IS MONEY NEUTRAL?
SOME EVIDENCE FOR ITALY

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Summary

The aim of this paper is to verify the hypothesis of money neutrality in the Italian experience. After a critical overview of the traditional techniques employed to verify this hypothesis, cointegration technique is used to verify: long-run neutrality, weak evidence of long-run superneutrality but absence of hyperneutrality. The absence of hyperneutrality implies that an acceleration of the growth rate of money affects real output.

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INTRODUCTION

The idea in what is generally known as "New Classical Economics" theory (henceforth NCE), is that anticipated growth in money does not influence real output. Phelps(1970) developed this idea in the context of decentralized markets where agents have limited information about other markets. In these kind of models, agents can misinterpret price signals and respond to shocks, which they would otherwise ignore. This idea had been developed in an important paper by Lucas(1973). In section one, we introduce this model explaining its implications together with its theoretical problems. Then, we review the proposed tests to verify real effects of anticipated and unanticipated money. We also consider a potentially serious identification problem of observational equivalence.

In section two, we propose an alternative approach to the topic using the univariate properties of the series and cointegration analysis.

SECTION 1

1.1 THE LUCAS MODEL

Lucas(1973) reports the results of an empirical study of real output-inflation trade-offs for eighteen countries over the years 1951-1967.

As he says: "These data are examined from the point of view of the hypothesis that average real output levels are invariant under changes in the time pattern of the rate of inflation, or that there exists a "natural rate" of real output".

In this model, Lucas makes three assumptions: (i) nominal output is demand determined; (ii) "rigidities", which dominate the short-run period, result from supplier's lack of information; and (iii) expectations are made rationally (or optimally), considering the stochastic character of the economy[1].

Using this model, we express output as a function of unanticipated changes in nominal money:

[1.1.1] $y_t=[b(1-\lambda)/(1+b(1-\lambda))][m_t - E(m_t \mid \Omega_{t-1})]$.

[1] Cf. Blanchard-Fischer(1989), pp.356-360.

Where b is the real output elasticity to unexpected change in prices, λ is the percentage of expected change in the general level of prices due to a unit change in the sector specific price, and Ω_{t-1} is the information set available at time t-1.Output responds to unanticipated changes in nominal money with an elasticity of $[b(1-\lambda)/(1+b(1-\lambda))]$. The Lucas model is subject to two serious criticisms: (i) the supply equation [1.1.1] is postulated rather than derived from a maximization problem, and (ii) the model has no dynamics (no rigidities are supposed to operate in the economy). Considering the Italian experience, there is an interesting paper of Guiso-Sestito(1987). In this paper the authors test the Lucas formulation for the supply function. Using OLS estimates for the sample period 1958-1985, they verify the hypothesis of positive correlation between the deviation of output from its trend and unanticipated inflation even though, in the short-run, they found that anticipated inflation influences output.

In this section, we review some procedures used to test the invariance proposition underlying [1.1.1].

1.2 TESTING THE INVARIANCE PROPOSITION

The monetary policy invariance proposition can be stated in the following statistical form:

[1.2.1] $E(y_t \mid I_t^m) = E(y_t \mid I_t)$.

Equation [1.2.1] states that the expectation of output, conditional on the information set containing the expected quantity of money (I_t^m), is equal to the expectation of output conditional on the same information set without the expected money (I_t). The next step is to specify an equation for the quantity of money. Let us suppose that the quantity of money is a linear function of its past values and a set of predetermined variables[2] plus a Gaussian white noise shock:

[1.2.2] $m_t = \mu(L)m_{t-1} + \rho(L)K_{t-1} + \varepsilon_{mt}$.

From [1.2.2], considering the rational expectations hypothesis, we can decompose money into two components: its expected values $m_t^e = E_{\mu,\rho}(m_t \mid I_t) = \mu(L)m_{t-1} + \rho(L)K_{t-1}$ and the "surprise" (ε_{mt}). Using [1.2.2], we can evaluate these two components using OLS:

$\hat{m}_t^e = E_{\hat{\mu},\hat{\rho}}(m_t \mid I_t^m) = \hat{\mu}(L)m_{t-1} + \hat{\rho}(L)K_{t-1}$ and $\hat{\varepsilon}_{mt} = m_t - E_{\hat{\mu},\hat{\rho}}(m_t \mid I_t^m)$[3].

Considering output and money simultaneously, we can test the invariance proposition as follows[4]:

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[2] In this context our variables are thought as deviations from their natural rate of growth.

[3] From now on we will consider $E_{\mu,\rho}(m_t \mid I^m_t) = m^e_t$.

[1.2.3] $y_t = \alpha(L)y_{t-1} + \beta(L)K_{t-1} + \pi_1(L)\,[m_t - m^e_t] + \pi_2(L)m^e_t + \varepsilon_{yt}$,

[1.2.4] $m_t = m^e_t + \varepsilon_{mt}$.

The FIML counterpart of system [1.2.3]-[1.2.4] is:

[1.2.3bis] $y_t = \tilde{\alpha}(L)y_{t-1} + \tilde{\beta}(L)K_{t-1} + \tilde{\pi}_1(L)\,[\,m_t - \tilde{\mu}(L)m_{t-1} - \tilde{\rho}(L)K_{t-1}] +$

$+ \tilde{\pi}_2(L)\,[\,\tilde{\mu}(L)m_{t-1} + \tilde{\rho}(L)K_{t-1}\,] + \tilde{\varepsilon}_{yt}$,

[1.2.4bis] $m_t = \tilde{\mu}(L)m_{t-1} + \tilde{\rho}(L)K_{t-1} + \tilde{\varepsilon}_{mt}$.

With $E(\varepsilon_t\varepsilon_t') = \Sigma$, where $\varepsilon_t = [\varepsilon_{yt}, \varepsilon_{mt}]'$. If we want to test the invariance proposition, we have to verify the hypothesis that $\tilde{\pi}_2(L)=0$. In order to test this hypothesis, we can use a likelihood ratio (LR) test:

[1.2.5] $-2[\log|\Sigma_r| - \log|\Sigma$ **Error! Bookmark not defined.** $_{ur}|\,]$ ~**Error! Bookmark not defined.** χ **Error! Bookmark not defined.** $^2_{(q)}$,

where Σ **Error! Bookmark not defined.** $_r$ and Σ **Error! Bookmark not defined.** $_{ur}$ are respectively the constrained and the unconstrained estimates of Σ **Error! Bookmark not defined.** and q is the number of restrictions.

If π **Error! Bookmark not defined.** $_1(L) = \pi$ **Error! Bookmark not defined.** $_2(L)$, [1.2.3] collapses to:

[1.2.6] $y_t = \alpha$ **Error! Bookmark not defined.** $(L)y_{t-1} + \beta$ **Error! Bookmark not defined.** $(L)K_{t-1} + \pi$ **Error! Bookmark not defined.** $(L)m_t + \varepsilon$ **Error! Bookmark not defined.** $_{yt}$,

equation [1.2.6] implies that unanticipated and anticipated money have identical effects on output. Further, to test the persistence of anticipated and unanticipated money a LR test is again appropriate. To see this, rewrite [1.2.3] as:

[1.2.7] $y_t = \beta$ **Error! Bookmark not defined.** $^*(L)K_{t-1} + \pi$ **Error! Bookmark not defined.** $_1^*(L)\,[m_t - m^e_t] + \pi$ **Error! Bookmark not defined.** $_2^*(L)m^e_t + \varepsilon$ **Error! Bookmark not defined.** $_{yt}^*$,

where: β **Error! Bookmark not defined.** $^*(L) = [1-\alpha$ **Error! Bookmark not defined.** $(L)L]^{-1}$ **Error! Bookmark not defined.** $\beta(L)$, π **Error! Bookmark not defined.** $_1^*(L) = [1-\alpha$ **Error! Bookmark not defined.** $(L)L]^{-1}\pi_1(L)$, π **Error! Bookmark not defined.** $_2^*(L) = [1-\alpha$ **Error!**

[4] This is the procedure proposed by Mishkin(1982).

Bookmark not defined.$(L)L]^{-1}\pi_2(L)$ and, ε**Error! Bookmark not defined.**$_{yt}{}^{*}=[1-\alpha$**Error! Bookmark not defined.**$(L)L]^{-1}\varepsilon_{yt}$[5].

The hypothesis that unanticipated money growth affects output in the short-run, can be performed by testing the null hypothesis π**Error! Bookmark not defined.**$_1{}^{*}(1)=0$[6]. π**Error! Bookmark not defined.**$_1{}^{*}(1)$ is the total multiplier of the unanticipated quantity of money on output. This is substantially the test procedure proposed by Mishkin[7]. The previous test procedure has been developed to overcome the inconsistent estimation (see Appendix 1) of the Sargent(1976a) procedure; this procedure consists of running a Granger causality test on the following model:

[1.2.8] $y_t = k_{11}(L)y_{t-1} + k_{12}(L)m_{t-1} + u_t,$

[1.2.9] $m_t=k_{21}(L)y_{t-1} + k_{22}(L)m_{t-1} + w_t$.

If $k_{12}(L)=0$, monetary policy has no real effects. In appendix A.1 we show that $k_{12}(L)=0$ is generally equivalent to both the ineffectiveness of monetary policy and that output is not affected by past monetary shocks (unless a special case occurs), while if $k_{12}(L)\neq$**Error! Bookmark not defined.**0 we cannot conclude anything about the effectiveness of the monetary policy. Even in the case of upper triangularity of the system, we cannot exclude that the causality is determined either by central banks pursuing accommodating policies (endogeneity of money)[8] or because most of the money stock is inside money, whose real volume adjusts to the level of economic activity[9]. As pointed out by Toda-Phillips(1993), the real problem of using a Wald test to verify this hypothesis is the presence of stochastic trends and cointegration in the system. They "......conclude [that] the empirical use of Granger causality tests in levels VAR's is not to be encouraged in general when there are stochastic trends and the possibility of cointegration" (p.1387).

1.3 THE OBSERVATIONAL EQUIVALENCE PROBLEM[10]

In this subsection we consider identification of the regime underlying the system. In equations [1.2.3] and [1.2.4], suppose π**Error! Bookmark not defined.**$_1(L)$ and π**Error! Bookmark not defined.**$_2(L)$ are different from zero, then

[5] Obviously, we need $[1-\alpha$**Error! Bookmark not defined.**$(L)L]$ to be invertible or, which is the same, that all the characteristic roots of $[1-\alpha$**Error! Bookmark not defined.**$(L)L]$ lie outside the unit circle of the complex plane.

[6] The same applies for m^e_t, (i.e. if π**Error! Bookmark not defined.**$_2{}^{*}(1)=0$).

[7] See also Barro(1978,1979) and Buiter(1983).

[8] Cf. Tobin(1970).

[9] Cf. King-Plosser(1984).

[10] A good reading on this topic is McCallum(1979).

model [1.2.3]-[1.2.4] has both unanticipated and anticipated money effects on output. If only unanticipated money influences output:

[1.3.1] $y_t=\alpha$**Error! Bookmark not defined.**$(L)y_{t-1}+\beta$**Error! Bookmark not defined.**$(L)K_{t-1}+$ π**Error! Bookmark not defined.**$_1(L)[m_t-m^e_t]+\varepsilon$**Error! Bookmark not defined.**$^+{}_{y,t}$,

[1.3.2] $m_t= \mu$**Error! Bookmark not defined.**$(L)\varepsilon$**Error! Bookmark not defined.**$^+{}_{m,t}$,

where ε**Error! Bookmark not defined.**$^+{}_{y,t}$ and ε**Error! Bookmark not defined.**$^+{}_{m,t}$ are independent white noise disturbance processes.

Although [1.3.1]-[1.3.2] appears to be Classical, Sargent(1976b) shows that it could be expressed in a form that makes it appear Keynesian. Expressing [1.3.2] in the following AR representation:

[1.3.2bis] μ**Error! Bookmark not defined.**$^{-1}(L)m_t=\varepsilon$**Error! Bookmark not defined.**$^+{}_{m,t}$,

and substituting it into [1.3.1] we obtain:

[1.3.1bis] $y_t=\alpha$**Error! Bookmark not defined.**$(L)y_{t-1}+\beta$**Error! Bookmark not defined.**$(L)K_{t-1}+\gamma$**Error! Bookmark not defined.**$(L)m_t+\varepsilon$**Error! Bookmark not defined.**$^+{}_{y,t}$, where: γ**Error! Bookmark not defined.**$(L)=$**Error! Bookmark not defined.**$\pi_1(L)\mu$**Error! Bookmark not defined.**$^{-1}(L)$.

As is well known, underidentification implies that different structural models can have the same reduced-form representation. Using a bivariate Granger causality test, if the polynomial matrix K(L) is a full matrix (see appendix A.1) an identification problem exists because we cannot distinguish if this is due to anticipated, unanticipated or both effects.

Accepting the Granger's "philosophy" of causality, we indirectly remove the observational equivalence problem. Suppose we tested the lower triangularity of K(L) but we do not entirely "believe" in Granger causality: again we would have a problem of identification (or observational equivalence); let us consider the bivariate VAR model for y_t and m_t[11]:

[1.3.3] $K(L)x_t=\varepsilon$**Error! Bookmark not defined.**$_t$, where $E(\varepsilon_t\varepsilon$**Error! Bookmark not defined.**$_t$**Error! Bookmark not defined.**$')=\Sigma$**Error! Bookmark not defined.**, $x_t=[y_t,m_t]'$ and ε **Error! Bookmark not defined.**$_t=[\varepsilon$**Error! Bookmark not defined.**$_{y,t}$, ε**Error! Bookmark not defined.**$_{m,t}]'$.

In this model the contemporaneity between variables is contained in the variance-covariance matrix Σ**Error! Bookmark not defined.**. We can consider model [1.3.3] as a reduced form of the "true" structural model:

[11] Where y_t and x_t are considered, for the moment, stationary.

[1.3.4] $Ax_t=K^*(L)x_{t-1} + Be_t$, where: diag(A)=I, $E(e_te_t')=I$ [12].
Pre-multiplying both sides of [1.3.4] by A^{-1} we obtain [1.3.3].

To achieve identification, we need to restrict coefficients on A and B and this fact implies that our results depend substantially on our view of the economy.

SECTION 2

2.1 A PRELIMINARY DISCUSSION ON COINTEGRATION THEORY

In this section we employ a different strategy to see whether monetary policy affects output. Instead of a structural model, we use an ARIMA model to describe the monetary rule; using this kind of model we do not have the problem of working with several time series and we have an unbiased estimation of m^e_t (for further details see appendix A.2).

Since we are working with non-stationary processes, we cannot use standard limiting distribution theory. Furthermore, we need to investigate the possibility of having cointegration in our model. Let us first introduce the concept of cointegration[13]. Consider the following VAR model:

[2.1.1] $x_t= \Pi$**Error! Bookmark not defined.**$(L)\ x_{t-1} + \varepsilon$**Error! Bookmark not defined.**$_t$,

where x_t is a n-dimensional vector; Π**Error! Bookmark not defined.**$(L) = \Pi$**Error! Bookmark not defined.**$_1+\Pi$**Error! Bookmark not defined.**$_2L+....+\Pi$**Error! Bookmark not defined.**$_kL^{k-1}$; x_{-k+1}, ... , x_0 are fixed and ε**Error! Bookmark not defined.**$_t$ ~**Error! Bookmark not defined.** i.i.d. $N(0,\Sigma$**Error! Bookmark not defined.**$)$. We can rewrite model [2.1.1] in the following form:

[2.1.2] Δ**Error! Bookmark not defined.**$x_t = A(L)\Delta$**Error! Bookmark not defined.**$x_{t-1} - Ax_{t-1} + \varepsilon$**Error! Bookmark not defined.**$_t$,

where: $A_i= -\sum_{h=i+1}^{k}\Pi_h$ and $A=I - \Pi$**Error! Bookmark not defined.**$_1$ - ....- Π**Error! Bookmark not defined.**$_k$. Cointegration analysis concerns the rank of the matrix A:
(a) ρ**Error! Bookmark not defined.**$(A) = n \Rightarrow$**Error! Bookmark not defined.** x_t ~**Error! Bookmark not defined.** $I(0)$;

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[12] e_t is the vector of structural disturbances (cf. Bernanke(1986)).
[13] Cf. Johansen(1988).

(b) ρ**Error! Bookmark not defined.**(A) = 0 $\Rightarrow$**Error! Bookmark not defined.** A = 0 and model [2.1.2] corresponds to a traditional VAR in which variables are expressed in their first differences;

(c) 0<ρ**Error! Bookmark not defined.**(A)=r<n $\Rightarrow$**Error! Bookmark not defined.** A=$\alpha\beta$**Error! Bookmark not defined.**', where α**Error! Bookmark not defined.** and β**Error! Bookmark not defined.** are matrices, of full column rank, of order n x r and β**Error! Bookmark not defined.**'x_t ~**Error! Bookmark not defined.** I(0). In this last case, model [2.1.2] can be considered as a multivariate error-correction model[14]. Without going deeply into cointegration theory, recall that α**Error! Bookmark not defined.** represents the loading matrix, or the velocity of adjustment toward the long-run equilibrium, while β**Error! Bookmark not defined.** contains, by column, the cointegration vectors representing the long-run equilibrium relations such that β**Error! Bookmark not defined.**'x_t ~**Error! Bookmark not defined.** I(0).

The first step in our analysis consists of investigating the order of integration of our set of variables. Instead of studying a bivariate model money-income, we prefer to consider a four-variate model, as in Friedman-Kuttner(1992), which includes: real income (y_t), nominal money (m_t), the level of prices (p_t), and an interest rate (r_t). While there is no debate about the order of integration of income and money expressed in real terms[15], a debate exists about the order of integration of the variables expressed in nominal terms; King et al.(1991) find evidence of I(2) series for US nominal money and nominal output data, implying p_t ~**Error! Bookmark not defined.** I(2)[16].

We intend to argue that the order of integration of prices must be the same as those of nominal money and income or that m_t and p_t are I(2) whereas y_t and (m_t-p_t) are I(1). Another issue is the order of integration of the interest rate; in some cases it has been verified to be (trend) stationary, while in some other cases, as in Italy, it appears stationary after differencing the series once.

With this introduction, we turn trying to verify the invariance proposition using cointegration analysis. Recall that the money-income correlation can arise in three different ways: (a) money is not neutral (H_{nn}); (b) the money-income correlation is due to price misperception (H_{pm}); (c) the money-income correlation is a case of reverse causality (H_{rc}). We will investigate H_0: $H_{nn} \mid \overline{H}_{rc}$, that is: money is not neutral conditional to the fact that the money-income correlation is not due to a case of reverse causality. Bergman-Warne(1993) describe the relation between Granger-causality and long-run neutrality: while Granger non-causality implies long-run neutrality, the reverse is not true. So as we are interested in testing the monetary policy neutrality, we prefer to investigate this hypothesis rather than Granger non-causality of money. In fact, the long-run non-neutrality of money implies that

[14] Engle-Granger(1987).

[15] They generally appear to be I(1).

[16] In fact, if two stochastic processes have different order of integration, then the order of integration of any linear combination among them (with coefficient different from zero) will be the highest of the single ones.

the level of money influences output[17]. Furthermore, if money appears to be neutral (in the long-run, i.e. no cointegration between money and income) and we consider the following bivariate model:

$$\begin{bmatrix}\Delta y_t \\ \Delta m_t\end{bmatrix}=\begin{bmatrix}a_{11}(L) & a_{12}(L) \\ a_{21}(L) & a_{22}(L)\end{bmatrix}\begin{bmatrix}\Delta y_{t-1} \\ \Delta m_{t-1}\end{bmatrix}+\begin{bmatrix}\varepsilon_{y,t} \\ \varepsilon_{m,t}\end{bmatrix};$$

money is superneutral[18] if $a_{12}(L)=0$ or long-run superneutral if $a_{12}(1)=0$.

Several authors prefer to consider a four-variate model[19] because, as pointed out in Friedman-Kuttner(1992): ".....unless the demand for money is interest insensitive or interest rates are [trend] stationary, neither of which assumption has received much support from the relevant empirical literature, money and income should not be expected to exhibit cointegration unless the cointegration equation also includes an interest rate" (p.487).

If there were cointegration, no conclusion about neutrality can be drawn because of the previously described identification problem. As an example, consider the following: A=αβ**Error! Bookmark not defined.**'; if we take any invertible matrix P of order r, it is easily verified that α**Error! Bookmark not defined.*** β**Error! Bookmark not defined.***', with α**Error! Bookmark not defined.*** = α**Error! Bookmark not defined.** Q^{-1} and β**Error! Bookmark not defined.*** = β**Error! Bookmark not defined.** Q', generate an "observationally equivalent" situation, that is: α**Error! Bookmark not defined.*** β*' =α**Error! Bookmark not defined.** β**Error! Bookmark not defined.**' .

2.2 TESTING NEUTRALITY IN INTEGRATED SYSTEMS

Consider the following representation of the integrated system Δ**Error! Bookmark not defined.**x_t=[Δ**Error! Bookmark not defined.**y_t, Δ**Error! Bookmark not defined.**m_t, Δ**Error! Bookmark not defined.**p_t, Δ**Error! Bookmark not defined.**r_t]'[20]:

[2.2.1] Δ**Error! Bookmark not defined.**x_t = A(L) Δ**Error! Bookmark not defined.**x_{t-1} -Ax_{t-1} + ε**Error! Bookmark not defined.**$_t$ [21],

where ε**Error! Bookmark not defined.**$_t$ ~**Error! Bookmark not defined.** i.i.d. N(0,Σ**Error! Bookmark not defined.**). Define the following matrices:

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[17] i.e. there exists a long-run equilibrium relation linking money and output.

[18] Superneutrality implies that the rate of change of money does note influence output.

[19] y_t, m_t, p_t, r_t.

[20] All the variables are assumed to be I(1).

[21] To simplify the problem we marginalize the analysis with respect to the possibility of having drift, trends and seasonal dummies.

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}; A(L) = \begin{bmatrix} a_{11}(L) & a_{12}(L) & a_{13}(L) & a_{14}(L) \\ a_{21}(L) & a_{22}(L) & a_{23}(L) & a_{24}(L) \\ a_{31}(L) & a_{32}(L) & a_{33}(L) & a_{34}(L) \\ a_{41}(L) & a_{42}(L) & a_{43}(L) & a_{44}(L) \end{bmatrix}.$$

Remembering how we defined Δ**Error! Bookmark not defined.**x_t, money is[22]:

(a) directly neutral (in the long-run) if $a_{12}=0$;
(b) indirectly neutral if $a_{32}=a_{42}=0$;
(c) totally neutral if (a) ∪**Error! Bookmark not defined.** (b);
(d) directly superneutral (in the long-run) if $a_{12}(1)=0$;
(e) indirectly superneutral if $a_{32}(1)=a_{42}(1)=0$;
(f) totally superneutral if (d) ∪**Error! Bookmark not defined.** (e);
furthermore, considering what we said about the relation linking the Granger-causality and neutrality, money does not help in predicting output:
(g) directly, if $a_{12}(L)=0$[23] and $a_{12}=0$;
(h) indirectly, if $a_{32}(L)=a_{42}(L)=0$ and $a_{31}=a_{42}=0$;
(i) totally, if (g) ∪**Error! Bookmark not defined.** (h).

Focusing on neutrality, we know that:

$$A = \alpha\beta\textbf{Error! Bookmark not defined.}' = \begin{bmatrix} \alpha_{1.} \\ . \\ . \\ \alpha_{r.} \end{bmatrix} . \begin{bmatrix} \beta_{.1} & . & . & \beta_{.r} \end{bmatrix}' .$$

For simplicity we consider two cases: r=1 and r=2.

<u>CASE 1</u>: r=1.

$$A = \alpha\beta\textbf{Error! Bookmark not defined.}' = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \end{bmatrix} \begin{bmatrix} \beta_1 & \beta_2 & \beta_3 & \beta_4 \end{bmatrix};$$

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[22] Given that all the coefficients not considered in the following are different from zero.
[23] In this case money is superneutral even in the short-run.

the hypothesis of neutrality is satisfied when (i) α**Error! Bookmark not defined.**$_1$=0 or (ii) β**Error! Bookmark not defined.**$_2$=0. The sub-case (ii) is the simplest one because it says that money does not enter the long-run relationship. When we will discuss necessary and sufficient conditions for identifying the cointegration space, it will be clear that this hypothesis can be tested since the unrestricted model is just identified, apart from a factor scale. The sub-case (i) implies that, even though a long-run relationship exists, it does not influence the output equation.

CASE 2: r=2.

$$A=\begin{bmatrix}\alpha_{11} & \alpha_{12}\\ \alpha_{21} & \alpha_{22}\\ \alpha_{31} & \alpha_{32}\\ \alpha_{41} & \alpha_{42}\end{bmatrix}\begin{bmatrix}\beta_{11} & \beta_{21} & \beta_{31} & \beta_{41}\\ \beta_{12} & \beta_{22} & \beta_{32} & \beta_{42}\end{bmatrix};$$

in this case the hypothesis of neutrality is satisfied when:

(i)α**Error! Bookmark not defined.**$_{11}\beta_{21}$+α**Error! Bookmark not defined.**$_{12}\beta_{22}$=0[24], (direct neutrality);
(ii)α**Error! Bookmark not defined.**$_{31}\beta_{21}$+α**Error! Bookmark not defined.**$_{32}\beta_{22}$=0, (indirect neutral, via prices);
(iii)α**Error! Bookmark not defined.**$_{41}\beta_{21}$+α**Error! Bookmark not defined.**$_{42}\beta_{22}$=0, (indirect neutral, via the interest rate).

Focusing our attention on direct neutrality (i) implies that the weighted effect of money on the output equation, via the two long-run relationships, is equal to zero. This can happen, for example, when the first, the third and the fourth rows of α**Error! Bookmark not defined.** are equal to zero or β**Error! Bookmark not defined.**$_{21}$=β**Error! Bookmark not defined.**$_{22}$=0. Obviously, the problem we deal with consists of identifying the cointegration space.

Johansen(1992) provides statistical tools that permit us to check identification. We can express our set of restrictions implicitly using $R_i\beta_i$=0[25], or explicitly β**Error! Bookmark not defined.**$_i$=$S_i\gamma_i$ **Error! Bookmark not defined.**[26], where R_iS_i=0. A necessary condition for identification is that the number of restrictions in each cointegrating vector is greater than or equal to the rank of cointegration minus one. Johansen(1992) reports necessary and sufficient conditions in the following theorem: "A necessary and sufficient condition that

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[24] This is a necessary but not sufficient condition.

[25] R_i is a matrix of order r_i x n with ρ**Error! Bookmark not defined.**(R_i)=r_i (r_i is the number of restrictions on the i-th equation).

[26] S_i and γ**Error! Bookmark not defined.**$_i$ are respectively of order n x (n-r_i) and (n-r_i) x 1, with ρ **Error! Bookmark not defined.**(S_i)=n-r_i.

a parameter value (β**Error! Bookmark not defined.**, Ω**Error! Bookmark not defined.**) is identified is that ρ**Error! Bookmark not defined.**$(R_i\beta)$=r-1, i=1,...,r" (p.2).

Equivalently, a necessary and sufficient condition for identification is that ρ**Error! Bookmark not defined.**$[R_i(S_{i_1}|....|S_{i_k})]\geq$**Error! Bookmark not defined.**k, for k=1,...,r-1 and 1$\leq$**Error! Bookmark not defined.**$i_1\leq...\leq$**Error! Bookmark not defined.**$i_k\leq r$. The i-th equation is overidentified if the strict inequality holds. If the overidentification case occurs, Johansen proved that we can test our set of restrictions using a χ**Error! Bookmark not defined.**^{2} statistics with $\sum_{i=1}^{r}\rho(R_i)$- [r (r-1)] degrees of freedom.

Now we have all we need to achieve identification in our model and to start from it to test restrictions in either the short-term and long-term structure (β**Error! Bookmark not defined.**). We can do more then this: instead of using the reduced form, we can consider the structural form and, as shown in Johansen-Juselius(1992), verify that the long-term structure is the same in both representation. Again, consider model [2.2.1]; after imposing a reduced rank (r) to A:

[2.2.1'] Δ**Error! Bookmark not defined.**x_t = A(L) Δ**Error! Bookmark not defined.**x_{t-1} -$\alpha\beta$ **Error! Bookmark not defined.**' x_{t-1} + ε**Error! Bookmark not defined.**$_t$;

the short-term structure is represented by the parameters A(L), α**Error! Bookmark not defined.** and Σ**Error! Bookmark not defined.**, while the long-term structure is contained in β**Error! Bookmark not defined.**. Since the instantaneous relationships are contained in the covariance matrix Σ**Error! Bookmark not defined.**, we have:

[2.2.2] Kε**Error! Bookmark not defined.**$_t$=Se_t,

where e_t ~**Error! Bookmark not defined.** i.i.d. N(0,I) are the structural disturbances and K and S are full rank square matrices. From [2.2.2] we obtain that $K^{-1}SS'(K^{-1})'=\Sigma$**Error! Bookmark not defined.**. If we let $S^{-1}K=B$ and pre-multiply [2.2.1'] by B, we obtain:

[2.2.3] BΔ**Error! Bookmark not defined.**x_t = BA(L)Δ**Error! Bookmark not defined.**x_{t-1} -B $\alpha\beta$**Error! Bookmark not defined.**'x_{t-1} +e_t;

defining $A^*(L)$=BA(L) and α**Error! Bookmark not defined.***=Bα**Error! Bookmark not defined.** and using a compact notation for [2.2.3], we obtain:

[2.2.3'] Q∇**Error! Bookmark not defined.**x =-$\alpha\beta$**Error! Bookmark not defined.**'x_{t-1} +e_t;

where Q=[B|-BA(L)] and ∇**Error! Bookmark not defined.**x=[Δ**Error! Bookmark not defined.**x_t|Δ**Error! Bookmark not defined.**x_{t-1}]'. The analysis of the long-term structure can be performed using either the structural or the reduced form; in fact while the structural short-run parameters now are contained in Q and α **Error! Bookmark not defined.***, the structural long-run parameters are again contained in β**Error! Bookmark not defined.**. Considering the short-term structure we can formulate identifying restrictions on $Q^{*}=[B|A^{*}(L)|\alpha$**Error! Bookmark not defined.**$^{*}]$. This is due to the fact that Σ contains the instantaneous relationships linking the x's which influence only the short-run coefficients.

For several reasons, it is not an easy task to test neutrality using cointegration analysis. First, we have to solve again a problem of identification, that is, we need to impose identifying restrictions on our parameter space. Second, the invariance proposition contains long-term as well as short-term meanings; furthermore, the problem becomes more and more complicated as the rank of cointegration increases and, last but not least, in our examples we considered I(1) processes even though, as we shall see, Italian data suggest that p_t ~**Error! Bookmark not defined.** I(2). In this last case, and if both real output and real money are I(1) processes, an important conclusion can be drawn: money is long-run neutral. In fact, being Δ**Error! Bookmark not defined.**m_t ~**Error! Bookmark not defined.** I(1), there could exist a long-run equilibrium relationship only between Δ**Error! Bookmark not defined.**m_t and real output; if it does not exist, money is superneutral in the long-run.

2.3 TESTING MONEY NEUTRALITY USING ITALIAN DATA

In this subsection, we test the hypothesis of money neutrality using Italian data. While, we do not claim that the monetary policy cannot affect real output, we find no evidence in favour of a long-run equilibrium relationship linking nominal money and real output.

Our variables are: real output (y), nominal money (m), the price level (p), and a nominal interest rate (i)[27]. As pointed out in the previous paragraph, we suspect nominal variables, excluding the interest rate, to be I(2) but we start our analysis assuming that y, m, p, and i are I(1) processes and then we evaluate the consistency of this hypothesis.

From the cointegration analysis[28] (cf. table 1) we obtain a strange result: choosing a significance level of 5%, we have a cointegration rank equal to 4 which means that our variables are stationary in levels. This is due to the problem of having some variables that are I(2), in fact the Johansen's test procedure is for a model in which the variables are I(1). As a further proof of working with I(2) processes, in figure 1 we plot the "most stationary" component of this model, which is evidently non stationary.

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[27] Sample period 1970:1-1988:4 (quarterly data); sources: the GDP deflator (p) is a Bank of Italy elaboration on ISTAT data; nominal money (m2), Bank of Italy; real GDP (y), ISTAT; the interest rate (i) is a three-months bond rate, Bank of Italy. All variables, apart the interest rate, are expressed in logarithms.

[28] We performed cointegration analysis using a RATS procedure (MALCOLM) of Rocco Mosconi.

We think that the problem of having I(2) series is due to prices. This series seems to be I(2)[29]: in fact, we tested the hypothesis of prices being I(2) against the alternative of I(1), using Dickey-Pantula(1987). We did not reject the null hypothesis. From this, we know that also nominal money and nominal output are I(2)[30], and we need to investigate the order of integration of real output which could be I(1); we test this last hypothesis considering an integrated model, where nominal variables and prices are expressed in their first difference, and testing the existence of a long-run equilibrium relation, with coefficients [1, -1],

FIGURE 1

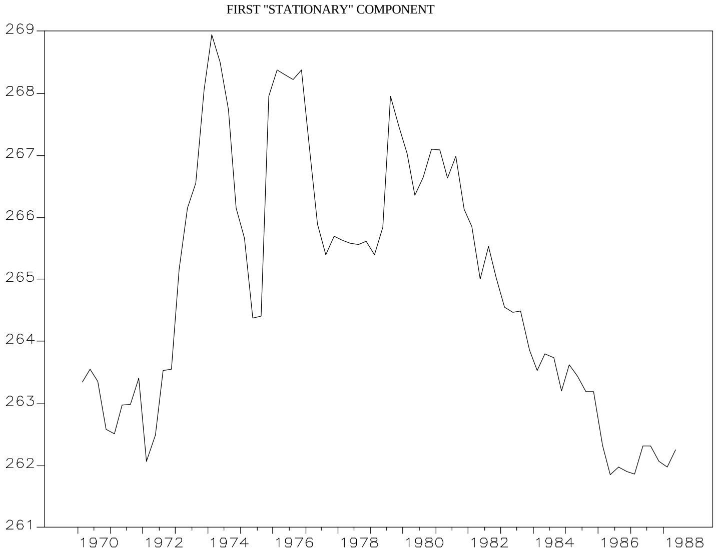

TABLE 1

SAMPLE PERIOD: 1970:1 1988:4
VARIABLES : y, m, p, i
MAXIMUM LAG : 2
MODEL : I(1)

TESTS for r

LAMBDA MAX TESTS:

r	Statistic	50%	80%	90%	95%	97.5%	99%
0	56.88	17.66	21.98	24.73	27.07	28.98	32.24
1	30.46	12.34	16.20	18.60	20.97	23.09	25.52
2	14.31	6.85	10.04	12.07	14.07	16.05	18.63
3	5.76	0.44	1.66	2.69	3.76	4.95	6.65

[29] Thinking in continuous time, this means that the velocity of prices (i.e. the quarterly inflation rate) is I(1) and the acceleration of prices is I(0).
[30] Unless real money and real output are I(2) processes (each) cointegrated CI(2,1) with prices with coefficients [1, 1] ($\Rightarrow y_t^r - m_t^r \sim I(1)$).

TRACE TESTS:

r	Statistic	50%	80%	90%	95%	97.5%	99%
0	107.40	33.60	40.15	43.95	47.21	50.35	54.46
1	50.52	18.70	23.64	26.79	29.68	32.56	35.65
2	20.06	7.55	11.07	13.33	15.41	17.52	20.04
3	5.76	0.44	1.66	2.69	3.76	4.95	6.65

between nominal output growth (Δ**Error! Bookmark not defined.**y_t^n) and the inflation rate[31]. We estimated the model[32] and we found two cointegrating relationships and we tested the hypothesis that one of them is Δ**Error! Bookmark not defined.**y_t^n-Δ**Error! Bookmark not defined.**p_t without rejecting the null hypothesis with a marginal significance level of 40%.

The fact that real output is I(1) while money is I(2), given that real money is I(1), implies that no equilibrium relationship exists between them and this is evidence of long-run money neutrality[33].

This result allows us to consider the following I(1) model:

[2.3.1] Δ**Error! Bookmark not defined.**$x_t=\mu$**Error! Bookmark not defined.**$_0+A(L)\Delta$**Error! Bookmark not defined.**x_{t-1} $-Ax_{t-1}$ $+\varepsilon$**Error! Bookmark not defined.**$_t$, where: $x_t=[y_t$, Δ**Error! Bookmark not defined.**m_t, Δ**Error! Bookmark not defined.**p_t, $i_t]'$;

using an LR test we choose a maximum lag of 5 for model [2.3.1].

At this point, we performed the Johansen's cointegration tests finding a rank of cointegration equal to one (cf. table 2) and the following estimates for α and β:

α'=[0.35225 -1.08009 0.48002 -5.17272];

β'=[0.04561 1.0000 -0.61142 0.000086].

TABLE 2

SAMPLE PERIOD: 1970:2 1988:4
VARIABLES : y, Δm, Δp, i
MAXIMUM LAG : 5
MODEL : I(1)

---

[31] When we are dealing with mixed processes, I(1) and I(2), it could be more interesting to use a model that takes this into account; in fact, using an I(1) model we cannot verify if an equilibrium relationship that links real output with real money. Cf. Johansen(1993).

[32] With no restrictions on the mean and no trend in the stationary part and a linear trend on the I(1) part.

[33] Cf. Fisher-Seater(1993), p.405.

TESTS FOR r

LAMBDA MAX TESTS:

r	Statistic	50%	80%	90%	95%	97.5%	99%
0	38.03	17.66	21.98	24.73	27.07	28.98	32.24
1	12.35	12.34	16.20	18.60	20.97	23.09	25.52
2	8.91	6.85	10.04	12.07	14.07	16.05	18.63
3	1.21	0.44	1.66	2.69	3.76	4.95	6.65

TRACE TESTS:

r	Statistic	50%	80%	90%	95%	97.5%	99%
0	60.51	33.60	40.15	43.95	47.21	50.35	54.46
1	22.47	18.70	23.64	26.79	29.68	32.56	35.65
2	10.13	7.55	11.07	13.33	15.41	17.52	20.04
3	1.21	0.44	1.66	2.69	3.76	4.95	6.65

The first problem we wanted to solve was the stability of our estimates, expecially if we consider that during this period the so-called "divorce" between the Government (Treasury) and the Central Bank occurs. Unfortunately, as far as we know, econometric literature does not consider structural breaks in cointegrated systems and the only thing we do is to try to have an idea of the stability of the cointegration rank, estimating the system recursively from 1981:1 until 1988:4. From this analysis, it is clear that the cointegration rank remains stable to one.

Now, we want to see whether β_2 or $\alpha_1 = 0$. We tested first the hypothesis of $\beta_2 = 0$ (cf. table 3) rejecting the null and then we tested $\alpha_1=0$ (cf. table 3) without rejecting the null. In this last case, our estimates of α and β are the following:

α'=[0 -1.20279 0.33149 -40.14549];

β'=[0.04616 1.0000 -0.63885 .000217043].

From these results we conclude that money results super-neutral (in the long-run), but we have to be careful because, as we previously said, things can change if we work with a structural model. In fact, in a structural model like [2.2.3], if the first row of the B matrix is not orthogonal to α we cannot claim long-run money neutrality[34].

[34] As we said, only the long-run relationships remain unchanged passing from a reduced form to a structural one.

TABLE 3

Testing $\beta_2 = 0$

LOG-LIKELIHOOD UNDER H0	:	1198.06855
LOG-LIKELIHOOD UNDER HA	:	1203.74821
NUMBER OF DEGREES OF FREEDOM	:	1
CHI SQUARE TEST	:	11.35933
SIGNIFICANCE LEVEL	:	7.50700e-004

Testing $\alpha_1 = 0$

LOG-LIKELIHOOD UNDER H0	:	1021.72331
LOG-LIKELIHOOD UNDER HA	:	1023.07295
NUMBER OF DEGREES OF FREEDOM	:	1
CHI SQUARE TEST	:	2.69929
SIGNIFICANCE LEVEL	:	0.10039

TABLE 4

Granger causality test: money vs real output

LOG-L	TEST (χ^2)	DGF	SIGNIF
1006.1	33.969	13	0.0012

The last step, is to test what we called the hyperneutrality hypothesis. This test can be easily performed using a Granger causality test in a ECM framework as in Mosconi-Giannini(1992). This test consists in testing restrictions on both short-run and long-run coefficients[35], and it provides evidence against this hypothesis (cf. table 4).

CONCLUSIONS

This paper, after reviewing some of the literature on the so called invariance proposition, studied money neutrality using Italian data. A number of difficulties were encountered, one of which was dealing with processes with different order of integration. In fact, prices and nominal money seem to be I(2) processes while real output and the nominal interest rate seem I(1). Due to this disparity, we attempted to overcome the problem using an I(1) model in which we have variables in their first and second differences.

[35] Again, Toda-Phillips(1993) suggest caution in using a Granger causality test is this kind of models. Anyway, their conclusion is in favour of Granger causality test in ECM' models rather then in level VAR' model expecially when the system is small (3-4 variables) and with at least 100 observations (cf. Toda-Phillips(1991)).

Due to the different order of integration of nominal money (I(2)) and real output (I(1)), given that real money is I(1), it follows that no equilibrium relationship exists between them and this is strong evidence of long-run nominal money neutrality. Following this finding, we went to investigate the hypothesis of superneutrality and hyperneutrality finding weak evidence for the first and no evidence at all for the second. Considering the fact that we found evidence of superneutrality via the loading matrix, a structural investigation is needed.

In conclusion, we state again that money neutrality is not evidence of monetary policy neutrality; furthermore, even in the case in which one finds strong evidence of this hypothesis, the case of reverse causality may occur.

TECHNICAL APPENDICES

APPENDIX A.1

In this section we will explain the partial usefulness of the Granger causality procedure in proving the invariance proposition. Before doing this we need to state the Granger causality concept; for this purpose we use the definition contained in Sims(1972): "...the time-series Y is said to "cause" X relative to the universe U (U is the vector time-series including X and Y as components) if, and only if, predictions of X(t) based on U(s) for all s<t are better than predictions based on all components of U(s) except Y(s) for s<t". Obviously the main problem of this definition of causality is that, even if Y does not Granger-cause X, it may be that it leads that variable or it determines X contemporaneously. We have seen that there are several theoretical explanation of the money versus income Granger causality even if output determines money.

As we said, we can use the Granger causality direction using the following VARMA representation[36]:

[A.1.1] $$\begin{bmatrix} a^{+}_{11}(L) & a^{+}_{12}(L) \\ a^{+}_{21}(L) & a^{+}_{22}(L) \end{bmatrix}\begin{bmatrix} y_t \\ m_t \end{bmatrix}=\begin{bmatrix} b_{11}(L) & b_{12}(L) \\ b_{21}(L) & b_{22}(L) \end{bmatrix}\begin{bmatrix} u_{yt} \\ u_{mt} \end{bmatrix};$$

[36] We consider y_t and m_t (respectively the natural logarithms of real output and money) to be a jointly covariance-stationary pair of stochastic processes; if their levels are I(1) processes, we can think y_t and m_t to be the rates of growth of the variables. In this last case, we exclude the presence of cointegration.

where: $u_t=[u_{yt}\ u_{mt}]'$ is NID[0,Σ**Error! Bookmark not defined.**][37]. Using the Granger concept of causality, m_t cause y_t if and only if: $a^+_{12}(L)$ and/or $b_{12}(L)\neq$**Error! Bookmark not defined.**0 and, $a^+_{21}(L)=b_{21}(L)=0$[38]. We can rewrite system [A.1.1] in the following way:

[A.1.1.b] $A^+(L)x_t=B(L)u_t$,

where $x_t=[y_t,m_t]'$ and, $u_t=[u_{yt},u_{mt}]'$. The [A.1.1.b] form is equivalent to:

[A.1.1.c] $x_t=A(L)x_t+B(L)u_t$, where $A(L)=I-A^+(L)$.

Sargent(1976) proposed to test the invariance proposition using the Granger causality framework using a simple VAR system of the following kind:

[A.1.2] $x_t=Q(L)x_{t-1}+u_t$.

The invariance proposition consists in testing the lower triangularity of the Q(L) matrix. As we previously said, this test procedure has been criticised for its supposed inconsistency in a situation in which passed monetary shocks affect output. In what follows, we will discuss this point.

Considering $A^+(L)$ and B(L) invertible polynomial matrices, we can transform the VARMA system [1] to a VAR system:

[A.1.3] $K(L)x_t=u_t$, where $K(L)=B(L)^{-1}A^+(L)$;

or to a VMA system:

[A.1.4] $x_t=F(L)u_t$, where $F(L)=A^+(L)^{-1}B(L)$.

Consider the first system [A.1.4]; using the Cramer's rule to invert $A^+(L)$ and considering [A.1.1] we can express [A.1.4] in its extended form:

[A.1.4bis] $x_t= d_a\begin{bmatrix}(a^+_{22}b_{11}-a^+_{12}b_{21}) & (a^+_{22}b_{12}-a^+_{12}b_{22})\\(a^+_{11}b_{21}-a^+_{21}b_{11}) & (a^+_{11}b_{22}-a^+_{21}b_{12})\end{bmatrix}\begin{bmatrix}u_{yt}\\u_{mt}\end{bmatrix}$;

where $d_a= 1/\det[A^+(L)]$ and $\xi_{ij}=\xi_{ij}(L)$, with $\xi=a^+,b$ and i,j=1,2.

Estimating a VMA system, its lower triangularity implies not only the invariance proposition [$a^+_{12}(L)=0$] but also that surprises do not affect output ($b_{12}(L)=0$, super neutrality of money). When $a^+_{22}b_{12}=a^+_{12}b_{22}$ the system can be lower triangular even if $a^+_{12}(L)$ and $b_{12}(L)$ are not equal to zero. In this particular case, it is interesting to note that the long run response of output to a

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[37] u_t is the non-structuralized vector of innovation. A way of testing the absence of contemporaneity consist of testing the diagonality of Σ**Error! Bookmark not defined.**.

[38] $a^+_{ii}(L)=1-a^+_{ii,1}L - a^+_{ii,2}L^2+......$; $b_{ii}(L)=1 + b_{ii,1}L + b_{ii,2}L^2+......$; $a^+_{ij}(L)$ and $b_{ij}(L)$, for i≠**Error! Bookmark not defined.**j, are polynomials in L not containing 1.

unitary monetary shock is ρ**Error! Bookmark not defined.** $\left[\frac{\det A^{+}(1)}{\det B(1)}\right]\frac{a^{+}_{22}(1)}{b_{22}(1)}$ where ρ**Error! Bookmark not defined.** = $E(u_{mt}u_{yt})/E(u_{mt})^2$. Therefore, if the shocks, u_{mt} and u_{yt}, are mutually uncorrelated, we have again long-run neutrality. Other cases of lower triangularity are ruled out when we imposed the invertibility of the $A^{+}(L)$ matrix and the stationarity of our processes. If our F(L) matrix is upper triangular, we cannot exclude the invariance proposition because we cannot exclude the case $a^{+}_{12}(L)=0$. Another possibility is that the F(L) matrix is full; in this case we cannot establish any direction of causality.

Consider now the VAR representation:

[A.1.3bis] $d_b\begin{bmatrix}(a^{+}_{11}b_{22}-a^{+}_{21}b_{12}) & (a^{+}_{12}b_{22}-a^{+}_{22}b_{12})\\(a^{+}_{21}b_{11}-a^{+}_{11}b_{21}) & (a^{+}_{22}b_{11}-a^{+}_{12}b_{21})\end{bmatrix}x_t=\begin{bmatrix}u_{yt}\\u_{mt}\end{bmatrix}$;

where $d_b=1/\det[B(L)]$. Again the lower triangularity of the model implies the invariance proposition. The same observation made for the VMA representation also applies for the VAR representation.

APPENDIX A.2

Consider the following ARX model as the "true" model of monetary policy[39]:

[A.2.1] $m_t=a(L)m_{t-1}+b(L)K_t+\varepsilon$**Error! Bookmark not defined.**$_{m,t}$, ε**Error! Bookmark not defined.**$_{m,t}$ ~ i.i.d. N(0,σ**Error! Bookmark not defined.**$^2_{\varepsilon}$),

where K_t is a vector of exogenous and/or pre-determined variables. This must be thought of as the model used to make expectations (rational), furthermore we are assuming m_t ~**Error! Bookmark not defined.** I(0). In [A.2.1]:

$m^e_t=a(L)m_{t-1}+b(L)E[K_t| I_t]$ ⇒**Error! Bookmark not defined.** $m_t-m^e_t=\varepsilon$**Error! Bookmark not defined.**$_{m,t}$[40]. m^e_t is the expected value of m_t conditioned on the information set I_t.

Instead of [A.2.1], consider a simple AR model of the following kind:

[A.2.2] $m_t=s(L)m_{t-1}+e_{m,t}$, $e_{m,t}$ ~**Error! Bookmark not defined.** i.i.d. N(0, σ**Error! Bookmark not defined.**^2_e);

[39] This must be thought of as the model used to make expectations (rational), furthermore we are assuming m_t ~**Error! Bookmark not defined.** I(0).
[40] The condition $E[K_t| I_t]=K_t$ is verified if K_t contains deterministic components and/or lagged variables.

in this case:

$m^e_t=s(L)m_{t-1} \Rightarrow s(L)m_{t-1}=a(L)m_{t-1}+b(L)E[K_t| I_t]$.

If $E[K_t| I_t]=K_t$ [41] we have that ε**Error! Bookmark not defined.**$_{m,t}=e_{m,t}$ and model [A.2.1] is equivalent to model [A.2.2] otherwise $Var(e_{m,t})>Var(\varepsilon$**Error! Bookmark not defined.**$_{m,t})$. In fact, from model [A.2.1]:

[A.2.3] $m_t-m^e_t=\varepsilon$**Error! Bookmark not defined.**$_{m,t}+b(L)(K_t-E[K_t| I_t])$.

Supposing K_t stationary and uncorrelated with ε**Error! Bookmark not defined.**$_{m,t}$:

$E[(m_t-m^e_t)^2]=Var(m_t)=Var(e_{m,t})=\sigma$**Error! Bookmark not defined.**^2_e and, using [A.2.3] :

[A.2.4]$Var(m_t)=Var(\varepsilon$**Error! Bookmark not defined.**$_{m,t})+b^2(1)Var(K_t)=\sigma$**Error! Bookmark not defined.**$^2_\varepsilon+b^2(1)\sigma$**Error! Bookmark not defined.**^2_k[42] **Error! Bookmark not defined.**$\Rightarrow \sigma^2_e=\sigma$**Error! Bookmark not defined.**$^2_\varepsilon+b^2(1)\sigma$**Error! Bookmark not defined.**^2_k .

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[41] For instance when we have K_{t-1} and so $E[K_{t-1}| I_t]=K_{t-1}$.

[42] Here $b^2(1)=b^2_1+b^2_2+...+b^2_p$, where p is the order of b(L).

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