Endogenous Acquisition of Information on Consumer Willingness to Pay in a Product Differentiated Duopoly*

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Abstract

We look at the incentives of two firms, who produce horizontally differentiated products, to acquire information of a certain quality on consumer willingness to pay. A firm who possesses such information can offer its product to different consumer groups at different prices (third degree price discrimination). We show that "acquiring information" and "price discriminating" is each firm's dominant strategy (for relatively low information costs) resulting in lower profit than when neither firm is engaged in price discrimination. Moreover, and given that firms price discriminate, equilibrium profits and average price exhibit a U-shape as a function of the information quality. Consumers are unambiguously better off under price discrimination as each one pays a lower price than the uniform non-discriminatory price.

Keywords: Price discrimination, information quality, information acquisition.

JEL classification: D43, L13, O30.

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1 Introduction

Information regarding customers' characteristics such as previous purchases, age group, income, tastes, education etc. is valuable to the firms for various reasons. Firms knowing the specific traits of a customer (or group of customers) can tailor their products to each customer's needs, develop and offer a new product, or even charge different prices (e.g. offer different discounts) to different buyers. The internet as a medium of communication and commerce has made the collection, analysis and application of such information feasible. There is ample evidence in the press that firms have recognized this opportunity and are trying to reap the benefits of the new technology.[1] Consumers can now be contacted by prospective sellers on an individual basis and offered various products at terms which are not uniform among all potential buyers. For example, Books.com, a books retailer, in early 1998 adopted a price discrimination strategy where different buyers were paying different prices for the same item, depending on their shopping behavior, Bailey (1998).[2] Bailey addresses the issue of why price discrimination is attractive to internet retailers and how they can implement such a system. He also shows how retailers are collecting large amounts of information about their customers' purchasing patterns with the intent to estimate each consumer's reservation price for the product. Knowing each customer's maximum willingness to pay will enable the seller to price discriminate more effectively. The rapid and constant improvement of internet and software technology also implies that the quality of information that can be collected (or acquired) by the firms is increasing. In the information technology

[1] New York Times in September 01, 2000 writes: "*Amazon.com has started to send e-mail advertising messages to its customers on behalf of other companies and...If the new measures fail to reverse the leading internet retailer's heavy losses and it is put up for sale, the buyer will acquire Amazon.com's customer data.* Furthermore, Video Business in February 21, 2000 writes: "*Video retailers are looking to use their databases of customer information as an additional revenue stream, by selling those names to marketers. According to some analysts each name in those databases could be worth as much as $700.*" Finally, Travel Agent in January 08, 1996 writes: "*Preferred Hotels & Resorts Worldwide plan to use computer-generated customer profiles to reach niche markets... With such information, a hotelier can develop a targeted package and send a letter to members of a 'cohort group', a group of people with similar interests.*"

[2] Ulph and Vulkan (2000) also discuss how technologies, such as an agent, who is a program that is authorized to act independently on behalf of its user [see Vulkan (1999)], can facilitate price discrimination by the firms.

(IT) era, therefore, it is important to look at the incentives of firms to adopt new technologies as well as the evolution of prices and profits as the technology becomes better.

We focus on the opportunities that are being available to the firms to price discriminate when they possess information of a certain quality.[3] In particular, we are interested in the following questions: 1) Is consumer information valuable to the firms and how much each firm is willing to pay to acquire it? 2) How does the quality of information affect i) each firm's willingness to pay for it and ii) the equilibrium prices and profits? and 3) How does the availability of consumer information and its quality affect the consumer welfare and the social welfare? We employ the standard Hotelling's model of horizontal product differentiation with two sellers which are located at the two endpoints of a unit interval. In addition to the two firms and the continuum of consumers there is an information intermediary (e.g. a marketing firm) who is in the business of collecting, analyzing and selling information about customers' market behavior to the two firms [we also allow for the firms themselves to collect information]. In the absence of such information each firm is forced to charge a uniform price. What the information does is to partition the unit interval into smaller subintervals and the firm who possesses it has a better idea about the location of each group of consumers. Thus the firm knows with higher precision the maximum willingness to pay of each consumer which enables it to third degree price discriminate. The information is of a higher quality if it partitions the interval into more segments. The intermediary, given the existing technology and the level of information quality, chooses the price at which he sells the information to the two firms. Each firm then decides whether to buy the information or not and the price(s) for its product without knowing the action of its rival.

Our paper builds upon two strands of literature: Price discrimination and endogenous information acquisition. Economists' interest in price discrimination dates back to the work of Robinson (1933) who studied the issue in a monopolistic environment. Later work on the subject was still confined in a one seller market, [e.g. Schmalensee (1981) and Varian (1985)]. In a monopolistic situation, price discrimination leads to higher profits for the monopolist

[3] Clearly, as we have already mentioned above, information can also be used by a firm to improve other aspects of its business. In this paper, we examine the effects of customer information on the equilibrium of the game when the firms use this knowledge with the intent to price discriminate only.

and to ambiguous consumer welfare results. This prediction however, does not necessarily carry over to imperfectly competitive markets. In an oligopolistic price discrimination model with symmetric firms Holmes (1989) shows that the results are similar to the ones derived from a monopoly market, i.e., consumer welfare for example is still ambiguous. Corts (1998) by relaxing the assumption of symmetric demand made by Holmes shows that unambiguous price and welfare results may arise, where all consumers become better off by having to pay a lower price. He also shows that price discrimination is a dominant strategy that results in lower equilibrium profits for the firms. Bester and Petrakis (1996) in a duopoly model with price discrimination obtain similar results, that is, the ability to charge different consumer segments different prices increases competition and reduces profits. Ulph and Vulkan (2000) study the incentives of two Hotelling firms to engage in perfect price discrimination. On the other hand, endogenous acquisition of information in markets with stochastic market demand has been studied quite extensively [e.g., Palfrey (1982), Vives (1988), Hwang (1993, 1995), Creane (1996) and Caglayan and Usman (2000)]. The information in these papers reduces the uncertainty in market demand and enables the firms to make more informed decisions regarding the production level. In our article market demand is not subject to exogenous uncertainty. The producers know the market demand (i.e., the distribution of the reservation prices), but they may potentially benefit by obtaining finer information on how much each consumer is willing to pay. Endogenous information acquisition has not only been confined to production economies. Allen (1986) studies a pure exchange general equilibrium economy with differentiated information where agents demand information along with other physical commodities.

A distinctive feature of our model is that it derives an equilibrium as a function of the information quality. By varying the quality of information, we can obtain all levels of price discrimination and in the limit (as the number of subintervals goes to infinity) the case of perfect discrimination. Hence, we offer predictions regarding equilibrium behavior in the market for any level of the IT, and therefore the evolution of prices and profits as the technology improves. In particular, we show that the game played between the two firms is a prisoner's dilemma game (consistent with the results in Bester and Petrakis and Corts) with "buying information and price discriminating" being the dominant strategy resulting in lower equilibrium profits (before the firms deduct the cost of information) than when both firms do not possess information and do not price discriminate. Moreover, given that the firms acquire information of some level of quality, the equilibrium profits (again before the firms deduct the cost of information) exhibit a U-shaped pattern as a function of the information quality. That is, initially for low quality of information, equilibrium profits decrease as the quality increases, but eventually higher information quality implies higher firm profits. Hence, as the technology improves and better information becomes available, the "surplus extraction effect" dominates the "intensified competition effect" [see Ulph and Vulkan where these two terms have been first introduced]. Consumers pay lower prices when firms price discriminate than when the option of price discrimination is not available. This indicates that consumers become unambiguously better off and resembles the results in Corts. Consumers would always prefer the firms to have the option to price discriminate since in this case each consumer is paying a lower price. The average equilibrium price exhibits a U-shape form (like equilibrium profits) as a function of the information quality, but is always lower than the uniform price. Thus, and given that information is available, consumers are relatively better off with a low information quality as in the case of higher quality the average price that they pay increases. There is a loss of efficiency associated with the availability of information which decreases as the information quality increases. The reason for the inefficiency is that the presence of information and the ability of the sellers to price discriminate results in an equilibrium where the transportation cost is not minimized. In the limit the efficiency loss equals to the transaction cost associated with collecting, analyzing and selling information.

Policymakers and regulators have raised concerns that the IT and the uncontrolled collection of information about consumers' shopping behavior may have detrimental effects on their welfare. This paper puts these issues in a proper perspective. Ulph and Vulkan, for instance, argue that the availability of consumer information makes (in the case of a linear transportation cost) all consumers better off. However, they only analyze the two extreme cases of no information and perfect information, ignoring the transition from one to the other which is likely to take place as the technology gradually improves. If the quality of information is taken into account, we see that although consumers become indeed better off when information first becomes available, after a certain level of information quality, better information makes them relatively worse off. Hence, as far as the consumers' welfare is concerned, one may want to put some limit to the extent and degree at which the information is collected and used, but not to prohibit it altogether nor to permit extensive utilization of it.

The rest of the paper is organized as follows. Section 2 provides a motivating example which captures the basic ingredients of our model. The model is presented in section 3 and section 4 contains the main analysis and the results in the two stage game we study. We summarize, discuss the assumptions of our model and possible extensions in section 5.

2 A motivating example

To fix ideas consider two firms who are selling two differentiated products. Each consumer's willingness to pay for each product is a function of certain variables such as age, gender, income, tastes, education, family size etc. Without any information on how these characteristics affect the willingness to pay and on the identity of each consumer, each firm charges a uniform price for its product. Now assume that a marketing firm (or the two sellers themselves) has designed a system which can collect and analyze a vast amount of consumer information and relate consumer characteristics to the maximum price that each one is willing to pay. The information can then be used by the firms to develop an optimal pricing strategy. This system can be of a certain quality depending on its ability to collect and analyze detailed data, and the quality and amount of data. Hence, at the early stage of its development this system may be able to unveil the willingness to pay of certain groups of consumers while as time passes and technology improves, each consumer's reservation price may be identified with higher precision.[4]

Consider a simple example where firm 1 produces product A and firm 2 product B. Each consumer's willingness to pay for each product depends on his age [senior citizen (S), or not

[4] Shaffer and Zhang (1995), for example, show how the advent of panel data on household purchase behavior and the statistical procedures to utilize this data has led firms to target coupons to selected households with considerable accuracy and cost effectiveness.

(NS)], gender [male (M), or female (F)] and a remaining set of variables which we call it X. To keep the example simple, assume that it is prohibitively expensive to collect information about X, but not about the age and gender which determine each consumer's type. Hence, there are four possible types under these assumptions. Let $V_i(T)$ denote type T's willingness to pay for product i, $i = A, B$. Further assume that $V_A(F, NS) \geq V_A(M, NS) \geq V_A(F, S) \geq V_A(M, S)$ and $V_B(F, NS) \leq V_B(M, NS) \leq V_B(F, S) \leq V_B(M, S)$.

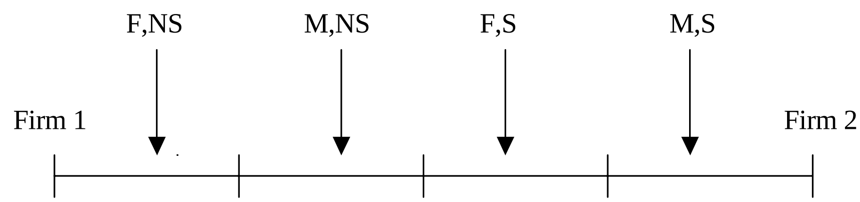

Figure 1: Location of the four groups of consumers

The closest a group of consumers is to one firm the higher it is willing to pay for that firm's product [see figure 1]. Since we have assumed that there is no information on how the set of variables in X affects the willingness to pay, firms do not know each consumer's reservation price with a 100% accuracy. The marketing firm by analyzing consumers' past market behavior for example, discovers the above mentioned ranking of each group's reservation price and also supplies the two firms with information on how to contact each group (e.g. supplies the two firms with a mailing list). If firms do not have such an information they have to charge a uniform price. An information of a minimal quality would be to realize that senior citizens have a relatively higher willingness to pay for firm B's product and non senior citizens have a relatively higher willingness to pay for firm A's product, without knowing anything about the other dimension which is the gender. Then, we should expect firm A charging non-senior citizens a higher price for its product than senior citizens and vice versa for firm B. The quality of information increases if now firms can differentiate each consumer's willingness to pay on the dimension of gender as well. In this case, we should expect a female non-senior consumer to pay the highest price for firm A' product, male non-senior the second highest and so on. Analogous predictions hold for firm B. The model we study next builds upon this example.

3 The model

We employ the standard Hotelling's model of horizontal product differentiation. Consumers are uniformly distributed on the interval $[0, 1]$ and two firms are located at the two endpoints of the interval. Each consumer buys either from firm 1 (which we assume that is located at 0), or from firm 2 (which is located at 1) or does not buy at all. We assume that each consumer derives a benefit equal to S if he buys a product from either one of the firms. Let p_1 and p_2 be the prices that firm 1 and firm 2 charge respectively. Both firms' marginal costs are normalized to zero. In addition, each consumer incurs a linear unit transportation cost denoted by $t > 0$. Therefore a consumer who is located at point $x \in [0, 1]$ on the interval and buys from firm 1 enjoys a surplus of $S - tx - p_1$. Likewise, if he buys from firm 2 his surplus is $S - t(1 - x) - p_2$. Each consumer buys the product which gives him the highest positive surplus. We assume that S is sufficiently high, ruling out the possibility of no sale.[5] Then the demand of each of the two firms' products is given by,

$$d_1 = \frac{p_2 - p_1 + t}{2t} \text{ and } d_2 = \frac{p_1 - p_2 + t}{2t}.$$

It can be easily shown [see e.g. Tirole (1988), p. 280] that in the equilibrium $p_1 = p_2 = t$, $d_1 = d_2 = \frac{1}{2}$ and $\pi_1 = \pi_2 = \frac{t}{2}$.

So far we have implicitly assumed that firms have no information regarding the location of each consumer. All they know is that consumers are uniformly distributed on the interval. Now assume that (some) information regarding the location of each consumer (or group of consumers) becomes available. This information partitions the interval into N sub-intervals (indexed by m, $m = 1, ..., N$) of equal distance and the firm who acquires this information has a better idea about the location of the consumers and their willingness to pay. In this case, a firm can charge different prices to different groups of consumers, though the price is the same within each group. We define an information to be of a higher quality if it partitions the interval into more sub-intervals. Thus higher N is synonymous to a higher information quality. An intermediary is in the business of collecting, analyzing consumer

[5] The assumption that the market is fully covered makes our model tractable. See section 5 for a discussion about this assumption.

information and selling it to the firms at price $\alpha(N)$.[6] The amount that a firm is willing to pay depends on the incremental profit that it will experience which in turn depends on whether the other firm has acquired the information or not. We assume that the current state of technology dictates the quality of information N and the players in our game take it as exogenously given. Hence, we do not consider the possibility where the intermediary (or the firms themselves) choose the quality of information.[7] Denote by S the set of firms which buys the information, and by $c(N)$ the cost of collecting information of quality N.[8] The intermediary's strategy for any given N is $\alpha(N)$ which is the price at which he sells the information of quality N to the two firms. The profit function of the intermediary is,

$$\pi_I(\alpha) = |S|\,\alpha(N) - c(N).$$

Each firm's strategy is (τ_i, p_i), $i = 1, 2$, where $\tau_i = 1$ if the firm buys the information from the intermediary and 0 otherwise;[9] and p_i is an $N \times 1$ vector of prices if $\tau_i = 1$ (since firm i can charge a different price in each segment) or a scalar if $\tau_i = 0$. We assume that a firm's pricing strategy is measurable with respect to its information. Let $d_{im}(p_{1m}, p_{2m})$, $m = 1, ..., N$, be the demand of firm i's product in the $m-$th segment (sub-interval). The profit function of firm i is therefore given by,

$$\pi_i(\tau_1, p_1, \tau_2, p_2) = \sum_{m=1}^{N} d_{im} p_{im} - \tau_i \alpha(N), \;\; i = 1, 2.$$

The game we consider unfolds as follows:

Stage 1: The intermediary, given N, decides about the price of information $\alpha(N)$ in order to maximize his profits $\pi_I(\alpha)$.

[6] We also allow the firms themselves to collect information. The main predictions do not vary significantly from the case where information is collected and sold by the intermediary [see Appendix].
[7] There are two reasons pertaining to why players in our game do not choose N. First, we believe that this would constitute an unrealistic assumption as the quality in our model is determined by the state of the technology which is more reasonable to assume that it is the outcome of a process outside the control of the two firms and the intermediary. Second, allowing N to be a choice variable, would enlarge the strategy space making the game we study intractable.
[8] We regard the cost of information $c(N)$ as exogenously given and we make no assumption about how it varies with N.
[9] In the case where the firms collect information themselves, $\tau_i = 1$ implies that firm i collects information while $\tau_i = 0$ implies that firm i decided not to collect information.

Stage 2: Given the quality of information and its price, firms decide simultaneously and independently whether to buy information or not and the prices that they will charge in order to maximize their profits $\pi_1(\tau_1, p_1, \tau_2, p_2)$ and $\pi_2(\tau_1, p_1, \tau_2, p_2)$, respectively.[10]

We begin by analyzing stage 2. Each firm has to decide whether to buy the available information of quality N or not and the prices that it will charge to different groups of consumers without knowing whether the other firm possesses the information or not. We find a Nash equilibrium for this subgame. In stage 1, the intermediary will set the price for his information.

In the next section, we look for a subgame perfect equilibrium (in pure strategies) of this game.

4 Analysis

We solve the game backwards starting from stage 2 and proceeding to stage 1. We begin by finding a Nash equilibrium in the game played between the two firms as a function of N.

4.1 Stage 2: The game between the two firms

We are going to find under what conditions (if any) the following set of strategies can be supported as a Nash equilibrium in this subgame: 1) both firms decide to purchase the information (B vs. B), 2) only one firm decides to purchase the information (B vs. NB) and 3) neither firm finds it profitable to acquire the information (NB vs. NB). In particular, we are looking for a $(\tau_1^*, \tau_2^*, p_1^*, p_2^*)$ such that any unilateral deviation by a firm is unprofitable. Since a firm's strategy entails to choose whether to buy information or not and its price(s), a deviation can be in both dimensions. There are some restrictions such as if a firm does

[10] Alternatively, if the firms collect information themselves, given N and the cost of collecting information $c(N)$, they decide simultaneously and independently whether to collect information and the prices that they will charge to maximize their profits $\pi_1(\tau_1, p_1, \tau_2, p_2)$ and $\pi_2(\tau_1, p_1, \tau_2, p_2)$, respectively. We study this in the Appendix.

not have any information then it must charge a uniform price, while a firm with information has the flexibility of charging each consumer group different prices. We begin by finding both firms' profits if they adhere to the above set of strategies and then we compute the profits from a unilateral deviation in each of the three cases mentioned above. Finally, in this subsection we do not deduct the information price ($\alpha(N)$) from the firms' profits. This is done in section 4.2.

4.1.1 Both firms buy information (B vs. B).

Since both firms buy consumer information, they know in which of the N segments each consumer is located and therefore they are able to charge different prices in each segment. Interval $[0, 1]$ is equally divided into N segments, each one having length of $1/N$. Segment m can be expressed as the interval $[\frac{m-1}{N}, \frac{m}{N}]$, where m is an integer between 1 and N [see figure 2 below].

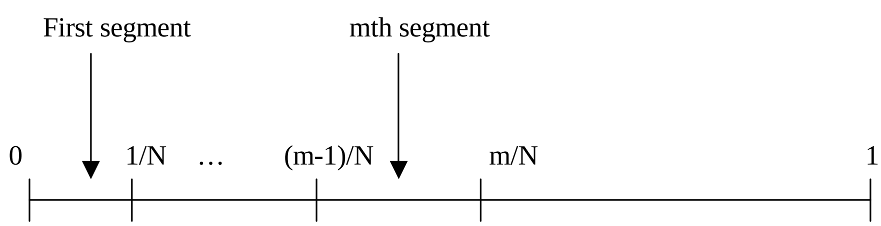

Figure 2

In segment m, firm 1 and 2 charge prices p_{1m} and p_{2m}, and the demands[11] of their products are

$$d_{1m} = \frac{p_{2m} - p_{1m} + t}{2t} - \frac{m-1}{N} \text{ and } d_{2m} = \frac{m}{N} + \frac{p_{1m} - p_{2m} - t}{2t}$$

and their profits are

$$\pi_{1m}(p_{1m}, p_{2m}) = p_{1m}d_{1m}, \text{ and } \pi_{2m}(p_{1m}, p_{2m}) = p_{2m}d_{2m}.$$

[11]Throughout the paper, demand in each segment must be within the interval $[0, 1/N]$.

Firm i's, problem is,

$$\max_{p_{im}\geq 0} \pi_{im}(p_{1m}, p_{2m}), \text{ for each } m,\ m = 1, ..., N, \text{ and } i = 1, 2.$$

The next proposition summarizes the solution to the above problem.

Proposition 1 *Assume that both firms acquire information and choose prices to maximize own profits. Then, for each N ($N \geq 2$), there exist two thresholds (integers) m_1 and m_2 (with $N + 1 \geq m_2 > m_1 \geq 0$) such that:*

i) Firm 1's equilibrium demand is equal to 1/N in all segments from 1 to m_1, i.e., firm 1 is a constrained monopolist in these segments. Firm 2's equilibrium demand in these segments is zero. Moreover, firm 1's prices are: $p^*_{1m} = t(N - 2m)/N$, while firm 2 sets $p^*_{2m} = 0$, $m = 1, ..., m_1$.

ii) Both firms share the demand in the segments from $m_1 + 1$ to $m_2 - 1$. Moreover, firm 1's prices are: $p^*_{1m} = t(N - 2m + 4)/3N$, and firm 2's prices are: $p^*_{2m} = t(2m - N + 2)/3N$, $m = m_1 + 1, ..., m_2 - 1$.

iii) Firm 2's equilibrium demand is equal to 1/N in all segments from $m_2 - 1$ to N, i.e., firm 2 is a constrained monopolist in these segments. Firm 1's equilibrium demand in these segments is zero. Moreover, firm 2's prices are: $p^*_{2m} = t(2m - N - 2)/N$, while firm 1 sets $p^*_{1m} = 0$, $m = m_2 - 1, ..., N$.

Moreover these two thresholds have the following properties. When N is an even subsequence, $m_1 = N/2 - 1$ and $m_2 = N/2 + 2$, whereas when N is an odd subsequence $m_1 = N/2 - 3/2$ and $m_2 = N/2 + 5/2$.

Finally, the equilibrium profits for both firms as a function of N are:

$$\pi_i^{BB}(N) = \frac{t(9N^2 - 18N + 40)}{36N^2},\ i = 1, 2,$$

when N is an even number, and

$$\pi_i^{BB}(N) = \frac{t(9N^2 - 18N + 43)}{36N^2},\ i = 1, 2,$$

when N is an odd number.

Proof. See appendix. ■

Numerical example

Suppose $N = 5$. Then $m_1 = 1$ and $m_2 = 5$, implying that firm 1 and 2 are monopolists in segments 1 and 5 respectively. In segments 2, 3 and 4 both firms have strictly positive demands. The prices that firm 1 charges, starting from segment 1 are: $p^*_{11} = 9t/15$, $p^*_{12} = 5t/15$, $p^*_{13} = 3t/15$, $p^*_{14} = t/15$ and $p^*_{15} = 0$. Firm 2's prices are symmetric with the highest price in segment 5 and price equal to zero in segment 1. The equilibrium profits are $\pi_1^{BB}(N = 5) = \pi_2^{BB}(N = 5) = .1977t$.

The availability of information and the associated price discrimination forces the two firms to compete more vigorously and as a result their profits drop compared to the standard benchmark no discrimination model. In the latter case the profit of each firm is $t/2$ while in the former, regardless of whether N is even or odd, profits are less than $t/2$ [see section 5 for a discussion]. Firms compete for customers in each segment. Although each firm knows the location of each customer more precisely than when no information is available and therefore can charge him a price according to his willingness to pay, the fact that the other firm has the exact same information makes the competition more intense.

4.1.2 Deviation from B vs. B

In order for B vs. B to be an equilibrium it must be the case that neither firm finds it profitable to deviate. A deviation in this case would be for a firm not to buy information, and charge a uniform price. Next, we find the profits of a deviating firm.

Suppose firm 1 is the deviating firm. We know from the previous analysis that (before deviation) firm 2 charges zero price in the segments from 1 to m_1; both firms charge positive prices in the segments from m_1+1 to m_2-1; and firm 1 charges a zero price in the segments from m_2 to N. It is quite obvious first of all that the deviating firm will have zero demand in the segments from m_2 to N. The reason is that firm 1 had a zero demand when it was charging a zero price (before deviation) and therefore when it deviates and charges a uniform strictly positive price its demand must be zero. Hence the problem of firm 1 is to find a price (p_1^d) to maximize its profits from segment 1 till segment $m_2 - 1$. The demand that firm 1 has in the first m_1 segments is,

$$d_1 = \frac{t - p_1^d}{2t}.$$

We can now distinguish between two cases. One is when N is an even number and the other when it is an odd number.

When N is even, $m_1 = N/2 - 1$ and $m_2 = N/2 + 2$, which implies that the two firms have both positive prices in just two segments. In these two segments firm 2 charges $p_{2,m_1+1} = 2t/3N$ and $p_{2,m_1+2} = 4t/3N$. Therefore the demand of the deviating firm in these two segments is:

$$d_{1,m_1+1} = \frac{2t/3N - p_1^d + t}{2t} - \left(\frac{1}{2} - \frac{1}{N}\right) \text{ and } d_{1,m_1+2} = \frac{4t/3N - p_1^d + t}{2t} - \frac{1}{2}.$$

The deviating firm solves the following constrained maximization problem,

$$\begin{aligned} \max_{p_1^d} \pi_1^d &= p_1^d \left(d_1 + d_{1,m_1+1} + d_{1,m_1+2}\right), \\ \text{subject to} &: \quad d_1 \in [0, 1/2 - 1/N] \text{ and } d_{1,m_1+1}, d_{1,m_1+2} \in [0, 1/N]. \end{aligned}$$

When N is odd, $m_1 = N/2 - 3/2$ and $m_2 = N/2 + 5/2$, which implies that the two firms have both positive prices in just three segments. In these two segments firm 2 charges $p_{2,m_1+1} = t/3N$, $p_{2,m_1+2} = 3t/3N$ and $p_{2,m_1+3} = 5t/3N$. Therefore the demand of the deviating firm in these two segments is:

$$\begin{aligned} d_{1,m_1+1} &= \frac{t/3N - p_1^d + t}{2t} - \left(\frac{N-3}{2N}\right), \; d_{1,m_1+2} = \frac{3t/3N - p_1^d + t}{2t} - \left(\frac{N-1}{2N}\right) \\ \text{and } d_{1,m_1+3} &= \frac{5t/3N - p_1^d + t}{2t} - \left(\frac{N+1}{2N}\right). \end{aligned}$$

The deviating firm solves the following constrained maximization problem,

$$\begin{aligned} &\max_{p_1^d} \pi_1^d = p_1^d \left(d_1 + d_{1,m_1+1} + d_{1,m_1+2} + d_{1,m_1+3}\right), \\ &\text{subject to} : \quad d_1 \in [0, (N-3)/2N] \text{ and } d_{1,m_1+1}, d_{1,m_1+2}, d_{1,m_1+3} \in [0, 1/N]. \end{aligned}$$

Proposition 2 *The profits of the deviating firm are constant for any N greater than or equal to 8, i.e., $\pi^d(N) = .125t$ for $N \geq 8$.*

Proof. See appendix. ■

For $N = 2, ..., 7$ it is more efficient to solve the problem for each N individually and the results are presented in the table below

N:	2	3	4	5	6	7
π^d :	$.2500t$	$.1975t$	$.1667t$	$.1334t$	$.1304t$	$.1327t$

Table 1

As the quality of information increases, a unilateral deviation from the set of strategies where both firms buy information becomes less profitable. The reason is that the deviating firm has to charge a uniform price and when N is high it becomes harder to steal demand from the non-deviating firm who has the flexibility to charge different prices in each segment. As the number of segments increases the deviating firm can only sell to consumers in its own territory (where it was the only seller before deviation). That is why after a critical level of information quality profits from deviation are constant.

Next, we analyze the case where one firm acquires the information but the other does not.

4.1.3 Only one firm buys information (B vs. NB)

Due to symmetry, we assume that only firm 1 has acquired information. Thus firm 2 will charge a uniform price p_2, while firm 1 charges p_{1m} in segment m. Then firms' demands in each segment are,

$$\begin{aligned} d_{1m} &= \frac{p_2 - p_{1m} + t}{2t} - \frac{m-1}{N}, \text{ for } m = 1, ..., N \\ d_{2m} &= \frac{m}{N} + \frac{p_{1m} - p_2 - t}{2t}, \text{ for } m = 1, ..., N, \end{aligned}$$

and the profit functions,

$$\pi_1^{BNB}(N, p_1, p_2) = \sum_{m=1}^{N} p_{1m} d_{1m} \text{ and } \pi_2^{BNB}(N, p_1, p_2) = p_2 \sum_{m=1}^{N} d_{2m}.$$

Firm 1 chooses its price in each segment taking the uniform price that the other firm charges given. Hence, firm 1's problem is,

$$\max_{p_{1m} \geq 0} \pi_{1m}^{BNB}(N, p_{1m}, p_2), \text{ for } m = 1, ..., N$$

and firm 2's problem is,

$$\max_{p_2 \geq 0} \pi_2(N, p_{11}, ..., p_{1m}, ...p_{1N}, p_2).$$

The Kuhn-Tucker conditions are:

$$\begin{aligned} \frac{\partial \pi_{1m}}{\partial p_{1m}} &\leq 0 \text{ for } m = 1, ..., N, \text{ with equality if } p_{1m} > 0 \\ \frac{\partial \pi_2}{\partial p_2} &\leq 0, \text{ with equality if } p_2 > 0, \\ \text{and } d_{2m} &\geq 0, \text{ for } m = 1, ..., N. \end{aligned}$$

Proposition 3 *When only one firm acquires information and $N > 2$, then no equilibrium (in prices) exists.*

Proof. See appendix. ■

Even if both firms had agreed that one would buy information and the other would not, they would not be able to coordinate their pricing strategies. In other words, there does not exist a set of prices (uniform for one firm and non-uniform for the other) such that no firm has an incentive to deviate from.

Proposition 4 *When $N = 2$, $p_{11}^* = .75t$, $p_{12}^* = .25t$, $p_2^* = .5t$, $\pi_1^* = .3125t$, $\pi_2^* = .25t$, can possibly be supported as an equilibrium.*

Proof. From the proof of the above proposition, we know that when $N = 2$, then the two firms share the demand in both segments, i.e., $m_1 = 0$ and $m_2 = 3$. The equilibrium prices are then derived by solving the first order Kuhn-Tucker conditions. ■

4.1.4 Deviation from B vs. NB

To support B vs. NB as an equilibrium it must be the case that no unilateral deviation is profitable. That is, firm 1 does not wish to deviate by not acquiring information and firm 2 finds it unprofitable to acquire information. We know that only $N = 2$ can possibly be supported as an equilibrium, and we give the results for $N = 2$.

We first study the deviation by firm 1 which does not buy information and solves the following maximization problem,

$$\max_{p_1^d} \pi_1^d = p_1^d \left(\frac{p_2^* - p_1^d - t}{2t} \right),$$

where p_2^* is given in proposition 4. The deviating firm takes the price of the other firm as given and charges a uniform price p_1^d equal to $.75t$ to maximize its profits. It can be shown that the profit is equal to $.28125t$.

Now we turn to the deviation by firm 2. Hence, firm 2 buys information and solves the following maximization problem,

$$\max_{p_{2m}^d} \pi_{2m}^d = p_{2m}^d \left(\frac{m}{2} + \frac{p_{1m}^* - p_{2m}^d - t}{2t} \right), \text{ subject to: } d_{2m} \in [0, 1/2],\ m = 1, 2$$

where p_{1m}^*'s are given in proposition 4. The deviating firm takes the price of the other firm as given and since it has now acquired information it charges a different price in each segment to maximize its profits. It can be easily shown that $p_{21}^d = .375t$, $p_{22}^d = .625t$, and $\pi_2^d = .265625t$.

Next, we look for the equilibrium outcome when neither firm has acquired information.

4.1.5 No firm buys information (NB vs. NB)

In section 2, we saw that when no firm has access to the information the equilibrium pricing strategy is: $p_1^* = p_2^* = t$ and the equilibrium profits: $\pi_1 = \pi_2 = t/2$.

4.1.6 Deviation from NB vs. NB

To support NB vs. NB as an equilibrium it must be the case that no firm wishes to deviate and acquire information. The problem that a deviating firm (say firm 1) solves is,

$$\max_{p_{1m}^d} \pi_{1m}^d = p_{1m}^d \left(\frac{t/2 - p_{1m}^d + t}{2t} - \frac{m-1}{N} \right), \text{ subject to: } d_{1m} \in [0, 1/N],\ m = 1, ..., N.$$

Proposition 5 *Suppose that firm 2 adheres to the NB vs. NB set of strategies, while firm 2 deviates and buys information. In this scenario, firm 1 will share the demand with firm 2 only in the last segment, i.e., segment N. In all other segments firm 2 is out of the market and the profit of the deviating firm is,*

$$\pi_1^d(N) = \frac{t(2N^2 - 2N + 1)}{2N^2}$$

Proof. See appendix. ■

4.2 Stage 1: Information price

In stage 1, the intermediary chooses the price of information, given N and the cost of collecting information $c(N)$, to maximize its profits. Alternatively, if the firms themselves collect information, then for a given cost $c(N)$, they decide whether to acquire information or not. In this case each firm makes the decision without knowing the decision of its rival firm. We put the latter case in the Appendix since it is similar to the case where the intermediary collects the information. Therefore, in the remaining of this section the firms purchase the information from the intermediary.

To calculate his profit, the intermediary must know how many firms will purchase the information for any given quality and price. Therefore, we find the equilibrium in the game played between the two firms for any information price and information quality. The price of information will dictate the equilibrium in the game played by the two Hotelling producers. If for example the information price is extremely high no firm will buy it and this is an equilibrium. In what follows, we assume that the intermediary charges a price for the information which equals the difference between the profits from (B vs. B) and the profits from deviation. This is clearly the maximum price for the information such that (B vs. B) can be sustained as an equilibrium. We assume that the intermediary has monopoly power which enables him to charge the highest price that the market can bear. This assumption is not so important as our purpose was to find the highest price that the two firms are willing to pay for the information as a function of its quality, and not the actual price that will prevail in the market. The latter will depend on aspects of the market that have not been modelled explicitly, such as the method at which the information is sold (e.g. posted price, bargaining etc.). Also, when the firms collect the information this issue disappears.

We begin by finding the range of information price that will support that both firms buy information (B vs. B) as an equilibrium. We assume that the cost of collecting information is the same regardless of whether the intermediary sells the information to one or to two firms. This is equivalent to assuming that there is a fixed cost associated with the initial collection of information and then the marginal cost is zero.

<u>Case 1:</u> **Both firms buy information (B vs. B).**

Recall that each firm's profit is $t(9N^2 - 18N + 40)/36N^2$ when N is even and $t(9N^2 - 36N + 43)/36N^2$ when N is odd. If one firm deviates while the other firm sticks to its strategies in B vs. B, the deviating firm's profit is $t/8$ for $N \geq 8$. Therefore, the maximum information price the intermediary can charge to support B vs. B as an equilibrium is the difference between the profits before and after deviation, which turns out to be

$$\alpha^{BB}(N) = \frac{t(9N^2 - 36N + 80)}{72N^2}$$

when N is even and

$$\alpha^{BB}(N) = \frac{t(9N^2 - 36N + 86)}{72N^2}$$

when N is odd ($N \geq 8$). Since the intermediary in this case sells the information to both firms, his revenue is $R^{BB}(N) = t(9N^2 - 36N + 80)/36N^2$ when N is even and $R^{BB}(N) = t(9N^2 - 36N + 86)/36N^2$ when N is odd ($N \geq 8$).

For $N < 8$ we can compute the maximum information price the intermediary can charge to support B vs. B as an equilibrium, and the intermediary's correspondent revenues, using table 1. The results are given in the table below.

N :	2	3	4	5	6	7
α^{BB} :	$.0278t$	$.0185t$	$.0278t$	$.0644t$	$.0671t$	$.0702t$
R^{BB} :	$.0556t$	$.0371t$	$.0555t$	$.1288t$	$.1343t$	$.1405t$

Table 2

Case 2: Only one firm buys information (B vs. NB)

Proposition 6 *B vs. NB will not be supported as an equilibrium.*

Proof. From proposition 3, we know that B vs. NB can possibly be supported as an equilibrium only when $N = 2$. From proposition 4 the profits of the two firms are, $(.3125t, .25t)$. From section 4.1.4 the profits from a unilateral deviation are $(.28125t, .2656t)$. Hence the maximum price that the intermediary can charge so that B vs. NB is an equilibrium is, $\alpha(N = 2) = .3125t - .28125t = .03125t$. Given this price the firm who has information is indifferent before and after deviation (and we assume that it does not deviate), and the firm who does not have information is strictly worse off by deviating and buying information. Notice from table 2 that when the intermediary wants to sell to both firms (B vs. B) the highest price that firms are willing to pay is $\alpha(N = 2) = .0278t$. Therefore, if the price is $.3125t$ only one firm buys and the intermediary's revenue is less than that when both firms buy, which is $.0278t \times 2 = .0556t$. Thus the intermediary finds it profitable not to support (B vs. NB) as an equilibrium. ■

Case 3: No firm buys information (NB vs. NB).

Recall that before deviation, each firm charges a uniform price t, and enjoys a profit of $t/2$. If one firm deviates while the other firm sticks to NB vs. NB, we show that the deviating firm's profit is $t(2N^2 - 2N + 1)/2N^2$. To support NB vs. NB as an equilibrium, information price must be greater than or equal to the difference between these two profits, i.e., $\alpha(N) \geq \alpha^{NBNB}(N) = t(N-1)^2/2N^2$, where α is the information price. Since no firm buys information, the intermediary has zero revenues.

Proposition 7 *The intermediary's revenue as a function of the quality of information (if he collects and sells information) is,*

$$R(N) = \frac{t(9N^2 - 36N + 80)}{36N^2},$$

when N is even and, $N \geq 8$, and

$$R(N) = \frac{t(9N^2 - 36N + 86)}{36N^2},$$

when N is odd and, $N \geq 8$ and in equilibrium both firms have acquired information. For $N < 8$ the intermediary's revenue function is given by the table below,

N	2	3	4	5	6	7
$R:$	$.0556t$	$.0371t$	$.0555t$	$.1288t$	$.1343t$	$.1405t$

Proof. The proof follows from the results in cases 1,2 and 3. ■

The intermediary maximizes his revenue by setting a price for the information such that no seller has an incentive to deviate by not purchasing the information. Given this information price both firms acquire information which gives them the "right" to price discriminate.

4.3 Perfect discrimination

In this section, we digress from our approach so far and we assume that firms have perfect information about each consumer and consequently can charge each one a different price. In the next section, we use the results from this section to compare the perfect discrimination case with the equilibrium in our game as $N \to \infty$.

Each consumer is paying a different price, depending on his location. Consider the consumer who is located at $x < 1/2$. Both firms know his exact location and the equilibrium prices are $p_1^* = (1 - 2x)$ and $p_2^* = 0$. Therefore, the consumer who is located at $x = 0$ pays $p_1^* = 1$ for the product of firm 1, while firm 2 charges a zero price. The consumer who is located at $x = 1/2$ pays a zero price. Hence the average price that firm 1 charges is $1/2$ and firm 1's profits equal to $1/4$. Since the problem is symmetric the same holds for firm 2.

5 Discussion and extensions

Whether we will observe both firms using information (B vs. B) or neither one using it (NB vs. NB) ultimately depends on the cost of collecting information, $c(N)$. If $c(N)$ exceeds the intermediary's revenue $R^{BB}(N)$, then clearly no information will be sold to the two sellers and the equilibrium is NB vs. NB. To make the problem interesting assume that this is not the case, i.e., $c(N) < R^{BB}(N)$ for all $N > 1$. The equilibrium then entails both firms using the available information and price discriminating with the equilibrium profits and prices given in proposition 1. Figure 3 graphs the equilibrium profits as a function of the quality of information. Notice that $.5t$ is each firm's profit when no information is available ($N = 1$). On the other extreme, as $N \to \infty$ the problem is equivalent to the perfect discrimination case of section 4.2. In this case each firm's profit tends to,

$$\lim_{N\to\infty} \pi^{BB}(N) = \frac{t(9N^2 - 18N + 40)}{36N^2} = .25t \text{ for } N \text{ even}$$

and

$$\lim_{N\to\infty} \pi^{BB}(N) = \frac{t(9N^2 - 18N + 43)}{36N^2} = .25t \text{ for } N \text{ odd,}$$

which is equal to firms' profits under perfect discrimination. For $N > 1$ the equilibrium profits initially decrease but quickly they recover and approach the perfect discrimination profits. This implies that as the quality of information increases, the "surplus extraction effect" dominates the "intensified competition effect."

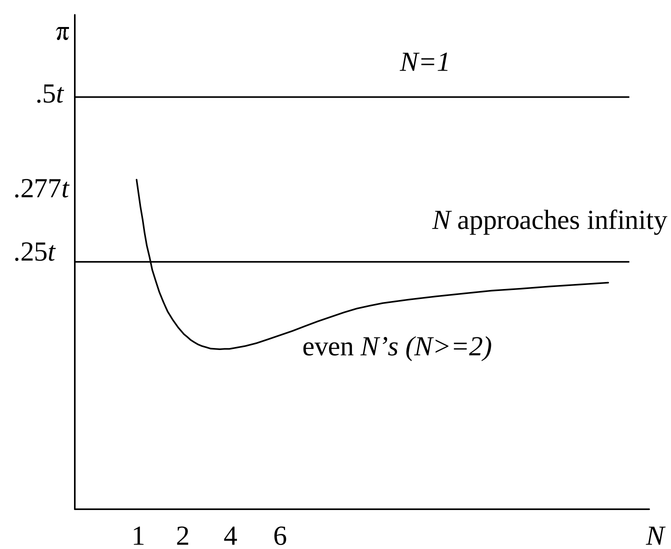

Figure 3: Equilibrium profit as a function of the information quality

The welfare implications are quite clear and unambiguous. Consumers become better off compared to the no discrimination case, as the prices that each firm charges are uniformly below t for any N. To see this consider for example firm 1 (the same holds for firm 2). When firm 1 is a constrained monopolist it charges a price equal to $t(N-2m)/N$, $m=1,...,m_1$, where $m_1 = N/2-1$ if N is an even subsequence and $m_1 = N/2-3/2$ if N is an odd subsequence. It can be easily checked that in the first segment, i.e., when $m=1$ the price is $t(1-2/N) < t$, whereas in the other extreme case when $m=m_1$ the price is equal to $t/N < t$. A similar result holds in the segments where the two firms share the customers. Therefore, the use of customer information by the firms makes the consumers better off. The average price[12] that consumers have to pay also exhibits a U-shape pattern similar to that in figure 3. That is, even though all consumers pay lower prices when the firms price discriminate, the average price increases as the quality of information increases.

On the other hand, both firms become worse off. When they charge a uniform price firms' profits are $t/2$, whereas when they possess information and price discriminate their profits are (before paying the information price) given by proposition 1 and are lower than $t/2$ for any $N > 1$. The market outcome when no information is available and firms do not price discriminate is Pareto optimal (given the locations of the firms).[13] Hence, the availability of information cannot improve upon this outcome. Actually for any $N > 1$ total welfare is lower than when $N=1$. The reason is twofold: first, the transportation cost is not minimized because as we show in proposition 1 the firms share the demand in the middle two (if N is even) or in the middle three (if N is odd) segments. For example, a consumer who is located at $x < 1/2$ (i.e., in firm 1's "back yard"), but in the right side of segment m_1+1, now is purchasing from firm 2 resulting in an unnecessary increase in transportation cost. This is due to the better information that both firms have about the locations of consumers and the ensuing price competition; second, the cost of collecting information is added to the welfare loss. However, as the quality of information improves the first effect vanishes as now the size of the intervals where the two firms share their demand shrinks and in the limit

[12] Due to symmetry, each firm gets half of the consumers. Therefore, the average price of each firm over the consumers who buy from this firm is 2 times its profit, i.e., $t(9N^2-18N+40)/18N^2$, if N is even; and $t(9N^2-18N+43)/18N^2$, if N is odd.

[13] An outcome is Pareto optimal when the transportation cost is minimized. When the locations of the two firms are exogenously fixed at 0 and 1 respectively, and no information is available, then each consumer buys from the nearest firm. However, when no information is available, the socially optimal locations are $1/4$ and $3/4$ [see Tirole (1988), p. 282].

(as $N \to \infty$) the size tends to zero. Nevertheless, the cost of collecting information is still positive and consequently the market outcome never becomes Pareto optimal. Despite this inefficiency, compared to the no information case, we believe it is important to point out that given that information is available, an increase in its quality leads to higher (relative) efficiency (ignoring the cost of collecting information).

Two of our modelling assumptions deserve further discussion.

The fact that we started our analysis assuming that the market is covered (i.e., all consumers buy either one of the goods) has contributed to the definite loss of efficiency when information becomes available. Suppose that firms have to charge a uniform price and some consumers in the middle of the unit interval are better off not buying from either one of the two firms. Hence, the utility that these consumers derive is zero. When information becomes available, and the firms can charge different prices to different consumer groups, some of the consumers who were not buying before can now be attracted by the firms by being offered the good at a lower price. Therefore, there is a welfare increasing effect due to information when the market is initially not fully covered, which has been ignored in our modelling approach. It would not be surprising in this case that, for some sufficiently high level of information quality, total welfare exceeds the one under uniform pricing.

One of the restrictive features of the Hotelling model is that if one consumer has high willingness to pay for one product (i.e., he is located close to the firm producing that product) then he automatically has a low willingness to pay for the other product. This is due to the fact that a consumer's willingness to pay for both goods is located on the one dimensional interval which generates this perfect negative correlation for the willingness to pay between the two goods. In a companion paper, we relax this assumption by allowing for the possibility that a consumer may have a high willingness to pay for both goods and not only for one. We then study cases where the correlation between the willingness to pay for each good ranges from perfectly negative to perfectly positive. The drawback of this model is that it does not allow us to model explicitly the different levels of the information quality as we have done in the present paper. Rather the entire analysis is performed on the two extreme cases of no information and perfect information.

Appendix

A) Firms collect information by themselves

The next proposition summarizes the main result of this case.

Proposition 8 *If firms collect information by themselves and there is no intermediary, then*

(i) Suppose $N = 2$. If the cost of information is between

1) $[0, .0156t)$ then both firms acquire information,

2) $[.0156t, .0278t]$ then there are two equilibria; one where both firms collect information and the other where only one firm collects,

3) $(.0278t, .0313t]$ then both firms acquire information,

4) $(.0313t, .125t)$ then there is no equilibrium and

5) $[.125t, \infty)$ neither firm acquires information.

(ii) Suppose $2 > N > 8$. Consider α^{BB} as given in table 2 and α^{NBNB} as given in section 4.2 case 3. Notice that $\alpha^{NBNB}(N) > \alpha^{BB}(N)$. If the cost of collecting information $c(N)$ is

1) less than $\alpha^{BB}(N)$, then both firms buy information is an equilibrium,

2) between $\alpha^{BB}(N)$ and $\alpha^{NBNB}(N)$, then there is no equilibrium and

3) greater than $\alpha^{NBNB}(N)$, then neither firm buying information is an equilibrium.

(iii) Finally suppose that $N \geq 8$. This is the same as case (ii) above if we replace the $\alpha^{BB}(N)$ of table 2 with the formula of $\alpha^{BB}(N)$ $(N \geq 8)$ from section 4.2 case 1.

Proof. To support B vs. NB as an equilibrium, we know it must be $N = 2$, we need to show that no firm wants to deviate unilaterally. For firm 1 not to find it profitable to deviate and do not collect information, we need the cost of collecting information to be less than or equal to $\pi_1^{BNB} - \pi_1^d = .3125t - .28125t = .03125t$. Similarly, for firm 2 not to find it profitable to deviate and collect information, we need the cost of collecting information to be greater than or equal to $\pi_2^d - \pi_2^{BNB} = .265625t - .25t = .015625t$. Therefore, if $N = 2$, and the cost of collecting information is between $.015625t$ and $.03125t$, B vs. NB can be supported as an equilibrium.

To support B vs. B as an equilibrium, since it is symmetric, we only look at firm 1's deviation. We need the cost of collecting information to be less than or equal to the difference between $\pi_1^{BB} - \pi_1^d$. Recall from $\pi_1^{BB}(N) = t(9N^2 - 18N + 40)/36N^2$ if N is even, and $\pi_1^{BB}(N) = t(9N^2 - 18N + 40)/36N^2$, if N is odd.

When $N > 8$, we have $\pi^d_{1,BB}(N) = t/8$. $\alpha^{BB}(N)$ is the difference between $\pi^{BB}_1(N)$ and $\pi^d_1(N)$ for even and odd N's.

When $8 > N \geq 2$, deviating profit is listed in Table 1. $\alpha^{BB}(N)$ is the difference between $\pi^{BB}_1(N)$ and $\pi^d_1(N)$ (in Table 1) for even and odd N's respectively, as in Table 2.

To support NB vs. NB as an equilibrium, since it is symmetric, we only look firm 1's deviation. We need the cost of collecting information to be greater than or equal to $\pi^{NBNB}_1 - \pi^d_1 = t(2N^2 - 2N + 1)/2N^2 - t/2 = t(N^2 - 2N + 1)/2N^2$. ■

The difference from the case where the intermediary collects and sells the information is that (B vs. NB) can now be an equilibrium when $N = 2$. Also for certain ranges of the cost of information, no equilibrium exists or multiple equilibria exist. If both firms agree to buy information one will find it profitable to deviate and not buy; and if they both agree not to buy one firm increases its profits by deviating and buying information. Equilibrium always exists when there is an intermediary, who by maximizing his profit, serves as a coordinating devise for the two firms' strategies.

B) Proofs of propositions

Proposition 1

Proof. The Kuhn-Tucker first order condition of firm i is:

$$\frac{\partial \pi_i}{\partial p_i} \leq 0, \text{with equality if } p_i > 0.$$

We will analyze the following three cases.

Case 1: Both firms charge strictly positive prices.

Ignoring the nonnegativity constraints and setting $\partial \pi_i / \partial p_i = 0$, $i = 1, 2$, we can the have following solutions for p_{1m} and p_{2m}:

$$p_{1m} = \frac{t(N - 2m + 4)}{3N} \text{ and } p_{2m} = \frac{t(2m - N + 2)}{3N}$$

We can see that p_{1m} is decreasing in m, and p_{2m} is increasing in m. This means that firm 1 may decide to charge a zero price and give the entire segment demand to firm 2 for segments that are in firm 2's territory. The same holds for firm 2 in segments that are in firm 1's territory. For segments in the middle of the interval both firms charge positive prices. Observe that $p_{2m} = t(2m - N + 2)/3N \leq 0$ for any $m \leq (N-2)/2$ and $p_{1m} = t(N - 2m + 4)/3N \leq 0$ for any $m \geq (N+4)/2$. Now define $m_1(N)$ to be the highest integer that is less than equal to $N/2 - 1$, and $m_2(N)$ to be the lowest integer that is higher than or equal to $N/2+2$. Observe at this point that when N is even, $m_1 = N/2-1$ and $m_2 = N/2+2$, whereas when N is an odd number $m_1 = N/2 - 3/2$ and $m_2 = N/2 + 5/2$. This will be used later in the proof. Hence, for any $m = m_1 + 1, ..., m_2 - 1$, both firms charge strictly positive prices and have strictly positive demand.

Case 2: Firm 1 charges strictly positive prices while firm 2 charges a zero price.

Following the analysis above, this case is valid for $m \leq m_1$. Begin by assuming that $p_{2m} = 0$. We will find firm 1's best response. We argue that p_{1m} must be such that firm 1's demand in each segment is equal to $1/N$. To see this first suppose that firm 1 charges prices that yield demand greater that $1/N$. Since firm 1's maximum demand in each segment is $1/N$, it can do better by increasing its price. Now suppose that its price is such that its demand is less than $1/N$. This means that firm 2 has a strictly positive demand and therefore firm 2 can increase its price infinitesimally from zero to increase its profits in these segments. But this contradicts the fact that firm 2 charges a zero price. Hence p_{1m} is the solution to $d_{1m}(p_{2m} = 0) = 1/N$, which yields $p_{1m} = t(N - 2m)/N$.

Case 3: Firm 2 charges strictly positive prices while firm 1 charges a zero price.

This case is valid for $m \geq m_2$. This case is symmetric to case 2. Firm 2's prices in these segments are: $p_{2m} = t(2m - N - 2)/N$.

Below we summarize the results:

The equilibrium prices and profits are,

if $m_1 + 1 \leq m \leq m_2 - 1$,

$$p_{1m} = \frac{t(N - 2m + 4)}{3N} \text{ and } p_{2m} = \frac{t(2m - N + 2)}{3N}$$
$$\pi_{1m}^{B,B}(N) = \frac{t(2m - N - 4)^2}{18N^2} \text{ and } \pi_{2m}^{B,B}(N) = \frac{t(2m - N + 2)^2}{18N^2},$$

if $m \leq m_1$,

$$p_{2m} = 0 \text{ and } p_{1m} = \frac{t(N-2m)}{N}$$
$$\pi_{1m}^{B,B}(N) = \frac{t(N-2m)}{N^2} \text{ and } \pi_{2m}^{B,B}(N) = 0,$$

and if $m \geq m_2$, then

$$p_{1m} = 0 \text{ and } p_{2m} = \frac{t(2m-N-2)}{N}$$
$$\pi_{1m}(N) = 0 \text{ and } \pi_{2m}(N) = \frac{t(2m-N-2)}{N}$$

Therefore, firms' profits for each N are:

$$\pi_1^{B,B}(N) = \sum_{m=1}^{m_1} \frac{t(N-2m)}{N^2} + \sum_{m=m_1+1}^{m_2-1} \frac{t(2m-N-4)^2}{18N^2}$$
$$\pi_2^{B,B}(N) = \sum_{m=m_1+1}^{m_2-1} \frac{t(2m-N+2)^2}{18N^2} + \sum_{m=m_2}^{N} \frac{t(2m-N-2)}{N^2}.$$

Now we need to express m_1 and m_2 as a function of N (see case 1 in this proof where we have the thresholds as a function of N for even and odd subsequences), so that we will be able to perform the summation. To this end, we distinguish between two cases: I) When N is an even number and II) when N is an odd number.

<u>N is even.</u> In this case the summation yields,

$$\pi_i^{B,B}(N) = \frac{t(9N^2-18N+40)}{36N^2}, \; i=1,2.$$

<u>N is odd.</u> In this case the summation yields,

$$\pi_i^{B,B}(N) = \frac{t(9N^2-18N+43)}{36N^2}, \; i=1,2.$$

■

Proposition 2

Proof. Let $p_1^*(N)$ and $p_2^*(N)$ be the equilibrium price vectors of the two firms prior to deviation [see proposition 1]. We divide the $N's$ into an even and an odd sub-sequence.

We first start with the even sub-sequence. In this case, $m_1 = N/2 - 1$ and $m_2 = N/2 + 2$, and the two firms share two segments, $N/2$ and $N/2 + 1$. Firm 2's prices in these two segments are $2t/3N$ and $4t/3N$.

Consider N_1 and N_2 with $N_2 > N_1$. We know that any consumer with location $\hat{x} > 1/2 + 1/N_2$, will not buy from firm 1 under $(p_1^*(N_2), p_2^*(N_2))$ before firm 1's deviation. It is obvious that after deviation, firm 1 will still get zero demand for $\hat{x} > 1/2 + 1/N_2$.

Furthermore, it can be shown that any consumer with location $\hat{x} \leq 1/2 + 1/N_2$ ($\leq 1/2 + 1/N_1$) faces a weakly lower price from firm 2 when $N = N_2$ than when $N = N_1$. Therefore, firm 1's profit is weakly lower when $N = N_2$ than when $N = N_1$. This proves that in the even sequence the profits of the deviating firm are weakly decreasing as N increases.

A similar argument will prove that the profits of the deviating firm are weakly decreasing in N, when N is an odd subsequence.

To complete the proof, we will show that the deviating firm cannot have profits lower than $.125t$. This in turn will imply that once the profits of the deviating firm are $.125t$ for some N they stay there for any greater N.

Firm 1 before deviation is a constrained monopolist in the segments up to m_1, with $d_1 = (t - p_1)/2t$ in these segments. Thus, firm 1 even if it gets no demand in any other segments, following its deviation, it can certainly maximize its profits in the segments up to m_1. The optimal deviating price is $p_1^d = t/2$, $d_1 = 1/4$ and $\pi_1^d = .125t$.

It turns out (by solving the deviating firm's problem) that when $N = 8$ and 9, $\pi_1^d = .125t$. Hence for any $N \geq 8$ the deviating firm experiences profits equal to $.125t$ ■

Proposition 3

Proof. In B vs. NB firm 1 charges a different price in each segment and firm 2 charges a uniform price. We claim that there exist m_1 and m_2 ($N + 1 \geq m_2 > m_1 > 0$) such that: A) from segment 1 until m_1, firm 1 exactly drives firm 2 out of market (left segments), B) from segment $m_1 + 1$ until $m_2 - 1$ both firms (strictly) share the demand (middle segments) and C) from segment m_2 until N, firm 1 charges a price equal to zero and is out of the market (zero demand) in each segment (right segments). Firm 2's segment profits are π_{2L} (case A),

π_{2M} (case B), and π_{2R} (case C).

Note that in the left segments (segment 1 until m_1) firm 1 exactly drives firm 2 out of the market, i.e., firm 2's demand is exactly zero. This is true since firm 1 has the flexibility of charging different prices in each segment and if firm 2's demand in these segments was not exactly zero (but rather strictly negative) then firm 1 could increase its profits by increasing its price. But this would not be an equilibrium. Now fix $p_2 > 0$ and find firm 1's best reply (p_{1m}) to firm 2's price. Firm 2's profit function π_{2L} is not differentiable at this pair (p_2, p_{1m}). To see this observe that firm 2 obtains zero profits in the left segments and by increasing its price profits remain zero, while by decreasing its price it enjoys strictly positive profits. It is also possible that at segment m_2+1 (when $m_2 \leq N-1$) firm 2 happens to drive firm 1 exactly out of market. However, π_{2M} is continuously differentiable. For any fixed $p_2 > 0$ firm 1's reaction functions are,

$$p_{1m} = \begin{cases} \frac{-2tm+p_2N+tN}{N}, & \text{if } m \leq m_1 \\ \frac{-2tm+p_2N+tN+2t}{2N}, & \text{if } m_2 > m > m_1 \\ 0, & \text{if } m \geq m_2 \end{cases}$$

Now we differentiate π_{2M} with respect to p_2 and we plug in the p_{1m}'s from the above equation to obtain the following:

$$\begin{aligned} \frac{\partial \pi_{2M}}{\partial p_2} &= \frac{-3m_2+3+3m_1}{4t}p_2 \\ &+ \frac{m_2 - m_2N + m_2^2 + Nm_1 + N - 3m_1 - m_1^2 - 2}{4N} \end{aligned}$$

We first show that $N \geq 4$ implies $m_1 > 0$, and $N \geq 6$ implies $m_1 > 1$.

Assume that both firms strictly share the demand in segment m. Firm 2 charges a uniform price $p_2 \geq 0$, and by the first order condition, we can find firm 1's best reply and firm 2's resulting demand in this segment, i.e.,

$$\begin{aligned} p_{1m} &= \frac{2t + Np_2 + tN - 2tm}{2N} \\ d_{2m} &= \frac{2tm + 2t - Np_2 - tN}{4tN} \end{aligned}$$

If $N \geq 4$, then $d_{21} \leq 0$ (plug in d_{2m} above $m = 1$) implies that $m_1 > 0$; if $N \geq 6$, then $d_{22} \leq 0$ (plug in d_{2m} above $m = 2$) implies that $m_1 > 1$.

For p_2 to be best reply to p_{1m}, in what follows we look at 4 cases. Since, as we pointed out above, π_2 is not continuously differentiable we use left and right hand side derivatives. For each case, we have two inequalities, $eq1$ (which is the left derivative of π_2 with respect to p_2)

$$\frac{\partial \pi_2(p_2-)}{\partial p_2} = eq1 = \frac{-m_1 p_2}{2t} + \frac{\partial \pi_{2M}}{\partial p_2} + \frac{N - m2 + 1}{N} \geq 0$$

which guarantees that firm 2 does not want to decrease price, and $eq2$ (which is the right derivative of π_2 with respect to p_2)

$$\frac{\partial \pi_2(p_2+)}{\partial p_2} = eq2 = \frac{\partial \pi_{2M}}{\partial p_2} + \frac{N - m2 + 1}{N} \leq 0$$

which guarantees that firm 2 does not want to increase price. When these two inequalities are both satisfied (in cases 2 and 3 below), they turn out to be equal to zero. Solving the equations, we can have p_2 (and thus p_{1m} as best reply to p_2) and d_{1m}, d_{2m} as a function of m_1 and m_2. By combining $d_{1m} = 0$ and $d_{2m} = 0$, we have the non-integer values of m_1 and m_2. Then we choose integer values around these non-integer values such that the resulting demand is consistent with our definition of m_1 and m_2.

Case 1: $m_1 > 0$ and firm 2 does not drive firm 1 exactly out of market at any segment (i.e., with $p_2 + \varepsilon$, firm 1 is still out of market at all segments $m \geq m_2$)

Since $m_1 > 0$ (by the assumption of this case), $p_2 > 0$ ($p_2 = 0$ obviously will not be best reply since firm 2 has strictly positive demand in segments $m \geq m_2$ and by increasing its price by ε its profits will also increase), and $t > 0$ it is implied that $eq1 < eq2$. Therefore, $eq1 \geq 0$ and $eq2 \leq 0$ will never hold simultaneously. Thus, there is no such an equilibrium.

Case 2: $m_1 > 0$ and firm 2 drives firm 1 exactly out of market at segment m_2.

Following the same logic as in case 1 above, we have

$$eq1 = \frac{-m_1 p_2}{2t} + \frac{\partial \pi_{2M}}{\partial p_2} + \frac{N - m2 + 1}{N} \geq 0$$

$$eq2 = \frac{\partial \pi_{2M}}{\partial p_2} + \frac{N - m2 + 1}{N} - \frac{p_2}{2t} \leq 0$$

The only time that $eq1 \geq 0$ and $eq2 \leq 0$ hold simultaneously is when $m_1 = 1$, and both inequalities become equalities (they are the same when $m_1 = 1$). We have shown that $N \geq 6$ implies that $m_1 > 1$. So we only need to check for N between 2 and 5. The results are listed as following:

$$\begin{aligned}
N &= 2, \text{ we find that } m_1 = 0, \text{ which is a controdiction} \\
N &= 3, \ m_1 = 1, \ m_2 = 4, \ \text{but } d_{13} = 0 \text{ (which implies } m_2 = 3), \ \text{contradiction} \\
N &= 4, \ m_1 = 1, \ m_2 = 4, \ \text{but } d_{14} < 0, \\
&\quad \text{firm 2 does not exactly drive firm 1 out of marekt at segment } m_2, \ \text{contradiction.} \\
N &= 5, \ \text{we find that } m_1 = 2, \ \text{which is a contradiction.}
\end{aligned}$$

We do not find such equilibrium.

Case 3: $m_1 = 0$ and firm 2 does not drive firm 1 exactly out of market at any segment (same as segment m_2 only)

$$eq1 = \frac{\partial \pi_{2M}}{\partial p_2} + \frac{N - m2 + 1}{N}$$
$$eq2 := \frac{\partial \pi_{2M}}{\partial p_2} + \frac{N - m2 + 1}{N}$$

We have shown that $N \geq 4$ implies that $m_1 > 0$. So we only need to check $N = 2$ and 3. We can see that $eq1$ and $eq2$ are the same, therefore both inequalities become equalities. For $N = 2$, $m_1 = 0$, $m_2 = 3$, and firm 2 does not drive firm 1 exactly out of market at any segment. Therefore, this is an equilibrium. When $N = 3$, we find that $m_1 = 1$ which is a contradiction.

Case 4: $m_1 = 0$ and firm 2 drives firm 1 exactly out of the market in segment m_2. We know that $m_1 = 0$ only if $N < 4$. In addition, since firm 2 drives firm 1 exactly out of the market in segment m_2, it must be that $N > 2$, then $N = 3$, and we find that $m_1 = 0$ and $m_2 = 3$. Firm 2 exactly drives firm 1 out of market at segment 3, and $p_{13} = 0$, implying that $p_2 = t/3$. Since firm 1 and 2 share segment 1 and 2, first order conditions are necessary and sufficient. We find $p_{11} = 2t/3$, but this implies $d_{21} = 0$, i.e., firm 2 is out of the market in segment 1 and $m_1 > 0$, which is a contradiction. Therefore, there is no such an equilibrium.

■

Proposition 5

Proof. If firm 1 deviates and chooses to buy information, since $p_2 = t$, it will never be out of the market in any segment, i.e., $d_{1m} > 0$ for any m and N. And if firm 1 wants, it can drive firm 2 out of the market in every segment, though this may not necessarily maximize its profit. We first find firm 1's segment price that can exactly drive firm 2 out of segment m to be $p_{1m} = -2t(-N + m)/N$ and $\pi_{1m} = -2t(-N + m)/N^2$. Assume that firm 1 and 2 will share the demand in segment m, firm 1's first order derivative of profit is

$$\frac{\theta\pi}{\theta p_{1m}} = -\frac{p_{1m}N - tN + tm - t}{tN}.$$

If we evaluate it at $p_{1m} = -2t(-N + m)/N$, we have

$$\frac{\theta\pi}{\theta p_{1m}}|_{p_{1m}=-\frac{2t(-N+m)}{N}} = \frac{-N + m + 1}{N}.$$

For firm 2 to have positive demand, this derivative must be positive, which implies $m = N$. This means that firm 2 will always have positive demand only in the last segment. Then firm 1 shares with firm 2 in segment N and $p_{1N} = t/N$, and $\pi_{1N} = t/2N^2$. In segments $m < N$, firm 1 charges $p_{1m} = -t(-N + m - 1)/N$ to exactly drive firm 2 out of market and $\pi_{1m} = -2t(-N + m)/N^2$. Therefore,

$$\pi_1^d(N) = \sum_{1 \le m < N} -\frac{2t(-N + m)}{N^2} + \frac{t}{2N^2} = \frac{t(2N^2 - 2N + 1)}{2N^2}$$

■

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