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Analysing the Sustainability of Fiscal Deficits in Developing Countries

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This paper surveys the recent literature analysing fiscal deficit sustainability, most of which focuses on the U.S. and other industrial countries, in an attempt to assess its potential usefulness in the developing country context. Both the *accounting approach* and the *present value constraint (PVC) approach* are considered. Typically, sustainability analyses for developing countries involve issues that are not particularly important in the industrial country context. Reliance on seigniorage to finance deficits is often quantitatively much more important, although its use varies widely across LDCs. The distinction between domestic and foreign-currency borrowing is central; concessional lending and grants may also make an important contribution to fiscal finance. We consider generalizations of the PVC approach to situations where money-financing of deficits is used and concessional financing is available. The simultaneous presence of domestic and foreign debt, which characterizes a growing number of LDCs, are also discussed.

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Analysing the Sustainability of Fiscal Deficits in Developing Countries

"Sustainability" is perhaps the most frequently used buzzword in economic policy making circles in the 1990s: sustainable development, sustainable environmental policies, sustainable debt and deficit levels. In the macroeconomic context, policy makers and analysts are frequently asked: Are current levels of fiscal deficits or levels of public-sector debt sustainable? Are renewed capital inflows to LDCs in the early 1990s sustainable?.[1] These issues are important for developed and developing countries alike.

This paper surveys the recent literature analysing fiscal deficit (and, in some cases, current account) sustainability, most of which focuses on the U.S. and other industrial countries, in an attempt to assess its potential usefulness in the LDC context. Two conceptual approaches have been used: the *accounting approach* the *present value constraint (PVC) approach.* Although we briefly summarize the former, our emphasis is the PVC approach. The starting point for both is the (temporal) financing constraint of the government or consolidated public sector, which is outlined in Section 1. The accounting approach to sustainability or macro policy consistency is discussed in Section 2.

The present value constraint or intertemporal budget constraint is derived in Section 3; the appropriate interpretation of PVC "test" in the literature is then discussed: What does it mean to "test" a budget constraint? We argue that the PVC tests should not be interpreted as tests of whether a government is "solvent" but rather as tests of whether its fiscal policy stance is sustainable. That is, could the past behaviour of key fiscal variables and the implied fiscal deficit or surplus, as captured by simple time series models in the econometric tests, be continued indefinitely without encountering resistance by lenders? For this to be feasible, the fiscal policy must not entail "Ponzi scheme" financing.

From this perspective, it is the behaviour (or "willingness") of the government's *creditors* that ultimately determines the sustainability of a fiscal policy. The no Ponzi game (NPG) condition employed in Section 3 can, in some circumstances, be derived from or is equivalent to the transversality condition in the lender's utility maximization problem. This

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[1]/ Of course, capital inflows to LDCs might be desirable even if they are unsustainable. Emphasizing this point, Max Corden has quipped, "The growth of a child is not sustainable, but desirable never the less!"

is taken up in Section 4, first in a deterministic setting and then in a more general stochastic environment. (This section, which is somewhat more technical, can be omitted on first reading without lose of continuity.)

Section 5 (and the spreadsheet examples in Appendix 2) provide some transparent examples of sustainable and unsustainable fiscal policy in the simple case where only domestic bond financing of fiscal deficits is possible. In reality, of course, fiscal rules are likely to be more complicated. Rather than trying to characterize these rules, the econometric literature testing the PVC focuses on time series properties of the primary surplus, debt, and in some cases, government spending and taxation, without explicitly relating them via an economic model to (presumably) endogenous variables like the real interest rate, GDP growth, inflation, etc.

Ideally, the unit root and/or cointegration-based tests of sustainability should employ long time series (say 50-100 annual observations) on various macroeconomic variables. For most LDCs, such long time series are typically not available, or are contaminated by one or more "regime shifts" which invalidate the assumption that the data are all from the same data generating process. We discuss the possibility of describing fiscal policy rules based on shorter time series combined with other country-specific information (such as the policy conditions or targets articulated in a country's IMF and/or World Bank programs). The methods in the literature can then be used to study whether the continuation of these hypothesized rules into the indefinite future is "sustainable" or whether this fiscal stance would ultimately require levels of financing that lenders would find objectionable.

Section 6 discusses various econometric methods used to test sustainability of fiscal policy. The empirical findings for U.S. fiscal policy are reviewed. As the tests are based on different auxiliary assumptions, they sometimes lead to different conclusions. These are highlighted and the empirical validity of the auxiliary assumptions is discussed. Section 6 concludes with Ahmed and Roger's (1995) extension of the PVC approach to the simultaneous sustainability of *current account* deficits and fiscal deficits.

Sustainability analyses for developing countries will in many cases involve issues that are not particularly important in the industrial country context. Reliance on seigniorage to finance deficits is often quantitatively much more important, although its use varies widely across LDCs. The distinction between domestic and foreign-currency borrowing is surely central; concessional lending and grants may also make an important contribution to fiscal finance. Section 7 considers generalizations of the PVC approach to situations where money-financing of deficits is used and concessional financing is available. The simultaneous presence of domestic and foreign debt, which characterizes a growing number of LDCs, are also discussed. The literature typically aggregates the two types of debt and considers a single NPG condition or PVC. (See, e.g., Agenor and Montiel (1996, Chapter 4) .) We argue that correct treatment of this situation involves two separate "no Ponzi game" conditions, one for domestic lenders and another for foreign lenders.

Section 8 concludes.

1. The Consolidated Public-Sector Financing Constraint

Analyses of fiscal policy sustainability as well as discussions about the mutual consistency of various macroeconomic objectives begin with the financing constraint of the consolidated public sector including the central bank. This constraint relates the *conventional* deficit, i.e. the *primary* deficit plus nominal interest payments, to increases in internal and external sources of financing, as follows:

$$\Delta\tilde{B}_t + S_t \, \Delta\tilde{B}_t^* + \Delta\tilde{M}_t = S\tilde{U}RP_t + i_t\tilde{B}_{t-1} + i_t^*\tilde{B}_{t-1}^* \qquad (1)$$

where B_t , B_t^*, and M_t are domestic-currency debt instruments ("bonds"), foreign-currency bonds, and the monetary base, respectively. The tilde (~) indicates nominal variables; i_t and i_t^* are nominal rates of return on domestic and foreign bonds.

For analytical work, it is more convenient to rewrite (1) in real terms:

$$\Delta B_t + \Delta(s_t B_t^*) + \Delta M_t = -SURP_t - \pi_t M_{t-1} + r_t B_{t-1} + (r_t^* + \epsilon_t)B_{t-1}^* \qquad (2)$$

where the absence of tildes on the financial stocks and the deficit indicates real magnitudes, i.e. nominal series deflated by the domestic GDP deflator.[2] r_t and r_t^* are real rates of return

[2] Discussions of deficit sustainability focus on the relationship between real primary surpluses and the real value of debt. For most countries, however, the bulk of debt is non-indexed *nominal* debt. In this context, Woodford (1995) points out that the determination of the price level may depend on the total quantity of nominal liabilities (monetary and nonmonetary)

on domestic and foreign bonds; s_t is the real exchange rate $s_t = S_t P_t^* / P_t$ (where P_t^* is the foreign price level). ϵ_t is the real rate of depreciation of the domestic currency.

From (2) , it is clear that any attempt to determine what level of (real) primary fiscal deficit (or surplus) is "sustainable" must involve assumptions about reliance on seigniorage, as well as assumptions about the relative importance of domestic and foreign sources of debt finance over time. The present value constraint approach to sustainable fiscal policy was initially developed to study industrial countries. It was assumed that seigniorage revenue was unimportant and all public sector debt was denominated in domestic currency. Under these assumptions the financing constraint simplifies to a simple dynamic equation relating the stock of debt carried forward from the previous period, inclusive of interest, and the primary surplus to next period's debt B_t :

$$B_t = (1+r_t)B_{t-1} - SURP_t \quad . \tag{3}$$

B_t is the outstanding debt at the end of period t and r_t equals the *ex post* return on government debt during period t. As with (1), equation (3) may be interpreted in nominal or real terms. On the other hand, the auxiliary assumptions required in the econometric tests (discussed below in Section 6) are more likely to be satisfied if we consider real debt (i.e. nominal debt divided by a same-currency price index such as the GDP deflator or CPI). Hence, r_t and $SURP_t$ are interpreted as the *real* interest rate and *real* primary surplus, respectively, in what follows.

Given time paths for r_t and $SURP_t$, the government financing constraint in (3) describes the time path of the stock of debt, i.e., the dynamics of debt accumulation or decumulation. Several things are apparent from (3):

- If the government runs a primary surplus equal to zero ($SURP_t = 0$), the stock of debt will grow at a rate equal to the interest rate:

$$\Delta B_t \equiv B_t - B_{t-1} = r_t B_{t-1} \quad . \tag{4}$$

- If the government runs a primary *deficit* ($SURP_t < 0$), the stock of debt will grow at a

rather than just the nominal money supply.

rate exceeding the interest rate.[3]

- If the government runs a primary surplus ($SURP_t > 0$), the stock of debt will grow more slowly than the interest rate. If the surplus more than offsets interest payments on existing debt (i.e. the conventional surplus, $SURP_t + r_t B_{t-1}$, is positive), then the debt will actually shrink over time.

Both the PVC tests of sustainability and the accounting approaches to the consistency of macro policy targets begin from (3), or more generally (1).

2. The Accounting Approach to Sustainability or Policy "Consistency"

The so-called accounting approach is sometimes viewed as an approach to fiscal sustainability. Other authors interpret it as a way to assess the mutual consistency among a number of macro policy targets. In any event, the approach focuses a particular debt *ratio*, typically debt to GDP, $b_t = B_t / Y_t$. Rewriting (3), which is in levels, in terms of the debt/GDP ratio yields:

$$\frac{B_t}{Y_t} = \frac{(1+r_t)B_{t-1}}{(1+g_t)Y_{t-1}} - \frac{SURP_t}{Y_t} \qquad (5)$$

or:

$$b_t = \frac{1+r_t}{1+g_t} b_{t-1} - surp_t \qquad (6)$$

where g_t is the growth rate in GDP between t-1 and t. Using (6), the change in the debt/GDP ratio equals:

$$\Delta b_t \equiv b_t - b_{t-1} = \frac{r_t - g_t}{1+g_t} b_{t-1} - surp_t \qquad (7)$$

where $surp_t = SURP_t/GDP_t$. It follows immediately that:

- If the primary surplus/GDP ratio is equal to zero, the debt/GDP ratio will grow (or

[3]/ For a constant deficit, however, the growth rate of the debt falls *asymptotically* toward r.

shrink) at the rate r-g.

- If the government runs a primary *deficit* (surplus), the debt/GDP ratio will grow at a rate exceeding (less than) r-g.

In the accounting approach, a primary deficit (or surplus) is defined as sustainable if it generates a constant (rather than ever-increasing) debt/GDP ratio, given a specified GDP growth target and constant real interest rate. Thus, in the simple case where seigniorage revenue and foreign borrowing are ignored, the sustainable primary surplus to GDP ratio is determined by setting the change in the debt/GDP ratio in (7) equal to zero:

$$surp_t = \frac{r_t - g_t}{1 + g_t}\, b \qquad . \qquad (8)$$

This is the level of the primary surplus that would be required each year to keep the debt/GDP ratio constant at its current level b.

Applications of the accounting approach invariably consider the possibility of using seigniorage revenue as a source of fiscal finance. In this case, ***surp*** in (8) should be interpreted as the primary surplus plus sustainable seigniorage revenue (as a ratio of GDP). The latter is calculated by assuming that the ratio of real high-powered money to GDP is a negative function of the inflation rate. The target inflation rate is then used to calculate the steady-state monetary base/GDP ratio and the resulting seigniorage. (See Anand and van Wijnbergen (1989) for a thorough discussion of this approach and an interesting application to Turkey.)

The accounting approach has also been used to assess the consistency among various macroeconomic policy targets.[4] Suppose the government has the following policy targets (denoted by *): (i) a constant debt/GDP ratio b*, (ii) a target GDP growth rate equal to g* and (iii) a primary surplus/GDP ratio equal to surp*. Are these policies targets mutually consistent? In addressing this question, the accounting approach typically assumes that

[4] Oftentimes the literature calls this an analysis of "*policy* consistency." This is potentially misleading in it is referring to the consistency of policy *targets*, such as GDP growth or the target debt ratio, rather than the policy *instruments,* which are directly controlled by the policy maker.

changes in the primary surplus will have no effect on either real interest rates or GDP growth. This is surely unrealistic. Presumably, the equilibrium real interest rate depends positively on the level of government spending and/or the amount borrowed. To answer the above question, one would ideally use a model that endogenously determines real interest rates and the GDP growth rate. It would then be possible to analyze how these key macro variables are affected by changes in fiscal policy variables.

Although the accounting approach focuses on steady-state debt ratios, the corresponding dynamic equation in (6) could be employed to ask various questions about transitional dynamics. For example, what time path of adjustment in the primary surplus over x years will result in a specified fall in the debt/GDP ratio? Would a debt write off be helpful in achieving this debt ratio target?

The primary shortcoming of the accounting approach is that it attempts to determine the "financable" fiscal deficit by making assumptions that liabilities can continue to grow at the growth rate of the economy's GDP, so that debt/GDP ratios remains constant. This leaves rather vague the role that *lenders* ultimately play in determining what debt strategies are "sustainable" and which are not. The PVC approach is more explicit in this regard.

3. The Present Value Constraint Approach

The PVC approach begins with the government financing constraint in level [i.e. (3) above] and iterates it forward N periods to get:

$$B_{t-1} = \sum_{j=0}^{N} \frac{SURP_{t+j}}{(1+r)^{j+1}} + \frac{B_{N+1}}{(1+r)^{N+1}} \quad . \qquad (9)$$

In deriving (9) from (3), it has been assumed for expositional simplicity that the (expected) real interest rate is constant over time. A generalization to time varying interest rates is considered below. Some of the econometric tests in Section 5 require the (strong) assumption that the expected real interest rate is constant; others require only the assumption that the real interest rate is stationary.[5]

At this point the co-called "no Ponzi game" (NPG) condition is invoked to argue that the last term in (9) goes to zero in the limit:

$$\lim_{N \to \infty} \frac{B^{N+1}}{(1+r)^{N+1}} = 0. \tag{10}$$

This condition states that the *present value* of the government's debt in the indefinite future converges to zero. For this to happen, real debt B (in the numerator) must grow more slowly than the real interest rate (which is the growth rate of the denominator).

The NPG condition is typically justified by arguing that lenders would presumably not be willing to allow the government to perpetually pay their entire current interest obligation merely by borrowing more. If lenders were willing to do this, (4) shows that the debt would grow at a rate equal to the interest rate. Hence the discounted debt in (10) would not converge to zero. Section 4 below discussed the NPG condition and its relationship to the transversality condition in the lender's intertemporal utility maximization problem in greater detail.

Imposing the NPG condition in (9) implies that the government debt at any point in time must equal the present value of its expected future primary surpluses:

[5] Even this assumption is not uncontroversial. See Rose (1988).

$$B_{t-1} = \sum_{j=0}^{\infty} \frac{SURP_{t+j}}{(1+r)^{j+1}} \quad . \tag{11}$$

Note that the real interest rate must be positive for the present value of future surpluses to be finite. David Wilcox (1989, pp. 291-2) elaborates in the importance of the PVC:

> Fiscal policy is constrained by the need to finance the deficit. Virtually any pattern of deficits would be sustainable if it were possible to borrow money, and pay the interest by borrowing more. Indeed, in some model economies it is possible for the government to do exactly that (Diamond 1965). In those economies, which are labeled dynamically inefficient, an increase in current debt has no implications for future surpluses. Governments in dynamically efficient economies, on the other hand, face a present-value borrowing constraint, so-called because it states that the current market value of the debt equals the discounted sum of expected future surpluses.

Interpreting Econometric Tests of the PVC or NPG Condition

The recent empirical literature, initiated by the seminal contribution of Hamilton and Flavin (1986), tests the validity of the PVC in (11) or equivalently the NPG condition (10). There is a question of how to interpret such tests. What does it mean to "test" a budget constraint? Some authors have interpreted tests of (10 or (11) as "solvency" tests. For example, Agenor and Montiel (1996, 123) argue that:

> The government is solvent if the expected present value of the future resources available to it for debt service is at least equal to the face value of its initial [i.e. current] debt stock. Under these circumstances, the government will be able to service its debt on market terms. Solvency thus requires that the government's prospective fiscal plans satisfy the present-value budget constraint...

On the other hand, an optimizing government should never plan to have a stream of future primary surpluses with a NPV strictly *in excess* of its current debt, because this would imply lower government spending and/or higher taxes than necessary to service the debt.

Other writers interpret the PVC tests as tests of the "sustainability" of current fiscal policy. Wilcox (1989, pp.293-4) contains an interesting discussion on how to interpret apparent violations of the PVC:

> Hamilton and Flavin view their tests as shedding light on whether the government must satisfy the borrowing constraint. If [the NPG condition in (10) or the PVC in (11)] were violated in the data, Hamilton and Flavin would conclude that the borrowing constraint need not be satisfied. By contrast, I regard the necessity of the present-value borrowing constraint in a dynamically efficient economy as established on theoretical grounds...This suggests a natural definition for the concept of sustainability: a sustainable fiscal policy is one that would be expected to generate a sequence of debt and deficits such that the present-value borrowing constraint would hold....Moreover, an unsustainable policy would be expected to change...
>
> If the present-value borrowing constraint does not hold, what will be the form of the violation?

Hakkio and Rush (1991, p.429) also interpret their PVC tests as tests of the sustainability of current fiscal policy:

> Is the [U.S.] budget deficit "too large?" Yes. Specifically, we find that recent spending and tax policies of the government -- if continued -- violate the government's intertemporal budget constraint. As a result government sending must be reduced and/or tax revenues must be increased.

The authors state explicitly that they are testing whether the NPG condition would be satisfied *if government revenue and expenditure continued to follow their past stochastic processes.*

I conclude that the PVC tests are appropriately viewed as tests of the sustainability of the current fiscal policy stance, as reflected in the historical times series data on government spending, revenue, deficits, and/or debt, not as *solvency* tests. An analysis of solvency would have to consider all conceivable government policies, and ask whether there is *any* economically and politically feasible policy stance that would satisfy the PVC, given the value of current debt. If there is none, then the government is insolvent.

In the U.S. context, it is reasonable to assume that government expenditure or tax policies would ultimately have to change in order to bring the projected stream of discounted future primary surpluses into line with the PVC in situations where the current fiscal policy is unsustainable. In the LDC context, on the other hand, inflationary surprises to wipe out debt obligations and/or debt repudiation (or threatening debt repudiation in order to secure more favorable terms from creditors) may be entertained as policy options. Presumably this arises when the political or economic costs of adjusting government expenditures or taxes become too high relative to the costs of debt repudiation. In short, LDC governments may become "unwilling to pay" before they reach the point where they are "unable to pay" or insolvent.

Sustainability Tests Involving Debt Ratios

In order to relate the PVC tests to the accounting approach below, it is interesting to recast NPG condition in (10) or equivalently the PVC in (11) in terms of ratios. Interestingly, an analysis based on ratios is often motivated by the argument that this is more appropriate for growing economies. For example, Hakkio and Rush (1991, p.430) note:

> In addition to examining real spending and revenue directly, we also normalize these variables by real GDP and population. This is an important extension beyond previous work since McCallum (1984), among others, deems these ratios -- per capita spending and revenue as a fraction of GNP -- as more pertinent for a growing economy.

I believe this statement is false when applied to GDP growth, but may be valid when population growth is involved.

Consider the government financing constraint (3) in levels. One can always rewrite this constraint with all variables expressed in terms of ratios to any variable that one might care choose, be it GDP, population, or whatever (i.e. the world output of bananas). Defining $b_t = B_t / Y_t$ as the relevant ratio (e.g., using the debt to GDP ratio in (5) above), the government financing constraint in (3) can be rewritten by dividing through by Y_t and using the identity:

$$Y_{t-1}(1+g_t) \quad \equiv \quad Y_t \qquad . \tag{12}$$

where g_t is the growth rate in GDP between t-1 and t. The result looks like (5) above. Assuming for simplicity that r and g are constant over time, recursive forward substitution in (3), using the definition in (13), yields an expression analogous to (9), but in ratio form:

$$b_0 \quad = \quad \sum_{j=0}^{N} [\frac{1+g}{1+r}]^{(j+1)} \frac{SURP_j}{Y_j} \; + \; [\frac{1+g}{1+r}]^{(N+1)} \; \frac{B_{N+1}}{Y_{N+1}} \qquad . \tag{13}$$

The NPG condition now appears to take the form:

$$\lim_{N\to\infty} \quad [\frac{1+g}{1+r}]^{(N+1)} \; \frac{B_{N+1}}{Y_{N+1}} \quad = \quad 0 \qquad . \tag{14}$$

Note that for the PVC or NPG conditions expressed in ratio terms as in (13) or (14), respectively, the appropriate discount factor (1+g)/(1+r) takes into account the growth rate of the variable is used in the denominator of the ratio. Hakkio and Rush (1991, p.430) explain:

> When variables are nominal, the discount factor is the nominal interest rate; when variables are real, it is the real interest rate; when variables are real per real GNP, it is the real interest rate minus the rate of growth of real GNP; and when variables are real per capita, it is the real interest rate minus the rate of population growth.

At first glance, the NPG condition (14) seems to require that r>g. In fact, this is not the case. The NPG condition in (14) is ***identical*** to that in (10). Using the identity in (12), it is easy to see that (14) does *not* depend on the growth rate in Y (regardless of whether Y is real income or some other variable). By similar reasoning, it can be shown that the PVC written in terms of ratios does not depend on g either:

$$b_{t-1} \quad = \quad \sum_{j=0}^{\infty} \frac{(1+r)^{-(j+1)} SURP_{t+j}}{(1+g)^{-(j+1)} Y_{t+j}} \quad = \quad \sum_{j=0}^{\infty} \frac{(1+r)^{-(j+1)} SURP_{t+j}}{Y_0} \quad . \qquad (15)$$

Given the above result that the conversion of the PVC and NPG conditions into ratio form leaves them unaffected, it is troublesome that empirical analyses sometimes arrive at different conclusions using level and ratio data. (See especially Hakkio and Rush (1991).) A key issue when implementing the PVC tests is the stationarity of the variables being used. For cointegration based tests, the variables in the cointegrating relationship must be I(1). It possible that level series are I(1), but the ratio series are I(0). Thus, the decision to transform the PVC into ratios may make the auxiliary assumptions associated with various tests (highlighted in Table 1 below) more or less plausible.

When *are* the relative magnitudes of the real interest rate and the growth rate of income relevant? Section 5 provides an illustration where the process determining primary surplus $SURP_t$ is assumed proportional to GDP. In this case, the present value constraint can be rewritten in terms of ratios to GDP and the difference between the real interest rate and the growth rate.

Before leaving the discussion of the debt/GDP ratio, it should be noted that in some of the literature on fiscal deficit sustainability, a different definition of sustainability is used. Fiscal policy is said to be sustainable if the time path of the debt/GDP ratio is bounded, i.e. does not continue to grow without limit.[6] Is this criterion stronger than the PV constraint? Under circumstances where the real interest rate is higher than the growth rate of GDP, the boundedness of debt to GDP is indeed a stronger criterion than the PV constraint. The PV constraint requires that the growth rate of debt be less than the real interest rate. This does not rule out the possibility that debt GDP ratio increases without bound. If the real interest rate is higher than GDP growth rate and the growth rate of debt is in between them, the PV

[6] Hakkio and Rush (1991), for example, test both the PV constraint and the boundedness of debt/GDP ratio.

constraint is satisfied but the debt GDP ratio explodes over time. On the other hand, if debt GDP ratio is bounded, the growth rate of debt has to be less than or at most equal to the GDP growth rate. Given the real interest rate is higher than the growth rate of GDP, PVC is certainly satisfied.

4. Optimal Lender Behavior and the No Ponzi Game Condition[7]

The purpose of this section is to discuss the situations where the NPG condition can be derived from the lender's utility optimization problem. The deterministic case with zero population growth is a benchmark case. The cases of positive population growth and a stochastic economy are then considered in turn.

McCallum (1984) considers a dynamical efficient economy in steady state equilibrium. Assuming a constant interest rate (as in (9) above), he shows that the NPG condition follows immediately from the transversality condition of the lender's utility maximization problem. That is, the NPG condition is an implication of optimal behavior by *lenders.* (See Appendix 1 for details). Thus, Ponzi financing should not be possible if lenders are rational in deterministic models with constant population.

Rational Ponzi Games

O'Connell and Zeldes (1988) discuss the possibility of *rational* Ponzi games. They explain how such schemes can arise when an economy's population is growing over time and where there is no (or, at least, not universal) intergenerational altruism:

> New agents are born into the economy and fend for themselves. The key point here is that while each individual will satisfy his own transversality condition, this will not suffice to rule out rational Ponzi games. Even though each individual's wealth will not be growing faster than the inverse discount factor, population growth may make it possible for aggregate desired wealth to grow at the rate of interest or faster. Ponzi games are therefore feasible in an economy with infinite-lived agents. (p.438)

Interestingly, they note that:

> when borrowers are running rational Ponzi schemes, this does not imply that lenders are in any sense losing out. In the models we study in this paper, rational Ponzi games are only feasible when the economy is in a dynamically Pareto inefficient equilibrium. The introduction of perpetually rolled over debt will never make the lending economy worse off and will in general make it better off relative to a world in which no Ponzi game is run. (p.433).

Presumably, however, the lending economy could make itself even better off by having its own government rather than foreigners run the Ponzi game!!

In sum, the O'Connell and Zeldes (1988) analysis concludes that when the relevant population of lenders is growing, a modest Ponzi scheme is feasible. The transversality

[7]This section may be omitted on first reading without loss of continuity.

condition of individuals will imply that, if the population is growing a rate n, say, the government debt must grow at a rate less than r+n, not a rate less than r as presumed in the various sustainability tests. The transversality condition for an individual lender in economies with growing populations equals:

$$\lim_{N\to\infty} \quad \frac{B_N}{(1+r)^N(1+n)^N} \quad = \quad 0 \qquad (16)$$

where B_N is the aggregate stock of debt and hence $B_N/(1+n)^N$ is debt per capita. The government must insure that debt *per capita* grows at a rate less than the interest rate in order to satisfy the transversality conditions of all lenders in the economy. Thus, when the lender population is growing, sustainability tests should be based on the *ratios* of debt and the primary surplus to the population. Taking ratios to GDP, on the other hand, is potentially confusing and in any event has no bearing on the PVC or NPG tests (as shown at the end of Section 3), regardless of whether GDP in either the debtor or creditor economy is growing.

<u>Transversality Conditions in Uncertain Environments</u>

Bohn (1995) notes that the following two empirical observations can not be reconciled in a deterministic setting: (i) the interest rates on U.S. government bonds have been significantly below the average rate of the economic growth and (ii) according to the empirical analysis in Abel et al (1989), the U.S. economy is dynamic efficient. He goes on to argue that "these two things can happen together only in a stochastic environment." Hence, it is more appropriate to examine the sustainability of fiscal policy by using tests that are derived from stochastic models of the economy.

The PVC in (11) and the NPG condition in (10) were derived under the assumption of constant real return on government debt. Here we relax that assumption by deriving the corresponding conditions for the case where the *ex post* return on government debt r_t is stochastic and hence may vary from period to period. With non-constant r_t, forward iteration of the government financing constraint in (3) and taking expectations yields:

$$B_{t-1} \quad = \quad E_t\sum_{j=0}^{\infty} q_{t+j}SURP_{t+j} \quad + \quad E_t q_{t+N}B_{t+N} \qquad (17)$$

where

$$q_{t+j} \equiv \Pi_{i=0}^{j}(1+r_{t+i})^{-1} \qquad \textbf{(18)}$$

is the compound discount factor relevant for period t+j cashflows. The NPG condition, therefore, equals:

$$\lim_{N\to\infty} E_t q_{t+N} B_{t+N} = 0 \quad . \qquad (19)$$

The resulting PVC is:

$$B_{t-1} = E_t \sum_{j=0}^{\infty} q_{t+j} SURP_{t+j} \quad . \qquad (20)$$

What is the general constraint on government borrowing in a stochastic setting? As Bohn (1995) shows, the lender's intertemporal budget constraint can be written as:

$$(1+r_t)B_{t-1} = \sum_{j=0}^{\infty} E_t(s_{t+j} SURP_{t+j}) + \lim_{N\to\infty} E_t[s_{t+N}(1+r_{t+N})B_{t+N-1}] \quad . \qquad \textbf{(21)}$$

where $s_{t+N} = \beta^N u'(c_{t+N})/u'(c_t)$ is the lender's marginal rate of intertemporal substitution between periods t and t+N. $(1+r_t)B_{t-1}$ is the lender's wealth (inclusive of interest earned between t-1 and t) going into period t. The transversality condition requires that the lender's present valuation of future government liabilities, the last term on the right-hand-side of (21), should go to zero in the limit:

$$\lim_{N\to\infty} E_t[s_{t+N}(1+r_{t+N})B_{t+N-1}] = 0 \quad . \qquad (22)$$

The resulting PVC, therefore, equals:

$$(1+r_t)B_{t-1} = \sum_{j=0}^{\infty} E_t[s_{t+j} SURP_{t+j}] \quad . \qquad (23)$$

Notice that (22) and (23) are different from (10) and (11) in that they use the marginal rate of intertemporal substitution instead of the real interest rate as the discount factor.

The marginal rate of substitution is closely related to the risk-free interest rate. The Euler equation from the consumer's optimization problem implies that at period t the risk-free rate r* is related to the one-period marginal substitution as follows:

$$\frac{1}{1+r_t^*} = E_t s_{t+1} \quad . \tag{24}$$

Under the assumptions that: (i) there is no correlation between s_t and s_{t+1} for all t and (ii) there is no correlation between s_{t+N} and B_{t+N},[8] (22) can be written as:

$$\lim_{N\to\infty} \; E_t \; [q_{t+N} \; B_{t+N}] \;=\; 0 \quad . \tag{25}$$

Under these assumptions, therefore, the appropriate discount factor is the risk-free interest rate from the lender's standpoint instead of the inverse of one plus the real interest rate on government debt. This suggests that if one is testing the NPG condition for the Brazilian government's external debt, say, the appropriate interest rate is LIBOR, not the actual real interest rate on Brazilian debt. This is because LIBOR is a reasonable estimate of the lender's risk-free rate. Thus, in evaluating the NPG in (10), the discount factor should be the lender's risk-free rate. The real interest rate faced by the borrower, on the other hand, is relevant for determining the time path of B_{t+N} (via the difference equation in (3)).

Under more general assumptions where there are significant correlations between s_t and s_{t+i} and between s_{t+i} and B_{t+i} , the transversality condition in (22) does not directly imply the a no Ponzi game condition like (25) holds.

When the NPG condition is not a direct implication of the lender's transversality condition, is it still an implication of optimal behaviour by lenders? The answer is yes. As long as the household utility function u is strictly increasing and bounded, violation of the NPG condition cannot be consistent with optimal household behavior. If the NPG condition, or equivalently the PVC, is violated, it can be shown that a reallocation of the household's consumption over time in a way that satisfies the NPG condition, will raise expected utility:

[8]This assumption implies that any risk associated with B_t is diversifiable risk, so that investors do not demand a risk premium in addition to the risk-free rate.

$$E_t \sum_{j=0}^{\infty} \beta^j u(c_{t+j}, m_{t+j}).$$

Hence, it can not be optimal for lender's to allow Ponzi game finance.

To prove the foregoing claim, first discount the period t+j household *temporal* budget constraint back to period t. This can be done by multiplying both sides of the period t+j temporal budget constraint by q_{t+j} as defined above for j=0,1,...,N. Add the N+1 discounted temporal budget constraints together and take expectations of both sides to obtain:

$$\begin{aligned} & E_t[c_t + q_{t+1}c_{t+1} + ... + q_{t+N}c_{t+N}] + E_t[q_{t+N}B_{t+N}] - (1+r_{t-1})B_{t-1} \\ = \; & E_t \sum_{j=0}^{N} q_{t+j}[f(k_{t+j}) - \tau_{t+j} - (1 + \pi_{t+j})M_{t+j+1} + M_{t+j} - k_{t+j+1} + k_{t+j}]. \end{aligned} \quad (26)$$

Now suppose for the sake of argument that the present value constraint in (26) did not hold. That is, there exists some large N such that $E_t[q_{t+N}B_{t+N}] > 0$. In this case, it is clear that the household can raise its consumption c_t without altering its consumption in the other periods before t+N. Under the assumption that the utility function u is strictly increasing and bounded, a sufficiently large N would make such a reallocation superior to the allocation where $E_t[q_{t+N}B_{t+N}] > 0$. Therefore, a violation of the present value constraint (26) is not consistent with the utility maximization by the lender.

Most of the literature testing the NPG condition has assumed that the appropriate discount rate is the real interest rate on government debt (rather than the risk-free rate). In light of the above discussion, these tests are valid even in a stochastic economy. On the other hand, for a stochastic economy the transversality condition is different from the NPG condition and also a constraint on government borrowing. Therefore, it appears that a valid test of sustainability can be based on (23) or (26).

5. An Illustrative Example of Sustainable Fiscal Policy

This section lays out simple examples of sustainable and nonsustainable fiscal policies. Consider an economy where GDP (Y) grows exogenously at rate g (independent of the level of government spending and income taxation).[9] The government's fiscal policy rule is a very simple one: (1) keep government spending G constant as a fraction of GDP, i.e. $\gamma = G_t/Y_t$ is constant; (2) tax all income with a flat tax at rate t = T/Y, with no other sources of revenue; (3) finance any resulting deficit resulting from the policies in (1) and (2) by issuing (domestic-currency) bonds at a constant real interest rate r. With this fiscal policy package, the resulting *primary* surplus equals:

$$SURP_t = (\tau - \gamma)\, Y_t \quad . \tag{27}$$

With our simple assumptions, SURP grows over time at rate g, the growth rate of real income.

$$SURP_t = (\tau - \gamma) Y_0 (1+g)^t \quad . \tag{28}$$

Given the outstanding debt from the previous period, B_{t-1}, the government financing constraint in (3) describes the dynamics of debt accumulation. Iterating (3) forward and inserting (28) for all of the $SURP_t$ terms to captures the stance of fiscal policy stance yields:

$$B_0 = \frac{SURP_0}{r-g} = \frac{(\tau-\gamma)Y_0}{r-g} \quad , \tag{29}$$

provided that the real interest rate r exceeds the growth rate g (r>g). When g>r, which implies that the economy is dynamically inefficient, the summation of discounted future surpluses in the PVC is infinite.

Is the above fiscal policy package "sustainable?" The calculation is straightforward. Find the NPV of the steam of future surpluses, defined by the right-hand-side of (29). If this NPV exceeds (falls short of) the current debt, the policy package is "sustainable" (unsustainable) in the sense that it does not violate the government's intertemporal budget constraint.[10]

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[9]/ This is obviously restrictive. The recent growth literature argues that fiscal policy can affect the economy's growth rate. See, e.g., the model in Barro (1991).

[10]Whether it is *politically* feasible to "sustain" this policy stance is, of course, a different matter and not the one being tested in the PVC literature.

If the PV of future surpluses is negative, the fiscal policy is unsustainable regardless of the current debt level. To reiterate, if the fiscal stance implies a perpetual primary *deficit*, no amount of debt reduction can make the fiscal stance sustainable. On the other hand, if the PV of future surpluses is positive but less than the value of current debt, a write-off of debt equal to the difference:

$$B_0 - \frac{(\tau-\gamma)Y_0}{r-g}$$

would restore fiscal policy sustainability. The ability to generate a primary surplus is a *precondition* for a successful debt reduction program.

Note that in situations where SURP is proportional to GDP and r and g are constant over time, (29) can be rewritten in terms of the debt/GDP *ratio*.[11] The fiscal stance is sustainable if:

$$b_0 \leq \frac{surp_0}{r-g} = \frac{\tau-\gamma}{r-g} \quad . \tag{30}$$

Here, in contrast to the discussion in Section 2, the choice of denominator for the ratio on the right-hand-side of (15) can not be chosen arbitrarily; it depends what variable SURP is hypothesized to be proportional to. Furthermore, in deriving (30) from (11) r must be greater than g (or the infinite sum in (11) will not converge).

The foregoing example is illustrative. For more complicated time paths of government spending and taxation and a prespecified initial debt level, it is straightforward to implement fiscal policy sustainability analysis using a standard spreadsheet program. See Appendix 2.

6. Econometric Tests of Sustainability

Several recent studies on fiscal deficit sustainability for U.S. fiscal policy are

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[11] Here's a slightly different application of the formula. Consider an EMS country wishing to keep its debt/GDP ratio at the Maastricht treaty target of 60%. What primary surplus to GDP ratio is required for the fiscal stance to be sustainable? If the real interest rate is 5% and the GDP growth rate is 2%, it is easy to show using (30) that the primary surplus ratio must be 1.8%.

summarized in Table 1. These studies are based on different empirical tests, which in turn depend on the validity of different auxiliary assumptions. Hence, they may yield (and in practice have yielded) different conclusions regarding fiscal sustainability. This section describes the various empirical approaches and attempts to explain and reconcile the various findings.

The tests are classified into two groups. The first involves tests derived from the NPG condition using the real interest rate as the discount rate. The second group of tests is based on the transversality condition using marginal rate of substitution as discount factor.

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Table 1

Recent Studies of Fiscal Policy Sustainability

Study	Country/ Sample	Data Requirements	Auxiliary Assumptions	Test Methods	Conclusions
Flavin-Hamilton (1986)	U.S. 1960-84	real primary surplus plus seigniorage, real stock of debt (at market value)		Test stationary of fiscal deficit and debt.	Both are stationary, implying sustainable fiscal policy.
Wilcox (1989)	U.S. 1960-84	Market value of gov't debt; ex-post real return on gov't debt.	Debt follows general ARIMA process	Test whether *discounted* debt series is stationary with mean zero.	Weak evidence that discounted debt is stationary, yet its mean is nonzero. Hence, fiscal policy is not sustainable.
Trehan and Walsh (1988)	U.S. 1890-1986	real government spending, real interest payments, real revenue, seigniorage	constant real interest rate. G, T may be stationary or nonstationary	Test whether fiscal deficit inclusive of real interest payments is stationary.	Reject nonstationarity, implying fiscal policy is sustainable.
Trehan and Walsh (1991)	U.S. 1960-84	Flavin-Hamilton dataset	Real return on gov't debt is strictly positive, but need not be constant.	Test for stationarity of (1) real deficit inclusive of real interest or (2) first-difference of debt.	First difference of stock of debt is stationary, implying fiscal policy is sustainable.

Hakkio-Rush (1991)	U.S. 1950:II-1988:IV	Real gov't revenue and spending inclusive of real interest.	Real interest rate is stationary. Gov't spending and revenue are difference-stationary.	Test whether gov't revenue and spending inclusive of interest are cointegrated.	A cointegration relationship is found for the entire sample, but not for the sub-period 1976:III-1988:IV. Hence, recent fiscal policy is not sustainable.
Corsetti-Roubini (1991)	OECD 1960-89	Net general gov't debt; ex post return on debt.		Test stationarity of discounted debt and the existence of positive drift or time trend.	Results are mixed for OECD countries studied.
Ahmed-Rogers (1995)	U.S. 1792-1992 U.K. 1692-1992	Real gov't tax revenue, expenditure, and real interest payments	$s_{t+j}\,G_{t+j}$ and $s_{t+j}\,T_{t+j}$ are difference stationary where s_{t+j} is the lender's marginal rate of substitution.	Test whether real gov't tax revenue, expenditure, and real interest payments are cointegrated with vector (-1,1,1).	For both the U.S. and the U.K., a cointegration relationship with vector (-1,1,1) is found, implying that U.S. fiscal policy is sustainable.

Testing NPG condition

The literature testing the NPG condition originated with the pioneering work of Hamilton and Flavin (1984). They tested a version of the NPG condition where the *ex post* real interest rate on government debt in each period r_t was replaced by the average real interest rate denoted by r. This involves rewriting the government budget constraint (3) as:

$$\begin{aligned} B_t &= (1+r)B_{t-1} - SURP_t + (r_t - r)B_{t-1} \\ &= (1+r)B_{t-1} - SURP_t + v_t \quad , \end{aligned} \tag{31}$$

where the error term v_t captures the deviation of the interest rate from its average value. Iterating forward yields:

$$B_{t-1} \;=\; \lim_{N\to\infty} B_{t+N}(1+r)^{-(N+1)} \;+\; \sum_{j=0}^{\infty} (1+r)^{-(j+1)} SURP_{t+j} \;+\; n_t \quad , \tag{32}$$

where

$$n_t \;\equiv\; \sum_{j=0}^{\infty} (1+r)^{-(j+1)} v_{t+j}$$

is assumed to be mean zero stationary. Taking expectations on both sides of (32) yields:

$$B_{t-1} \;=\; E_t \, \lim_{N\to\infty} \; (1+r)^{-(N+1)} B_{t+N} \;+\; E_t \sum_{j=0}^{\infty} (1+r)^{-(j+1)} SURP_{t+j} \quad . \tag{33}$$

Hamilton and Flavin test[12] whether

$$E_t \lim_{N\to\infty} \quad (1+r)^{-(t+N)} B_{t+N} \;=\; 0 \quad . \tag{34}$$

or, equivalently:

$$B_{t-1} \;=\; E_t \sum_{j=0}^{\infty} (1+r)^{-(j+1)} SURP_{t+j} \quad . \tag{35}$$

For situations where the expected rate of return on government debt is constant (i.e., $E_t r_{t+i} = r$, for i=1,2,...) and r_t is uncorrelated with $SURP_s$ and B_s for all t and s,[13] the PVC in (20) becomes (35).[14]

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[12] Equations (34) or (35) are also used in sustainability tests by Trehan and Walsh (1988,1991), Hakkio and Rush (1991), and Tanner and Liu (1994).

[13] The assumption of no correlation between real interest rates and deficits or debt is a very strong one. Presumably, increases in government spending that lead to higher borrowing will cause an increase in real interest rates (except in the case of a small open economy with perfect capital mobility).

[14]Here the PVC is expressed as the same form as the transversality condition derived in Appendix 1. Asymptotically it is no different than the expression in (34). Because of the

Hamilton and Flavin (1984) consider the class of debt processes satisfying

$$E_t \lim_{N\to\infty} B_{t+N}(1+r)^{t+N} \quad = \quad A_0, \tag{36}$$

where A_0 can be any constant. For this class of debt processes, the government budget constraint in (32) can be rewritten as

$$B_{t-1} \quad = \quad A_0(1+r)^t \;+\; \sum_{j=0}^{\infty} (1+r)^{-(j+1)} SURP_{t+j} \;+\; n_t \qquad . \tag{37}$$

Among this class of debt processes, only those with $A_0 = 0$ satisfy the PVC. Testing the PVC amounts to testing whether A_0 is zero or not.[15] From (37), it can be seen that if $\sum_{j=0}^{\infty} (1+r)^{-(j+1)} SURP_{t+j}$ is stationary, A_0 is equal to zero if and only if B_{t-1} is also stationary. Therefore, if both $SURP_t$ and B_t are stationary time series processes, the PVC in (35) will necessarily be satisfied.

Employing augmented Dicky Fuller (ADF) tests for the presence of unit roots, Hamilton and Flavin (1984) reject the null hypotheses that $SURP_t$ and B_t are nonstationary. Therefore, they conclude that the PVC holds, implying that U.S. fiscal policy is sustainable. Kremers (1988), however, argues convincingly Hamilton and Flavin's ADF regressions were misspecified by not including sufficient lagged differences of the dependent variable to eliminate serially correlation in the residuals. He claims that the addition of a second lagged dependent variable produces a correctly specified regression. With this specification, the ADF test indicates that the debt series B_t is nonstationary due to the presence of a unit root.

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algebraic manipulation in (31), however, the value of B_t in the two expressions will differ somewhat at finite horizons.

[15]Note that these tests have the same form as those testing for the presence of (deterministic) speculative bubbles in financial asset pricing literature.

He therefore concludes that the PVC does not hold, overturning Hamilton and Flavin's conclusion that U.S. fiscal policy is sustainable.

The Hamilton and Flavin method is limited in a couple of respects. First, it assumes the debt process is in the class satisfying (36). Hence it is important to determine whether this assumption is too restrictive. Second, the Hamilton-Flavin test does not handle situations where $SURP_t$ is nonstationary, yet this is not necessary for fiscal sustainability. Finally, it is desirable to allow for stochastic expectations of the real rate of return in the sustainability tests.

Trehan and Walsh (1991) extend the Hamilton and Flavin method in these two respects. First, they prove that the Hamilton and Flavin method is valid as long as the debt series can be characterized as a general autoregressive moving average (ARIMA) process. It does not have to satisfy (36) with A_0 constant. Trehan and Walsh demonstrate that if $SURP_t$ is stationary, the simplified PVC (35) holds *if and only if* B_t is also stationary.

Trehan and Walsh also construct sustainability tests for situations where $SURP_t$ happens to be a nonstationary time series process. They do this by using the restriction that (35) imposes on the time series properties of B_{t-1} and $SURP_t$. The particular proposition derived in Trehan and Walsh (1991, proposition 1, p.209) is:

> If the evolution of B_t is given by (3) with $E(r_{t+i} \mid I_{t+i}) = r$ for all $i \geq 1$ [a constant expected interest rate] and $(1-\lambda L)SURP_t$ is a mean zero stationary with [i.e. for some λ in the range] $0 \leq \lambda < (1+r)$, then [the NPG condition] holds *if and only if* there exists a linear combination of $SURP_t$ and B_{t-1} that is stationary.

The procedure for testing for the presence of this linear combination is as follows. First, determine the value or range of values of λ that make $(1-\lambda L)SURP_t$ a mean-zero stationary series. If $SURP_t$ itself is stationary, then $(1-\lambda L)SURP_t$ must also be stationary for all λ in the range [0,1]. In this case, (35) holds if and only if B_{t-1} is also stationary. This is precisely the Hamilton-Flavin test. In the situation where $SURP_t$ is *difference* stationary (i.e., $\lambda = 1$), the PVC in (35) holds if and only if the conventional deficit, $rB_{t-1} - SURP_t$, is stationary. This implies that B_{t-1} and $SURP_t$ are cointegrated with cointegration vector (r, -1). Finally, for the case where the largest root in the $SURP_t$ process exceeds unity but is less than 1+r ($1< \lambda < (1+r)$), (35) holds if and only if $SURP_t$ and B_{t-1} are cointegrated. In this case, the cointegrating vector need not be (r,-1). The later is called the "Trehan-Walsh cointegration test" below.

Trehan and Walsh (1991) applied their cointegration test to U.S. debt and primary deficits over the period 1964-1984, the same period employed in Hamilton and Flavin (1984). Their statistical tests indicate that $SURP_t$ is stationary (i.e., $0 \leq \lambda < 1$), but B_t is not. Therefore they conclude that the government budget process is not sustainable. This result is consistent with the conclusion in Kremer's (1988) reconsideration of the Hamilton-Flavin analysis.

The government temporal budget constraint (3) depends only on SURP, not its decomposition into, say, non-interest government spending G and tax revenue T. By decomposing SURP in this way, however, it is possible to determine the restriction that the PVC imposes on the time series properties of government spending and revenue. Hakkio and Rush (1991) propose testing the PVC in (34) by checking whether the government expenditure *inclusive* of interest payment $G_t + r_tB_{t-1}$ is cointegrated with T_t and whether the cointegration vector is (1,-1). The validity of their test depends on several auxiliary assumptions: (i) r_t is stationary with unconditional mean r and (ii) both $G_t + (r_t\text{-}r)B_{t-1}$ and T_t follow unit root processes (i.e. the $\lambda = 1$ case above).

Hakkio and Rush apply their test to U.S. data for the period 1950:II to 1988:IV as well as the sub-samples: 1964:I-1988:IV and 1976:III-1988:IV. For the later two periods, they conclude there is no cointegration between T_t and $G_t + r_tB_{t-1}$. Like Trehan and Walsh and Kremers, therefore, they reach the conclusion that recent US fiscal policy is not sustainable once the evidence from the 1980s is included in the dataset. The similarity in the Trehan-Walsh and Hakkio-Rush findings is no coincidence. If the auxiliary assumptions two approaches are satisfied, the two tests are equivalent. That is, if $G_t + r_tB_{t-1}$ is cointegrated with T_t and the cointegration vector is (1,-1), then

$$+1(G_t + r_tB_{t-1}) - 1T_t = r_tB_{t-1} - SURP_t = (r_t - r)B_{t-1} + rB_{t-1} - SURP_t \qquad (38)$$

must be stationary. The cointegration of $G_t + r_tB_{t-1}$ and T_t with a cointegration vector (1,-1) means that the left-hand side of (38) is stationary. Under the assumption that r_t is stationary with unconditional mean r, $r_t - r$ and hence $(r_t - r)B_{t-1}$ are stationary. Therefore, $G_t + r_tB_{t-1} - T_t$ is stationary if and only if $rB_{t-1} - SURP_t$ is stationary.

What if the assumption of a constant expected real rates of return on government debt is not provide a good characterization of the data?[16] The Hakkio-Rush and Trehan-Walsh cointegration tests, as well as the Hamilton-Flavin test, are no longer valid.

Wilcox (1989) presents a stationarity test that does not require expected real interest rates to be constant. In the presence of stochastic expected real rates, one can use the *ex post* real rates to discount the government debt outstanding in period t to a fixed point, say, period 0. Then the time series characteristics of the discounted debt series $q_{t+i}B_{t+i}$ can be examined. If this series is stationary with mean zero, then the NPG condition (25) must hold. Applying

[16] Using data for eighteen OECD countries, Rose's (1988) econometric examination of nominal interest rates and inflation rates implies that *real* interest rate are, in fact, nonstationary. Even if the real rate was stationary, the assumption of a constant expected rate implies the absence of serial correlation in the series, which is clearly contradicts the facts for U.S. T-bill rates.

this test to US fiscal policy for the period 1960-1984, Wilcox finds that the discounted debt series is not (mean) stationary. We can not be sure from this evidence that recent US fiscal policy is not sustainable, however, since mean-zero stationary of $q_{t+i}B_{t+i}$ is only a *sufficient*, not a necessary, condition for the PVC to hold. Suppose, for example, that $q_{t+i}B_{t+i}$ is exponentially decaying toward zero, the PVC clearly holds even though the discounted debt series is nonstationary.

Trehan and Walsh (1991, proposition 2) also present a test that allows for time-varying expected real rates. They show that, if r_t is a stochastic process strictly bounded below by $\delta>0$, a sufficient condition for the present value constraint to hold is that $(1-L)B_t$ is stationary.

The reasoning behind this result is straightforward. If B_t is difference stationary, it can contain a time trend of order no greater than one. Thus, B_t grows at most *linearly* with the time. If real rates are strictly positive, the discount factor q_t will decay *exponentially*. Therefore the present value of B_t , $q_t B_t$, must go to zero as t goes to infinity.

Applying this test to US fiscal policy for the period 1960 to 1984, Trehan and Walsh found the first difference of debt to be stationary. As this is a *sufficient* condition[17] for the PVC to hold, they conclude that the recent US fiscal policy is sustainable. Note that this conclusion contradicts the results of their cointegration test discussed above. Trehan and Walsh "interpret this finding to imply that the deficit process is consistent with sustainability, but that the assumption of a constant expected real rate is a bad approximation to the data."

[17] Note that both the Wilcox test and the Trehan-Walsh stationarity test revolve around *sufficient* conditions for the debt process to satisfy the present value constraint. If these sufficient conditions are violated, therefore, the two tests are inconclusive regarding fiscal policy sustainability.

Testing the transversality condition in a stochastic economy

Ahmed and Rogers (1995) have recently derived the testable implications of the PVC or NPG conditions in a stochastic economy [(22) or (23) above]. They take the first-difference of the intertemporal budget constraint (21) and simplify the resulting expression using the following form of the government's temporal budget constraint in (3):

$$(1+r_t)B_{t-1} - (1+r_{t-1})B_{t-2} = r_t B_{t-1} - SURP_{t-1}$$

to get:

$$r_t B_{t-1} - SURP_{t-1} - \sum_{j=0}^{\infty} [\Delta E_t(s_{t+j} SURP_{t+j})] = \lim_{N\to\infty} \Delta E_t[s_{t+N}(1+r_{t+N})B_{t+N-1}] \quad . \quad (39)$$

The transversality condition (22) implies that the limit term on the right-hand side of (39) is zero. Provided that the first difference terms on the left-hand side are stationary, therefore, $r_t B_{t-1}$ - $SURP_t$ must also be stationary for the transversality condition (22) to hold. Decomposing $SURP_t$ into T_t-G_t yields the implication that a necessary condition for the transversality condition (22) to hold is the presence of cointegration among G_t, $r_{t-1}B_{t-1}$, and T_t with cointegrating vector (1,1,-1). Equivalently, $\Delta B_t = G_t + r_{t-1}B_{t-1} - T_t$ is stationary.

Ahmed and Rogers also show that this cointegrating relationship is also a *sufficient* condition for the transversality condition (22) to hold provided the debt process falls within the following class of time series processes:

$$B_t = \mu + B_{t-1} \lambda^t + \varepsilon_t \quad .$$

Ahmed and Rogers apply their cointegration test to U.S. (U.K.) fiscal policy over the very long sample period 1792 (1692) to 1992. They are unable to reject the hypothesis that G_t, $r_{t-1}B_{t-1}$, and T_t are cointegrated with vector (1,1,-1). They conclude, therefore, that the

PVC holds for the long period. Their tests allows for changes in the dynamic relations among variables during major wars, where *a priori* one might anticipate structural breaks to occur.

As discussed above, under certain assumptions the transversality condition (22) can be transformed into (25) where the discount factor becomes the risk-free interest rate. Following the same argument as in Wilcox (1989), stationarity of the discounted debt series (with mean zero) is sufficient for (25) to hold. Notice, however, the discount factor here should be risk-free rate instead of *ex post* real interest rate on government debt. In a deterministic economy this does not make any difference. In a stochastic economy, the risk-free rate can be very different than the real return on government debt. Therefore, for a stochastic economy, to test (22) or equivalently (25), one can test whether the discounted debt (using the risk-free rates) is stationary and, if so, whether its mean is zero.

"Structural" Breaks

Several papers in the sustainability literature note that the time series characteristics of fiscal variables may vary over time, exhibiting apparent "structural breaks" from time to time. Wilcox (1989), for example, finds that the series of discounted debt $q_{t+n}B_{t+N}$ for the United States was stationary prior to 1974 (1960-1974), but became non-stationary thereafter (1975-1984). Tanner and Liu (1994) re-do the Hakkio and Rush (1991) test, adding a level-shift dummy post 1982:I to the cointegration relationship involving tax revenue and government expenditure (inclusive of interest). Their objective is to "to capture a shift in the fiscal process in the first Reagan administration" (p.511). They find that the Hakkio-Rush conclusions regarding sustainability of U.S. fiscal policy are reversed once the break is taken into account. That is, the deficit appears to be stationary and so potentially sustainable.[18]

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[18]/ The unit root with break-point tests they employ allow for a one-time shift in the long-run (cointegration) relationship but not the short-run dynamics involving government revenues

Note that in the context of U.S. fiscal policy, the issue is whether fiscal policy has shifted from sustainable to unsustainable in recent years. In many LDCs, in contrast, we will be looking for instances where unsustainable polies were replaced with sustainable ones.

Do such breaks in the time series characteristics of these variables reflect fundamental changes in the government budget process? Was this change necessitated by the unsustainable nature of fiscal policy prior to the "data break?" Or is the change in the time series properties of the data merely reflecting changes in short-run dynamics rather than long-run relationships among fiscal variables? When there are data breaks, we should presumably focus on the more recent government budget process instead of the government budget process over the whole period if we want to assess the sustainability of current fiscal policy. The entire data sample would be used in assessing the sustainability of fiscal policy, past and present, allowing for shifts in policies over time (perhaps caused by the desire to insure sustainability).

Ahmed and Roger's (1995) analysis of sustainability of fiscal and current accounts in the United States and the United Kingdom includes annual data from 1792 and 1692, respectively. The U.S. sample, therefore, includes major wars such as the civil war, World War I and World War II. The U.K. sample includes the wars of Spanish Succession, Austrian Succession, the Seven Years' War, American Independence, wars with France, the Crimean and Boer wars, and World Wars I and II. The authors carry out a thorough analysis of the cointegration relationship implying sustainability taking into account the above-mentioned break points. While the U.K. results are somewhat inconclusive, they conclude that: "despite the recent U.S. twin deficits problem, the currently expected future course of fiscal policy

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and expenditures. This is fairly restrictive as a way of introducing shifts in fiscal policy.

might plausibly be regarded as sustainable. However, it is clear that to formally conclude this with any substantial degree of confidence--given the nature of the tests involved and the need for a long span of data that we have argued for--must await the availability of much more data" (p.18). In the LDC context, of course, the econometrician Oliver Twist would certainly be chastised for calling out "More, Sir" after being given three hundred years of data. Over much shorter time spans, structural breaks are often frequent. Thus, the sustainability tests proposed by Ahmed and Rogers are unlikely to produce definitive conclusions about the sustainability of fiscal or current account imbalances in developing countries.

7. The Application of the PVC Tests to Fiscal Policy in LDCs

Besides the disadvantage of demanding time-series data requirements, fiscal sustainability tests make several assumptions that make them less than ideal for application in developing countries. Seigniorage is often a significant source of financing for fiscal deficits. For poorer LDCs, grants and concessional lending are may be an important source of funding for both fiscal and net export imbalances. A large fraction of public debt may be denominated in foreign currency.[19]

Furthermore, the interesting empirical questions may be somewhat different. Rather than asking whether a country's current fiscal policy stance has become unsustainable (which has been the focus in the debate over U.S. fiscal deficits in the last decade or so), we may want to know whether changes in a country's fiscal regime have moved it from a path that

[19] An additional complexity is that debt prices are often much less than 100, because of default risk. In contrast, fluctuations in the market value of U.S. debt is primarily due to interest rate changes.

proved to be unsustainable to a sustainable fiscal stance. In principle, sustainability analysis be used to see whether sustainability has been restored by considering periods before and after supposed changes in fiscal regime. (See “Structural Breaks” above.)

Accounting for Seigniorage Revenue in Sustainability Analyses

The illustrative example of sustainable and nonsustainable fiscal policies in Section 5 ignored the possibility of financing the fiscal deficit at least in part via money creation. Hamilton and Flavin (1986, p.815), as well as many but not all of the subsequent studies, define SURP as the fiscal surplus plus any seignorage revenue collected by the consolidated public sector (i.e. government cum central bank):

$$SURP_t = T_t - G_t + \pi_t \frac{M_t}{P_t} \qquad (40)$$

where π_t is the inflation rate and M/P is the real monetary base. Once the possibility of money financing of deficits is introduced, a fiscal policy that implies primary fiscal deficits, rather than surpluses, may become sustainable for governments with outstanding debt.

The treatment of seigniorage in the literature on the accounting approach to policy consistency seems more thorough than that in the literature testing PVCs or NPG conditions. In particular, the accounting approach (e.g., see Anand and van Wijnbergen (1989) and van Wijnbergen (1990)) specifies inflation targets, and then uses estimates of the inflation semi-elasticity of money demand to calculate the resulting seigniorage revenues.

The Presence of Concessional Debt

Governments in many developing countries receive grants or subsidized loans from official (bilateral or multilateral) institutions. How can the availability of such financing be incorporated into sustainability analyses? An ambitious approach would be to attempt to model the decision making processes of these institutions as they attempt to allocated available resources among client (borrower) countries. A more empirical approach would be to assume that the time series characteristics of past concessional financing to an LDC government would prevail into the indefinite future. Alternatively, if one has specific conjectures about the likely availability of future concessional financing, those financing flows can be subtracted from the primary deficit, just as seigniorage revenue is, in order to come up with an adjusted primary SURP measure for use in PVC or NPG tests of the sort already discussed.

In the case of official lenders, any of the above approaches seem preferable to invoking an assumption of rationality of the lenders, and then attempting to test the implied transversality condition. Presumably, a no Ponzi game condition does not apply to the granting of aid flows or highly subsidized loan financing.

To apply tests on the present value constraint in the presence of concessional debt, we propose separating the concessional debt and grants (which are just concessional debt with an interest rate of -100%) and nonconcessional debt. Define concessional debt as B_t^c and, as before, let B_t be nonconcessional debt. Their respective rates of return as r_t and r_t^c. The government financing constraint in (3) now becomes:

$$B_t - (1+r_{t-1})B_{t-1} = - SURP_t - B_t^c + (1+r_{t-1}^c)B_{t-1}^c \qquad (41)$$

where the right-hand-side now equals the primary deficit inclusive of the *net* inflow of the concessional debt financing. Using (41) to describe the debt dynamics, sustainability tests using the PVC can now be applied to the public sector's nonconcessional borrowing.

Lending Foreign and Domestic Financing of Fiscal Deficits

Unlike the illustration in Section 5, the government financing constraint typically reflects more than a single source of financing. Suppose that the government follows the fiscal policy rule in Section 3 augmented by the following financing rule. Use the proportion x (0<x<1) of the primary *surplus* to pay down domestic debt and the remainder (1-x) to pay down foreign debt. Conversely, finance primary *deficits* using the same proportion: x (1-x) percent of any primary deficit should be financed by borrowing domestically (abroad). This fiscal policy stance, which describes the time path of primary surpluses, and the following debt accumulation equations for domestic (B) and foreign debt (B*), respectively, can be written as follows:

$$B_t = (1+r_t)B_{t-1} - x \; SURP_t \tag{42}$$

$$B_t^* = (1+r_t^*)B_{t-1}^* - (1-x) \; SURP_t \quad . \tag{43}$$

Substituting for SURP using (28) and substituting forward, it is straightforward to determine the amount of domestic and foreign debt that will be outstanding at any future date if the government adheres to its current fiscal policy and financing is forthcoming.[20] We then ask, do these time paths for debt satisfy the NPG conditions for the domestic and foreign lenders, respectively. The only tricky issue is what discount rate to use for each category of lender in order to reflect their respective marginal rates of substitution in consumption.

Regarding the appropriate transversality conditions, O'Connell and Zeldes (1988) emphasize that:

[20]The spreadsheet developed in section 5 above can presumably be extended to incorporate this distinction between the two types of debt.

> the feasibility of a rational Ponzi game depends on some key characteristics of the economy whose agents are going to hold the debt. For the case of external debt, this means that the characteristics of the borrower's economy are irrelevant to the feasibility of perpetual debt rollover. With regard to the Third World debt situation, it follows that the feasibility of perpetual rollover of debt depends on the performance of the economy of the lenders -- in particular, on the relationships between real interest rates, population growth rates, and growth in per capita income -- and not on that of the borrowing countries. (pp.431-2).

Reiterating:

> conditions in the borrower's economy are irrelevant to the feasibility of Ponzi game equilibria. As our analysis has shown, the feasibility of perpetual rollover of debt [their so-called "rational Ponzi games"] depends entirely on conditions in the *lender's* economy. To ensure the existence of an equilibrium in which U.S. lenders forever hold Argentinean debt that is growing at rate r, we need the growth rate of the U.S. (Not the Argentine) economy [or, rather, the population] to exceed r. (P.446)

They go on to ask:

> what happens if more than one individual or government tries to run a rational Ponzi game?...If the U.S. government can run a rational Ponzi game in the U.S., what prevents the government of Argentina or Nigeria from running its own Ponzi game in the U.S." (pp.446-7.)

Perhaps competition among those borrowers wishing to use Ponzi game financing will result in the bidding up of U.S. interest rates to the point where the Ponzi scheme financing no longer exists.

8. Conclusion

This paper considers two perspectives on the sustainability of fiscal policy: accounting approach and the present value constraint approach (PVC) approach. Parallel approaches have recently been developed in the literature for assessing current account sustainability (see, e.g. Sheffrin and Woo (1990), Trehan and Walsh (1991), Husted (1992), Wickens and Uctum (1993), and Ahmed and Rogers (1995)). The accounting approach focuses on steady-states and assumes that a fiscal deficit (or surplus) that leads to unchanging debt/GDP ratios over time is sustainable. The data requirements to apply this approach are relatively modest.

The PVC approach begins from the premise that the sustainability of fiscal policy ultimately depends on what level of deficit is financeable. This, in turn, depends on the behaviour of *lenders*. Recent empirical implementations of this approach concentrate on various methods for testing whether maintenance of the current fiscal policy stance, as captured by historical time series on government spending, revenue, and debt, violates the present value constrain or, equivalently, the no Ponzi game condition. In order to use the econometric methods used in this literature (e.g. tests for the presence of unit roots and cointegration), one needs long time series over a constant fiscal "regime." These data requirements may be demanding indeed in many LDC applications.

A possible compromise was outlined in Section 5. Rather than using time series techniques to describe constant fiscal regimes, one can specify fiscal rules into the foreseeable future based on country-specific information on fiscal targets (perhaps as stated in IMF stabilization programs). The implied time path for domestic and foreign debt, given current debt levels as initial conditions, can then be calculated. Using this hypothesized time path for debt, one can then ask whether it satisfies the no Ponzi game condition. If it does, fiscal policy is -- by this definition -- sustainable. If the NPG condition is violated, fiscal policy is unsustainable.

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Appendix 1:
Deriving the No Ponzi Game (NPG) Condition
from the Lender's Transversality Condition in a Deterministic Economy

For an economy can be represented by the following deterministic, aggregative, flexible price equilibrium model, McCallum (1984) has shown that, at the steady state, the NPG condition becomes the transversality condition for the household's optimization. A sketch of his model follows. The economy is composed of a large number of similar households, each of which seeks at period t to maximize the intertemporal utility function:

$$\sum_{t=1}^{\infty} \beta^{t-1} u(c_t, m_t),$$

where u is strictly increasing in both c and m (and u is bounded). Here c_t is household's consumption at period t, and $m_t = M_t/P_t$ is the real money balance held by the household at period t. β equals 1/(1+δ), where δ>0 is the time preference parameter.

Each household has access to a constant return to scale production function using two factors, labor and capital. Since labor is fixed, the production function can be written as $f(k_t)$, where k_t is the stock of capital held at the start of period t. The function f(k) is assumed to satisfy Inada conditions so that the economy will reach a steady state. There are three types of assets, namely, money, government bonds, and physical capital.

The household's budget constraint in real terms at period t can be written as:

$$f(k_t) - \tau_t = c_t + (1+\pi_t)m_{t+1} - m_t + b_t - (1+r_{t-1})b_{t-1} + k_{t+1} - k_t; \qquad (1a)$$

where τ_t is the lump-sum tax, π_t is the inflation rate, r_t is the *ex post* real interest rate on government debt b_t.

Given this setup, the Lagrangian for the household's optimization problem is:

$$L = \sum_{t=1}^{\infty} \beta^{t-1}(u(c_t,m_t)$$
$$+ \lambda_t[f(k_t) - \tau_t - c_t - (1+\pi_t)m_{t+1} + m_t - b_t + (1+r_{t-1})b_{t-1} - k_{t+1} + k_t]).$$

The first-order Euler conditions for the household's optimization are:

$$u_1(c_t,m_t) - \lambda_t = 0; \qquad (2a)$$

$$\beta u_2(c_{t+1},m_{t=1}) - \lambda_t(1+\pi_t) + \beta\lambda_{t+1} = 0; \qquad (3a)$$

$$b_t[-\lambda_t + \beta\lambda_{t+1}(1+r_t)] = 0; \tag{4a}$$

$$-\lambda_t + \beta\lambda_{t+1}(1+r_t) \leq 0; \tag{5a}$$

$$-\lambda_t + \beta\lambda_{t+1}[f^{/}(k_{t=1}) + 1] = 0; \tag{6a}$$

Corresponding to the three state variables m, b, and k, there are three transversality conditions:

$$\lim_{t\to\infty} m_{t+1}\beta^{t-1}\lambda_t(1+\pi_t) = 0; \tag{7a}$$

$$\lim_{t\to\infty} k_{t+1}\beta^{t-1}\lambda_t = 0; \tag{8a}$$

$$\lim_{t\to\infty} b_t\beta^{t-1}\lambda_t = 0. \tag{9a}$$

Conditions (1a)-(6a) are necessary for optimality while conditions (1a)-(9a) are jointly sufficient.

The government budget identity expressed in per capita real terms is:

$$(1+\pi_t)m_{t+1} - m_t + b_t - (1+r_{t-1})b_{t-1} = g_t - \tau_t; \tag{10a}$$

where g_t is the government expenditure. The right-hand side is the primary deficit, and the left-hand side shows that the primary deficit can be financed either by issuing bonds or printing money. The policy variables selected by the government are nominal money supply, M_t, government expenditure g_t, and lump-sum tax τ_t. Given these policy variables, the conditions (1a)-(6a), and (10a) describe the equilibrium paths for c_t, k_t, b_t, π_t, r_t, λ_t, and m_t.

It can be shown that at the steady state the lender's transversality condition (9a) is equivalent to the government's PVC in (11). To see this, the steady state of the economy needs to be derived first. At the steady state, inflation π_t is zero, k_t, c_t and m_t are constant. From condition (2a), this implies that λ_t is also constant and positive. Suppose that the government debt b_t is positive at the steady state, as this is the case where sustainability becomes relevant. Then (6a) implies that r_t is constant and equals to the time preference parameter δ. At the steady state (9a) can be written as:

$$\lim_{t\to\infty} b_t(1+r)^{-(t-1)} \quad = \quad 0;$$

which is the NPG condition used in the text.

Appendix 2

A Spreadsheet Implementation of Fiscal Sustainability Analysis

The foregoing analysis of the sustainability of fiscal policy uses very simple rules: spending and taxation are both assumed to be proportional to GDP. This Appendix shows how to implement fiscal sustainability analysis for arbitrary (i.e. nonconstant) time paths of government spending and taxation and a prespecified initial debt level using a standard spreadsheet program. Whether the specified fiscal policy satisfies the present value constraint (PVC) can be determined by examining NPVSURP-Debt (cell C23), which calculates the net present value (using the specified time path for real interest rates) of fiscal surpluses minus the initial stock of debt. If this value is nonnegative, fiscal policy is sustainable. Otherwise, it is nonsustainable. An equivalent test based on Wilcox (1989) is that the discounted value of future debt must be non-positive in the limit as time goes to infinity (here, the year 2240). See row 24, which shows the debt in each period discounted back to the initial year 1990.

Table A2.1 shows an situation where the economy has real GDP and public-sector debt both equal to 100 in 1990. The primary fiscal deficit is 8 percent of GDP, as the share of government (non-interest) spending is 30 of GDP, while revenues are 22 percent of GDP. Clearly, this situation is unsustainable: NPVSURP-Debt(1990) = -$427. The debt is growing faster than the (average) interest rate, implying that the discounted debt (cell IS24) is growing, not approaching zero. This suggests that Ponzi game financing would be needed to sustain this fisdcal regime.

Table A2.2 shows the impact of a complete debt write-off by setting Debt(1990)=0. The fiscal policy remains unsustainable, illustrating the point in the text that primary surpluses (at some point) are a precondition for a successful debt deruction program.

Returning to case where initial debt is 100, one can ask whether a specified fiscal adjustment program would restore sustainability of fiscal policy. Suppose the tax ratio to GDP is raised by 1 percentage point per year from 22 percent until it reaches 31 percent, nine years later. Table A2.3 shows that this large, albeit gradual, adjustment program is not sufficient to restore sustainability.

Table A2.4 considers the same fiscal adjustment program, but accompanies it with debt reduction. Interestingly, even a 100% debt write-off is insufficient to restore sustainability. This is because the fiscal adjustment program is so slow that debt increases dramatically during the early years of the program. By 2000 when the fiscal deficit has been eliminated, the debt has grown from zero to almost $53, more than half of its pre-write-off amount.

Table A2.5 shows the effect of rapid fiscal adjustment, with the primary deficit being reduced 2 percent of GDP per year until a surplus of 2 percent is achieved five years later. Still, fiscal is unsustainable, given the level of the initial debt (100). The NPVSURP-Debt value is still negative (-$46.6). This tell us that the debt would have to be written down from

100 to no more than 53.4 (100-46.6), in conjunction with the fiscal adjustment, to achieve sustainability. Then the program is *just* sustainable, leaving no leeway for adverse shocks in the future.