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Lanouar CHARFEDDINE* et Dominique GUEGAN**

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*Université Paris II Panthèon Assas, Centre de Recherche ERMES, Doctorant associès à l'équipe MORA-ENS Cachan
12 Place de la Panthèon, 75006 Paris.
E-mail: lanouar_charf@yahoo.fr.
**Ecole Normale Supérieure de Cachan, Senior Academic Fellow de l'IEF, Département d'Economie et Gestion
61, avenue du Président Wilson - 94235 CACHAN Cedex
Tél. : + 33 (1) 47 40 55 75 - Fax : + 33 (1) 47 40 68 42
guegan@ecogest.ens-cachan.fr

Can the $SupLR$ test discriminate between different switching regressions models: Applications to the U.S GNP and the US/UK exchange rate?

Lanouar CHARFEDDINE * Dominique GUEGAN †

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Abstract

In this paper we study, using the $SupLR$ test, the possibility of discrimination between two classes of models: the Markov switching models, Hamilton (1989) and the Threshold Auto-Regressive Models (TAR), Lim and Tong (1980). This work is motivated by the fact that generally practicians, in applications, use switching models without any statistical justification. We show using simulation experiments that it is very difficult to discriminate between MSAR and SETAR models specially using large samples. this means that, when the null hypothesis is rejected it appears that different switching models are significant. The power of the $SupLR$ test seems to be sensitive to the mean, the noise variance and the delay parameter which appear in the previous models. Finally, we apply this methodology to the US GNP growth rate and the US/UK exchange rate. We shall retain Markov switching process for US GNP and US/UK exchange rate (monthly data) and a random walk for US/UK exchange rate (quarterly data).

JEL classification: C12;C15;F31

Keywords: Switching Models, $SupLR$ test, Empirical power

*Université Paris II Panthèon Assas, Centre de Recherche ERMES, 12 Place de la Panthèon, 75005 Paris. Doctorant associès à l'équipe MORA-ENS Cachan, lanouar_charf@yahoo.fr

†Ecole Normale Supérieure, Cachan, Département d'Economie et Gestion, 61, avenue du Président Wilson, 94230 Cachan.

1 Introduction:

A huge literature on non linear econometric models have been developed in the last two decades. Researchers have largely used non linear time series models to explain specific phenomena observed in many macroeconomic and financial series.

A specific attention is attributed to Markov Switching Auto-Regressive models (MSAR afterhere). Applications of these models have extensively increased since the seminal paper of Hamilton (1989). Indeed, Markov switching models are good candidates to explain changes in economic and financial times series. For example, if we consider the evolution of the dollar in the last three decades, it rises in the early 1980s, falls afterward, rises again in the lastly 1980 and so on. These swings can be explained by the effects of monetary and fiscal policy (Jeffery and Frankel, 1988). Moreover, the intervention of monetary authority on the exchange market is another source of this movement. Thus, all these effects provoke dollar's shifts between several regimes. Similarly, the behavior of real interest rate is non constant and fluctuates according to monetary (monetary expansions and restrictions) and fiscal policies. Thus, many authors employed switching models in order to study the aggregate outputs (Hamilton, 1990), the annual growth rate of consumption in an asset-pricing (Cecchetti et *al.*, 1990), the behavior of foreign exchanges rates (Engle and Hamilton, 1989), the effect of federal reserve actions on interest rates (Hamilton 1988, Garcia and Perron 1989), financial panics (Schwert, 1989,1990) and the behavior of option prices (Bollen et *al* 1989), for instance.

Another class of nonlinear time series models which is also quite popular in the literature is the Threshold Auto-Regressive Models (TAR). It has been introduced by Tong and Lim (1980) and it is now used in several applications to model changes observed in economic and financial data, see Potter (1995), Hansen (1997), Proietti (1998) and Ferrara and Guégan (2005) for instance. Contrary to the MSAR models, for which changes in regimes is unobserved, the changes between states in the threshold autoregressive models occur when an observed variable z passes a certain threshold parameter r.

These two models have the advantage of being able to modelize and capture asymmetry, sudden changes and irreversibility time observed in many economic and financial time series. Despite these similarities and common points, these models have been involved, in the literature, largely independently. While the estimation methods is well established for these two classes of models, Coslett and Lee (1984), Hamilton (1988, 1989), Tong (1990) and Hansen (1997, 2000), testing between different types of switching regression models are seldom explored by researchers and only a few papers in the economic literature, deal with this problem, Carrasco (2002), and Kopp and Potter (1999, 2001).

Thus, it appears important to define robust tests which permit to discriminante between these two classes of models : SETAR and Markov switching processes. Generally, before applying a regime switching model, one should test the null hypothesis of no shift against several states. Building such tests is problematic because of the presence of nuissance parameters in the models. For instance, the probabilities of transition p_{00} and p_{11} for the MSAR models and the threshold r and the delay d parameters for the SETAR models are not identified under the null hypothesis. Moreover, in case of Markov switching models another problem occurs through the score function which is identically zero under the null. As a result, the Likelihood Ratio (LR), the Lagrange Multiplicateur (LM) and the Wald tests (W) have no standard asymptotic distribution, Davies (1977, 1987), Hansen (1992,1996), Gong and Mariano (1997), and Garcia (1998) for more details.

In this paper we study the empirical power of the *SupLR* test, which is the most widely used test, see Davies (1977, 1987), Hansen (1992), Gong and Mariano (1997) and Garcia (1998). Precisely, we assess the ability of *SupLR* test proposed by Garcia (1998) to reject the alternative when the Data Generated Process (DGP) is a SETAR model. This is motivated by the fact that the majority of practitioners used switching models without any statistical justification and the model is selected in an *ad hoc* way. Indeed, when the null hypothesis is rejected it appears that different switching models are significant [1]. Moreover, we investigate the sensitivity of the power of the *SupLR* test with respect to the number of lags, the mean parameters and the noise volatility which appear in the expression of the SETAR process. The critical values of the *SupLR* test are determined by the algorithm proposed in Garcia (1998). Finally, we show that the *SupLR* test cannot distinguish between switching models and SETAR processes. This distinction appears impossible as soon as the data set's sample size is large.

The remain of the paper is organized as follows: in the next section we present the two models and the *SupLR* test. We specify its property under the null AR process and the alternative MSAR process. In section 3, we report the simulations results and specify the novelty of this work and its interest for applications. In section 4, we apply the *SupLR* test to three exchange rates series. Section 5 concludes.

[1] Hamiton (1989) applied a Markov switching model, with autoregressive order equal to four, to the growth rates of U.S GNP but Hansen (1992) and Garcia (1998) show, based on *SupLR* test, that the null hypothesis of $AR(4)$ cannot be rejected. Potter (1995) applied SETAR model to the U.S GNP but Hansen (1996) doubts about the evidence for the SETAR model.

2 Models and Testing procedure:

In this section, we introduce the Markov switching and the SETAR models on which we work and the $SupLR$ test.

2.1 The Models

We consider the stationnary Markov switching model (y_t) defined by:

$$y_t = \phi_{0,1} + (\phi_{0,2} - \phi_{0,1})s_t + z_t \tag{1}$$
$$\text{with } z_t = \theta_{s_t} z_{t-1} + u_t,$$

where (u_t) is a Gaussian strong white noise N(0,σ_u). We assume that the state s_t is independent of y_t. The parameters $\phi_{0,1}, \phi_{0,2}$ and θ_{s_t} take values in $\Re$. The state (s_t) is an unobserved Markov chain whose transition probability is defined by:

$$p(s_t = j | s_{t-1} = i) = p_{ij}, \qquad i, j = 0, 1, \tag{2}$$

with $0 \leq p_{ij} \leq 1$ and $\sum_{j=0}^{1} p_{ij} = 1, i = 0, 1,$ and the transition matrix is

$$\mathbf{P} = \begin{pmatrix} p_{00} & 1 - p_{00} \\ 1 - p_{11} & p_{11} \end{pmatrix}.$$

We denote π_i the unconditional probabilities for the process (s_t) to be in each regime : $\pi_i = P(s_t = i), i = 0, 1$. These unconditional probabilities are equal to :

$$\pi_0 = P(s_t = 0) = \tfrac{1-p_{11}}{2-p_{00}-p_{11}} \text{ and } \pi_1 = P(s_t = 1) = \tfrac{1-p_{00}}{2-p_{00}-p_{11}}.$$

In the Markov switching model (1) the 'state' or 'regime' plays an important role. Indeed, Hamilton (1989) suggests that the existence of discrete 'regimes' explain the nonlinearity in GNP growth rates. The first 'state' will correspond to a fast growth and the second one to a slow growth. Here we assume that the MSAR model is strictly stationnary and β-mixing. Guégan and Rioublanc (2005) show that a sufficient condition for strict stationarity for Markov switching model is $(1 - p_{11}) log\,|\theta_0| + (1 - p_{00}) log\,|\theta_1| \leq 0$, see also Yao and Attali (2000) for geometric ergodicity of MSAR Models.

We consider also the stationary first-order threshold autoregressive process (y_t) Tong (1990), given by:

$$y_t = \begin{cases} \phi_{0,1} + \theta_0 y_{t-1} + \epsilon_t & \text{if } y_{t-d} \leq r \\ \phi_{0,2} + \theta_1 y_{t-1} + \epsilon_t & \text{if } y_{t-d} > r, \end{cases} \tag{3}$$

where r is the threshold parameter and d the delay parameter. Chen and Tsay (1991) showed that the necessary and sufficient condition for the geometrical ergodicity of (y_t) in model (3) is $\theta_0 \leq 1$, $\theta_1 \leq 1$, $\theta_0\theta_1 \leq 1$,

$\theta_0^{s(d)}\theta_1^{t(d)} \leq 1$, $\theta_0^{t(d)}\theta_1^{s(d)} \leq 1$, where $s(d)$ and $t(d)$ are nonnegative integers depending on d. They are odd and even numbers, respectively. In the following, we assume that only the means shift between states, thus $\theta_0 = \theta_1 = \theta$, $d = 1, 2$ and $r = 0$. In this simple case, the necessary and sufficient condition for stationarity for the model (1) is $\theta \ \leq 1$.

The SETAR model is intimately related to the MSAR model because, in both models, the change is permanent. The SETAR model is also a special case of Markov switching model when $\theta = 0$ although in the latter case the Markov chain $I\{y_{t-1} \leq r\}$ is not exogenous. see Hamilton (1989) and Carrasco (2002).

We provide some representations of MSAR and SETAR models, in Figures 1-3. We can observe the similarity of the trajectories. Figure 4 gives the scatterplots of y_t versus y_{t-1} for the previous two models: we do not observe any difference between these scatterplots. Thus, a graphical analysis can lead to misspecification. As a result of this misspecification, problems concerning predictions and forecastings can be occur. To handle this problem, it is important to develop a testing procedure allowing researchers to discriminate between different types of switching models.

2.2 The *SupLR* test for Markov Switching Model

The literature on testing, when nuissance parameters are present under the alternative hypothesis, has growing rapidely and a variety of statistical tests have been developed. Most of the tests adopt the approach developed by Davies (1977,1987), which proposes a *SupLR* test. The weakness of his test lies on the fact that we do not know the asymptotic distribution of this test under the alternative.

Other tests have also been developed. Hansen (1992) proposes the likelihood ratio statistic. He considers the likelihood function as a function of unknown parameters, and he gets a bound for the asymptotic distribution of the test. The Hansen's approach (1992) is time consuming and makes it inappropriate in applications. In another hand, this test provides only bounds in terms of decision theory, it does not provide critical values. Andrews (1993) and Andrews and Ploberger (1994) proposed the *SupLM* test and a class of average exponential LM, Wald and LR tests. They showed that they are optimal in terms of weighted average power. Andrews and Ploberger (1995) show that the *SupLR* test is asymptotically admissible. It is the best test against alternatives that are sufficiently distant from the null hypothesis.

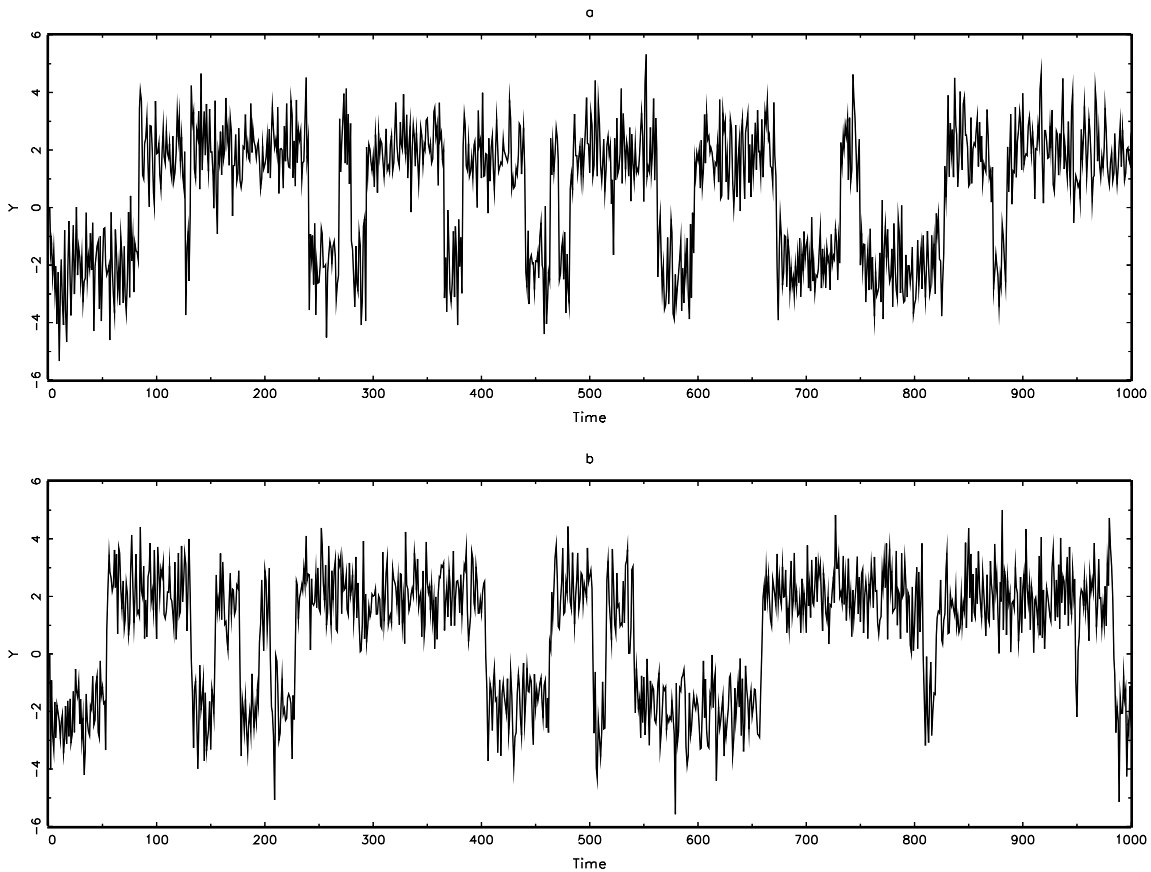

Figure 1: Trajectories of (a) MSAR model (1) with $p_{00} = 0.98$, $p_{11} = 0.98$ and $\phi_{0,1} = -\phi_{0,2} = 2$ and (b) SETAR model (3) with $r = 0$, $d = 1$, $\phi_{0,1} = -\phi_{0,2} = 2$.

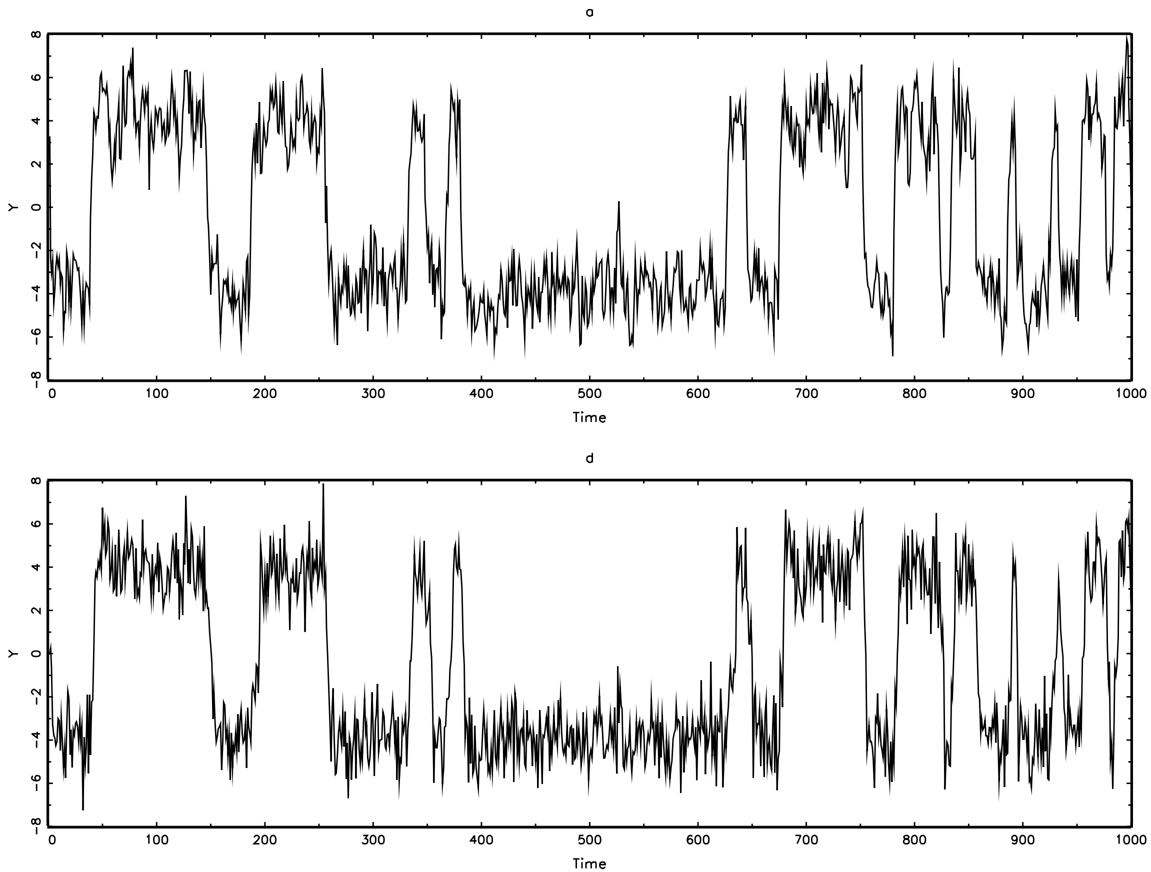

Figure 2: Trajectories of (c) MSAR model (1) with $p_{00} = 0.98$, $p_{11} = 0.98$ and $\phi_{0,1} = -\phi_{0,2} = 2$ and $\theta = -0.5$ and (d) SETAR model (3)with $r = 0$, $d = 1$, $\phi_{0,1} = -\phi_{0,2} = 2$ and $\theta = -0.5$.

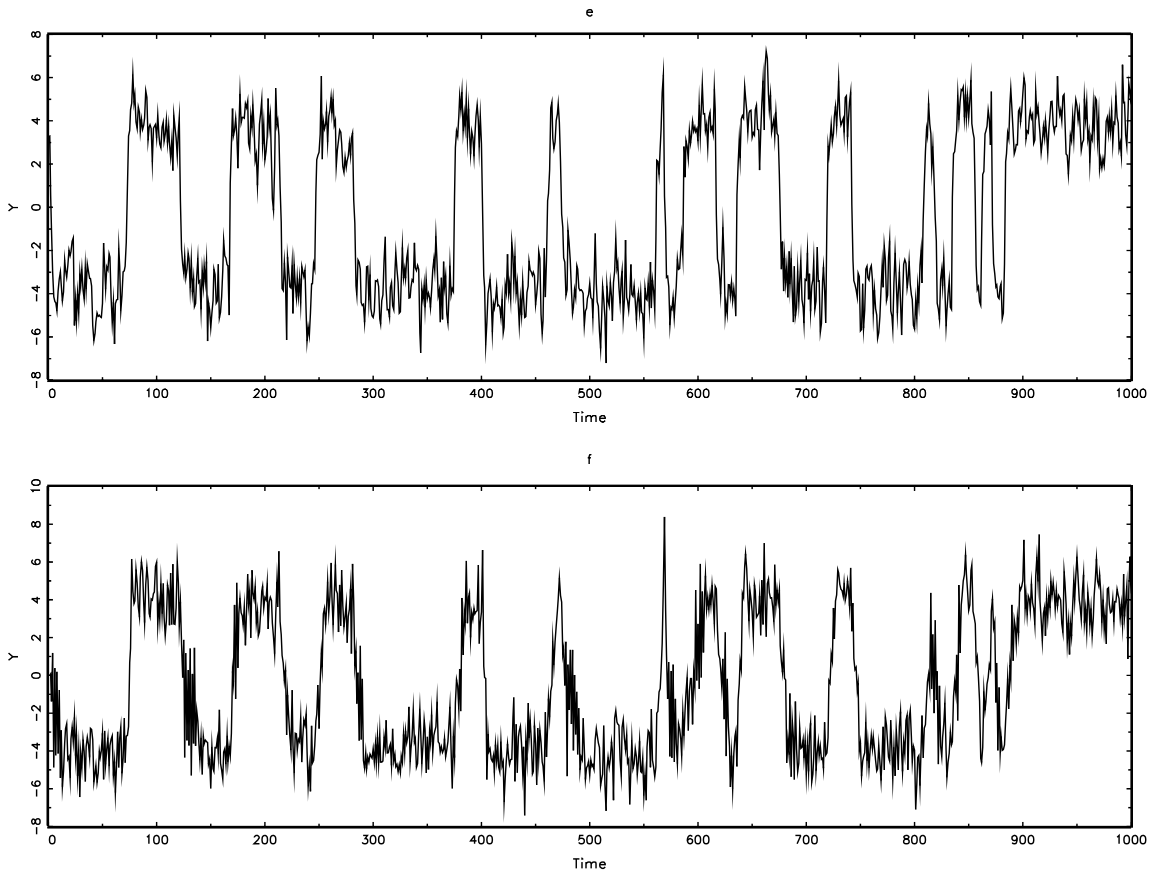

Figure 3: Trajectories of (e) MSAR model (1) with $p_{00} = 0.95$, $p_{11} = 0.95$ and $\phi_{0,1} = -\phi_{0,2} = 2$ and $\theta = -0.5$ and (f) SETAR model (3) with $r = 0$, $d = 2$, $\phi_{0,1} = -\phi_{0,2} = 2$ and $\theta = -0.5$.

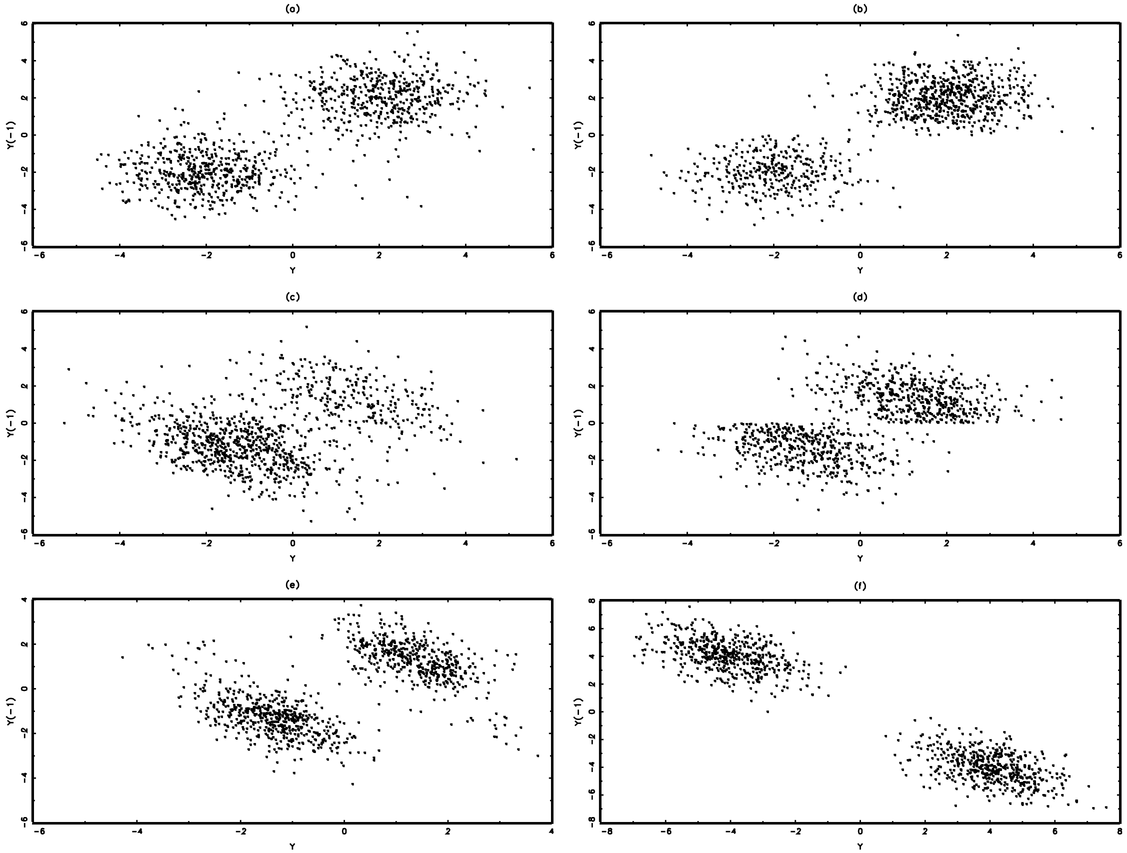

Figure 4: Scatterplots (a) MSAR model (1) with $p_{00} = 0.98$, $p_{11} = 0.98$ and $\phi_{0,1} = -\phi_{0,2} = 2$ and $\theta = -0.5$,
(b) SETAR model (3)with $r = 0$, $d = 1$, $\phi_{0,1} = -\phi_{0,2} = 2$ and $\theta = -0.5$,
(c) MSAR model (1) with $p_{00} = 0.98$, $p_{11} = 0.98$ and $\phi_{0,1} = -\phi_{0,2} = 2$ and $\theta = -0.5$,
(d) SETAR model (3) with $r = 0$, $d = 2$, $\phi_{0,1} = -\phi_{0,2} = 2$ and $\theta = -0.5$,
(e) MSAR model (1) with $p_{00} = 0.95$, $p_{11} = 0.95$ and $\phi_{0,1} = -\phi_{0,2} = 2$ and $\theta = -0.5$
and (f) SETAR model (3) with $r = 0$, $d = 2$, $\phi_{0,1} = -\phi_{0,2} = 2$ and $\theta = -0.5$.

Gong and Mariano (1997) developed two statistic tests for Markov switching models: the Difference Test DT_N (analogous to the LR test) and the LM test in the frequency domain. They derive their exact asymptotic null distributions under the condition of unidentified nuissance parameters. They show that, they only have to face to the problem of unidentified nuissance parameters in nonlinear context because the singularity problem disapears.

As shown in the literature the likelihood ratio statistics behave relatively smoothly when a nuissance parameter is present under the alternative contrary to Wald tests. The $SupLR$ test is the most widely used and suggested to be the best test under such irregularity. Thus, in this paper we focus on this test. We provide below a brief review of his testing procedure.

Let be the model (1). We test

$$H_0 : \phi_{0,1} = \phi_{0,2} \quad \text{against the alternative} \quad H_1 : \phi_{0,1} \neq \phi_{0,2}. \tag{4}$$

This means that we test a linear AR model under (H_0) against a MSAR model under (H_1). Following the works of Davies (1977, 1987), Hansen (1992, 1996) and Garcia (1998), we use the following statistic:

$$LR = 2[L(\hat{\beta}) - L(\tilde{\beta})], \tag{5}$$

where $L(.)$ represents the log-likelihood function, $\beta = (\phi_{0,1}, \phi_{0,2}, \theta_0, \theta_1, \sigma_u^2)$ and the vector of nuissance parameter is given by $\gamma = (p_{00}, p_{11})$. For the statistic LR, $\hat{\beta}$ is the maximum likelihood estimator of β under the alternative of Markov switching model, and $\tilde{\beta}$ is the estimated value of β under the null hypothesis (H_0). The critical values of this test are given by Garcia (1998). They are equal to 10.89 for the MSAR with non-autoregressive order variable and 8.68 for the MSAR (1) model.

To avoid the possibility of locals maxima we have used a lot of starting values in order to be sure that the maximum obtained is a global one.

3 Simulation Experiment

In this section, in a first part, we investigate the size and the empirical power of the test introduced in (5) under the assumptions (H_0) and (H_1). This permits to calibrate the test and to specify its sensitivity in presence of different parameters. In a second part, we simulate 1000 series of SETAR processes defined by

$$y_t = \begin{cases} \phi_{0,1} + \theta y_{t-1} + \epsilon_t & \text{if } y_{t-d} \leq 0 \\ \phi_{0,2} + \theta y_{t-1} + \epsilon_t & \text{if } y_{t-d} > 0 \end{cases} \tag{6}$$

with different samples in order to analyse the capability of the test (5) to reject this last model.

3.1 The size and the empirical power of the $SupLR$ test

To explore the size and the empirical power of this test, we generate 1000 series of sample size 100, 300 and 1000. First, to assess the size of the test we simulate a linear Gaussian AR(1) model which corresponds to the hypothesis (H_0). Rejection's percentages of $AR(1)$ processes are reported in table 1. Three combinations of mean parameters and noise's variances are considered, with $\theta = -0.5$.

Table 1: Size of the $SupLR$ test when the DGP is AR(1) for nominal size 5%

mean parameters	σ^2	100	300	1000
	0.36	0.028	0.029	0.025
$\phi_{0,2} = -\phi_{0,1} = 0.5$	1	0.027	0.020	0.021
	4	0.019	0.018	0.018
	0.36	0.024	0.017	0.020
$\phi_{0,2} = -\phi_{0,1} = 1$	1	0.026	0.020	0.020
	4	0.023	0.021	0.022
	0.36	0.021	0.018	0.021
$\phi_{0,2} = -\phi_{0,1} = 2$	1	0.023	0.019	0.020
	4	0.021	0.019	0.019

Table 1's results suggest that the size of the test (5) is not sensitive neither to the mean nor to the noise's variance. The test provides a size around 2% which means that the test underreject (H_1), whatever the sample size.

Now, to assess the power of the test we generate a two states Markov switching model (1) with $\theta = -0.5$, $p_{00} = 0.95$, $p_{11} = 0.95$ and several means and noise's variances. The results are reported in table 2. For small samples, the power depends on the noise's variance. The empirical power is higher than 0.8 except in three cases. When we use samples size greater than 300, the test has an empirical power close to 1. Table 2 shows that the power of this test depends also on the ratio,

$$|(\phi_{0,2} - \phi_{0,1})| / \sigma^2. \tag{7}$$

For instance, if we take $\phi_{0,2} = -\phi_{0,1} = 0.5$ with $\sigma = 1$ or $\phi_{0,2} = -\phi_{0,1} = 1$ with $\sigma = 2$ and $\phi_{0,2} = -\phi_{0,1} = 2$ with $\sigma = 0.6$ we observe that the ratio (7) belongs to the interval $[1, 10]$ and the empirical power of the test (5) is close to 1 and smaller in the other cases. When $\phi_{0,2} = -\phi_{0,1} = 0.5$ and $\sigma = 2$ for N=100 and N=300 the power is smaller. In this latter case, the volatility of the data is high.

This empirical study shows that the test (5) is able to recognize a Markov switching model even using small samples sizes. Nevertheless, we observe through the articles of Garcia (1998), Gong and Mariano (1997) and Coe

Table 2: Empirical power of the $SupLR$ test when the DGP is a MSAR(1) process

$(\phi_{0,1}, \phi_{0,2})$	σ^2	100	300	1000
	0.36	97	100	100
(-0.5, 0.5)	1	79.3	100	100
	4	15.5	60.1	99.3
	0.36	98.4	100	100
(-1, 1)	1	98.3	100	100
	4	78	100	100
	0.36	87.6	98.2	100
(-2, 2)	1	95.2	99.8	100
	4	99	100	100

(2002), that using real data, the results are no so evident.

In a simulation study, not reported here, our results concerning the size and the empirical power of the $SupLR$ test are close to those of Carrasco (2002) when under (H_0) we use a strong white noise.

3.2 Capability of the $SupLR$ test to reject SETAR model

Now, we simulate the SETAR process (6) in order to study the ability of the test (5) to detect if the shifts come from a MSAR model (1) or not. We use 1000 Monte Carlo realizations from samples whose sizes are equal to N=100, 200, 300, 400, 500 and 1000. We set the threshold parameter and the initial value y_0 equal to zero. Six combinations for the parameters $(\phi_{0,1}, \phi_{0,2})$ are considered. The value of the noise's variance is equal respectively to $\sigma = 0.6, 1$ and 2. All combinations are chosen in order to be sure that we have enough points in each regime. To minimise the influence of starting values we have discarded the first 200 observations. In this study, we do not investigate the influence of the autoregressive parameter θ, which is setted equal to -0.5 [2]. In all the empirical study we use a significant level 5%.

In the following paragraph, we analyse the results, reported in tables 3-6, for the empirical powers, the percentage of rejections of the null assumption, with respect of different values of the delay parameter d.

3.2.1 d=1 in (6)

1- *Results for SETAR(0)*

When the data generating process follows the SETAR process (6) with $r = 0$, $d = 1$ and $\theta = 0$, the results are reported in table 3.

[2]Garcia (1998) shows that the distribution of the test is not sensitive to the value of the autoregressive parameters.

Table 3: Power of $SupLR$ test when the DGP is SETAR (0) with d=1

$(\phi_{0,1}, \phi_{0,2})$	σ^2	100	200	300	400	500	1000	p_{00} and p_{11}
	0.36	99.3	100	100	100	100	100	$p_{00} = 0.86$, $p_{11} = 0.86$
(-0.5, 0.5)	1	74.3	97.5	100	100	100	100	$p_{00} = 0.80$, $p_{11} = 0.80$
	4	13.8	38.5	59.1	74.6	99.6	100	$p_{00} = 0.74$, $p_{11} = 0.72$
	0.36	100	100	100	100	100	100	$p_{00} = 0.95$, $p_{11} = 0.95$
(-1, 1)	1	98.3	100	100	100	100	100	$p_{00} = 0.88$,$p_{11} = 0.88$
	4	66.8	97.7	100	100	100	100	$p_{00} = 0.80$, $p_{11} = 0.80$
	0.36	45.5	69.2	82	90.5	94.5	100	$p_{00} = 0.99$, $p_{11} = 0.99$
(-1.5, 1.5)	1	99.4	100	100	100	100	100	$p_{00} = 0.94$, $p_{11} = 0.94$
	4	97.8	100	100	100	100	100	$p_{00} = 0.84$, $p_{11} = 0.84$
	0.36	3.4	9	10.2	15.7	18.4	26.4	$p_{00} = 0.999$, $p_{11} = 0.77$
(-2, 2)	1	88.7	98.5	100	100	100	100	$p_{00} = 0.98$, $p_{11} = 0.97$
	4	34.7	64.3	84.7	93.8	96.3	100	$p_{00} = 0.88$, $p_{11} = 0.88$
	0.36	99.5	100	100	100	100	100	$p_{00} = 0.93$, $p_{11} = 0.85$
(-0.8, 0.5)	1	89.4	100	100	100	100	100	$p_{00} = 0.89$, $p_{11} = 0.84$
	4	29.4	65.5	88.6	96.5	99.3	100	$p_{00} = 0.88$, $p_{11} = 0.84$
	0.36	13.8	32.4	43.3	54	63.2	82.5	$p_{00} = 0.68$, $p_{11} = 0.95$
(-1.5, 0)	1	52.6	83.5	96.8	99.3	99.7	100	$p_{00} = 0.69$, $p_{11} = 0.94$
	4	28.4	68	89.6	96.5	99.6	100	$p_{00} = 0.70$, $p_{11} = 0.82$

For a sample size larger than 200, the $SupLR$ test correctly rejects the null hypothesis. Under the alternative the $SupLR$ test is unable to recognize this SETAR process. It accepts nearly always the SETAR process although we use a statistic built to recognize the MSAR process (1). Garcia (1998, appendix.3, p.785) shows that the LR test has the same asymptotic distribution under (H_0) whatever the process that we consider under the alternative: MSAR(0) or SETAR(0). This empirical work is in phase with these results.

Now, we estimate the probabilities to be in one regime. We observe that the estimate values $\hat{p}_{00}$ and $\hat{p}_{11}$ have identical values when the means ($\phi_{0,1}$ and $\phi_{0,2}$) are symmetrical with respect to the threshold r. These value $\hat{p}_{00}$ and $\hat{p}_{11}$ increase with ($\phi_{0,1}$, $\phi_{0,2}$) and decrease with σ. These behaviours depend on the expressions $p_{00} = \Phi(r - \phi_{0,1}/\sigma)$ and $p_{11} = \Phi(r - \phi_{0,2}/\sigma)$, where $\Phi(.)$ is the c.d.f (cumulative distributive function) of the standard Gaussian distribution, see Appendix for more details.

When $\phi_{0,2} = -\phi_{0,1} = 2$ and $\sigma = 0.6$, we get a very small empirical power for the $SupLR$ test. Amongt the 1000 series generated only one hundred series present changes from one regime to another one. This is due to the important great value of the mean and the small value of the noise's variance. For these series, $\hat{p}_{00} = 0.999$ which indicates that the probability to change from one regime to another one is very small.

2- *Results for SETAR(1)*

Table 4 gives the results of the empirical power of the *SupLR* test for different sample sizes when the data are generated under model (6) using $d = 1$, $r = 0$ and different values of the autoregressive parameters. For large sample sizes, the test has no ability to differentiate between SETAR(1) and MSAR(1) models. We analyse below in more details the behavior of this test under the alternative.

Table 4: Power of *SupLR* test when the DGP is a SETAR (1) process with d=1

$(\phi_{0,1}, \phi_{0,2})$	σ^2	100	200	300	400	500	1000	p_{00} and p_{11}
(-0.5, 0.5)	0.36	22.4	36.6	50	55.6	60.4	64.2	$p_{00} = 0.79$, $p_{11} = 0.70$
	1	5.5	10.1	11.9	12.2	15.5	21.4	$p_{00} = 0.80$, $p_{11} = 0.65$
	4	2.3	3.7	4	4.9	5.7	6.7	$p_{00} = 0.82$, $p_{11} = 0.54$
(-1, 1)	0.36	96.1	100	100	100	100	100	$p_{00} = 0.85$, $p_{11} = 0.85$
	1	33.4	66.6	84.7	90.6	94.5	100	$p_{00} = 0.73$, $p_{11} = 0.73$
	4	6.2	9.3	14.7	16.3	21	25.8	$p_{00} = 0.79$, $p_{11} = 0.50$
(-1.5, 1.5)	0.36	99.9	100	100	100	100	100	$p_{00} = 0.92$, $p_{11} = 0.93$
	1	87	98.5	100	100	100	100	$p_{00} = 0.82$, $p_{11} = 0.82$
	4	16.6	33.3	45.3	57.6	70.7	93.2	$p_{00} = 0.67$, $p_{11} = 0.66$
(-2, 2)	0.36	100	100	100	100	100	100	$p_{00} = 0.97$, $p_{11} = 0.97$
	1	93.8	99.5	100	100	100	100	$p_{00} = 0.88$, $p_{11} = 0.88$
	4	34.7	64.3	84.7	93.8	96.3	100	$p_{00} = 0.73$, $p_{11} = 0.73$
(-0.8, 0.5)	0.36	35	52	57.2	58.7	59.6	65.9	$p_{00} = 0.78$, $p_{11} = 0.78$
	1	11.9	15.8	18.3	22.4	26.9	30.5	$p_{00} = 0.80$, $p_{11} = 0.70$
	4	3.6	4	4.2	5.6	8.6	9.6	$p_{00} = 0.84$, $p_{11} = 0.55$
(-1.5, 0)	0.36	54	79.7	88.8	94.6	96.1	100	$p_{00} = 0.52$, $p_{11} = 0.93$
	1	17.2	30.8	43.1	57	66.9	87.9	$p_{00} = 0.52$, $p_{11} = 0.85$
	4	6.6	7.6	8.9	11.6	15.6	20.5	$p_{00} = 0.60$, $p_{11} = 0.64$

- Assume that σ is fixed:

When the value of $\phi_{0,2}$ and $\phi_{0,1}$ are large, $\phi_{0,2} = -\phi_{0,1} = 1.5$ or $\phi_{0,2} = -\phi_{0,1} = 2$, the *SupLR* test rejects the null with high empirical power. This means that if the data are generated from SETAR(1) this test builded to recognize a MSAR(1) models accepts, with a very high probability, the SETAR(1) as a MSAR(1) model. Again, we observe that the test cannot discriminate between the two models, in particular when the data are less noisy ($\sigma = 0.6$ or $\sigma = 1$). In another hand, when the difference between the two means decreases, for instance when $\phi_{0,1}$ and $\phi_{0,2}$ are small, the test can distinguish between the two models (1) and (6). The empirical power to reject the SETAR(1), for $\phi_{0,2} = -\phi_{0,1} = 1.5$ and $\phi_{0,2} = -\phi_{0,1} = 2$ with $\sigma = 2$ is equal to 83.4% and 65.3% respectively, using N=100 observations.

- Assume that $\phi_{0,1}$ and $\phi_{0,2}$ are fixed:

Now we make varying the noise's variance σ, the results are reported in table 4. When the data are very noisy the ability of the test to discriminate between the two models (1) and (3) increases. For example, when $\phi_{0,1} = -0.8, \phi_{0,2} = 0.5$ and N=100, the rejection of the alternative of SETAR(1) model increases from 65% ($\sigma = 0.6$) to 88.1% ($\sigma = 1$) and 96.4% ($\sigma = 2$). This means that when the noise variance σ increases the data set becomes very noisy and $p_{00} + p_{11}$ decreases. Thus, the switches between the two states increase[3]. In that latter case, the process appears similar to an $AR(1)$ process. In such a case, the test rejects the alternative in most of the cases, despite the presence of changes in regimes. For example, for N=100 with $\phi_{0,1} = -0.8, \phi_{0,2} = 0.5$ we observe that $p_{00} + p_{11}$ decreases from 1.56 when ($\sigma = 0.6$) to 1.5 when ($\sigma = 1$) and then to 1.39 when ($\sigma = 2$).

3.2.2 d=2 in (6)

The table 5 gives the empirical powers of the $SupLR$ test when the Data Generated Process is a SETAR(0) model with $d = 2$ in (6).

The results show that when the sample size is large (N=1000) the test has a reasonable empirical power, except for a few combinations. When the value of the mean increases, the power of the test changes: when the noise's variance is fixed ($\sigma = 1$ or $\sigma = 2$) and the mean increases, the power of the test increases too. These latter simulations show that the ability of the test to discriminate between a MSAR(0) and SETAR(0) models is larger in small samples, specially when the data set is very noisy.

Two series seem to have a specific behavior, when $\phi_{0,2} = -\phi_{0,1} = 0.5$ with $\sigma = 2$ and $\phi_{0,2} = -\phi_{0,1} = 2$ with $\sigma = 0.6$. For these two processes the empirical power is very low and the ratio (7) attains respectively its minimum and maximum values. Thus, when the ratio (7) is inside the interval [1, 10], the power is high, but as soon as the ratio attains the boundary of the interval [1, 10], the frequency of rejections of the null hypothesis becomes very small.

The table 6 provides the empirical power of the $SupLR$ test for the Monte Carlo simulations when the data are generated with a SETAR(1) model using d= 2. Here, the test has a good empirical power even if the sample size is small. This means that we are not able to discriminate between the two switching models.

[3] p_{00} and p_{11} are the probabilities of staying in the same regime

Table 5: Power of *SupLR* test when the DGP is a SETAR (0) model with d=2

$(\phi_{0,1}, \phi_{0,2})$	σ^2	100	200	300	400	500	1000	p_{00} and p_{11}
(-0.5, 0.5)	0.36	36.3	72.1	87.9	96.7	99.2	100	$p_{00} = 0.94$, $p_{11} = 0.94$
	1	14.3	34.3	50.8	67.7	80.4	99	$p_{00} = 0.91$, $p_{11} = 0.91$
	4	4.1	6.7	11.1	17.4	24.8	53.1	$p_{00} = 0.88$, $p_{11} = 0.64$
(-1, 1)	0.36	59.7	90.9	97.7	98.4	99.1	100	$p_{00} = 0.94$, $p_{11} = 0.94$
	1	45.4	79.7	93.4	98.7	99.7	100	$p_{00} = 0.97$, $p_{11} = 0.96$
	4	13.3	34.3	55.2	69.6	79.1	99	$p_{00} = 0.90$, $p_{11} = 0.90$
(-1.5, 1.5)	0.36	28.7	51	64.6	74.3	92.5	100	$p_{00} = 0.73$, $p_{11} = 0.57$
	1	57.7	86	94	98.8	99.5	100	$p_{00} = 0.97$, $p_{11} = 0.96$
	4	34	64.3	83.2	91.6	97.1	99.6	$p_{00} = 0.93$, $p_{11} = 0.93$
(-2, 2)	0.36	3.7	8.2	12.8	23.3	29	38.2	$p_{00} = 0.69$, $p_{11} = 0.85$
	1	57.7	87	97.5	100	100	100	$p_{00} = 0.64$, $p_{11} = 0.53$
	4	45.6	79.9	93	98.1	100	100	$p_{00} = 0.95$, $p_{11} = 0.91$
(-0.8, 0.5)	0.36	44.4	79.3	95.5	98.1	99.9	100	$p_{00} = 0.93$, $p_{11} = 0.95$
	1	20.8	52.7	73.3	87.1	96.2	100	$p_{00} = 0.90$, $p_{11} = 0.93$
	4	5.8	14.6	22.2	30.6	44.2	78	$p_{00} = 0.89$, $p_{11} = 0.88$
(-1.5, 0)	0.36	5.5	12.3	23.4	29.7	30.8	54.6	$p_{00} = 0.73$, $p_{11} = 0.98$
	1	10.6	32.5	47.4	63.8	75.3	99	$p_{00} = 0.84$, $p_{11} = 0.96$
	4	12.6	14	27.3	36.6	43.9	82.1	$p_{00} = 0.86$, $p_{11} = 0.90$

For small sample size, N=100 observations, the power is not sensitive to the mean and the noise's variance σ. The empirical power is around 60%, except for the two series that have the lower and higher values of ratio $|(\phi_{0,2} - \phi_{0,1})| / \sigma^2$. In these two cases the power is 16% and 47.7% respectively.

When $p_{00} + p_{11} > 1$ the probability to shift from one regime to another one is low. Then we stay more time in the same regime and the empirical power should be high. Table 6 shows an opposite result: when the estimated value of $p_{00} + p_{11}$ is around 1, the data change often from one regime to another one.

4 Applications

In economic and financial domains, two particular series have been widely examined by researchers to justify the existence of shifts and change between one regime to another one: The growth rate U.S GNP and the US/UK exchange rate.

Hamilton (1989) proposes a MSAR(4) model to modelize the growth rate U.S GNP. Hansen (1992) doubts in Hamilton MSAR(4) model and proposes a constrained model in which he allows the intercept, slope parameters, and error variance to shift between the 'states'. Garcia (1998) shows that there is no evidence for MSAR(4) model for the U.S GNP growth rate. In another

Table 6: Power of $SupLR$ test when the DGP is SETAR (1) with d=2

$(\phi_{0,1}, \phi_{0,2})$	σ^2	100	200	300	400	500	1000	p_{00} and p_{11}
	0.36	73.4	96.7	99.5	100	100	100	$p_{00} = 0.60$, $p_{11} = 0.60$
(-0.5, 0.5)	1	50.1	81	96.1	98.6	99.9	100	$p_{00} = 0.67$, $p_{11} = 0.67$
	4	16	36.4	51.9	68.5	79.9	98.2	$p_{00} = 0.66$, $p_{11} = 0.64$
	0.36	66.8	92.5	97.8	98.1	99.4	100	$p_{00} = 0.65$, $p_{11} = 0.47$
(-1, 1)	1	74.4	95.5	99.5	99.9	100	100	$p_{00} = 0.55$, $p_{11} = 0.55$
	4	50	83.9	94.6	99.8	99.9	100	$p_{00} = 0.66$, $p_{11} = 0.6$
	0.36	66.3	90.7	95.2	98	99.2	100	$p_{00} = 0.39$, $p_{11} = 0.37$
(-1.5, 1.5)	1	66.6	94	99.1	99.6	99.8	100	$p_{00} = 0.49$, $p_{11} = 0.46$
	4	66.9	96.1	99.4	99.8	99.9	100	$p_{00} = 0.61$, $p_{11} = 0.61$
	0.36	47.7	77.6	93.3	97.9	100	100	$p_{00} = 0.39$, $p_{11} = 0.37$
(-2, 2)	1	65.3	92.1	96.5	99.5	100	100	$p_{00} = 0.43$, $p_{11} = 0.40$
	4	73	96.8	99.7	100	100	100	$p_{00} = 0.57$, $p_{11} = 0.57$
	0.36	74.7	97	98.2	99.9	100	100	$p_{00} = 0.62$, $p_{11} = 0.55$
(-0.8, 0.5)	1	63.4	93	99.3	99.8	100	100	$p_{00} = 0.68$, $p_{11} = 0.66$
	4	29.2	53.8	74.3	86.4	95	100	$p_{00} = 0.66$, $p_{11} = 0.65$
	0.36	76.7	99.9	100	100	100	100	$p_{00} = 0.94$, $p_{11} = 0.88$
(-1.5, 0)	1	71.5	98.1	99.7	100	100	100	$p_{00} = 0.76$, $p_{11} = 0.70$
	4	33.1	65.6	81.2	93	97.2	100	$p_{00} = 0.68$, $p_{11} = 0.66$

hand, this former series have been modelized by nonlinear models whose the SETAR model. For instance, Potter (1995) proposes a SETAR(5) model without the third and fourth lags and Hansen (1996) doubts that this SETAR model can capture changes in this U.S GNP growth rate series.

The US/UK exchange rate has been studied by Engle and Hamilton (1990). They propose a MSAR model without autoregressive order. Engle and Kim (1994), Bollen, Gray and Whaley (2000) and Cheung and Erlandsson (2005) amongt others explore also the possibility of adopting MSAR model for these series.

In economic and financial theory these non-linearities can be explained by many facts. For U.S GNP growth rate a lot of researchs argue the presence of asymmetry behavior and suggest that during great depression shocks to GNP are more persistent. For the US/UK exchange rates, the presence of non-linearities and the adoption of switching model is justified mainly by the heterogeneity of the participants in the foreign exchanges markets, transaction costs, and the differences between domestic and foreign monetaries and fiscal policies. Also, the rigidities of certain markets and the presence of chartists and fundamentalists in the foreign exchanges markets induce differences in opinions and in expectations.

Here, we propose to use the previous test (5) to see if it possible to justify the use of these two models for these series.

4.1 The data

We use a quarterly data set for the U.S real GNP, over the period january 1947 to april 2005. The use of the quarterly data provides a sample of 232 points. For the US/UK exchange rates we use two frequency data sets, monthly and quarterly data. In the first case we have 176 points, from january 1986 to september 2000, and 58 points in the latter case from january 1986 to october 2000. The GNP data set is provided from the website of Federal Reserve Bank at St Louis in USA and the exchange rate data set comes from Datastream Base. In order to make the data stationnary we use the following transformation $100*[log(X_t)-log(X_{t-1})]$, where X_t represents the observed data.

4.2 Results

First, we start estimating a SETAR process for the GNP growth rate and the US/UK exchange rate data. The results are provided in table 8, when we use d=1 in model (6).

Here we are interested by the value of the expression (7) given in the last line of the table 7. In all cases, the value of the ratio is outside of the interval [1,10]. Following the results given in paragraph 3, this means that the test has to discriminate between the two models: SETAR or MSAR. This means that the Sup LR test rejects the SETAR process and accepts a linear model for these series.

Table 7: Estimates of SETAR model of the U.S GNP growth rate (Quarterly data) and the US/UK exchange rate (Monthly and Quarterly data)

Parameter	$U.S\ GNP$	$US/UK(M)$	$US/UK(Q)$
	SETAR(1)	SETAR(0)	SETAR(0)
$\phi_{0,1}$	0.614	-0.551	0.044
	(0.1046)	(0.362)	(0.801)
$\phi_{0,2}$	1.172	0.446	-1.384
	(0.119)	(0.300)	(2.163)
θ	0.598	-	-
	(0.049)		
σ^2	0.931	9.447	33.607
	(0.92)	(0.082)	(0.973)
r	0.740	0.049	3.953
d	1	1	1
$\lvert(\phi_{0,2}-\phi_{0,1})\rvert/\sigma^2$	0.599	0.105	0.042

Now we simulate, using the previous estimated model, a SETAR processes for the GNP and US/UK (M,Q). We do 1000 realizations using N=100, 200, 300, 400 and 500 samples sizes and we compute the expected empirical power of the $SupLR$ test for each experiment. The results are given in table 8. In all cases, we reject the alternative with a high power. This means that a SETAR(1) model with d=1 is innappropriate for these data sets. Thus now, we try to adjust a Markov switching process for these data sets.

Table 8: Power of the $SupLR$ test when the DGP is the estimates parameter of real data ($U.S\ \ GNP$, $US/UK(M)$ and $US/UK(Q)$)

	100	200	300	400	500
$U.S\ \ GNP$	3.4	3.6	4.5	5.4	6.4
$US/UK(M)$	6.3	14.4	22.3	35.3	44.7
$US/UK(Q)$)	1.8	3.4	4.3	7.4	8.6

In table 9, colums 2 and 3, we provide an estimate of the real growth rate GNP data using an AR(1) linear model and an MSAR(1) model. We provide also the log-likelihood function under each model. The value of the likelihood ratio statistic is 8.938, this value is greater than Garcia's 95% asymptotic critical value of 8.68 for the US GNP. Thus, we accept the alternative of MSAR(1) model against the null of linear AR(1) model. The value of the ratio in (7) is between 1 and 10 for this series. This means that, if we use the estimate value of the U.S real GNP to generate artificial data under an MSAR(1) model, the empirical power of the $SupLR$ test will be close to 1. Simulation experiments results confirm this intuition. Their power is equal to 80.2%, 99.8% and 100% for a sample size equal to N=100, N=200 and N=300 respectively, when the DGP is the estimated model for the U.S real GNP provided in table 9. These results induce to retain an MSAR(1) model for the U.S real GNP data for the period under study.

Table 9: Estimates of MSAR model for the U.S GNP growth rate (Quarterly data) and the US/UK exchange rate (Monthly and Quarterly data)

Parameter	$U.S\ \ GNP$		$US/UK(M)$		$US/UK(Q)$	
	AR(1)	MSAR(1)	AR(0)	MSAR(0)	AR(0)	MSAR(0)
$\phi_{0,1}$	0.563	-1.214	-0.018	-8.296	-0.0743	-13.084
	(0.095)	(1.214)	(0.231)	(1.865)	(0.739)	(13.68)
$\phi_{0,2}$	-	2.165	-	8.567	-	13.855
		(0.1021)		(1.679)		(12.40)
θ	0.332	0.391	-	-	-	-
	(0.073)	(0.071)				
p_{00}	-	0.264	-	0.967	-	0.950
		(0.222)		(0.040)		(-)
p_{11}	-	0.9618	-	0.205	-	0.227
		(0.023)		(1.101)		(4.97)
σ^2	0.938	0.668	9.42	6.687	31.764	20.813
	(0.092)	(0.082)	(0.325)	(0.214)	(1.231)	(0.973)
Log-Likelihood	-101.205	-96.736	-284.002	-274.483	-129.267	-125.823

In the table 9, colums 4-7, we provide an estimate of the monthly and quarterly US/UK exchange rate data. Under the null the series are modelized by a random walk and by a Markov switching model under the alternative. The log-likelihood function is provided in the last line. The statistic of the likelihood ratio for the monthly data is equal to 19.38. This value is greater than the asymptotic critical value, 10.89. Thus, for this US/UK series we accept a Markov switching model against a random walk process. Using quarterly data (58 observations) the likelihood ratio statistic is equal to 6.9. This value does not excess the Garcia's 95% asymptotic critical value, 10.89. Thus, we accept a random walk specification (linear model) against a Markov switching specification for the quarterly data. We are not surprised by this result because the sample size and the frequency of the data plays an important role for the choice of the model.

Now, we propose a simulation using the estimated MSAR(0) for US/UK exchange rate, whose parameters are given in table 7. We make 1000 realisations for different sample sizes, N=100, 200, 300, 400, 500, in order to study the influence of the sample size on the empirical power of the *SupLR* test. The simulations results show that the power of the test is equal to 85% when N=100, and close to 1 when the sample size is $N \geq 200$. We remark also that there is small difference between the empirical power for the monthly and the quarterly data set. This comes from the values obtained for the ratio (7). This ratio belongs to [1,10] and the parameter $\phi_{0,1}$ and $\phi_{0,2}$ are symmetric around 0. The distinction between the two regimes are evident despite of the large value of the noise's.

In fine, we retain a MSAR(1) model for US GNP growth rate, a MSAR(0) model for the US/UK monthly data and when we use the quarterly data set for the US/UK we retain a random walk.

5 Conclusion

In this paper, we assess the power of *SupLR* test (5), constructed for a MSAR model, when the underlying simulated process is a SETAR model. We explore the sensitivity of the empirical power with respect to the mean, the variance and the delay parameter d of the model (6). The results show that it is very difficult to discriminate between the SETAR model and the MSAR model, in particular using large sample sizes. For small sample sizes the *SupLR* test allows to doubt about the nature of the changes in regimes. It seems that the higher and the lower value of the ratio $|(\phi_{0,2} - \phi_{0,1})| / \sigma^2$ reduce the power of the *SupLR* test for small samples, but in large samples this test still has good power.

Tow specific cases are of interest, when the ratio is lower than 1 and when it is greater than 10. For these two extreme cases the test has a correct power to discriminate between different types of switching models. We apply this approach to the US GNP growth rate and the US/UK exchange rate and we show that a Markov switching specification is more appropriate for the US GNP growth rate and for the US/UK exchange rate (Monthly data). For the US/UK Quarterly data we accept a random walk specification because we have only a few observations, 58 points.

Here, we only consider applications of the $SupLR$ test for models governed by their means. Other extensions can be, for instance, developed using other switching models, like the so-called sign model and the smooth threshold autoregressive model (STAR), Granger and Teräsvirta (1993,1999).

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Appendix

Consider the simple first-autoregressive SETAR model:

$$y_t = \phi_{0,1} + (\phi_{0,2} - \phi_{0,1})s_t + z_t \tag{8}$$

with $z_t = \theta z_{t-1} + u_t$, and

$$s_t = I\{y_{t-d} \leq r\} = \begin{cases} 0 & \text{if } y_{t-1} \leq r \\ 1 & \text{if } y_{t-1} > r \end{cases}$$

This model rewrites as:

$$y_t = \phi_{0,1} + (\phi_{0,2} - \phi_{0,1})I\{y_{t-1} \leq r\} + \theta(y_{t-1} - \phi_{0,1} + (\phi_{0,2} - \phi_{0,1})I\{y_{t-2} \leq r\}) + u_t$$

Let be

$Pr(s_t = 0|s_{t-1}, Y) = Pr(y_{t-1} \leq r|s_{t-1}, Y)$

$= Pr(\phi_{0,1} + (\phi_{0,2} - \phi_{0,1})I\{s_{t-1} = 0\} + \theta(y_{t-2} - \phi_{0,1} + (\phi_{0,2} - \phi_{0,1})I\{s_{t-2} = 0\}) + \epsilon_{t-1} \leq r)$

$= Pr(\epsilon_{t-1} \leq r - \phi_{0,1} - (\phi_{0,2} - \phi_{0,1})I\{s_{t-1} = 0\} - \theta(y_{t-2} - \phi_{0,1} - (\phi_{0,2} - \phi_{0,1})I\{s_{t-2} = 0\})$

$= \Phi(\frac{r - \phi_{0,1} - (\phi_{0,2} - \phi_{0,1})I\{s_{t-1}=0\} - \theta(y_{t-2} - \phi_{0,1} - (\phi_{0,2} - \phi_{0,1})I\{s_{t-2}=0\})}{\sigma})$

$$= Pr(s_t = 0|s_{t-1}, y_{t-2}) \neq Pr(s_t = 0|s_{t-1})$$

Therefore for the SETAR(1) the probability $Pr(s_t = 0|s_{t-1}, Y)$ depends on y_{t-2} then the process $I\{y_{t-d} \leq r\}$ is not a Markov chain.

- When $\theta = 0$ we see that he model (8) becomes a particular case of Markov Switching model because $Pr(s_t = 0|s_{t-1}, Y) = Pr(s_t = 0|s_{t-1})$. For model (8) we get:

$$\mathbf{P} = \begin{bmatrix} p_{00} & 1 - p_{00} \\ 1 - p_{11} & p_{11} \end{bmatrix} = \begin{bmatrix} \Phi(\frac{r-\phi_{0,1}-\theta(y_{t-2}-\phi_{0,1})}{\sigma}) & \Phi(\frac{\phi_{0,1}+\theta(y_{t-2}-\phi_{0,1})-r}{\sigma}) \\ \Phi(\frac{r-\phi_{0,2}-\theta(y_{t-2}-\phi_{0,2})}{\sigma}) & \Phi(\frac{\phi_{0,2}+\theta(y_{t-2}-\phi_{0,2})-r}{\sigma}) \end{bmatrix}.$$

- When $\theta = 0$ and $\phi_{0,1} = -\phi_{0,2}$ in (8), this induces to:

$$\mathbf{P} = \begin{bmatrix} \Phi(\frac{-\phi_{0,1})}{\sigma}) & \Phi(\frac{\phi_{0,1})}{\sigma}) \\ \Phi(\frac{\phi_{0,1}}{\sigma}) & \Phi(\frac{-\phi_{0,1})}{\sigma}) \end{bmatrix}.$$

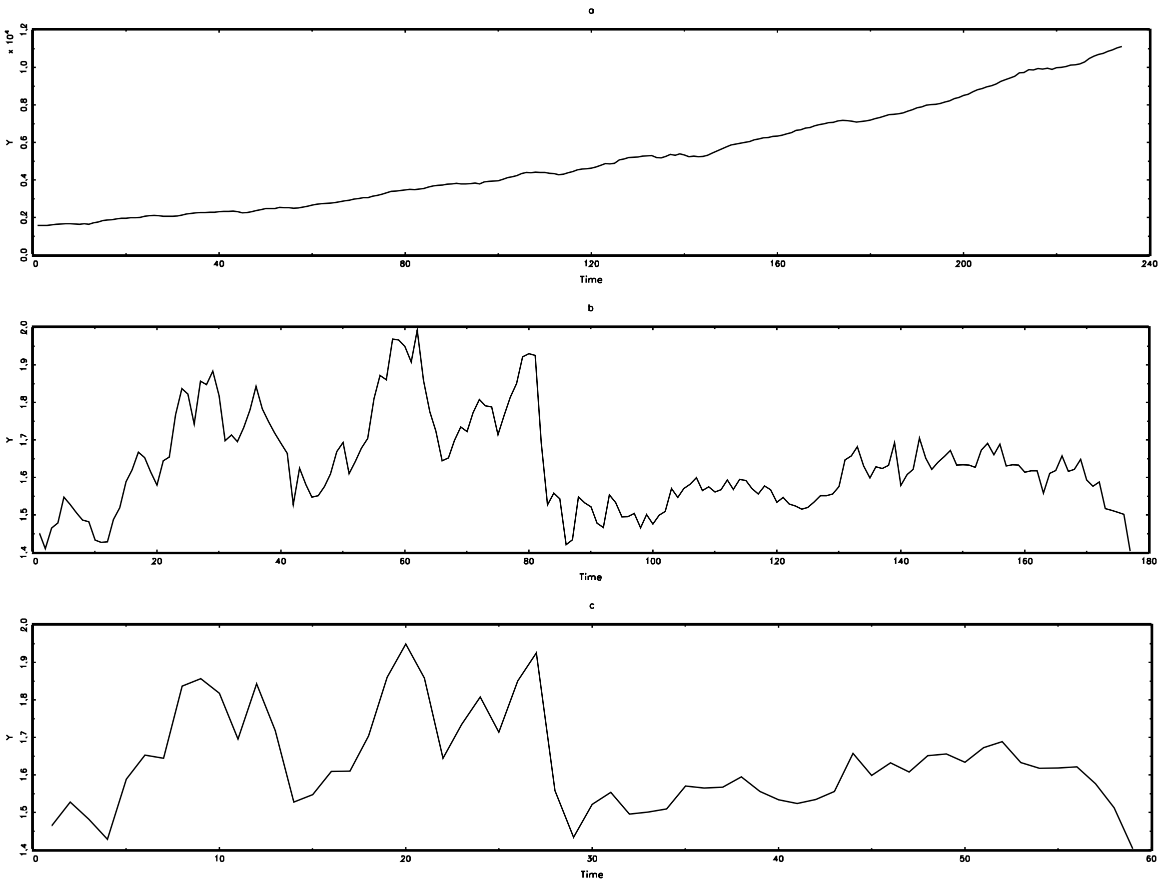

Figure 5: Trajectories of (a) U.S real GNP data, (b) Exchange rate of the US/UK monthly data, (c) Exchange rate of the US/UK quarterly data.