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Empirical Contributions to Optionpricing

analyzing Black and Scholes and other Models

by Gerhard Schroeder

A. Introduction

There are two major approaches in option pricing which will be referenced in the following paper: One approach analyzes the past and tries to project from into the future. The other one proceeds from mathematical[2] financial market models like the models based on the Brownian Motion or more generally on kinetic processess described by the Wiener processes mathematically.

Both beginnings try to describe and above all predict also far refined corollary assumptions and techniques the real movements of the financial markets.

A general evaluation model is presented, which - ex post - allows a comprehensive evaluation of verious options and the examination of evaluation models.

Some samples analyzed during 1998 may be still of interest: Figure 11: SFR-Options, Strike 1,23 DEM e.g. is a rare sample where the course crosses four times the line of the strike price. The results of analyzing fictitious options (Figure 7: Evaluation errors of 469 Options (1998 until 9/1997)) is valid at all times[3]. Test datas are sent gladly by email: gschroeder@foni.net.

B. Suggested Model Systematics

The most important rules are discussed and used for the construction of fictitious realistic courses like they can be observed in reality. The Stochastic Volatility (SV) seems to be an important rule to ensure realistic formats. A sparing use of MR[4] allows to generate precisely a volatility intended.

B. I. The Brownian Motion as a Model Blueprint

Professional microscopes allow to observe the Brownian Motion. The surrounding molecules of the liquid preparing a particle for examinations under the microscope move in random directions and are knocking the pollen grain in random directions. Since it is uniformly surrounded by this bombardment, the effects tend to neutralize over time. The idea is that stock orders create similar impulses regarding the course[5] being the equivalent to the particle however in an one dimensional process[6] over time while the original Brownian motion is two dimensional.

Financial markets processes with special timing and phases for order gathering and course fixing are different - not to mention special rules for opening and closing. The order size varies[7] and doesn't correspond to the homogenuous molecules of the preparation liquid.

The Brownian motion allows jump processes also within the range of the lognormal distribution that can be proven for this cinetic phenomenon. However, in case of financial markets jumps are probably reactions to news and exogene foundet respectively. Actual courses thus are better represented by leptocurtic courses than by the lognormal distribution.

To describe the „laws“ of financial markets in theoretical modell it requires obviously stochastic subprocesses with the SV processes beeing the most important ones in controlling the volatility and the bandwidth.

B. II. Suggested Systematics of advanced Capital Market Models

To cover all known properties a model should include the following stochastic properties:

Figure 1 : Model Sytematics

B&S	**Base Distribution Process**	lognormal, exp.-hyperbolical, historical or any other		**Sign Bias Effects**
	Subprocesses:			
	Stochastic Volatility	excessive Volatility non excessive	Clustering ↓ Mean Reversion | Non MR ↑ Volatiliy within Mean Range	
	Jump Diffusion	Conformity	Non Conformity	
	Stalagmites	Formats	„White Noise“	
	Parameters or Subprocesses			
B&S	**Interest Rates**	Could be as complex as to Volatility		
	Bandwidth	stochastic limit, related to Volatility or not		
	Drift / Trend	Only linear trends make sense		
	Saisonal Effects	as a yearly, monthly, weekly pattern		
	Trading Rules	Rules covering the price fixing process stock exchange rules and index definitions		

The Black and Scholes Model (B&S) covers the base process and interest rates only. This may explain the limited power in representing real markets.

SV, Clustering and Jump Processes are important model contributions and could be considered as 'overtones'. Jump subprocesses have a smoothing effect, while during Stalagmite phases returns above normal can be observed. Both refer primarely to the underlying process.

They are subject of the advanced models yet until today there is no mathematical model able to handle all these phenomena in common. The implementation of two or three rules already causes quite some mathematical complexity. To avoid unrealsistic courses a bandwidth control based also on the volatility and reflecting trends (drifts) in case of indices should be included.

This is a chance for experimental research to test and optimize a model concept even befor a mathematical formula is available.

B. III. Stochatics of Model Properties

Except the volatility per centages all other figures indicate how many weeks out of 52 weeks relate to the particular effect. Except the non-excess figures all other figures are not exclusive. I. e. a bandwidth violation would very probably coincide with a jump etc.

Figure 2: Profile of Subprocesses

	Jump	Double	MR	Clustering	Bandwidth	Non Excess	Average
Definition	**> 2,8%**	Jump	**> 0,25%**	**< 0,25%**	Xt-26 +hVol	**2,00%**	Volatility
USD	**1,4**	**0,35**	**1,8**	**4,5**	**6,7**	**10,1**	**10,1%**
DOW	**3,4**	**1,65**	**2,1**	**4,2**	**5,9**	**8,7**	**14,5%**
NIKKEI	**4,3**	**2,17**	**2,8**	**4,8**	**6,3**	**6,2**	**15,7%**
DAX	**4,8**	**2,25**	**2,5**	**4,2**	**6,9**	**7,8**	**15,8%**
NASDAQ	**2,5**	**1,38**	**2,1**	**2,3**	**4,5**	**5,0**	**16,4%**

C. Test Methodology

By experimental economic research is meant in addition to stochastic analysis the introduction of fictitious options and *cloning* of artificial financial values. All derived values such as volatility, evaluation results, exercise results etc. are computed in a rolling procedure during the entire period from 1988 to 1997. The last values in 1997 change thereby because of the missing advance gradually from actual, ex post determined values into historical values.

C. I. Introduction of fictitious Options

C. I. along historical (weekly) quotations fictitious options are emitted, which have the respective spot as basis (thus "at the money"). These options are exercised when expiring and the present value of the results determined. Interest rate and running time can be varied.

For calls (puts) "in the money" the proportional deviation of the option basis from the spot can be set to - 5% (+ 5%). ("out the money" in reverse: with + 5% (- 5%)). With the comparison with actual options, each demanded moneyness in per cent can be entered by the course. As evaluation result and yardstick for the comparison the average value of practice proceeds from approximately 500 options (including the unsuccessful) was specified. The hit rate was indicated at the same time. The means of proceeds is called empirical value in the future.

The model of Black and Scholes, presupposing options of European type was examined according to "European" rules . It becomes however with the option examples in section 0 ignored that the theoretical models suffer partially very restrictive acceptance, which would partly forbid application to shares, indices, rate of exchange etc. or option types (American or European) (e. g. "fair value"[8]). It can be agreed upon to rate only the cases as hits with those the change of course a threshold value, about one per cent, exceeds.

C. II. Test of Evaluation Methods

To test a hypothesis, that a given distribution fits into a normal distribution in general techniques like the X^{2}-Test can be used. Then the potential range of an underlying is split into classes and their frequencies compared with correspondent classes e. g. of lognormal function. A test value is computed allowing a statement that hypothesis that a distribution can or cannot be considered lognormal on a level of confidence set to 0,1, 0,05 or 0,01 typically. This type of test can be used to eliminate different aproximations. However, ist doesn't contribute to option pricing for two reasons:

1. The classes have to be designed in a way that the frequencies do not fall below 5. This would mean to have large ranges for classes for the exerior flanks of the distribution that on the other side contain the most profitable cases to evaluate the options.
2. The key question is: What impact does a particular distribution have on the related option prices?

It's a matter of taste to say both distributions are different or remarkably close. In terms of option pricing they differ significantly:

The value of fictitious options emitted weekly are determined here using the formula of Black and Scholes. Then they are compared with results of all 500 options when exercised. The volatility was determined ex post over 13 weeks and scaled on one year[9].

Some real options used in addition as comparison are American type. Practically they are exercised rarely. Usually all observed options - even if "deeply in the money" - show courses slightly higher than their theoretical exercise results.

Different interest rates can be taken to determine the present value when exercised and for the respective method, in order to be able to differentiate between interest rates without risk and others. The interest rate was selected in such a way - without consideration of the modelltheorischen defaults - that in case of deviations a minimum was obtained. Thus resulted, 6 per cent "interest without risk" in the case of DAX and DOW as well as 3 per cent in the case of fictitious dollar rate of exchange options, 4.5 per cent in the case of the log-normal distributions. With actual currency options with zero per cent the best approximation was obtained.

One could object, that a quotation beeing a parameter of most of the known option pricing models[10], undergoes reasonnable changes and in case of indexes like DJ and DAX follows a a trend over a period of several years. However, these effects are taken into account by comparing them synchronuously. In section „Comparison: Underlying vs Option Chances“ a possibility standardize options will be suggested (100 units, 1 DEM, Points etc.).

C. III. Measurement of Autocorrelation Frequencies

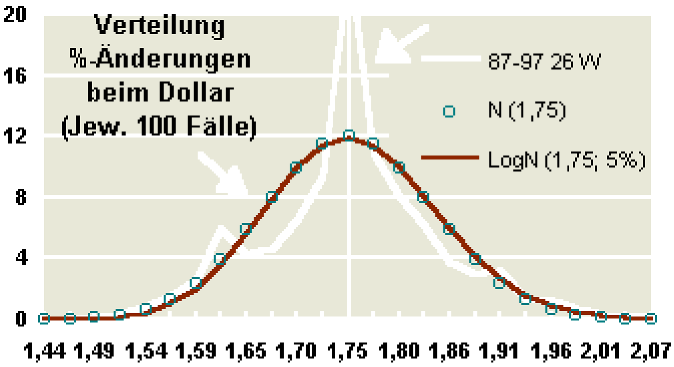
By extensive computing the coerrelation between a x-week period of quotation and evrey other x-week period will be measured and the number of cases counted if the coerrelation exceeds a given limit - e.g. 0,949.

The first set of the rolling starts at week 1 and moves forward through the complete sample of 500 weeks in this case until the last week minus x. In a corresponding row the number of hits will be saved[11].

C. IV. Measurements of Distribution Differences

To measure the phenomenon of deviating real quotations with respect to ideal type assumed mathematical distributions in statistics Curtosis[12] , Skewness as well as Chi-Square-Tests are offered. However, the context of these analysis rather suggests to use the pricing quality itself as a comparison method and either accept evaluation differences or disapprove them. The lognormal distribution (black) in Figure 3 corresponds with the distribution of weekly Dollar exchangerates (white) as far as they were computed using 26 weeks (7,1% / Squareroot(2). The symmetric normal distribution (O) shows light right oriented srewness of the lognomal distribution (dark bold line). Multiplieing the probailities with the present values of individual exercise returns of the sample above (Options with a exercise price 1,75 DEM shows the leverage effect:

Figure 3: Evaluation Effects of Distributions



While the deviation seems to be rather marginal wenn looking at Figure 3 the effect on option pricing ist significant: Comparing the normal and lognormaler distribution a 9 per cent under-(10 per cent over-)-evaluationv of calls (puts) can be observed. The deviation between the historical and the lognormal distribution in the sample is an overevalution of calls arround 10 per cent % and an unerevaluation of puts around 3 per cent.

Figure 4: Comparisons of Distributions via the Price

Distribution 26 Weeks	Calls Euro	Puts Euro	Hits Calls	Puts	w/o Return	Curt-osis	Skrew-ness	cross correlation t.	ln.	n.
historical	**3,45**	**3,22**	40	37	23	7,24	2,38	1	0,78	0,85
lognormal	**3,79**	**3,12**	46	42	12	-1,32	0,49		1	0,93
normal	**3,45**	**3,45**	44	44	12	-1,35	0,52			1

The significant peak around the mid point (curtosis) of the historical Dollar distributiom results in lower returns on the left side lowering put prices consequently.

The skewness supports higher call prices. For option pricing both terms are not productive.

The hypothesis, the Dollar distribution would be lognormal verteilt, is not supported by a Chi-square test at a significance niveau of $\alpha = 0,1$.

Dollar returns and lognormal correlate positive with a coefficient r = 0,78. However, it is suggested to speak of strong ("stramm") correlation only if r > 0,95.

C. V. Fictitious Courses with a given Distribution

A course over period of 520 weeks (in one case 52 weeks) is to be generated. It is done by drawing randomly and applieing returns of perfectly generated lognormal return numbers. The fictituous course starts with its mean which might be an actual course.

It was also made certain that on 10 years the log-normal distribution counted was not changed by further 13 quotations needed for the advance. For realistic prognosis one would use more plausible distributions - for instance the own distribution of returns of a quotation over the last years, corrected by specific trends.

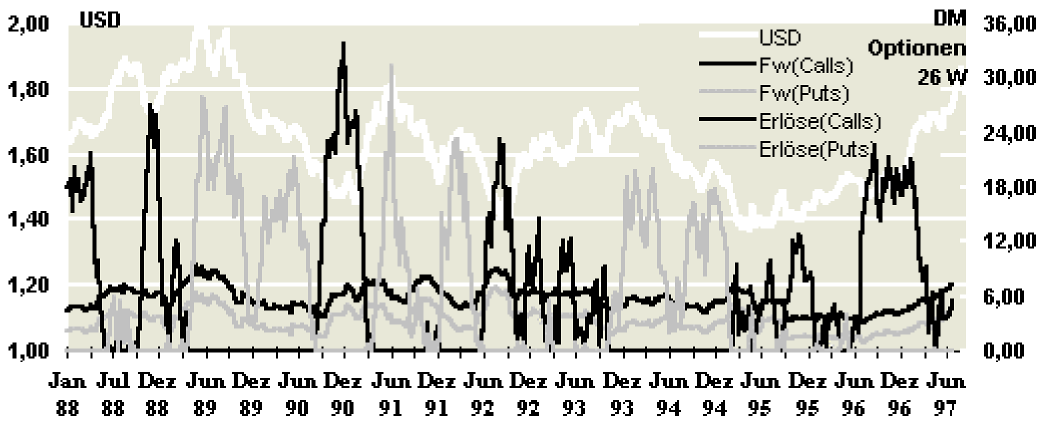
D. Empirical Results Part 1

Fictitious options permit a more comprehensive analysis of evaluation beginnings, than it would be possible with Optionssamples.

D. I. Fictituous Dollar Options

Figure 5 shows, how the fair value stands after Black and Scholes (lines within the range of 0 to 6 DM) in relation to individual practice proceeds and to their average value. The deviation between the average value of all "fair values" to that of practice proceeds is selected as yardstick for the correctness of an evaluation method waiting for comparison

Figure 5: Fictitious Options - Dollar



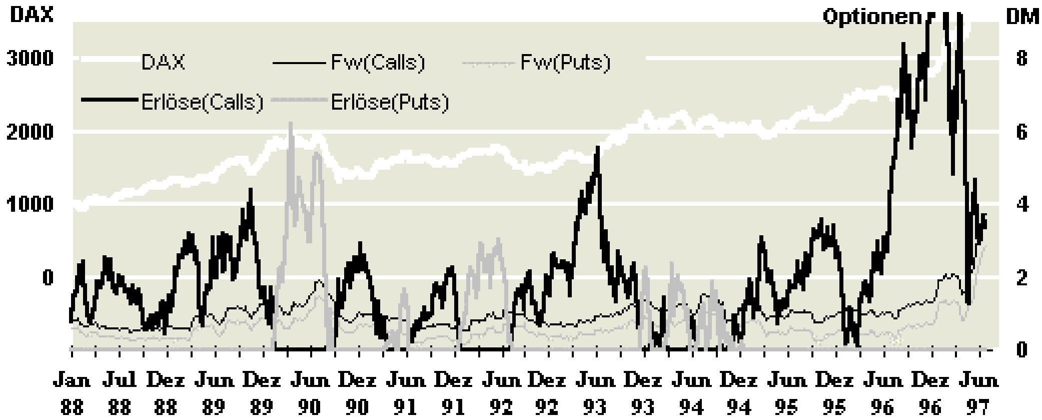
The phenomenon of positive average yields (or trends of the Underlying) is considered in the model by Black and Scholes only in the context of the log-normal distribution. The middle "fair value" for calls, (black line, puts grey) amounts to here 5.70 DM (3.30 DM), that from proceeds (strongly deflecting lines) 5.35 DM (5.42 DM). There are phases of up to one and a half years, in which with dollar calls or puts (nearly) no practice proceeds could be obtained, while the fair values vary also in these periods only slightly around their average value.

D. II. Fictitious DAX Options

With DAX options the same notes are to be made as for the formation of means i.e. of the expectancy value. However the average yield of many years DAX of over 11 per cent causes an increase of the hits with calls (69% to 31%). The "fair values" underestimate calls (-43%) and overestimate puts (+29%). (to further measurements s. tab. 2).

Volatilitypeaks (maxima of the „Fair-Value"-Curves (e. g. July 1990, volatility at 30%), show rather the change of call- and put chances instead of their actual chances. During mid of 89 until beginning of 91 the volatility approaches a niveau of 22% (Ø = 15%). This period includes profitable DAX-Puts in the mid range only, however,worthless DAX calls in the same time.

Figure 6: Fictive Optionen - DAX



The increasing call returns during Dec. 96 are a consequence of the increasing DAX-returns. The volatility here is beyong mean. (See paragraph „Comparison: Underlying vs Option Chances, page 15 ") also. Put returns can be observerd in 1990 (1.-3. Quarter), 1991 (3. Q.) 1992 and 1994. The maximum returns of calls around June 1993 is not reflected by the volatility reflektiert (here max 15% = ∅).

D. III. Examinations at quoted Options (DAX, USD, SRF)

Four options analyzed from expenditure to practice with running times of ½ year (DAX), 1 and 2 years (USD) as well as 2½ years (SFR) [13].

The following DAX option corresponds to the fictitious DAX options (above) in the running time. The DAX course was illustrated without scale.

Call prices are first closely at the "fair values" like empirical values. In the last four months however overestimation increases with the prices as constant at the "fair values". The put prices are close to the "fair values" until two months before maturity exceeding them and the "empirical values" in accordance with section C. I. .

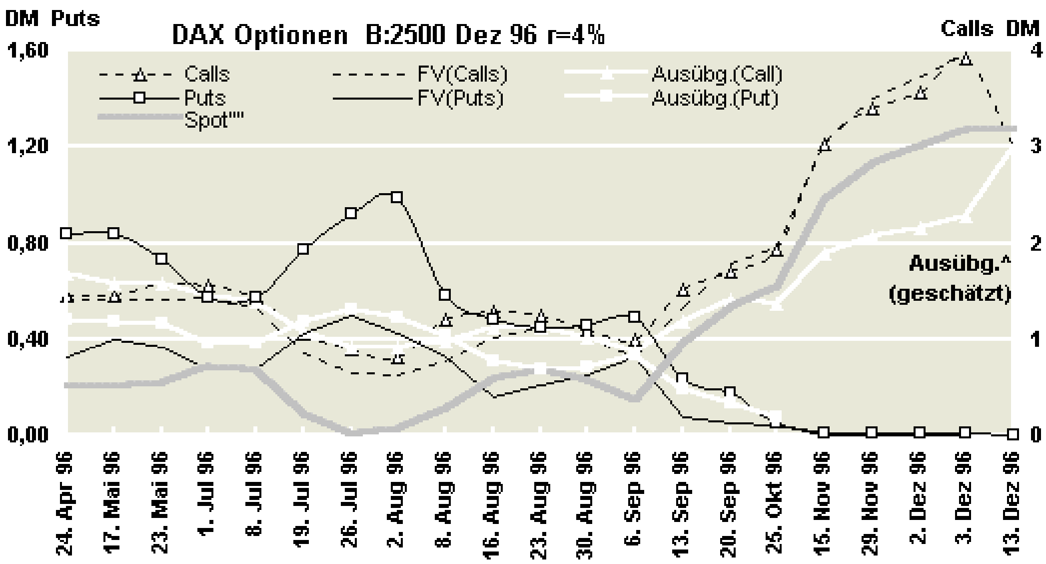
(continued page 10 following the next table)

Figure 7: Evaluation errors of 469 Options (1998 until 9/1997)

Errors B-S	I'rate	Volatility	Wochen	Moneyness	Calls	Puts
USD	3%	Ø 9,6%	13	in	9%	-16%
Hits			26	in	12%	-25%
- 5% 75 : 25			13	at	-1%	-34%
0% 49 : 51			26	at	7%	-39%
+5% 23 : 77			13	out	-21%	-47%
			26	out	-7%	-53%
Ln(usd 1,5%)	3%	Ø 6,8%	13	in	7%	-25%
Hits			26	in	-4%	-38%
- 5% 79 : 21			13	at	-16%	-55%
0% 51 : 49	(see upper curve		26	am	-13%	-61%
+5% 26 : 74	page 13		13	aus	-85%	-78%
			26	out	-49%	-53%
DAX	4.5%	Ø 15,3%	13	in	-22%	33%
Hits			26	in	-35%	47%
- 5% 86 : 14			13	at	-29%	35%
0% 73 : 27			26	at	-43%	29%
+5% 55 : 45			13	out	-34%	20%
			26	out	-48%	5%
Ln(dax 3%)	3%	Ø 14,3%	13	in	-13%	-16%
Hits			26	in	-21%	-25%
- 5% 65 : 35			13	at	-29%	-29%
0% 50 : 50	(see page 8)		26	at	-35%	-40%
+5% 35 : 65			13	out	-51%	-45%
			26	out	-49%	-53%
DOW	4,5%	Ø 12,1%	13	in	-27%	79%
Hits			26	in	-38%	174%
- 5% 96 : 4			13	at	-37%	156%
0% 82 : 18			26	at	-47%	278%
+5% 50 : 50			13	out	-40%	167%
			26	out	-54%	411%
NASDAQ	4,5%	Ø 12,5%	13	in	-5%	-13%
Hits			26	in	-11%	-12%
- 5% 70 : 30			13	at	-12%	-25%
0% 56 : 44			26	at	-19%	-19%
+5% 40 : 60			13	out	-20%	-38%
			26	out	-28%	-25%
NIKKEI 95-97	4,5%	Ø 16,6%	13	in	-31%	40%
Hits			26	in	-40%	83%
- 5% 87 : 13	**100**	**Weeks**	13	at	-45%	31%
0% 73 : 27		**only**	26	at	-50%	74%
+5% 54 : 46			13	out	-58%	3%
			26	out	-59%	58%

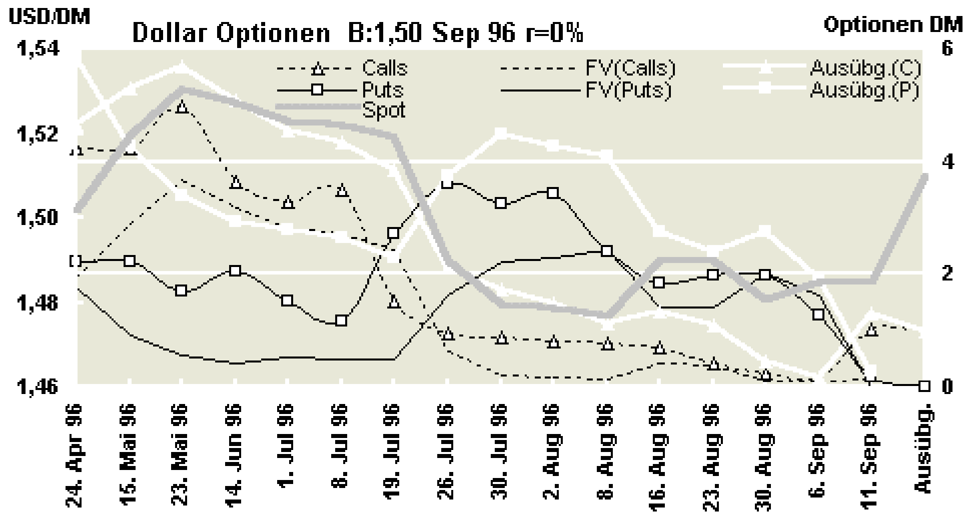
The white lines "practice" for calls and puts represent the expectancy value after proceeds ex post according to the method represented in section „Test of Evaluation Methods“ . The times between the measuring points on Weeks became rounded. It concerns thus empirical values, which would have resulted for options with same properties in the last 10 years. The changed moneyness was again adjusted thereby for each time. In the months April/May such as July/August. 1996 follows the put prices for instance the line of the fair values only around 30 to 40 DPf more highly. That could be because of the use superelevated implicit Volatilitaetswerte with the purchase decisions. Starting from September prices and fair ones of values of the calls lie however closely together continuous over the empirical values. The Volatility nearly constantly rose in 1996 up to 7,4. puts are starting from October from the money.

Figure 8: DAX-Options, Strike 2500



Call- as well as put prices of the following Dollar option exceed the fair values mot of the time.

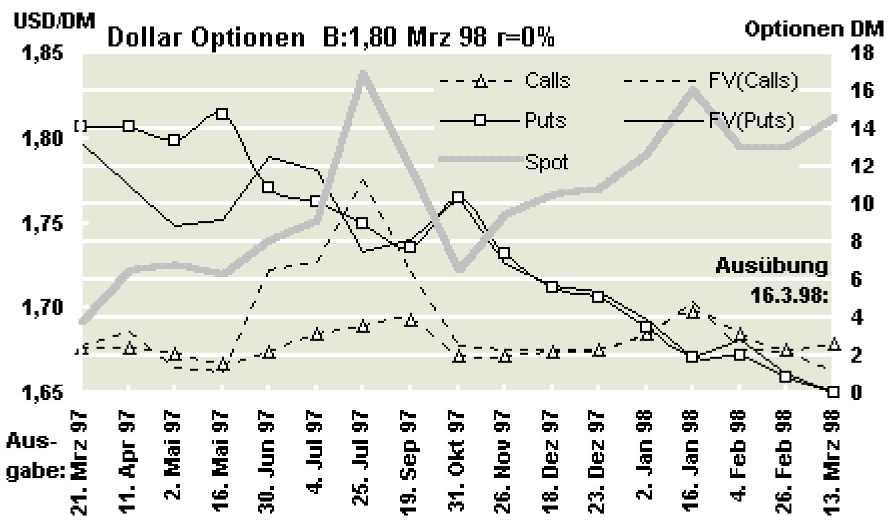
Figure 9: Dollar Options, Strike 1,50 DEM



The approximation of the evaluation after Black and Scholes, however with the interest rate 0%, is relatively successful. The Volatility was determined ex post, as 13-week volatilities were converted with the factor square root(4) = 2 to year volatilities. For the following illustration the volatility was smoothed by halving. The higher valuation in the place 25 July based itself by that there ex post determined high value for the Volatility (16% instead of average value 11%). Die Volatility wurde ex post festgestellt, indem 13-wöchige Volatilityen mit dem Faktor Wurzel(4)=2 auf Jahresvolatilityen umgerechnet wurden.

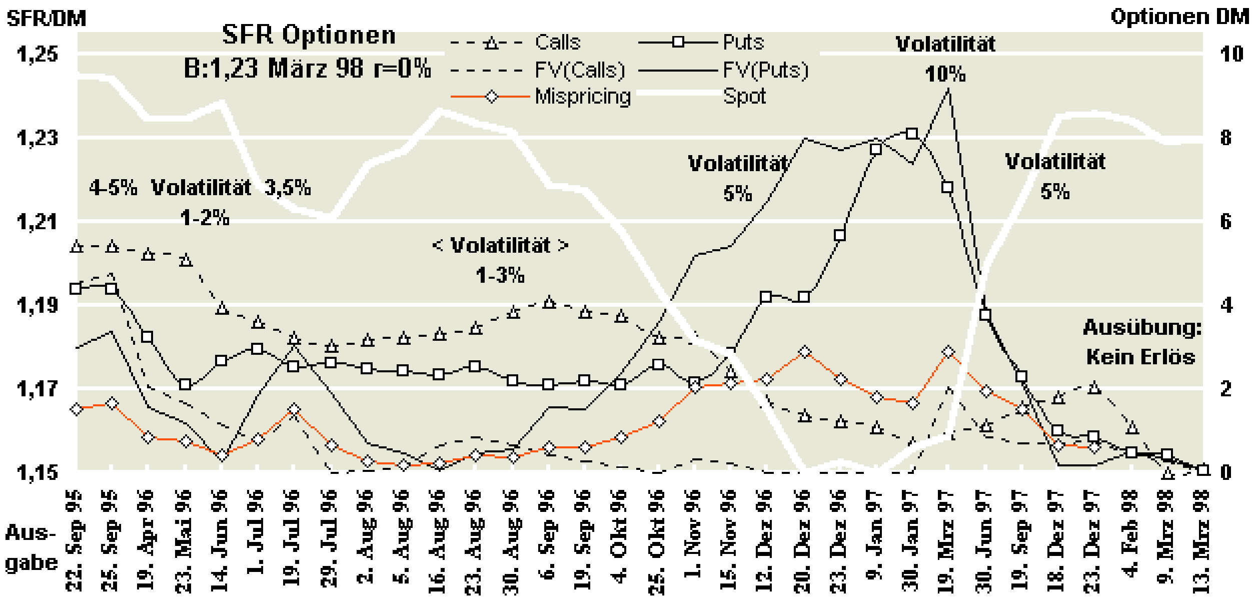
The volatility in the following diagram is smoothed by halving. The higher valuation at July 25 is a result of the high value for the Volatility measured ex post (16% instead of average value 11%).

Figure 10: Dollar-Options, Strike 1,80 DEM



The wrong value is related to the actual, current option prices. The Dollar rate reaches nearly this height only later at the beginning of of 1998.

Figure 11: SFR-Options, Strike 1,23 DEM

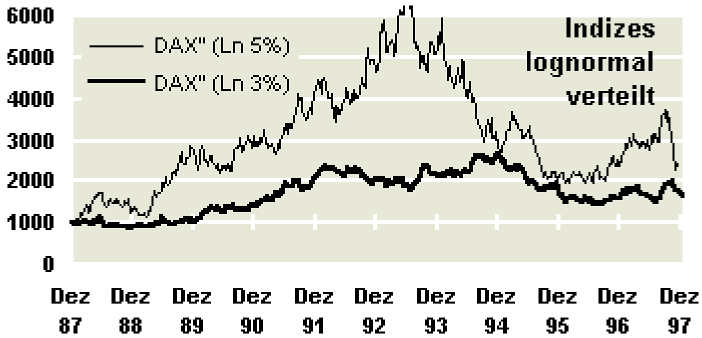


The SFR puts register a high price increase in the first half of 1997 with 5 per cent on December20. the coming increase seems to suggest 96 highly failing volatility (average 3%). However, the evaluation that should result from it is not reached by the prices. The volatility excess at the Sept 19th, 1997 lead to an easy rise of the false estimate (line with lozenge). Around the turn of the year 97/98 the call prices develop optimistically again. The volatility was calculated here by two weeks base (otherwise 13 weeks). (the "Mispricing line": "◊" consists of call and PUT deviations.)

D. IV. Fictitious Courses with given Distributions

Fictitious courses for examining evaluation beginnings can be produced also for prognosis purposes with the help of given return distribution. To examine the formula of Black and Scholes a log-normal distribution is computed, a corresponding DAX quotation selected as starting point and oriented at the standard deviation of the DAX index in the time of 1988 to 1997 (approx. 8%) and with 12 months rolling,

Figure 12: Artificially generated Courses (DAX-Type)



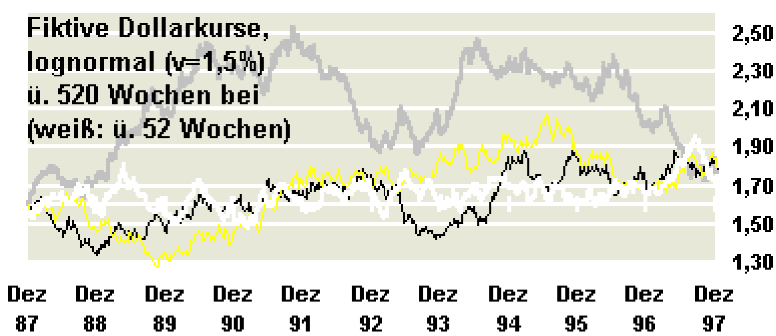
The log-normal indices cannot copy the DAX index from 87 to 97. While the DAX in that quadruples itself the years from 1988 to 1997, log-normal courses reach 1.8 (2,5)-fache increase during a standard distribution of 3 per cent (5 per cent) only.

A higher standard distribution would lead to unrealistic (not observed in 16 years) excursions. One of the following courses (type of dollar, white) is already lognormally distributed over 52 weeks. One sees that with a reduced supply of only 52 this course strews closely around a meant straight line accordingly.

The larger dollar movements (standard deviation 4%) of the past cannot be formatted thereby. The evaluation error is here - with accepted interest from 0% to 5.9% for calls (puts) with -30 to +14 per cent (-23% to -53%). In the most favorable case with log-normally distributed course (type of dollar) with 13 Weeks time an applicable evaluation was determined and with 26 weeks 12% (10%) overestimation with calls (Puts). The error by rounding on integral frequency was with 1 per cent.

Although all artificial dollar rates were produced after a log-normal distribution with 1,5 per cent of standard deviation, its volatility lies with up to 7 per cent.

Figure 13: Artificially generated Courses (USD-Type)



Evaluations of different quotations (Figure Figure 3) of DAX“ and Dollar“-courses are significat.

D. V. Actual Volatility DAX vs VDAX

End-of- Month-values [x] of the socalled Deutsche Volatilitysindex (VDAX), published by the DTB, were compared with the real volatility computed ex post (solid line).

Figure 8 show the results of B-S evaluations but using the VDAX index (instead of real values) compared with exercise results ex post. The values for exercised calls are slightly lower, ending with 1,50 DEM, than in figure 5, ending with 1,40. This is due to a different time frame as the VDAX started in 1992. Figure Figure 9 relates to options „at-the-money“:

Actual volatility quotes are compared Figure 14 with the German futures exchange VDAX values ex post available since 1992. Values were converted for 13 weeks and scaled to the year (square root(4)). Starting from 1994 the VDAX values are appropriate for values (15.3% to 13.6 %) on average over the values determined ex post.

Both data rows differ significantly, they don’t correlate (coeffizient r = -0,03, with -1 > r < +1). Thus the implied volatility is rather casual indicating the real volatility and for option pricing of no use . The evaluation error on „hot“ days („Hexensabbat“, due dates) ranges up to 30 per cent. The VDAX indicates rather a temporary overpricing than relevant impact on future option values.

Figure 14: VDAX vs Ex post measured

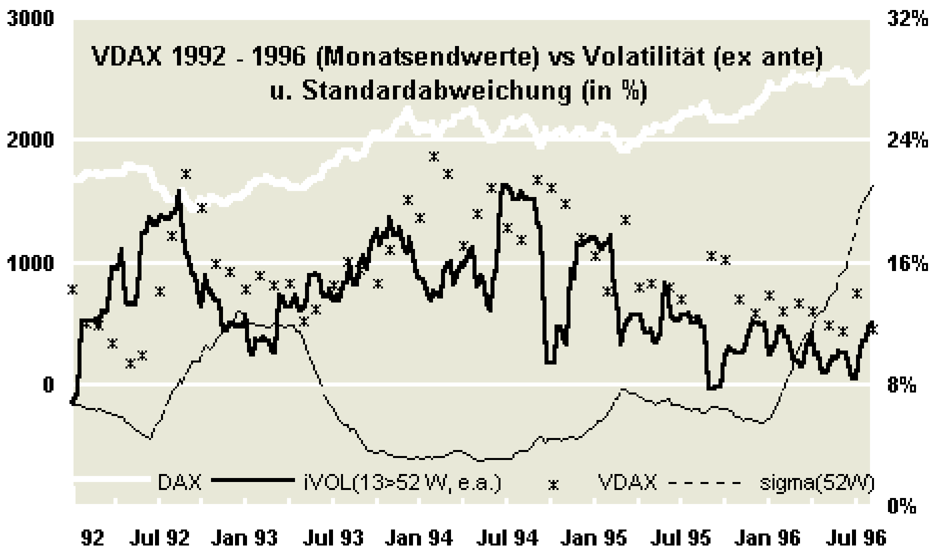


Figure 15 resembles time and total numbers from Figure 14 compares the stronger evaluation effect of actual volatility values and VDAX values with fictitious options. The tendentious underestimation of DAX calls by Black and Scholes is weakened by the higher VDAX values. Overestimation around DAX puts is strengthened.

Figure 15: Pricing Errors

Time	Weeks	Volatility	Basis =	Calls	Puts
88-96	13	real	course + 10%	*-10%*	*26%*
	26	real	course + 10%	*-41%*	*50%*
	13	real	course	*-24%*	*44%*
	26	real	course	*-38%*	*37%*
	13	real	course - 10%	*-15%*	*-18%*
	26	real	course - 10%	*-26%*	*-6%*
92-95	13	real	course	*-8%*	*21%*
	26	real	course	*-38%*	*37%*
	13	VDAX	course	*0%*	*37%*
	26	VDAX	course	*-15%*	*43%*

For a better fit calls (puts) the real volatility ex post should be multiplied by 1,2 to retrieve equal evaluations according to the B-S-Model compared with results when exercised.

For calls (puts) in-the-money the basis was assumed as - 10% (+ 10%) of the actual course. (Out-of-the-money reverse)

Figure 16: VDAX vs Volatility ex post (Interest Rate 4,5%)

B-S Errors			VDAX		DAX'92-'95		DAX'88-'97	
Hits:	Volatility:		15,3%		13,7%		15%	
'92 - '95	Weeks	Money-ness	1992-1996		1992-1996		1988-1997	
- 5% 85:15			Calls	Puts	Calls	Puts	Calls	Puts
0% 75:25	13	in	-30%	66%	-32%	58%	-22%	33%
+5% 65:35	26	in	-47%	115%	-49%	100%	-35%	47%
'88 - '97	13	at	-37%	110%	-43%	85%	-29%	35%
- 5% 90:10	26	at	-55%	130%	-60%	102%	-43%	29%
0% 70:30	13	out	-43%	169%	-54%	107%	-34%	20%
+5% 45:55	26	out	-63%	155%	-69%	104%	-48%	5%

D. VI. Comparison: Underlying vs Option Chances

To the comparison of option chances with different quotations the course volume on in each case 100 points or DM was standardized. Thus the influence of the kurs-niveaus and the reference relationship is neutralized. It concerns again Fictive options "at the money" with 13 weeks time.

Figure 17: Price Comparison of norm. Underlying

in DEM	I'rate	Vola	Method	Calls	Puts
Dollar	3%	9,6%	B-S	2,30	1,57*
Dollar			actual	2,40	2,29
DOW	4,5%	12,1%	B-S	3,00	1,91
DOW			actual	4,34	0,86
DAX	4,5%	15,3%	B-S	3,63	2,53
DAX			actual	5,23	1,77
NASDAQ	4,5%	12,5%	B-S	3,09	1,99
NASDAQ			actual	5,73	1,66

Maxima are emphasized in the table. Based on actual practice proceeds NASDAQ calls (dollar puts) have - ceteris paribus - the highest value. The evaluation after Black and Scholes, which turns off to the volatility, estimate DAX calls and puts most highly. With all calls and with dollar puts the evaluation is after Black and Scholes under the expectancy value after proceeds. (here only the "option propensity" concerns to a quotation without consideration of rate of exchange risks for DEM investors!)

D. VII. „Definition“ of the Dollar Exchange Rate

A model for the Dollar exchange rates can be defined in the following way:

A weekly USD-DEM exchange rate is generated when based on an approximated hyperbolic distribution returns are drawn randomly while applying the following rules:

1. When the volatility is excessive those returnes are selected and applied for the following n_1 weeks that ensure mean reversion.
2. When the bandwidth is excessive those returnes are selected and applied for the following n_2 weeks that reduce the bandwidth best.
3. The n_3 weeks after an excessive return (jump) make a Poisson process.

The application of the rules is not determined strongly but rather „soft“ and controlled randomly how often and how long they should be applied in a particular situation. The rules require the following parameter:

Bandwidth, number of repetitive draws in case of excessivity, mean (expected value) of volatility (being a measure of Bandwidth also), Factor for sign bias effects and duration parameters: For simplification $n_1 = n_2 = n_3$ can be assumed.

For tuning purposes the rules and parameters can be tried out with the history quotations. In case of forecasting weekly seasonal effekts should be considered.

That’s a way to describe but also to create or clone a fictitious USD-DEM exchange rates or important indexes. By increasing or decreasing positive returns (and negative ones vice versa) hausse and baisse effecs can be enabled.

D. VIII. Experimental Approach - a Suggestion

To analyze the impact of volatility and performance on evaluation the original DAX index is suggested to transform the DAX into an index that developes „horizontal“ without a positive performance. This cannot be achieved by just increasing the volatility by a factor since this would leave the index quotations and thus the ealuations of exercise results untouched.

- Step 1: Adding a negative linear function by substracting 3,8 points per week.
- Step 2: Smoothing the significantly increased Volatility by using 2 week means (gliding means) of DAX quotations.
- Step 3: By using 5 week means a volatility of 15,5% is achieved that is close to the „natural“ or long term volatility of 16,1%.

- Step 4: By using 10 week means a volatility of 10,5% is achieved that is close to the volatility of the Dollar exchange rate.

The approach hadn't be performed yet.

E. **Empirical Results Part 2**

The dynamics of autocorrelation are analyzed with respect to GARCH beginnings.

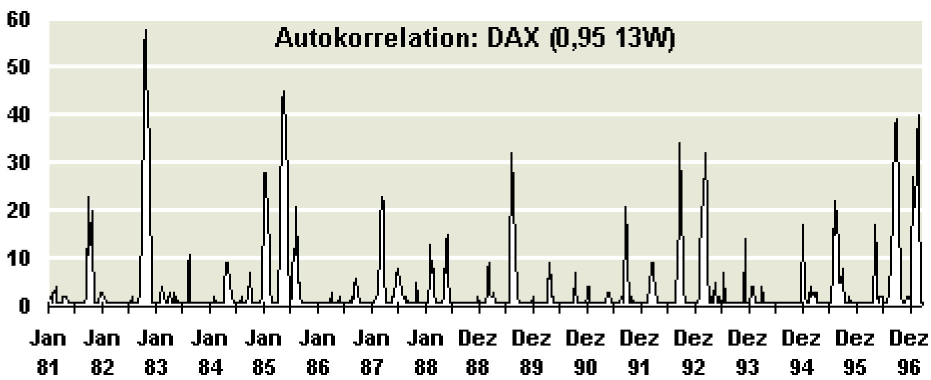
E. I. Diskontinuität der Autokorrelation

With the DAX as with other indices and the dollar rate of exchange a phenomenon is to be observed that the frequency of similar course sequences can rise at certain times, on average once in the year, precipitously and then again to a low level drops back. This effect leads in the following illustration to curve developments, which look like "stalagmiteS".

The form supply of distance sections with 13 points achieves astronomical numbers after the combinatorial mathematics. The numbers do not reduce noticeably, if all variations became to go through. Turned around the observation of repetitions means a characteristic only because of the practically unlimited form variety.

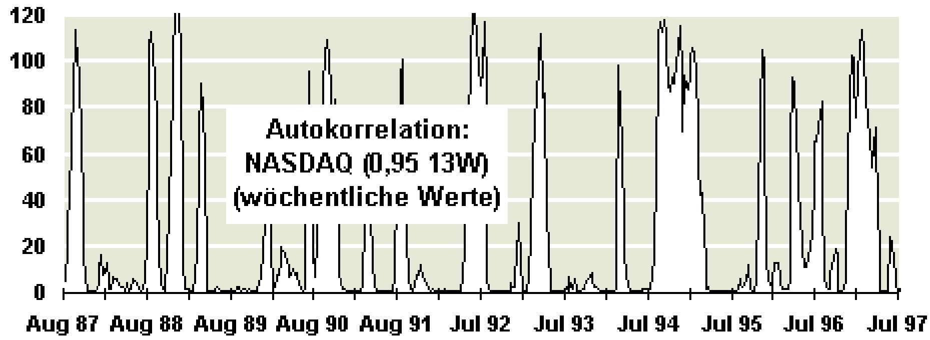
The number of similar 13 weeks periods is enough with the DAX to 60, with the DOW and dollar rate of exchange to 40 times (DOW and dollar without fig.). In 55 per cent of the cases are present to 3 repetitions with each the three quotations in 16 years: The occurrence of more "strong" autocorrelation is to a considerable degree synchronous with the DAX.

Figure 18: Autocorrelation DAX

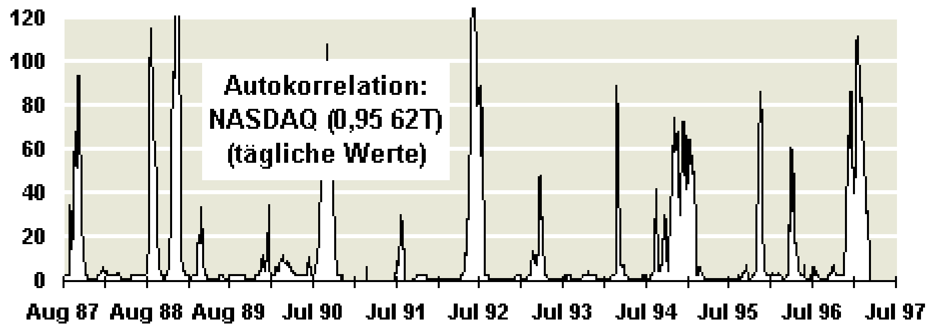


The next two diagrams compare NASDAQ autocorrelation effects on a

Figure 19: w+d NASDAQ - weekly vs daily Values



weekly



daily

on a 13 weeks and 62 days basis (both making aproximately a quarterof a year). The discontinuity effect with both data types runs to a considerable degree synchronously and runs on the same level over remarkable 120 repetitions. Autocorrelation steps up with shorter distances, e. g. 8 weeks, as expected more frequently. The phenomenon of the precipitous change remains. Only with sequences under 6 weeks the effect is becoming diffuse.

E. II. White Noise

Using these criteria two clases of „temperature“ can be defined. Figure 20 shows returns over 13 weeks in relation to periods of autoregression.

Figure 20: 13 Weeks Returns following Autocorrelation

	DAX	DAX	DOW	USD
Sample	87-97	81-7/97	81-7/97	81-7/97
Years	*11*	*16,6*	*16,6*	*16,6*
totalt	16%	14,5%	13,6%	-0,1%
„white“	**13%**	**13%**	**12%**	**-0,1%**
„black“	**22%**	**15%**	**20%**	**-0,1%**
„>30 x“	21%	17%	30%	- /+5,0%

Periods with sequences over 13 weeks without any repetition of formats can be qualified as "white noise".

Periods with intensive correlation are symbolized by "black". The last line contains 13-weeks-periods with formats that repeat more than 30 times within the sample..

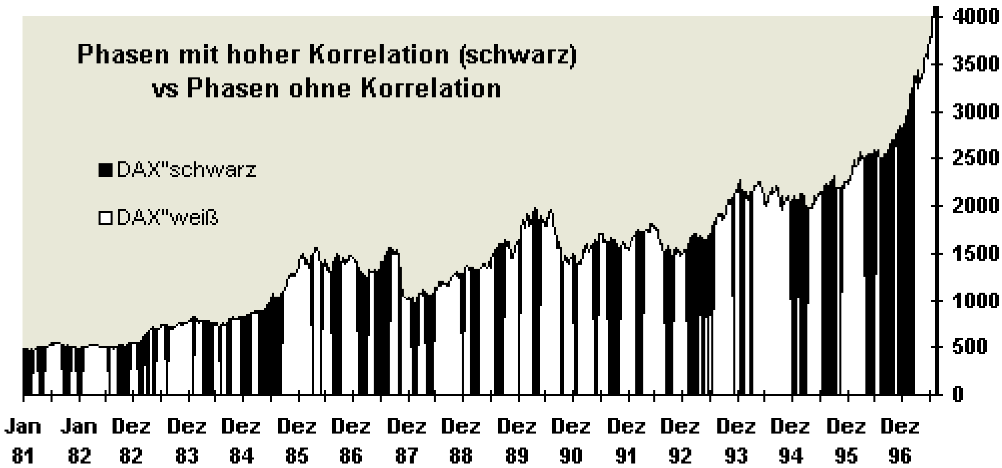
With DAX and DOW the repetitions incline with the rise of the returns, with the dollar rate of exchange significant returns rises changes in both directions. (the dollar is not subject also like the indices to a constant return expectation). The measured returns depend - less than with DAX and DOW - on the coincidental choice of the time period. The curves run with DAX (see illustration), DOW and dollar to a considerable degree synchronously. "white (noise)" phases, which constitute two thirds of the observation time of 81 to 97, point with the indices by (with the dollar rate of exchange by a middle) a return below average over 13 weeks. (A paradoxon: The week returns of the black periods are below average!)

„Black" periods can be interpreted in such a way that in them global relevant influences on groups of quotations or the financial markets work altogether at the same time. In "black" phases 13-week sequences with strong correlation on average repeat themselves nine times.

An increase of the Volatility approximately over the average value of 11 per cent is not an indication for reaching a phase with high correlation. The average values of the volatility levels from in each case all "white" and "black" sections are alike up to fractions of per cent, while the individual values in both section vary from 5 to 24 per cent.

Figure 21: Autocorrelation DAX

Intensive correlation (black) vs lower correlation (white)



E. III. Volatility - Stochastic vs Implied

Studying SV has implications for the usage of implied but recent volatility as a predictor for the future volatility. The only rational is based on the clustering effect. But this is not a continuous effect. In 75 per cent of all cases the volatility related to week x+1 changes less than 0.25 per cent from the volatility in week x. During 210 weeks with excessive volatility (2.5% deviation from mean) MR happens in 47 weeks - in 20 weeks the volatility is becoming more excessive. But excessive values suggest predictions being too high.

The existence of SV disallows the application of the formula required to convert a daily volatility into a yearly figure:

$V_{year} = V_{day}$ * SQUARE ROOT (1/365) (1/252, ifn counting banking days only) [X]

The equation requires a stable volatility![14] Thus the daily or even hourly DAX volatility computations are methodolically outrageous The volatility is an price indicator but not predictor of a future development.

F. **Consequences and Hypthesis**

Why the difference? One could think that the B-S-model doesn't „recognize“ the long-time performance which in case of the Dollar/DEM exchange rate is -1% (per year) while it is for the DAX index +11,5% (DOW index 13,8%). However, the B-S-evaluation doesn't change significantly when e.g. the DAX-curve is turned down by adding a negative linear funktion with the performance lowering to zero. The reason for the different evaluation is the different standard deviation or volatility.

The Black-Scholes-Model has many deficiencies:

1. The assumtion that courses are distributed lognormally cannot be proved for the Dollar exchange rates and prominent indexes like the DOW, NASDAQ or DAX.
2. The B-S-formula implies using the **strike price** instead of the most recent quotation as the **center** of the distribution curve. Due to the logarithmic component of the B-S-formula this position is the **maximum** of the distribution while empirical distributions prove that the **most recent course** is the „best predictor of any future price“, (Giddy I. H. / Dufey G. 1975).
3. The B-S-formula is applicable for european type options. However, that doesn't hinder financial newspapers and investment bankers to apply the model on all types of options. This explains the most important evaluation error. Less time to maturity means not only a reduced present value but a much smaller spread of the distribution and lower returns.

The Black-Scholes-results depend mainly on the volatility which has to be estimated also. This is questionnable pe se. It is even more questionnable when the estimation is done by using the reverse form of the Black and Scholes formula: the Volatility is requested that could have led to the current prices. The VDAX Volatility quotations this is done first for individual options that are merged while they are adjusted to cover different lead times, strike prices etc. These volatility figures can reach more than 30 Per cent while real Volatility measured ex post did not exceed 18 % during 1992 til mid of 1996. It is to be demonstrated due to the empirical results, which restrictions of the prognostic quality of the different models are to be made

F. I. Violation of assumed Lognormal Distribution

The underevaluation of puts by „Fair Values“ in Figure 5: Fictitious Options - Dollar is caused by the skewness of the lognormal distribution. However, even lognormal distributed, artificially generated indexes are subject of erroneous evaluation according to Black and Scholes.

A significant part of the literature over financial models is concerned with the idea (hope?), financial data and their returns could be attributed to the normal distribution, in order to be able to develop prognoses to it. From mathematician Bachelier (1900) comes one of the first contributions in this area. (the log-normal distribution is in this connection a normal distribution of the relative-reciprocal returns: resembles frequency for halving and duplication etc.)

That creates a statistic dilemma: The log-normal distribution for instance in the model of Black and Scholes can be justified nor rejected alone theoretically for stock exchange courses neither compellingly. While no empirical vouchers for log-normal distribution are present, since beginning of the 80's increasingly were objections reported (overviews with Galai, 1983, and Kolb, 1995/stone burner, 1996) that the acceptance "log-normally" rarely (never?) apply. That often came already in the title to the expression: "Skewness, kurtosis and Black Scholes Mispricing (Geske, Touros, 1991) or" failure of the Gaussian Hypothesis" (Peters, 1994). During the evaluation early theoretical models it should be considered that the data availability is not to be compared with the today's possibilities. Vouchers also in this work for "deviations" can affirm but for all conceivable quotations of the future not exclude only doubts.

There have been daubts from the beginning and also critics like: „Skewness, Kurtosis and Black-Scholes **Mispricing** (Geske,Touros, 1991), or the „**failure of the Gaussian Hypothesis**“ (E. E. Peters, 1994) stating finanicial trading consists of „games with memory“ and not all variables can be assumed to be random.

On the other hand the formula of Black und Scholes as it relates to the lognormal distribution is globally the most frequently used formula for option pricing - now with the blessing of the Nobel Prize of 1997. The results based on Black and Scholes formula are called fair values. However, the reversed form is used to predict the volatility also as well as the computing of beta-, gamma, omega, theta and rho- etc. and other indicators all relate to the Black and Scholes formula (S. Doll, 1992).
The practical unlimited availability of financial data...
A new method is introduced as a new approach to validate current option pricing methods. Samples contain quotations of weekly Dollar/DEM exchange rates and of DAX, DOW as well as NASDAQ indexes during 1988 til 1997. However, any other historical data could be used also. Since 52 future notations are required to calculate ex post the

real exercise results as well as the real ex ante volatility values 469 weekly evaluations are compared covering 1988 til 1996. (1997 data are used only as a forecast stack ex post.)

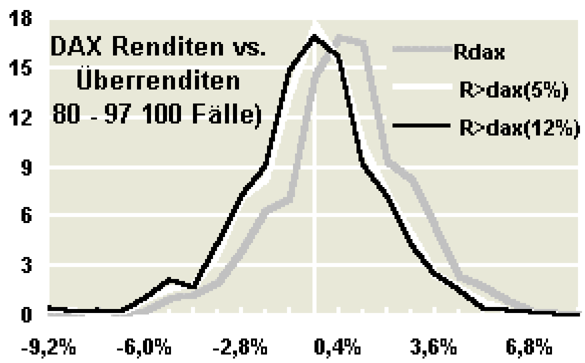
Distributions cannot be log-normally distributed whenever their net yield exceeds the more appropriate log-normal distributions, as applies with important indices and their values. Otherwise Volatilityswerte would have to be subordinated, which would result in unrealistic exchange rate fluctuations. "horizontal" course movements can be considered only with an interest of r = 0%.

F. II. Distributions and Excess Returns

GARCH-beginnings[15] try the returns and other characteristics of a quotation in one for these typical to divide fixed value and into a residual value with the intention that the residues can be better described then with the normal distribution or at least with a simpler distribution. The following representation, for which with the DAX the performance of many years was out-counted of approximately 12%, shows this effect. The residues are then nearly symmetrically distributed around the 0%-Linie. Only 5 per cent, out-counted - thus a value around the interest charges without risk - arises still no satisfying symmetry. In the place -5% even an unevenness is strengthened by considering the residuals. The curtosis is remaining pronounced too strongly for a normal distribution.

For prognosis considerations the following objection is to be attached: The returns of the last 16 years vary strongly from -30 to +60 per cent. Only sliding averages over 8 years oscillate weakly around the average total net yield:

Figure 22: Excess Returns beyond 5 or 12 %



That is for prognoses over short distances a too long period. That is called a constant, which is introduced, in order to harmonize a determined distribution by allocation in "basic returns" and "overreturns", is even one most "volatile" variable.

If this effect is reduced by the fact that the smaller interest rate for creation without risk is selected, the harmonization effect falls more weakly out (white curve). The actual goal of finding about a Basic Law moderateness like the normal distribution with the residues is not achieved. Which sense is a regularity of a several times derived size (if it would exist) to have retroactively for the Underlying? Already the quotation is an abstraction through means between supplies and demand demands and rounding on decimal places or fractions. Average value, standard deviation and returns are a further derivative. Volatility as standard deviation log arithmierten returns again a derivative of higher degree of etc.. Another idea, sign bias effects [16] (stronger volatility changes with course decreases) approximately for a prognosis in the model to consider, was to be determined not constantly: The effect can even change with strong course movements (examines with the dollar)

Excursion: The railway ticket view.

It's like a railway ticket. Assuming tickets are sold always for the full distance a train is driving. When one enters the train later he might get a deduction for the mileage not used. If one leaves the train before the final target he gets again another deduction for the mileage unused.

A test FV (full distance) should be FV (stage 1) + FV (stage 2) if d = s1 + s2
The impact of time length on the B-S model is higher than it is on the present value.

F. III. Mispricing

There is only one overall tendency: The estimation error is increasing with the time to maturity.

When options are deeper in the money or when volatility figures are applied higher the real volatiliy ex post effects of misevalaution depend on the stochastic characteristics of the underlying.

Considering 26 week time to maturity the B-S-formular is under-evaluating **calls** (< -20%) and under-evaluating **puts** (> + 40%). This is again using the real ex post volatility. Assuming the estimated and published so called implied volatility figures are often higher (see DAX comments.)

Thus increasing the real volatilitiy by the factor 1,25 the effect is changing to > + 40% for calls and < - 20% for puts. (see table 1 containing correspondent pairs of deviation figures for 13 and 26 weeks.

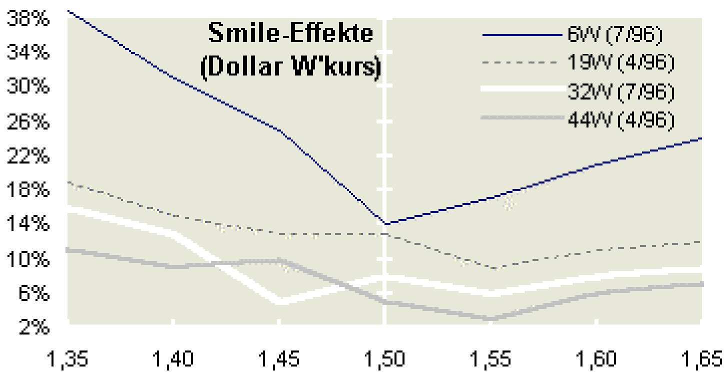
Out-of-the-money

The individual evaluation differences made of table 2 - false evaluations, if one accepts the suggested method - from +20 to -60 per cent with the DAX are substantial. (the designation "fair value" for the results after Black and Scholes is rather euphemistisch). The deviations seem to follow also no systematics.

a) The hypothesis (Steinbrenner[17] 1996 a. o.) in the reversal that the implicit Volatility, counted on prices "at the money", which smallest distortion exhibits (Smile effect), is not proven with fictitious and actual options. (see in addition also section „Actual Volatility DAX vs VDAX")

The original smile effect relates to implied volatility (G. Murphy 1994, O. Pilz 1994.Volatility values show a minimum, a „smile" [☺], **when computed based** on prices of calls „at-the-money".

Figure 23: Smile Effects



While the original smile effect means a minimun „at-the-money" (1,49/1,52) minima can be observerd around 1,55.

Instead of the implicit volatility values computed by the reverse Black und Scholes formula the actual volatility ex post was 7,1% - 7,2%.

The actual volatility of the DAX index ex post was 7,1% - 7,2%. end of July 1996 and 22% mid of January 1998, i.e. the implicit volatility is higher significantly

An explanation may be another observation: In case of the DAX the error of Black und Scholes is minimal around position 0% showing some kind of „smile" [☺]

The fact that the Black and Scholes evaluation is equal to the exercising result around position 9% (13%) for options with time to maturity of 13 (26) weeks seems rather arbitrary. and

However for the Dollar exchange rate there is a maximum indicating for situations „at the money" and a rather gloomy [☹] situation.

The Black and Scholes evaluation is equal to the exercising result around position 10% only for options with time to maturity of 13 weeks seems also arbitrary.

In both cases options hight „out-of-the-money" deviations reach extremeous per cen-

b) With running times up tol 130 weeks it can be shown that the underestimation settles down with "fair values" for DAX calls (hit 75 from 100) on -30 per cent. Overestimation with DAX puts increases. With dollar calls against it overestimation constantly increases, lasts themselves the underestimation with -50 per cent settles down starting from running times of 26 weeks.

c) A cause for the substantial evaluation errors lies primarily in the fact that the actual return distributions of the accepted, theoretical distributions of the models deviate.

d) With the formula after Black and Scholes it is added that this accepts the future, subordinated course in fact as around the base value strewing. The empirical experience, which is observed and divided also by many authors, shows however that courses strew rather around the last value, "best predictor". It explains that usually the false evaluation for options precipitates "from the money" most highly (exception DOW, see Figure 7).

e) From the right-inclined of the subordinated log-normal distribution resulting tendentious overestimation of calls is not sufficient for compensation: At the indices the net yield of many years the right-inclined of the log-normal distribution exceeds and leads DAX, DOW and NASDAQ therefore also to underestimations of calls[18].

f) Artificially produced indices with lognormaler(!) Over 10 years (an example over 1 year) are subject to distribution - unexpectedly - to a false evaluation. The errors are with log-normal Dollar"courses (DAX") between +7 and -85 per cent (-13 to -53%). puts are appropriate here on the average for a stronger underevaluations in addition also that.

g) A contradiction[19] with the application of the formula after Black and Scholes consists of the fact that the model assumes homoscedasticity, i.e. constant variance over the remaining time of an option which cannot be observed in reality. (see below in addition also the section for the "Nature of Volatility“). There are course formats and thus variance and volatility formats favouring calls or puts.

One could object that the predominant underestimation with calls after Black and Scholes is not a disadvantage. It is pointed out by different authors that at fair values with addition of the transaction costs and taxes no profits are to be obtained "above normally".

“ examination at noted options (DAX, USD, SRF)" show that the "fair values" seem to have quite a coining effect.

Man könnte einwenden, daß die überwiegende Unterbewertung bei Calls nach Black und Scholes kein Nachteil sei[20]. Von verschiedenen Autoren[21] wird darauf hingewiesen, daß bei Fairen Werten mit Hinzurechnung der Transaktionskosten und Steuern keine Gewinne „above normal“ zu erzielen seien. Die Beispiele im Abschnitt „zeigen, daß die „Fairen Werte“ durchaus eine prägende Wirkung zu haben scheinen.

Since the Brownian Motion has metaphorical meaning for courses only and the log-normal distribution subordinated thereby is not to be proven, the proper price evaluation still waits with the financial markets for a rational explanation. The following section examines in addition a fundamental objection whether the "fair values" represent at all the actual process of the proceeds curves.

F. IV. Improved results by reducing Volatility

For a fit calls (puts) the real volatility ex post should be multiplied by 0,58 (1,55) to retrieve equal evaluations according to the B-S-Model compared with results when exercised.

Considering 26 week time to maturity the B-S-formular is over-evaluating **calls** (> + 30%) and under-evaluating **puts** (< - 40%). This is using the real ex post volatility. Assuming the estimated and published so called implied volatility figures are often higher (see DAX comments.) Thus increasing the real volatilitiy by the factor 1,25 the effect is changing to > + 40% for calls and < - 20% for puts. (see table 1 containing correspondent pairs of deviation figures for 13 and 26 weeks.

For the Dollar/DEM exchange rate **calls „in-the-money“** the B-S-formular is over-evaluating **calls** (+ 30%) and under-evaluating corresponding **puts** (< - 55%). This is using the real ex post volatility. Assuming the estimated and published so called implied volatility figures are often higher (see DAX comments.) Thus increasing the real volatilitiy by the factor 1,25 the effect is changing to > 40% for calls and < - 20% for puts.

F. V. Law of Great Numbers

In „Figure 5: Fictitious Options - Dollar“ it becomes clearly like the expectations of profit with options - equal only over a long time, here 10 years, form - directly to the appropriate illustration to the DAX whether one pushes away to the average value of proceeds or the "fair value". (months-long phases of calls or puts without practice proceeds do not exclude short term profits with the daily trade with actual options and during in reciprocal effect with the Volatility developing market prices.)

The statistical contribution to this problem is the "law of the large number": "with sufficient large extent n of the observation series the relative frequency becomes only rare deviating from her corresponding the probability (...) by more than a given, arbitrarily small amount." (Anderson, 1957) for prognoses thereby an additional, problematic acceptance must be always added: The future values, which were not with the statement of the collective characteristics of the examined observation series involved, depend nevertheless on the same (or subordinated) valid distribution etc.

Black and Scholes stated: „we assumed.., that **over a finite time interval** the returns on a common stock are lognormally distributed[22]". To describe a discret log-normal Distribution realisticly, about 40 classes and 100 Cases[23] are required. A distributionn can be represented only when more than 100 times (weeks, days etc) Options funished similarely will be bought. Or differently said: A distribution, accepted for n times - thus for a temporal profile - cannot be applied to the temporal cross section.

Emitters can in such a way determine with distributions over years the risks of emission strategies, but not punctual investors. Whereby a strategy lies correctly only if the results approach on the average increasingly the forecast. The picture, a "fair value" can be attached as it were as value proof of an individual option, is not to be justified statistically according to this model.

The Brownian Motion has influenced the theory of financial markets[24]. In diesem kinetischem Modell werden Erwartungswerte für Millionen von sich gleichzeitig bewegenden Moleküle (die bei gegebener Temperatur einen bestimmten Druck bilden) bestimmt. Die Übertragung auf die Preisbildung des Marktes fällt im Detail schwer. Da vor allem die lognormale Verteilung nicht zutrifft, wird in aktuellen Ansätzen[25] versucht, durch zusätzliche Modellbedingungen (Filterregeln, Handelsregeln wie der „Bid-Ask Spread" und Bedingungen für Exzeßvolatility) Asymmetrien, Leptokurtose der Renditen etc., „Marktanomalien" abzubilden.

Ein anderer Gedanke liegt in der Frage, ob die Optionen (noch) Derivate sind oder ob ihre Preisentwicklung und ihr Handelsvolumen nicht eher den Kurs des Underlying beeinflussen. Kolb[26] wirft diesen Gedanken mit der These mit der (Eintages-) „Lead-Lag"-Relationship auf. Vermutet wird dieses Phänomen besonders vor dem Verfallstermin. Es werden Modelle diskutiert, die umgekehrt aus der Optionspreisentwicklung Gesetzmäßigkeiten für das Underlying ableiten (Rady[27], 1995).

F. VI. Nature of Volatility

In „Figure 5: Fictitious Options - Dollar" it was shown that the actual volatility does not have an indicator function for option chances. It can be shown also with the artificially produced log-normal courses, against the acceptance of different models not constant for the duration of short option running times. It is said, that both sides in an option sale agree on the level of the future voaltility. However this is wrong. They may see a price they didn't expect instead[28].

The examples in chapter D. III. Examinations at quoted Options (DAX, USD, SRF) show that "fair values" continue to remove with high volatility (ex post) from the actual prices. Similarly work the superelevated VDAX values also, which however reduce the false evaluation with calls (puts) according to Black and Scholes with the DAX something (increase). A reason for the interest in the volatility variable consists of simplifying the task of prognosis for the underlying on one central parameter. In practice is rekkoned back with actual daily changing option prices, so-called implicit volatility under reversal of the formula by Black and Scholes or - more correctly - "estimated" again. It is statistically doubtful, that these daily values are taken as prognosis for annual values and nevertheless are assumed - model-conformal - as constant for the remaining running time. Implicit Volatility can induce unreal market tendencies in the change with superelevated prices. The actual Volatility includes against it by definition an averaging, with individual values the only weak influence on the average for several days or weeks.

The systematic question reads whether price tendencies essentially from that the systematic question reads itself, whether preistendenzen can be essentially justified from the Volatility. In the evaluation comparison actual like artificially produced courses on the chances of options in preceded the sections it could be shown that the Volatility is not the only regulation factor: (in chapter "Empirical Results part 1") about shows that increased Volatilityswerte does not affect coining/shaping actual proceeds. With course movements similar to a shaft about also the relation between wavelength and running time is relevant. Also the standardized "Comparison: Underlying vs option Chances "shows that the value does not depend on after proceeds - at least not exclusive - the volatility. The systematic question reads of whether price tendencies can essentially be justified from the volatility. Into the evaluation comparison actual like artificially produced courses on the chances of options into preceded the sections it could shown that the volatility is emergency the only regularization factor: Figure 11: SFR option, Strike 1.23 (in chapter "Empirical Results part 1") about shows that increased volatility levels does ######## emergency affect coining/shaping actual proceeds. With course movements similar tons of A shaft about thus the relation between wavelength and running time is relevantly. Thus the standardized "Comparison: Underlying vs option Chances "shows that the VALUE of does emergency depend on after proceeds - RK leases emergency exclusive - the Volatility The meaning (implicit) of the volatility as A cent ral of parameter (the prognosis of the volatility is a branch of research for itself!) is on difficult this background tons explain. It is as implicit Volatility from current market prices if necessary at indicator for the option prices, which may indicate "heating" or "cooling". The tendency of the market, which with A separate evaluation and weighting can be determined according to call and put of prices, hardly finds interest.

F. VII. Discontinuity of Autocorrelation

The phenomenon of the correlation with financial data rows among themselves and in particular the phenomenon of the autocorrelation forms the basis (auto -) of the involution models - "own data history" for prognoses is used, how others (Hindler, 1983) to say. In addition, the statement of strongly varying autocorrelation levels is not new. Dourat et al. (1996) refer even to intra day analyses ("notoriously unstable") to it. Autocorrelation at financial values, also the often subordinated "stramme" correlation between DAX, DOW and dollar is subject to strong dynamics (of r 0.95 to r.

The representation of the phenomenon for longer periods and in particular the realization that the autocorrelation level changes practically between two conditions, "white noise" and autocorrelation on high level, is interesting both for the supply of ideal Test-samples for the experimental research and for the prognosis models. Autocorrelation with coincidence-generated log-normal courses (without illustration) precipitates substantially smaller than for instance with the DAX, although those is strongly reduced form variety with only 25 different net yield values. The causes for autocorrelation are a field for further investigations. Also, why the effect precipitates with the DOW relatively small (until 40 repetitions), with the DAX is more strongly pronounced (until 60) and with the NASDAQ twice as strongly precipitates - both with weekly like daily quotations. To ask it would be whether there is a factor bundle that autocorrelation or evenly nevertheless (strangely -) correlation to exogenous factors provokes. Two theses are offered for it:

1. Autocorrelation is here an illusory effect. Autocorrelation - approximately over one year - does not mean actual connection of the current courses with the old courses as for instance in the population statistics between marriage ceremonies and births. Rather a course (and the market participants interested in it) behaves to similar external impulses, the current market draws homogeneously from the "repertoire" of historical market reactions.

2. An index like the DOW is in its reaction to impulses more dominanter, more independently, than for instance the DAX. A beginning for the analysis could consist of looking for comparable conditions for frequently arising profiles

F. VIII. Discontinuity and Predicting

For prognoses the interesting beginning results to reproach separate strategies, "regime Switching Models", for different status forms (defined like here as correlation levels) from the diskontinuitaet. For the quality of prognoses on the basis of autoregressiver beginnings as for instance with the GARCH models, is to be asked however, how this dynamics for prognoses can be considered satisfyingly. This information cannot be derived from the "own" past. Approximately 17-fache repetitions arose within a yearly or only after two years. This question cannot be eliminated with "out of sample" tests also.

Simplified expressed, is the characteristic of a value forms in times of high autocorrelation out. Their duration amounts to on average 5 weeks, with peak values only 3 weeks. The change between the different phases runs abruptly and without recognizable advance of course formations, which one could use as "triggers" for "if then"-prognoses. Formats, which repeat themselves particularly frequently, could thereupon to be examined, to what extent they run with other course sequences or exogenous factors synchronously.

Summary

By analyzing fictitious options significant mispricing due to the formula of Black and Scholes can be shown systematically and independent from market distortion. Without any evidence of lognormal distributions in reality even options based on fictitious and lognormally distributed courses are not valued properly.

According to the **Law of Large Numbers** pricing models based on time distibutions should be applied to strategies rather than to single option trading.

The discontinuity of autocorrelation has impact on forecasting models. The future will belong to those models able to include exogene factors like relevant information and changes of the economic scenario. Promising aproaches that could cope with these influences are „if then“-predictions, regime switching models, neuronal and „learning“ approaches. The current impact of volatility on option pricing is not justified.

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Contents

Figures / Tables

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[1] In case of section difficult to understand I am prepared to "retry" and would appriciate any syggestion.

[2] In the department „civil ("bürgerliche") mathematics“ (This term includes simple as well as applied mathematics and mathematics for economical science) of the Flensburg joined university library the term volatiliy doesn't exist. The classical statstistic doesn't know the term „Volatility“ (or „Vola“ as the banking personell say). Typical characteristic are the standard deviation σ (sigma) or the variance with t σ^2 respectively. Generally with

$$\sigma 2 = 1/n \sum (X_i - \varnothing(X)) \quad \text{(for i = 1 til N)}$$

defined. In option pricing formulas this form is used also:

$$\sigma\text{“}2 = 1/n \sum [\ln(X_i / X_{i-1}) - \mu] \quad \text{(for i = 1 til N)}$$

[3] See also: Schroeder, G., Stochastic Pricing, 2005

[4] The Clustering rule should be seen complementary to the Mean Reversion rule since once the volatile returns are consumed Clustering results in an smoothing effect also.

[5] The Brownian Motion is well described along with the model of B&S. However the details of stock exchange procedures and OTC are not covered. An exemption is Aït-Sahalia (1998).

[6] The Brownian Motion is three dimensional. Considering the room between the preparation glasses as a plane it is two dimensional.

[7] NASDAQ’s trading system Opti-Mark allows to split large orders into different volumes at different prices to make them less visible. (FAZ, 1.10.1999, P. 25)

[8] Following Hull (1997) a / the standard textbook for option pricing.

[9] Nach der Formel: $V_{year} = V_{13} * \sqrt{52/13} = 2 * V_{13\ weeks}$. Die Umrechnung auf die Laufzeit findet in der Bewertungsformel statt.

[10] Das Underlying (S) geht mit in die Formel von Black und Scholes ein:

Mit C = Value of the call option
S σ = Spot
r = Standard deviation, volatility
t = riskless interest rate
L = (Residual-) time
N = Strike Price
= Normal distribution

$$C = SN(d) - Le^{-rt} N(d - \sigma\sqrt{t}$$

(reg. „d“ see note 18.)
SN (d) is interpreted as expected value Under lying, the second expression is interpreted as the expected costs of the option exercised at maturity, leaving the difference of both expressions as the option price.

[11] The values could also the centers of the sections (with 13 Weeks i.e. the 7th week etc..) are added. If the found values are analyzed, an appropriate lag should be granted...

[12] Curtosis (when leptocurtic, slim, exaggerates or as platykurtisch, flat) and inclined ones (skewness) are statistic terms, with which these observations are generally described, with those in addition, im-

plies hope along, it are still correct but defective forms of the Gaussian curve, leaving the hypothesis of normaldistributed returns valid still.

[13] Options of Citibank N.A., New York, Branch Frankfurt/Main :

	WKN	Typ	Strike	from:	due:		WKN	Typ	Strike	from:	due:
SFR/DEM	815543	CA	1,23	9/95	3/98	Dollar/DM	815277	CA	1,50	3/96	9/96
SFR/DEM	815544	PU	1,23	9/95	3/98	Dollar/DM	815278	PU	1,50	3/96	9/96
DAX	815506	CA	2500	3/96	12/96	Dollar/DM	818434	CA	1,80	3/97	3/98
DAX	815507	PU	2500	3/96	12/96	Dollar/DM	818435	PU	1,80	3/97	3/98

These options are subject of German law.

[14] ODE / PDE: Abbreviation for ordinary (gewöhnliche) / partial (partielle) differential equations.

[15] Zu einem Vorschlag aus einer unveröffentlichten Manuskript (1997), das von der ZfB zu Verfügung gestellt wurde.

[16] Following Engle/Ed. (1993) Der Hinweis darauf stammt aus der gleichen Arbeit (Anm. 15). Zur Überprüfung wurde hier der Dollarwechselkurs 1988 bis 1997 analysiert..

[17] The ideal type format of the Smile-Effects is described in Steinbrenner (1996) P. 291

[18]

The Variable „d“ in the B-S formula has with „L“ the strike price in the denominator (and not the actual spot t S_n!).

$$d = \frac{ln\frac{S}{L} + (r + \frac{\sigma^2}{2})t}{\sigma\sqrt{t}}$$

[19] The idea leading to this point was suggested by the referee. An extensive discussion of this dieser model deviation can be found already in Galai (in Brenner 1983), Pages. 65-68.

[20] Galai (1983, in M. Brenner, 19983) P. 46, sees herein a sign of market efficiency , that market prices equal with the sum out of (undervalued) „Fair Values“ including transaction costs (and taxes). Thus a buy at fair value would not allow margins „above-normal“.

[21] Galai S. 46 (in Brenner 1983)

[22] Reg. the assumption of the lognormal Distribution see: Black/Scholes (1972) S. 400

[23] According the Chi-square test, requiring a minimum of 5 cases per classification, even 5 x 40 = i.e. more than 200 cases would be required.

[24] According to an Internet-Research 1998 intstitutes like the LSE Finanancial Markets Group (see Rady 1995), the Freiburger Zentrum der für Datenanalys und Modellbildung, the Institut für numerische und angewandte Mathematik, Göttingen, the Institut für medizinische Statistik in Berlin (FU), as well as typically the Applied Mathematics / Statistics faculties of the Universities in Cambridge, Harvard, Konstanz, Leiden, Paris (IV), Siegen, Sydney, and Warschau are researchin in Marcovian and Martingale-Processes, Brownian Motion (or more general Wiener Processes) in relation to market modelling.

[25] Aït-Sahalia (1998) Pages 94f. and 119f

[26] Reg. the Hypothesis „Lead-Lag-Relationship“ see Kolb (1996) P. 185

[27] Rady (1995), Pages. 1 and 11

[28] The wrong position is quoted from Beike/Schulz (1996) P. 629f.