy. The reason the distribution of shares matters is that when the firm has market power it can alter the prices in such a way that redistribution among shareholders occurs, depending on the shareholders' endowments. If shareholders differ in their endowments they would support different production plans. The distribution of endowments would affect the identity of the median voter in the firm, and therefore affecting the firm's behavior.

Roemer (1993), in a related paper examining the role of distribution, models a situation in which a firm's production causes a negative externality. All individuals have the same preferences but differ in share endowments. The firm's production decisions are taken through shareholder voting. He shows that the more right-skewed the distribution of share ownership is, i.e., the poorer the median voter is relative to the average, the more production and the more of the externality the firm produces. Another related paper analyzing the distribution of share ownership is by Renstrom and Roszbach (1998). They analyze wage setting by a monopoly union, when union members own shares in the firm. Union members vote on the wage rate and the firm is a price taker. They reach similar conclusions to Roemer, that the more right-skewed the distribution of share ownership among union members the higher is the demanded wage rate and the higher is unemployment.

Most of the literature analyzes situations where share ownership is exogenous and there is no trade in shares. An exception is Geraats and Haller (1998) who have conducted a study to analyze the outcome of a single majority voting among shareholders of a single firm with one dimensional production decision. The asset market is effective by assumption and the safe asset is chosen to be the numéraire. As a result of their assumption on a stock market economy, a shareholder voting equilibrium (i.e., a median voter outcome in before-trade voting) exists and is essentially unique. They find that no sophisticated shareholder supports the production plan which maximizes the net market value of the firm. An investor's preferred production plan depends, as a rule, on his (initial or final) share holding and his risk aversion. Distributional assumptions regarding initial shareholdings and risk aversion parameters prove crucial for the median voter outcome.

Our paper differs from the previous papers in that we shall analyze the consequences of distribution and market structure for behavior and efficiency of a monopoly firm. The economy consists of a two-sector, three-good economy with Cobb-Douglas preferences. Heterogeneity among individuals are due to differences in shareholdings and initial endowments with labor and, at a later stage, in labor productivity.

First, we model a benchmark case when the monopoly firm acts as a perfect competitor, i.e., ignores its market power and behaves as a price taker, which we label the Competitive Economic Equilibrium (CEE). The CEE allocation is Pareto-optimal. We then consider two cases where one or the other consumption good serves as numéraire and also convex combinations of these two price normalizations. We show that the CEE allocation is not obtained in either of these cases, if the monopolist realizes its market power [Proposition 3.2].

As for consumers, a consumer prefers the monopolist to choose a higher/ equal/ lower output than the CEE level if and only if that consumer's endowment of shares is lower/ equal/ higher than her relative endowment of labor [Proposition 4.2]. This has an immediate implication for the case when the median voter determines the monopolist's production decision [Proposition 4.3]. In particular, if consumers are identical in their labor endowments and public ownership, then the CEE results [Proposition 4.4]. Two more results are derived when variation in labor endowments is replaced by variation in labor productivity.

Thus, when the shareholders realize that the firm has market power, rational voting may imply overproduction as well as underproduction, relative to the CEE. For a certain distribution of shares the CEE allocation is obtained. These are results for an exogenous distribution of shares. Finally, we deal with the issue of opening up the stock market, allowing individuals to trade their endowments. Since we have no risk present, the only reasons for trading share endowments are either to purchase a share that offers higher return than another or to strategically gain voting rights to influence the political outcome of the monopoly firm. This raises questions of to what extend individuals perceive themselves changing the decision of the firm, and what financial positions that are allowed. We therefore analyze two situations, one in which individuals do not recognize their influence on the political equilibrium in the firm when they trade shares, and one in which they do. When they do not recognize their influence, then any distribution of shares in an equilibrium. This is because share prices will be such that no one has an incentive to trade given the expectations about the voting outcome [Proposition 5.1]. When individuals recognize that trading in shares will alter the distribution of share ownership, and consequently the voting outcome, then in the absence of short-selling constraints the only equilibrium is the CEE allocation and all shareholders agree [Proposition 5.2]. Individuals then hold portfolios to match their endowments of labor and initial wealth. This result changes when short-selling constraints are introduced, and we are more likely to get underproduction in the monopoly sector [Proposition 5.3]. This leads us to a conclusion that it is not market power itself that causes underproduction, and consequently overpricing, but the inability to trade the rights to market power. These conclusions are in line with the *Coase Theorem* established by Coase (1960), which did not say anything about non-competitive economies, however.

The remainder of the paper is organized as follows. Section 2 provides the formal assumptions of the model, the equilibrium concept, and discusses the general strategy for modelling imperfect competition in a general equilibrium setting. Section 3 deals with the Pareto efficient equilibrium allocations (the CEE), the benchmark for the analysis in this paper. Section 4 endogenizes the objective of the firm through shareholder voting. Section 5 allows trade in shares prior to the voting stage. Finally, Section 6 offers some concluding remarks.

2. The Model

Consider an economy in which firms are distinguished between two types, perfect competitors and a single monopoly, so that there are two sectors, $(k = 1, 2)$. The former takes prices as given, while the monopoly firm observes that it can influence the price system in a given market. In the perfectly competitive sector a single commodity is produced by a continuum of firms, indexed by $j \in [0, 1]$. The aggregate output from the perfectly competitive sector will be denoted by y_1. The monopoly firm produces y_2. The profit in each sector is measured by

$$p_k y_k - \omega l_k,$$

where p_k is the price of commodity k, l_k is the labor used in sector k, and ω is the wage rate. We shall assume that labor, the only factor of production, is elastically supplied to the production sectors by consumers at their competitive prices.

There is continuum of heterogeneous consumers, indexed by $h \in [0, 1]$. Consumer h consumes x_1^h unit of commodity 1 and x_2^h unit of commodity 2. Each consumer is assumed to be a shareholder in both sectors. The fraction of the competitive sector owned by consumer h is denoted by θ_1^h and her share of the monopoly firm is given by θ_2^h. The consumers derive income from labor and the share ownership.

Assumption 2.1 (Consumer Characteristics)**.** *The consumers' preferences over consumption and labor are given by the Cobb-Douglas utility function, that is,*

$$u^h = x_1^h \left(x_2^h\right)^a \left(L^h - l^h\right)^b, \tag{2.1}$$

where $a, b > 0$ *and* L^h *is the total time available for each consumer. The budget constraint for consumer* h *is then given by*

$$p_1 x_1^h + p_2 x_2^h = \theta_1^h \left(p_1 y_1 - \omega l_1\right) + \theta_2^h \left(p_2 y_2 - \omega l_2\right) + \omega l^h. \tag{2.2}$$

Assumption 2.2 (Firm Characteristics). *Firm* j *in the competitive sector has a production function of the form:*

$$\begin{aligned} y_1^j &= F^j\left(l_1^j\right) = A^j\left(l_1^j\right)^{\alpha},\ l_1^j \geq \varepsilon_1^j \\ y_1^j &= 0,\ l_1^j < \varepsilon_1^j \end{aligned} \tag{2.3}$$

where $0 < \alpha \leq 1$ *and* $\varepsilon_1^j > 0$. *The production technology for the monopoly firm is given by*

$$\begin{aligned} y_2 &= G\left(l_2\right) = B\left(l_2\right)^{\beta},\ l_2 \geq \varepsilon_2 \\ y_2 &= 0,\ l_2 < \varepsilon_2, \end{aligned} \tag{2.4}$$

where $0 < \beta < 1$, *and* $\varepsilon_2 > 0$.

We have assumed minimum production levels in both sectors. For instance, if a firm wishes to produce less than $A^j(\varepsilon_1^j)^{\alpha}$ it must produce zero, i.e., close down. This is to avoid prices going to infinity in the limit. Exact conditions on ε_1 will be stated later on in Lemma 4.1.

2.1. **Economic Equilibrium for Given Monopoly Behavior.** Maximizing Equation (2.1) subject to (2.2) and taking prices as given yields the consumer's optimal decisions, that is,

$$p_1 x_1^h = \frac{1}{1+a+b}\left[\theta_1^h \left(p_1 y_1 - \omega l_1\right) + \theta_2^h \left(p_2 y_2 - \omega l_2\right) + \omega L^h\right], \tag{2.5}$$

$$p_2 x_2^h = \frac{a}{1+a+b}\left[\theta_1^h \left(p_1 y_1 - \omega l_1\right) + \theta_2^h \left(p_2 y_2 - \omega l_2\right) + \omega L^h\right], \tag{2.6}$$

$$\omega\left(L^h - l^h\right) = \frac{b}{1+a+b}\left[\theta_1^h \left(p_1 y_1 - \omega l_1\right) + \theta_2^h \left(p_2 y_2 - \omega l_2\right) + \omega L^h\right]. \tag{2.7}$$

Because of the Cobb-Douglas utility characterization, consumer h's expenditure share for each commodity is independent of income. This in turn implies *Linear Engel Curves*, which is a convenient property when dealing with consumer heterogeneity.

The market clearing prices in the economy are conveniently solved for by the market clearing conditions, that is,[2]

$$\int l^h \mathbf{d}h = l_1 + l_2 = l, \tag{2.8}$$

[2]Note that the symbol $\int$ always stands for aggregates, not for means.

$$\int x_1^h \mathbf{d}h = y_1, \tag{2.9}$$

$$\int x_2^h \mathbf{d}h = y_2. \tag{2.10}$$

Substituting Equations (2.8)-(2.10) into (2.5)-(2.7), one can obtain the market clearing prices:

$$\frac{p_2}{p_1} = a\frac{y_1}{y_2}, \tag{2.11}$$

$$\frac{\omega}{p_1} = b\frac{y_1}{L-l}, \tag{2.12}$$

$$\frac{\omega}{p_2} = \frac{b}{a}\frac{y_2}{L-l}, \tag{2.13}$$

where $L = \int L^h \mathbf{d}h$ is the total aggregate time available.

Therefore, the relative prices in the economy will be a function of the aggregate quantities produced and the aggregate labor used in the production process. A competitive firm would take these prices as given when making its production decisions, while the monopoly firm would realize that it can influence these prices. Using the price system in the exchange equilibrium the consumers' consumption decisions as a function of the produced quantities may be obtained. Hence, Equations (2.5) and (2.6) together with (2.11) yield

$$x_1^h = \psi^h\left(l_1, l_2\right) y_1, \tag{2.14}$$

$$x_2^h = \psi^h\left(l_1, l_2\right) y_2, \tag{2.15}$$

and Equation (2.7) together with (2.11) yield

$$L^h - l^h = \psi^h\left(l_1, l_2\right)\left(L - l\right), \tag{2.16}$$

where

$$\psi^h\left(l_1, l_2\right) = \frac{\theta_1^h\left(1 - b\frac{l_1}{L-l}\right) + \theta_2^h\left(a - b\frac{l_2}{L-l}\right) + b\frac{L^h}{L-l}}{1+a+b} \tag{2.17}$$

Remark 2.3. We shall note that this economy has linear sharing rules, where ψ^h is consumer h's share of each of the aggregate goods.

Since firm j is a price taker, it maximizes profits and solves

$$\max_{l_1^j} p_1 A^j \left(l_1^j\right)^\alpha - \omega l_1^j. \tag{2.18}$$

Remark 2.4. It is evident that there is no interior solution unless $\alpha < 1$. However, we can allow for the case when $\alpha = 1$. If $\alpha = 1$ the equilibrium wage must be $w/p_1 = \max_j A^j$. Only the firms with the largest productivity will operate. We still assume that there is a large number of those firms. When $\alpha = 1$ the wage rate normalized by Sector 1 price will be a constant and cannot be affected by the monopolist firm's decision. Our results for the rest of the paper still remain unchanged. This is important to notice, because our results do not come from manipulating w/p_1.

The aggregate labor demand and production in the competitive sector will then be[3]

$$l_1 = \frac{\alpha}{b+\alpha}\left(L - l_2\right), \tag{2.19}$$

$$y_1 = \left(\frac{\alpha}{b+\alpha}\right)^{\alpha} \tilde{A}\left(L - l_2\right)^{\alpha}, \tag{2.20}$$

where

$$\tilde{A} = \left(\int \left(A^j\right)^{\frac{1}{1-\alpha}} \mathbf{d}j\right)^{1-\alpha}. \tag{2.21}$$

If $\alpha = 1$ we take $\tilde{A}$ to be the aggregate productivity of the firms with $\max_j A^j$. In the rest of the analysis $\alpha = 1$ is possible.

The monopoly firm indirectly affects output and employment in the competitive sector by means of the variable l_2, the labor used in the imperfectly competitive sector.

3. Economic Equilibria under Exogenous Objectives of the Monopoly Firm

If the behavior of the monopoly firm is such that it chooses l_2 so as to maximize its profit, then there will be two benchmark cases. In the first case, the monopoly firm acts as a competitive firm and takes the price system in the economy as given. In the other case, the monopoly firm realizes its influence on the market prices and takes that into account when profit maximizing.

3.1. Monopoly as a Competitive Firm.

This case yields a Pareto efficient equilibrium outcome and will be the benchmark for the analysis in this paper. We shall label it *Competitive Economic Equilibrium.* The monopoly firm then chooses l_2 to solve

$$\max_{l_2} p_2 B \left(l_2\right)^{\beta} - \omega l_2. \tag{3.1}$$

Therefore, the economic equilibrium is given by [4]

$$l_1^* = \frac{\alpha}{b+\alpha+a\beta} L, \tag{3.2}$$

[3]See Appendix A.
[4]See Appendix B.

$$l_2^* = \frac{a\beta}{b+\alpha+a\beta}\, L, \tag{3.3}$$

$$y_1^* = \left(\frac{\alpha}{b+\alpha+a\beta}\right)^{\alpha} \tilde{A} L^{\alpha}, \tag{3.4}$$

$$y_2^* = \left(\frac{a\beta}{b+\alpha+a\beta}\right)^{\beta} B L^{\beta}. \tag{3.5}$$

Hence, Equations (3.2)-(3.5) completely describe the real equilibrium outcome in the economy.

Remark 3.1. The Competitive Economic Equilibrium is independent of the distribution of shares. This aggregation result follows from the linearity in the Engel curves. This holds for any additively separable or multiplicative HARA utility characterization.

3.2. **Monopoly Power.** When the monopoly firm is profit maximizing it does matter which price is used as a numéraire, that is, in which good profits are measured. If profits are measured in terms of good 1, then the following profit function is obtained:

$$\pi_1 = \frac{p_2}{p_1}\, y_2 - \frac{\omega}{p_1}\, l_2 \tag{3.6}$$

and if profits are measured in term of good 2, then the profit function becomes

$$\pi_2 = y_2 - \frac{\omega}{p_2}\, l_2. \tag{3.7}$$

Consider now a firm objective as a linear combination of π_1 and π_2, that is,

$$\max_{l_2} \lambda\pi_1 (l_2) + (1-\lambda)\, \pi_2 (l_2)\, . \tag{3.8}$$

Proposition 3.2. *When the monopoly firm realizes its influence on the price system, then there exists no weighted profit maximization rule (3.8) such that the Competitive Economic Equilibrium is reached.*[5]

Proof. Here we shall not give a formal proof of the Proposition (3.2), but rather sketch the proof.

Both functions π_1 and π_2 are concave in l_2. It is then sufficient to show that

$$\lambda\pi_1' (l_2^*) + (1-\lambda)\, \pi_2' (l_2^*) < 0,$$

where l_2^* is the competitive quantity as described by Equation (3.3).

[5]We shall note that there are other profit maximization rules rather than those of the form (3.8). First of all, labor could be used as a numéraire. Secondly, there are price normalization rules which are not convex combinations of the numéraire rules.

Since the monopoly firm takes into account the endogeneity of prices, Equations (2.11)-(2.13) and the profit functions may be written as

$$\pi_1 = y_1 \left(a - b \, \frac{l_2}{L-l} \right),$$

$$\pi_2 = \frac{y_2}{a} \left(a - b \, \frac{l_2}{L-l} \right).$$

Considering the behavior of the competitive sector characterized by Equations (2.19) and (2.20), the profit functions become

$$\pi_1 \left(l_2\right) = \left(\frac{\alpha}{b+\alpha} \right)^{\alpha} \tilde{A} \left(L - l_2\right)^{\alpha} \left(\frac{aL - \left(a+b+\alpha\right) l_2}{L - l_2} \right),$$

$$\pi_2 \left(l_2\right) = \frac{B}{a} \, l_2^{\beta} \left(\frac{aL - \left(a+b+\alpha\right) l_2}{L - l_2} \right).$$

Next, $\pi_1' \left(l_2^*\right) < 0$ and $\pi_2' \left(l_2^*\right) < 0$, which follow by evaluating the derivatives at the CEE quantity of l_2 in Equation (3.3). This completes the sketch of the proof. □

Remark 3.3. The economic equilibrium when the monopoly firm maximizes profit is independent of the distribution of shares. Hence, the aggregation property remains unchanged even when the monopoly firm uses its influence on the equilibrium prices.

Notice that when $\lambda = 1$ in (3.8), the maximizing $l_2 = \varepsilon_2 > 0$, the assumed minimum production level. Obviously, one may ask whether there is a non-linear price index, p_0, such that when

$$\pi_0 = \frac{y_2 p_2}{p_0} - \frac{l_2 w}{p_0}$$

is maximized the CEE is reached. It is easy to verify that for $p_0 = p_1^{1+\eta} p_2^{-\eta}$, where

$$\eta = \frac{(1-a)\alpha + (1+\alpha)\beta a + b}{(1-\beta)(b+\alpha+\alpha a)},$$

this is the case. It is clear that the monopolist needs to recognize the influence on p_0 with respect to l_2 for this to work. If an objective of maximizing profits in terms of p_0 is specified, individual shareholders will generally disagree upon which p_0 to use. The ideal price index for each individual does not take the simple form $p_0 = p_1^{1+\eta} p_2^{-\eta}$, so we cannot simply ask shareholders to express preferences over η. Instead, we will ask shareholders to express preferences over l_2, recognizing the general equilibrium price consequences. Eventually, we will define a shareholder voting equilibrium where l_2 will be determined. This is the topic of next section.

4. Endogenous Firm Objectives

Substituting the competitive sector's quantities (2.19) and (2.20) into (2.14)-(2.16), we obtain consumer h's consumption in terms of the monopoly firm activity l_2. If we substitute these quantities into (2.1), we obtain consumer h's indirect utility, that is,

$$V^h(l_2) = \Phi\left(\psi^h(l_2)\right)^{1+a+b}(l_2)^{a\beta}(L-l_2)^{\alpha+b}, \tag{4.1}$$

where

$$\psi^h(l_2) = \frac{\theta_1^h(1-\alpha) + \theta_2^h\left(a - (\alpha+b)\frac{l_2}{L-l_2}\right) + (\alpha+b)\frac{L^h}{L-l_2}}{1+a+b}. \tag{4.2}$$

We shall note that l_2 affects the consumers share of the aggregate output through two channels. First, through her share in the monopoly firm and second, through her time endowment. A decrease in l_2 increases profits but decreases the wage. The net effect will depend upon consumer h's endowment of shares θ_2^h relative to her endowment of potential work time. It can be easily seen that a decrease in l_2 plays a role of a *wage tax*. In order to make the net effect explicit we shall take the derivative of equation (4.2) with respect to l_2, which yields

$$\psi^{h'}(l_2) = \left(\frac{L^h}{L} - \theta_2^h\right)\frac{\alpha+b}{1+a+b}\frac{L}{(L-l_2)^2}. \tag{4.3}$$

Therefore, a change in consumer h's share ψ^h is increasing/ constant/ decreasing in Sector 2 activity if her share θ_2^h in monopoly firm is less/ equal/ greater than the population average.

In order to proceed further we need to know the properties of the individuals' indirect utilities (4.1). Define

$$m \equiv \frac{a\beta}{1+a+b},$$

$$n \equiv \frac{\alpha+b}{1+a+b},$$

$$\tilde{\theta}^h \equiv \frac{\theta_1^h(1-\alpha) + \theta_2^h(\alpha+a+b)}{1+a+b},$$

$$\Delta^h \equiv \frac{n}{\tilde{\theta}^h}\left(\frac{L^h}{L} - \theta_2^h\right),$$

and

$$\lambda_{1,2} \equiv \frac{n\frac{1+\Delta^h}{\Delta^h} - (1+m) \pm \sqrt{\left(n\frac{1+\Delta^h}{\Delta^h} - r_1\right)\left(n\frac{1+\Delta^h}{\Delta^h} - r_2\right)}}{2(1-n)},$$

where

$$r_{1,2} = 1 - m + 2\frac{m}{n} \pm \sqrt{4m\frac{(1-n)(m+n)}{n^2}}.$$

Notice that both m and n are positive and smaller than one, for $\alpha \leq 1$ and $\beta < 1$. Also, since indirect utility is only for individuals that can afford consuming at all, we only look at budgets that allow positive consumption. That is, we only consider θ_1^h, θ_2^h, and L^h such that $\psi^h(0) > 0$. This is equivalent to $\Delta^h > -1$, which is the lowest level of Δ^h to be considered in the analysis. Hence, the indirect utility (4.1) has the following properties:

Lemma 4.1. (**1**) *For $\Delta^h < 0$, $V^h(l_2)$ reaches a global maximum at $\frac{l_2}{L-l_2} = \lambda_1 < \frac{m}{n}$;* (**2**) *For $\Delta^h = 0$, $V^h(l_2)$ reaches a global maximum at $\frac{l_2}{L-l_2} = \lambda_1 = \frac{m}{n}$;* (**3**) *For $0 < \Delta^h \leq \frac{n}{r_1-n}$, $V^h(l_2)$ reaches a local maximum at $\frac{l_2}{L-l_2} = \lambda_2 > \frac{m}{n}$;* (**4**) *For $\Delta^h > \frac{n}{r_1-n}$, $V^h(l_2)$ is always increasing in l_2;* (**5**) *For $\Delta^h > 0$, as $l_2 \to L$, $V^h(l_2) \to +\infty$;* (**6**) *If*

$$\varepsilon \geq \frac{\alpha}{b+\alpha}\,\frac{2(1-n)L}{1-m-n+r_1-n}$$

then for $0 < \Delta^h \leq \frac{n}{r_1-n}$, $V^h(l_2)$ reaches a global maximum at $\frac{l_2}{L-l_2} = \lambda_2$.

Proof. See Appendix C. □

Lemma 4.1 characterizes the curvature of the indirect utility function for individuals with different share endowments, relative to their time endowments, Δ^h. For individuals with $\Delta^h < 0$ or $\Delta^h = 0$ the indirect utility function is concave and has one maximum, i.e., the individual have one ideal point (Parts 1 and 2 of Lemma 4.1). The only potential problem is for individuals with $\Delta^h > 0$, i.e., for individuals that have greater time endowments than their share endowments. Then the utility function approaches infinity as the monopoly firm employs virtually all labor available in the economy (Part 5 of Lemma 4.1). The reason for this is that the competitive sector's production approaches zero, which drives the price of the good produced by the competitive sector to infinity. This is due to the Cobb-Douglas preference specification. In order to have a well defined problem we introduced a smallest production unit in Sectors 1 and 2, (Equations 2.3 and 2.4, respectively). This makes it possible only to choose l_2 either in an interval, i.e., $l_2 \in [\varepsilon_2, L - \varepsilon_1]$, or zero.[6]

[6]That single peakedness fails for some individuals is not due to profits turning negative in the monopoly firm. To see this we proceed as follows. Profits are non-negative as long as (3.6), or alternatively (3.7), is non-negative. Using (2.11) and (2.12) in (3.6) gives non-negative profits if $a \geq bl_2/(L-l)$. Using this in sector 1's labor demand (2.19), and substituting for l_2 this condition can be written as $l_1 \geq \alpha L/(a+b+\alpha)$. Obviously, as $l_1 \to 0$, profits in Sector 2 turns negative. However, requiring non-negative profits does not rule out the possibility of very large prices in Sector 1. We can be in a situation where $l_1 = \varepsilon_1 > \alpha L/(a+b+\alpha)$, that is, profits in Sector 2 does not turn negative but the

If a shareholder has an interest in increasing production in the monopoly firm to drive the competitive sector to zero production, she has to consider either driving l_1 to zero or to its smallest unit ε_1. As utility is zero at $l_1 = 0$ (since there is no consumption of good 1), utility will be larger at $l_1 = \varepsilon_1$. If ε_1 is large enough, satisfying the condition in Lemma 4.1, then there are some shareholders with $\Delta^h > 0$ that will have a well defined global maximum (Part 6 of Lemma 4.1).

Proposition 4.2. *Consumer h prefers higher/ equal/ lower Sector 2 production than the Competitive Economic Equilibrium level if and only if her endowment of shares in the monopoly firm that is lower/ equal/ higher than her relative time endowment L^h/L.*

Proof. If the share endowment is smaller/ equal/ larger than the relative time endowment, then

$$\Delta^h > 0\ ,\ \Delta^h = 0\ ,\ \Delta^h < 0, \tag{4.4}$$

respectively. If $\Delta^h < 0$ or $\Delta^h = 0$, then V^h has a global maximum at

$$\frac{l_2}{L - l_2} = \lambda_1 < \frac{m}{n}\ ,\ \frac{l_2}{L - l_2} = \lambda_1 = \frac{m}{n}, \tag{4.5}$$

respectively, that is, lower than/ equal to the competitive equilibrium level $\frac{m}{n}$. If $\Delta^h > 0$ and ε_1 satisfies the condition in Lemma 4.1, then V^h has a global maximum at

$$\frac{l_2}{L - l_2} = \lambda_2 > \frac{m}{n}. \tag{4.6}$$

If ε_1 is smaller than the condition in Lemma 4.1, then some or all individuals with $\Delta^h > 0$ prefers

$$l_2 = L - \varepsilon, \tag{4.7}$$

that is, larger than the competitive equilibrium level. □

Proposition 4.2 emphasizes the distributional conflict in the economy. It is only when the consumer's share of the aggregate quantities is unaffected by the production level in Sector 2, she wishes the competitive outcome. In all other cases the consumer gains from redistributive consequences of using monopoly power.

Rather than voting directly on the firm's production decision, we will assume that shareholders vote on candidates taken from the group of shareholders, and the majority elected candidate will implement her preferred production decision. We then truncate the policy space to values of l_2 that

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constraint on smallest production unit is binding. Requiring non-negative profits is not enough to guarantee single-peaked preferences.

are ideal points of shareholders. This is necessary because preferences are not single peaked in l_2 for individuals with $\Delta^h > 0$ (see Lemma 4.1). However, preferences are single peaked in l_2 when l_2 is restricted to be an ideal point of *some* shareholders and the assumption regarding ε_1 in Part 6 of Lemma 4.1 holds (see Lemma D.1 in Appendix D).

Now we are almost in the position where we can apply the median-voter theorem. There is one more complication, however. If the monopoly firm is not a co-operative, then voting rights are typically proportional to the number of shares a shareholder owns. This implies that the median voter is not the individual with the median Δ. This causes no problem, because differential voting rights just alter the distribution. For example, take an initial distribution of Δ^h. We can then find a new distribution over Δ^h to identify the median voter in the following way. An individual with n times as many shares as another individual will enter the new distribution n times. In this way a median voter is found as the individual cutting the new distribution in half. We can then apply the median-voter theorem, since the candidate preferred by the median voter in the firm cannot lose against any other candidate. We will take this median-voter equilibrium as our political equilibrium.

We shall therefore define a *Shareholder Voting Equilibrium* as the production decision taken by a candidate decision maker who cannot lose against any other candidate in a binary election (the electorate being the shareholders), assuming that all shareholders costlessly can stand as candidates. The candidate decision maker, whose production decision is implemented, is referred to as the *Median Voter*.

Proposition 4.3. *Suppose all consumers have the same time endowment and that the restriction on ε_1 in Lemma 4.1 holds, then in a Shareholder Voting Equilibrium, the production in Sector 2 is higher/ equal/ lower than in the Competitive Economic Equilibrium if the Median Voter owns a proportion of shares in the monopoly firm that is less/ equal/ higher than the inverse of the population size.*

Proof. The firm's production decision will be taken by the median shareholder. The rest follows from Proposition 4.2. □

We shall now give examples of distributions of shares for which we have underproduction as well as overproduction relative to the Competitive Economic Equilibrium. First, consider a continuous differentiable distribution function, $\Gamma(\theta_2)$, given the number of individuals owning a share θ_2 or less. Suppose for simplicity that the individuals own shares between 0 and $\hat{\theta}_2$, then $\Gamma\left(\hat{\theta}_2\right) = L$ according to our notation. In the aggregate all of the firm is owned, therefore

$$\int_0^{\hat{\theta}_2} \theta_2 \Gamma'(\theta_2) d\theta_2 = 1.$$

If voting rights are proportional to the number of shares an individual owns, then the voter distribution is not the same as the ownership distribution.[7] It is not necessary to specify the number of votes each share carries, instead we can normalize the total number of votes equal to unity. The median voter would then be the individual with endowment where the voting distribution is cut in half, that is, the individual with holding θ_2^d such that

$$\int_0^{\theta_2^d} \theta_2 \Gamma'(\theta_2) d\theta_2 = \frac{1}{2}.$$

This tend to make the median voter having a greater share than the median in the ownership distribution.

There are distributions of shares for which *underproduction* occurs. The simplest case, when the median voter owns more shares than the population average, is when $\Gamma(\theta_2) = \upsilon\theta_2$, where υ is a positive constant. Then, $\Gamma(\hat{\theta}_2) = L$ implies $\hat{\theta}_2 = L/\upsilon$. Furthermore, that all shares sum to unity implies

$$1 = \int_0^{\hat{\theta}_2} \theta_2 \upsilon d\theta_2 = \frac{\upsilon}{2}(\hat{\theta}_2)^2,$$

so that $\upsilon = L^2/2$. The median voter is the individual with holding θ_2^d such that

$$\int_0^{\theta_2^d} \theta_2 \left(\frac{L^2}{2}\right) d\theta_2 = \frac{1}{2},$$

that is, $\theta_2^d = 2^{1/2}L^{-1} > L^{-1}$. That is, the median voter has a share greater than the population average. ◇

There are also distributions of shares for which *overproduction* occurs. We shall specify the following threshold values

$$0 < \theta_2^0 < \theta_2^d < \hat{\theta}_2 < 0.5,$$

and consider the following distribution function

$$\Gamma(\theta_2) = \begin{cases} 0 & \text{for } \theta_2 < \theta_2^0 \\ a(\theta_2 - \theta_2^0) & \text{for } \theta_2^0 \leq \theta_2 \leq \theta_2^d \\ b(\theta_2 - \theta_2^d) + \Gamma(\theta_2^d) & \text{for } \theta_2^d < \theta_2 \leq \hat{\theta}_2 \end{cases},$$

where

$$a = \frac{1}{(\theta_2^d)^2 - (\theta_2^0)^2} \quad \text{and} \quad b = \frac{1}{(\hat{\theta}_2)^2 - (\theta_2^d)^2}.$$

[7]The median in ownership is θ_2^m such that $\Gamma(\theta_2^m) = L/2$.

For the distribution function above, the decisive individuals is the one endowed with θ_2^d (see Appendix E). Evaluating the distribution function at $\hat{\theta}_2$, and pre-multiplying by θ_2^d gives

$$\theta_2^d \Gamma(\hat{\theta}_2) = \frac{\theta_2^d}{\theta_2^d + \theta_2^0} + \frac{\theta_2^d}{\hat{\theta}_2 + \theta_2^d}.$$

Since $\Gamma(\hat{\theta}_2) = L$, then $\theta_2^d < L^{-1}$ if (and only if) the right-hand side in the equation above is smaller than one, that is, if and only if

$$\theta_2^d < \sqrt{\hat{\theta}_2 \theta_2^0}.$$

Obviously, there is a whole range of parameter values that satisfy this inequality (together with $0 < \theta_2^0 < \theta_2^d < \hat{\theta}_2 < 0.5$). As a numerical example, let $\theta_2^0 = 0.1$, $\theta_2^d = 0.15$, and $\hat{\theta}_2 = 0.4$, then $\Gamma(\hat{\theta}_2) = 5.8181... = L$. Thus, $L^{-1} = 0.171875 > \theta_2^d$. $\diamond$

We will next consider a publicly owned monopoly, i.e., a nationalized monopoly. We assume that there is no other function of the government, for the sake of simplicity. A publicly owned monopoly would be characterized by equal ownership. Hence, the profits of the firm is assumed to be handed out lump-sum to the population. The government is modelled as a representative democracy. We will also assume that any individual can stand as a candidate, and analogously to our definition of Shareholder Voting Equilibrium, we shall define a *Politico-Economic Equilibrium* as the production decision taken by a candidate government representative who cannot lose against any other candidate in a binary election (the electorate being the entire population), assuming all individuals in the economy costlessly can stand as candidates. The candidate decision maker, whose production decision is implemented, is referred to as the Median Voter.

Proposition 4.4. *Suppose that all consumers have the same time endowment, then a publicly owned monopoly in a democracy performs as a competitive firm in Politico-Economic Equilibrium.*

Proof. When the firm is nationalized all consumers will have the same proportion of the firm which implies $L^h/L = \theta_2^h$ for all h, and all consumers will support the production choice. This in turn implies Competitive Economic Equilibrium. □

The above result depends upon the assumption on equal wage for all consumers. Suppose now that consumers differ linearly in terms of productivity. The wage for consumer h is then given by

$$\omega^h = \gamma^h \omega, \tag{4.8}$$

where γ^h is a productivity parameter normalized so that

$$\int \gamma^h \mathbf{d}h = 1. \tag{4.9}$$

We shall now focus on the changes of the key equations. Equation (2.8), (2.16), and (4.2) will then be modified in the following way:

$$\int \gamma^h l^h \mathbf{d}h = l_1 + l_2 = l, \tag{4.10}$$

$$\gamma^h \left(L^h - l^h\right) = \psi^h \left(l_1, l_2\right) \left(L - l\right), \tag{4.11}$$

$$\psi^h \left(l_2\right) = \frac{\theta_1^h \left(1 - \alpha\right) + \theta_2^h \left(a - \left(\alpha + b\right) \frac{l_2}{L - l_2}\right) + \left(\alpha + b\right) \frac{\gamma^h L^h}{L - l_2}}{1 + a + b}, \tag{4.12}$$

where $L = \int \gamma^h L^h \mathbf{d}h$, that is, total aggregate time in efficiency units. We shall note that the economy still has linear sharing rules, so that ψ^h is the consumer's share of each of the aggregate goods.

The analysis of the indirect utility function in Lemma 4.1 and Lemma D.1 in Appendix D applies here with Δ^h redefined, i.e.,

$$\Delta^h \equiv \frac{n}{\tilde{\theta}^h} \left(\frac{\gamma^h L^h}{L} - \theta_2^h \right),$$

Proposition 4.5. *Suppose that consumers differ in productivity as in* (4.8)*, but not in time endowments, and that the monopoly firm production decision is taken by shareholder voting, then the Competitive Economic Equilibrium is reached if the median voter in the firm has a share equal to her relative productivity divided by the population size.*

Proof. Given (4.8) and (4.9), Equation (4.7) becomes

$$V^{h'} \left(l_2\right) = \frac{V^h \left(l_2\right)}{\psi^h \left(l_2\right) \left(L - l_2\right) l_2} \left[\frac{\left(\alpha + b\right) L}{\left(L - l_2\right)^2} \left(\frac{\gamma^h L^h}{L} - \theta_2^h \right) + a\beta \left(1 - \eta\right) L \right], \tag{4.13}$$

which is zero for $\eta = 1$. This implies $l_2 = l_2^*$ if and only if

$$\frac{\gamma^h L^h}{L} = \theta_2^h.$$

This completes the proof. □

Proposition 4.5 has important implications. It says that Pareto efficient outcome can be reached even with a right skewed distribution of shares, if the relatively more productive consumers are endowed with relatively larger proportions of shares in the monopoly firm. However, when the firm is publicly owned we have the following property:

Proposition 4.6. *Suppose that all consumers have the same time endowment, then a publicly owned monopoly acts as a competitive firm in Politico-Economic Equilibrium if the distribution of skills is symmetric. If the distribution of skills is right/ left skewed the publicly owned monopoly will under/ over produce.*

Proof. When the firm is publicly owned all individuals have the same proportions of the firm. The median voter will then be the individual endowed with median skill, γ_{median}. This individual's choice will be characterized by Δ evaluated at $\gamma = \gamma_{median}$. If the distribution of skills is symmetric the median coincides with the mean and $\Delta = 0$. If

$$\gamma_{median} < \gamma_{mean} = 1,$$

then $\Delta < 0$, and if

$$\gamma_{median} > \gamma_{mean} = 1,$$

then $\Delta > 0$. The rest follows from Proposition 4.2. □

5. Trade in Shares

First we look at a situation when individuals do not recognize their influence on the decision of the monopoly firm. Then, only the returns of shares will matter. The relative share prices would in equilibrium be such that nobody has incentive to trade. Second, we look at the situation when all individuals are strategic, i.e., they realize that when trading (thus changing their ownership) they will influence the decision taken by the monopoly firm. Finally, we will look at the effects of constraints on trading (short-selling and credit constraints), when individuals are strategic.

In all cases we begin with an initial distribution of shares, then we allow the individuals to trade, and we examine which distribution of share constitute an equilibrium. The economy is still as in the previous sections, just that individuals prior to the voting stage can trade their shares.

We will denote the individual h's initial share ownership by $\bar{\theta}_1^h$ and $\bar{\theta}_2^h$, and the prices of shares by q_1 and q_2, respectively. For simplicity, we will treat share ownership in the competitive sector as an index portfolio. We could price the competitive firms individually[8], but to save on notation we allow individuals to trade in the index only.

We shall allow for the most general case where individuals may differ in both time endowments and in labor productivities. The objective of an

[8]In that case the relative share price between two firms, j and k, would be their profit ratio, i.e., $q_1^j/q_1^k = \pi_1^j/\pi_1^k = \left(A^j/A^k\right)^{1/(1-\alpha)}$.

individual is to maximize her indirect utility (4.1) and (4.12), subject to

$$q_1\theta_1^h + q_2\theta_2^h = q_1\bar{\theta}_1^h + q_2\bar{\theta}_2^h. \tag{5.1}$$

Since the relative share price, q_2/q_1, is a function of firm 2's profits, which in turn is a function of firm 2's decision, l_2, we may write $q_2/q_1 = Q(l_2)$. Equation (5.1) allows us to write $\bar{\theta}_1^h + (q_2/q_1)(\bar{\theta}_2^h - \theta_2^h)$, then taking the derivative of the individual h's indirect utility (4.1) with respect to θ_2^h gives the first-order condition to her portfolio decision.

$$-\frac{\partial V^h}{\partial \theta_1^h}\frac{q_2}{q_1} + \frac{\partial V^h}{\partial \theta_2^h} + \left[\frac{\partial V^h}{\partial \theta_1^h}(\bar{\theta}_2^h - \theta_2^h)\frac{\partial (q_2/q_1)}{\partial l_2} + \frac{\partial V^h}{\partial l_2}\right]\frac{\partial l_2}{\partial \theta_2^h}. \tag{5.2}$$

The first two terms in Equation (5.2) reflect the direct effect of trading shares, i.e., the marginal utility of giving up θ_1^h for θ_2^h. The terms within square brackets reflect the strategic effect of trading (since trading changes the political equilibrium in the monopoly firm). The first of those terms is the marginal utility of changing share prices (due to the change in the monopoly firm's decision). The second term is the direct effect of changing the monopoly firm's decision (which was the focus of section 4 in this paper).

We now turn to examine the various consequences of trading in shares.

5.1. **Non-strategic Investors.** If no individual realizes that trading in shares changes the political equilibrium in the monopoly firm when changing the ownership, we have the following result.

Proposition 5.1. *Assume that the restriction on ε_1 in Lemma 4.1 holds. If investors do not recognize their influence on the decision of the monopoly firm when trading shares, then any initial distribution of shares can constitute a Shareholder Voting Equilibrium.*

Proof. For non-strategic investors the square brackets of (5.2) is ignored. Then the first-order condition (5.2) becomes

$$\frac{q_2}{q_1} = \frac{\partial V^h}{\partial \theta_2^h}\Big/\frac{\partial V^h}{\partial \theta_1^h} = \frac{\pi_2}{\pi_1} = \frac{a - (\alpha + b)\frac{l_2}{L - l_2}}{1 - \alpha}, \tag{5.3}$$

where the second and third equalities follow from (4.1) and (4.12). All investors face the same prices, q_2/q_1, and are indifferent trading their initial portfolios. □

Thus, in this case, opening the stock market does not change anything of the previous analysis, and share ownership can be treated as exogenous. The reason is that equilibrium prices of shares are such that no individual has an incentive to trade. Notice also that Proposition 5.1 does not depend on the specific functional forms of the utility and production functions. Any function would have the property that the first two equalities hold, and consequently q_2/q_1 is independent of individual characteristics.

5.2. **Strategic Investors.** If investors recognize that when purchasing / selling shares of the monopoly firm they change the identity of the decisive individual, there are two consequences. First, individuals may purchase additional shares (deviating from the initial distribution) to acquire voting rights and affect the decision in their desired direction. Second, by purchasing / selling shares, the individuals also affect the equilibrium prices of shares. These incentives are captured by the terms within square brackets in Equation (5.2). The strategic effect drastically reduces the number of possible equilibria. In fact we have the following result.

Proposition 5.2. *If all investors realize their influence on the decision of the firm when trading shares, and if there are no restrictions on trading, then given any initial distribution of shares, the equilibrium distribution is characterized by*

1. $\frac{\gamma^h L^h}{L} = \theta_2^h$, *$\forall h$;*
2. *Shareholder unanimity;*
3. *Competitive equilibrium [Equations (3.2) - (3.5)].*

Proof. Investors being strategic implies that the whole of (5.2) must be taken in to account. Dividing (5.2) with respect to $\partial V^h/\partial\theta_1^h$, and use (4.1) and (4.12), we get

$$-\frac{q_2}{q_1} + \frac{a-(\alpha+b)\frac{l_2}{L-l_2}}{1-\alpha} + \left[(\bar{\theta}_2^h - \theta_2^h)\frac{\partial(q_2/q_1)}{\partial l_2} + \frac{\frac{\partial V^h}{\partial l_2}}{\frac{\partial V^h}{\partial \theta_1^h}}\right]\frac{\partial l_2}{\partial \theta_2^h} \tag{5.4}$$

A decisive individual, where $\bar{\theta}_2^h$ is such that $\partial V^h/\partial l_2 = 0$, is in equilibrium only if the first two terms of (5.4) cancel, i.e., if (5.3) holds. This implies that the term within square brackets must be zero for all h. This in turn implies that $\partial V^h/\partial l_2 = 0$ for all h, i.e., *shareholder unanimity*. Suppose $\partial V^h/\partial l_2 > 0$ (or $\partial V^h/\partial l_2 < 0$) for some h, then this individual wishes to increase (or reduce) l_2, and wish to purchase more θ_2^h in order to affect the voting outcome in the desired direction, that is, $\partial l_2/\partial\theta_2^h > 0$ (or $\partial l_2/\partial\theta_2^h < 0$), since if the individual remains at $\bar{\theta}_2^h$ the first order variation (5.4) is positive. This implies that all individuals in the economy must have $\partial V^h/\partial l_2 = 0$. It follows that all individuals hold shares so as to satisfy $\theta_2^h = \gamma^h L^h/L + C^h$, where

$$C^h \equiv \frac{a\beta}{\alpha+b}\frac{L-l_2}{l_2}(1-\eta)\psi^h. \tag{5.5}$$

Integrating over population

$$\int \theta_2^h f(h)\mathbf{d}h = \int \gamma^h \frac{L^h}{L} + C^h f(h)\mathbf{d}h = 1 + \int C^h f(h)\mathbf{d}h, \tag{5.6}$$

where the second equality follows from the definition of L. However, the shares must sum to unity, therefore C^h must be zero for all h, in turn implying $\eta = 1$, that is, the *competitive equilibrium.* □

If we were in a situation, with an initial distribution of shares, such that a non-competitive equilibrium was reached, with the resulting inefficiency, shareholders always have the incentive to trade their shares until the inefficiency is eliminated, i.e., until the competitive equilibrium is reached. This result is very close to the Coase conjecture if property rights are well defined. Trade in those property rights would ensure that any inefficiencies are internalized. Our result suggests that when stock markets are well functioning, any inefficiency due to market power would be eliminated. However, our equilibrium may require some individuals to go short in the competitive sector, i.e., take a negative position in the competitive sector in order to purchase a share $\theta_2^h = \gamma^h L^h/L$.[9] Alternatively, the individual can write a debt contract in terms of commodity 1 (the good produced by the competitive sector). If short sales are not allowed (or alternatively if there are credit constraints), the equilibrium in Proposition 5.2 may not be reached. This leads us to investigate short-selling constraints in the next section.

5.3. **Short-Selling Constraints.** If investors cannot go short in the competitive sector, the share distribution $\theta_2^h = \gamma^h L^h/L$ may be infeasible. In such a situation some investors will be constrained, and their first order variation of indirect utility with respect to θ_2^h will not be equal to zero. However, among investors for whom the short selling constraint does not bind, i.e., they own initial positions large enough, we have the following result.

Proposition 5.3. *If all investors realize their influence on the decision of the firm when trading shares, and if short-selling is not allowed (or if debt contracts are not allowed), then for investors with endowments large enough for the short-selling constraint not to be binding, the following hold.*

(1) $\gamma^h L^h/L = \theta_2^h$;
(2) *Unanimity with regard to the choice of* l_2.

The equilibrium is not necessarily the competitive equilibrium.

[9] In the extreme case when an individual owns no shares initially she must take a short position of

$$\theta_1^h = -\frac{q_2}{q_1}\frac{\gamma^h L^h}{L} = -a\frac{1-\beta}{1-\alpha}\frac{\gamma^h L^h}{L}.$$

Proof. Follows first part of the proof of Proposition 5.2. □

To go any further, we need to know the initial distribution of shares. The way in which short-selling constraints bind depends critically on the initial distribution of shares and how this may be correlated with productivity / time endowment. We will consider a number of examples.

First,we will investigate a situation where a fraction, δ, of the population own no shares at all, the other fraction, $1-\delta$, own equal amount of shares (both in competitive sector and in the monopoly firm), and no correlation between productivity and share ownership. Among the individuals owning no shares, all types must be represented according to the population distribution. Those owning shares will trade in such a way that shareholder unanimity is reached. Among share owners

$$\int \frac{\gamma^h L^h}{L} + C^h f(h)\mathbf{d}h = 1-\delta + \int C^h f(h)\mathbf{d}h,$$

therefore (5.6) implies

$$\delta = \int C^h f(h)\mathbf{d}h, \tag{5.7}$$

where C^h is defined in (5.5). Using (4.12) $\int \psi^h f(h)h = 1$, then Equation (5.7) becomes

$$\delta = \frac{a\beta}{\alpha+b}\frac{L-l_2}{l_2}(1-\eta) = \left[1+(1-\eta)\frac{a\beta}{\alpha+b}\right]\frac{1-\eta}{\eta}. \tag{5.8}$$

We see that the larger δ the smaller η. When a fraction of the population owns no shares initially, and there are short selling constraints, in any equilibrium post trade in shares they will still own no shares. If there is no correlation between share ownership and the underlying heterogeneity (productivity and time endowments), the larger the fraction without shares, the smaller is the production of the monopoly firm (further away from the competitive equilibrium).

Furthermore, from Equation (5.8) we can verify Proposition 5.2 when δ is zero. Note also that as $\delta \to 1$, $\eta > 0$. In the extreme case, as in the limit an infinitely small fraction own the monopoly (and the competitive sector), production is strictly positive. Thus, even in the extreme case, we cannot reproduce the firm objective suggested by the traditional industrial organization literature. This strengthens our view that the traditional firm objectives are inconsistent with rationality of the owners.

6. Conclusion

We have endogenized the objective of a monopoly firm through shareholder voting, in a simple two-sector general equilibrium model. In this way we ensured that the firm's objective is consistent with the preferences of the owners, which it would fail to be under traditional profit maximization. When the shareholders realize that the firm has market power, we showed that rational voting may imply overproduction as well as underproduction, relative to the CEE. For certain distribution of shares the CEE allocation was obtained. We characterized the properties of the underlying distribution of shares for either case to be generated. We also found that a nationalized monopoly, when all individuals own the same amount of shares, may underproduce relative to the CEE.

Finally we endogenized share ownership by allowing trade in shares. If investors are myopic in the sense that they do not recognize their influence on the voting outcome, and thereby on the share prices, when they trade, then any distribution of shares could constitute an equilibrium. If individuals realize their influence on the voting outcome when trading, and if individuals are allowed to sell short their shares, then trade occurs until the distribution of shares is such that the voting outcome supports the CEE. This result is close to the Coase Theorem, in the sense that the economy trades itself to efficiency. If individuals are not allowed to sell short their shares then we showed that the equilibrium is such that all shareholders agree on the production decision, but it typically involves underproduction relative to the CEE. We conclude that it is not market power itself which causes underproduction, but the inability to perfectly trade the rights (i.e., shares) in the economy.

Appendix A

Firm j solves Equation (2.18) taking prices as given. Then each firm's labor demand and production will be given by

$$l_1^j = \left(\alpha A^j\right)^{\frac{1}{1-\alpha}} \left(\frac{\omega}{p_1}\right)^{\frac{1}{\alpha-1}}, \tag{A.1}$$

$$y_1^j = \alpha^{\frac{\alpha}{1-\alpha}} \left(A^j\right)^{\frac{1}{1-\alpha}} \left(\frac{\omega}{p_1}\right)^{\frac{\alpha}{\alpha-1}}, \tag{A.2}$$

Aggregating over all firms in the competitive sector, we obtain

$$l_1 = \alpha^{\frac{1}{1-\alpha}} \int \left(A^j\right)^{\frac{1}{1-\alpha}} \mathbf{d}j \left(\frac{\omega}{p_1}\right)^{\frac{1}{\alpha-1}}, \tag{A.3}$$

$$y_1 = \int \left(A^j\right)^{\frac{1}{1-\alpha}} \mathbf{d}j \, \alpha^{\frac{\alpha}{1-\alpha}} \left(\frac{\omega}{p_1}\right)^{\frac{\alpha}{\alpha-1}}, \tag{A.4}$$

Substituting for the price ω/p_1 in equation (2.12) into (A.3) and (A.4), and rearranging, we obtain

$$l_1 = \frac{\alpha}{b}\left(L - l\right), \tag{A.5}$$

$$y_1 = \left(\frac{\alpha}{b}\right)^{\alpha} \left(\int \left(A^j\right)^{\frac{1}{1-\alpha}} \mathbf{d}j\right)^{1-\alpha} \left(L - l\right)^{\alpha}. \tag{A.6}$$

Note that since $l = l_1 + l_2$ by definition, Equation (A.5) becomes (2.19) and (A.6) becomes (2.20).

Appendix B

The monopoly firm chooses l_2 to solve Equation (3.1), which gives demand for labor and production, respectively.

$$l_2^* = \left(\beta B\right)^{\frac{1}{1-\beta}} \left(\frac{\omega}{p_2}\right)^{\frac{1}{\beta-1}}, \tag{B.1}$$

$$y_2^* = \beta^{\frac{\beta}{1-\beta}} B^{\frac{1}{1-\beta}} \left(\frac{\omega}{p_2}\right)^{\frac{\beta}{\beta-1}}, \tag{B.2}$$

Substituting for the price ω/p_2 in Equation (2.13) into (B.1) and (B.2), and rearranging, we obtain

$$l_2^* = \beta \, \frac{a}{b}\left(L - l^*\right), \tag{B.3}$$

$$y_2^* = B\left(\beta \, \frac{a}{b}\right)^{\beta} \left(L - l^*\right)^{\beta}. \tag{B.4}$$

Combining (A.5) and (B.3) yields (3.2), and combining (2.19) and (3.2) yields (3.3). Furthermore, combining (3.2) and (3.3) together with (2.20) yields (3.4). Substituting (3.4) into (3.1) yields (3.5).

Appendix C

Proof of Lemma 4.1. Define

$$z \equiv \frac{L}{L - l_2}$$

then (4.2) together with the definitions for $\tilde{\theta}^h$ and Δ^h gives

$$\psi^h(z) = \tilde{\theta}^h(1 + \Delta^h z)$$

and (4.1) then gives

$$\frac{\ln V^h(z)}{1 + a + b} = \ln \tilde{\theta}^h + \ln(1 + \Delta^h z) + m \ln(z - 1) - (m + n) \ln z.$$

First-order variation (FOV) with respect to z:

$$\begin{aligned} FOV &= \frac{\Delta^h}{1 + \Delta^h z} + \frac{m}{z - 1} - \frac{m + n}{z} \\ &= \frac{\Delta^h (z - 1) z + [m - n(z - 1)] (1 + \Delta^h z)}{(1 + \Delta^h z)(z - 1) z} \\ &= \frac{(1 - n)\Delta^h}{(1 + \Delta^h z)(z - 1) z} \\ &\left[(z - 1)^2 + \left(\frac{1 + m}{1 - n} - \frac{n}{1 - n} \frac{1 + \Delta^h}{\Delta^h} \right)(z - 1) + \frac{m}{1 - n} \frac{1 + \Delta^h}{\Delta^h} \right] \\ &= \frac{(1 - n)\Delta^h}{1 + \Delta^h z)(z - 1) z} (z - 1 - \lambda_1)(z - 1 - \lambda_2), \end{aligned}$$

where $\lambda_{1,2}$ are defined in Lemma 4.1. Since $0 < n < 1$ and $0 < m < 1$, the roots $r_{1,2}$ are real and satisfy

$$r_1 > 1 - m + 2\frac{m}{n} > r_2 \geq n > 0.$$

Then, for $\lambda_{1,2}$ to be real we must have (if $\Delta^h > 0$)

$$n \frac{1 + \Delta^h}{\Delta^h} \geq r_1.$$

This condition is equivalent to

$$\Delta^h \leq \frac{n}{r_1 - n}.$$

When

$$0 < \Delta^h \leq \frac{n}{r_1 - n}$$

we have two roots satisfying the first-order condition (FOV=0).

For $z-1<\lambda_2$, $FOV>0$ (since $\lambda_2<\lambda_1$); for $z-1=\lambda_2$, $FOV=0$; for $\lambda_2<z-1<\lambda_1$, $FOV<0$; for $z-1=\lambda_1$, $FOV=0$; and for $\lambda_2\leq z-1$, $FOV>0$. Thus, going from the smaller root to the larger decreases utility. In fact, λ_1 is a local minimum, and we can concentrate on λ_2.

λ_2 is a local maximum for an individual with

$$0<\Delta^h\leq\frac{n}{r_1-n}.$$

That $\lambda_2>\frac{m}{n}$ is straightforward to verify.

In the case $-1<\Delta^h<0$, $\lambda_{1,2}$ are always real and only λ_1 is of interest (since $\lambda_2<0$ here). λ_1 is the global maximum since when $z-1<\lambda_1$, $FOV>0$ and when $z-1>\lambda_1$, $FOV<0$. That $\lambda_1<\frac{m}{n}$ is straightforward to verify.

Finally, when $\Delta^h=0$,

$$\begin{aligned}FOV&=\frac{m}{z-1}-\frac{m+n}{z}\\&=n\frac{\frac{m}{n}-(z-1)}{(z-1)z}.\end{aligned}$$

Then $\frac{m}{n}$ is the global maximum since when $z-1<\frac{m}{n}$, $FOV>0$ and when $z-1>\frac{m}{n}$, $FOV<0$.

When $l_2\to L$, $z\to+\infty$. The objective may be written as

$$\begin{aligned}V^h(z)^{\frac{1}{1+a+b}}&=\tilde{\theta}^h(1+\Delta^h z)(z-1)^m z^{-(m+n)}\\&=\tilde{\theta}^h\left(z^{-n}+\Delta^h z^{1-n}\right)\left(1-\frac{1}{z}\right)^m.\end{aligned}$$

Hence,

$$\lim_{z\to+\infty}V^h(z)^{\frac{1}{1+a+b}}\to+\infty.$$

When the smallest unit of production in Section 1 is reached at $l_1=\varepsilon$, an individual with preferences for a corner solution has to compare $l_1=0$ against $l_1=\varepsilon$. However, at $l_1=0$, $V^h=0$, thus $l_2=L$ can never be preferred. When ε equals the constraint in Lemma 1, employment in Sector 2 cannot be larger than the local maximum for an individual with

$$\Delta^h=\frac{n}{r_1-n}.^{10}$$

A individual with $\Delta^h<\frac{n}{r_1-n}$ will prefer her own local maximum to the local maximum of $\Delta^h=\frac{n}{r_1-n}$. Thus, each individual's local maximum is a global maximum. □

[10] To see this, consider such an individual with a local maximum at $\lambda=\frac{r_1-(1+m)}{z(1-n)}$. This implies $L-l_2=\frac{z(1-n)L}{1-m-n+r_1-n}$. Next, $l_1=\frac{\alpha}{\alpha+b}(L-l_2)$. Replacing l_1 by ε gives the condition.

Appendix D

Denote a candidate with superscript c. Since a candidate's most preferred l_2 is a function of her Δ^c, we can replace l_2 with its function of Δ^c in an individual's (say individual $h \neq c$) indirect utility function, to obtain an indirect utility function in Δ^c. Then preferences of shareholder h will be single peaked in Δ^c, with the maximum reached at $\Delta^c = \Delta^h$.

Lemma D.1. *Assume ε_1 satisfies the inequality in Lemma 4.1, and that candidates are shareholders, then individual shareholders' preferences over candidates are single peaked.*

Proof. We know from Lemma 4.1 that an individual with $\Delta^h < 0$ or $\Delta^h = 0$ has single peaked preferences over all l_2. The only potential problem is for individuals with $\Delta^h > 0$. We know from Appendix C that the utility function for such a shareholder has a local minimum at $\frac{l_2}{L-l_2} = \lambda_1$. We must then make sure that no potential candidate would implement $\frac{l_2}{L-l_2} > \lambda_1$, for any λ_1. Thus we must ensure that the individual with the smallest λ_1 has λ_1 larger than (or equal to) the maximum possible $l_2/(L-l_2)$. The maximum possible $l_2/(L-l_2)$ is when l_1 is driven to ε_1. This implies that $l_2/(L-l_2)$ cannot be larger than the maximum for an individual with $\Delta^h = \frac{n}{r_1-n}$ (see end of Appendix C). We then need to find the smallest λ_1. From the definition of λ_1 we see that it is increasing in Δ^h. therefore the smallest λ_1 is reached for an individual with $\Delta^h = \frac{n}{r_1-n}$. This is when $\lambda_1 = \lambda_2$ and we have an inflexion point. Thus for any candidate with $-1 < \Delta^h \leq \frac{n}{r_1-n}$, the candidate's most preferred $l_2/(L-l_2)$ will never reach the region where any other individual's indirect utility reaches beyond its eventual local minimum. This is also true for a candidate with $\Delta^h > \frac{n}{r_1-n}$, since she prefers the same level of $l_2/(L-l_2)$ as an individual with $\Delta^h = \frac{n}{r_1-n}$ to the level where l_1 is driven to zero (because utility reaches zero at that level). Thus any individual's preferences are single peaked over l_2 that are restricted to be optima for some other shareholders. Since a candidate will implement her most preferred l_2, shareholders have single peaked preferences over the types of the candidates, i.e., over Δ^c. □

Appendix E

To show that the decisive individual has a share equal to θ_2^d, we have to demonstrate that

$$\frac{1}{2} = \int_{\theta_2^0}^{\theta_2^d} a\theta_2 d\theta_2 = \frac{a}{2}\left[(\theta_2^d)^2 - (\theta_2^0)^2\right]$$

Inserting the definition of a gives the result. To show that the number of outstanding shares is equal to unity, we have to demonstrate that

$$1 = \int_{\theta_2^0}^{\theta_2^d} a\theta_2 d\theta_2 + \int_{\theta_2^d}^{\hat{\theta}_2} b\theta_2 d\theta_2 = \frac{a}{2}\left[(\theta_2^d)^2 - (\theta_2^0)^2\right] + \frac{b}{2}\left[(\hat{\theta}_2)^2 - (\theta_2^d)^2\right]$$

Inserting the definitions of a and b gives the result. Finally

$$\Gamma(\hat{\theta}_2) = \int_{\theta_2^0}^{\theta_2^d} a d\theta_2 + \int_{\theta_2^d}^{\hat{\theta}_2} b d\theta_2 = a\left(\theta_2^d - \theta_2^0\right) + b\left(\hat{\theta}_2 - \theta_2^d\right).$$

Inserting the definitions of a and b gives

$$\Gamma(\hat{\theta}_2) = \frac{\theta_2^d - \theta_2^0}{(\theta_2^d)^2 - (\theta_2^0)^2} + \frac{\hat{\theta}_2 - \theta_2^d}{(\hat{\theta}_2)^2 - (\theta_2^d)^2} = \frac{1}{\theta_2^d - \theta_2^0} + \frac{1}{\hat{\theta}_2 - \theta_2^d}.$$

The result follows. $\diamond$

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