QUALITY OF INFORMATION AND OLIGOPOLISTIC PRICE DISCRIMINATION*

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Abstract

We investigate how the acquisition of information, of a certain quality level, on consumers' willingness to pay (location) affects the equilibrium prices and welfare in a spatial price discrimination model. By varying the information quality we are able to obtain the equilibrium in the game for all levels of price discrimination and in the limit the case of perfect price discrimination. This gives us insights about equilibrium behavior in markets where: 1) Information on consumer characteristics is used by the firms to estimate each customer's degree of loyalty, which facilitates price discrimination and 2) The quality of information is constantly improving due to advances of the information technology (IT).

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1 Introduction

Information regarding customers' characteristics such as previous purchases, age group, income, tastes, education etc. is valuable to firms for various reasons. Firms knowing the specific traits and preferences of a customer (or group of customers) can customize their products, develop and offer a new product, or charge different prices to different buyers. The development and widespread use of information technology (IT), and especially the internet as a medium of communication and commerce has made the collection, storage, analysis and application of such information feasible. There is ample evidence in the press that firms have recognized this opportunity and are trying to reap the benefits of the new technology.[1]

Consumers can now be contacted by prospective sellers on an individual basis and offered various products at terms which are not uniform among all potential buyers. For example, Books.com - a books retailer - adopted in early 1998 a price discrimination strategy where different buyers were paying different prices for the same item depending on their shopping behavior [see Bailey (1998)].[2] Bailey addresses the issue of why price discrimination is attractive to internet retailers and how they can implement such a system. He also shows how retailers are collecting large amounts of information about their customers' purchasing patterns with the intent to estimate each consumer's reservation price for the product. Knowing each customer's maximum willingness to pay will enable the seller to price discriminate more effectively. Shaffer and Zhang (1995) discuss how the advent of panel data on household purchase behavior and the statistical procedures to utilize this data has led firms

[1] *New York Times* in September 01, 2000 writes: "Furthermore, Amazon.com has started to send e-mail advertising messages to its customers on behalf of other companies and...If the new measures fail to reverse the leading internet retailer's heavy losses and it is put up for sale, the buyer will acquire Amazon.com's customer data." *Video Business* in February 21, 2000 writes: "Video retailers are looking to use their databases of customer information as an additional revenue stream, by selling those names to marketers. According to some analysts each name in those databases could be worth as much as $700." Finally, *Travel Agent* in January 08, 1996 writes: "Preferred Hotels & Resorts Worldwide plan to use computer-generated customer profiles to reach niche markets... With such information, a hotelier can develop a targeted package and send a letter to members of a 'cohort group', a group of people with similar interests."

[2] Ulph and Vulkan (2000) also discuss how technologies, such as an agent, which is a program authorized to act independently on behalf of its user [see Vulkan (1999)], can facilitate price discrimination by the firms.

to price discriminate, through the distribution of coupons which are targeted to selected households, with considerable accuracy and cost effectiveness. The rapid and constant improvement of internet and software technology also implies that the quality of information that can be collected (or acquired) by the firms is increasing. The purpose of this paper is to study the incentives of firms to price discriminate as a function of the quality of information that it is available, as well as the evolution of equilibrium profits, prices and welfare as the quality improves.

We employ the standard Hotelling's model of horizontal product differentiation with two sellers who are located at the two endpoints of a unit interval. Firms can acquire costly information about each consumer's *degree of loyalty*. In practice, firms can obtain such information from a number of different sources, such as: i) directly through past transactions with the customers, ii) via a telemarketing or direct-mail survey, iii) from credit card reports, or iv) from a marketing firm [see Shaffer and Zhang (2000, 2001) for a more extensive discussion and more references on this issue]. We assume that the information partitions the unit interval into smaller subintervals allowing the firms to learn exactly which subinterval each consumer is located in.[3] Thus, a firm knows with higher *precision* (compared to the no-information case) each consumer's degree of loyalty which facilitates third degree price discrimination.[4] The information is of a higher *quality* if it partitions the interval into more segments. Each firm, given the available information of a certain quality, decides: 1) whether to price discriminate (i.e., whether to acquire information) or not and 2) about the price(s) for its product without knowing the action of its rival.

Economists' interest in price discrimination dates back to the work of Robinson (1933) who studied the issue in a monopolistic environment. Later work on the subject was still confined to a one seller market, [e.g. Schmalensee (1981) and Varian (1985)]. In a monopolistic

[3] Alternatively, one can think of the information as an imperfect signal s on consumer's true location ℓ, with conditional density $f(s|\ell)$ on $[0,1]$ satisfying the monotone likelihood ratio property. The partition approach, however, is a more realistic approximation of reality since it allows each consumer to be categorized in a specific group depending upon his relative strength of preferences for a firm's product.

[4] Since consumers buy at most one unit, third degree price discrimination coincides with the first degree price discrimination.

situation, price discrimination leads to higher profits for the monopolist and to ambiguous consumer welfare results. This prediction does not necessarily carry over to imperfectly competitive markets. In an oligopolistic price discrimination model with symmetric firms Holmes (1989) shows that the consumer welfare result is still ambiguous. However, Corts (1998) and Shaffer and Zhang (2000) by relaxing the assumption of symmetric demand made by Holmes show that unambiguous price and welfare results may arise, where all consumers become better off, as a result of being charged a lower price (prisoners' dilemma). They also show that price discrimination is a dominant strategy that results in lower equilibrium profits for the firms. Bester and Petrakis (1996) in a duopoly model with price discrimination obtain similar results, that is, the ability to charge different consumer segments different prices increases competition and reduces profits. Ulph and Vulkan study the incentives of two Hotelling firms to engage in perfect price discrimination.

The literature on price discrimination in spatial models has mainly assumed that firms: i) either have the ability to identify the location of each consumer perfectly [e.g. Anderson and de Palma (1988), Bhaskar and To (2001), Lederer and Hurter (1986), Shaffer and Zhang (2001) and Ulph and Vulkan], or ii) discriminate between only two groups of consumers [e.g. Bester and Petrakis, Fudenberg and Tirole (2000) and Shaffer and Zhang (1995, 2000)]. Our innovation in this paper is to investigate what happens in between (as well as the above mentioned two extreme cases), i.e., when the firms have some information about the location of each consumer, which is not "perfect," but on the other hand may allow them to separate the consumers into more than two groups. By varying the quality of information (in the form of comparative statics), we can obtain all levels of price discrimination and in the limit (as the number of subintervals goes to infinity) the case of perfect discrimination.[5] This is a more flexible way of modeling price discrimination and fits reality better, where the quality of consumer information that firms are using to develop their pricing strategies is far from perfect, but is constantly improving due to advances of the IT. As Shaffer and Zhang (2001)

[5]Technically speaking, our limit (as the number of subintervals goes to infinity) is not the same as the perfect price discrimination, where each one of the continuum of consumers pays a different price. In our limit we can generate all rational numbers in $[0, 1]$, but it is not the case that each consumer pays a different price. We do however, obtain the same results in terms of average price, profits, average transportation cost, consumer and social welfare as in the perfect discrimination paradigm.

put it,

> In practice, of course, we would expect firms to be more or less certain about the loyalty of each consumer, depending on the quality and quantity of their data on individuals' past purchasing behavior.

The question which arises is whether the firms will use the information against the consumers or against each other and how these considerations are affected by the quality of information.[6] Our framework allows us to analyze the consequences of price discrimination from two different reference points. First, one can look at the transition from no price discrimination to price discrimination based on information of a certain quality, which we call an *absolute transition*. Second, one can look at the transition from price discrimination based on a given level of information quality to that based on a better information, which we call a *relative transition*. The latter case seems more realistic since it can be argued that many firms already have an ability to price discriminate and they are contemplating to expand this practice over more consumer groups through the acquisition of a more refined information. While results pertaining to an absolute transition when price discrimination is practiced only against two consumer groups or every individual consumer are by now well understood, we do not know anything about the implications of a general absolute transition and a relative transition.

Using our game-theoretic framework, we derive each firm's willingness to pay for the information as a function of its quality. For relatively low values of the information price "price discrimination" is each firm's dominant strategy. In this case equilibrium profits are lower, for any level of the information quality, than the profits under no price discrimination. Firms' profits (before we subtract the information price) exhibit a U-shape pattern as a

[6] Essentially there are two effects at work here. One is the "surplus extraction" effect and the other is the "intensified competition" effect [see Ulph and Vulkan where these two terms have been first introduced]. The first effect refers to the possibility that information will enable the firms to extract more surplus from each consumer and the second to the possibility that since both firms know more about each consumer's willingness to pay, competition will become more vigorous.

function of the information quality. This implies, that after a certain threshold, higher quality translates into higher firm profits. Thus, when the game is viewed from a relative transition perspective it is not a prisoners' dilemma.[7] All consumers are paying lower prices when the firms engage in price discrimination, compared to the uniform price. The average consumer welfare, however, has an inverse U-shape as a function of the information quality. That is, moderate levels of information quality yield the highest consumer welfare and, after a quality threshold, some consumer groups pay relatively higher prices when the firms refine their ability to price discriminate. Interestingly, a price equilibrium, in pure strategies, where only one firm engages in price discrimination does not exist for high levels of the information quality.

Policymakers and regulators have raised concerns that the increasing collection of information about consumers' shopping behavior may have detrimental effects on consumer welfare. Consumer groups and organizations are also concerned about the way that personal information collected from consumers or about consumers by third parties is used.[8] This paper attempts to highlight some of these issues.[9] What we show is that moderate information is beneficial for the consumers. Hence, as far as the consumers' welfare is concerned, one should neither prohibit the collection and application of consumer information by the firms completely, nor permit an extensive utilization of such information.

The rest of the paper is organized as follows. The model is presented in section 2 and section 3 contains the main analysis which deals exclusively with an absolute transition. The main results are presented in section 4, where we also show how the basic framework can be used to study the implications of a relative transition. In section 5, we extend our analysis

[7] Shaffer and Zhang (2001) look at one-to-one promotions based on perfect information under the assumption that firms are *asymmetric*, and they show that the game (relative to the one price strategy) is *not* a prisoners' dilemma.

[8] See, for instance, the Electronic Privacy Information Center's [www.epic.org, August 10, 2001] complaint against Microsoft, concerning Windows XP and Microsoft's ability to collect a huge amount of personal information which is allegedly unfair and leads to deceptive trade practices.

[9] Of course, the whole debate about consumer privacy and the issue of the proprietary rights of consumer information and how this information should be used is far more general than the specific approach we have taken in this paper.

by investigating the consequences of relaxing the assumption that the market is always fully covered and we show that the game need not be a prisoners' dilemma. We summarize and provide directions for future research in section 6.

2 The description of the model

We employ the standard Hotelling's model of horizontal product differentiation. Consumers are uniformly distributed on the interval $[0, 1]$ and two firms are located at the two endpoints of the interval. Each consumer buys a single unit either from firm 1 (which we assume that is located at 0), or from firm 2 (which is located at 1), or does not buy at all. We assume that each consumer derives a benefit equal to V if he buys a product from either one of the firms. Let p_1 and p_2 be the prices that firm 1 and firm 2 charge respectively. Both firms' marginal costs are normalized to zero. In addition, each consumer incurs a linear unit transportation cost denoted by $t > 0$.[10] Therefore a consumer who is located at point $x \in [0, 1]$ on the interval and buys from firm 1 enjoys a surplus of $V - tx - p_1$. Likewise, if he buys from firm 2 his surplus is $V - t(1 - x) - p_2$. Each consumer buys the product which gives him the highest positive surplus. We assume that V is sufficiently high, ensuring that each consumer will buy.[11] Then the demand of each of the two firms' products is given by,

$$d_1 = \frac{p_2 - p_1 + t}{2t} \text{ and } d_2 = \frac{p_1 - p_2 + t}{2t}.$$

It can be easily shown [see e.g. Tirole (1988), p. 280] that in the equilibrium $p_1 = p_2 = t$, $d_1 = d_2 = 1/2$ and $\pi_1 = \pi_2 = t/2$.

So far we have implicitly assumed that firms have no information regarding the location of each consumer. All they know is that consumers are uniformly distributed on the unit interval. Now assume that (some) information regarding the location of each consumer

[10] When firms are located at 0 and 1 respectively, their demand and profit functions are the same whether the transportation cost is linear or quadratic.

[11] The assumption that the market is fully covered makes our model tractable. See section 5 where this assumption is relaxed.

becomes available. This information partitions the interval into N sub-intervals (indexed by m, $m = 1, ..., N$) of equal distance and the firm who acquires this information has a better idea about the location of the consumers and their willingness to pay. In this case, a firm can charge different prices to different groups of consumers, though the price is the same within each group. We assume that arbitrage between consumers is not feasible.

Definition: *An information which partitions the unit interval into N sub-intervals is of a higher quality than the one which partitions the interval into N' sub-intervals, if and only if $N > N'$.*

Therefore, each consumer's location can be identified with a higher precision if the information is of a higher quality. Notice that the above definition is not the same as that of an information refinement. An information refinement implies higher quality, but not the other way around. Hence, our definition is more general.

We assume that an information of quality N is available to both firms at a cost equal to $\alpha(N)$. The amount that a firm is willing to pay for the information depends on the incremental profit that it will experience, which in turn depends on whether the other firm has acquired the information or not. We assume that the current state of technology dictates the quality of information N and the firms take it as exogenously given.

Each firm's strategy is (τ_i, p_i), $i = 1, 2$, where $\tau_i = 1$ if the firm acquires the information and 0 otherwise; and p_i is an $N \times 1$ vector of prices if $\tau_i = 1$ (since firm i can charge a different price in each segment) or a scalar if $\tau_i = 0$. We assume that a firm's pricing strategy is measurable with respect to its information. Let $d_{im}(p_{1m}, p_{2m})$, $m = 1, ..., N$, be the demand of firm i's product in the $m-$th segment (sub-interval). The profit function of firm i is therefore given by,

$$\pi_i(\tau_1, p_1, \tau_2, p_2) = \sum_{m=1}^{N} d_{im} p_{im} - \tau_i \alpha(N), \ i = 1, 2.$$

The game we consider unfolds as follows:

Stage 1: Nature chooses N and the information is available to both firms at cost $\alpha(N)$.

Stage 2: Given the quality of information and its price, firms decide simultaneously and independently whether to acquire information or not and about the prices they will charge in order to maximize their profits $\pi_1(\tau_1, p_1, \tau_2, p_2)$ and $\pi_2(\tau_1, p_1, \tau_2, p_2)$, respectively.

In the next section, we look for a subgame perfect equilibrium (in pure strategies) of this game.

Remark 1: Although the issue of information provision is very interesting, in this paper we focus primarily on stage 2 of our game and therefore we do not study in detail the stage where the cost of information is determined. As a consequence, we treat the information price as exogenously given. Our model can be augmented to incorporate a stage where the information price is endogenously determined. This, however, will critically depend upon the assumptions we make about the information provision market structure and will add considerably to the length and complexity of our paper. Also, the firms in our game cannot influence the information quality, which is entirely dictated by the state of the technology which is outside the control of the two firms. In reality firms (or the information providers) can improve, up to a certain extent, the information quality by investing more resources, but technology clearly imposes an upper bound to this process. For tractability, we implicitly assume a very tight bound and an infinite cost for any deviation from it.

Remark 2: We assume that the firms have not observed each other's decision to buy information or not when they set their prices, justifying the simultaneous move game after the information of quality N becomes available. We believe that "acquiring information" is a very flexible choice with no element of commitment. This should be contrasted with capacity or quality games where firms may observe the choice (i.e., level of capacity or quality design) of their rivals when they set their prices. Capacity or quality, however, can be safely assumed to be publicly observable and once their levels have been chosen it becomes prohibitively costly for the firms to modify them. This is less likely to be the case with the information, which may entail just the purchase of software or a mailing list. Furthermore, a sequential move game, where firms first decide whether to acquire information or not and then, after the first stage decision is observed by both players, set their prices, does not have an equilibrium in pure strategies. In particular, the subgame where only one firm acquires information does not have a price equilibrium [see section 3.1.3] and therefore the whole game cannot be studied. A simultaneous move game, in addition to being in our opinion more realistic, overcomes this shortcoming.

3 Analysis

We solve the game backwards starting from stage 2 and proceeding to stage 1. We begin by finding a Nash equilibrium in the subgame played between the two firms as a function of N. Each firm has to decide whether to acquire the available information of quality N or not and the prices that it will charge to different groups of consumers without knowing whether the other firm possesses the information or not.

3.1 Stage 2: The game between the two firms

We are going to find under what conditions (if any) the following set of strategies can be supported as a Nash equilibrium in this subgame: 1) both firms decide to buy[12] the information (B vs. B), 2) only one firm decides to buy the information (B vs. NB) and 3) neither firm finds it profitable to buy the information (NB vs. NB). In particular, we are looking for a $(\tau_1^*, \tau_2^*, p_1^*, p_2^*)$ such that any unilateral deviation by a firm is unprofitable. Since a firm's strategy entails to choose whether to purchase the information or not and a price vector, a deviation can be in both dimensions. We begin by finding both firms' profits if they adhere to the above set of strategies and then we compute the profits from a unilateral deviation in each of the three cases mentioned above. Finally, in this subsection we calculate firms' gross profits before deducting the information price ($\alpha(N)$). In section 3.2., we find

[12] The word "buy" should not be interpreted literally. It signifies that a firm possesses the information. It could be the case that a firm itself has collected the information.

the range of $\alpha(N)'$s such that (B vs. B), (B vs. NB) and (NB vs. NB) can be supported as an equilibrium respectively.

3.1.1 Both firms buy information (B vs. B)

Since both firms buy consumer information, they know in which of the N segments each consumer is located and therefore they are able to charge different prices for different segments. Interval $[0,1]$ is equally divided into N segments, each one having length of $1/N$. Segment m can be expressed as the interval $[(m-1)/N, m/N]$, where m is an integer between 1 and N [see figure 1 below].

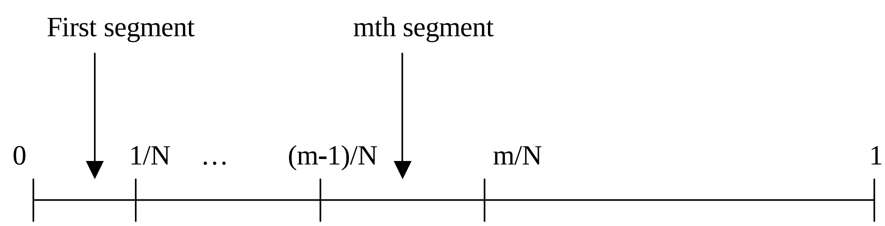

Figure 1

In segment m, firm 1 and 2 charge prices p_{1m} and p_{2m}, the demands[13] of their products are,

$$d_{1m} = \frac{p_{2m} - p_{1m} + t}{2t} - \frac{m-1}{N} \text{ and } d_{2m} = \frac{m}{N} + \frac{p_{1m} - p_{2m} - t}{2t},$$

and their profits are,

$$\pi_{1m}(p_{1m}, p_{2m}) = p_{1m} d_{1m}, \text{ and } \pi_{2m}(p_{1m}, p_{2m}) = p_{2m} d_{2m}.$$

Firm i's, problem is,

$$\max_{p_{im} \geq 0} \pi_{im}(p_{1m}, p_{2m}), \text{ for each } m, \ m = 1, ..., N, \text{ and } i = 1, 2.$$

The next proposition summarizes the solution to the above problem.

[13] Throughout the paper, demand in each segment must be within the interval $[0, 1/N]$.

Proposition 1 *Assume that both firms acquire information and choose prices to maximize own profits. Then, for each N ($N \geq 2$), there exist two thresholds (integers) m_1 and m_2 (with $N+1 \geq m_2 > m_1 \geq 0$) such that:*

i) Firm 1's equilibrium demand is equal to 1/N in all segments from 1 to m_1, i.e., firm 1 is a constrained monopolist in these segments. Firm 2's equilibrium demand in these segments is zero. Moreover, firm 1's prices are: $p^*_{1m} = t(N-2m)/N$, while firm 2 sets $p^*_{2m} = 0$, $m = 1, ..., m_1$.

ii) Both firms share the demand in the segments from $m_1 + 1$ to $m_2 - 1$. Moreover, firm 1's prices are: $p^*_{1m} = t(N-2m+4)/3N$, and firm 2's prices are: $p^*_{2m} = t(2m-N+2)/3N$, $m = m_1+1, ..., m_2-1$.

iii) Firm 2's equilibrium demand is equal to 1/N in all segments from $m_2 - 1$ to N, i.e., firm 2 is a constrained monopolist in these segments. Firm 1's equilibrium demand in these segments is zero. Moreover, firm 2's prices are: $p^*_{2m} = t(2m-N-2)/N$, while firm 1 sets $p^*_{1m} = 0$, $m = m_2-1, ..., N$.

Moreover, these two thresholds have the following properties. When N is an even number, $m_1 = N/2 - 1$ and $m_2 = N/2 + 2$, whereas when N is an odd number, $m_1 = N/2 - 3/2$ and $m_2 = N/2 + 5/2$.

Finally, the equilibrium profits for both firms as a function of N are:

$$\pi_i^{BB}(N) = \frac{t(9N^2 - 18N + 40)}{36N^2}, \ i = 1, 2,$$

when N is an even number, and

$$\pi_i^{BB}(N) = \frac{t(9N^2 - 18N + 43)}{36N^2}, \ i = 1, 2,$$

when N is an odd number.

Proof. See appendix. ■

Numerical example

Suppose $N = 5$. Then $m_1 = 1$ and $m_2 = 5$, implying that firm 1 and 2 are constrained monopolists in segments 1 and 5 respectively. In segments 2, 3 and 4 both firms have strictly positive demands. The prices that firm 1 charges, starting from segment 1 are: $p_{11}^* = 9t/15$, $p_{12}^* = 5t/15$, $p_{13}^* = 3t/15$, $p_{14}^* = t/15$ and $p_{15}^* = 0$. Firm 2's prices are symmetric with the highest price in segment 5 and price equal to zero in segment 1. The equilibrium profits are $\pi_1^{BB} = \pi_2^{BB} = .1977t$.

The availability of information and the associated price discrimination force the two firms to compete more vigorously and as a result their profits drop compared to the standard benchmark no discrimination model. In the latter case the profit of each firm is $t/2$ while in the former, regardless of whether N is even or odd, profits are less than $t/2$ [see section 4 for a discussion of the results]. Firms compete for customers in each segment. Although each firm knows the location of each customer more precisely than when no information is available and therefore can charge him a price according to his willingness to pay, the fact that the other firm has the exact same information makes the competition more intense, i.e., the intensified competition effect dominates the surplus extraction effect. Our model becomes equivalent to that in Shaffer and Zhang (2000) if we set $N = 2$ in our model and $\theta = 1/2$, $l_\alpha = l_\beta = t$ in theirs [e.g. compare the equilibrium prices after Lemma 2, p.409 in Shaffer and Zhang (2000) with our results in Proposition 1].

3.1.2 Deviation from B vs. B

In order for B vs. B to be an equilibrium it must be the case that neither firm finds it profitable to deviate. A deviation in this case would be for a firm not to buy information, and charge a uniform price. To see if a unilateral deviation is profitable, we begin by determining the profit of a deviating firm.

Suppose firm 1 is the deviating firm. We know from the previous analysis that (before deviation) firm 2 charges a zero price in the segments from 1 to m_1; both firms charge positive prices in the segments from $m_1 + 1$ to $m_2 - 1$; and firm 1 charges a zero price in the segments from m_2 to N. It is quite obvious first of all that the deviating firm will have zero demand in the segments from m_2 to N. The reason is that firm 1 had a zero demand when it was charging a zero price (before deviation) and therefore when it deviates and charges a uniform strictly positive price its demand must be zero. Hence the problem of firm 1 is to find a price (p_1^d) to maximize its profits from segment 1 to segment $m_2 - 1$. The demand that firm 1 has in the first m_1 segments is,

$$d_1 = \frac{t - p_1^d}{2t}, \text{ with } d_1 \in \left[0, \frac{m_1}{N}\right].$$

For the segments which are above m_1, we can distinguish between two cases. One is when N is an even number and the other when it is an odd number.

When N is even, $m_1 = N/2 - 1$ and $m_2 = N/2 + 2$, which implies that the two firms have both positive prices in just two segments. In these two segments firm 2 charges $p_{2,m_1+1} = 2t/3N$ and $p_{2,m_1+2} = 4t/3N$. Therefore the demand of the deviating firm in these two segments is,

$$d_{1,m_1+1} = \frac{2t/3N - p_1^d + t}{2t} - \left(\frac{1}{2} - \frac{1}{N}\right) \text{ and } d_{1,m_1+2} = \frac{4t/3N - p_1^d + t}{2t} - \frac{1}{2}.$$

The deviating firm solves the following constrained maximization problem,

$$\begin{aligned} \max_{p_1^d} \pi_1^d &= p_1^d \left(d_1 + d_{1,m_1+1} + d_{1,m_1+2}\right), \\ \text{subject to} &: \quad d_1 \in \left[0, \frac{m_1}{N}\right] \text{ and } d_{1,m_1+1}, d_{1,m_1+2} \in \left[0, \frac{1}{N}\right]. \end{aligned}$$

When N is odd, $m_1 = N/2 - 3/2$ and $m_2 = N/2 + 5/2$, which implies that the two firms have both positive prices in just three segments. In these three segments firm 2 charges $p_{2,m_1+1} = t/3N$, $p_{2,m_1+2} = 3t/3N$ and $p_{2,m_1+3} = 5t/3N$. Therefore the demand of the deviating firm in these three segments is,

$$\begin{aligned} d_{1,m_1+1} &= \frac{t/3N - p_1^d + t}{2t} - \left(\frac{N-3}{2N}\right), d_{1,m_1+2} = \frac{3t/3N - p_1^d + t}{2t} - \left(\frac{N-1}{2N}\right) \\ \text{and } d_{1,m_1+3} &= \frac{5t/3N - p_1^d + t}{2t} - \left(\frac{N+1}{2N}\right). \end{aligned}$$

The deviating firm solves the following constrained maximization problem,

$$\max_{p_1^d} \pi_1^d = p_1^d \left(d_1 + d_{1,m_1+1} + d_{1,m_1+2} + d_{1,m_1+3} \right),$$

$$\text{subject to} \quad : \quad d_1 \in \left[0, \frac{m_1}{N}\right] \text{ and } d_{1,m_1+1}, d_{1,m_1+2}, d_{1,m_1+3} \in \left[0, \frac{1}{N}\right].$$

The next proposition summarizes the solution to the above problem.

Proposition 2 *The profits of the deviating firm are constant for any N greater than or equal to 8, i.e., $\pi^d(N) = .125t$ for $N \geq 8$.*

Proof. See appendix. ■

For $N = 2, ..., 7$ it is more efficient to solve the problem for each N individually and the results are presented in the table below,

N	2	3	4	5	6	7
π^d	$.2500t$	$.1975t$	$.1667t$	$.1334t$	$.1304t$	$.1327t$

Table 1

As the quality of information increases, a unilateral deviation from the set of strategies where both firms buy information becomes less profitable. The reason is that the deviating firm has to charge a uniform price and when N is high it becomes harder to steal demand from the non-deviating firm who has the flexibility to charge different prices in each segment. As the number of segments increases the deviating firm can only sell to consumers in its own territory (where it was the only seller before deviation). That is why after a critical level of information quality profits from deviation are constant.

Next, we analyze the case where one firm acquires the information but the other does not.

3.1.3 Only one firm buys information (B vs. NB)

Due to symmetry, we assume that only firm 1 has acquired information. Thus, firm 2 will charge a uniform price p_2, while firm 1 charges p_{1m} in segment m for $m = 1, ..., N$. Firms' demands in each segment are,

$$\begin{aligned} d_{1m} &= \frac{p_2 - p_{1m} + t}{2t} - \frac{m-1}{N}, \text{ for } m = 1, ..., N \\ d_{2m} &= \frac{m}{N} + \frac{p_{1m} - p_2 - t}{2t}, \text{ for } m = 1, ..., N, \end{aligned}$$

and the profit functions are,

$$\begin{aligned} \pi_1^{BNB}(N, p_{11}, ..., p_{1m}, ...p_{1N}, p_2) &= \sum_{m=1}^{N} p_{1m} d_{1m} \text{ and} \\ \pi_2^{BNB}(N, p_{11}, ..., p_{1m}, ...p_{1N}, p_2) &= p_2 \sum_{m=1}^{N} d_{2m}. \end{aligned}$$

Firm 1 chooses its price in each segment taking the uniform price that the other firm charges given. Hence, firm 1's problem is,

$$\max_{p_{11}, ..., p_{1m}, ...p_{1N}} \pi_1^{BNB}(N, p_{11}, ..., p_{1m}, ...p_{1N}, p_2),$$

$$\text{subject to: } d_{1m} \in \left[0, \frac{1}{N}\right], \text{ for } m = 1, ..., N$$

and firm 2's problem, taking firm 1's vector of prices as given, is,

$$\max_{p_2} \pi_2(N, p_{11}, ..., p_{1m}, ...p_{1N}, p_2),$$

$$\text{subject to: } d_{2m} \in \left[0, \frac{1}{N}\right], \text{ for } m = 1, ..., N.$$

We claim that there exist m_1 and m_2 ($N + 1 \geq m_2 > m_1 \geq 0$) such that: A) from segment 1 until m_1, firm 1 exactly drives firm 2 out of the market (left segments), B) from segment $m_1 + 1$ until $m_2 - 1$ both firms (strictly) share the demand (middle segments) and C) from segment m_2 until N, firm 1 is out of the market in each segment (right segments). Firm 2's segment profits are π_{2L} (case A), π_{2M} (case B), and π_{2R} (case C). Observe that if

$m_2 = N + 1$, then firm 1 is never out of the market, while if $m_1 = 0$, firm 2 is never out of the market. Hence, we allow for the possibility that the two firms share the market in all segments. The structure that we impose is that if firm 1 is out of the market in some segment m then it must also be out in any $m' > m$. Furthermore, if firm 2 is out of the market in some segment m, then it is out in any $m' < m$. The reason is clear and intuitive and we do not present the proof.

Note that in the left segments (segment 1 until m_1) firm 1 exactly drives firm 2 out of the market, i.e., firm 2's demand is exactly zero. This is true since firm 1 has the flexibility of charging different prices in each segment and if firm 2's demand in these segments was not exactly zero (but rather strictly negative) then firm 1 could increase its profits by increasing its price. But this would not be an equilibrium. Now fix $p_2 > 0$ and find firm 1's best reply (p_{1m}) to firm 2's price. Firm 2's profit function in the left segments π_{2L} is not differentiable at this pair (p_2, p_{1m}). To see this observe that firm 2 obtains zero profits in the left segments and by increasing its price profits remain zero, while by decreasing its price it enjoys strictly positive profits. The same may be true in the right segments where firm 1 is out of the market. However, π_{2M} is differentiable. Therefore, an equilibrium set of prices $(p_{11}^*, ..., p_{1N}^*, p_2^*)$ must satisfy, for firm 2, the following conditions,

$$\frac{\partial \pi_2(p_2^*-)}{\partial p_2} = \frac{-m_1 p_2^*}{2t} + \frac{\partial \pi_{2M}(p_2^*)}{\partial p_2} + \frac{N - m_2 + 1}{N} \geq 0, \tag{1}$$

which is the left derivative and guarantees that firm 2 does not want to decrease its price. The first term on the right hand side of Eq.(1) is what firm 2 would gain (in absolute terms) by lowering its price in the left segments when it is driven exactly out of the market in each segment m, $m = 1, ...m_1$. A marginal decrease in p_2 would increase firm 2's profits in each segment by $p_2/2t$ and consequently the whole increase in profits would be equal to $m_1 p_2/2t$. The second term on the right hand side of Eq.(1) represents the marginal change in firm 2's profits from the middle segments where demand is strictly shared with firm 1. In this case payoffs are continuously differentiable. The last term on the right hand side of Eq.(1) depicts the decrease in firm 2's profits in the right segments when it lowers its price. In the right segments, by definition, firm 2's demand is $(N - m_2 + 1)/N$ (i.e., it captures the entire market) and a marginal decrease in p_2 lowers the profits by $(N - m_2 + 1)/N$.

The next equation ensures that firm 2 has no incentive to increase its price, provided that firm 1 is not exactly out of the market, i.e., in the right segments firm 1's demand is strictly negative,

$$\frac{\partial \pi_2(p_2^*+)}{\partial p_2} = \frac{\partial \pi_{2M}(p_2^*)}{\partial p_2} + \frac{N - m_2 + 1}{N} \leq 0. \tag{2}$$

This is the right derivative of firm 2's profit function. In the left segments this derivative is now zero because an increase in p_2 yields no additional profits to firm 2 since it is already out of the market in these segments. The first term on the right hand side of Eq.(2) is the same as in Eq.(1) since in the middle segments the left and the right derivatives are the same. The last term of Eq.(2) is what firm 2 gains by increasing its price in the right segments, since at these prices firm 1's demand is strictly negative. Potentially, this can occur since firm 2 charges only a uniform price and does not have the flexibility to adjust its strategy in each market (segment).

If on the other hand, firm 1 is exactly out of the market in segment m_2, then $p_{1m_2}^* = 0$ (otherwise we can derive a contradiction; see above) and in all other segments to the right of m_2, firm 1 must be strictly (not exactly) out of the market. In this case the condition that must be satisfied is,

$$\frac{\partial \pi_2(p_2^*+)}{\partial p_2} = \frac{\partial \pi_{2M}(p_2^*)}{\partial p_2} + \frac{N - m_2 + 1}{N} - \frac{p_2^*}{2t} \leq 0, \tag{2$'$}$$

where the extra term is added because a marginal increase in firm 2's price lowers demand in segment m_2 by $1/2t$. We use Eqs.(1, 2 and $2'$) extensively to show the non-existence of a pure strategy price equilibrium for $N > 2$ when firm 1 has acquired information and firm 2 has not.

Proposition 3 *When only one firm acquires information and $N > 2$, no price equilibrium in pure strategies exists.*

Proof. See appendix. ■

Even if both firms had agreed that one would buy information and the other would not, they would not be able to coordinate their pricing strategies. In other words, there does not exist a set of prices (uniform for one firm and non-uniform for the other) such that no firm has an incentive to deviate from.[14] The non-existence of equilibrium in pure strategies does *not* depend on the linear transportation cost assumption. It follows from the proof of the above Proposition that as long as firm 1 is a constrained monopolist in at least the first two segments (i.e., $m_1 > 1$), then equilibrium in pure strategies does not exist independent of the transportation cost. Clearly for sufficiently high N's, $m_1 > 1$ holds. For example, with a quadratic transportation cost, $m_1 > 1$ if $N \geq 6$.[15]

Remark 3: Our model becomes equivalent to that in Shaffer and Zhang (2001), if we set $N = \infty$ and they set $\ell_A = \ell_B = t$ and a zero targeting cost, i.e., $z = 0$. They analyze a two stage game, where in the first stage the firms can commit to regular uniform prices P_A and P_B and in the second stage the firms, after having observed each other's regular prices, choose the discounts $(d_A(\ell), d_B(\ell))$ that they offer to each consumer ℓ. They show that a price equilibrium when only one firm targets (this is like our B vs. NB case) exists [see Proposition 2 b) in their paper]. This should be contrasted with our game where, in general, such an equilibrium does not exist (see the proposition 3 in our paper). It will be instructive at this point to highlight the differences between the two modeling approaches. Let's focus on the case in the Shaffer and Zhang game where only one firm targets (say firm A) and let $(\tilde{P}_A, \tilde{P}_B)$ be a proposed equilibrium pair of pricing strategies. If firm B deviates, in stage 1, by changing its price firm A will not be able to respond with a new regular price, but it will

[14] When the firms charge uniform prices but are located inside the unit interval, payoff functions may be discontinuous and nonconcave yielding a problem which is not well behaved. In this case, an equilibrium in pure strategies may not exist [see d' Aspremont et al. (1979)]. In our paper payoff functions are continuous, but the payoff function of the firm which has no information is not quasiconcave. The non-quasiconcavity of firm 2's payoff function stems from the *unbalanced* distribution of information and the resulting ability of firm 1 to charge many prices while firm 2 can charge only a uniform price. This results in a firm 2's payoff function which may have more than one peak depending upon the markets that firm 2 serves.

[15] A mixed strategy equilibrium will exist, since the payoff functions are continuous [see Dasgupta and Maskin (1986)]. However, computation of mixed strategy equilibria in this multi-dimensional continuous strategy space setting is very difficult, if not impossible.

be able to adjust, in the second stage, its level of discounts, which lessens the gain from a price cut. In this case firm B' marginal profit is the same whether P_B increases or decreases from $\tilde{P}_B$, i.e., firm B's payoff function is differentiable. In our game, a price deviation by the firm who charges a uniform price (say firm B) does not lead to any pricing adjustment by the firm who price discriminates (due to the simultaneity of the pricing decisions). This makes a price cut more profitable than in the sequential game and leads to the non-differentiability of firm B's payoff function and the ultimate non-existence of price equilibrium. The issue then is whether the firms can commit or not to a regular price. If they cannot commit, then the simultaneous pricing game is more appropriate, while if commitment is feasible (as it is assumed in Shaffer and Zhang) then the sequential game makes more sense.

Proposition 4 *When $N = 2$, $p_{11}^* = .75t$, $p_{12}^* = .25t$, $p_2^* = .5t$, $\pi_1^* = .3125t$, $\pi_2^* = .25t$, can possibly be supported as an equilibrium.*

Proof. From the proof of the above proposition, we know that when $N = 2$, the two firms share the demand in both segments, i.e., $m_1 = 0$ and $m_2 = 3$. The equilibrium prices are then derived by solving the first order Kuhn-Tucker conditions. ■

3.1.4 Deviation from B vs. NB

To support B vs. NB as an equilibrium it must be the case that no unilateral deviation is profitable. That is, firm 1 does not wish to deviate by not acquiring information and firm 2 finds it unprofitable to acquire information. We know that only $N = 2$ can possibly be supported as an equilibrium, and we give the results for $N = 2$.

We first look at firm 1 which deviates by not buying information, and solves the following maximization problem,

$$\max_{p_1^d} \pi_1^d = p_1^d \left(\frac{p_2^* - p_1^d - t}{2t} \right),$$

where p_2^* is given in proposition 4. The deviating firm takes the price of the other firm as given and charges a uniform price p_1^d equal to $.75t$ to maximize its profits. It can be shown that the profit is equal to $.28125t$.

Now we turn to the deviation by firm 2, which buys information and solves the following maximization problem,

$$\max_{p_{2m}^d} \pi_{2m}^d = p_{2m}^d \left(\frac{m}{2} + \frac{p_{1m}^* - p_{2m}^d - t}{2t} \right), \text{ subject to: } d_{2m} \in \left[0, \frac{1}{2} \right], \ m = 1, 2$$

where p_{1m}^*'s are given in proposition 4. The deviating firm takes the price of the other firm as given and since it has now acquired information it charges a different price in each segment to maximize its profits. It can be easily shown that $p_{21}^d = .375t$, $p_{22}^d = .625t$, and $\pi_2^d = .265625t$.

Next, we look for the equilibrium outcome when neither firm has acquired information.

3.1.5 No firm buys information (NB vs. NB)

In section 2, we saw that when no firm has access to the information the equilibrium pricing strategy is: $p_1^* = p_2^* = t$ and the equilibrium profits: $\pi_1^* = \pi_2^* = t/2$.

3.1.6 Deviation from NB vs. NB

To support NB vs. NB as an equilibrium it must be the case that no firm wishes to deviate and acquire information. The problem that a deviating firm (say firm 1) solves is,

$$\max_{p_{1m}^d} \pi_{1m}^d = p_{1m}^d \left(\frac{t/2 - p_{1m}^d + t}{2t} - \frac{m-1}{N} \right), \text{ subject to: } d_{1m} \in \left[0, \frac{1}{N} \right], \ m = 1, ..., N.$$

Proposition 5 *Suppose that firm 2 adheres to the NB vs. NB set of strategies, while firm 1 deviates and buys information. In this scenario, firm 1 will share the demand with firm 2* *only in the last segment, i.e., segment N. In all other segments firm 2 is exactly out of the market and the profit of the deviating firm is,*

$$\pi_1^d(N) = \frac{t(2N^2 - 2N + 1)}{2N^2}$$

Proof. See appendix. ■

3.2 Stage 1: Information price

The price of information $\alpha(N)$ will dictate the equilibrium played by the two Hotelling producers. If for example the information price is extremely high no firm will acquire it and this is an equilibrium. We begin by finding the range of information price that will support that both firms buy information (B vs. B) as an equilibrium.

<u>Case 1:</u> **Both firms buy information (B vs. B).**

Recall that each firm's profit is $t(9N^2 - 18N + 40)/36N^2$ when N is even and $t(9N^2 - 36N + 43)/36N^2$ when N is odd. If one firm deviates while the other firm sticks to its strategies in B vs. B, the deviating firm's profit is $t/8$ for $N \geq 8$. Therefore, the maximum information price such that B vs. B is an equilibrium is the difference between the profits before and after deviation, which turns out to be,

$$\alpha^{BB}(N) = \frac{t(9N^2 - 36N + 80)}{72N^2},$$

when N is even and

$$\alpha^{BB}(N) = \frac{t(9N^2 - 36N + 86)}{72N^2},$$

when N is odd ($N \geq 8$).

For $N < 8$ we can compute the maximum information price that supports B vs. B as an equilibrium, using table 1. The results are given in the table below.

N	2	3	4	5	6	7
α^{BB}	$.0278t$	$.0185t$	$.0278t$	$.0644t$	$.0671t$	$.0702t$

Table 2

Hence, $\alpha^{BB}(N)$ can be viewed as each firm's willingness to pay for an information of quality N.

<u>Case 2:</u> Only one firm buys information (B vs. NB)

Proposition 6 *B vs. NB may be supported as an equilibrium when* $N = 2$.

Proof. From proposition 3, we know that B vs. NB can possibly be supported as an equilibrium only when $N = 2$. From proposition 4 the profits of the two firms are, $(.3125t, .25t)$. From section 3.1.4 the profits from the two separate unilateral deviations are $(.28125t, .2656t)$. Thus, for any $\alpha(N = 2) \in [.0156t, .03125t]$, B vs. NB is an equilibrium. ■

Hence, when $N = 2$, there are two equilibria B vs. B and B vs. NB when the information price $\alpha(N = 2)$ is between $.0156t$ and $.0278t$. Recall that, in this case, when $N > 2$ there is no pure strategy price equilibrium.

<u>Case 3:</u> No firm buys information (NB vs. NB).

Recall that before deviation each firm charges a uniform price t and enjoys a profit of $t/2$. If one firm deviates while the other firm sticks to NB vs. NB, we show that the deviating firm's profit is $t(2N^2 - 2N + 1)/2N^2$. To support NB vs. NB as an equilibrium, the information price must be greater than or equal to the difference between these two profits, i.e., $\alpha(N) \geq \alpha^{NBNB}(N) = t(N-1)^2/2N^2$.

It can be easily checked by looking at cases 1 and 3 that $\alpha^{BB}(N) < \alpha^{NBNB}(N)$. It then follows, that as long as the information price is less than $\alpha^{BB}(N)$, it is each firm's *dominant strategy* to acquire information and price discriminate making B vs. B the equilibrium of the game; except when $N = 2$ where the game has two equilibria. In section 4, we assume that $\alpha(N) < \alpha^{BB}(N)$ and that the firms play B vs. B (even when $N = 2$) and we study the evolution of the equilibrium of the game as N varies.

3.3 Perfect discrimination

In this section, we digress from our approach so far and we assume that firms have perfect information about each consumer and consequently can charge each one a different price. In the next section, we use the results from this section to compare the perfect discrimination case with the equilibrium in our game as $N \to \infty$.

Each consumer is paying a different price, depending on his location. Consider the consumer who is located at $x < 1/2$. Both firms know his exact location and the equilibrium prices are $p_1^* = t(1-2x)$ and $p_2^* = 0$. Therefore, the consumer who is located at $x = 0$ pays $p_1^* = t$ for the product of firm 1, while firm 2 charges a zero price. The consumer who is located at $x = 1/2$ pays a zero price. Hence the average price that firm 1 charges is $t/2$ and firm 1's profits equal to $t/4$. Since the problem is symmetric the same holds for firm 2.

4 Main results

Below, we discuss the equilibrium properties and the implications for the firms, the consumers and the social welfare.

IMPLICATIONS FOR FIRMS

It can be easily seen that both firms become worse off under B vs. B compared to NB vs. NB. When they charge a uniform price firms' profits are $t/2$, whereas when they possess information and price discriminate their profits (before paying the information price), as given in proposition 1, are lower than $t/2$ for $N > 1$. Figure 2 graphs the equilibrium profits (for even $N's$ and before $\alpha(N)$ is deducted from the firm's profit) as a function of the quality of information. As $N \to \infty$ the problem is equivalent to the perfect discrimination case of section 3.3, i.e.,

$$\lim_{N\to\infty} \pi^{BB}(N) = \lim_{N\to\infty}\left[\frac{t(9N^2-18N+40)}{36N^2}\right] = \frac{t}{4} \text{ for } N \text{ even}$$

and

$$\lim_{N\to\infty} \pi^{BB}(N) = \lim_{N\to\infty} \left[\frac{t(9N^2 - 18N + 43)}{36N^2}\right] = \frac{t}{4} \text{ for } N \text{ odd.}$$

For $N > 1$ the equilibrium profits initially decrease but quickly they recover and approach the perfect discrimination profits. This implies that as the quality of information increases and after a certain threshold, the "surplus extraction effect" becomes less dominated by the "intensified competition effect."

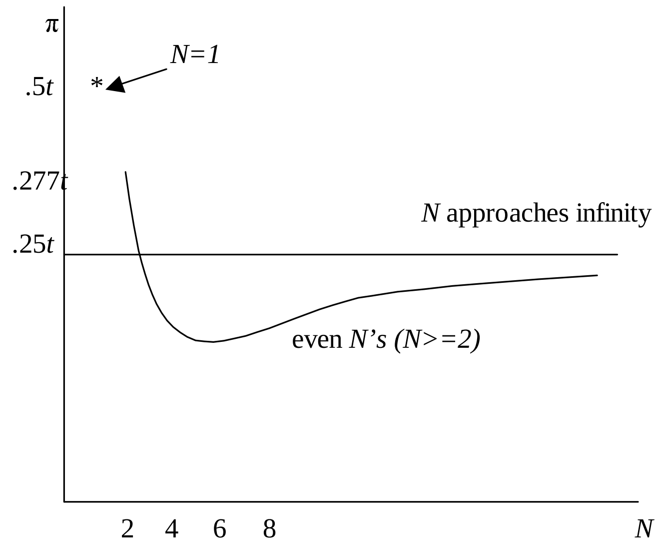

Figure 2: Equilibrium profit as a function of the information quality

Now if we assume that firms are already price discriminating and are positioned on the increasing part of the U-shape curve, they will have an incentive to move to a higher information quality (relative transition). Hence, the game played between the two firms, in this setting, is not a prisoner's dilemma. This will be true even if we combine the even with the odd N's. Due to space limitations, we do not examine the existence or non-existence of an asymmetric equilibrium (B vs. NB) in this setting, or the deviation from B vs. B. Under some assumptions about the evolution of information, it can be shown in a similar to the proof of proposition 3 manner, that in general an equilibrium under B vs. NB does not exist. Also, it can be easily proved that for sufficiently low information cost B vs. B is an equilibrium.

IMPLICATIONS FOR CONSUMERS

The welfare implications for the consumers are as follows. Ignoring for the moment the transportation cost, consumers become better off compared to the no discrimination case, as the prices that each firm charges are uniformly below the non-discriminatory price t for any N. To see this consider for example firm 1 (the same holds for firm 2). When firm 1 is a constrained monopolist it charges a price equal to $t(N-2m)/N \leq t$, $m = 1, ..., m_1$. A similar result holds in the segments where the two firms share the customers. The reason is that markets are treated asymmetrically by the two firms. Firm 1's strongest market (i.e., the group of consumers closest to firm 1) is firm 2's weakest market and so on. This is called *best-response asymmetry* which is a necessary condition for price discrimination to yield unambiguous price effects [see Corts and Shaffer and Zhang (2000)].

The average price of each firm over the consumers who buy from this firm is 2 times its profit, i.e.,

$$AVP(N) = \frac{t(9N^2 - 18N + 40)}{18N^2},$$

if N is even and

$$AVP(N) = \frac{t(9N^2 - 18N + 43)}{18N^2},$$

if N is odd. Therefore, the average price that consumers have to pay also exhibits a U-shape pattern similar to that in figure 2. That is, even though all consumers pay lower prices when the firms price discriminate, the average price (after a given threshold) increases as the quality of information increases. In addition to the price, consumers also incur the transportation cost, which is calculated below.

Transportation cost: Due to symmetry we only calculate the average transportation cost of the consumers who buy from firm 1. The calculation is done as follows. In each segment m we find the transportation cost of the middle consumer among the ones who buy from firm 1, we multiply it by firm 1's demand in that segment, and we sum up over all segments that firm 1 has a strictly positive demand. By combining this with the average transportation cost incurred by the consumers who buy from firm 2, we obtain,

$$AVTC = \frac{t(9N^2 + 8)}{36N^2},$$

if N is even and,

$$AVTC = \frac{t(9N^2 + 14)}{36N^2},$$

if N is odd.[16]

Hence as $N \to \infty$ the average transportation cost approaches $.25t$ which is the same as that under NB vs. NB. That is, in the limit every consumer buys from the closest firm.

Combining the average transportation cost with the average price $AVP(N)$ we can compute the average consumer surplus (given a sufficiently high V), which is,

$$AVCS = V - \frac{t(27N^2 - 36N + 88)}{36N^2},$$

for even $N's$, and,

$$AVCS = V - \frac{t(27N^2 - 36N + 100)}{36N^2},$$

for odd N's .

Consumer surplus exhibits an inverse U-shape as a function of N, implying that moderate levels of information yield the highest surplus. Moreover, when no information is available the average consumer surplus is $V - 1.25t$, which can be easily seen that it is always less than that when information is available for any N. Therefore, consumers are better off when firms use information than when no information is used, but given that information is used there is an optimal (for the consumer welfare) level which is finite. After the peak of the consumer welfare, some consumers pay higher prices as information quality increases.

SOCIAL WELFARE

We assume that the social cost of generating information of quality N is sunk. Therefore, the average social welfare is $V - AVTC$, i.e., the value that consumers derive from buying

[16]The file with the calculations is available upon request.

a good minus the average transportation cost. The market outcome when no information is available and firms do not price discriminate is Pareto optimal (given the locations of the firms).[17] Hence, the availability of information cannot improve upon this outcome. Actually, for any $N > 1$ total welfare is lower than when $N = 1$. The reason is that the transportation cost is not minimized because as we show in proposition 1 the firms share the demand in the middle two (if N is even) or in the middle three (if N is odd) segments. For example, a consumer who is located at $x < 1/2$ (i.e., in firm 1's "back yard"), but in the right side of segment $m_1 + 1$, now is purchasing from firm 2 resulting in an unnecessary increase in transportation cost. This is due to the finer information that both firms have about the locations of consumers and the ensuing price competition. However, if we look at a relative transition, and as the quality of information improves the "transportation loss" gradually vanishes as now the size of the intervals where the two firms share their demand shrinks and in the limit (as $N \to \infty$) the size tends to zero. In the next section, we consider an extension of our basic model where in the absence of information the market is not fully covered. The introduction of information and the ability of firms to price discriminate may yield a Pareto improving equilibrium outcome where all consumers are served.

5 Extensions (Uncovered market)

The fact that we started our analysis assuming that the market is covered (i.e., all consumers buy either one of the goods) has contributed to the definite loss of efficiency when information becomes available. Suppose instead that firms have to charge a uniform price and some consumers in the middle of the unit interval are better off not buying from either one of the two firms. Hence, the utility that these consumers derive is zero. When information becomes available, and the firms can charge different prices to different consumer groups, some of the consumers, who were not buying before, can now be attracted by the firms by being offered the good at a lower price. Therefore, there is a welfare increasing effect due

[17] An outcome is Pareto optimal when the transportation cost is minimized. When the locations of the two firms are exogenously fixed at 0 and 1 respectively, and no information is available, then each consumer buys from the nearest firm.

to information when the market is initially not fully covered, which has been ignored in our modelling approach. It would not be surprising in this case that, for some sufficiently high levels of information quality, total welfare exceeds the one under uniform pricing. To illustrate this possibility, we consider a range of $V's$ where when $N = 1$ the market is not covered, while for any $N \geq 2$ the market is covered.

A) $N = 1$

The next Proposition summarizes the equilibrium in the game played by the two Hotelling producers for all positive V's (not just for sufficiently high V's that we have implicitly assumed so far) when no information is available.

Proposition 7 *Suppose firms do not possess information (NB vs. NB). Then when,*

i) $V \geq 1.5t$, *there is a unique pure strategy equilibrium with* $p_1^* = p_2^* = t$ *and* $\pi_1^* = \pi_2^* = .5t$,

ii) $1.5t > V > 1.2t$, *there is a continuum of pure strategy equilibria such that for any* $p_1 \in [1.333V - t, .6667V]$, $p_2 = 2V - t - p_1$ *and* $\pi_1 = p_1(V - p_1)/t$, $\pi_2 = (2V - t - p_1)(p_1 + t - V)/t$,

iii) $1.2t \geq V > t$, *there is a continuum of pure strategy equilibria such that for any* $p_1 \in [.5V, 1.5V - t]$, $p_2 = 2V - t - p_1$ *and* $\pi_1 = p_1(V - p_1)/t$, $\pi_2 = (2V - t - p_1)(p_1 + t - V)/t$,

iv) $t \geq V > 0$, *there is a unique pure strategy equilibrium with* $p_1^* = p_2^* = .5V$ *and* $\pi_1^* = \pi_2^* = V^2/4t$.

In i), ii) and iii) all consumers are served (covered market), while in iv) only a fraction of the consumers buys from either firm (uncovered market).

The proof is omitted for brevity.[18]

Since we only care about uncovered market, we focus on case iv) above. It can be easily calculated in this case that the social welfare is,

$$SW = \frac{3V^2}{4t}. \tag{3}$$

[18] Omitted proofs are available upon request.

B) $N \geq 2$

Now assume that both firms have acquired information. The next proposition gives a range of $V's$ which guarantee that the market will be fully covered for the associated levels of information quality.

Proposition 8 *For any*

$$V \geq \underline{V} = \frac{(N+2)t}{2N}$$

the market is covered in the B vs. B case when the quality of information is N.

Proof. See appendix. ■

We would like to find a range of $V's$ such that the market is not fully covered when no information is available (NB vs. NB), but it is always covered when both firms have information (B vs. B). The existence of such a range will highlight the social welfare increasing properties of consumer information which have been assumed away in the main part of the paper. In particular, we show that information of sufficiently high quality is beneficial for the social welfare. The next Proposition, which is only concerned with an absolute transition, states this result.

Proposition 9 *For any $V \in [(N+2)t/2N, t)$ there exists an $N^*(V)$ such that for any $N \geq N^*(V)$ social welfare under B vs. B is strictly greater than that under NB vs. NB.*

Proof. Since the price that consumers pay is just a transfer, we can focus on the transportation cost. When firms possess information, the transportation cost is not minimized but it tends to its minimum value $t/4$ as $N \to \infty$ [see section 4 where we introduced the average transportation cost formula]. Also, and given Propositions 7 and 8, the market under B vs. B is fully covered when $V \geq (N+2)t/2N \geq t/2$, but not covered under NB vs. NB when $V < t$. Let $N \to \infty$. Then, the social welfare under B vs. B which is,

$$V - \frac{t}{4},$$

is strictly greater [when $V \in (.5t, t)$] than that under NB vs. NB which is [see Eq.(3)],

$$\frac{3V^2}{4t}.$$

Therefore there exists some finite N^* such that for every level of quality greater than N^* the availability of information strictly improves social welfare. ■

It should be pointed out that when $V \in [t(N+2)/2N, t)$ the profits of the two firms under B vs. B are different from those when V is sufficiently high as it is implicitly assumed in Proposition 1. The next proposition presents the equilibrium of the game that is played between the two firms for the range of $V's$ that we considered in Proposition 9.

Proposition 10 *Suppose that $V \in [t(N+2)/2N, t)$ and $N \geq 4$ (If $N = 2$ or 3, then firms share the demand in every segment, and the result is the same as in Proposition 1). Then there exists the lowest integer n ($1 \leq n \leq m_1 + 1$) that is greater than or equal to $(tN - VN)/t$, such that under B vs. B when,*

i) $1 \leq m < n$ the marginal consumer is indifferent between buying from firm 1 and not buying at all. In this case, $p^*_{1m} = -(tm - VN)/N$, $p^*_{2m} = 0$.

ii) $n \leq m \leq m_1$ the marginal consumer is indifferent between buying from firm 1 and firm 2. Firm prices are the same as in proposition 1.

iii) $m_1 < m < m_2$ same as the middle segments in proposition 1.

The rest follows from symmetry.

Finally, the equilibrium profits for both firms as a function of N are:

$$\pi_i^{BB}(N) = \frac{40t + 36VNn + 18tn^2 + 18tn - 18tN + 9tN^2 - 36tNn}{36N^2}, \ i = 1, 2,$$

when N is even and

$$\pi_i^{BB}(N) = \frac{43t + 36VNn + 18tn^2 + 18tn - 18tN + 9tN^2 - 36tNn}{36N^2}, \ i = 1, 2,$$

when N is odd.

We omit the proof for brevity.

Clearly, the set of conditions under which B vs. B is an equilibrium when $V \in [t(N+2)/2N, t)$ are different from the ones of section 3.2, case 1. We do not pursue this issue further and we simply assume that B vs. B is an equilibrium. Moreover, we do not examine at all the case of an asymmetric equilibrium B vs. NB.

We now show, by means of a numerical example, how some of our results in the main body of the paper depend critically on the value of V. In particular, the game played between the two firms is no longer a prisoners' dilemma and some consumers, when the firms price discriminate, pay higher prices than the uniform non-discriminatory price.

Numerical example

Suppose $V = .8t$ and $N = 6$. From proposition 10 we know that $n = 2$ and from proposition 1 that $m_1 = 2$ and $m_2 = 5$. We focus on firm 1. The prices, under B vs. B, that firm 1 charges are $p_{11}^* = .6333t$, $p_{12}^* = .3333t$, $p_{13}^* = .2222t$, $p_{14}^* = .1111t$, $p_{15}^* = p_{16}^* = 0$ and its profit is $\pi_1^* = .192t$. Notice that under NB vs. NB, as in proposition 7, when $V = .8t$ then $p_1^* = .4t$ and $\pi_1^* = .16t$. Hence, some consumers pay higher prices when the firms price discriminate and equilibrium profits are higher when the firms buy information, if its cost is sufficiently low.

Most of the existing literature has analyzed price discrimination under the assumption that the market is covered, e.g. Corts and Shaffer and Zhang (2000, 2001). When we allow for an uncovered market, however, and even under an absolute transition, it is possible that many standard results are reversed. As the above numerical example illustrates, the game played between the two firms may no longer be a prisoners' dilemma, and some consumers may end up paying higher prices when firms price discriminate.

6 Concluding remarks and future research

In this paper, we investigated the role of information about consumer characteristics on equilibrium prices, profits, and welfare. We employed Hotelling's model of horizontal product differentiation with the assumption that the firms acquire information with the intention to price discriminate. We allowed the information to be of various levels of quality, trying to emulate reality where the quality of information about consumer characteristics increases as the IT constantly improves. Our results are summarized as follows.

■ Absolute transition

1) We show that for very low levels of information quality ($N = 2$) there are two equilibria. A symmetric one where both firms buy information and price discriminate and an asymmetric one where only one firm decides to purchase information and engage in price discrimination. For high levels of information quality ($N > 2$), however, the asymmetric equilibrium ceases to exist. Focusing on the symmetric equilibrium, the game played between the two firms is a prisoners' dilemma with "buying information and price discriminating" being the dominant strategy resulting in lower equilibrium profits than when both firms do not possess information and do not price discriminate. Moreover, the equilibrium profits (before the firms deduct the cost of information) exhibit a U-shape pattern as a function of the information quality. That is, initially for low quality of information, equilibrium profits decrease as the quality increases. Eventually, firms' profits increase with information quality, but are still lower than the profits under no price discrimination.

2) Consumers pay lower prices when firms price discriminate than when the option of price discrimination is not available. This indicates that consumers are made unambiguously better off, reminiscent of the results in Corts and Shaffer and Zhang (2000). Consumer welfare is maximized at moderate levels of information quality.

3) There is a loss of efficiency associated with the availability of information which decreases as the information quality increases. This result hinges upon the assumption we

make in the main body of the paper that the market is fully covered. If the market is not covered when no information is available, then the presence of information may lead to a Pareto improvement where all consumers are being served. Therefore, with finer information about the consumers' willingness to pay the firms, through price discrimination, can reach out to new consumers, something which was not possible under a uniform pricing strategy. In addition, the game need not be a prisoners' dilemma and some consumers may end up paying higher discriminatory prices than the uniform one.

■ Relative transition

Suppose firms already engage in price discrimination based on information of a given quality level which exceeds a certain threshold.

1) It is each firm's dominant strategy to acquire information of a higher quality, provided that its cost is low, which also yields higher profits. Hence, the game played between the two firms when it is analyzed from this perspective is not a prisoners' dilemma.

2) Since consumer welfare is maximized at moderate levels of information quality, better information makes the consumers, on average, worse off. Therefore, relatively speaking, some consumers pay higher prices, while others pay lower prices.

3) Social welfare increases as the information quality increases, since as mentioned above, the loss from transportation cost keeps decreasing.

The model we used is very specific and more research needs to be done on this issue before one can derive general insights and policy suggestions. This work can be extended at least in the following four directions.

First, the duopoly assumption can be relaxed to allow for n firms (circular model) in order to study the effects of the quality of information on entry decisions and ultimately on social welfare; second, the demand may be assumed to be elastic [e.g., Rath and Zhao

(2001)]; third, the present model can be modified to address the same issues in a vertically differentiated market; and fourth, we can address the issue of collusion both on prices and on information acquisition.

APPENDIX

Proof of proposition 1

Proof. We will analyze the following three cases.

Case 1: Both firms charge strictly positive prices.

Ignoring the nonnegativity constraints and setting $\partial\pi_i/\partial p_i = 0$, $i = 1, 2$, we can the have following solutions for the prices,

$$p_{1m} = \frac{t(N - 2m + 4)}{3N} \text{ and } p_{2m} = \frac{t(2m - N + 2)}{3N}.$$

Using these prices we obtain the demands,

$$d_{1m} = \frac{-2m + N + 4}{6N} \text{ and } d_{2m} = \frac{2m - N + 2}{6N}.$$

We can see that d_{1m} is decreasing in m, and d_{2m} is increasing in m. This means that firm 1 may decide to charge a zero price and give the entire segment demand to firm 2 for segments that are in firm 2's territory. The same holds for firm 2 in segments that are in firm 1's territory. For segments in the middle of the interval both firms charge positive prices. Observe that $d_{2m} = (2m - N + 2)/6N \leq 0$ for any $m \leq N/2 - 1$ and $d_{1m} = (N - 2m + 4)/6N \leq 0$ for any $m \geq N/2 + 2$. Now define $m_1(N)$ to be the highest integer that is less than equal to $N/2 - 1$, and $m_2(N)$ to be the lowest integer that is higher than or equal to $N/2 + 2$. Observe at this point that when N is even, $m_1 = N/2 - 1$ and $m_2 = N/2 + 2$, whereas when N is an odd number $m_1 = N/2 - 3/2$ and $m_2 = N/2 + 5/2$. This will be used later in the proof. Hence, for any $m = m_1 + 1, ..., m_2 - 1$, both firms charge strictly positive prices and have strictly positive demand.

Case 2: Firm 1 charges strictly positive prices while firm 2 charges a zero price.

Following the analysis above, this case is valid for $m \leq m_1$. Then $d_{2m} \leq 0$. This implies that $d_{2m} = 0$ and $d_{1m} = 1/N$. This further implies that $p_{2m} = 0$, and p_{1m} is the solution to $d_{1m}(p_{2m} = 0) = 1/N$, which yields $p_{1m} = t(N - 2m)/N$.

Case 3: Firm 2 charges strictly positive prices while firm 1 charges a zero price.

This case is valid for $m \geq m_2$. This case is symmetric to case 2. Firm 2's prices in these segments are: $p_{2m} = t(2m - N - 2)/N$.

Below we summarize the results:

The equilibrium prices and profits are,

if $m_1 + 1 \leq m \leq m_2 - 1$,

$$\begin{aligned} p_{1m} &= \frac{t(N-2m+4)}{3N} \text{ and } p_{2m} = \frac{t(2m-N+2)}{3N} \\ d_{1m} &= \frac{-2m+N+4}{6N} \text{ and } d_{2m} = \frac{2m-N+2}{6N} \\ \pi_{1m}^{BB}(N) &= \frac{t(2m-N-4)^2}{18N^2} \text{ and } \pi_{2m}^{BB}(N) = \frac{t(2m-N+2)^2}{18N^2}, \end{aligned}$$

if $m \leq m_1$,

$$\begin{aligned} p_{1m} &= \frac{t(N-2m)}{N} \text{ and } p_{2m} = 0 \\ d_{1m} &= \frac{1}{N} \text{ and } d_{2m} = 0 \\ \pi_{1m}^{BB}(N) &= \frac{t(N-2m)}{N^2} \text{ and } \pi_{2m}^{BB}(N) = 0, \end{aligned}$$

and if $m \geq m_2$, then

$$\begin{aligned} p_{1m} &= 0 \text{ and } p_{2m} = \frac{t(2m-N-2)}{N} \\ d_{1m} &= 0 \text{ and } d_{2m} = \frac{1}{N} \\ \pi_{1m}^{BB}(N) &= 0 \text{ and } \pi_{2m}^{BB}(N) = \frac{t(2m-N-2)}{N^2}. \end{aligned}$$

Therefore, firms' profits for each N are:

$$\begin{aligned} \pi_1^{BB}(N) &= \sum_{m=1}^{m_1} \frac{t(N-2m)}{N^2} + \sum_{m=m_1+1}^{m_2-1} \frac{t(2m-N-4)^2}{18N^2}, \\ \pi_2^{BB}(N) &= \sum_{m=m_1+1}^{m_2-1} \frac{t(2m-N+2)^2}{18N^2} + \sum_{m=m_2}^{N} \frac{t(2m-N-2)}{N^2}. \end{aligned}$$

Now we need to express m_1 and m_2 as a function of N (see case 1 in this proof where we have the thresholds as a function of N for even and odd numbers), so that we will be able to perform the summation. To this end, we distinguish between two cases: I) When N is an even number and II) when N is an odd number.

N is even. In this case the summation yields,

$$\pi_i^{BB}(N) = \frac{t(9N^2 - 18N + 40)}{36N^2}, \ i = 1, 2.$$

N is odd. In this case the summation yields,

$$\pi_i^{BB}(N) = \frac{t(9N^2 - 18N + 43)}{36N^2}, \ i = 1, 2.$$

■

Proof of proposition 2

Proof. Let $p_1^*(N)$ and $p_2^*(N)$ be the equilibrium price vectors of the two firms prior to deviation [see proposition 1]. We divide the $N's$ into an even and an odd sub-sequences.

We first start with the even sub-sequence. In this case, $m_1 = N/2 - 1$ and $m_2 = N/2 + 2$, and the two firms share two segments, $N/2$ and $N/2 + 1$. Firm 2's prices in these two segments are $2t/3N$ and $4t/3N$.

First we show that the deviating firm's (firm 1) profit decreases as N increases. To prove this we need the following two observations (A and B). Consider N_1 and N_2 with $N_2 > N_1$.

A) We know that any consumer with location $\hat{x} > 1/2 + 1/N_2$, will not buy from firm 1 under $(p_1^*(N_2), p_2^*(N_2))$ before firm 1's deviation. After deviation, firm 1 will still get zero demand for $\hat{x} > 1/2 + 1/N_2$. This is true since before deviation firm 1 was charging zero prices in these segments having a zero demand. An optimal deviating price must be strictly positive and therefore firm 1's demand in these segments must still be zero. As N increases the length for which firm 1 gets zero demand increases, i.e., $\hat{x} > 1/2 + 1/N_1 > 1/2 + 1/N_2$.

B) Furthermore, any consumer with location $\hat{x} \leq 1/2 + 1/N_2$ $(\leq 1/2 + 1/N_1)$ faces a weakly lower price from firm 2 when $N = N_2$ than when $N = N_1$ [see firm 2's prices above].

Combining A and B, it can be easily seen that firm 1's profit is weakly lower when $N = N_2$ than when $N = N_1$. This proves that in the even sequence the profits of the deviating firm weakly decrease as N increases.

A similar argument will prove that the profits of the deviating firm are weakly decreasing in N, when N is an odd subsequence.

Second, we will show that the deviating firm cannot have profits lower than $.125t$. This in turn will imply that once the profits of the deviating firm are $.125t$ for some N they stay there for any greater N.

Firm 1 before deviation is a constrained monopolist in the segments up to m_1, with $d_1 = (t - p_1)/2t$ in these segments. Thus, firm 1 even if it gets no demand in any other segments, following its deviation, it can certainly maximize its profits in the segments up to m_1. The optimal deviating price is $p_1^d = t/2$, $d_1 = 1/4$ and $\pi_1^d = .125t$.

Finally, it turns out (by solving the deviating firm's problem) that when $N = 8$ and 9, $\pi_1^d = .125t$. Hence for any $N \geq 8$ the deviating firm experiences profits equal to $.125t$ ■

Proof of proposition 3

Proof. What we would like to show is that there does not exist a set of prices $(p_{11}, ..., p_{1N}, p_2)$ which satisfy Eqs.(1 and 2) or Eqs.(1 and $2'$). We tackle this by showing that for any vector of prices that firm 1 charges and are best response to some p_2, firm 2 would always have an incentive to deviate from this p_2.

We begin by proving that:

$$i)\ N \geq 4 \Longrightarrow m_1 > 0 \text{ and } ii)\ N \geq 6 \Longrightarrow m_1 > 1.$$

Firm 1's reaction functions in the middle segments are,

$$p_{1m} = \frac{-2tm + p_2 N + tN + 2t}{2N}, \text{ for } m_2 > m > m_1. \tag{A1}$$

Assume that both firms strictly share the demand in segment m. Firm 2 charges a uniform price $p_2 > 0$, and by plugging Eq.(A1) into firm 2's demand $d_{2m} = m/N + (p_{1m} - p_2 - t)/2t$, we obtain

$$d_{2m} = \frac{2tm + 2t - Np_2 - tN}{4tN}$$

To prove i) suppose by way of contradiction that $N \geq 4$ but $m_1 = 0$. This implies that both firms share the demand in the first segment, i.e., $1/N > d_{21} > 0$. By setting $m = 1$ (this is equivalent to $m_1 = 0$), we obtain,

$$d_{21} = \frac{4t - Np_2 - tN}{4tN},$$

which is always negative if $N \geq 4$. Contradiction. Similarly, we can prove ii).

To prove the main claim of this proposition we look at the following 4 cases.

Case 1: $m_1 > 0$ and firm 2 does not drive firm 1 exactly out of market in any segment (i.e., with $p_2 + \varepsilon$, firm 1 is still out of market in all segments $m \geq m_2$).[19]

Since $m_1 > 0$ (by the assumption of this case), $p_2 > 0$ ($p_2 = 0$ obviously will not be best reply since firm 2 has strictly positive demand in segments $m \geq m_2$ and by increasing its price by ε its profits will also increase), and $t > 0$ it is implied that Eq.(1)<Eq.(2). Therefore, Eq.(1)≥ 0 and Eq.(2)≤ 0 will never hold simultaneously. Thus, there is no such an equilibrium.

Case 2: $m_1 > 0$ and firm 2 drives firm 1 exactly out of market in segment m_2.

Now the relevant conditions for firm 2 are given by Eq.(1) and Eq.($2'$). The only time that Eq.(1)$\geq$0 and Eq.(2)$\leq$ 0 hold simultaneously is when $m_1 = 1$, and both inequalities become equalities. We have shown that $N \geq 6$ implies that $m_1 > 1$. So we only need to check for N between 2 and 5. The contradiction in this case is obtained as follows. We differentiate firm 2's profit function in the middle segments,

$$\pi_{2M} = p_2 \sum_{m=m_1+1}^{m_2-1} d_{2m} = p_2 \sum_{m=m_1+1}^{m_2-1} \left(\frac{m}{N} + \frac{p_{1m} - p_2 - t}{2t} \right)$$

with respect to p_2, plug in Eq.(A1) and then sum up over m from m_1+1 to m_2-1 to obtain,

$$\frac{\partial \pi_{2M}}{\partial p_2} = \frac{-3m_2 + 3 + 3m_1}{4t} p_2$$

[19] If firm 1 is not out of the market in any segment, then $m_2 = N+1$, which is implicitly allowed for in all of the cases that we investigate.

$$+\frac{m_2 - m_2 N + m_2^2 + N m_1 + N - 3m_1 - m_1^2 - 2}{4N}.$$

We set $m_1 = 1$ and examine $N = 2, 3, 4$ and 5. We plug $\partial \pi_{2M}/\partial p_2$ from above into Eq.(1) and we solve Eq.(1) $= 0$ for $p_2(m_2)$, which we plug it into Eq.(A1). This gives us a candidate equilibrium prices $(p_{1m}(m_2), p_2(m_2))$, which we plug them back into both firms' demand functions in the middle segments, $m = m_1 + 1, ..., m_2 - 1$. Observe that the only free parameter now is m_2. Next, we try to find an m_2 such that the equilibrium prices and $(m_1 = 1, m_2)$ constitute an outcome which is consistent with the assumptions of this case. However, for all the $N's$ that we examined we did not find such a consistent outcome.[20]

Cases 1 and 2 show that when firm 2 is out of the market in at least one left segment, then there does not exist a price equilibrium in pure strategies.

The next two cases, deal with the situation where both firms share the demand in all the left segments, i.e., firm 2 is never out of the market in any segment.

Case 3: $m_1 = 0$ and firm 2 does not drive firm 1 exactly out of market in any segment

In this case Eq.(1) becomes,

$$\frac{\partial \pi_2(p_2-)}{\partial p_2} = \frac{\partial \pi_{2M}}{\partial p_2} + \frac{N - m_2 + 1}{N} \geq 0,$$

while Eq.(2) remains the same, i.e.,

$$\frac{\partial \pi_2(p_2+)}{\partial p_2} = \frac{\partial \pi_{2M}}{\partial p_2} + \frac{N - m_2 + 1}{N} \leq 0.$$

We have shown that $N \geq 4$ implies that $m_1 > 0$. So we only need to check $N = 2$ and 3. We can see that the left and right derivatives are the same, therefore both inequalities become equalities. We follow the same procedure as in case 2 above. For $N = 2$, $m_1 = 0$, $m_2 = 3$, and firm 2 does not drive firm 1 exactly out of market in any segment. Therefore, this is an equilibrium. When $N = 3$, we find a contradiction.

Case 4: $m_1 = 0$ and firm 2 drives firm 1 exactly out of the market in segment m_2.

We know that $m_1 = 0$ only if $N < 4$. In addition, since firm 2 drives firm 1 exactly out of the market in segment m_2, it must be that $N > 2$. This is true since if $N = 2$ for firm 1

[20] The file with the computations is available upon request.

to be out of the market in segment 2 it must be that firm 2 charges a zero price; otherwise firm 1 would have some demand in the second segment. But $p_2 = 0$ gives firm 2 zero profits and cannot be an equilibrium. Thus, $N = 3$, and we find that $m_2 = 3$. Firm 2 exactly drives firm 1 out of the market in segment 3, and $p_{13} = 0$, implying that $p_2 = t/3$. Since firm 1 and 2 share segments 1 and 2, first order conditions are necessary and sufficient. We find $p_{11} = 2t/3$, but this implies that $d_{21} = 0$, i.e., firm 2 is out of the market in segment 1 and $m_1 > 0$, which is a contradiction. Therefore, there is no such an equilibrium. ■

Proof of proposition 5

Proof. If firm 1 deviates and chooses to buy information, since $p_2 = t$, it will never be out of the market in any segment, i.e., $d_{1m} > 0$ for any m and N. And if firm 1 wants, it can drive firm 2 out of the market in every segment, though this clearly will not maximize its profit in the last segment. We first find firm 1's segment price that can exactly drive firm 2 out of segment m to be $p_{1m} = 2t(N-m)/N$ and $\pi_{1m} = 2t(N-m)/N^2$. Assuming that firm 1 and 2 will share the demand in segment m, firm 1's first order derivative of profit is,

$$\frac{\partial \pi_{1m}}{\partial p_{1m}} = -\frac{p_{1m}N - tN + tm - t}{tN}.$$

If we evaluate it at $p_{1m} = 2t(N-m)/N$, we have,

$$\left.\frac{\partial \pi_{1m}}{\partial p_{1m}}\right|_{p_{1m}=\frac{2t(N-m)}{N}} = \frac{-N+m+1}{N}.$$

For firm 2 to have positive demand, this derivative must be positive, which implies $m = N$. This means that firm 2 will always have positive demand only in the last segment. Then firm 1 shares with firm 2 in segment N and $p_{1N} = t/N$, and $\pi_{1N} = t/2N^2$. In segments $m < N$, firm 1 charges $p_{1m} = 2t(N-m)/N$ to exactly drive firm 2 out of market and $\pi_{1m} = 2t(N-m)/N^2$. Therefore,

$$\pi_1^d(N) = \sum_{1 \le m < N} \frac{2t(N-m)}{N^2} + \frac{t}{2N^2} = \frac{t(2N^2 - 2N + 1)}{2N^2}.$$

■

Proof of proposition 8

Proof. First consider the middle segments as given in Proposition 1. Using the equilibrium prices we can find the lowest V such that the market in the middle segments is still covered, and the equilibrium as we find in proposition 1 does not change. All we need to do is to make sure that the marginal consumer in each segment (who is the one with the highest transportation cost and the lowest surplus) gets surplus at least zero, which is guaranteed if,

$$V \geq \underline{V}_M = \frac{t(N+2)}{2N}.$$

Now let's consider the left segments where firm 1 is a constrained monopolist and the marginal consumer is indifferent between buying from firm 1 and firm 2. First consider the equilibrium that is given in Proposition 1 where firm 2 charges a zero price in every left segment and firm 1 has all the demand. Take the marginal consumer in the first segment (who is indifferent between firm 1 and 2) and find the V such that his surplus is exactly zero. This is $V = t(N-1)/N$. Notice now that all the other marginal consumers in the left segments, given that V, receive surplus greater than zero. This is true since firm 2 still charges a zero price and these marginal consumers are indifferent between firm 1 and 2 but are located closer to firm 2 than the marginal consumer of the first left segment. Hence any

$$V \geq \underline{V}_{L1} = \frac{t(N-1)}{N},$$

guarantees that the left segments are covered.

Now assume that the marginal consumer is indifferent between buying from firm 1 and not buying at all. Consider a left segment m such that the marginal consumer is indifferent between buying from firm 1 and not buying at all and firm 1's demand is $1/N$ (covered market). The marginal consumer's utility (whose location is $x = m/N$) is $V - p_{1m} - tm/N = 0$ which implies that the price must be $p^*_{1m} = V - tm/N$. Now we have to make sure that firm 1 does not have incentive to increase its price, i.e., we need $\partial\pi_{1m}/\partial p_{1m} \leq 0$. To this end, we find the firm's profit function in segment m,

$$\pi_{1m} = \left(\frac{(V - p_{1m})\, p_{1m}}{t} - \frac{m-1}{N} \right) p_{1m},$$

which we differentiate with respect to price and then plug in the $p^*_{1m} = V - tm/N$. Then, we solve

$$\left.\frac{\partial \pi_{1m}}{\partial p_{1m}}\right|_{p_{1m}=p^*_{1m}} \leq 0,$$

for V to obtain,

$$V \geq \frac{t(m+1)}{N}.$$

Since it is increasing in m, it is sufficient to use m_1 (from Proposition 1) which is the highest m in the left segments. This yields,

$$V \geq \underline{V}_{L2even} = \frac{t}{2},$$

if the sequence is even and,

$$V \geq \underline{V}_{L2odd} = \frac{t(N-1)}{2N},$$

if the sequence is odd.

Notice that $\underline{V}_{L1} \geq \underline{V}_{L2even}$ and $\underline{V}_{L1} \geq \underline{V}_{L2odd}$ for all N. To find a tighter bound for V in what follows we ignore $\underline{V}_{L1}$. Hence observe that,

$$\underline{V}_M = \frac{t(N+2)}{2N} = \max\left\{\underline{V}_M = \frac{t(N+2)}{2N},\ \underline{V}_{L2even} = \frac{t}{2},\ \underline{V}_{L2odd} = \frac{t(N-1)}{2N}\right\},$$

which implies that as long as $V \geq \underline{V} = t(N+2)/2N$, the market in the B vs. B case is covered for any segment. However, we cannot use the equilibrium prices of Proposition 1 since we do not know whether V is greater than $\underline{V}_{L1}$ or not. ■

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