Are Real Exchange Rates Nonlinear or Nonstationary? Evidence From A New Threshold Unit Root Test

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Summary

We analyze the post-float real exchange rates for a group of OECD countries using the newly developed threshold test and tests for unit roots by Caner and Hansen (2001). These tools help us disentangle the nonlinearity from the nonstationarity rigorously for the first time in the literature. We find evidence for non-linearity of exchange rates. Specifically real exchange rates behave like a unit root in a band and when the depreciation or appreciation of the currency against $US exceeds the boundary of the band, the real exchange rates are mean-reverting. The threshold value is treated as unknown and estimated in the model.

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1 Introduction

Recently, a lot of attention has been paid to whether real exchange rates are nonlinear or not . It has been argued in a seminal paper by Taylor (2001) that fixed and variable trading costs can result in a "band of inaction" where there is no arbitrage. The mean reversion begins when the prices move a lot. This idea also dates back to Heckscher (1916). It has been argued that there are two regimes. One shows slow or nonconvergence when the price differences in two countries are small and the other one shows rapid convergence when the price difference exceeds transaction costs (Obstfeld and Taylor, 1997). In another paper it has been pinpointed that for nontradables sector we expect unit root behavior in their prices and for the tradables mean reversion is expected (Engel, 2000). Other theoretical explanations in this respect generating non-linear exchange rates are also given.(Dumas, 1992, Michael *et al.,* 1997) . They analyze the general equilibrium model with separated countries and differing transportation costs and these models generate a nonlinear real exchange rate. Other studies use smooth threshold models (Taylor and Peel, 2000, Kilian and Taylor, 2001).

Even in those studies which use nonlinear exchange rates , there is no clear way of differentiating nonlinearity from the unit root behavior by the existing unit root tests. The existing unit root tests are linear in such a way that the null of a linear unit root is tested against a linear stationary model. One major problem with the existing unit root tests is their low power against nonlinear but stationary alternatives (Caner and Hansen, 2001, Pippenger and Goering, 1993). Even though unit root tests have low power against linear alternatives the power is lower when the alternative is nonlinear. This has important implications since if the real exchange rates are nonlinear the existing unit root tests might not reject the false unit root null. The former literature do not find strong evidence for nonlinearity of exchange rates (Taylor and Peel, 2000). The econometric techniques developed in this literature generally assumes normality of errors, stationarity of real exchange rates , also the limit for unit root tests against a stationary threshold autoregressive alternative does not exist.

These are related to two important problems encountered in econometrics as well as PPP literature. These are the existence of unidentified threshold parameter under the null hypothesis and nonstationarity of the data. These problems have not been simultaneously tackled before. Caner and Hansen (2001) provides the set of tools to solve this problem rigorously. They derive the limit of threshold tests (nonlinearity tests) in a nonstationary model and also develop the limit for unit root tests when the alternative is a two-regime threshold autoregression. Two bootstrap methods are used to get better finite sample results. It has been shown that these bootstrap procedures have excellent size and good power (Caner and Hansen, 2001). In this literature, this helps us to differentiate between nonlinearity and nonstationarity of real exchange rates.

Specifically , we examine a two-regime threshold autoregressive model with an autoregressive unit root. Within this model, we study Wald tests for a threshold effect (for nonlinearity), and Wald/t tests for unit roots (for nonstationarity). We allow for general autoregressive orders, and do not artificially restrict the coefficients across regimes.

The contribution of this article is twofold. First we are able to test whether the real exchange rates are non-linear or not . Then we conduct two unit root tests such that null of unit root (linear or non-linear) is tested against a non-linear stationary or a partial unit root process which displays random walk behavior in one regime and mean reverting one in the other. We evaluate this two-regime symmetric threshold model . We do not impose the threshold value , this is unknown and estimated in the model.

Our approach of allowing threshold to be on currency appreciation (or depreciation) is also novel. Usual practice in the nonlinear exchange rate modeling allows the level of real exchange rate to be the threshold. The typical justification for this approach (Dumas, 1992, Obstfeld and Taylor, 1997), which is based on international trade theory, is the presence of transaction costs that keep arbitrage on hold until prices in two countries depart sufficiently above a threshold of inaction. Nevertheless, the price indices used for real exchange rate calculations incorporate prices of non-traded goods as well as traded goods. This argument has also been made by Engel (2000) in order to draw attention to the theoretical possibility of a non-stationary real exchange rate when the international relative prices of non-traded goods are non-stationary.

Models of real exchange rate that are based on international finance theory (e.g. Dornbush, 1976, Obstfeld and Rogoff, 1995, Lane and Milessi-Ferretti, 2000) predict a connection between the net asset position of a country and the real exchange rate. This connection works through the relative prices of non-traded goods. Indebted countries tend to have a persistently depreciated real exchange rate to be able to generate a sustained trade surplus that is needed to cover interest payments. The absence of any major change in the net asset position of countries, imply persistence (i.e. a unit root) in the real exchange rate. A major appreciation/depreciation, in contrast, indicates a major extraordinary capital inflow/outflow and hence may trigger a mean-reversion in the real exchange rate process through changes in long term targeted net asset positions. This corrective switch may be a result of central bank interventions as well as private capital flow reversals triggered by the large movement in the exchange rate. Our approach is novel in allowing for such real exchange rate dynamics within a univariate model.

The approach here is univariate. The econometric technique only allows for the univariate analysis. However, the reason for panel studies stem mainly from the low power of standard linear unit root tests. The threshold test and the threshold unit root tests have good power against various linear and non-linear alternatives by Caner and Hansen (2001). The threshold variable is stationary unlike the literature and this is mainly due to the restrictions imposed by the econometric technique. Also we think that having a nonstationary threshold variable results in a time-varying band and may result in low power for the unit root tests.

By using the Wald test in Caner and Hansen (2001) we find strong evidence for nonlinearity of real exchange rates. 14 out of 17 OECD countries have non-linear exchange rates. Then we analyze whether the real exchange rates have unit root or not. Our exercise is more informative compared to the previous literature since our alternative is a symmetric two-regime threshold autoregressive model. After our empirical analysis, we find real exchange rates display unit root behavior when recent historical depreciation or appreciation of a currency is within a band. When there is a large depreciation or appreciation then real exchange rates display mean reverting behavior. We also provide evidence for this model in an out-of sample forecasting exercise.

There are related studies to this paper in the literature. First one is by Bec, Ben-Salem and Carrasco (2003). They provide a unit root test in threshold models with a nonstationary threshold model. This model has economic justification and important to analyze from the real exchange rate literature. The test tests a linear unit root against a discrete threshold model, but unlike our study the threshold can be the level of real exchange rate. The delay parameter is set to be one. They analyze real exchange rates for Canada, UK, Germany, France and Italy against US dollar for monthly data covering 1973.9-2000.9. They find that the data follow the null of nonstationarity linear system. They also analyze European currencies against Deutsche Mark and find evidence for a three regime model where the middle regime is random walk and the outer regimes display mean-reverting behavior. They also use a test for threshold due to Hansen (1996). But that test works only under stationary random variables, whereas the bootstrap modeling of the threshold test developed in Caner and Hansen (2001) is robust to stationary/nonstationary behavior of the real exchange rates.

In our paper, we find evidence for non-linear but stationary behavior for the behavior of European currencies against US dollar, this is unlike the Bec, Ben-Salem and Carrasco (2003) study and it is more in line with Taylor's (2001) explanation of the real exchange rate behavior. The other study that is done in nonlinear-nonstationary framework is by

Kapetanios and Shin (2002). They analyze a constrained threshold model, whereas ours is not constrained. The data determines the type of the model in our case. There is also a study done by Pesaran and Potter (1997) analyzing the US GNP. They use a threshold model and discover asymmetries in US GNP behavior.

Section 2 introduces our econometric model. Section 3 analyzes the data. Section 4 concludes. The programs used to generate the results can be obtained from Mehmet Caner. "$\Longrightarrow$" represents weak convergence and $\Longrightarrow_p$ represents weak convergence in probability for bootstrap distributions (as defined in Gine and Zinn, 1990).

2 The Model

We use the following model for our study:

$$\Delta q_t = \theta_1' x_{t-1} + e_t \quad if\, |q_{t-1} - q_{t-m-1}| < \lambda \tag{1}$$

$$\Delta q_t = \theta_2' x_{t-1} + e_t \quad if\, |q_{t-1} - q_{t-m-1}| \geq \lambda \tag{2}$$

where q_t is the natural logarithm of the real exchange rate, $x_{t-1} = (q_{t-1}, 1, \Delta q_{t-1}, \cdots, \Delta q_{t-k})$ for $t = 1, 2, \cdots, T$. e_t is iid error term. m represents the delay parameter and $1 \leq m \leq k$. As can be seen from equations (1)-(2) the threshold variable is the absolute value of $q_{t-1} - q_{t-m-1}$. Threshold value λ is unknown and takes the value in the compact interval $\Lambda = [\lambda_1, \lambda_2]$ where these values are picked according to $P(|q_{t-1} - q_{t-m-1}| \leq \lambda_1) = .15, P(|q_{t-1} - q_{t-m-1}| \leq \lambda_2) = .85$. Note that the threshold variable satisfies the stationarity and ergodicity described in section 2 of Caner and Hansen (2001). Threshold variable is defined in changes rather than levels since the econometric theory developed does not allow levels. Also , the theory crucially depends on the discrete nature of the jump in the model. A better model will involve a smooth transition, however in our case the econometric properties of the tests developed are not known in the smooth case and possibly very difficult to handle. Next , decompose the coefficients

$$\theta_1 = \begin{pmatrix} \rho_1 \\ \beta_1 \\ \alpha_1 \end{pmatrix} \quad \theta_2 = \begin{pmatrix} \rho_2 \\ \beta_2 \\ \alpha_2 \end{pmatrix}. \tag{3}$$

where $\rho_1, \rho_2, \beta_1, \beta_2$ are scalar and α_1, α_2 are $k \times 1$ vectors. ρ's represents the slopes on q_{t-1}, βs are the intercept coefficients and α's are the slopes on dynamic regressors. Equations

(1)-(2) can be written in a single equation format as follows:

$$\Delta q_t = \theta_1' x_{t-1} 1_{\{|q_{t-1} - q_{t-m-1}| < \lambda\}} + \theta_2' x_{t-1} 1_{\{|q_{t-1} - q_{t-m-1}| \geq \lambda\}} + e_t \tag{4}$$

where $1_{\{.\}}$ is the indicator function. We impose the following assumption on the model.

Assumption 1. *e_t is an iid mean zero sequence with a bounded density function and $E|e_t|^{2\gamma} < \infty$, for some $\gamma > 2$. The following parameter restrictions apply $\rho_1 = \rho_2 = 0$, and $|\alpha_1' \iota| < 1$ and $|\alpha_2' \iota| < 1$ where ι is a k vector of ones.*

2.1 Estimation

We estimate (4) by concentrated Least Squares (LS). For each $\lambda \in \Lambda$ (4) is estimated by OLS so

$$\Delta q_t = \hat{\theta}_1' x_{t-1} 1_{\{|q_{t-1} - q_{t-m-1}| < \lambda\}} + \hat{\theta}_2' x_{t-1} 1_{\{|q_{t-1} - q_{t-m-1}| \geq \lambda\}} + \hat{e}_t(\lambda) \tag{5}$$

where $\hat{e}_t(\lambda)$ represents the residual from the OLS given λ.

Then

$$\hat{\sigma}^2(\lambda) = T^{-1} \sum_{t=1}^{T} \hat{e}_t(\lambda)^2$$

be the OLS estimate of σ^2 (variance of the errors) for fixed λ . The LS estimate of the threshold parameter is

$$\hat{\lambda} = argmin_{\lambda \in \Lambda} \hat{\sigma}^2(\lambda).$$

In order to find estimates of slope parameters we plug in $\hat{\lambda}$ to have the slope estimates : $\hat{\theta}_1 = \hat{\theta}_1(\hat{\lambda}), \hat{\theta}_2 = \hat{\theta}_2(\hat{\lambda}),$. Next the estimated model is

$$\Delta q_t = \hat{\theta}_1' x_{t-1} 1_{\{|q_{t-1} - q_{t-m-1}| < \hat{\lambda}\}} + \hat{\theta}_2' x_{t-1} 1_{\{|q_{t-1} - q_{t-m-1}| \geq \hat{\lambda}\}} + \hat{e}_t(\hat{\lambda}) \tag{6}$$

Set $\hat{\sigma}^2 = T^{-1} \sum \hat{e}_t^2$ to be the residual variance from the LS estimation. The estimates in (4) are used for testing.

2.2 Test for a Threshold

In the model (4) we want to see whether there is nonlinearity in the exchange rates due to the threshold effect. We set up the null as :

$$H_0 : \theta_1 = \theta_2$$

So, under the null hypotheses there is no threshold effect and the model is linear. The following sup Wald test is used for testing the null against the threshold model

$$\sup_{\lambda \in \Lambda} W_T(\lambda) = \sup_{\lambda \in \Lambda} T(\frac{\hat{\sigma}_0^2}{\hat{\sigma}^2(\hat{\lambda})} - 1). \tag{7}$$

where $\hat{\sigma}_0^2$ is the residual variance from simple OLS estimation of the null linear model. $\hat{\sigma}^2(\hat{\lambda})$ is explained immediately after equation (5).

We analyze the limit distribution of the test statistic in (7). The following limit law for the sup Wald test for threshold effect , in the case of unit roots, is Theorem 4 of Caner and Hansen (2001).

Theorem 1. *Under Assumption 1 , $H_0 : \theta_1 = \theta_2$, the Wald statistic W_T in (7) is distributed as*

$$W_T \Longrightarrow T = \sup_{\pi_1 \leq u \leq \pi_2} T(u)$$

where

$$T(u) = Q_1(u) + Q_2(u)$$

and $Q_1(u), Q_2(u)$ are the independent stochastic processes that will be explained below.

Unfortunately , the asymptotic distribution in Theorem 1 is nonpivotal and depends upon nuisance parameters, since the dependence on the data structure is quite complicated , critical values cannot be tabulated. The details of the limit processes are in section 4.2 of Caner and Hansen (2001). In the following we discuss a bootstrap method to approximate the null distribution of W_T.

2.3 Bootstrap for Threshold Test

Here, following section 4.3 of Caner and Hansen (2001) we introduce two bootstrap approximations . The bootstrap p-values are reported in Table 2. We report the conservative of the two (larger one) in the Table.

We set $\mu = 0$, as the distribution of the test is invariant to level shifts. So under the null of $\theta_1 = \theta_2$ the model is $\Delta q_t = \rho q_{t-1} + \alpha' \tilde{\Delta} q_{t-1} + e_t$, where $\tilde{\Delta} q_{t-1} = (\Delta q_{t-1}, \cdots, \Delta q_{t-k})'$. F is the distribution function of the errors e. Since this is entirely determined by ρ, α, F we can use a model based bootstrap.

1.Unrestricted Bootstrap :

Let $(\tilde{\rho}, \tilde{\alpha}, \tilde{F})$ be the estimates of (ρ, α, F). The bootstrap distribution of the Wald is determined as follows. Let e_t^b be a random draw from $\tilde{F}$ and let q_t^b be generated as $\Delta q_t^b = \tilde{\rho} q_{t-1}^b + \tilde{\alpha}' \tilde{\Delta} q_{t-1}^b + e_t^b$, where $\tilde{\Delta} q_{t-1}^b = (\Delta q_{t-1}^b, \cdots, \Delta q_{t-k}^b)'$. Initial values for the recursion

can be set to sample values of the demeaned series . The distribution of q_t^b is the bootstrap distribution of the data. Let W_T^b be the threshold Wald test calculated from the series q_t^b. The distribution of W_T^b is the bootstrap distribution of the Wald test. Its bootstrap p-value is $p_T = P(W_T^b > W_T | I_T)$. Conditioning on I_T denotes that this probability is conditional on observed data. Typically the bootstrap p-value is calculated by simulation, where a large number of independent Wald tests W_T^b are simulated , and the p-value p_T is approximated by the frequency of simulated W_T^b that exceeds W_T. To implement this bootstrap we need estimates $(\tilde{\rho}, \tilde{\alpha}, \tilde{F})$. For (ρ, α) we need an estimate that imposes the null ,so we regress q_t on x_t (x_t is defined immediately after (2) and represents the regressors) to get those estimates. An estimator for F is the empirical distribution of the OLS residuals.
If the time series has a unit root , however, unrestricted bootstrap will not work, so we also analyze the constrained bootstrap.

2.Constrained Bootstrap:

We can achieve the correct asymptotic distribution by imposing the true unit root. This is done by imposing the constraint $\rho = 0$. This can be done by setting the estimates of (ρ, α, F) to be $(0, \tilde{\alpha}, \tilde{F})$, where $(\tilde{\alpha}, \tilde{F})$ are defined above. Generate random samples q_t^b from the model $\Delta q_t^b = \tilde{\alpha}' \tilde{\Delta} q_{t-1}^b + e_t^b$ with e_t^b drawn randomly from $\tilde{F}$. These samples are unit root processes. For each sample q_t^b , calculate the test statistic W_T^b . The estimated bootstrap p-value is the percentage of simulated W_T^b that exceed the observed W_T.

If the true order of integration is unknown then it appears prudent to calculate the bootstrap p-values both ways, and base inference on the more conservative p-value. Note that the usage of the conservative bootstrap does result in very small loss of power as observed in the simulations in section 4.4 of Caner and Hansen (2001) . Both bootstraps have also excellent size as shown in Table 1 of Caner and Hansen (2001). Both of the bootstrap p-value are first-order asymptotically correct.

Regarding the choice of delay parameter "m", two methods are used. First, "m" is fixed and bootstrap is run for fixed m. We tried this for $m = 1, 2, \cdots, 6$. Then we chose the one with the minimal p-value. In the second method, since "m" is generally unknown , we made the selection of m endogenous. Since the Wald test is a monotonic function of the residual variance we select the least squares estimate of "m". This is equivalent to selecting m as the value that maximizes the Wald test. So , in this method, the bootstrap p-value is calculated along with estimation of m (simultaneous choice of $\hat{m}, \hat{\lambda}$).

In our case , with both methods we obtained the same "m". We preferred to report the bootstrap p-value from the first method since the econometric theory explicitly models fixed m in Caner and Hansen (2001).

2.4 Testing Against Other Nonlinear Alternatives

In this section we test whether real exchange rates are linear or display ESTAR (Exponential Self Exciting Threshold) type of behavior. We also analyzed a test against LSTAR type of behavior, but the results are the same and ESTAR has a more general structure so we concentrate on ESTAR type of alternative. Recently, a Wald type of test is suggested by Kilic (2003) in nonstationary framework to test linearity of real exchange rates versus an ESTAR model. This is a simple extension of the test developed by Luukkonen et.al (1988) and Granger and Terasvirta (1993) to the nonstationary framework. The model is:

$$q_t = \phi_1 q_{t-1} \left[\frac{exp(-\gamma(z_t - c))}{1 + exp(-\gamma(z_t - c))} \right] + \phi_2 q_{t-1} \frac{1}{1 + exp(-\gamma(z_t - c))} + u_t,$$

where u_t is iid with $(0, \sigma^2)$, z_t denotes the transition variable, $\gamma > 0$ and c is an unknown constant. The null of linearity is formulated as $H_0 : \gamma = 0$. To simplify these models Kilic (2003) takes a Taylor series expansion around $\gamma = 0$, then the ESTAR model can be rewritten as in Granger and Terasvirta (1993) (imposing $z_t = q_{t-1}$)

$$q_t = \beta_1 q_{t-1} + \beta_2 q_{t-1}^2 + \beta_3 q_{t-1}^3 + \eta_t,$$

where η_t is a composite error consisting of u_t, q_{t-1}, γ, c. This error $\eta_t = u_t$ under the null. Theorem 1 of Kilic (2003) develops the limit theory for the Wald test under the null of $H_0 : \beta_2 = \beta_3 = 0$. The limit is nonstandard and consists of functionals of standard Brownian Motions and the critical values are tabulated in Table 1 of Kilic (2003). We will also check the viability of the simple ESTAR against a linear null in section 3.

However, one important fact is that all the linearity tests depend on the stationary or nonstationary behavior of the random variables. In that respect the bootstrap approach developed by Caner and Hansen (2001) is not affected by this type of behavior, the results of the bootstrap threshold test is agnostic about stationary or nonstationary behavior. Also we should note that the tests for ESTAR type of nonlinearity developed by Kilic (2003) is simple and it is not clear how that may be connected with estimation and unit root testing of ESTAR models. This is another advantage area of Caner and Hansen (2001) threshold test, since it takes into account weak dependency in data and well connected with estimation and with unit root testing. So we proceed with the nonlinear model that is developed in section 2.1.

2.5 Unit Root Tests

Here we set up two tests . Our null is the same but alternatives change . We set up the null as :

$$H_0 : \rho_1 = \rho_2 = 0$$

Under the null by equation (4) we see that real exchange rates have unit root.

Next we set meaningful alternatives , first a major case of interest is when the real exchange rates follow a stationary threshold autoregressive pattern. In the model (4) when $\rho_1 < 0, \rho_2 < 0, (1+\rho_1)(1+\rho_2) < 1$ then the real exchanges are stationary (Chan and Tong, 1985) , so the alternative of interest is :

$$H_1 : \rho_1 < 0, \quad \rho_2 < 0$$

Another case in between H_0 and H_1 is the case of partial unit root

$$H_2 : \left\{ \begin{array}{ll} \rho_1 < 0 & and\, \rho_2 = 0 \\ \rho_1 = 0 & and\, \rho_2 < 0 \end{array} \right\}$$

So if H_2 holds then real exchange rate is nonstationary but not a classic unit root (Caner and Hansen, 2001). Now we set up the test statistics for testing H_0 vs H_1 and H_0 vs H_2.

Since H_1 is one-sided we chose to have a simple one-sided Wald as our test statistic. In (4) we test $H_0 : \rho_1 = \rho_2 = 0$ with 1-sided Wald

$$R_{1T} = t_1^2 1_{\{\hat{\rho}_1 < 0\}} + t_2^2 1_{\{\hat{\rho}_2 < 0\}} \tag{8}$$

as in section 5 of Caner and Hansen (2001). t_1, t_2 are the t-ratios for $\hat{\rho}_1$, $\hat{\rho}_2$ respectively from the estimated model in (6).

However the R_{1T} given in (8) cannot discriminate between the stationary case H_1 and the partial unit root case H_2. So in order to test H_0 vs H_2 we use the negative of the the t statistics $-t_1$,$-t_2$.

For both unit root tests Caner and Hansen (2001) obtained the limit distributions and the critical values are tabulated in Table 3 of their article. The limit for the unit root tests are derived for both when the threshold is unidentified ($\theta_1 = \theta_2$) and when the threshold is identified ($\theta_1 \neq \theta_2$) are derived. The test for unit root in the case of identified threshold does not have good small sample properties, so we do not use it here. We now give the limit when there is an unidentified threshold . This is Theorem 5 of Caner and Hansen (2001).

Theorem 2. *Under Assumption 1, and $\theta_1 = \theta_2$, then*

$$(t_1, t_2) \Longrightarrow (t_1(u^*), t_2(u^*)) \leq \sup_{u \in [\pi_1, \pi_2]} (t_1(u), t_2(u))$$

and

$$R_{1T} \Longrightarrow t_1(u^*)^2 1_{\{\rho_1<0\}} + t_2(u^*)^2 1_{\{\rho_2<0\}} \leq \sup_{u\in[\pi_1,\pi_2]} [t_1(u)^2 1_{\{\rho_1<0\}} + t_2(u)^2 1_{\{\rho_2<0\}}]$$

where

$$u^* = arg \max_{u\in[\pi_1,\pi_2]} T(u),$$

$T(u)$ is defined in Theorem 1.

$$t_1(u) = \frac{\int_0^1 W^*(s)dW(s,u)'}{(u\int_0^1 W^*(s)^2 ds)^{1/2}},$$

$$t_2(u) = \frac{\int_0^1 W^*(s)(dW(s,1) - dW(s,u))}{((1-u)\int_0^1 W^*(s)^2 ds)^{1/2}}$$

where $W(s,u)$ is a two parameter Brownian motion, $W(s) = W(s,1)$ and $W^*(s)$ is the demeaned standard Brownian motion.

Even though the unit root tests have the asymptotic bound distribution , we can benefit from a bootstrap in finite samples.

3 Empirical Evidence

In this section we use the framework given in section 2 and try to understand whether this new econometric methodology will further our knowledge in understanding the behavior of real exchange rates. The real exchange rate data are constructed from the IMF's International Financial Statistics data base on CD-ROM. They are based on the end-of period nominal US dollar spot exchange rates and the US and foreign consumer price indices. The data set comprises monthly data for 1973.1-1997.4 (292) observations for 17 countries including Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Italy, Japan, The Netherlands, Norway, Portugal, Spain, Sweden, Switzerland and the United Kingdom.

First we test for threshold in a nonstationary model, if we find threshold existence then we test for unit roots against a stationary threshold autoregressive model and the partial unit root model. This methodology is also used for analyzing the post war unemployment rate in US in Caner and Hansen (2001).

We begin with the analysis by fitting lag orders for the various countries. We use a sequential t-test to fit the lags and we used the model in (4) with $\theta_1 = \theta_2, \rho = 0$ restrictions (linear unit root). Such an analysis is recommended by Stock (1994). Since the lag order does not change under the null and various alternatives we used this specific null. In this analysis we fit first an upper bound of 8 lags to each series and tested sequentially. The resulting optimal lag orders are in Table 2 and the footnote in Table 1.

Before beginning the tests we consider Augmented Dickey Fuller (ADF) tests for unit roots against linear stationary alternatives. In Table 1 , all of the countries have a unit root in real exchange rates according to ADF. This is not surprising since ADF tests have almost no power when the alternative is non-linear, for further information and simulations see Table 6 in Caner and Hansen (2001).

3.1 Threshold Test

Here we try to find whether the real exchange rates are nonlinear or not . For this reason we benefit from the Wald test described in section 2. Since the limit law for the test depends on nuisance parameters , a bootstrap is advised in Caner and Hansen (2001). This bootstrap procedure has excellent size and good power in small samples.

At Table 2 we test for a threshold in real exchange rates in 17 countries in the post-float period. We report the bootstrap p-values and the corresponding optimal delay parameter along with the percentage of the observations that fall into each regime. In Table 2 , we find very strong evidence for threshold effect hence nonlinearity of exchange rates at 10% level. 14 out of 17 countries (except Greece, Japan, and Portugal) have nonlinear exchange rates. To give an example in the case of Germany, the Wald test rejects the null of a linear real exchange rate at p-value of .006. Then we also find that if the change in the log real exchange rate in the past 6 months (i.e $|q_{t-1} - q_{t-7}|$) are above .127 in absolute terms we switch to the second regime. The first regime (inside the band) has 82% and the second regime (outside the band of .127) has 18% of the observations. In the case of Germany this band of .127 corresponds to 13.5% in the real exchange rate level. When we analyze the table we see that first regime is dominant in all of the countries that exhibit nonlinearity . Specifically, the share of the“inside the band “ regime fluctuates between 67% to 85% of the observations in the countries analyzed . Next, by using the unit root tests described in section 2.2 we analyze the behavior of these two regimes in each country.

We also checked whether the real exchange rates follow ESTAR model by the test developed by Kilic (2003) and described in section 2.3. We find that the null of linearity is rejected except from Switzerland. So these results are not too different from the previous results.

3.2 Threshold Unit Root Test

In this section we analyze the countries that display nonlinearity in their real exchange rates. We use the one-sided Wald test (R_{1T}) and t_1, t_2 tests for unit roots to determine whether these regimes are nonstationary or not. For these tests critical values are tabulated in Table 3 of Caner and Hansen (2001) , however we use the bootstrap procedure described in section 5.3 of Caner and Hansen (2001) and repeated in section 2.4 here since this seems to work better in finite samples. The bootstrap p-values correspond to the m that is calculated in Table 2. So bootstrap p-values are reported for one-sided Wald and the t-ratio tests in Table 3 as well as the coefficient on the lagged real exchange rates in two regimes.

We reach two main conclusions after analyzing Table 3 . First , there is evidence for partial unit root regime which consists of one nonstationary and one stationary regimes. In a band the real exchange rates follow unit root behavior and then if the real exchange rate appreciates or depreciates a lot against $ US they display mean reversion. (See Tables 2 and 3) The band is already calculated in Table 2 under the threshold estimate column.

In Table 3 the one-sided Wald tests which tests unit root against a two-regime stationary nonlinear model, is rejected in 9 out of 14 countries (except Canada, Denmark, Norway, Switzerland and Sweden) However when we also look at closely the results of the individual t-ratio tests for we see that in Denmark, Norway the results for the one-side Wald unit root test is due to the fact that the first regime (inside the band) has unit root whereas the second regime has mean reverting behavior and the first regime is dominant (see, also Table 2). With that analysis in hand we can say that in 11 out of 14 countries that is analyzed real exchange rates have a unit root inside the band and outside the band we see mean reversion.

To give a specific example , in the case of Germany we reject the null of unit root (linear or 2 nonlinear unit root regimes) in favor of two stationary exchange rate regimes with a p-value of .016 (one-sided Wald test) . However this rejection is due to the second regime (outside the band) where p-value for t test for the second regime is .005 and for the first regime the p-value is .746.

3.3 Out-of Sample Forecasting Exercise

We compare the linear AR(k) model with threshold model in (4) in an out of sample forecasting exercise. First we cut the original sample of 292 observations into two. We take the first 280 monthly observations (1973.1-1996.4) and obtain 1, 3, 6 period forecasts for the real exchange rate q_t. So we proceed exactly as in section 3: we test for a threshold and then carry unit root tests for 280 observations and estimate the model in (4). The results can be obtained from the authors on demand, but they are qualitatively the same as the ones reported in Tables 1-3 for the original sample. But we use 1000 iterations for the bootstrap this time.

Then we compute the forecast error (root mean squared error) for one period, three period and sixth period ahead forecasts. (For 1996.5, 1996.7 and 1996.10 respectively) RMSE is obtained by:

$$RMSE = \left(\frac{1}{p} \sum_{j=1}^{p} (q_{t+j} - \hat{q}_{t+j})^2 \right)^{1/2},$$

where $p = 1, 3, 6$. In the threshold model $\hat{q}_{t+j}$ is computed by using the estimators in section 2.1 $(\hat{\theta}, \hat{\lambda})$. We begin with computing $\hat{q}_{t+1}$ and then recursively generate $\hat{q}_{t+2}$ and other periods forecasts by using the same $\hat{\theta}, \hat{\lambda}$ found in the in-sample estimation of data. This is a naive threshold forecast. It does not use any advantages of threshold modeling after the results for 280 observations (1973.1-1996.4). The linear model is estimated as AR(k) model. The ADF tests show that there are unit roots for each country's currency with a sample of monthly observations in the period of 1973.1-1996.4.

Table 4 reports the results. We only consider the countries that we find nonlinear regimes with our threshold test for the period of 1973.1-1996.4. Results show that for one and three period forecasts threshold model does better than the linear model in most of the countries analyzed. Only for Austria, Belgium, Germany, Sweden and Netherlands linear model does better than the naive threshold model clearly. For the six period ahead forecasts the results are mixed. Note that the lag length (k) and delay factor in threshold model (m) changed in some countries compared to the full sample of 1973.1-1997.4 that is used in sections 3.1 and 3.2. For Canada $k = 11, m = 3$, Denmark $k = 6, m = 3$, Finland $k = 6, m = 3$, Spain $k = 7, m = 1$, United Kingdom $k = 11, m = 4$ in the first 280 observations of the data. Also we try to see whether the initial forecast period was a factor in results, we tried a different initial regime point for Austria and the results did not change.

4 Conclusion

We analyzed real exchange rates in OECD countries in the post-float period. The new econometric tools developed by Caner and Hansen (2001) are used . First we test for nonlinearity in exchange rate system . We find strong evidence for nonlinearity . Next two unit root tests , one-sided Wald test and t-ratio test are used. We find evidence for a two -regime threshold model in which the real exchange rates display unit root behavior inside a band of real appreciation or depreciation of about 10% and then outside this band the real exchange rates are mean reverting.

The implications of our findings can be summarized as follows. There is a strong indication of non-linearity in real exchange rates. Nature of non-linearity requires the need for incorporating international finance theory based explanations like Dornbusch (1976) as well.

. As a further study tradables and nontradables sectors can be analyzed with these tests to get a better understanding of the real exchange rate. Multiregime threshold models can be analyzed for the future work using the tools developed by Gonzalo and Pitarakis (2002).

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Table 1: ADF TESTS

Countries	ADF tests
Austria	-1.52
Belgium	-1.51
Canada	-1.13
Denmark	-1.59
Finland	-1.91
France	-1.90
Germany	-1.68
Greece	-1.44
Italy	-1.92
Japan	-1.64
Netherlands	-1.73
Norway	-1.83
Portugal	-2.01
Spain	-1.85
Switzerland	-1.87
Sweden	-1.59
United Kingdom	-2.10

Note : The 10% asymptotic critical value for ADF test is -2.57. By a sequential t-test Canada has $k = 8$ *lags , Greece has 4 , Japan has 1 , Spain has 5 and all the other countries has 6 lags in the linear null model.*

Table 2: THRESHOLD TESTS

Countries	p-value	Threshold estimate	k	m	Regime 1	Regime 2
Austria	.049*	.121	6	6	81	19
Belgium	.000*	.112	6	6	76	24
Canada	.049*	.030	8	3	81	19
Denmark	.001*	.061	6	3	67	33
Finland	.077*	.109	6	6	85	15
France	.035*	.109	6	6	78	22
Germany	.006*	.127	6	6	82	18
Greece	.178	L	4	L	L	L
Italy	.010*	.101	6	5	84	16
Japan	.811	L	1	L	L	L
Netherlands	.002*	.131	6	6	85	15
Norway	.001*	.110	6	6	85	15
Portugal	.187	L	6	L	L	L
Spain	.002*	.042	5	1	84	16
Switzerland	.001*	.111	6	4	81	19
Sweden	.048*	.070	6	3	80	20
United Kingdom	.048*	.101	6	4	84	16

Note : p-value is the bootstrap p-value calculated using the bootstrap method in section 4.3 of Caner and Hansen (2001). Threshold estimate is $\hat{\lambda}$ in our model. k represents the lag order in (3)and m represents the optimal delay parameter. Under the columns of Regime 1 and Regime 2 we have the percentage of the values that fall into each regime . L represents the linear model so threshold is not estimated and there is only one regime. 5000 bootstrap repetitions are used."" denotes significance at 10% level.*

Table 3: UNIT ROOT TESTS

Countries	R_{1T}	t_1	t_2	ρ_1	ρ_2
Austria	.081*	.752	.028*	-.003	-.081
Belgium	.002*	.862	.001*	.004	-.097
Canada	.621	.649	.319	-.006	-.033
Denmark	.228	.788	.090*	-.001	-.052
Finland	.026*	.577	.009*	-.012	-.117
France	.005*	.704	.005*	-.006	-.113
Germany	.016*	.746	.005*	-.003	-.098
Italy	.016*	.534	.006*	-.013	-.109
Netherlands	.010*	.725	.002*	-.004	-.120
Norway	.178	.486	.098*	-.020	-.067
Spain	.060*	.333	.042*	-.017	-.074
Switzerland	.249	.427	.160	-.017	-.060
Sweden	.586	.330	.593	-.019	-.023
United Kingdom	.083*	.600	.036*	-.013	-.082

Note : R_{1T}, t_1, t_2 are unit root tests described in section 2, specifically t_1, t_2 are tests for H_0 versus H_2 and R_{1T} is for testing H_0 versus H_1. Under their respective columns we report the bootstrap p-values. The columns under ρ_1, ρ_2 represent the coefficients of the lagged real exchange rate in each regime. 5000 bootstrap repetitions are used ."" represents significance at 10% level.*

Table 4: FORECAST ERRORS

	Threshold Model			Linear Model		
Countries	Period 1	Period 3	Period 6	Period 1	Period 3	Period 6
Austria	.0392	.0394	.0350	.0372	.0361	.0289
Belgium	.0350	.0404	.0369	.0340	.0370	.0299
Canada	.0021	.0079	.0165	.0022	.0086	.0149
Denmark	.0304	.0325	.0269	.0298	.0307	.0235
Finland	.0392	.0252	.0307	.0433	.0275	.0270
France	.0231	.0225	.0202	.0217	.0217	.0182
Germany	.0373	.0383	.0344	.0353	.0338	.0270
Italy	.0038	.0189	.0283	.0068	.0230	.0324
Netherlands	.0313	.0428	.0392	.0275	.0377	.0302
Norway	.0207	.0171	.0140	.0253	.0230	.0175
Spain	.0144	.0157	.0209	.0184	.0262	.0204
Switzerland	.0303	.0366	.0296	.0364	.0468	.0378
Sweden	.0066	.0072	.0058	.0021	.0051	.0045
United Kingdom	.0010	.0251	.0307	.0073	.0153	.0222

Note : Periods represents the forecast after the last period 1996.4 in the forecasting data set. For example Period 3 represents the forecast error made when making a forecast for 1996.7